A-002
Regarding Fillets

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When parts need their sharp corners removed: tradesmen, craftsmen and engineers; will either chamfer or fillet the corner. A chamfer is a straight cut created at an angle between two adjoining sides. Whereas, a fillet is simply the rounding-off of a part's interior or exterior corners. Fillets can be concave or convex arcs, usually at the intersection of two straight lines or edges. A rounded-off table corner, a beaded weld between steel plates, a vehicle's path when turning at an intersection, a curved railroad track adjoining two straight sections; are all examples of fillets.

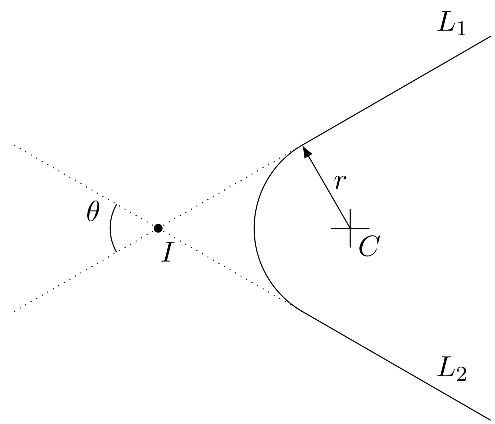

Figure 1: Fillet C is tangent to lines L_1 and L_2

1 Problem

Given:

- As shown in Figure 1:
- Two lines, L_1 and L_2, that intersect at an angle θ, at some point I,
- *and* a fillet C, of radius r, that is tangent to both L_1 and L_2;

Find:

1. The location of the center of C relative to I,
2. *and* the locations where the C is tangent to L_1 and L_2,
3. *and* the location on C closest to I.

2 Solution

1. See Figure 2. The solution here is similar to the solution found in prior work.[1]
2. Draw axes, placing the origin at point I. Orient the x-axis such it bisects L_1 and L_2, and it passes thru the center of C.
3. A line tangent to a circle will always intersect radius line r at right angles.[2] The tangent point P will lie on circle C.
4. Let α be the angle formed by points CIP. Since the x-axis bisects angle θ, the angle α is equal to $\theta/2$.
5. Let Δx be the distance from point I to C. Note: Δx is the hypotenuse of right-triangle IPC.

[1] A-001, Regarding Circles and Tangent Lines, Vincent W. Finley, January 2022

[2] ISBN 0-8311-2575-6, Machinery's Handbook, page 46

6. Let d be the distance from point I to P.
7. Since points IPC form a right triangle, fundamental trigonometric functions can be used to find Δx and d.

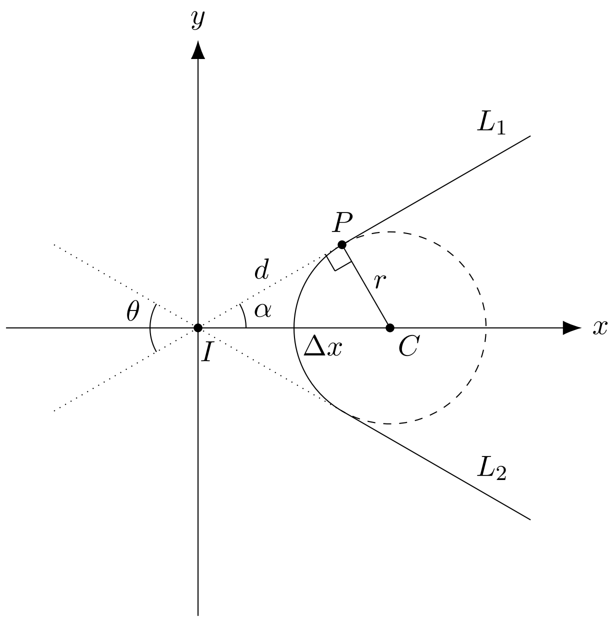

Figure 2: IPC is a right-triangle

From:

$$\sin(\alpha) = \frac{r}{\Delta x}$$
$$\tan(\alpha) = \frac{r}{d}$$

it is found,

$$\Delta x = \frac{r}{\sin(\alpha)} \tag{1}$$
$$d = \frac{r}{\tan(\alpha)} \tag{2}$$

and since:

$$\alpha = \frac{\theta}{2} \tag{3}$$

substituting equation 3 into equations 1 and 2 yields:

$$\Delta x = \frac{r}{\sin\left(\frac{\theta}{2}\right)} \tag{4}$$
$$d = \frac{r}{\tan\left(\frac{\theta}{2}\right)} \tag{5}$$

The location of the center of C is simply:

$$C_{x,y} = (\Delta x, 0)$$

$$\boxed{C_{x,y} = \left(\frac{r}{\sin\alpha}, 0\right)} \tag{6}$$

Furthermore, the locations where L_1 and L_2 are tangent to C are simply the polar and Cartesian coordinates of P, *and* $P's$ reflection about the x-axis:

$$P_{d,\alpha} = (d, \pm\alpha) \tag{7}$$
$$P_{x,y} = (d \cdot \cos\alpha, \pm d \cdot \sin\alpha) \tag{8}$$

Finally, the location where C crosses the x-axis nearest to I is:

$$\boxed{C_I = (\Delta x - r, 0)} \tag{9}$$

Q.E.D.

3 Example

Given:

- A fillet whose radius is 3.5 units,
- *and* whose legs intersect at a 45° included angle.

Find:

- The center of the fillet relative to the point where the fillet legs intersect,
- *and* the location where the fillet legs are tangent to the fillet arc,
- *and* the point on the fillet nearest to where the legs intersect.

Steps:

1. First, from equation 3, α is found to be 22.5°
2. Converting α to radians, and using equation 1, Δx is calculated to be:

$$\Delta x = \frac{3.5}{\sin 0.3927}$$
$$\Delta x = 9.1459$$

3. Converting α to radians, and applying equation 2, d is found:

$$d = \frac{3.5}{\tan 0.3927}$$
$$d = 8.4497$$

4. Substituting r and α into equation 6, the center of C is located:

$$C_{x,y} = (9.1459, 0)$$

5. The coordinates where L_1 and L_2 are tangent to the curved part of the fillet are discovered by substituting into equations 7 and 8:

$$P_{d,\alpha} = (8.4497, \pm 22.5^\circ)$$
$$P_{x,y} = (7.8065, \pm 3.2336)$$

6. Finally, from 9 the point on C nearest to where the fillet legs intersect is located:

$$C_I = (9.1459 - 3.5, 0)$$
$$C_I = (5.6459, 0)$$