Vincent $\to K = 7$

Exam 1

① Compute $\langle 3, 2, -1\rangle \times \langle -5, 4, 7\rangle$

$$\begin{array}{cc} a & b \end{array}$$

$$a \times b = \begin{vmatrix} i & j & k \\ 3 & 2 & -1 \\ -5 & 4 & 7 \end{vmatrix} = \begin{vmatrix} 2 & -1 \\ 4 & 7 \end{vmatrix} \hat{i} - \begin{vmatrix} 3 & -1 \\ -5 & 7 \end{vmatrix} \hat{j} + \begin{vmatrix} 3 & 2 \\ -5 & 4 \end{vmatrix} k$$

$$a \times b = (14 - (-4))\hat{i} - (21 - 5)\hat{j} + (12 - (-10))k$$
$$= 18\hat{i} - 16\hat{j} + 22k$$

② Consider the surface $z = 5x - 2y^2$

a) Describe the traces parallel to the xz-plane

y-traces $\quad z = 5x - 2(7)^2 \quad$ traces are lines

$K=7 \quad z = 5x - 98$

b) Describe the traces parallel to the yz-plane

x-traces

$K=7 \quad z = 5(7) - 2y^2 \quad$ traces are parabolas

$$= 35 - 2y^2$$

(3) Compute the rate of change of the function $f(x,y) = \ln(xy)$ in the direction of $\langle 1,7 \rangle$ at point $(-1,1)$

$$f_x = \frac{1}{xy} \cdot y = \frac{1}{x} \qquad f_x(-1,1) = -1$$

$$f_y = \frac{1}{xy} \cdot x = \frac{1}{y} \qquad f_y(-1,1) = 1$$

$$\vec{\nabla f} = \langle \tfrac{1}{x}, \tfrac{1}{y} \rangle$$

$$\vec{\nabla f}(-1,1) = \langle -1, 1 \rangle$$

Rate of change

$$\nabla f \cdot \vec{v}$$
$$= \langle -1,1 \rangle \cdot \langle 1, 7 \rangle$$
$$= (-1)(1) + (1)(7) = 6$$

(4) Find any one line perpendicular to the plane given by $x - 7y + 6z = 6$

$\vec{n} = \langle 1, -7, 6 \rangle$ Assume $P: (1,1,1)$

Symmetric equations: $\dfrac{x-1}{1} = \dfrac{y-1}{-7} = \dfrac{z-1}{6} = t$

$$x = t+1, \quad y = 1-7t, \quad z = 6t+1$$

$$r(t) = (1+t)\hat{i} + (1-7t)\hat{j} + (6t+1)\hat{k}$$

5) The point $(1,1)$ is a critical point for $f(x,y) = x^2 + 2y^2 - 2x - 4y + 1$.

Determine whether $(1,1)$ is a saddle point, local min/max, or if there is

not enough information to continue

$f(x,y) = x^2 + 2y^2 - 2x - 4y + 1$

$f_x = 2x - 2 \quad f_y = 4y - 4$

$f_{xx} = 2 \quad f_{yy} = 4$

$f_{xy} = 0$

$D_{\vec{v}}(x,y) = f_{xx} f_{yy} - (f_{xy})^2$

$= (2)(4) - 0^2$

$= 8$

$f_{xx} = 2$

$f(1,1) = 1^2 + 2(1^2) - 2(1) - 4(1) + 1$

$= 1 + 2 - 2 - 4 + 1$

$= -2$

Because $D_{\vec{v}}(1,1) = 8 > 0$ and $f_{xx}(1,1) = 2 > 0$, there is a local minimum at point $(1,1)$. That value of $f(1,1)$ is -2.

(6) Find the maximum value of $f(x,y) = 2x+8y$ under the condition $g(x,y) = x^2+y^2 = 17$

$f(x,y) = 2x+8y$ $\quad$ $g(x,y) = x^2+y^2=17$

$f_x = 2$ $\quad$ $g_x = 2x$

$f_y = 8$ $\quad$ $g_y = 2y$

$\vec{\nabla f} = \langle 2, 8 \rangle$ $\quad$ $\vec{\nabla g} = \langle 2x, 2y \rangle$

Lagrange Eq

$2 = 2x\lambda$ $\quad$ $8 = 2y\lambda$

$\lambda = \frac{1}{x}$ $\quad$ $\lambda = \frac{4}{y}$

$\frac{1}{x} = \frac{4}{y}$ $\quad$ $y = 4x$

$x = \frac{y}{4}$

$x^2+y^2 = 17$

$\left(\frac{y}{4}\right)^2 + y^2 = 17$

$\frac{1}{16}y^2 + y^2 = 17$

$\left(\frac{16}{17}\right)\frac{17}{16}y^2 = 17\left(\frac{16}{17}\right)$

$\sqrt{y^2} = \sqrt{16}$

$y = \pm 4$

$y = 4, x = 1$

$y = -4, x = -1$

Critical Points

$(1,4)$, $(-1,-4)$

$f(1,4) = 2(1) + 8(4)$
$= 2 + 32$
$= 34$

$f(-1,-4) = 2(-1) + 8(-4)$
$= -34$

The absolute maximum of $f(x,y)$ is at $(1,4)$ with the value of 34.