

# A-002

## Regarding Fillets

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When parts need their sharp corners removed: tradesmen, craftsmen and engineers; will either chamfer or fillet the corner. A chamfer is a straight cut created at an angle between two adjoining sides. Whereas, a fillet is simply the rounding-off of a part's interior or exterior corners. Fillets can be concave or convex arcs, usually at the intersection of two straight lines or edges. A rounded-off table corner, a beaded weld between steel plates, a vehicle's path when turning at an intersection, a curved railroad track adjoining two straight sections; are all examples of fillets.

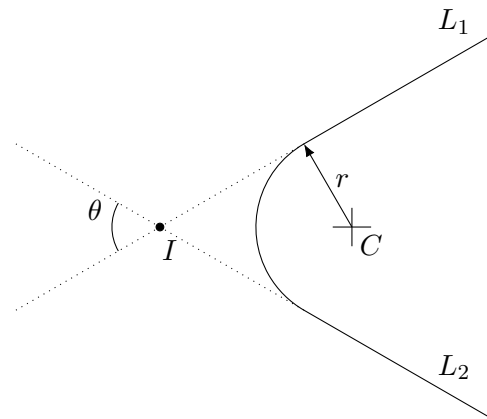


Figure 1: Fillet  $C$  is tangent to lines  $L_1$  and  $L_2$

## 1 Problem

Given:

- As shown in Figure 1:
- Two lines,  $L_1$  and  $L_2$ , that intersect at an angle  $\theta$ , at some point  $I$ ,
- and a fillet  $C$ , of radius  $r$ , that is tangent to both  $L_1$  and  $L_2$ ;

Find:

1. The location of the center of  $C$  relative to  $I$ ,
2. and the locations where the  $C$  is tangent to  $L_1$  and  $L_2$ ,
3. and the location on  $C$  closest to  $I$ .

## 2 Solution

1. See Figure 2. The solution here is similar to the solution found in prior work.<sup>1</sup>
2. Draw axes, placing the origin at point  $I$ . Orient the x-axis such it bisects  $L_1$  and  $L_2$ , and it passes thru the center of  $C$ .
3. A line tangent to a circle will always intersect radius line  $r$  at right angles.<sup>2</sup> The tangent point  $P$  will lie on circle  $C$ .
4. Let  $\alpha$  be the angle formed by points  $CIP$ . Since the x-axis bisects angle  $\theta$ , the angle  $\alpha$  is equal to  $\theta/2$ .
5. Let  $\Delta x$  be the distance from point  $I$  to  $C$ . Note:  $\Delta x$  is the hypotenuse of right-triangle  $IPC$ .

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<sup>1</sup>A-001, Regarding Circles and Tangent Lines, Vincent W. Finley, January 2022

<sup>2</sup>ISBN 0-8311-2575-6, Machinery's Handbook, page 46

6. Let  $d$  be the distance from point  $I$  to  $P$ .
7. Since points  $IPC$  form a right triangle, fundamental trigonometric functions can be used to find  $\Delta x$  and  $d$ .

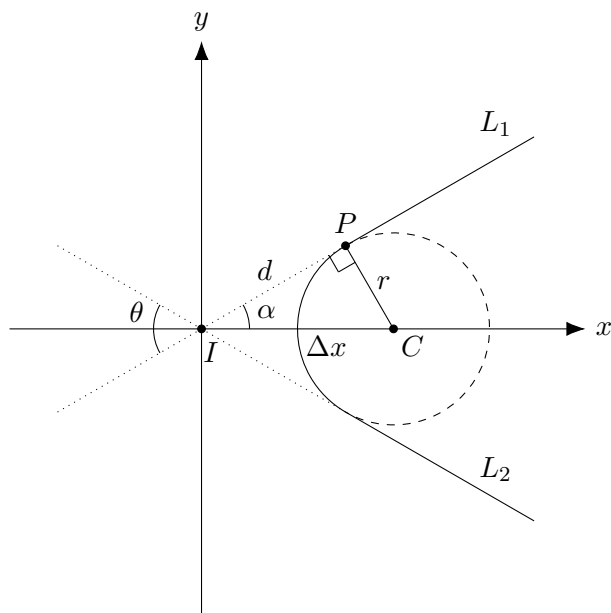


Figure 2:  $IPC$  is a right-triangle

From:

$$\sin(\alpha) = \frac{r}{\Delta x}$$

$$\tan(\alpha) = \frac{r}{d}$$

it is found,

$$\Delta x = \frac{r}{\sin(\alpha)} \quad (1)$$

$$d = \frac{r}{\tan(\alpha)} \quad (2)$$

and since:

$$\alpha = \frac{\theta}{2} \quad (3)$$

substituting equation 3 into equations 1 and 2 yields:

$$\Delta x = \frac{r}{\sin\left(\frac{\theta}{2}\right)} \quad (4)$$

$$d = \frac{r}{\tan\left(\frac{\theta}{2}\right)} \quad (5)$$

The location of the center of  $C$  is simply:

$$C_{x,y} = (\Delta x, 0)$$

$$C_{x,y} = \left(\frac{r}{\sin \alpha}, 0\right) \quad (6)$$

Furthermore, the locations where  $L_1$  and  $L_2$  are tangent to  $C$  are simply the polar and Cartesian coordinates of  $P$ , and  $P$ 's reflection about the x-axis:

$$P_{d,\alpha} = (d, \pm\alpha) \quad (7)$$

$$P_{x,y} = (d \cdot \cos \alpha, \pm d \cdot \sin \alpha) \quad (8)$$

Finally, the location where  $C$  crosses the x-axis nearest to  $I$  is:

$$C_I = (\Delta x - r, 0) \quad (9)$$

Q.E.D.

### 3 Example

Given:

- A fillet whose radius is 3.5 units,
- and whose legs intersect at a  $45^\circ$  included angle.

Find:

- The center of the fillet relative to the point where the fillet legs intersect,
- and the location where the fillet legs are tangent to the fillet arc,
- and the point on the fillet nearest to where the legs intersect.

Steps:

1. First, from equation 3,  $\alpha$  is found to be  $22.5^\circ$
2. Converting  $\alpha$  to radians, and using equation 1,  $\Delta x$  is calculated to be:

$$\Delta x = \frac{3.5}{\sin 0.3927}$$

$$\Delta x = 9.1459$$

3. Converting  $\alpha$  to radians, and applying equation 2,  $d$  is found:

$$d = \frac{3.5}{\tan 0.3927}$$

$$d = 8.4497$$

4. Substituting  $r$  and  $\alpha$  into equation 6, the center of  $C$  is located:

$$C_{x,y} = (9.1459, 0)$$

5. The coordinates where  $L_1$  and  $L_2$  are tangent to the curved part of the fillet are discovered by substituting into equations 7 and 8:

$$P_{d,\alpha} = (8.4497, \pm 22.5^\circ)$$

$$P_{x,y} = (7.8065, \pm 3.2336)$$

6. Finally, from 9 the point on  $C$  nearest to where the fillet legs intersect is located:

$$C_I = (9.1459 - 3.5, 0)$$

$$C_I = (5.6459, 0)$$