$\boxed{HW5}$ VB2182 1

first ≡ first pick by contestant
car ≡ car is hidden
host ≡ host opened the door

(1) Doors X, Y, Z :-)

(I) $\boxed{P(car = X) = 1/3;\ P(car = Y) = 1/3;\ P(car = Z) = 1/3}$

(II) $P(car = a \mid first = b) = P(car = a)$
$\forall$ a, b belong to $\{X, Y, Z\}$ sample size
car and first pick are independent

Say I picked door "X" and host opened door "Z"

$P(host = Z \mid car = Z, first = X) = 0$
$P(host = Z \mid car = Y, first = X) = 1$
$P(host = Z \mid car = X, first = X) = 1/2$

By Bayes thm,

$$P(car = X \mid host = Z, first = X) = \frac{P[host = Z \mid car = X, first = X] \times P[car = X \mid first = X]}{\sum_{m=1}^{3} P(host = Z \mid car = m, first = X) \times P(car = m, first = X)}$$

Using (II)

$$= \frac{P(host = Z \mid car = X, first = X) \times P(car = X)}{\sum_{m=1}^{3} P(host = Z \mid car = m, first = X) \times P(car = m)}$$

$\left[P(car = m) = 1/3 = P(car = X)\right]$

$$= \frac{P(host = Z \mid car = X, first = X)}{\sum_{m=1}^{3} P(host = Z \mid car = m, first = X)}$$

$$\boxed{P(car = X \mid host = Z, first = X) = \frac{1/2}{0 + 1/2 + 1} = \frac{1/2}{3/2} = 1/3}\ \text{(IV)}$$

$\Rightarrow P(car = Y \mid host = Z, first = 1) = 1 - \frac{1}{3} = 2/3$ (V)

As (V) > (IV), I will get better chance by switching to door Y.

From definition of Bayesian Network, and using Bayes Ball rules.

(2) As per Bayesian network

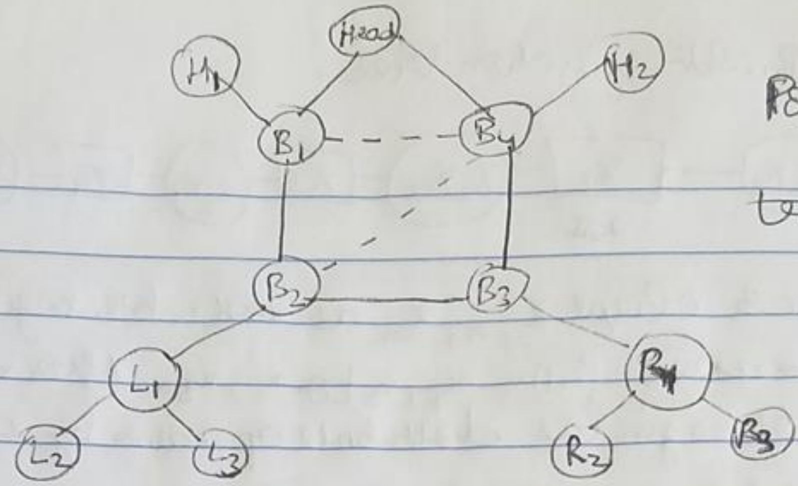
$$P(x_1, x_2, x_3, x_4, x_5) = P(x_1)\, P(x_2|x_1)\, P(x_3)\, P(x_4|x_1, x_3)\, P(x_5|x_2, x_4)$$

Using Bayes Balls

1. FALSE (goes through 1, hence dependent)
2. FALSE (goes through 5, hence dependent)
3. TRUE (No go through x_i ~~such that~~ for dependency)
4. FALSE (goes through 3-4-1-2-5, hence dependent)
5. TRUE (No go through x_i for dependency)
6. FALSE (goes through 1-2-5-4-3, hence dependent)
7. TRUE (No go through x_i for dependency)
8. TRUE (No go through x_i for dependency)
9. FALSE (goes through 3-4-5-2, hence dependent)
10. FALSE (goes through 3-4-1-2, hence dependent)

