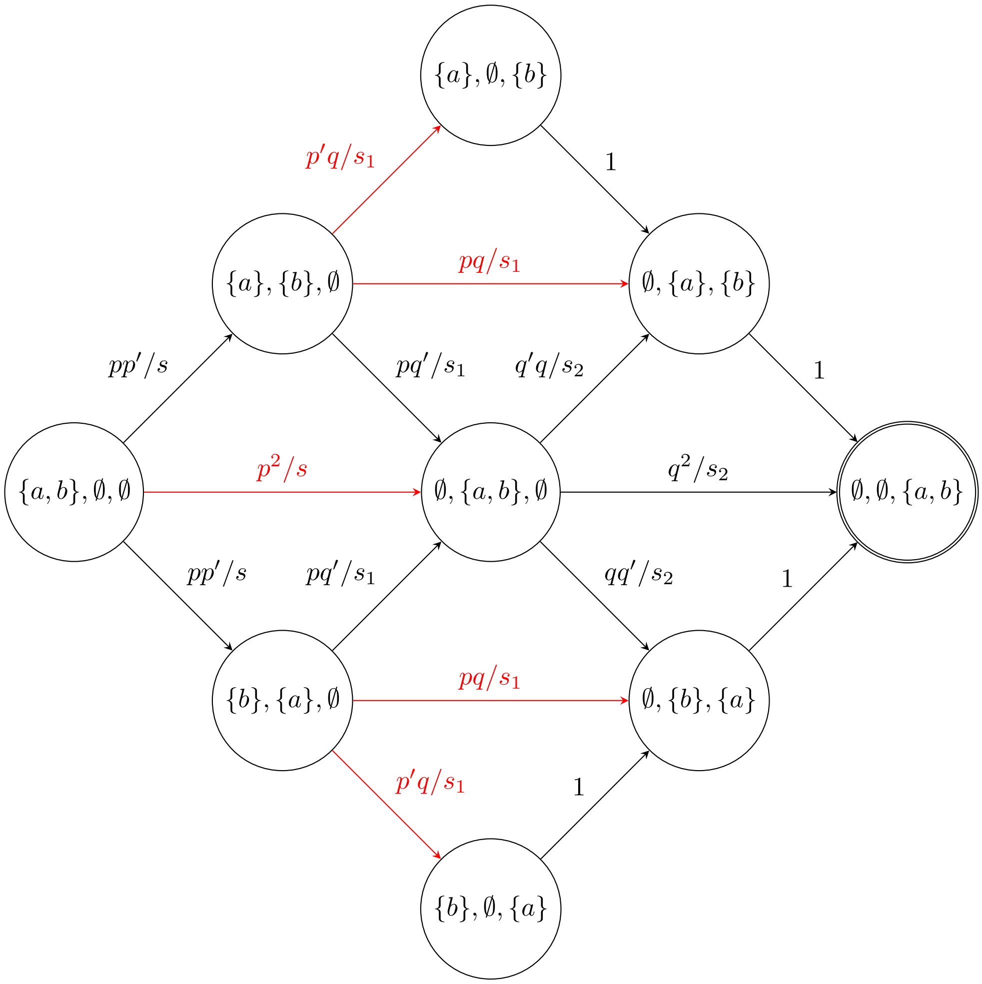
Transition probabilities by conditionalization Born with prob p, die with q, and divide by sum for conditional probability, stipulating change. This rules out loops.



where $p' := 1-p$, $q' := 1-q$ and

$$s := 2pp' + p^2 \qquad s_1 := pq' + p'q + pq \qquad s_2 := 2qq' + q^2.$$

If $p = q$ then $s = s_1 = s_2$, reducing to $s = 3p^2$ if $p = p'$ and $2pp'$ as $p \to 0$

Allen relation	probabilities	$p = q$	$p = q = p'$	$p = q \to 0$
`eq`	p^2q^2/ss_2	p^4/s^2	$1/9$	0
`b,bi`	pp'^2q/ss_1	$p^2p'^2/s^2$	$1/9$	$1/4$
`m,mi`	$p^2p'q/ss_1$	p^3p'/s^2	$1/9$	0
`s,si`	p^2qq'/ss_2	p^3p'/s^2	$1/9$	0
`o,oi`	$p^2p'qq'^2/ss_1s_2$	$p^3p'^3/s^3$	$1/27$	$1/8$
`d,di`	$p^2p'qq'^2/ss_1s_2$	$p^3p'^3/s^3$	$1/27$	$1/8$
`f,fi`	$p^2p'q^2q'/ss_1s_2$	$p^4p'^2/s^3$	$1/27$	0

Remarks

1. The exponent on s, separating Suliman's two classes $(1/3^2, 1/3^3)$, is the number of (genuine) choices along the way from the initial to the final state. A path is given by 2 choices iff it pass through a red arc.

2. Suliman's 1/9 class includes representatives from Fernando-Vogel's long, medium and short strings. Note also that `s` and `f` get different probabilities (above), contra Fernando-Vogel where the direction of time does not matter, and the difference between `s` and `f` dissolves (interchangeable by reversing direction).

3. For comparison with birth-death process (Wikipedia) in queueing theory, let $p, q \to 0$ to eliminate simultaneous events.

4. Doubling p (whilst remaining ≤ 1) reduces $p' = 1 - p$ by varying degrees (depending on p): solve for k in $1 - 2p = k(1 - p)$ to get
$$k = \frac{1-2p}{1-p}.$$

5. There is no distinction between the old and young in a birth-death process. In the probability table above, overlap `o` and during `d` are equiprobable.

6. If the old die before the young, then `o` should be more probable than `d`. This suggests raising q (if not p) with time. Does the inertial mechanism degrade (wear out) over time? Why is it not enough that $(1-q)^n \to 0$ as $n \to \infty$?

7. The previous item suggests p and q differ. One way is to let there be only one state of being unborn, but many of being alive (a precondition for dying), leading to a single p but many q_i's (for age i). Inasmuch as any run uses only finitely many i's, finite automata will do.

8. Birth-death processes suggest for this case, two born rates λ_0, λ_1 and two death rates μ_1, μ_2. But it is not immediately obvious how these relate to p and q. Do rates (given in units of time) come before probabilities p, q (which can be specified without mentioning time)?

9. If $p = 2q \to 0$, then $ss_1 \to 2pp'(pq' + pp'/2)$ and so prob(`b`) $\to$
$$\frac{p^2 {p'}^2/2}{2pp'(pq' + pp'/2)} = \frac{p'}{4(q' + p'/2)} = \frac{1}{4(q'/p' + 1/2)} \to 1/6$$
and $ss_1s_2 \to 2p^2p'q'(pq' + pp'/2)$ so prob(`o`) $\to$
$$\frac{p^3 p' {q'}^2/2}{2p^2p'q'(pq' + pp'/2)} = \frac{q'}{4(q' + p'/2)} = \frac{1}{4(1 + p'/2q')} \to 1/6$$