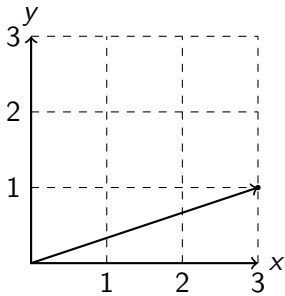


6. Vectors and Matrices

The Plan

- ▶ Vectors
- ▶ Matrices
- ▶ Matrix multiplication
- ▶ Matrices and transformations
- ▶ Inverse of a matrix
- ▶ Gaussian elimination

$$\begin{bmatrix} v_0 \\ v_1 \\ \vdots \\ v_{n-1} \end{bmatrix}$$



$$v = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\|v\| = \sqrt{3^2 + 1} = \sqrt{10}$$

$$A = \begin{bmatrix} 3 & -2 & 0 & 1 \\ 1 & 0 & 2 & -1 \\ -5 & 2 & 1 & 0 \\ -1 & -1 & 3 & -1 \end{bmatrix}$$

$$a_{13} =$$

$$a_{11} =$$

$$a_{32} =$$

$$a_{03} =$$

$$A = \begin{bmatrix} 3 & -2 & 0 & 1 \\ 1 & 0 & 2 & -1 \\ -5 & 2 & 1 & 0 \\ -1 & -1 & 3 & -1 \end{bmatrix}$$

$$a_{13} =$$

$$a_{11} =$$

$$a_{32} =$$

$$a_{03} =$$

$$a_{13} = -1$$

$$a_{11} = 0$$

$$a_{32} = 3$$

$$a_{03} = 1$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix} \quad \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a \times e + b \times g & a \times f + b \times h \\ c \times e + d \times g & c \times f + d \times h \end{bmatrix}$$

$$\begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ 3 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & 2 \\ -1 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \\ -2 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 - 1 - 2 & 2 + 0 - 1 & 3 - 1 + 1 \\ 2 + 1 - 4 & 4 + 0 - 2 & 6 + 1 + 2 \\ -1 + 1 - 2 & -2 + 0 - 1 & -3 + 1 + 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 3 \\ -1 & 2 & 9 \\ -2 & -3 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -1 \\ -6 & 1 \end{bmatrix} \times \begin{bmatrix} -3 & 0 \\ 1 & -2 \end{bmatrix} =$$

$$\begin{bmatrix} 2 & 1 & -1 \\ 3 & 2 & 0 \\ -7 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & -2 \\ 1 & 1 & -1 \\ -2 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} 4 & -1 \\ -6 & 1 \end{bmatrix} \times \begin{bmatrix} -3 & 0 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} -13 & 2 \\ 19 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & -1 \\ 3 & 2 & 0 \\ -7 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & -2 \\ 1 & 1 & -1 \\ -2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 1 & -6 \\ 5 & 2 & -8 \\ -9 & 0 & 15 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$\begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}^{-1} = \frac{1}{2} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \times \frac{1}{2} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & 1 & 2 \\ 1 & 2 & 3 & 1 \\ 1 & 1 & -1 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & 1 & 2 \\ 0 & \frac{5}{2} & \frac{5}{2} & 0 \\ 0 & 0 & -3 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$