

# A-006

## Regarding Tangent Ogives

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Ogives are tapered solids of revolution. They are often used for aircraft/missile nose-cones and firearm bullets.

The **tangent ogive** is one of several ogive family members. It is a simple shape, easily turned with an engine lathe.

### 1 Problem

Given:

- Some semi-circle of radius  $r$ ,
- *and* a vertical line, of length  $c$ , inscribed on the semi-circle.

Find:

1. A procedure to construct tangent ogives.
2. Relationships among tangent ogive elements.

### 2 Solution

#### 2.1 (Stage 1) Inscribing ogive section on semi-circle

Refer to Figure 1 (Stage 1). Let a radius  $r$  sweep out a semi-circle centered about a point  $P$  which coincides with the origin.

The semi-circle intersects the x-axis at a point  $P_b$  located on the x-axis at distance  $r$  away from point  $P$ .

Let  $c$  be the **inscribed** vertical line which intersects the semi-circle at some point  $P_c$ , and

intersects the x-axis, at right angles, at a point  $P_a$ . Point  $P_a$  lies on the x-axis some distance  $a$  away from  $P$ . Finally, let  $b$  be the line-segment between points  $P_a$  and  $P_b$ .

Select the area bounded by sides  $b$ ,  $c$ , and the arc between  $P_b$  and  $P_c$ . This area will be used in subsequent stages of the construction.

Distances between points can be listed as:

$$\overline{PP_a} = a$$

$$\overline{P_aP_b} = b$$

$$\overline{P_aP_c} = c$$

$$\overline{PP_b} = r$$

Furthermore, locations of the points can also be listed:

$$P = (0, 0) \tag{1a}$$

$$P_a = (a, 0) \tag{1b}$$

$$P_b = (r, 0) \tag{1c}$$

$$P_c = (a, c) \tag{1d}$$

Because points  $P$ ,  $P_a$  and  $P_b$  are collinear, the distance  $\overline{PP_b}$  is just the sum of  $\overline{PP_a}$  and  $\overline{P_aP_b}$ .

$$\boxed{r = a + b} \tag{2}$$

Now since points  $P$ ,  $P_a$  and  $P_c$  form a right triangle, the Pythagorean theorem can be used to relate hypotenuse  $r$  with sides  $a$  and  $c$ .

$$\boxed{r = \sqrt{a^2 + c^2}} \tag{3}$$

To find length of side  $a$  given lengths of  $b$  and  $c$ , first equate variable  $r$  in equations 2 and 3,

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then reduce.

$$\begin{aligned}
a + b &= \sqrt{a^2 + c^2} \\
(a + b)^2 &= a^2 + c^2 \\
a^2 + 2ab + b^2 &= a^2 + c^2 \\
2ab + b^2 &= c^2 \\
2ab &= c^2 - b^2 \\
\boxed{a = \frac{c^2 - b^2}{2b}} & \quad (4)
\end{aligned}$$

Next, from equation 3, it is convenient to express  $r^2$  in terms of  $a^2$  and  $c^2$

$$\begin{aligned}
r &= \sqrt{a^2 + c^2} \\
r^2 &= a^2 + c^2
\end{aligned}$$

Then to substitute  $a = r - b$  from equation 2.

$$r^2 = (r - b)^2 + c^2 \quad (5)$$

Using equation 5,  $b$  can be found in terms of  $r$  and  $c$ .

$$\begin{aligned}
r^2 &= (r - b)^2 + c^2 \\
(r - b)^2 &= r^2 - c^2 \\
\sqrt{(r - b)^2} &= \sqrt{r^2 - c^2} \\
r - b &= \sqrt{r^2 - c^2} \\
\boxed{b = r - \sqrt{r^2 - c^2}} & \quad (6)
\end{aligned}$$

Next, by expanding equation 5 and simplifying,  $c$  can be found in terms of  $r$  and  $b$ .

$$\begin{aligned}
r^2 &= (r - b)^2 + c^2 \\
r^2 &= r^2 - 2rb + b^2 + c^2 \\
c^2 &= 2rb - b^2 \\
\boxed{c = \sqrt{2rb - b^2}} & \quad (7)
\end{aligned}$$

## 2.2 (Stage 2) Translate ogive half-section to y-axis

The ogive section is easily translated to the origin and y-axis by subtracting length  $a$  from the  $x$ -coordinates in points  $P$  through  $P_c$  from equa-

