The authors discuss a methodology to setup equations of motion in terms of Euler parameters and so-called quasi-velocities that coincide up to a constant factor with the angular velocity.

The material fits definitely into the scope of this journal. It is presented in a tutorial like style with numerical test results for a double pendulum that illustrate the approach.

The reviewer is not fully convinced that the material has the level of novelty and originality that one would expect from a paper in a top-ranked international journal but the final decision is up to the Editorial board anyway.

The numerical test scenario (two body system, error bounds of size 10^{-9} / 10^{-10}) is not really representative for applications in (industrial) multibody system simulation.

Recommendation

The approach should be tested on more complex model problems representing, e.g., the motion of an industrial robot in 3D. Error tolerances should be chosen in a more typical range like, e.g., $10^{-4} \ldots 10^{-6}$ and the residuals in the normalization constraint (see Figs. 4 and 5) should be checked carefully. For the double pendulum, the residuals in Fig. 5 exceed the error tolerances by a factor close to 1000 which would of course not be acceptable for error tolerances in the size of 10^{-4} or 10^{-5}.

Classical stabilization techniques from the field of DAE time integration could be used to reduce the drift-off effect, i.e., to avoid the permanently growing residuals in the normalization constraints $\mathbf{e}^\top\mathbf{e} = 1$, see Figs. 4 and 5. Adding, e.g., an extra term $-\alpha\,(\mathbf{e}^\top\mathbf{e} - 1)\,\mathbf{e}$ with a constant $\alpha > 0$ to (12_1), one would get

$$\frac{\mathrm{d}}{\mathrm{d}t}\big(\mathbf{e}^\top\mathbf{e} - 1\big) = 2\,\mathbf{e}^\top\dot{\mathbf{e}} = 2\,\underbrace{\mathbf{e}^\top\big(\mathbf{E}_{1:3,:}\big)^\top}_{=\,\mathbf{0}}\mathbf{u}_{1:3} - 2\,\alpha\,(\mathbf{e}^\top\mathbf{e} - 1)\,\underbrace{\mathbf{e}^\top\mathbf{e}}_{\approx\,1} \approx -2\,\alpha\,(\mathbf{e}^\top\mathbf{e} - 1)$$

and any non-zero residuals $\mathbf{e}^\top\mathbf{e} - 1$ will be damped out with a rate of $\exp(-2\alpha(t - t_0))$, see [1, 2].

Minor comments

1. Please check carefully if there is any strong reason to introduce quasi-speeds $\mathbf{u}_{1:3}$ that coincide up to a constant factor with the classical angular velocity. At the end, the equations of motion are just given in terms of the Euler parameters, the angular velocity vectors and the residuals $u_4 = \mathbf{e}^\top\dot{\mathbf{e}}$ in the time derivative of the normalization constraint $\mathbf{e}^\top\mathbf{e} = 1$. All these quantities are well known in this field of research. No need to introduce new terms like quasi-speed, quasi-velocity.

2. The material is carefully prepared and well presented but other authors start earlier (with algorithms to setup the equations of motion *directly* in terms of Euler parameters and angular velocities [3]) or go one step further (using stabilization techniques to avoid or to reduce the drift-off effect, see above).

3. The numerical test results for the double pendulum illustrate that Matlab's built-in `ode45` solver does what it is expected to do. In a quadruple precision implementation with error bounds in the size of 10^{-20}, similar results could perhaps also be obtained for longer time intervals. But `ode45` is definitely not designed for such long-term simulations and other integrators (energy conserving, symplectic, …) would perhaps be much more appropriate.

4. Bibliographic information for refs. [2], [6], [9] is missing or incomplete.

5. line 4 in the right column of page 4: the performance OF the simulation

References

[1] U.M. Ascher, H. Chin, and S. Reich. Stabilization of DAEs and invariant manifolds. *Numer. Math.*, 67:131–149, 1994.

[2] U.M. Ascher, H. Chin, L.R. Petzold, and S. Reich. Stabilization of constrained mechanical systems with DAEs and invariant manifolds. *J. Mech. Struct. Machines*, 23:135–158, 1995.

[3] M. Géradin and A. Cardona. *Flexible Multibody Dynamics: A Finite Element Approach.* John Wiley & Sons, Ltd., Chichester, 2001.