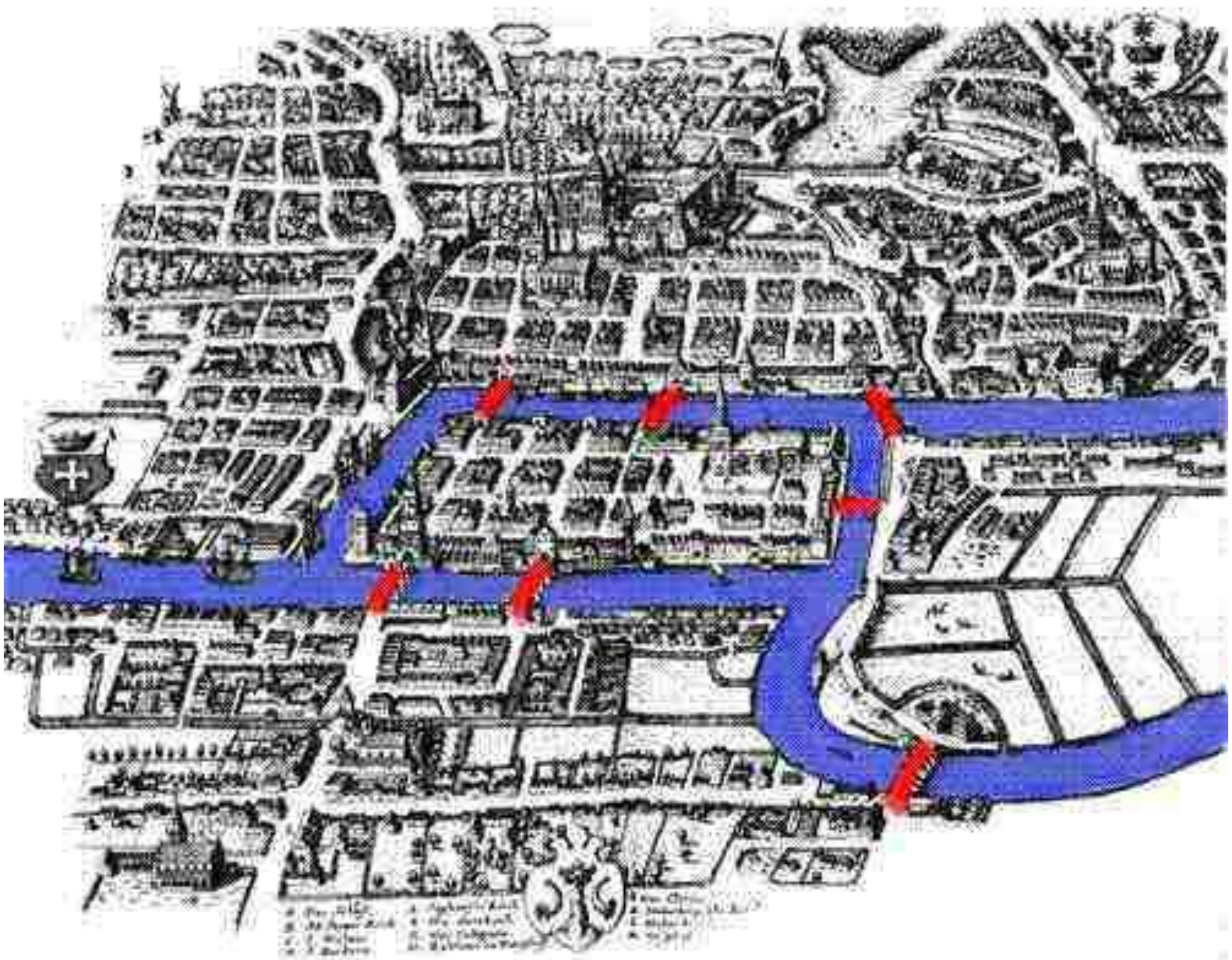
8. Graphs

Our plan

- Euler and the Königsberg Bridges
- Graphs
- Paths and Cycles
- Subgraphs
- Isomorphism
- Euler circuits and when they exist

Königsberg Bridges



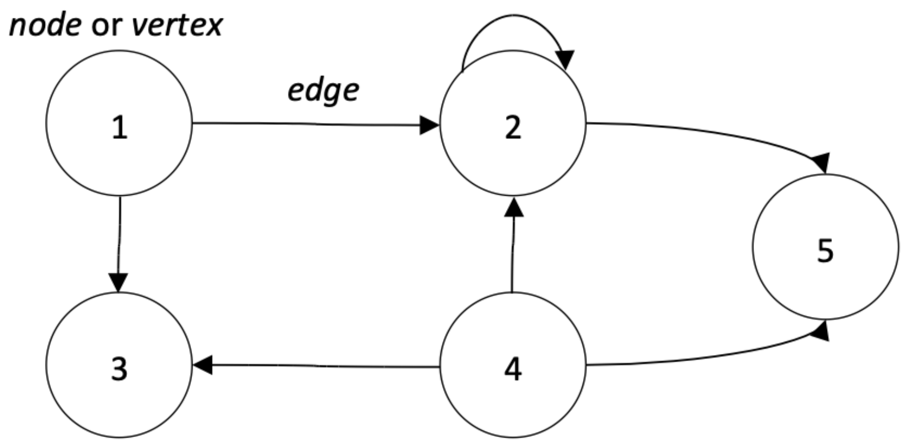
http://www-history.mcs.st-andrews.ac.uk/Extras/Konigsberg.html

