Total Least Squares

Total Least Squares is considered an "errors-in-variables" model as in its usage both the numerical features (dependent variables) and the labels (independent variables) are corrupted by noise. Total Least Squares is also sometimes referred to as orthogonal regression.

In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
from numpy.linalg import matrix_rank
%matplotlib inline
from sklearn.datasets import make_regression
```

In [2]:
```
# Total least squares assumes corruption in all variables, i.e both in input and
output data
#a great way to solve for the solution is by manipulating the k+1th singluar val
ue of the SVD decomposition of [A,b]
import numpy as np
def TLS(A,b):
    svd_matrix= np.vstack((A.T,b)).T
    U,sigma,V_transpose= np.linalg.svd(svd_matrix)
    V= V_transpose.T
    split= A.shape[1]
    Vyy= V[split:,split:]
    Vxy= V[:split,split:]
    sol_tls= -Vxy/Vyy
#now we can also return the denoised A and b as well
    V_split= V[:,split:]
    A_error= np.matmul(svd_matrix,V_split)
    A_error= -np.matmul(A_error,V_split.T)
    A_e= A_error[:,:split]
    A_tls= A + A_e
    b_tls= np.matmul(A_tls,sol_tls)
    return sol_tls,A_tls,b_tls
```

In [3]:
```
#this method is an ordinary least square solvers, good for datasets with uniform
data, no need for reguralization
def OLS(A,b):
    #solving the equation Ax=b
    if (matrix_rank(np.matmul(A.T,A)) != A.shape[1]):
        print("Matrix times its tranpoise is not full rank")
    lhs= np.matmul(A.T,A)
    rhs= np.matmul(A.T,b)
    x=np.matmul(np.linalg.inv(lhs),rhs)
    return x
```

Creating sample data courtesy of https://towardsdatascience.com/total-least-squares-in-comparison-with-ols-and-odr-f050ffc1a86a (https://towardsdatascience.com/total-least-squares-in-comparison-with-ols-and-odr-f050ffc1a86a)

In [4]:
```
x, y, coefficients = make_regression(
    n_samples=50,
    n_features=1,
    n_informative=1,
    n_targets=1,
    noise=5,
    coef=True,
    random_state=1
)
```

We will now create an error function that will help us calculate the error between our predicted labels, and the true labels

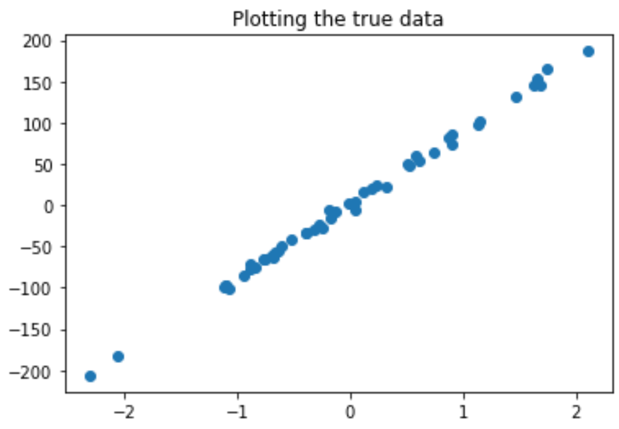
Note on Mean sqared error (MSE)

The MSE is a measure of the quality of an estimator. It calculates the mean squared difference across two quantities. The mean squared error is never negative, and the closer to zero, the better the estimate

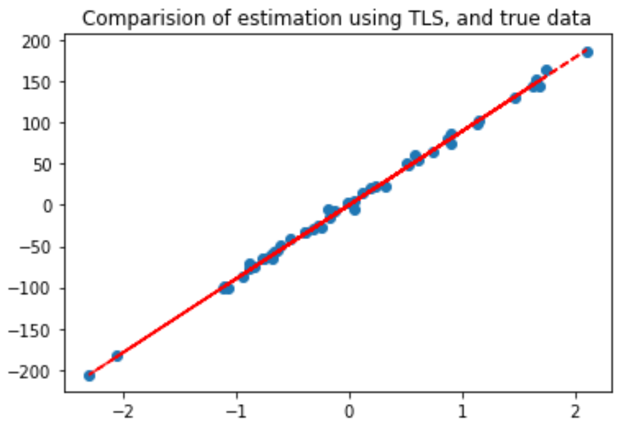
In [5]:
```
def MSE(y_true,y_predicted):
    return (1/len(y_true))*np.sum((np.subtract(y_true,y_predicted))**2)
```

To begin with, let's visualize our data

In [6]:
```
plt.scatter(x,y)
plt.title("Plotting the true data")
plt.show()
```



In [7]:
```
weights_predicted,A,b= TLS(x,y)
y_predicted= np.matmul(x,weights_predicted)
error= MSE(y,y_predicted)
plt.scatter(x,y)
plt.plot(x,y_predicted,linestyle='dashed',c='red')
plt.title("Comparision of estimation using TLS, and true data")
plt.show()
mse= MSE(y,y_predicted)
print("The mean squared error is " + str(mse) )
```

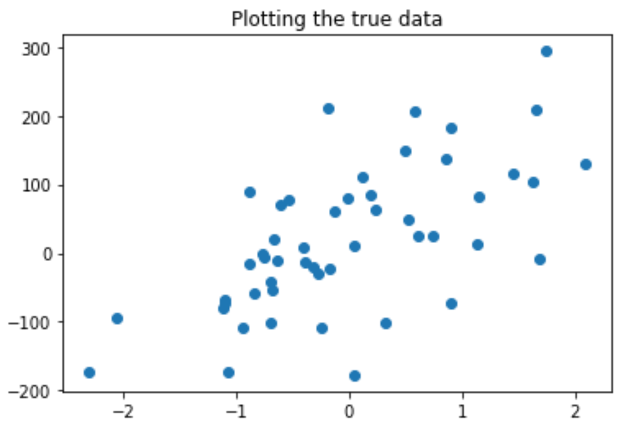


