B-001
Regarding NMRA® RP-25

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November 2024

Abstract: RP-25 is the NMRA® recommended practice for manufacture of model train wheels. The RP-25 document recommends wheel contours and dimensions.

A procedure for drawing the RP-25 wheel contour is presented here. Furthermore, contour geometry is analyzed and critical points are tabulated.

1 Introduction

The RP-25 document[1] is perplexing. It could be improved by:

- Discussing the procedure to accurately draw the wheel contour.
- Providing critical information, like the location of circle/arc centers.
- Describing how recommended circle/arc radii values were determined.
- Suggesting reasonable manufacturing tolerances.

When reviewing dimensions defined by adjacent RP-25 wheel codes, it becomes obvious that the published values progress disproportionally from one code to the next. Answers to the following questions would help explain the published values.

- Were dimensions derived from a mathematical formula?
- Were dimensions based upon common practices at the time a particular rail/wheel/gauge combination was developed?
- Were dimensions a compromise amongst competing equipment manufacturers?

Rigorous mathematical treatment of the RP-25 wheel contour in an official NMRA® publication would be helpful. Unfortunately, historical NMRA®: bulletins, magazines, standards and recommended practices; lack mathematical basis for the contour. Alternative sources, like the article in Model Railroader magazine[2], only discusses history of wheel profile development and restates technical information found in the RP-25 document; but does not discuss derivation of profile geometry.

Please understand, specific contour profile of a wheel is relatively unimportant. Over the years many wheel contours have been proposed and tried with varying success. In fact you are free to invent your own contours. What is truly important is that a wheelset (pair of wheels mounted on an axle):

- Conforms to NMRA® standard S-4.2 for wheelsets.[3]
- Passes inspection with a NMRA® RP-2 test gauge.[4]

Remember, RP-25 is only a recommended practice rather than a standard. Nonetheless, it still would be helpful to understand what that recommended practice is, and how to apply it.

The good news is: if you can draw a RP-25 contour accurately, then you can understand the ideal you are attempting to achieve. The following sections describe an approach to:

- Draw the RP-25 contour
- Analyze the RP-25 contour mathematically
- Calculate and tabulate critical points

2 Problem

How does one draw the RP-25 wheel contour? Don't think in terms of wheel: treads, fillets and flanges! The trick to solving this problem is by framing it geometrically.

Simplify! Reduce the problem to its most fundamental mathematic ideas.

The problem of drawing the RP-25 contour is really just a problem of drawing three circles in the correct locations!

Given:

- Three circles: $C1$, $C2$, $C3$; whose radii are: $R1$, $R2$, $R3$; respectively...
- where $0 < R1 < R2 = R3$

Find: Circle center locations: p_1, p_2 and p_3; such that:

1. $C1$ is tangent to a sloped line passing some distance P above the origin
2. with, $C1$ and $C2$ mutually tangent to each other, at the origin
3. and with, $C2$ and $C3$ intersecting at some location: $P - d'$ (not D') below, and $T/2$ to right of; the origin.

3 Geometric Theorems

Three helpful plane geometry theorems are declared here. They will be used within the solution. They are quoted from their original source, which does provide proofs. A curious reader will find the source referenced in the endnotes section.

Theorem 1: Points on a circle

> A circle is a plane figure bounded by a line called the circumference, all points of which are equidistant from a point within called the center. [5]

Theorem 2: Tangent line perpendicular to radius

> A straight line perpendicular to a radius (of a circle) at its extremity is tangent to the circle; conversely, the tangent at the extremity of a radius is perpendicular to the radius. [6]

Theorem 3: Mutually tangent circles

> If two circles are tangent to each other externally or internally, the line of centers passes through the point of tangency. [7]

4 Strategy

The RP-25 contour is difficult to draw accurately because a dependency chain exists amongst contour points. We would like to draw gaging point p_g at a coordinate system origin, however; p_g depends on point p_1. Furthermore, all point locations are dependant on slope angle θ_s.

If you draw the points in the wrong order, your attempt will fail. If you try to immediately locate p_g at a coordinate system origin, your attempt will fail.

Everything depends on your choice of slope angle θ_s. As we shall see, the chain of dependencies is: $p_d \to p_3 \to p_2 \to p_g \to p_1 \to p_s \to \theta_s$.

The Trick: Ideally, we want all points to have single dependencies upon a single point at the origin.

To draw the RP-25 contour successfully, we will employ a trick. Trick: Rather than establishing a coordinate system and locating points relative to the origin, we will instead wait to find gaging point p_g and only then establish a coordinate system at p_g as the origin.

To apply our trick we must draw tread slope angle θ_s first!

5 Constructive Solution

The wheel profile can be drawn either:

- Manually, with paper, pencil and drafting tools; or,
- Electronically, with a computer aided drafting (CAD) program.

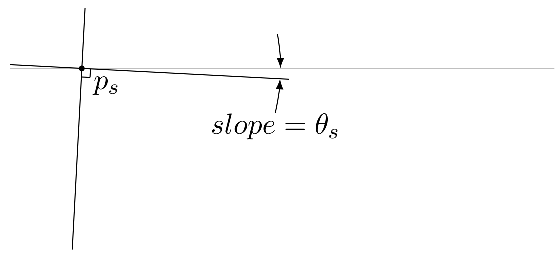


Figure 1:
Step 1: Establishing Point p_s

If drawing manually, you will want to scale up all dimensions by a ratio of 20 : 1 or so. Also when drawing manually, a dial caliper or machinist's ruler, graduated in tenths and hundredths of an inch, will be needed.

5.1 Step 1: Establishing Point p_s

Begin the drawing as shown in Figure 1.

On a blank screen or page, draw a horizontal line. Next draw a second line at some slope, from 0° to 3°, to the horizontal line. To the left of the intersection, the sloped line becomes the wheel tread.

Mark the point where the horizontal line and wheel tread line intersect. Label the point p_s. Point p_s is the point where the wheel fillet will end and the wheel tread begins.

Figure 2:
Step 2: Drawing Point p_1

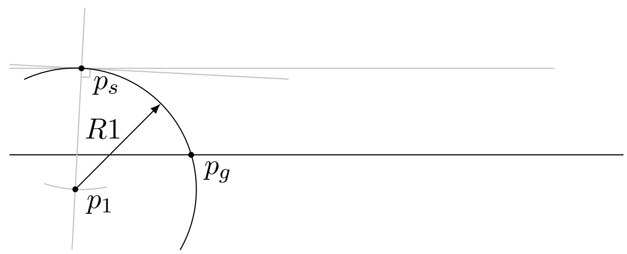


Figure 3:
Step 3: Drawing Point p_g

Now assign label θ_s, to the slope angle between the horizontal line and the wheel tread line.