Königsberg Bridges

In Königsberg, Germany, a river ran through the city which also consisted of two islands and was connected by seven bridges. People often wondered (so the story goes) whether it was possible to walk a circuit round Königsberg, taking in all seven bridges which were crossed exactly once and ending up back in the same place – except that no-one seemed able to find a route that achieved this. This led Leonard Euler to discover a new branch of mathematics – graph theory – a topic that is not often taught at school level, but is highly important in Computing.

http://www-history.mcs.st-andrews.ac.uk/Extras/Konigsberg.html

Consider a road map, the tube map, an electrical circuit, a chemical structure – all of these may be represented diagrammatically by means of points and lines.



The points 1,2,3,4,5 in the diagram above are called *vertices* (sometimes these may be labelled, P,Q,R,…), the lines (or arrows) connecting them are called *edges* and the whole diagram is called a *graph*.

Exercise: what is the best way to represent this?

Nodes = {1, 2, 3, 4, 5}

Edges == Nodes X Nodes

Map = ??

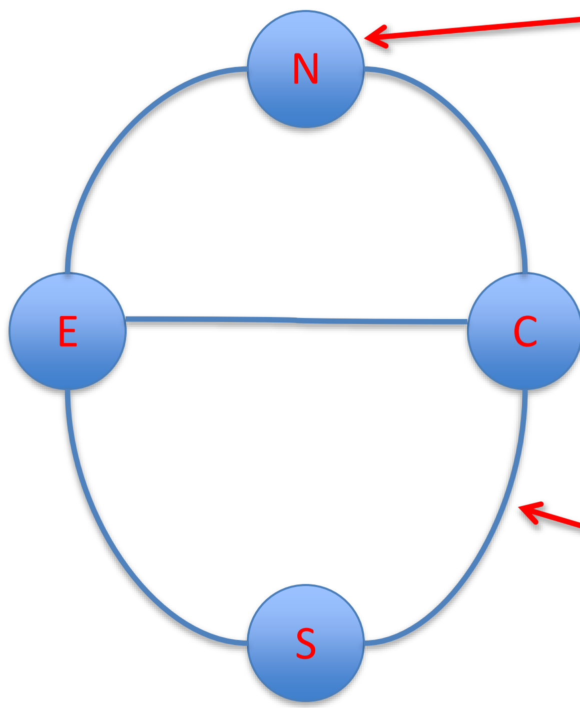
Exercise: what is the best way to represent this?