```
The mean squared error is 751409.7183623331
```

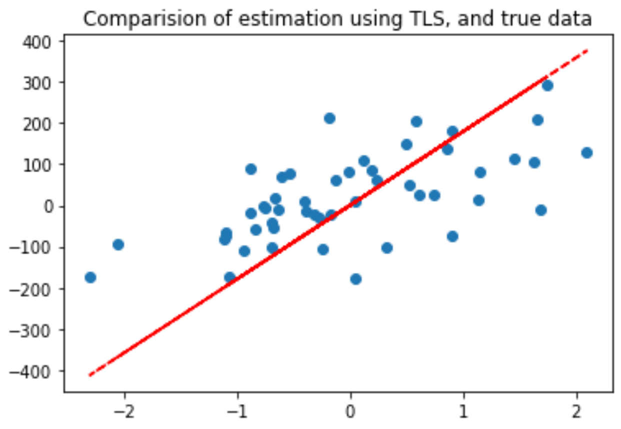
Let us up the noise and see what happens

In [9]:
```
x, y, coefficients = make_regression(
    n_samples=50,
    n_features=1,
    n_informative=1,
    n_targets=1,
    noise=100,
    coef=True,
    random_state=1
)
```

In [10]:
```
plt.scatter(x,y)
plt.title("Plotting the true data")
plt.show()
```

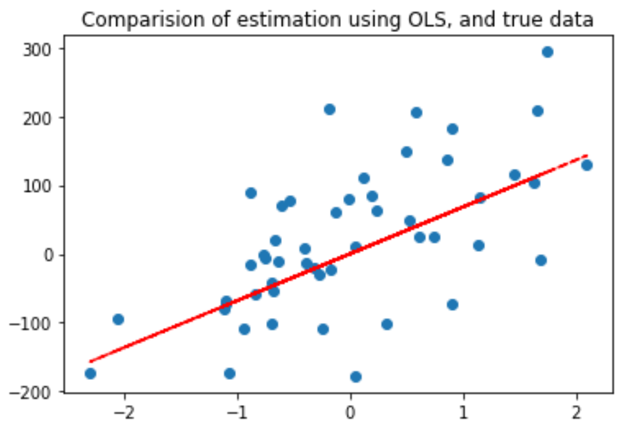


In [11]:
```
weights_predicted,A,b= TLS(x,y)
y_predicted= np.matmul(x,weights_predicted)
error= MSE(y,y_predicted)
plt.scatter(x,y)
plt.plot(x,y_predicted,linestyle='dashed',c='red')
plt.title("Comparision of estimation using TLS, and true data")
plt.show()
```



Comparing to ordinary least squares:

In [12]:
```
weights_predicted= OLS(x,y)
y_predicted= np.matmul(x,weights_predicted)
error= MSE(y,y_predicted)
plt.scatter(x,y)
plt.plot(x,y_predicted,linestyle='dashed',c='red')
plt.title("Comparision of estimation using OLS, and true data")
plt.show()
```


Comparision of estimation using OLS, and true data