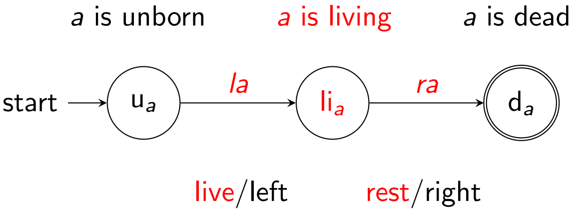
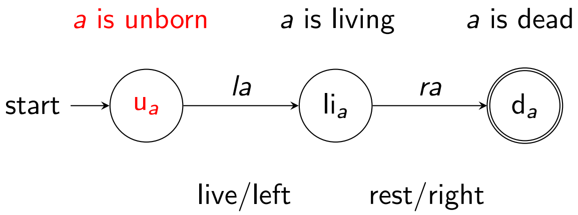
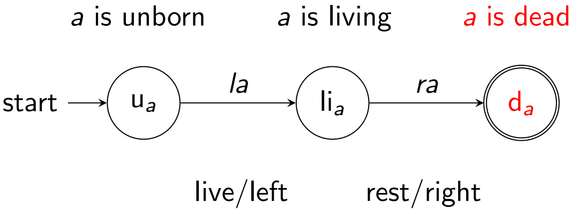
Bounding life by two events



