1. MLE

a) $\mathcal{L}(\lambda, x) = P_\lambda(X=x) \quad P_\lambda(X=x_i) = \dfrac{\lambda^{x_i} e^{-\lambda}}{x_i!}$

$$\mathcal{L}(\lambda, x) = \prod_{i=1}^{n} \frac{\lambda^{x_i} e^{-\lambda}}{x_i!}$$

The x_i s are independent & therefore can all be just multiplied together

b) $$\mathcal{L}(\lambda, x) = \prod_{i=1}^{n} \frac{\lambda^{x_i} e^{-\lambda}}{x_i!}$$

$$\log \mathcal{L}(\lambda, x) = \sum_{i=1}^{n} \log \frac{\lambda^{x_i} e^{-\lambda}}{x_i!}$$

$$= \sum_{i=1}^{n} (x_i \log\lambda - \lambda \log e - \log(x_i!))$$

ln instead of log doesn't make a difference

$\Updownarrow$

$$\ln \mathcal{L}(\lambda, x) = \sum_{i=1}^{n} (x_i \ln\lambda - \lambda - \ln(x_i!))$$

$$\hat{\lambda}_{MLE} = \underset{\lambda}{\operatorname{argmax}}\, \mathcal{L}(\lambda, x) = \underset{\lambda}{\operatorname{argmax}}\, \ln \mathcal{L}(\lambda, x)$$

Take derivative, set to 0

$$\frac{d}{d\lambda}\left(\sum_{i=1}^{n}(x_i \ln\hat{\lambda} - \hat{\lambda} - \ln(x_i!))\right) = 0$$

$$\sum_{i=1}^{n}\left(\frac{x_i}{\hat{\lambda}} - 1\right) = 0$$

$$\sum_{i=1}^{n} \frac{x_i}{\hat{\lambda}} = n$$

$$\hat{\lambda} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

2. Poisson Counts

a) $H_0: \quad \lambda_0 = \dfrac{1}{2n} \displaystyle\sum_{i=1}^{n} (x_i + y_i)$

$H_a: \quad \lambda_x = \dfrac{1}{n} \displaystyle\sum_{i=1}^{n} x_i$

$\lambda_y = \dfrac{1}{n} \displaystyle\sum_{i=1}^{n} y_i$

b) Jupyter Notebook

c) $$\mathcal{L}_0 = \prod_{i=1}^{n} \frac{\lambda_0^{x_i} e^{-\lambda_0}}{x_i!} \cdot \frac{\lambda_0^{y_i} e^{-\lambda_0}}{y_i!}$$

$$\mathcal{L}_a = \prod_{i=1}^{n} \frac{\lambda_x^{x_i} e^{-\lambda_x}}{x_i!} \cdot \frac{\lambda_y^{y_i} e^{-\lambda_y}}{y_i!}$$

d) Jupyter Notebook

e) χ^2 value for 1 dof: 3.84

Reject if ratio > 3.84

Ratio = 0.296

so we can't reject the null hypothesis