Cryptocurrency-portfolios in a mean-variance framework

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Abstract

By the end of 2017, 27 cryptocurrencies topped a market capitalization of one billion USD. Bitcoin is still shaping market and media coverage, however, recently we faced a vibrant rise of other currencies. As a result, 2017 has also witnessed the advent of a large number of cryptocurrency-funds. In this paper, we use Markowitz' mean-variance framework in order to assess risk-return-benefits of cryptocurrency-portfolios. We relate risk and return of different portfolio strategies to single cryptocurrency investments. In an out-of-sample analysis accounting for transaction cost we find that combining cryptocurrencies in a portfolio enriches the set of 'low'-risk cryptocurrency investment opportunities.

Keywords: cryptocurrencies, portfolio optimization, Markowitz, naive diversification

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1. Introduction

The market of cryptocurrencies (CC henceforth) is undoubtedly dominated by Bitcoin, a decentralized digital currency, introduced by Satoshi Nakamoto in 2009. However, alternative currencies (altcoins) among which Ethereum, Ripple, Cardano, Litecoin and Stellar are the most popular, boldly gained attention as well as market shares. The focus of CC investors is therefore no longer directed towards Bitcoin alone and as a result, 2017 has witnessed the advent of a considerable number of cryptocurrency-funds.

There is an ongoing debate on whether Bitcoin and other CC shall be viewed as a currency or rather a speculative asset, though, Bitcoin based evidence points at the use of this CC as the latter (cf. Baur et al. (2017b)). Several authors therefore examined statistical properties of individual CC traded in a growing number of emerging marketplaces (i.e., exchanges like Bitfinex, Bitstamp, GDAX and hundreds of others; for a discussion of stylized facts of the Bitcoin market see Bariviera et al. (2017)). Early empirical results find Bitcoin to be (at least partly) 'inefficient' (in Fama's sense of efficient markets and predictability of returns; see Urquhart (2016); Bariviera (2017)), whereas more recent work on CC asserts efficiency (cf. Nadarajah & Chu (2017); Tiwari et al. (2018)). In a recent paper, Brauneis & Mestel (2018) study the cross-section of over 70 CC and find efficiency of single CC to be closely and positively related to their liquidity.

Academic efforts addressing CC as a portfolio constituent so far exclusively consider Bitcoin as a hedge for traditional asset classes (e.g. Baur et al. (2017a); Bouri et al. (2017a,b); Dyhrberg (2016a,b)). The paper at hand is the first to empirically evaluate benefits from forming CC portfolios in a Markowitz mean-variance framework and relates risk and returns thereof to single currency portfolio benchmarks. Hence, we aim at closing the research gap of risk-return effects of diversified, CC-only investments. The remainder of the text is structured as follows. Section 2 introduces the dataset and the methodology, section 3 presents results and discussion and section 4 concludes.

2. Data and Methodology

We use daily market data, covering a period of 3 years from 01/01/2015 to 12/31/2017, freely available from www.coinmarketcap.com. This website collects CC data from several exchanges worldwide and publishes volume weighted prices. The data comprises 1,096 individual price and trading volume observations for each of the 500 most capitalized CC as of December 2017. Descriptive statistics of discrete daily returns point at high mean and high volatility, further we observe skewed and fat tailed return distributions. Furthermore, we find evidence for non-normality, serial correlation, and heteroscedasticity (detailed descriptives upon request). Interestingly, it turns out that despite sharing common properties, individual CC returns are weakly correlated. We find almost 98% of all pairwise correlations among 500 CC to fall within the range from -0.10 to 0.20, suggesting a considerable extent of diversification effects in a risk-return framework.

In the attempt to address and quantify portfolio effects in the cryptocurrency investment universe we rely on the traditional mean-variance framework as proposed by Markowitz in 1952 (Markowitz (1952)). We are aware, though, of a considerable body of literature proposing alternatives to mean-variance optimization when returns are not normal. However, several authors like Levy & Markowitz (1979) or Kroll et al. (1984) derive almost equivalence of the mean-variance criterion to expected utility maximization under non-normality. In addition, this paper aims at revealing preliminary evidence on portfolio effects, i.e. properties of multiple CC investments, which is why we opt for using the basic mean-variance framework in determining CC portfolio structures.