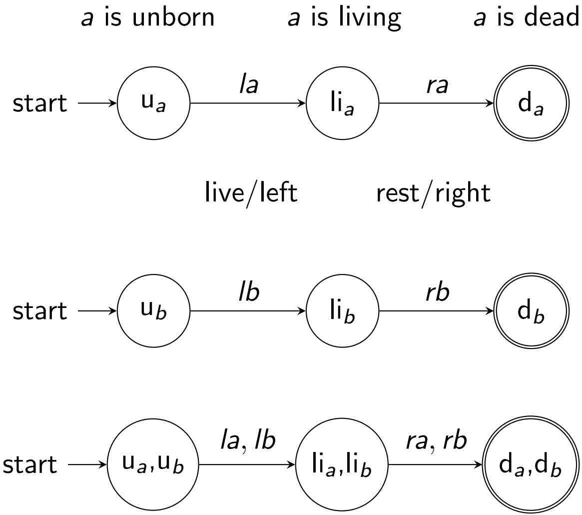
Bounding life by two events

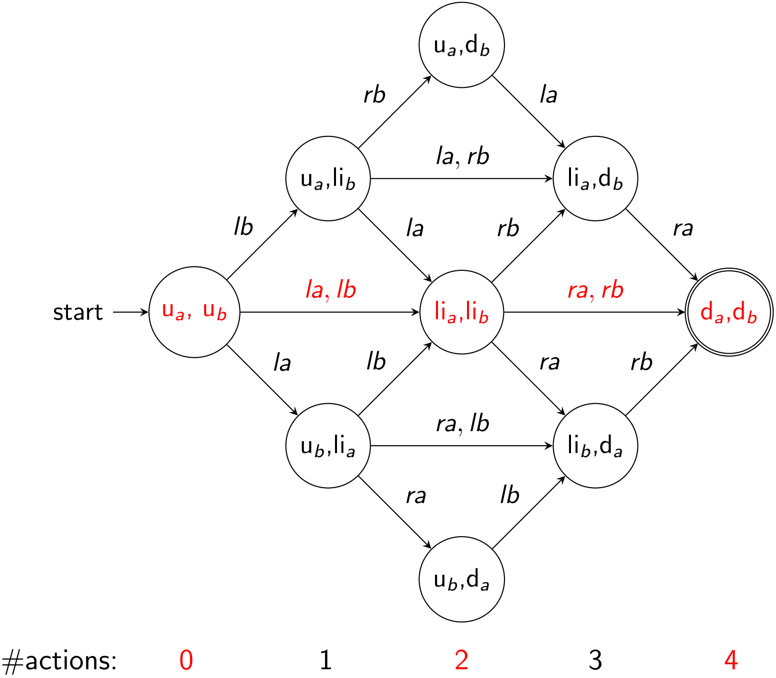


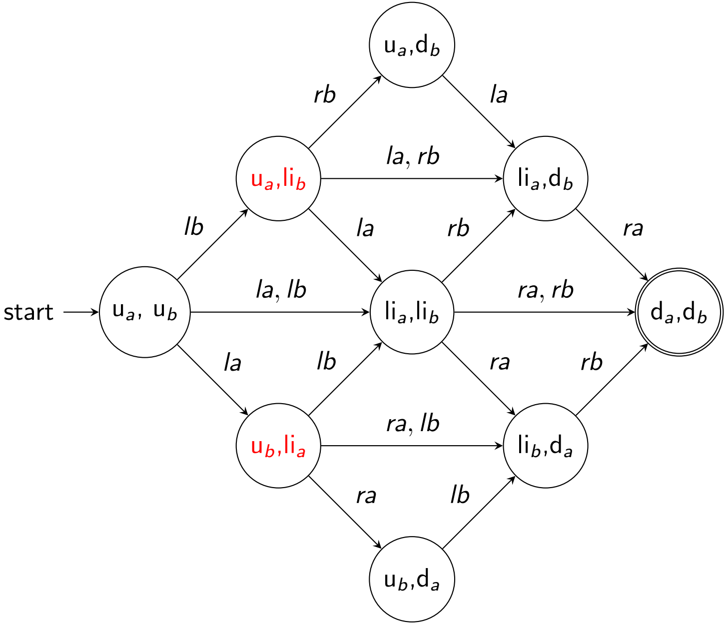
Bounding life by two events



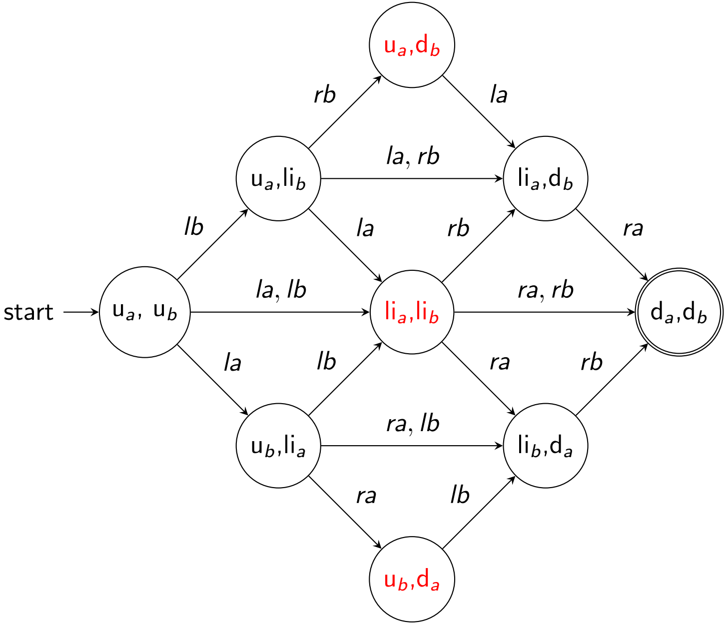
Bounding life by two events



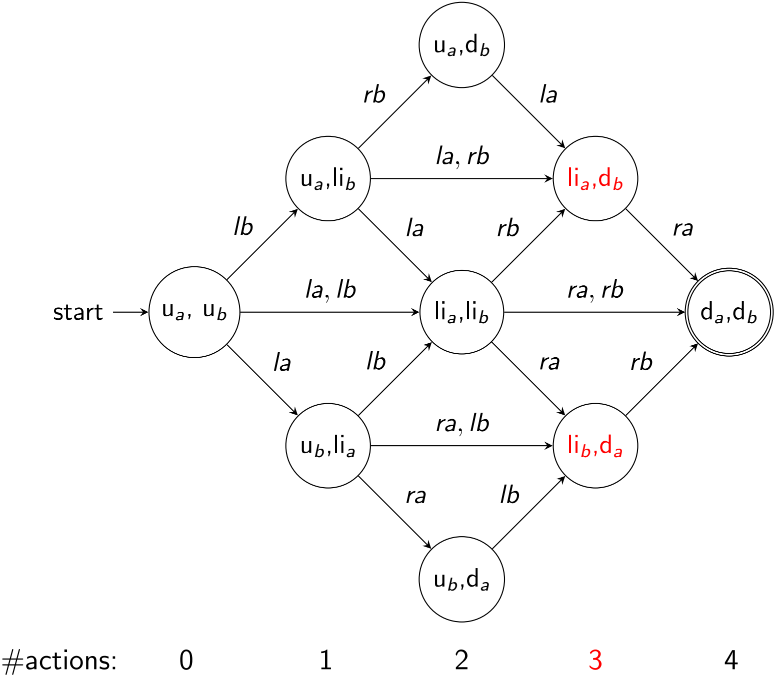

start
u_a, u_b
u_a,li_b
u_b,li_a
u_a,d_b
li_a,li_b
u_b,d_a
li_a,d_b
li_b,d_a
