# A-001
# Regarding Circles and Tangent Lines

Vincent W. Finley*

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Occasionally, one may need to locate the point where a straight line is tangent to a circle. Scenarios such as: preparing a mechanical drawing, designing a belt and pulley system, or planning a highway; come to mind.

## 1 Problem

Given:

- Circle $C$ of radius $r$,
- *and* line $L$ sloped at an angle $\alpha$ with respect to the horizontal,
- *and* point $P$ where $L$ is tangent to $C$;

Find: The location of $P$ relative to the center of $C$.

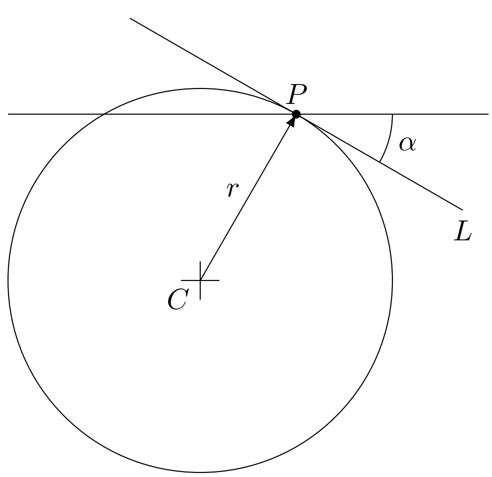


Figure 1: Line $L$ tangent to circle $C$



## 2 Solution

1. Draw axes, placing the origin at center of $C$, such that point $C = (0, 0)$.
2. Project point $P$ vertically downwards to a point $I$ on the x-axis. Points: $C$, $I$ and $P$; form a right-triangle $CIP$ having sides: $\Delta x$, $\Delta y$ and $r$.

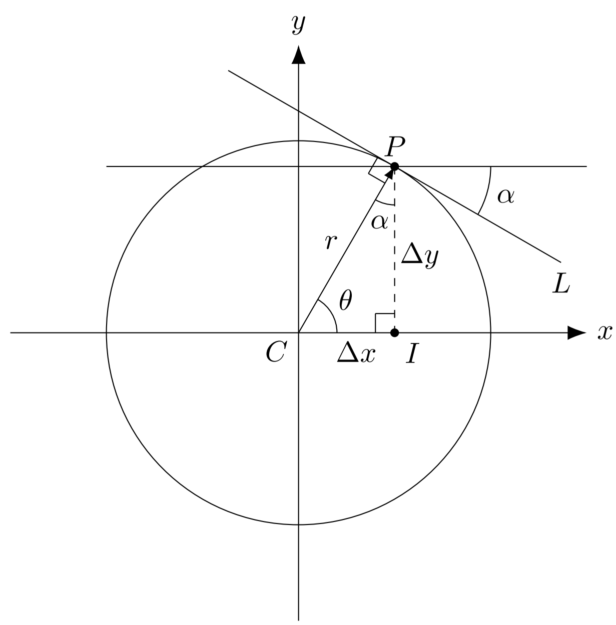


Figure 2: Point $P$ projected onto x-axis

3. All points on a circle lie a distance $r$ from the center, the distance $r$ being the radius of the circle. Furthermore, a point on the circle will lie at some angle $\theta$ from the positive x-axis. In other words, point $P$ on the circle will be located at polar coordinate $P = (r, \theta)$.

4. A line tangent to a circle will always intersect radius line $r$ at right angles.[1]. Which means, $L$ and $r$ will intersect at right angles at point $P = (r, \theta)$.

5. Since $L$ is at an angle $\alpha$ with respect to the x-axis (or parallel to x-axis), *and* $r$ is right angles to $L$, then $r$ will be at an angle $\alpha$ with respect to the y-axis (or parallel to y-axis).

