Vincent Xie

Feb 8, 2024

HW3

3.30

Several ovens in a metal working shop are used to heat metal specimens. All the ovens are supposed to operate at the same temperature, although it is suspected that this may not be true. Three ovens are selected at random, and their temperatures on successive heats are noted. The data collected are as follows:

Oven	Temperature					
1	491.50	498.30	498.10	493.50	493.60	
2	488.50	484.65	479.90	477.35		
3	490.10	484.80	488.25	473.00	471.85	478.65

a) Is there a significant variation in temperature between ovens? Use alpha = 0.05.

The design for this dataset is a one way random effects model. The null and alternative hypothesis are:

$$H0: \sigma^2_\tau = 0 \text{ vs. } Ha: \sigma^2_\tau > 0$$

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	2	594.530250	297.265125	8.62	0.0048
Error	12	413.812083	34.484340		
Corrected Total	14	1008.342333			

From this output, we see that the F-value = 8.62 and the p-value = 0.0048. Because the p-value is less than the alpha value, we reject the null hypothesis and say that there is a significant variation in temperature between the three ovens.

b) Estimate the components of variance for this model.

σ^2_e = MSE = 34.484

Type 1 Estimates	
Variance Component	**Estimate**
Var(Oven)	53.26638
Var(Error)	34.48434

Estimating the variance component for ovens, σ^2_τ:

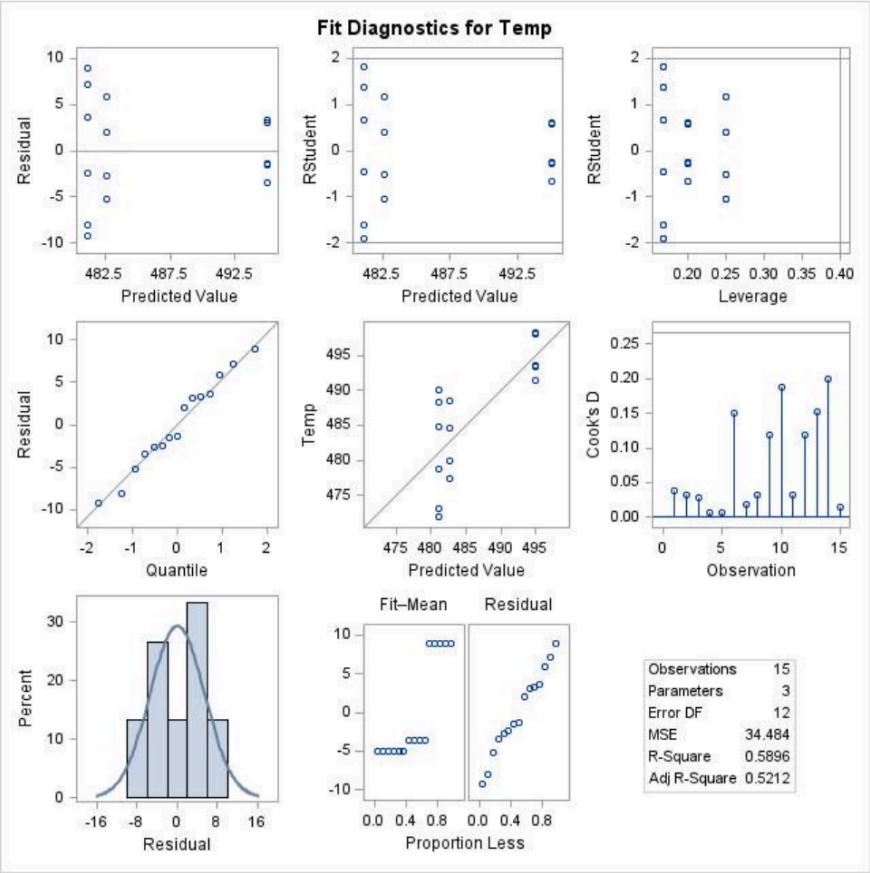
$$\sigma^2_\tau = MS_{Treatment} - MSE / n_0 = (297.265-34.484) / n_0$$

$$N_0 = \tfrac{1}{2}(15 - (25+16+36)/15) = 4.93$$

$$\sigma^2_\tau = (297.265-34.494) / 4.93 = 53.27$$

Proportion of variability due to ovens: p = 53.27/(53.27+34.484) = 0.61

c) Analyze the residuals from this experiment and draw conclusions about model adequacy.



Residual vs Quantile: There is a straight pattern which indicates the normality assumption has not been violated.
Cook's D vs Observation: All values are less than 1, indicating that there are no influential data points.
RStudent vs Leverage: All values are between 2 and -2, so there are no outliers.
Histogram: The curve resembles a bell curve which also proves normality.