As a starting point for mean-variance optimization, we calculate daily log-returns r_{it} for CC i at time t, derived from close prices P according to $r_{it} = \log[P_{it}/P_{it-1}]$. We estimate the vector of mean returns $\mu_{t,f}$ and the covariance matrix $\mathbf{\Sigma}_{t,f}$ at time t using data of f past trading days. We restrict the investment universe to the K most liquid out of 500 available CC, i.e. those with the highest mean dollar trading volume over the period from $(t-f)$ to t. We only allow for long positions in a portfolio, hence, the actual number of constituents in a particular portfolio may well be lower than K. We neither consider a cash position (i.e. a riskless asset) implying that at any time all funds are entirely invested in CC. Mean-variance optimal portfolios are formed and held for h days, subsequently, at time $(t + h)$ CC holdings are being rebalanced.

Besides buy-and-hold single CC investments, we pursue 8 different long only portfolio strategies given the time t CC investment universe. First, as a trivial portfolio, we apply the naively diversified $1/N$ strategy which may also be seen as a benchmark for data-driven optimized portfolios (cf. DeMiguel et al. (2009) for a discussion of the relative performance of $1/N$ portfolios to optimal portfolios); second, the maximum return portfolio given Bitcoin's return volatility (denoted as $BTC\mu_{opt}$); third, the tangency portfolio (PFT; alias the 'market portfolio' using CAPM's terminology) assuming a zero risk free rate; and fourth to eighth, 5 efficient frontier portfolios including the minimum variance portfolio (MVP) and the maximum return CC ($maxR$) as well as 3 equally spaced target-return portfolios in between these two (referred to as $PF1$, $PF2$, $PF3$; suppose MVP yields 1% and $maxR$ yields 5%, then the equally spaced target returns would be 2% ($PF1$), 3% ($PF2$), and 4% ($PF3$) respectively and the weights of these portfolios are set such that the corresponding risk is minimal). In addition and as a second benchmark portfolio, we take into account the CRIX, a market capitalization weighted CC index with freely available data on http://crix.hu-berlin.de (for details see Trimborn & Härdle (2016)).

Figure 1 exemplarily depicts an in-sample representation of these portfolios for $t = 9/25/2016$, with $f = 183$ and $K = 20$.

In the next section we empirically evaluate the performance of these portfolios out-of-sample accounting for transaction cost and different holding periods, i.e. different time spans for which the portfolio-weights remain unchanged.

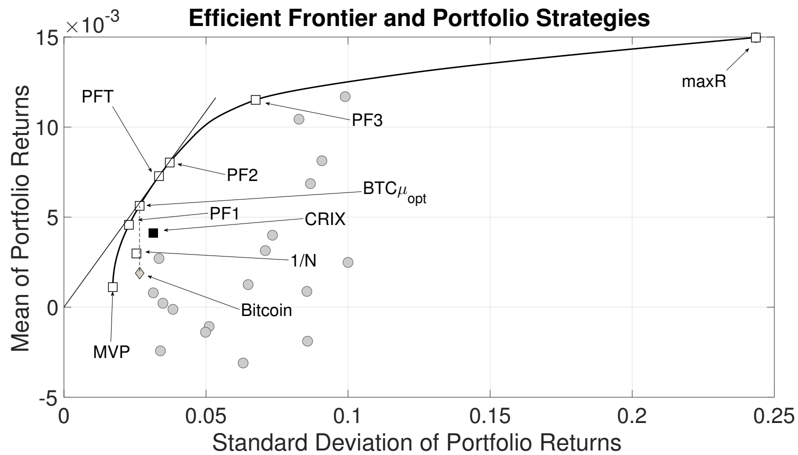

Figure 1: Location of the $K = 20$ most liquid individual CC in a risk-return-framework as of September 25, 2016. Mean and standard deviation are based on $f = 183$ daily past observations. In addition, the plot shows the location of Bitcoin, the $1/N$ portfolio, 7 mean-variance optimal portfolios and the CC benchmark index CRIX.

