Homework Set #3 Problem 1

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0.1 Find the classes

First, we set up the hypothesis test. Let

$$
\begin{aligned}
H_0 &: f_X(x) = f_o(x) \\
H_1 &: f_X(x) \neq f_o(x)
\end{aligned}
$$

where $X \sim N(157, 13.7^2)$ denotes the total cholesterol values for individuals. Then, we know

$$\frac{X - 157}{31.7} \sim Z$$

where Z is the standard normal variable. To figure out the first class, we essentially need to find a z-score z_1 such that

$$P(Z \leq z_1) = 1/6$$

We could just observe the table and eyeball the value, but Python provides a function to approximate it:

```
In [4]: from scipy.stats import norm

        norm.ppf(1/6)

Out[4]: -0.96742156610170105
```

This means that

$$P(\frac{X - 157}{31.7} = Z \leq -0.967) = 1/6$$

Then, we could find the threshold of the first class. Namely, the lower threshold should be $-\infty$ and the upper threshold should be $ (-0.9674)(31.7) + 157 = 126.33$. Similarly, we could find the threshold for other classes.

```
In [5]: thresholds = [
            norm.ppf(0, loc=157, scale=31.7),
            norm.ppf(1/6, loc=157, scale=31.7),
            norm.ppf(2/6, loc=157, scale=31.7),
            norm.ppf(3/6, loc=157, scale=31.7),
            norm.ppf(4/6, loc=157, scale=31.7),
            norm.ppf(5/6, loc=157, scale=31.7),
            norm.ppf(6/6, loc=157, scale=31.7),
        ]

        print(thresholds)

[-inf, 126.33273635457607, 143.34594461233399, 157.0, 170.65405538766601, 187.66726364542393, in
```

Therefore, the classes should be

$$\begin{array}{ll} r_1 = (-\infty, 126.33] & r_2 = (126.33, 143.34] \\ r_3 = (143.34, 157.0] & r_4 = (157.0, 170.65] \\ r_5 = (170.65, 187.66] & r_6 = (187.66, \infty) \end{array}$$

0.2 Build a table

```
In [6]: # Find the observed frequency
        import numpy

        data = [
            95, 129, 136, 143, 152, 165, 175, 197,
            108, 129, 139, 144, 152, 166, 180, 204,
            108, 131, 140, 144, 155, 171, 181, 220,
            114, 131, 142, 145, 158, 172, 189, 223,
            115, 135, 142, 146, 158, 173, 192, 226,
            124, 136, 143, 148, 162, 174, 194, 230
        ]

        counts, bins = numpy.histogram(data, bins=thresholds)
        counts

Out[6]: array([ 6, 13,  8,  5,  7,  9], dtype=int64)
```

```
In [7]: # Build a table
        import pandas as pd

        df = pd.DataFrame(
            index = ['r1', 'r2', 'r3', 'r4', 'r5', 'r6'],
            data={
                'Obs.Freq.': counts,
                'Prob.': [1/6] * 6,
                'Exp.Freq.': [1/6*48] * 6
            }
        )
        df[['Obs.Freq.', 'Prob.', 'Exp.Freq.']]
```

```
Out[7]:     Obs.Freq.      Prob.  Exp.Freq.
        r1          6  0.166667        8.0
        r2         13  0.166667        8.0
        r3          8  0.166667        8.0
        r4          5  0.166667        8.0
        r5          7  0.166667        8.0
        r6          9  0.166667        8.0
```

0.3 Calculate the test statistic

The test statistic is

$$D = \sum_{i=1}^{6} = \frac{(X_i - np_i)^2}{np_i} \sim X^2_{6-1}$$

Now calculate it:

```
In [8]: d = 0

        for index, row in df.iterrows():
            d += ((row['Obs.Freq.'] - row['Exp.Freq.']) ** 2) / row['Exp.Freq.']

        d
```

```
Out[8]: 5.0
```

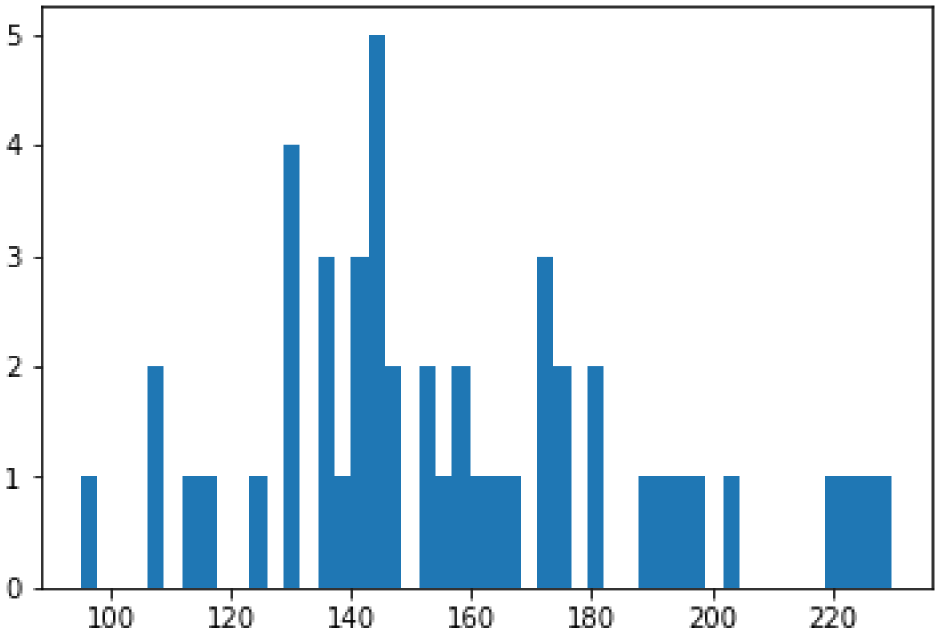
Suppose the level of significance $\alpha = 0.05$, then since

$$X^2_{0.90,5} = 9.236 \geq 5 = d$$

We don't have enough evidence to reject the null hypothesis. Given the histagram of the data, we find this conclusion to be reasonable.

```
In [9]: import matplotlib.pyplot as plt
        %matplotlib inline

        counts, bins, graph = plt.hist(data, bins=48)
```


5
4
3
2
1
0
100
120
140
160
180
200
220