Exercise 6.4.4 Page 373

March 21, 2018

0.1 Problem

Construct a power curve for the $\alpha = 0.05$ test of $H_0 : \mu = 60$ vs $H_1 : \mu \neq 60$ if the data consist of a random sample of size 16 from a normal distribution having $\sigma = 4$

0.2 Solution

First, we have to figure out the critical values:

$$\begin{aligned} P(\text{ reject } H_0 \mid H_0 \text{ is true}) &= \alpha = 0.05 \\ &= 2P(\bar{Y} \geq \bar{y}_u^* \mid \mu = 60) \\ &= 2P(Z \geq \frac{\bar{y}_u^* - 60}{4/\sqrt{16}}) \end{aligned}$$

which means

$$\frac{\bar{y}_u^* - 60}{4/\sqrt{16}} = 1.96 \quad \bar{y}_u^* = 61.96$$

Similarly, we could get $\bar{y}_l^* = 58.04$. Now, Suppose the true mean is x, then the power of the test is defined as

$$\begin{aligned} 1 - \beta &= P(\text{ reject } H_0 \mid H_0 \text{ is false}) \\ &= P(\text{ reject } H_0 \mid \mu = x\ ) \\ &= P(\bar{Y} \geq 61.96 \mid \mu = x) + P(\bar{Y} \leq 58.04 \mid \mu = x) \\ &= P(Z \geq \frac{61.96 - 60}{4/\sqrt{16}}) + P(Z \leq \frac{58.04 - 60}{4/\sqrt{16}}) \end{aligned}$$

Now we can express this equation in Python, so that we may plot the power curve

In [11]:
```
import numpy as np
import math
import scipy.stats
import matplotlib.pyplot as plt
%matplotlib inline

def power_curve(x: float, x_upper: float, x_lower: float, sigma: float, n: int):
    """
    x is the true $\mu$
    """
    left_z_score = (x_lower - x) / (sigma / math.sqrt(n))
    right_z_core = (x_upper - x) / (sigma / math.sqrt(n))
    normal = scipy.stats.norm(0, 1)

    return normal.cdf(left_z_score) + normal.cdf(-right_z_core)

x = np.arange(53, 67, 0.05) # x range from 56 to 62, with step of 0.05
y = [power_curve(item, 61.96, 58.04, 4, 16) for item in x]

plt.plot(x, y)
```

Out[11]: [<matplotlib.lines.Line2D at 0x2544f93fa58>]