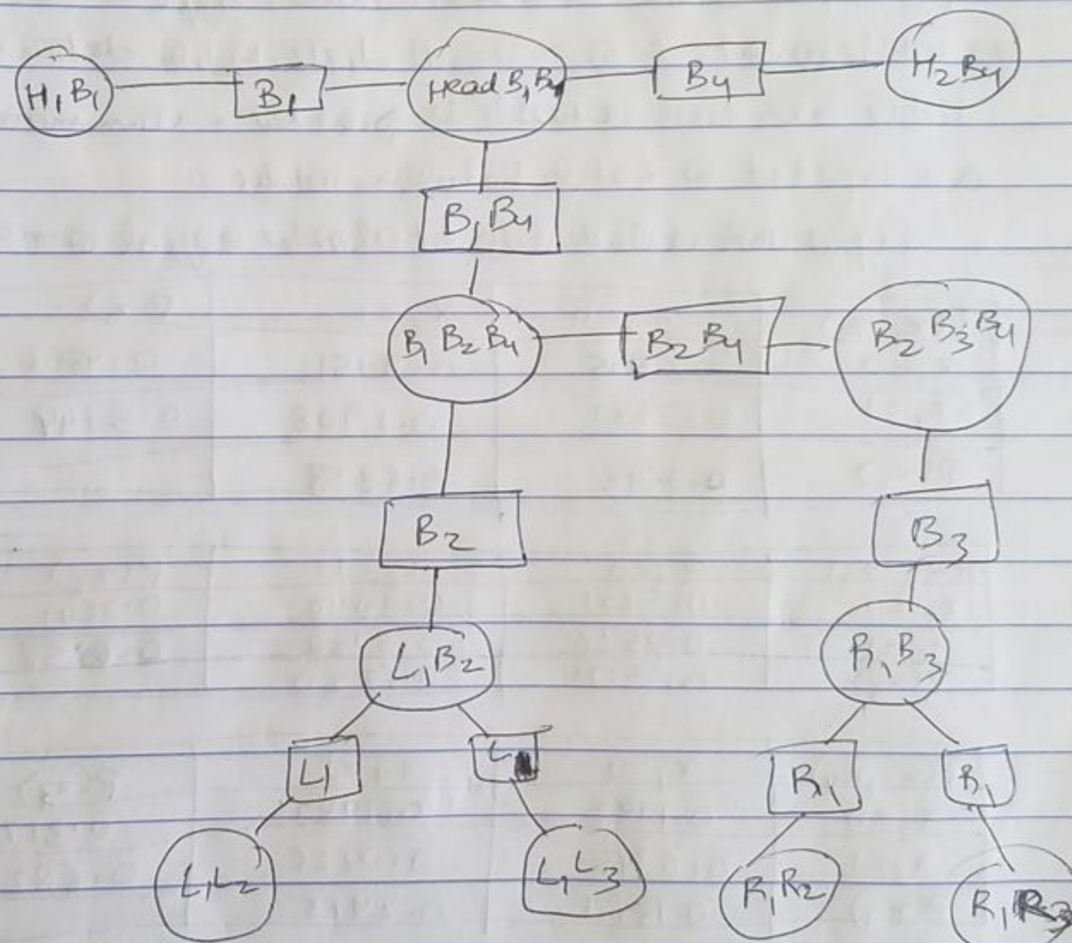
(3)-

3



Post moralization and triangulation

constructed Junction tree:



(4) Building junction tree:

$(X_1X_2) - \boxed{X_2} - (X_2X_3) - \boxed{X_3} - (X_3X_4) - \boxed{X_4} - (X_4X_5)$

Lets pick clique x_{n-1}, x_n as root. We begin w/ sending messages from x_1, x_2 to x_2, x_3. After reaching the root, we begin the distribute operation by sending the information from x_{n-1}, x_n to x_{n-2}, x_{n-1}.