d_a,d_b
lb
la
rb
la, rb
la, lb
ra, lb
ra
rb
ra, rb
#actions: 0 1 2 3 4

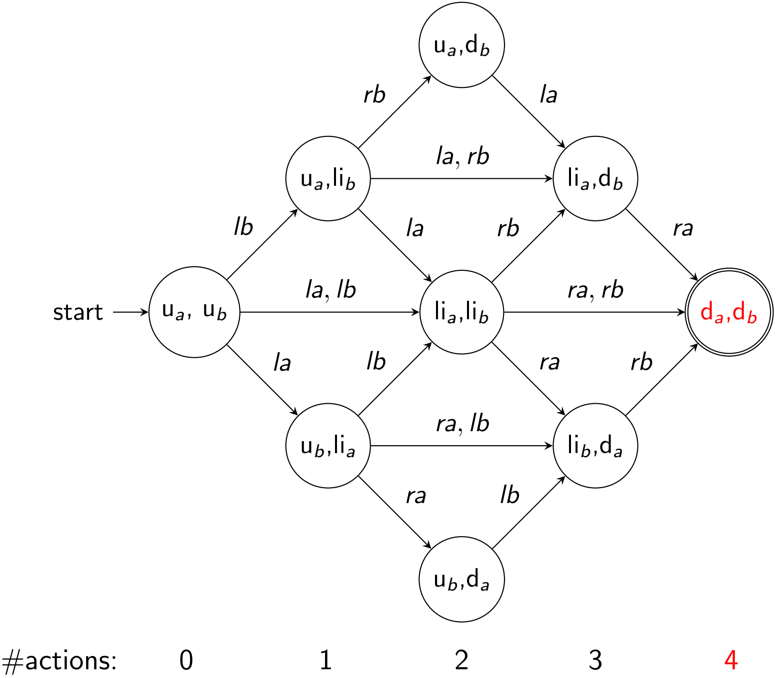


#actions: 0 1 2 3 4

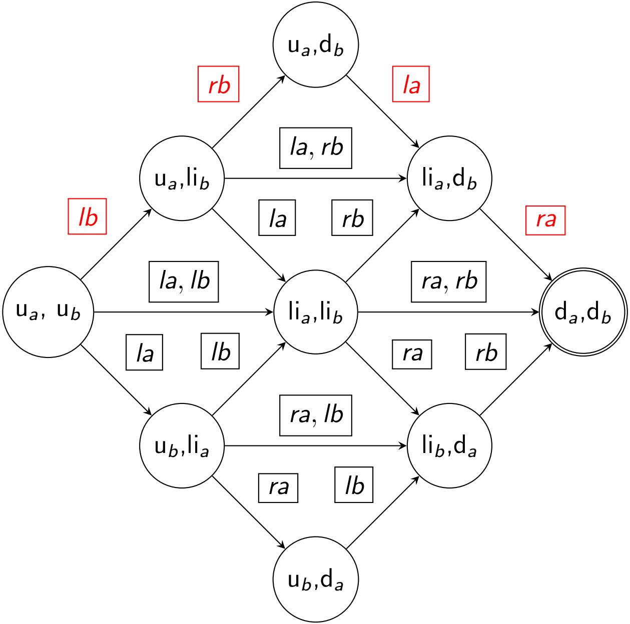


#actions: 0 1 2 3 4


start
u_a, u_b
u_a,li_b
u_b,li_a
u_a,d_b
li_a,li_b
u_b,d_a
li_a,d_b
li_b,d_a
d_a,d_b
lb
la, lb
la
rb
la, rb
la
lb
ra, lb
ra
la
rb
ra, rb
ra
rb
ra
rb
lb
#actions: 0 1 2 3 4

