课程名

Course Name

姓名

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第 1 章　章节名

1.1　例子

中文正文测试 test。

定义 1.1.1 Limit of Sequence in $\mathbb{R}$

Let $\{s_n\}$ be a sequence in $\mathbb{R}$. We say

$$\lim_{n\to\infty} s_n = s$$

where $s \in \mathbb{R}$ if $\forall$ real numbers $\epsilon > 0$ $\exists$ natural number N such that for $n > N$

$$s - \epsilon < s_n < s + \epsilon \text{ i.e. } |s - s_n| < \epsilon$$

解. We have to take its complement and check whether that set is a open set i.e. if it is a union of open balls. We have to take its complement and check whether that set is a open set i.e. if it is a union of open balls We have to take its complement and check whether that set is a open set i.e. if it is a union of open balls We have to take its complement and check whether that set is a open set i.e. if it is a union of open balls We have to take its complement and check whether that set is a open set i.e. if it is a union of open balls

测试

注

We will do topology in Normed Linear Space (Mainly $\mathbb{R}^n$ and occasionally $\mathbb{C}^n$)using the language of Metric Space.

断言 1.1.1 Topology

Topology is cool

例 1.1.1 (Open Set and Close Set)

Open Set:
- ϕ
- $\bigcup_{x \in X} B_r(x)$ (Any $r > 0$ will do)
- $B_r(x)$ is open

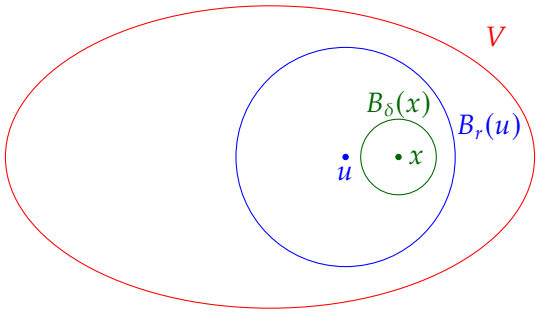
Closed Set:
- $X,\ \phi$
- $\overline{B_r(x)}$

$x-\text{axis} \cup y-\text{axis}$

定理 1.1.1

If $x \in$ open set V then $\exists\ \delta > 0$ such that $B_\delta(x) \subset V$

证明. By openness of V, $x \in B_r(u) \subset V$



Given $x \in B_r(u) \subset V$, we want $\delta > 0$ such that $x \in B_\delta(x) \subset B_r(u) \subset V$. Let $d = d(u, x)$. Choose δ such that $d + \delta < r$ (e.g. $\delta < \frac{r-d}{2}$)

If $y \in B_\delta(x)$ we will be done by showing that $d(u, y) < r$ but

$$d(u, y) \leqslant d(u, x) + d(x, y) < d + \delta < r$$

□

推论 1.1.1

By the result of the proof, we can then show...

引理 1.1.1

Suppose $\vec{v}_1, \ldots, \vec{v}_n \in \mathbb{R}^n$ is subspace of $\mathbb{R}^n$.

命题 1.1.1

$1 + 1 = 2$.

1.2 Random

定义 1.2.1 Normed Linear Space and Norm $\|\cdot\|$

Let V be a vector space over $\mathbb{R}$ (or $\mathbb{C}$). A norm on V is function $\|\cdot\| V \to \mathbb{R}_{\geqslant 0}$ satisfying

① $\|x\| = 0 \iff x = 0 \; \forall \, x \in V$

② $\|\lambda x\| = |\lambda|\|x\| \; \forall \, \lambda \in \mathbb{R}(\text{or } \mathbb{C}), x \in V$

③ $\|x + y\| \leqslant \|x\| + \|y\| \; \forall \, x, y \in V$ (Triangle Inequality/Subadditivity)

And V is called a normed linear space.