6. Aside from having a right angle, triangle $CIP$ has angles $\theta$ and $\alpha$. The sum of $\theta$ and $|\alpha|$ must be $\pi/2$. However since the actual value of $\alpha$ can be negative with respect to the horizontal, $\theta - \alpha = \pi/2$ is used here. Furthermore $\theta = \pi/2 + \alpha$ .

7. Using the fundamental trigonometric functions, polar coordinates can be easily converted to Cartesian coordinates, such that $P = P_{r,\theta} = (r, \theta)$ and $P = P_{x,y} = (\Delta x, \Delta y)$, as follows:

From:

$$\cos(\theta) = \frac{\Delta x}{r}$$
$$\sin(\theta) = \frac{\Delta y}{r}$$

it is found,

$$\Delta x = r \cdot \cos(\theta) \tag{1}$$
$$\Delta y = r \cdot \sin(\theta) \tag{2}$$

and since:

$$\theta - \alpha = \frac{\pi}{2}$$
$$\theta = \frac{\pi}{2} + \alpha \tag{3}$$

substituting equation 3 into equations 1 and 2 yields:

$$\Delta x = r \cdot \cos\left(\frac{\pi}{2} + \alpha\right)$$
$$\Delta y = r \cdot \sin\left(\frac{\pi}{2} + \alpha\right)$$

Therefore, from above:

$$P = (\Delta x, \Delta y)$$

$$\boxed{P = \left(r \cdot \cos\left(\frac{\pi}{2} + \alpha\right), r \cdot \sin\left(\frac{\pi}{2} + \alpha\right)\right)} \tag{4}$$

Q.E.D.

[1] ISBN 0-8311-2575-6, Machinery's Handbook, page 46

# 3 Example

Given:

- A line $L$ tangent to circle $C$ whose radius $r = 3.75$,
- *and* $L$ is at an angle $\alpha = -12.5°$ from the horizontal;

Find: The polar and cartesian coordinates of tangent point $P$.

Steps:

1. First, $\alpha$ must be converted from degrees to radians:

$$\alpha_{rad} = \alpha_{deg}\left(\frac{\pi}{180°}\right)$$
$$\alpha_{rad} = -12.5°\left(\frac{\pi}{180°}\right)$$
$$\alpha_{rad} = -0.218166$$

2. Substituting values for $r$ and $\alpha$ yields:

$$P_{r,\theta} = (r, \theta)$$
$$P_{r,\theta} = \left(r, \frac{\pi}{2} + \alpha\right)$$
$$P_{r,\theta} = \left(3.75, \frac{\pi}{2} + -0.218166\right)$$
$$P_{r,\theta} = \left(3.75, \frac{\pi}{2} - 0.218166\right)$$
$$P_{r,\theta} = (3.75, 1.570796 - 0.218166)$$
$$P_{r,\theta} = (3.75, 1.352630)$$

3. Substituting values into equation 4 yields:

$$P_{x,y} = \left(r \cdot \cos\left(\frac{\pi}{2} + \alpha\right), r \cdot \sin\left(\frac{\pi}{2} + \alpha\right)\right)$$

$$P_{x,y} = (r \cdot \cos\left(\frac{\pi}{2} + -0.218166\right),$$
$$r \cdot \sin\left(\frac{\pi}{2} + -0.218166\right))$$

$$P_{x,y} = (r \cdot \cos\left(\frac{\pi}{2} - 0.218166\right),$$
$$r \cdot \sin\left(\frac{\pi}{2} - 0.218166\right))$$

$$P_{x,y} = (r \cdot \cos(1.570796 - 0.218166),$$
$$r \cdot \sin(1.570796 - 0.2181662))$$

$$P_{x,y} = (r \cdot \cos(1.352630), r \cdot \sin(1.352630))$$

$$P_{x,y} = (3.75 \cdot \cos(1.352630), 3.75 \cdot \sin(1.352630))$$

$$P_{x,y} = (3.75 \cdot 0.216440, 3.75 \cdot 0.976296)$$

$$P_{x,y} = (0.811649, 3.661110)$$