1. $m(a+bX) = a + b \times m(X)$

$$m(X) = \frac{1}{N}\sum_{i=1}^{N} x_i \Rightarrow$$

$$m(a+bX) = \frac{1}{N}\sum_{i=1}^{N} (a+bx_i) \rightarrow$$

$$\frac{1}{N}\sum_{i=1}^{N} a + \frac{1}{N}\sum_{i=1}^{N} bx_i \Rightarrow a + b\frac{1}{N}\sum_{i=1}^{N} x_i$$

$$\Downarrow$$

a and b are single values

so (1) a = a, (1) b = b

They stay the same

$$a + b \times m(X)$$

2. $cov(X, a+bY) = b \times cov(X,Y)$

$$cov(X,Y) = \frac{1}{N}\sum_{i=1}^{N} (x_i - m(X))(y_i - m(Y)) \Rightarrow$$

$$cov(X, a+bY) = \frac{1}{N}\sum_{i=1}^{N} (x_i - m(X))((a+by_i) - m(a+bY))$$

$$\frac{1}{N}\sum_{i=1}^{N} (x_i - m(X)) \cdot ((a+by_i) - m(a+bY))$$

From 1. $m(a+bY) = a + b \times m(Y)$

$$\frac{1}{N}\sum_{i=1}^{N} (x_i - m(X))(\cancel{a} + by_i - \cancel{a} + b \times m(Y))$$

$$\frac{1}{N}\sum_{i=1}^{N} (x_i - m(X))(b(y_i + m(Y)))$$

b will remain constant so we can move it

$$b \times \frac{1}{N}\sum_{i=1}^{N} (x_i - m(X))(y_i + m(Y)) = b \times cov(X,Y)$$

3. $cov(a+bX, a+bX) = b^2 cov(X,X)$

$$\Downarrow$$

$$\frac{1}{N}\sum_{i=1}^{N}\left(a+bx_i - m(a+bX)\right)\left(a+bx_i - m(a+bX)\right)$$

$$\frac{1}{N}\sum_{i=1}^{N}\left(\cancel{a}+bx_i - \cancel{a}+bm(X)\right)\left(\cancel{a}+bx_i - \cancel{a}+bm(X)\right)$$

$$\frac{1}{N}\sum_{i=1}^{N}\left(b(x_i - m(X))\right)\left(b(x_i + m(X))\right)$$

constants

$$b \cdot b \underbrace{\frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))(x_i + m(X))} = b^2 \underbrace{cov(X,X)}$$

$$\Rightarrow \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))^2 = s^2$$

4. say $X = \{2, 3, 5, 7, 10\}$

$$g(X) = \{12, 17, 27, 37, 52\}$$

$$g(5) = 2+5(5) = \underline{27}$$

yes the median of the transformed variable is the same data pain as the original non-decreasing transformation. This would apply to any quantile, as well as the IQR and range, because these are all based on order which is preserved in transformations like these

5. No the mean doesn't always hold for non-decreasing transformations

$m(g(X)) \neq g(m(X))$

because the transformation can distort the distribution, for example log():

$X = \{1, 10, 100\}$

$g(m(X)) = \log(37) \approx 1.57$

$m(X) = \frac{111}{3} \approx 37$

$1.57 \neq 1$

$\log(X) = \{0, 1, 2\}$ $m(g(X)) = \frac{3}{3} = 1$