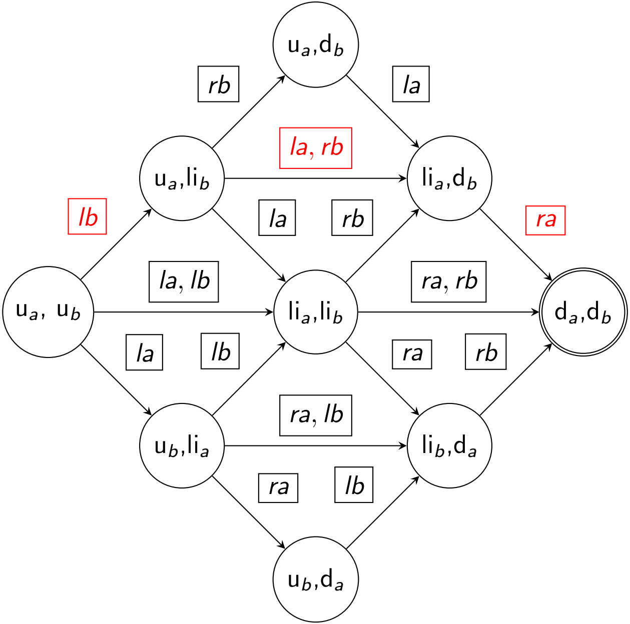

start
u_a, u_b
u_a,li_b
u_b,li_a
u_a,d_b
li_a,li_b
u_b,d_a
li_a,d_b
li_b,d_a
d_a,d_b
lb
la
la, *lb*
rb
la, *rb*
la
rb
lb
ra, *lb*
ra
la
ra, *rb*
ra
rb
lb
ra
rb
#actions: 0 1 2 3 4