3. Results and Discussion

This section presents results from CC mean-variance optimal portfolios, benchmark portfolios ($1/N$ and CRIX) as well as single CC investments. Note that all calculations are done on a daily basis, i.e. for each portfolio strategy we derive a time series of daily returns over the course of the time period under consideration. Our baseline setting uses $f = 183$ past days in order to estimate mean returns and covariances. Further, we restrict the investment universe to the $K = 20$ most liquid CC and hold portfolios for $h = \{1, 7, 30\}$ days before rebalancing portfolio weights. In particular, the first portfolios are formed at time $t = f$ (the first point in time for which f days of data are available for estimating μ and $\mathbf{\Sigma}$), with a net investment of one dollar respectively. Generally, when portfolios need to be rebalanced at time t, for each of the 8 portfolio strategies, a new Markowitz optimal weight vector $\mathbf{w}_t^\theta$ (θ representing the respective strategy) is derived. Current holdings in strategy θ at time t based on last optimal portfolio weights $\mathbf{w}_{t-h}^\theta$ are rebalanced according to the new target weights. The net portfolio turnover is subject to transaction cost which we assume to amount to 25 basispoints (this is taken from bitstamp.com, one of the world's biggest CC exchanges). No changes to the portfolios' weights will be carried out from day t forward until day $t + h$. Overall, taking eight different portfolio strategies and three holding periods, we analyze a total of 21 mean-variance optimized strategies, three versions of the $1/N$ benchmark portfolio, the CRIX, as well as single CC investments. In the baseline parameterization from the beginning of 2015 to the end of 2017 an aggregate of 59 single CC at least once enter a portfolio as one of the constituents (of which only 31 CC have a full time series of data from the very beginning though: for instance, some of the top capitalized and most liquid CC like Ethereum, Nem and Iota are first available well after 1/1/2015). The final set of investments thus comprises 25 portfolios and 59 single CC.

Note that we also performed alternative parameterizations of our analysis with shortened and extended estimation data ($f = \{93, 365\}$), and additional choices for the holding period of portfolios ($h = \{5, 15, 60, 100\}$). We find all our results to remain qualitatively unchanged (details upon request), which is why we opt to present detailed results only for the baseline setup. However, a broader investment universe ($K = \{30, 50, 100\}$) alters results towards a higher mean return at no additional risk but linked to substantially reduced liquidity of constituents CC on the downside.

Panel (A) of Table 1 reports mean daily returns and corresponding standard deviations for our 24 strategies. Panel (B) shows results for a version which excludes highly volatile CC. We define as a cutoff point a historical daily volatility of 0.15, i.e. all CC exceeding this volatility level over f past days. The rationale behind this is to rule out potential biases stemming from extreme risk CC. Effectively, 8 out of 59 CC in the baseline setup do not meet this criterion. However, overall results for the low-risk set of 51 CC are virtually identical with those of the full sample. We take this as a strong hint that results might not be driven by single high-risk CC. Note that over the same time period we observe for the CRIX benchmark mean daily returns of 0.5750% with a standard deviation of 3.60%.

Panel (A): Unrestricted volatility of CC

mean(r)	$1/N$	$BTC\mu_{opt}$	PFT	MVP	$PF1$	$PF2$	$PF3$	$maxR$
$h = 1$	0.6858	0.6085	0.6055	0.6064	0.6087	0.5844	0.5613	0.3023
$h = 7$	0.6816	0.5598	0.5268	0.6067	0.5767	0.5481	0.5764	0.2538
$h = 30$	0.6861	0.4920	0.4160	0.6000	0.5432	0.4499	0.3619	-0.5317
std(r)								
$h = 1$	4.40	4.38	5.09	3.48	3.96	5.41	7.64	12.83
$h = 7$	4.38	4.15	4.83	3.74	3.79	4.95	7.16	12.48
$h = 30$	4.35	4.07	4.57	3.85	3.99	4.71	6.07	9.04

Panel (B): Restricted volatility of constituent CC, $\sigma_f < 0.15$

mean(r)	$1/N$	$BTC\mu_{opt}$	PFT	MVP	$PF1$	$PF2$	$PF3$	$maxR$
$h = 1$	0.6946	0.5767	0.5780	0.6103	0.6083	0.5944	0.5331	0.4345
$h = 7$	0.6822	0.5429	0.5162	0.6110	0.5987	0.5748	0.5892	0.2483
$h = 30$	0.6995	0.5614	0.4722	0.6080	0.5943	0.5476	0.5231	0.2555
std(r)								
$h = 1$	4.39	4.28	5.07	3.51	3.78	4.66	6.03	10.43
$h = 7$	4.40	4.20	4.92	3.78	3.85	4.74	6.75	10.72
$h = 30$	4.37	4.28	4.73	3.89	3.99	4.55	5.79	9.34

Table 1: Percentage net mean daily strategy returns ('mean(r)') and standard deviation of net daily strategy returns ('std(r)'). Transaction cost is set to 25 basispoints. Columns present different strategies, h refers to the holding period after which the portfolio is rebalanced. Panel (A) reports results for an unrestricted selection of CC whereas panel (B) reports results for portfolios with constituent CC, estimation period from $(t - f)$ to t returns not exceeding a daily volatility of 15%. Overall, in comparison with the CRIX (mean(r) = 0.5750, std(r) = 3.60), virtually all portfolios feature higher mean returns and higher risk.