Finally, draw a line thru p_s and at right angles to the tread line.

5.2 Step 2: Drawing Point p_1

Refer to Figure 2.

From Theorem 1 we know the center p_1 of a circle with radius $R1$ is some distance $R1$ away from a point p_s on the circle. Furthermore, from Theorem 2 we know that line through circle center p_1 and point p_s is perpendicular to a tangent line. In this case the tangent line is the tread line that is at a slope angle θ_s with respect to the horizontal.

Sweep an arc, of radius $R1$, centered at point p_s. The arc will intersect the line, that passes through p_s and is at right angles to the tread. Mark the intersection and label it point p_1.

5.3 Step 3: Drawing Point p_g

Refer to Figure 3.

Draw a new horizontal line. The new horizontal line should be parallel to the original horizontal line from Figure 1. It should be a distance P below the original horizontal line. The Distance P is given by the RP-25 document.

Now sweep an arc, of radius $R1$, centered at point p_1. The arc should pass thru point p_s and intersect the new horizontal line. Mark the location where the arc intersects the new horizontal line. Label the intersection point p_g.

5.4 Step 4: Establishing axes

Refer to Figure 4.

Let's apply our "Trick" from Section 4.

Draw an arrowhead on the right end of the horizontal line (drawn in previous step) that

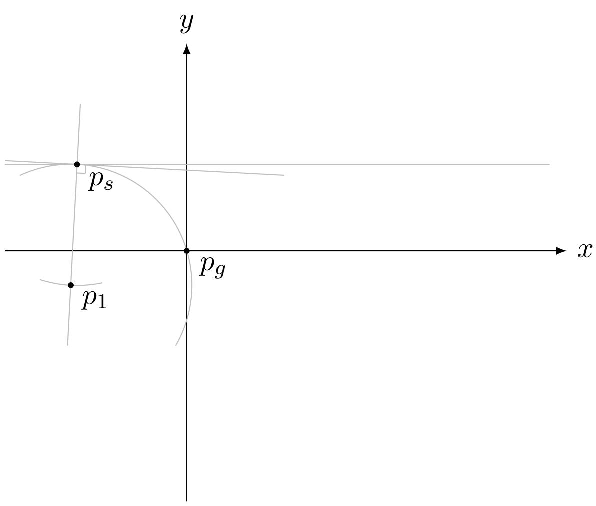


Figure 4:
Step 4: Establishing axes

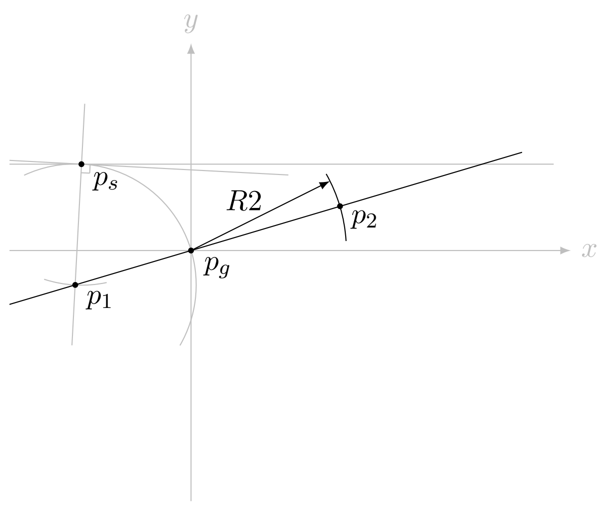


Figure 5:
Step 5: Drawing p_2

passes through p_g. Mark it with label x. The horizontal line is now the x-axis.

Draw a vertical line thru p_g. Draw an arrowhead at the upper end of the vertical line and mark it with label y. The vertical line is now the y-axis.

If you are using a CAD drawing program on a computer, either:

1. Use the CAD program's feature to move the coordinate system origin to p_g,

2. or, select all the: points, lines, arcs and labels; drawn so far, then select p_g as the relative insertion point, then drag and snap p_g to the coordinate system origin. Verify all: points, lines, arcs and labels; follow p_g and maintain their relative positions.

5.5 Step 5: Drawing p_2

Refer to Figure 5.

The RP-25 document says the circles of radius $R1$ and $R2$ are tangent to each other at the gagging point p_g. From Theorem 3 we know that the point of tangency p_g is colinear with the circle centers, p_1 and p_2.

Extend a line through points p_1 and p_g, and slightly beyond.

Now sweep an arc, of radius $R2$, centered at p_g. Mark the location where the arc crosses the line, that extends through p_1 and p_g, and label it point p_2.

5.6 Step 6: Drawing circle of radius $R2$

Refer to Figure 6.

Sweep an arc, of radius $R2$, centered at point p_2. The arc should pass through point p_g, and

