An Introduction to Mathematical Modeling in the Life Sciences
Problem Set 1
Due Week #4, 3/13, before lecture

You can work individually or in small groups (of 2-4). If you do work as a group, hand in one copy of the solutions.

For each problem, upload all code and/or all relevant plots, together with analysis and explanation, in a single file, to the course website before lecture time on the due date. For our fellow THU students, please email everything (in a single file) to mmls@pku.edu.cn

Please label your files (and emails) with your group members' names, and on the first or cover page of your submitted file

例如，邮件标题："PS1 汪淼、史强....."；邮件附件命名为：代码（文件夹）"PS1 code 汪淼、史强....."以及文档说明（pdf 或者 word）"PS1 assignment 汪淼、史强....."

This problem set spans 2 pages

1) Reading: EG Chapters 1 & 2
2) Work through Sections 1-9 in the EG Lab Manual. Hand in solutions to Exercises 6.1 and 9.3
3) Fibonacci sequence: The Fibonacci sequence is

   1, 1, 2, 3, 5, 8, 13, 21, 34, …

   and is used to "model" the number of rabbits but appears in many biological phenomena. The sequence can be defined recursively as

   $$f_{n+2} = f_{n+1} + f_n .$$

   Define a state vector with two elements

   $$\vec{x}_n = \begin{pmatrix} f_{n+1} \\ f_n \end{pmatrix}$$

   Write down an equation involving matrix multiplication that computes $\vec{x}_{n+1}$ by multiplying $\vec{x}_n$ by a 2 x 2 matrix A.

   Write a Matlab program that computes the first 100 elements in the Fibonacci sequence. Using the program, compute the ratio of subsequent elements in the sequence $r_n = \frac{f_{n+1}}{f_n}$.

   Plot r_n vs. n. What happens as n gets larger?

   Explain your observation in terms of the eigenvalues and eigenvectors of the matrix A.

   Derive a general formula for f_n not involving previous terms in the sequence. Hint. You will need to use the eigenvalues and eigenvectors of the matrix A.

4) EG Exercise 2.6.

5) EG Exercise 2.7.

6) EG Exercise 2.11. Hint: The matrices are 4 x 4 and should be non-negative (negative entries are not possible physical solutions). You may want to simulate the population dynamics from your non-power-positive projection matrix to answer the last part of this exercise.

7) EG Exercise 2.13. You may want to check your results with the sensitivity formula EG [2.36]. What do you find?

8) EG Exercise 2.14.

9) EG Exercise 2.15.

10) *Optional Exercise:* Derive Equation [2.36] in EG.