From Table 1 we observe a tendency that, despite higher transaction cost, frequent rebalancing within a given portfolio strategy results in higher mean daily returns without leading to higher standard deviations. Irrespective of the examined holding period, the $1/N$ portfolios yield the highest returns.

Interestingly, however, and apart from the $maxR$ portfolio (which actually is one single CC), results for distinctly risky strategies almost denote the same out-of-sample performance. With respect to the mean-variance framework, this implies that CC portfolios gather in the southwest of a risk-return-plot and augment investment opportunities for CC investors seeking for low-risk (provided that low-risk is an appropriate objective in the highly volatile CC markets). Figure 2 reveals the risk-return pattern of single CC (all CC that at least once entered a portfolio) and the unrestricted volatility portfolios in our baseline setting.

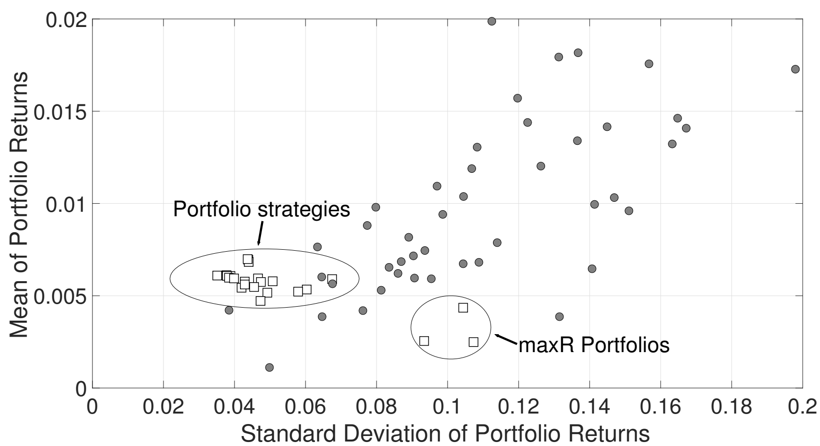

Figure 2: A risk-return perspective of portfolios and constituents CC. Squares represent CC portfolios, whereas gray circles correspond to individual CC.

From Figure 2 one may conclude that under the chosen setting, any arbitrary portfolio strategy in the CC market substantially reduces risk. To investigate whether this result applies generally or is owed to our model specification, we implement an alternative parameterization of our setting by extending the investment universe to the $K = \{30, 50, 100\}$ most liquid CC for both, the unrestricted and restricted volatility version in selecting constituents of CC portfolios. That is, for $f = 183, h = \{1, 7, 30\}$ we derive mean returns and standard deviations and depict them in a shared risk-return-plot (again, accounting for transaction cost of 25 basispoints). Figure 3 shows a total of 144 mean-variance-optimal portfolios (6 optimized portfolio strategies excluding the $maxR$ strategy, 4 different choices of K, 3 choices for h and 2 choices for volatility (restricted/unrestricted)), 24 differently parameterized $1/N$ portfolios and the CRIX.

Overall, we find that our extended CC investment universe does not lead to qualitatively altered results. Mean returns and standard deviations mostly

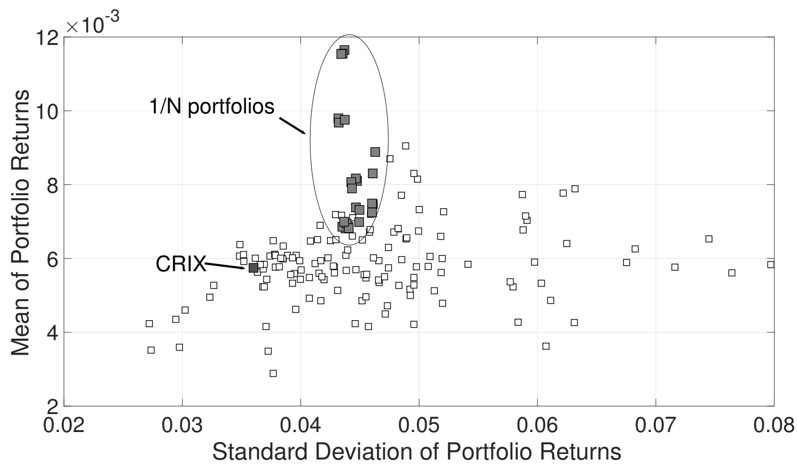

Figure 3: Risk and return of the baseline CC portfolios for 24 different parameterizations respectively. The section of the risk-return space plotted in this figure roughly corresponds to the circled region of portfolios in Figure 2.