3.31

An article in the *Journal of the Electrochemical Society* (Vol. 139, No. 2, 1992, pp. 524-532) describes an experiment to investigate the low-pressure vapor deposition of polysilicon. The experiment was carried out in a large-capacity reactor at Sematech in Austin, Texas. The reactor has several wafer positions, and four of these positions are selected at random. The response variable is film thickness uniformity. Three replicates of the experiment were run, and the data are as follows:

Wafer Position	Uniformity		
1	2.76	5.67	4.49
2	1.43	1.70	2.19
3	2.34	1.97	1.47
4	0.94	1.36	1.65

a) Is there a difference in the wafer positions? Use alpha=0.05.

The design for this dataset is also a one way random effects model. The null and alternative hypotheses for this model are:

$$H0: \sigma^2_\tau = 0 \text{ vs. } Ha: \sigma^2_\tau > 0.$$

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	16.21982500	5.40660833	8.29	0.0077
Error	8	5.21746667	0.65218333		
Corrected Total	11	21.43729167			

From the SAS output, the F-statistic = 8.29 with a p-value = 0.0077. This p-value is less than the alpha value so we can reject the null hypothesis and say that there is significant variation in the uniformity between the four wafer positions.

b) Estimate the variability due to wafer positions.

Type 1 Estimates			
Variance Component	**Estimate**	**95% Confidence Limits**	
Var(wf)	1.58481	0.26150	24.81078
Var(Error)	0.65218	0.29755	2.39363

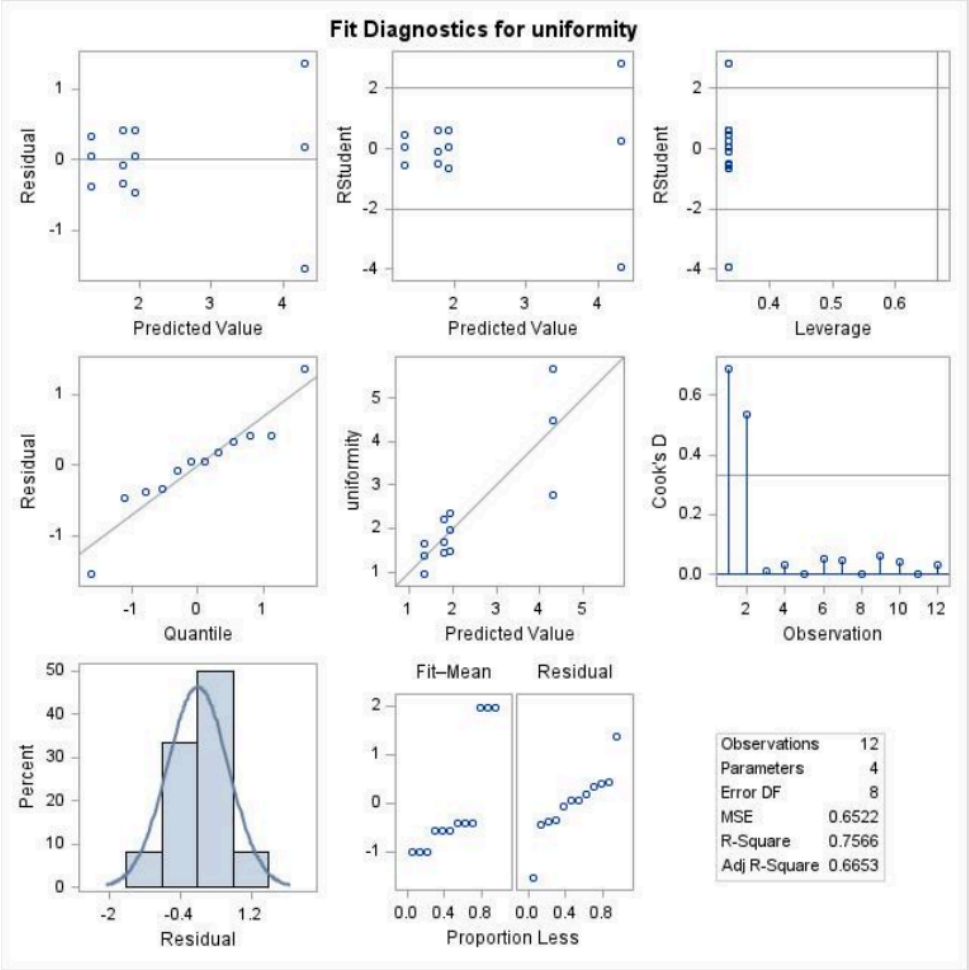
N_0 = 3

σ^2_τ = MS_Treatment - MSE / n_0 = 5.406-0.652/3 = 1.5848

c) Estimate the random error component.

Random error component = MSE = 0.652

d) Analyze the residuals from this experiment and comment on model adequacy.



Residual vs Quantile: The data points are pretty close to the trend line in the middle. The tail seems to have a slight curvature.
Cook's D vs Observation: All of the values are less than 1, indicating that there are no influential observations.
RStudent Leverage: There is a moderate outlier at -4.
Residual vs Predicted Value: There is a slight pattern which shows that the variances of the random errors may increase.
Histogram: The trendline also resembles a bell curve so the normality assumption is not badly violated.