Allen relations



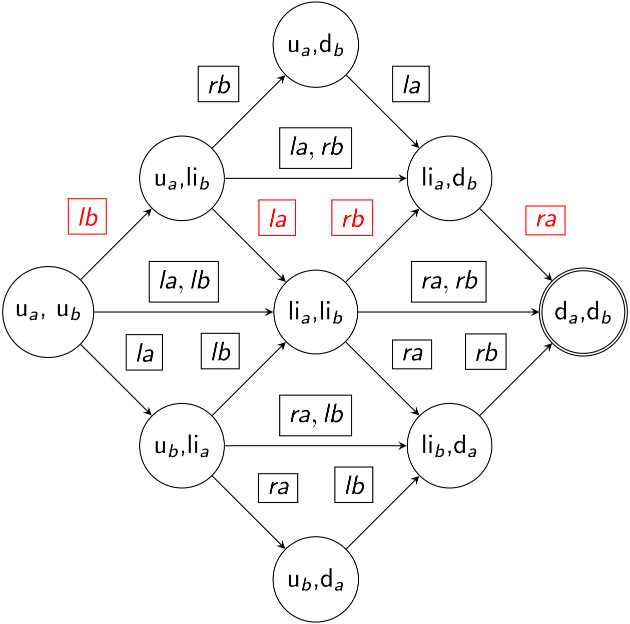
$>$ | *lb* | *rb* | *la* | *ra* |

Allen relations



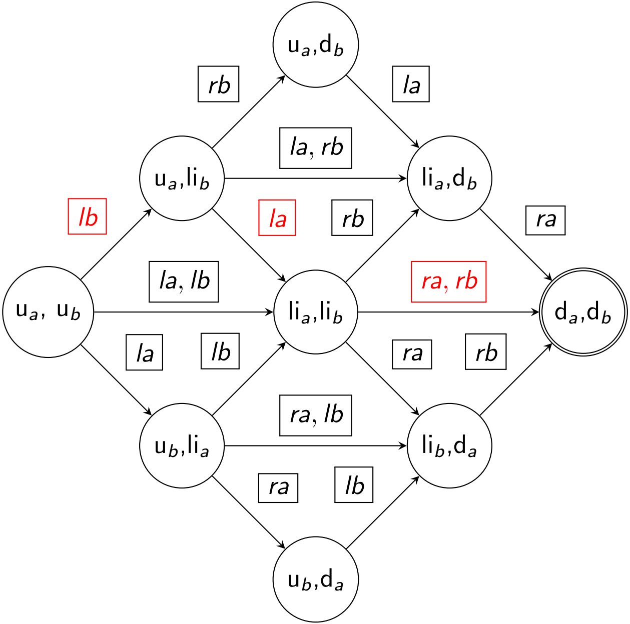
>	*lb*	*rb*	*la*	*ra*
mi	*lb*	*la, rb*	*ra*	

Allen relations



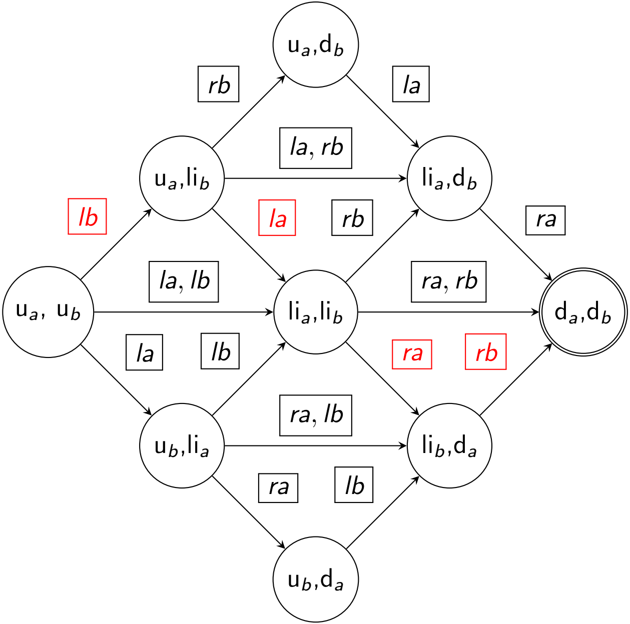
$>$	*lb*	*rb*	*la*	*ra*
mi	*lb*	*la, rb*	*ra*	
oi	*lb*	*la*	*rb*	*ra*

Allen relations



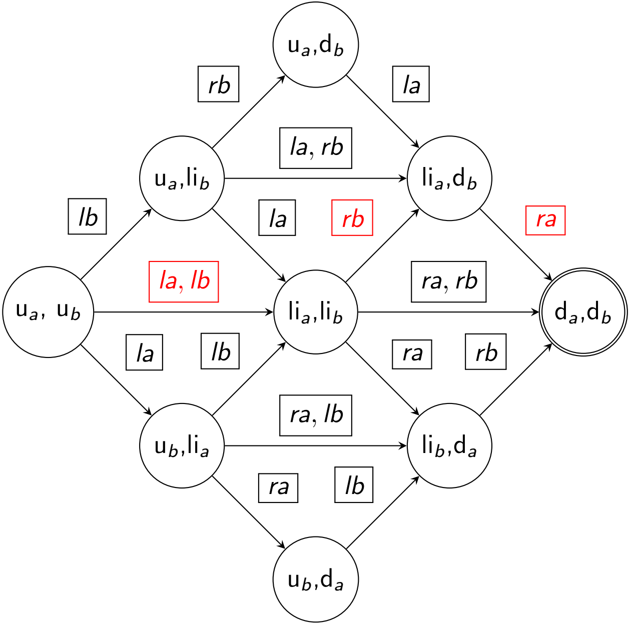
>	*lb*	*rb*	*la*	*ra*
mi	*lb*	*la, rb*	*ra*	
oi	*lb*	*la*	*rb*	*ra*
f	*lb*	*la*	*ra, rb*	