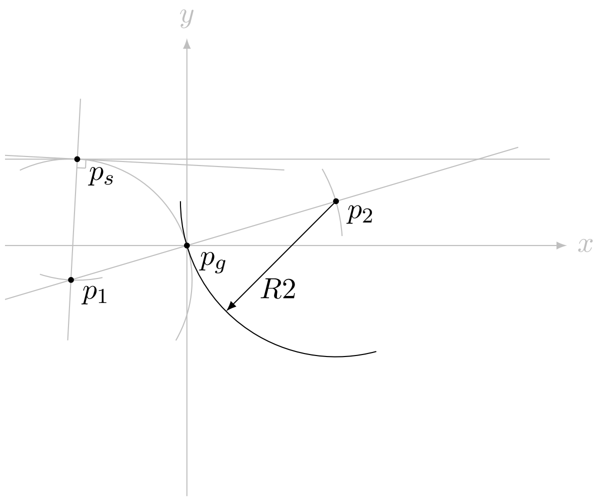


Figure 6:
Step 6: Drawing circle of radius $R2$

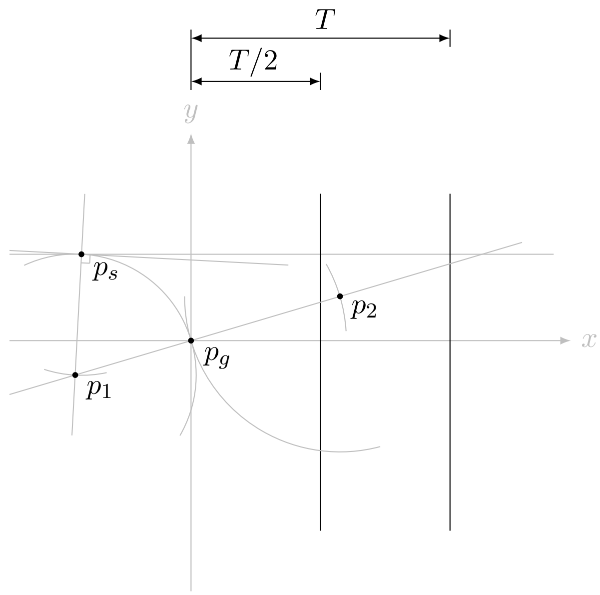


Figure 7:
Step 7: Drawing vertical guidelines

extend below and to the right.

5.7 Step 7: Drawing vertical guidelines

Refer to Figure 7.

The distance T is given by the RP-25 document.

Draw a vertical line a distance T to the right of the origin.

Now draw a vertical line a distance $T/2$ to the right of the origin, where $T/2$ is half the given distance T.

5.8 Step 8: Drawing p_3

Refer to Figure 8.

Draw a horizontal line through point p_2. Measure the distance from p_2 to the vertical guideline at $T/2$.

Sweep an arc, with the radius found in the measurement above, centered at the intersection

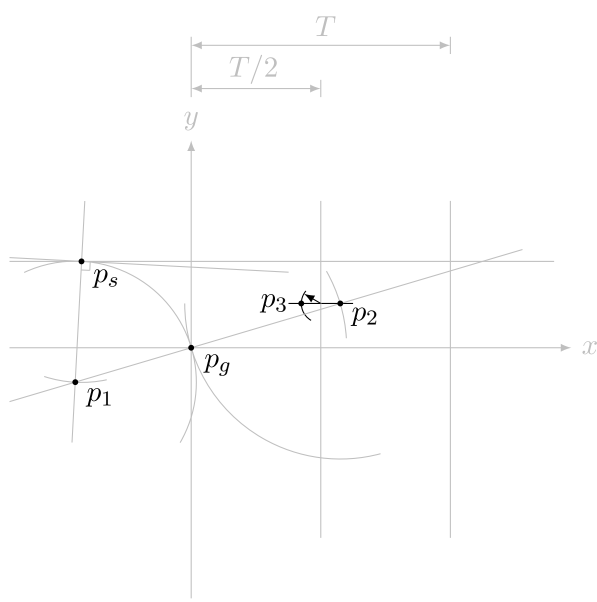


Figure 8:
Step 8: Drawing p_3

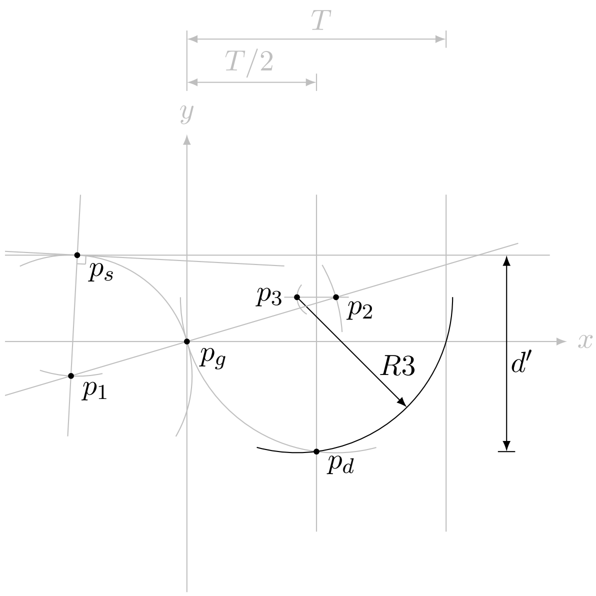


Figure 9:
Step 9: Drawing p_d

where the horizontal line crosses the $T/2$ vertical guideline.

Mark the point where the arc crosses the horizontal line, and label it p_3.

This process above is equivalent to reflecting point p_2 across the $T/2$ vertical guideline.

5.9 Step 9: Drawing p_d

Refer to Figure 9.

Sweep an arc, with radius $R3 = R2$, centered at p_3. Mark the point where the arc intersects the $R2$ radius arc at the $T/2$ vertical guideline. Label the point p_d.

Observe that the arcs of radius $R2$ and $R3$ form a tangent ogive [8] cross section.

Note: p_d is at some distance d' (not D') below p_s.

5.10 Step 10: Completing the Drawing

Refer to Figure 10.

Extend the wheel tread line to the left.

Label the drawing with: D', N', P and W; add the vertical and horizontal guidelines, and dimensions given in the RP-25 document.

Finally, break the sharp corner, shown in the RP-25 document, with a fillet (round off) or chamfer (clip). How much you break the corner is not critical.

The drawing is now complete!

6 Analytical Solution

Discovering the equations for critical points will enable us to calculate precise locations. Our strategy will be to:

1. Identify a series of right triangles that contain pairs of critical points.
2. Apply trigonometry to find the sides of those right triangles.
3. Use the right triangle sides to locate cartesian coordinates for the critical points.

We will be referring to Figure 11 during our analysis. You should note that in Figure 11 the slope angle θ_s has been exaggerated for clarity. Figure 11 has also been scaled up slightly to allow more space for labels.

6.1 Step a: Locating p_1

Figure 11a shows a right triangle containing vertices at points p_s and p_1. Point p_s is the location where the wheel tread begins, and p_1 is the center of the fillet arc whose radius is $R1$.

Let's label the lengths of the horizontal and vertical sides, x_s and y_s respectively.

