Relationship of popular distributions

$$\frac{Z}{\sqrt{V/\nu}}$$

Student t

$$\frac{1}{2^{k/2}\Gamma(k/2)}\, x^{k/2-1} e^{-x/2}\, \mathbf{1}_{\{x \geq 0\}}$$

chi-squared

Z is Standard Gaussian
v degrees of freedom

Gaussian
$N(0,1)$

$(x-\mu)/\sigma$

$k = \frac{v}{2}, \theta = 2$

$$\frac{\beta^\alpha x^{\alpha-1} e^{-\beta x}}{\Gamma(\alpha)} \quad \text{for } x > 0 \quad \alpha, \beta > 0$$

$$\begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

Exponential

Gaussian
Mixture

Gaussian

$N(\mu, \sigma^2)$

Gamma

α=1

$$\frac{Gam(\alpha,1)}{Gam(\alpha,1) + Gam(\beta,1)} = Beta(\alpha,\beta)$$

multiplication

$$\frac{n!}{x_1! x_2! \cdots x_k!} p_1^{x_1} p_2^{x_2} \cdots p_k^{x_k}$$

$$\binom{n}{k} p^k (1-p)^{n-k}$$

Dirichlet

MultiNomial

Binomial

Beta

$$\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\, x^{\alpha-1}(1-x)^{\beta-1}$$

$$\Gamma(n) = (n-1)!$$

$$\frac{1}{B(\boldsymbol{\alpha})} \prod_{i=1}^{K} x_i^{\alpha_i - 1}$$

$$B(\boldsymbol{\alpha}) = \frac{\prod_{i=1}^{K} \Gamma(\alpha_i)}{\Gamma\left(\sum_{i=1}^{K} \alpha_i\right)}$$

Multi-Bernoulli
(Categorical)

Bernoulli

α=1, β=1

$$\prod_{i=1}^{k} p_i^{x_i} \quad \sum_i p_i = 1$$

$$p^k (1-p)^{1-k} \quad \text{for } k \in \{0,1\}$$

Uniform(0,1)

$$\begin{cases} \frac{1}{b-a} & \text{for } a \leq x \leq b, \\ 0 & \text{for } x < a \text{ or } x > b \end{cases}$$

Legend:
- ← → Conjugate
- → Multi-Class
- → N Times
- ⇢ Limit
- → Special Relationship
- *(Conditional, etc)*