Allen relations



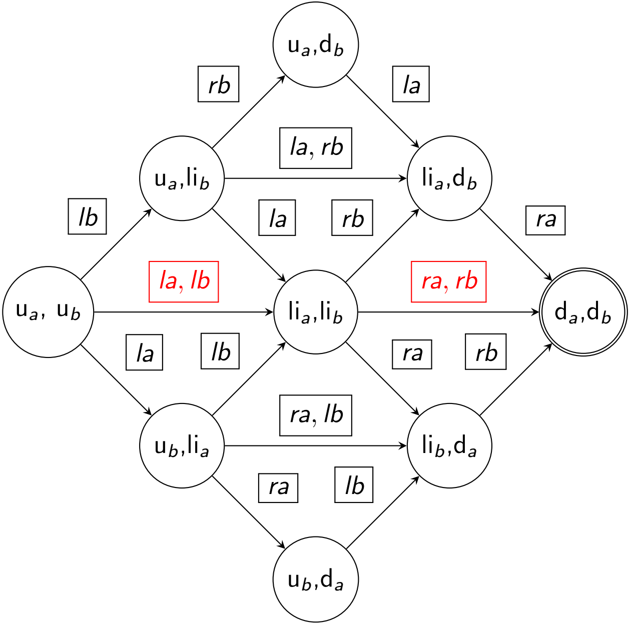
$>$	*lb*	*rb*	*la*	*ra*
mi	*lb*	*la, rb*	*ra*	
oi	*lb*	*la*	*rb*	*ra*
f	*lb*	*la*	*ra, rb*	
d	*lb*	*la*	*ra*	*rb*

Allen relations



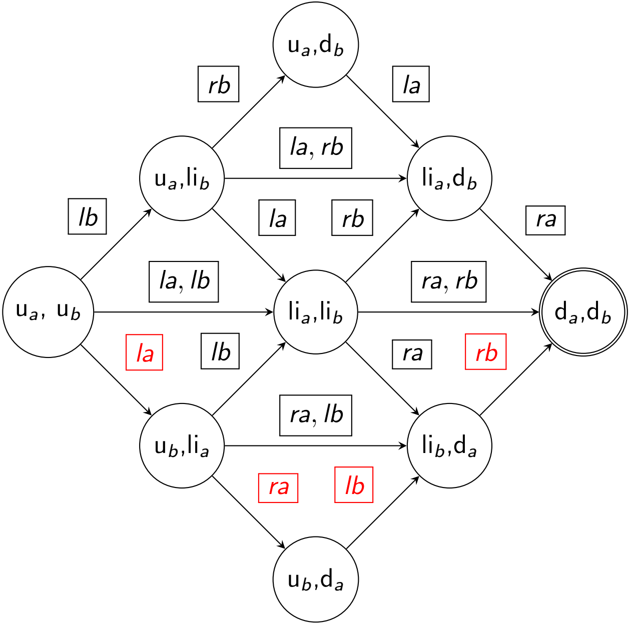
$>$	*lb*	*rb*	*la*	*ra*
mi	*lb*	*la, rb*	*ra*	
oi	*lb*	*la*	*rb*	*ra*
f	*lb*	*la*	*ra, rb*	
d	*lb*	*la*	*ra*	*rb*
si	*la, lb*	*rb*	*ra*	

Allen relations

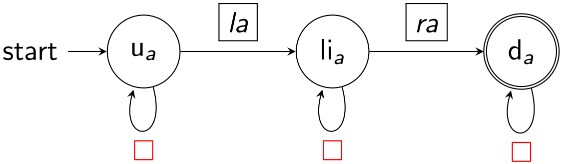


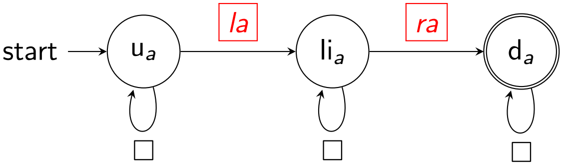
$>$	*lb*	*rb*	*la*	*ra*
mi	*lb*	*la*, *rb*	*ra*	
oi	*lb*	*la*	*rb*	*ra*
f	*lb*	*la*	*ra*, *rb*	
d	*lb*	*la*	*ra*	*rb*
si	*la*, *lb*	*rb*	*ra*	
$=$	*la*, *lb*	*ra*, *rb*		