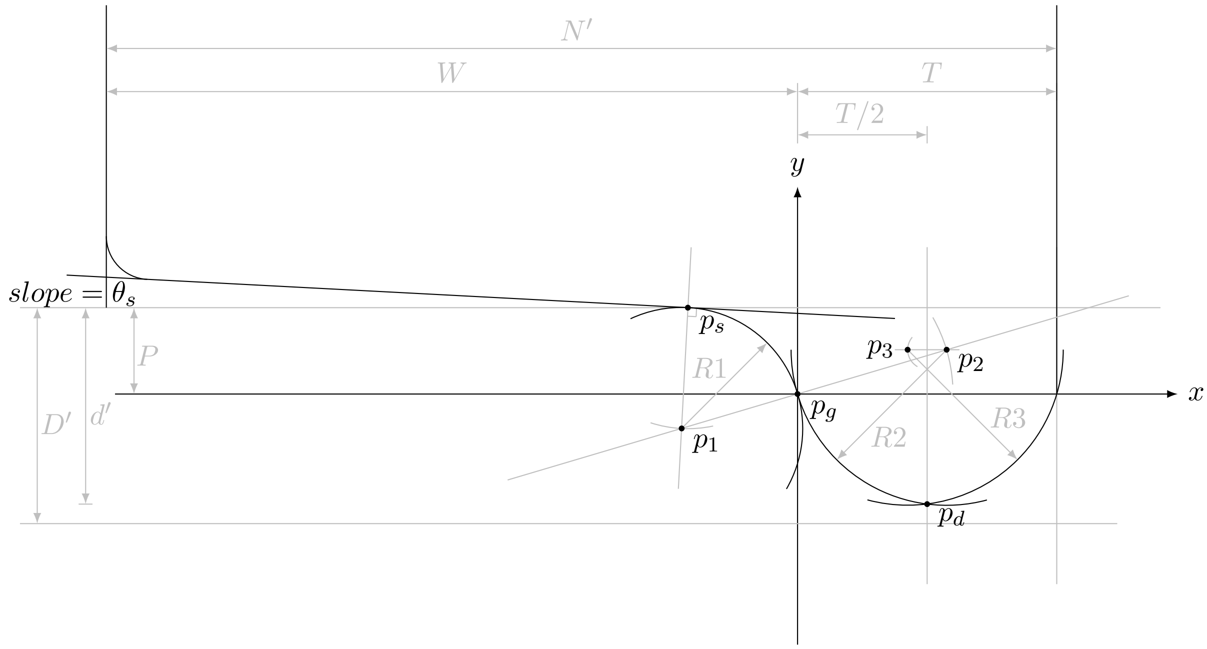


Figure 10:
Step 10: Completing the Drawing

Since the hypotenuse of length $R1$ is at right angles to the tread slope, the angle between vertical side y_s and hypotenuse $R1$ is the slope angle θ_s.

Therefore, the lengths of sides x_s and y_s can be found as follows:

$$x_s = R1 \cdot \sin(\theta_s) \tag{1}$$

$$y_s = R1 \cdot \cos(\theta_s) \tag{2}$$

6.2 Step b: Locating p_g

Figure 11b shows a right triangle containing vertices at points p_1 and p_g. Point p_g is the gaging point. Furthermore, p_1 is the center of the fillet arc whose radius is $R1$.

The hypotenuse of the triangle has length $R1$. Let's label the lengths of the horizontal and vertical sides, x_g and y_g respectively.

Returning to figure 11a we see p_1 is a distance y_s below point p_s, where the wheel tread begins. Since point p_g is a distance P below point p_s, it can be seen that $y_s = y_g + P$ and:

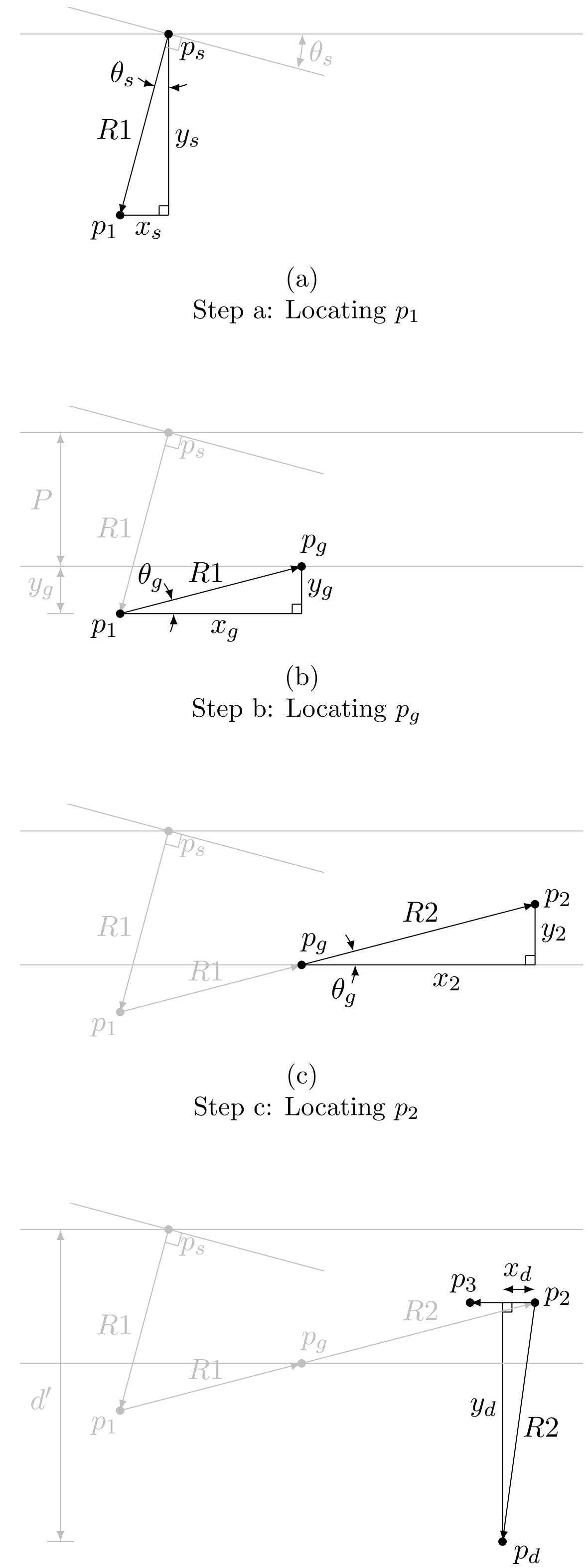
$$y_g = y_s - P \tag{3}$$

Now that y_g is known, we can use the Pythagorean theorem to find x_g:

$$x_g = \sqrt{(R1)^2 - y_g^2} \tag{4}$$

Let's assign the label θ_g to the angle between side x_g and hypotenuse $R1$. The angle θ_g can be found easily with:

$$\boxed{\theta_g = \arccos\left(\frac{x_g}{R1}\right)} \tag{5}$$



(a)
Step a: Locating p_1

(b)
Step b: Locating p_g

(c)
Step c: Locating p_2

(d)
Step d: Locating p_3 and p_d

Figure 11: Locating critical points analytically

6.3 Step c: Locating p_2

Figure 11c shows a right triangle containing vertices at points p_g and p_2.