• Same definition works with V a vector space over $\mathbb{C}$ (again $\|\cdot\| \to \mathbb{R}_{\geqslant 0}$) where ② becomes $\|\lambda x\| = |\lambda|\|x\|$ $\forall \, \lambda \in \mathbb{C}, x \in V$, where for $\lambda = a + ib$, $|\lambda| = \sqrt{a^2 + b^2}$

例 1.2.1 ($p-$Norm)

$V = \mathbb{R}^m$, $p \in \mathbb{R}_{\geqslant 0}$. Define for $x = (x_1, x_2, \cdots, x_m) \in \mathbb{R}^m$

$$\|x\|_p = \left(|x_1|^p + |x_2|^p + \cdots + |x_m|^p\right)^{\frac{1}{p}}$$

(In school $p = 2$)

Special Case $p =$ 1: $\|x\|_1 = |x_1| + |x_2| + \cdots + |x_m|$ is clearly a norm by usual triangle inequality.

Special Case $p \to \infty$ ($\mathbb{R}^m$ with $\|\cdot\|_\infty$): $\|x\|_\infty = \max\{|x_1|, |x_2|, \cdots, |x_m|\}$

For $m = 1$ these $p-$norms are nothing but $|x|$. Now exercise **解. For Property ③ for norm-2**

When field is $\mathbb{R}$:

We have to show

$$\sum_i (x_i + y_i)^2 \leqslant \left(\sqrt{\sum_i x_i^2} + \sqrt{\sum_i y_i^2}\right)^2$$

$$\implies \sum_i (x_i^2 + 2x_i y_i + y_i^2) \leqslant \sum_i x_i^2 + 2\sqrt{\left[\sum_i x_i^2\right]\left[\sum_i y_i^2\right]} + \sum_i y_i^2$$

$$\implies \left[\sum_i x_i y_i\right]^2 \leqslant \left[\sum_i x_i^2\right]\left[\sum_i y_i^2\right]$$

So in other words prove $\langle x, y\rangle^2 \leqslant \langle x, x\rangle\langle y, y\rangle$ where

$$\langle x, y\rangle = \sum_i x_i y_i$$

注

- $\|x\|^2 = \langle x, x\rangle$
- $\langle x, y\rangle = \langle y, x\rangle$

- $\langle \cdot, \cdot \rangle$ is $\mathbb{R}-$linear in each slot i.e.

$$\langle rx + x', y \rangle = r\langle x, y \rangle + \langle x', y \rangle \text{ and similarly for second slot}$$

  Here in $\langle x, y \rangle$ x is in first slot and y is in second slot.

Now the statement is just the Cauchy-Schwartz Inequality. For proof

$$\langle x, y \rangle^2 \leqslant \langle x, x \rangle \langle y, y \rangle$$

expand everything of $\langle x - \lambda y, x - \lambda y \rangle$ which is going to give a quadratic equation in variable λ

$$\begin{aligned} \langle x - \lambda y, x - \lambda y \rangle &= \langle x, x - \lambda y \rangle - \lambda \langle y, x - \lambda y \rangle \\ &= \langle x, x \rangle - \lambda \langle x, y \rangle - \lambda \langle y, x \rangle + \lambda^2 \langle y, y \rangle \\ &= \langle x, x \rangle - 2\lambda \langle x, y \rangle + \lambda^2 \langle y, y \rangle \end{aligned}$$

Now unless $x = \lambda y$ we have $\langle x - \lambda y, x - \lambda y \rangle > 0$ Hence the quadratic equation has no root therefore the discriminant is greater than zero.

When field is $\mathbb{C}$:

Modify the definition by

$$\langle x, y \rangle = \sum_i \overline{x_i} y_i$$

Then we still have $\langle x, x \rangle \geqslant 0$

1.3 Algorithms

Algorithm 1: what

Input: This is some input

Output: This is some output

```
/* This is a comment */
```

1 some code here;
2 $x \leftarrow 0$;
3 $y \leftarrow 0$;
4 **if** $x > 5$ **then**
5 x is greater than 5 ; // This is also a comment
6 **else**
7 x is less than or equal to 5;
8 **foreach** *y in 0..5* **do**
9 $y \leftarrow y + 1$;
10 **for** y *in* $0..5$ **do**
11 $y \leftarrow y - 1$;
12 **while** $x > 5$ **do**
13 $x \leftarrow x - 1$;
14 **return** Return something here;