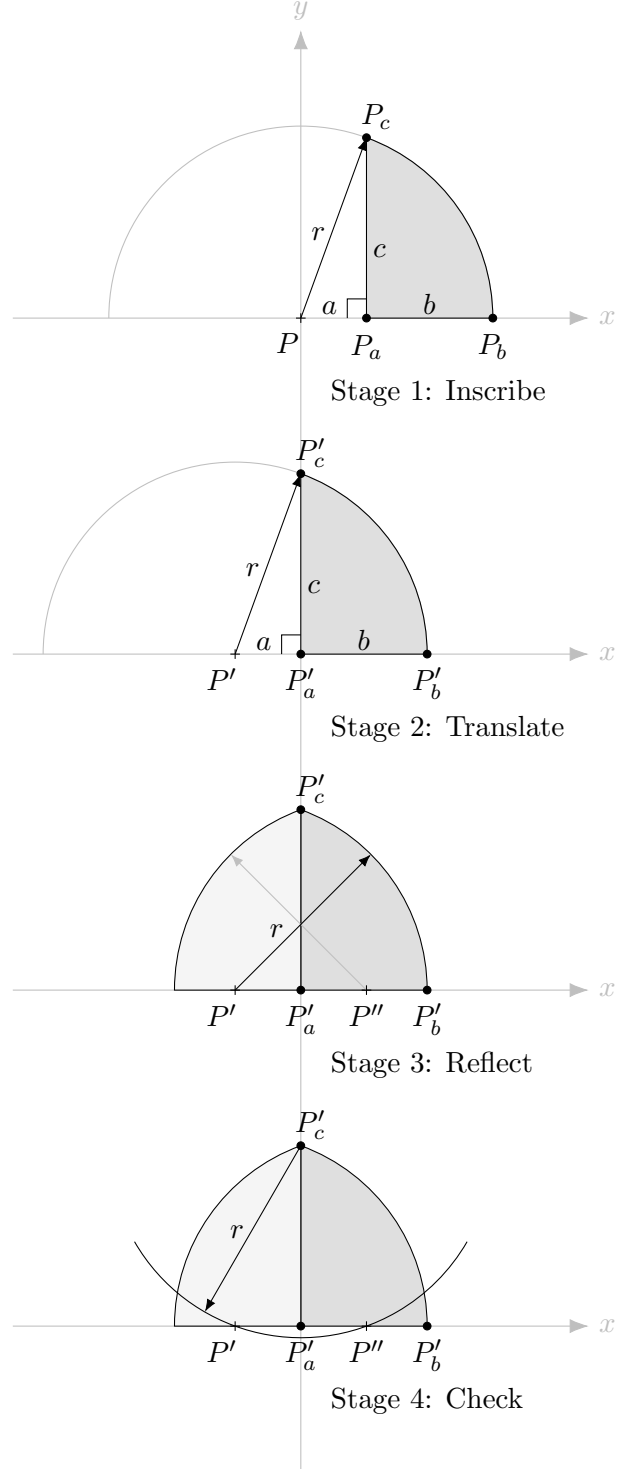


Figure 1: Construction of Tangent Ogive

tions 1a through 1d, as follows:

$$P' = (-a, 0) \quad (8a)$$

$$P'_a = (0, 0) \quad (8b)$$

$$P'_b = (b, 0) \quad (8c)$$

$$P'_c = (0, c) \quad (8d)$$

Prime notation is used here to show how the new points are derived from the original points.

### 2.3 (Stage 3) Reflect ogive half-section about y-axis

Reflecting the right ogive half-section across the y-axis is easily accomplished by applying a negative to all  $x$ -coordinates in points  $P'$  through  $P'_c$  from equations 8a through 8d, as follows:

$$P'' = (a, 0) \quad (9a)$$

$$P''_a = (0, 0) \quad (9b)$$

$$P''_b = (-b, 0) \quad (9c)$$

$$P''_c = (0, c) \quad (9d)$$

Double-prime notation is used here to show how the new points are derived from the original points. Reflecting the half-section about y-axis yields a full ogive cross-section.

Note: points  $P''_a$ ,  $P''_b$  and  $P''_c$  are omitted for clarity in Figure 1 (Stage 3). However point  $P''$  is shown.

### 2.4 (Stage 4) Check ogive cross-section for accuracy

Once the ogive cross-section has been calculated and constructed, checking for correctness and accuracy is recommended.

Checking the completed cross-section is a simple matter of recognizing the relationship between:  $P'_c$ ,  $P'$  and  $P''$ . Refer to Figure 1 (Stage 3). The lines centered at either  $P'$  or  $P''$ , drawn to point  $P'_c$ , have length  $r$  and sweep arcs of radius  $r$ . The arcs meet at a common point  $P'_c$ .

Refer to Figure 1 (Stage 4). Lines centered at  $P'_c$  drawn back, in the opposite direction, to either  $P'$  or  $P''$  will also have length  $r$ . So, an arc of radius  $r$ , centered at point  $P'_c$ , will include  $P'$  and  $P''$ .