Point p_g is the gaging point. Point p_2 is at the center of the wheel flange arc whose radius is $R2$. Let's label the horizontal and vertical sides x_2 and y_2 respectively.

The hypotenuse is length $R2$.

The angle between side x_2 and hypotenuse $R2$ is the same θ_g value found by equation 5, such that:

$$x_2 = R2 \cdot \cos(\theta_g) \tag{6}$$

And:

$$y_2 = R2 \cdot \sin(\theta_g) \tag{7}$$

6.4 Step d: Locating p_3 and p_d

Figure 11d shows a right triangle containing vertices at points p_2 and p_d.

Point p_2 is the center of the flange arc whose radius is $R2$. Point p_d is the bottom of the flange where the flange halves intersect. Let's label the lengths of the horizontal and vertical sides x_d and y_d respectively.

The hypotenuse is of length $R2$.

From figures 10 and 11c we can see the horizontal distance x_2, found in equation 6, is just a distance x_d to the right of $T/2$. In other words: $x_2 = T/2 + x_d$

Therefore we can write:

$$x_d = x_2 - T/2 \tag{8}$$

Using the Pythagorean theorem we can find the vertical distance y_d as follows:

$$y_d = \sqrt{(R2)^2 - x_d^2} \tag{9}$$

Now the horizontal distance of p_3 from p_2 is just twice the distance x_d:

$$x_3 = 2 \cdot x_d \tag{10}$$

Since p_3 is at the same vertical coordinate as y_2, the vertical distance y_3 is zero.

$$y_3 = 0 \tag{11}$$

6.5 Locating point coordinates

We have found the series of right-triangles that connect the points. In doing so we have found distances between points on the triangles. However, the point locations are only relative to each other.

Ultimately we desire to reference all points to a common origin. By referencing all points to a common origin, calculating the actual point locations on a coordinate system becomes possible.

Let's begin by choosing the gaging point p_g to be the origin, such that p_g will always be at coordinate (0, 0).

$$\boxed{p_g = (0, 0)} \tag{12}$$

From Figure 11b we see point p_1 is left of and below p_g distances of x_g and y_g respectively.

$$\boxed{p_1 = (-x_g, -y_g)} \tag{13}$$

In Figure 11a point p_s is relative to p_1 a distance x_s to the right, and distance y_s above. Since, in equation 13, p_1 is relative to the origin at $(-x_g, -y_g)$, p_s shall be relative to the origin at:

$$\boxed{p_s = (x_s - x_g, y_s - y_g)} \tag{14}$$

In Figure 11c it is seen that point p_2 is right of p_g a distance x_2, and above p_g a distance y_2. Therefore relative to p_g at the the origin, p_2 is located at:

$$\boxed{p_2 = (x_2, y_2)} \tag{15}$$

Figure 11c shows point p_d to the left of and below p_2 distances of x_d and y_d respectively. Relative to p_g at the origin, point p_d is located at:

$$\boxed{p_d = (x_2 - x_d, y_2 - y_d)} \tag{16}$$

The point p_3 shown in figure 11c is a distance $x_3 = 2 \cdot x_d$ to the left of p_2. Its y-coordinate is the same as the y-coordinate of p_2. Therefore relative to p_g at the origin, p_3 is at:

$$\boxed{p_3 = (x_2 - x_3, y_2)} \tag{17}$$

Figure 11d shows length d' ghosted. Length d' is the true flange depth. It is just the vertical distance from point p_s, where the tread slope begins, down to the flange bottom at p_d.

Length d' is equal to the distance P (vertical distance down from p_s to p_g), minus distance y_2 (vertical distance up from p_g to p_2), plus distance y_d (vertical distance down from p_2 down to p_d).

$$\boxed{d' = P - y_2 + y_d} \tag{18}$$

Compare d' against the D' value given by the RP-25 document. Keep in mind the D' is the maximum flange depth allowed by RP-25.

7 Example

We now know how to algebraically locate all the features of the RP-25 wheel profile.

As a test we should apply what we know to a concrete example. Let's calculate the critical: points, angles and distances; for a code 110 wheel profile with a 3° tread slope.

Given:

- A wheel profile where $code = 110$, and
- Wheel tread with $slope = \theta_s = 3°$

Find:

- Published values for: N', T, W, D', P, $R1$, $R2$ and $R3$
- Calculated values for: x_s, y_s, x_g, y_g, θ_g, x_2, y_2, x_d, y_d, x_3 and y_3
- Calculated values for: p_g, p_1, p_s, p_2, p_d, p_3 and d'

Note: When using trigonometric functions you may need to convert angles from degrees to radians and back again. This will depend upon your calculator, spreadsheet, math programming library, etc.

- To convert from degrees to radians:

$$\theta_{rads} = \theta_{deg} \cdot \left(\frac{\pi}{180°}\right)$$

- To convert from radians to degrees:

$$\theta_{degs} = \theta_{rad} \cdot \left(\frac{180°}{\pi}\right)$$

Steps:

1. From the RP-25 document lookup the row with published values for code = 110 :

$$\begin{aligned} N' &= 0.110\ inch \\ T &= 0.030\ inch \\ W &= 0.080\ inch \\ D' &= 0.025\ inch \\ P &= 0.010\ inch \\ R1 &= 0.014\ inch \\ R2 &= 0.018\ inch \\ R3 &= 0.018\ inch \end{aligned}$$

2. The value of the tread slope was given as:

$$slope = \theta_s = 3°$$

3. Substitute values for $R1$ and $slope = \theta_s$ into equation 1 to find x_s.

$$\begin{aligned} x_s &= R1 \cdot \sin(\theta_s) \\ x_s &= 0.014 \cdot \sin(3°) \\ x_s &= 0.014 \cdot 0.0523359562 \\ x_s &= 0.0007327034\ inch \end{aligned}$$