remain within the limit values illustrated in Figure 2. In addition, Figure 3 indicates that the naively diversified $1/N$ portfolios exhibit very similar standard deviations; however, mean returns differ substantially.

The figure also points out that the $1/N$ portfolios seem to have superior risk-return patterns compared to the optimized portfolios. Following DeMiguel et al. (2009), we further investigate this point by comparing the Sharpe ratios as well as the certainty equivalent returns of all portfolio strategies in the different parameterizations outlined above. The Sharpe ratio SR is defined as the average return earned in excess of the risk free rate per unit of total risk. Assuming a zero risk free rate, the Sharpe ratio for any investment i therefore equals $SR_i = \widehat{\mu}_i / \widehat{\sigma}_i$. CC i's certainty equivalent return (CEQ) is defined as $CEQ_i = \widehat{\mu}_i - \frac{\gamma}{2} \cdot \widehat{\sigma}_i^2$, where γ reflects risk aversion which is set to one as in DeMiguel et al. (2009).

Table 3 presents our results. In line with our previous considerations the $1/N$ portfolios outperform mean-variance optimal portfolios as well as individual CC. It turns out that the minimum Sharpe ratio of all $1/N$ portfolios exceeds the Sharpe ratios of more than 75% of the optimized portfolios. The average Sharpe ratio of the $1/N$ investments of .1865 is also higher than that of the

CRIX ($SR_{CRIX} = .1596$). The same conclusions may be drawn from certainty equivalent returns, $1/N$ portfolios on average outperform at least 75% of mean-variance optimal portfolios and constituent CC according to the chosen measure CEQ. We take these results as empirical evidence that an equally weighted portfolio might be the best choice in building a CC-only portfolio.

Panel (A): Sharpe ratios for sets of CC investments

	M	mean	q1	median	q3	min	max	std
$1/N$	24	0.1865	0.1580	0.1717	0.2075	0.1549	0.2667	0.0377
optimized	144	0.1301	0.1113	0.1342	0.1534	0.0596	0.1852	0.0285
constituent CC	226	0.0928	0.0806	0.0920	0.1073	0.0236	0.1775	0.0246

Panel (B): Certainty equivalent returns for sets of CC investments with $\gamma = 1$

	M	mean	q1	median	q3	min	max	std
$1/N$	24	0.0073	0.0060	0.0067	0.0083	0.0059	0.0107	0.0016
optimized	144	0.0047	0.0040	0.0048	0.0054	0.0018	0.0079	0.0011
constituent CC	226	-0.0336	-0.0134	0.0006	0.0041	-1.6790	0.0225	0.1654

Table 2: Descriptive statistics for the Sharpe ratios (panel (A)) and the certainty equivalent returns (Panel (B)) for the set of differently parameterized $1/N$ strategies, mean-variance optimal portfolios [optimized] and all constituent CC [constituents]. The table reports the mean of Sharpe ratios in each set, 25%-quantile (q1), median, 75%-quantile (q3), the minimum and maximum Sharpe ratio within a set and standard deviation. M denotes the number of investment portfolios and single CC, respectively. Notice that the CRIX as the second benchmark portfolio besides $1/N$ strategies features a Sharpe ratio of 0.1596 and a CEQ of 0.51%.

4. Conclusion

In the recent past, a growing number of cryptocurrency funds have found their way into the cryptocurrency asset class and hence the investment universe. We provide a first study investigating the effects of diversified cryptocurrency investments in a traditional Markowitz mean-variance framework. While previous studies exclusively focus on the diversification effect of adding one single CC (typically Bitcoin) to a portfolio containing traditional asset classes, we find substantial potential for risk reduction when several CC are mixed. Applying different parameterizations of mean-variance investments, our study identifies naively diversified portfolios to derive risk-adjusted outperformance when compared to mean-variance optimized portfolios.

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