Therefore, a quick algebraic check for correctness is to measure the distance of lines  $\overline{P'_c P'}$  and  $\overline{P'_c P''}$ . Both should equal  $r$ .

A corresponding geometric check is to sweep an arc of radius  $r$  centered at  $P'_c$ . If points  $P'$  and  $P''$  lie on the arc, then the ogive cross-section is correctly constructed.

## 2.5 Restrictions and valid values

The three most important variables in equations 2 through 7 are:  $a$ ,  $b$  and  $r$ . They serve as inputs to the ogive geometry and are subject to some restrictions. These equations are undefined for values outside the valid ranges.

Restrictions:

- $r > 0$  : Otherwise the ogive is undefined.
- $0 < a < r$  : Otherwise the ogive is undefined
- $0 < b < r$  : Otherwise the ogive is undefined
- $0 < c < r$  : Otherwise the ogive is undefined
- Equation 2 must be satisfied. Otherwise, the ogive is undefined.

Q.E.D.

## 3 Example

Given:

- A tangent ogive whose radius is 4.5 units and height is 2.3 units.

Find:

- Values for  $a$  and  $b$ .
- Locations of points  $P$  through  $P_c$ .
- Locations of points  $P'$  through  $P'_c$  for the right half.
- Locations of points  $P''$  through  $P''_c$  for the left half.
- Distances of  $\overline{P'_c P'}$  and  $\overline{P'_c P''}$ .

Steps:

1. Let:

$$r = 4.5$$

$$c = 2.3$$

2. From Equation 6, line-segment  $b$  is found by substituting values for  $r$  and  $c$ :

$$b = r - \sqrt{r^2 - c^2}$$

$$b = 4.5 - \sqrt{4.5^2 - 2.3^2}$$

$$b = 4.5 - \sqrt{20.25 - 5.29}$$

$$b = 4.5 - \sqrt{14.96}$$

$$b = 4.5 - 3.8678159212$$

$$\boxed{b = 0.6321840788}$$

3. The value for  $a$  is easily found by rearranging Equation 2, solving for  $a$ , and substituting values for  $r$  and  $b$ .

$$r = a + b$$

$$a = r - b$$

$$a = 4.5 - 0.6321840788$$

$$\boxed{a = 3.8678159212}$$

4. Points  $P$  through  $P_c$  are found by substituting values of  $a$ ,  $b$ ,  $c$  and  $r$  into Equations 1a through 1d.

$$P = (0, 0)$$

$$P_a = (3.8678159212, 0)$$

$$P_b = (4.5, 0)$$

$$P_c = (3.8678159212, 2.3)$$

5. Points  $P'$  through  $P'_c$  are found by substituting values of  $a$ ,  $b$ ,  $c$  and  $r$  into Equations 8a through 8d.

$$P' = (-3.8678159212, 0)$$

$$P'_a = (0, 0)$$

$$P'_b = (0.6321840788, 0)$$

$$P'_c = (0, 2.3)$$

6. Points  $P''$  through  $P''_c$  are found by substituting values of  $a$ ,  $b$ ,  $c$  and  $r$  into Equations 9a through 9d.

$$P'' = (3.8678159212, 0)$$

$$P''_a = (0, 0)$$

$$P''_b = (-0.6321840788, 0)$$

$$P''_c = (0, 2.3)$$

7. Use the method of Section 2.4 to check the correctness of the calculations.

The distance of  $\overline{P'_c P'}$  is just the distance between point  $P'_c$  and point  $P'$ .

From above:

$$P'_c = (0, 2.3)$$

$$P' = (-3.8678159212, 0)$$

By using the Pythagorean theorem, the distance is found to be:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\overline{P'_c P'} = \sqrt{(P'_x - P'_{cx})^2 + (P'_y - P'_{cy})^2}$$

$$\overline{P'_c P'} = \sqrt{(-3.8678159212 - 0)^2 + (0 - 2.3)^2}$$

$$\overline{P'_c P'} = \sqrt{14.96 + 5.29}$$

$$\overline{P'_c P'} = \sqrt{20.25}$$

$$\overline{P'_c P'} = 4.5$$

As expected, the distance  $\overline{P'_c P'}$  is equal to  $r$  thereby confirming correctness of calculations.

Furthermore, absolute values  $|P'_x| = |P''_x|$ , and  $P'_x$  or  $P''_x$  are squared when either is substituted.

$$\overline{P'_c P'} = \overline{P'_c P''} = r = 4.5$$

8. Use Equation 2 as another check.

$$r = a + b$$

$$r = 3.8678159212 + 0.6321840788$$

$$r = 4.5$$

Since all restrictions of Section 2.5 are satisfied, correctness of the calculations is confirmed.