4. Calculate y_s by substituting values for $R1$ and $slope = \theta_s$ into equation 2.

$$\begin{aligned} y_s &= R1 \cdot \cos(\theta_s) \\ y_s &= 0.014 \cdot \cos(3°) \\ y_s &= 0.014 \cdot 0.9986295348 \\ y_s &= 0.0139808135\ inch \end{aligned}$$

5. Now plug in the calculated value of y_s and the published value for P into equation 3 to

find y_g.

$$y_g = y_s - P$$
$$y_g = 0.0139808135 - 0.010$$
$$y_g = 0.0039808135 \ inch$$

6. Equation 4 with values substituted in for $R1$ and y_g yields x_g.

$$x_g = \sqrt{(R1)^2 - y_g^2}$$
$$x_g = \sqrt{(0.014)^2 - (0.0039808135)^2}$$
$$x_g = \sqrt{0.000196 - 0.000015846876015}$$
$$x_g = \sqrt{0.000180153123985}$$
$$x_g = 0.0134221132 \ inch$$

7. To calculate angle θ_g simply apply values for x_g and $R1$ to equation 5.

$$\theta_g = \arccos\left(\frac{x_g}{R1}\right)$$
$$\theta_g = \arccos\left(\frac{0.0134221132}{0.014}\right)$$
$$\theta_g = \arccos(0.9587223747)$$
$$\theta_g = 16.5196295818^\circ$$

8. Distance x_2 can be found with equation 6 and the values for $R2$ and θ_g.

$$x_2 = R2 \cdot \cos(\theta_g)$$
$$x_2 = 0.018 \cdot \cos(16.5196295818^\circ)$$
$$x_2 = 0.018 \cdot 0.9587223747$$
$$x_2 = 0.0172570027 \ inch$$

9. Likewise y_2 is calculated with $R2$, θ_g and equation 7.

$$y_2 = R2 \cdot \sin(\theta_g)$$
$$y_2 = 0.018 \cdot \sin(16.5196295818^\circ)$$
$$y_2 = 0.018 \cdot 0.2843438205$$
$$y_2 = 0.0051181888 \ inch$$

10. To calculate x_d, substitute the value x_2 and the published T value into equation 8.

$$x_d = x_2 - T/2$$
$$x_d = 0.0172570027 - (0.030)/2$$
$$x_d = 0.0172570027 - 0.015$$
$$x_d = 0.0022570027 \ inch$$

11. Distance y_d is calculated with equation 9 and the values for $R2$ and x_d.

$$y_d = \sqrt{(R2)^2 - x_d^2}$$
$$y_d = \sqrt{(0.018)^2 - (0.0022570027)^2}$$
$$y_d = \sqrt{0.000324 - 0.000005094061390}$$
$$y_d = \sqrt{0.0003189059}$$
$$y_d = 0.0178579377 \ inch$$

12. From equation 10, find x_3 by plugging in the value for x_d.

$$x_3 = 2 \cdot x_d$$
$$x_3 = 2 \cdot 0.0022570027$$
$$x_3 = 0.0045140055 \ inch$$

13. Equation 11 simply states y_3 will always be zero.

$$y_3 = 0 \ inch$$

14. Equation 12 states gaging point p_g will al-

ways be at the origin by definition.

$$p_g = (0, 0)$$

15. To locate point p_1, plug in x_g and y_g into equation 13.

$$p_1 = (-x_g, -y_g)$$
$$p_1 = (-0.0134221132, -0.0039808135)$$
$$p_1 = (-0.0134, -0.0040)$$

16. Point p_s is found by applying values for: x_s, x_g, y_s and y_g; to equation 14.

$$p_s = (x_s - x_g, y_s - y_g)$$
$$p_s = (0.0007327034 - 0.0134221132, \quad 0.0139808135 - 0.0039808135)$$
$$p_s = (-0.0126894098, 0.01)$$
$$p_s = (-0.0127, 0.0100)$$

17. Use equation 15 and the values for x_2 and y_2 to find point p_2.

$$p_2 = (x_2, y_2)$$
$$p_2 = (0.0172570027, 0.0051181888)$$
$$p_2 = (0.0173, 0.0051)$$

18. Now use equation 16 and values: x_2, x_d, y_2 and y_d; to yield point p_d.

$$p_d = (x_2 - x_d, y_2 - y_d)$$
$$p_d = (0.0172570027 - 0.0022570027, \quad 0.0051181888 - 0.0178579377)$$
$$p_d = (0.015, -0.0127397489)$$
$$p_d = (0.015, -0.0127)$$

19. The location of point p_3 can by discovered by substituting values for: x_2, x_3 and y_2; into equation 17.

$$p_3 = (x_2 - x_3, y_2)$$
$$p_3 = (0.0172570027 - 0.0045140055, \quad 0.0051181888)$$
$$p_3 = (0.0127429972, 0.0051181888)$$
$$p_3 = (0.0127, 0.0051)$$

20. The distance d' can easily be calculated with equation 18 and values for: P, y_2 and y_d.

$$d' = P - y_2 + y_d$$
$$d' = 0.010 - 0.0051181888 + 0.0178579377$$
$$d' = 0.022739748$$
$$d' = 0.0227\ inch$$

8 Conclusion

In general, any of the RP-25 wheel profiles can be found by:

- Looking up published values, in the RP-25 document, for a desired wheel code.
- Choosing a desired tread slope θ_s.
- Calculating distance and angle values with equations 1 through 11.
- Calculating point locations with equations 12 through 18.

Table 1 shows calculated values: p_g, θ_g, p_s, p_1, p_2, p_3, p_d and d'; found from equations 1 through 18. The values are tabulated by wheel code and indexed by tread slopes, θ_s, from 0° to 3°.

Published values from RP-25 and tabulated values from Table 1 can be used together to draw the profile shown in Figure 10. To draw a wheel profile:

- First draw x and y axes at the desired origin.