After sending all the info, we normalize the table. Joint probability distributions over clique's are given in the tables below. Also marginal distributions determined from table over each variable is provided. Common marginals are expected to match between tables.

Separator values should also be equal to marginals.

$P(x_1, x_2)$	$x_2=0$	$x_2=1$	$P(x_1)$
$x_1=0$	0.0405	0.4451	0.4856
$x_1=1$	0.3237	0.1908	0.5144
$P(x_2)$	0.3642	0.6358	

$P(x_2, x_3)$	$x_3=0$	$x_3=1$	$P(x_2)$
$x_2=0$	0.2601	0.1040	0.3641
$x_2=1$	0.0578	0.5780	0.6358
$P(x_3)$	0.3179	0.6820	

$P(x_3, x_4)$	$x_4=0$	$x_4=1$	$P(x_3)$
$x_3=0$	0.1192	0.1987	0.3179
$x_3=1$	0.6395	0.0426	0.6821
$P(x_4)$	0.7587	0.2413	

$P(x_4, x_5)$	$x_5=0$	$x_5=1$	$P(x_4)$
$x_4=0$	0.5690	0.1897	0.7587
$x_4=1$	0.0603	0.1810	0.2413
$P(x_5)$	0.6293	0.3707	

Code:

```
clc; clear variables;

values_of_p = cell(4, 1);
values_of_p{1} = [0.1, 0.7; 0.8, 0.3];
values_of_p{2} = [0.5, 0.1; 0.1, 0.5];
values_of_p{3} = [0.1, 0.5; 0.5, 0.1];
values_of_p{4} = [0.9, 0.3; 0.1, 0.3];
marg = values_of_p;
n = size(marg,1);
seprt = ones(n-1,2);
%Left To Right
for i=1:n-1
   seprt(i,:) = sum(marg{i});
   marg{i+1} = marg{i+1}.*(seprt(i,:)'*[1,1]);
end
%Right to Left
for i=1:n-1
  seprts_old = seprt(n-i,:);
   seprt(n-i,:) = sum(marg{n-i+1},2)';
   marg{n-i} = marg{n-i}.*([1;1]*(seprt(n-i,:)./seprts_old));
end
%Normalizing
for i=1:n
 marg{i} = marg{i}/sum(sum(marg{i}));
end
disp('Marginal Values Calculated:');
for i=1:n
    val = marg{i};
    disp(val);
end
```

```
Marginals Values Calculated:
    0.0405    0.4451
    0.3237    0.1908

    0.2601    0.1040
    0.0578    0.5780

    0.1192    0.1987
    0.6395    0.0426

    0.5690    0.1897
    0.0603    0.1810
```

Q5: Use ArgMax Junction Tree Algorithm (JTA), where we replace sums with max in JTA, find the biggest separators. The emotional states are most likely here:

Day 1	Day 2	Day 3	Day 4	Day 5
Happy	Angry	Angry	Angry	Angry

Code:

```
clc; clear variables;
emis_p = [0.4, 0.1, 0.3, 0.2; 0.1, 0.4, 0.2, 0.3];
init = [1; 0];
obs_st = [1, 4, 2, 2, 3];
trans_p = [0.8, 0.2; 0.2, 0.8];
t_size = size(trans_p, 1);nsize = size(obs_st, 2);
val1 = zeros(t_size, t_size, nsize);val2 = zeros(t_size, nsize);
val2(:, 1) = init;
% L to R
for i = 2:nsize
   val = obs_st(1, i);
   val1(:, :, i) = diag(val2(:, i - 1)) *trans_p * diag(emis_p(:,val));
    val2(:, i) = max(val1(:, :, i));
end
% R to L
for i = nsize-1:-1:1
   val2_new = max(val1(:, :, i + 1), [], 2);
   val1(:, :, i) = val1(:, :, i) * diag(val2_new ./ val2(:, i));
    val2(:, i) = val2_new;
end
[~, V] = max(val2);
```

```
>> V

V =

     1     2     2     2     2
```