Allen relations

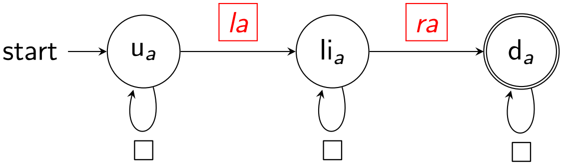


$>$	lb	rb	la	ra
mi	lb	la, rb	ra	
oi	lb	la	rb	ra
f	lb	la	ra, rb	
d	lb	la	ra	rb
si	la, lb	rb	ra	
$=$	la, lb	ra, rb		
$\vdots$				
$<$	la	ra	lb	rb

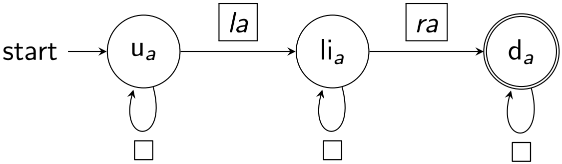

start
u_a
la
li_a
ra
d_a



start
u_a
la
li_a
ra
d_a



$$\boxed{}^{*}\,\boxed{\color{red}{la}}\,\boxed{}^{*}\,\boxed{\color{red}{ra}}\,\boxed{}^{*} = \boxed{\color{red}{la}}\,\boxed{\color{red}{ra}} + \boxed{}\,\boxed{la}\,\boxed{ra} + \cdots$$



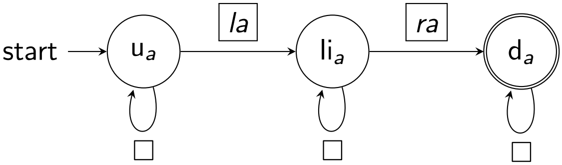
$$\boxed{}^{*}\boxed{la}\boxed{}^{*}\boxed{ra}\boxed{}^{*} = \boxed{la}\boxed{ra} + \boxed{}\boxed{la}\boxed{ra} + \cdots$$

Componentwise union $\sqcup$ between strings of the same length

$$\boxed{la}\boxed{ra} \sqcup \boxed{lb}\boxed{rb} = \boxed{la, lb}\boxed{ra, rb} \qquad =$$

$$\boxed{}\boxed{la}\boxed{ra} \sqcup \boxed{lb}\boxed{rb}\boxed{} = \boxed{lb}\boxed{la, rb}\boxed{ra} \qquad mi$$

$$\vdots$$



$$\boxed{}^{*}\boxed{la}\boxed{}^{*}\boxed{ra}\boxed{}^{*} = \boxed{la}\boxed{ra} + \boxed{}\boxed{la}\boxed{ra} + \cdots$$

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$$\boxed{la}\boxed{ra} \sqcup \boxed{lb}\boxed{rb} = \boxed{la, lb}\boxed{ra, rb} \qquad\qquad =$$

$$\boxed{}\boxed{la}\boxed{ra} \sqcup \boxed{lb}\boxed{rb}\boxed{} = \boxed{lb}\boxed{la, rb}\boxed{ra} \qquad\qquad mi$$

$$\vdots$$

After $\sqcup$, undo $\square$-loop insertion

$$depad(s) \;:= s \text{ with } \square \text{ deleted}$$

Superposing automata (over an alphabet of sets)

Given finite automata $M = \langle \rightarrow, F, q_0 \rangle$

$$M' = \langle \rightarrow', F', q'_0 \rangle$$

let $M\&M'$ be $\langle \rightsquigarrow, F \times F', (q_0, q'_0) \rangle$ with transitions $\rightsquigarrow$ generated alongside a set P of pairs of states including (q_0, q'_0) as follows

$$\text{(cu)} \quad \frac{P(q,q') \qquad q \xrightarrow{\alpha} r \qquad q' \overset{\alpha'}{\rightarrow}{}' r' \qquad \alpha \cap A' \subseteq \alpha' \qquad \alpha' \cap A \subseteq \alpha}{P(r,r') \qquad (q,q') \overset{\alpha \cup \alpha'}{\rightsquigarrow} (r,r')}$$