Code	θ_s (degs) slope	p_g (inches) Eq# 12	θ_g (degs) Eq# 5	p_s (inches) Eq# 14	p_1 (inches) Eq# 13	p_2 (inches) Eq# 15	p_3 (inches) Eq# 17	p_d (inches) Eq# 16	d' (inches) Eq# 18
175	0.0	(0.0000, 0.0000)	23.5782	(-0.0229, 0.0150)	(-0.0229, -0.0100)	(0.0257, 0.0112)	(0.0223, 0.0112)	(0.0240, -0.0168)	0.0318
175	1.0	(0.0000, 0.0000)	23.5687	(-0.0225, 0.0150)	(-0.0229, -0.0100)	(0.0257, 0.0112)	(0.0223, 0.0112)	(0.0240, -0.0168)	0.0318
175	2.0	(0.0000, 0.0000)	23.5401	(-0.0220, 0.0150)	(-0.0229, -0.0100)	(0.0257, 0.0112)	(0.0223, 0.0112)	(0.0240, -0.0168)	0.0318
175	3.0	(0.0000, 0.0000)	23.4925	(-0.0216, 0.0150)	(-0.0229, -0.0100)	(0.0257, 0.0112)	(0.0223, 0.0112)	(0.0240, -0.0168)	0.0318
145	0.0	(0.0000, 0.0000)	22.3927	(-0.0194, 0.0130)	(-0.0194, -0.0080)	(0.0213, 0.0088)	(0.0177, 0.0088)	(0.0195, -0.0142)	0.0272
145	1.0	(0.0000, 0.0000)	22.3833	(-0.0191, 0.0130)	(-0.0194, -0.0080)	(0.0213, 0.0088)	(0.0177, 0.0088)	(0.0195, -0.0142)	0.0272
145	2.0	(0.0000, 0.0000)	22.3549	(-0.0187, 0.0130)	(-0.0194, -0.0080)	(0.0213, 0.0087)	(0.0177, 0.0087)	(0.0195, -0.0142)	0.0272
145	3.0	(0.0000, 0.0000)	22.3078	(-0.0183, 0.0130)	(-0.0194, -0.0080)	(0.0213, 0.0087)	(0.0177, 0.0087)	(0.0195, -0.0142)	0.0272
126	0.0	(0.0000, 0.0000)	19.4712	(-0.0170, 0.0120)	(-0.0170, -0.0060)	(0.0198, 0.0070)	(0.0162, 0.0070)	(0.0180, -0.0139)	0.0259
126	1.0	(0.0000, 0.0000)	19.4620	(-0.0167, 0.0120)	(-0.0170, -0.0060)	(0.0198, 0.0070)	(0.0162, 0.0070)	(0.0180, -0.0139)	0.0259
126	2.0	(0.0000, 0.0000)	19.4342	(-0.0163, 0.0120)	(-0.0170, -0.0060)	(0.0198, 0.0070)	(0.0162, 0.0070)	(0.0180, -0.0139)	0.0259
126	3.0	(0.0000, 0.0000)	19.3880	(-0.0160, 0.0120)	(-0.0170, -0.0060)	(0.0198, 0.0070)	(0.0162, 0.0070)	(0.0180, -0.0140)	0.0260
116	0.0	(0.0000, 0.0000)	12.3736	(-0.0137, 0.0110)	(-0.0137, -0.0030)	(0.0176, 0.0039)	(0.0134, 0.0039)	(0.0155, -0.0140)	0.0250
116	1.0	(0.0000, 0.0000)	12.3647	(-0.0134, 0.0110)	(-0.0137, -0.0030)	(0.0176, 0.0039)	(0.0134, 0.0039)	(0.0155, -0.0140)	0.0250
116	2.0	(0.0000, 0.0000)	12.3379	(-0.0132, 0.0110)	(-0.0137, -0.0030)	(0.0176, 0.0038)	(0.0134, 0.0038)	(0.0155, -0.0140)	0.0250
116	3.0	(0.0000, 0.0000)	12.2932	(-0.0129, 0.0110)	(-0.0137, -0.0030)	(0.0176, 0.0038)	(0.0134, 0.0038)	(0.0155, -0.0140)	0.0250
110	0.0	(0.0000, 0.0000)	16.6015	(-0.0134, 0.0100)	(-0.0134, -0.0040)	(0.0172, 0.0051)	(0.0128, 0.0051)	(0.0150, -0.0127)	0.0227
110	1.0	(0.0000, 0.0000)	16.5924	(-0.0132, 0.0100)	(-0.0134, -0.0040)	(0.0173, 0.0051)	(0.0127, 0.0051)	(0.0150, -0.0127)	0.0227
110	2.0	(0.0000, 0.0000)	16.5651	(-0.0129, 0.0100)	(-0.0134, -0.0040)	(0.0173, 0.0051)	(0.0127, 0.0051)	(0.0150, -0.0127)	0.0227
110	3.0	(0.0000, 0.0000)	16.5196	(-0.0127, 0.0100)	(-0.0134, -0.0040)	(0.0173, 0.0051)	(0.0127, 0.0051)	(0.0150, -0.0127)	0.0227
93	0.0	(0.0000, 0.0000)	0.0000	(-0.0090, 0.0090)	(-0.0090, 0.0000)	(0.0170, 0.0000)	(0.0090, 0.0000)	(0.0130, -0.0165)	0.0255
93	1.0	(0.0000, 0.0000)	-0.0087	(-0.0088, 0.0090)	(-0.0090, 0.0000)	(0.0170, -0.0000)	(0.0090, -0.0000)	(0.0130, -0.0165)	0.0255
93	2.0	(0.0000, 0.0000)	-0.0349	(-0.0087, 0.0090)	(-0.0090, 0.0000)	(0.0170, -0.0000)	(0.0090, -0.0000)	(0.0130, -0.0165)	0.0255
93	3.0	(0.0000, 0.0000)	-0.0785	(-0.0085, 0.0090)	(-0.0090, 0.0000)	(0.0170, -0.0000)	(0.0090, -0.0000)	(0.0130, -0.0165)	0.0255
88	0.0	(0.0000, 0.0000)	14.4775	(-0.0116, 0.0090)	(-0.0116, -0.0030)	(0.0145, 0.0038)	(0.0105, 0.0038)	(0.0125, -0.0111)	0.0201
88	1.0	(0.0000, 0.0000)	14.4685	(-0.0114, 0.0090)	(-0.0116, -0.0030)	(0.0145, 0.0037)	(0.0105, 0.0037)	(0.0125, -0.0111)	0.0201
88	2.0	(0.0000, 0.0000)	14.4415	(-0.0112, 0.0090)	(-0.0116, -0.0030)	(0.0145, 0.0037)	(0.0105, 0.0037)	(0.0125, -0.0111)	0.0201
88	3.0	(0.0000, 0.0000)	14.3964	(-0.0110, 0.0090)	(-0.0116, -0.0030)	(0.0145, 0.0037)	(0.0105, 0.0037)	(0.0125, -0.0111)	0.0201
79	0.0	(0.0000, 0.0000)	15.8266	(-0.0106, 0.0080)	(-0.0106, -0.0030)	(0.0135, 0.0038)	(0.0095, 0.0038)	(0.0115, -0.0100)	0.0180
79	1.0	(0.0000, 0.0000)	15.8176	(-0.0104, 0.0080)	(-0.0106, -0.0030)	(0.0135, 0.0038)	(0.0095, 0.0038)	(0.0115, -0.0100)	0.0180
79	2.0	(0.0000, 0.0000)	15.7903	(-0.0102, 0.0080)	(-0.0106, -0.0030)	(0.0135, 0.0038)	(0.0095, 0.0038)	(0.0115, -0.0101)	0.0181
79	3.0	(0.0000, 0.0000)	15.7450	(-0.0100, 0.0080)	(-0.0106, -0.0030)	(0.0135, 0.0038)	(0.0095, 0.0038)	(0.0115, -0.0101)	0.0181
72	0.0	(0.0000, 0.0000)	11.5370	(-0.0098, 0.0080)	(-0.0098, -0.0020)	(0.0118, 0.0024)	(0.0082, 0.0024)	(0.0100, -0.0095)	0.0175
72	1.0	(0.0000, 0.0000)	11.5281	(-0.0096, 0.0080)	(-0.0098, -0.0020)	(0.0118, 0.0024)	(0.0082, 0.0024)	(0.0100, -0.0095)	0.0175
72	2.0	(0.0000, 0.0000)	11.5013	(-0.0095, 0.0080)	(-0.0098, -0.0020)	(0.0118, 0.0024)	(0.0082, 0.0024)	(0.0100, -0.0095)	0.0175
72	3.0	(0.0000, 0.0000)	11.4568	(-0.0093, 0.0080)	(-0.0098, -0.0020)	(0.0118, 0.0024)	(0.0082, 0.0024)	(0.0100, -0.0095)	0.0175
54	0.0	(0.0000, 0.0000)	7.1808	(-0.0079, 0.0070)	(-0.0079, -0.0010)	(0.0089, 0.0011)	(0.0051, 0.0011)	(0.0070, -0.0077)	0.0147
54	1.0	(0.0000, 0.0000)	7.1720	(-0.0078, 0.0070)	(-0.0079, -0.0010)	(0.0089, 0.0011)	(0.0051, 0.0011)	(0.0070, -0.0077)	0.0147
54	2.0	(0.0000, 0.0000)	7.1456	(-0.0077, 0.0070)	(-0.0079, -0.0010)	(0.0089, 0.0011)	(0.0051, 0.0011)	(0.0070, -0.0077)	0.0147
54	3.0	(0.0000, 0.0000)	7.1016	(-0.0075, 0.0070)	(-0.0079, -0.0010)	(0.0089, 0.0011)	(0.0051, 0.0011)	(0.0070, -0.0077)	0.0147

Table 1: Tabulated Values for RP-25 Profile

- Then mark the tabulated locations for points: p_g, p_s, p_1, p_2, p_3 and p_d.
- Next draw arcs of radius $R1$, $R2$ and $R3$ centered at points p_1, p_2 and p_3.
- From point p_s draw the tread slope at an angle θ_s.
- Last, complete the drawing with vertical lines a distance N' apart at the left and right sides of the profile.

Drawings for adjacent wheel codes will have different relative spacing between points. This is expected and is due to the discrete values published in the RP-25 document.

Upon inspecting Table 1 you will notice the angle θ_g varies greatly for different wheel profile codes. For example θ_g for *Code* 110 at $\theta_s = 3°$ is 16.5196°, whereas it is $-0.0785°$ at $\theta_s = 3°$ for *Code* 93.

In Table 1 you will notice that changing the slope angle θ_s within any particular wheel code has an effect upon location of p_s. On the other hand, the effect on θ_g is slight. Furthermore, changing θ_s has negligible effect upon other point locations. Table 1 is deliberately redundant to show that except p_s, when rounded to four decimal places, point locations are effectively independent of tread slope.

When using a CAD program to draw profiles from the tabulated data, you may notice some slight misalignments when zooming in closely. This is due to the tabulated data being rounded to 4 decimal places. If you need greater precision you can always perform calculations with equations 1 through 18 to whatever precision you require.

Also, if you need point locations for an intermediate value of slope angle, say for $\theta_s = 1.5°$, you should perform calculations with equations 1 through 18.

Furthermore, you can propose wheel profiles for new wheel codes that follow the RP-25 form. To do this, propose new values for N', T, W, D', P, $R1$, $R2$ and $R3$. Then declare the code number of the new profile based upon the value of N'. Next use equations 1 through 18 to determine the proposed profile: distances, angles and points.

As a final thought, the location of p_d is at flange depth d', which is well short of the flange depth D' given in the RP-25 document. The RP-25 document misleadingly draws the intersection of arcs $R2$ and $R3$ at flange depth D'. You should realize D' is the maximum flange depth specified by NMRA Standard S-4.2, therefore locating p_d at depth d' meets the standard.

Endnotes

[1]Recommended Practice RP-25, July 2009, Copyright 1986-2009, National Model Railroad Association, Inc., Designed by: Olesen, Mortimer and Bradley, Updated by D.A. Voss, https://www.nmra.org

[2]Article: NMRA Recommended Practice Wheel contour - RP25, Model Railroader Magazine, January 1962, page 69, Kalmbach Publishing

[3]Standard S-4.2, January 2019, National Model Railroad Association, Inc., https://www.nmra.org

[4]Recommended Practice RP-2, Standards Gauge, January 15, 2024, National Model Railroad Association, Inc., https://www.nmra.org

[5] Practical Shop Mathematics, Volume 1, Definition 35, Page 126, John H. Wolfe and Everett R. Phelps, Copyright 1939, McGraw-Hill Book Company, Inc.

[6] Practical Shop Mathematics, Volume 1, Proposition 36, Page 132, John H. Wolfe and Everett R. Phelps, Copyright 1939, McGraw-Hill Book Company, Inc.

[7] Practical Shop Mathematics, Volume 1, Proposition 43, Page 144, John H. Wolfe and Everett R. Phelps, Copyright 1939, McGraw-Hill Book Company, Inc.

[8] https://github.com/vwfinley/regarding/blob/main/A/A-006/A-006.pdf, A-006, Regarding Tangent Ogives, Vincent W. Finley, September 2022