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$$\text{(d1)} \quad \frac{P(q,q') \quad q \overset{\alpha}{\rightarrow} r \quad \alpha \cap A' = \square}{P(r,q') \quad (q,q') \overset{\alpha}{\rightsquigarrow} (r,q')}$$

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$$\text{(cu)} \quad \frac{P(q,q') \quad q \overset{\alpha}{\rightarrow} r \quad q' \overset{\alpha'}{\rightarrow'} r' \quad \alpha \cap A' \subseteq \alpha' \quad \alpha' \cap A \subseteq \alpha}{P(r,r') \quad (q,q') \overset{\alpha \cup \alpha'}{\rightsquigarrow} (r,r')}$$

$$\text{(d1)} \quad \frac{P(q,q') \quad q \overset{\alpha}{\rightarrow} r \quad \alpha \cap A' = \Box}{P(r,q') \quad (q,q') \overset{\alpha}{\rightsquigarrow} (r,q')}$$

$$\text{(d2)} \quad \frac{P(q,q') \quad q' \overset{\alpha'}{\rightarrow'} r' \quad \alpha' \cap A = \Box}{P(q,r') \quad (q,q') \overset{\alpha'}{\rightsquigarrow} (q,r')}$$

Superposing automata (over an alphabet of sets)

Given finite automata $M = \langle \rightarrow, F, q_0 \rangle$

$$M' = \langle \rightarrow', F', q_0' \rangle$$

let $M\&M'$ be $\langle \rightsquigarrow, F \times F', (q_0, q_0') \rangle$ with transitions $\rightsquigarrow$ generated alongside a set P of pairs of states including (q_0, q_0') as follows

$$(\mathrm{cu}) \quad \frac{P(q,q') \quad q \overset{\alpha}{\rightarrow} r \quad q' \overset{\alpha'}{\rightarrow'} r' \quad \alpha \cap A' \subseteq \alpha' \quad \alpha' \cap A \subseteq \alpha}{P(r,r') \quad (q,q') \overset{\alpha \cup \alpha'}{\rightsquigarrow} (r,r')}$$

$$(\mathrm{d1}) \quad \frac{P(q,q') \quad q \overset{\alpha}{\rightarrow} r \quad \alpha \cap A' = \Box}{P(r,q') \quad (q,q') \overset{\alpha}{\rightsquigarrow} (r,q')}$$

$$(\mathrm{d2}) \quad \frac{P(q,q') \quad q' \overset{\alpha'}{\rightarrow'} r' \quad \alpha' \cap A = \Box}{P(q,r') \quad (q,q') \overset{\alpha'}{\rightsquigarrow} (q,r')}$$

(d1) and (d2) follow from componentwise union (cu) assuming

$$q \overset{\Box}{\rightarrow} q \quad \text{and} \quad q' \overset{\Box}{\rightarrow'} q'.$$

Superposing languages

Given a set X, an *S-string over X* is a string s of non-empty subsets of X.

Let $\mathcal{S}(X)$ be the set of S-strings over X

$$\mathcal{S}(X) \ = \{\mathit{depad}(s) \mid s \in (2^X)^*\}.$$

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Let $\mathcal{S}(X)$ be the set of S-strings over X

$$\mathcal{S}(X) \ = \{depad(s) \mid s \in (2^X)^*\}.$$

Assuming M and M' accept S-strings over A and A' respectively

$$\mathcal{L}(M) \subseteq \mathcal{S}(A)$$
$$\mathcal{L}(M') \subseteq \mathcal{S}(A')$$

the language $\mathcal{L}(M\&M')$ accepted by $M\&M'$ consists of S-strings s over $A \cup A'$ such that

$$depad(\rho_A(s)) \in \mathcal{L}(M) \ \text{ and } \ depad(\rho_{A'}(s)) \in \mathcal{L}(M')$$

where $\rho_X(s)$ is s intersected componentwise with X

$$\rho_X(\alpha_1 \cdots \alpha_n) \ := (\alpha_1 \cap X) \cdots (\alpha_n \cap X).$$