Nodes = {1, 2, 3, 4, 5}

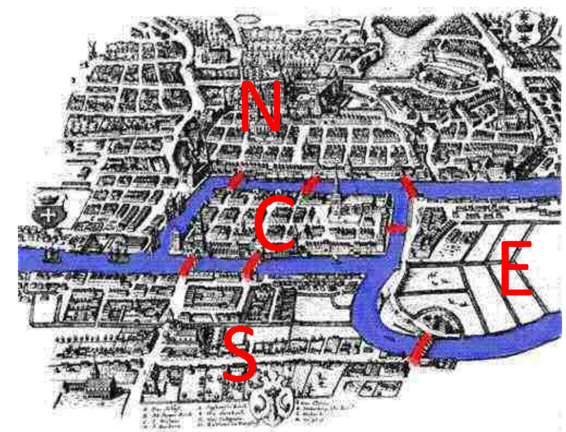
Edges == Nodes X Nodes

Map = { (1,2), (1,3), (2,2), (4,3), (4,3),
(2,5), (4,5), }

Königsberg Bridges: a Graph



Vertex or Node

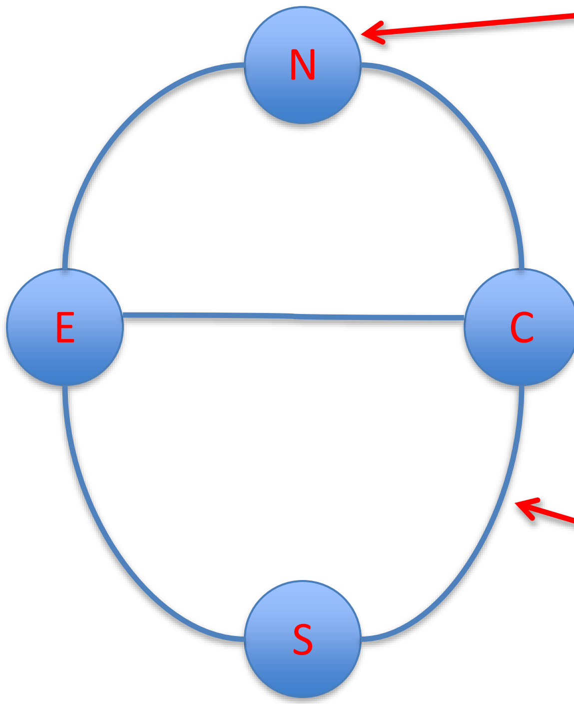


Edge

V(G) = {North , Centre , East ,South } = { N, C, S, E}

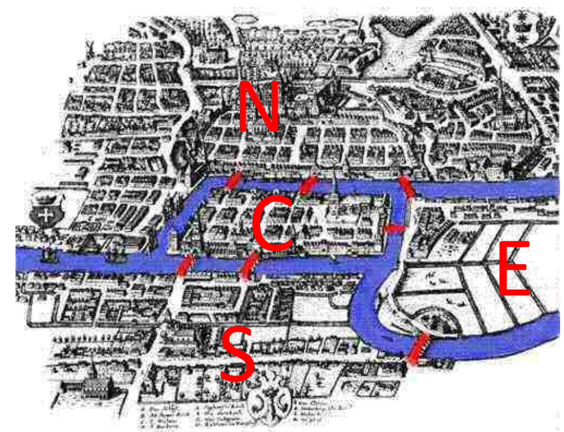
E(G) = ?

Königsberg Bridges: a Graph



Vertex or Node

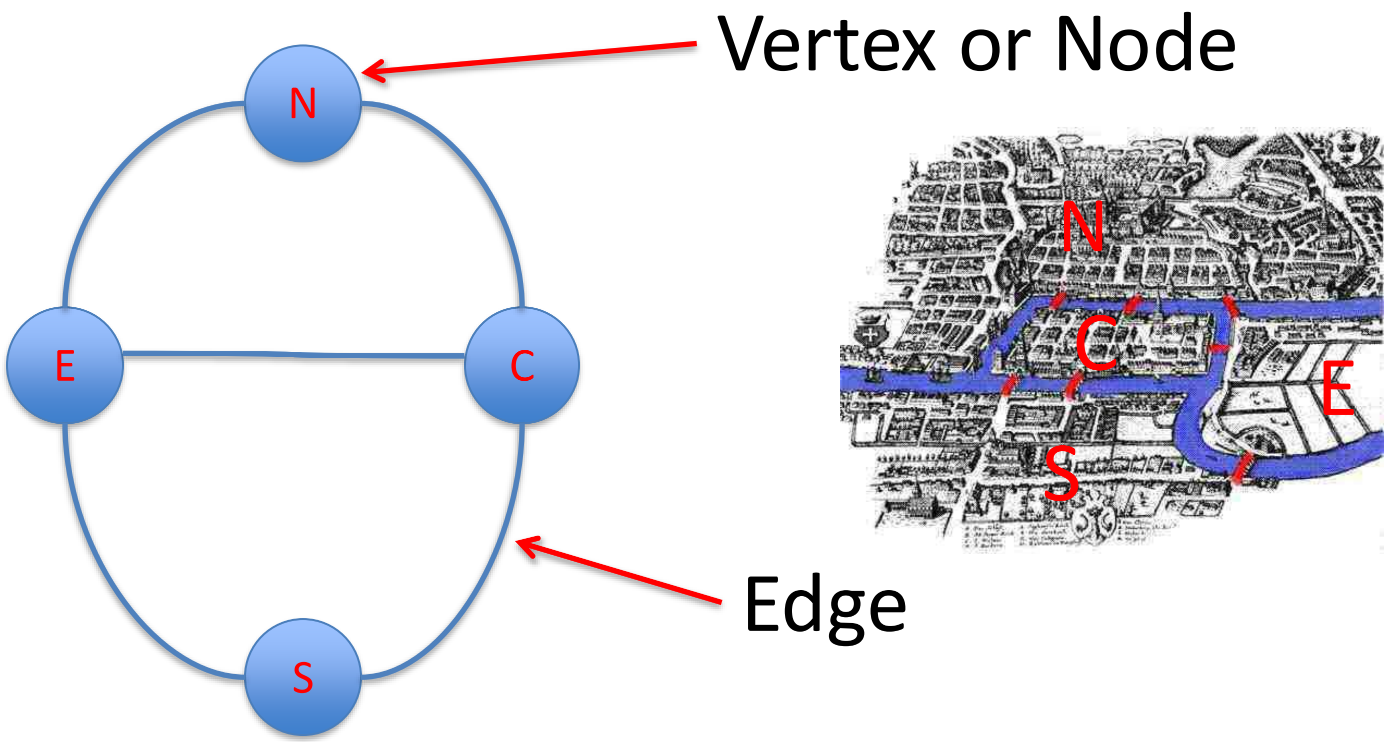
Edge



V(G) = {North , Centre , East ,South } = { N, C, S, E}

E(G) = { (N,C), (C,N), (N,E), (E,N), (E,S), (S,E), (S,C), (C,S) }

Königsberg Bridges: a Graph

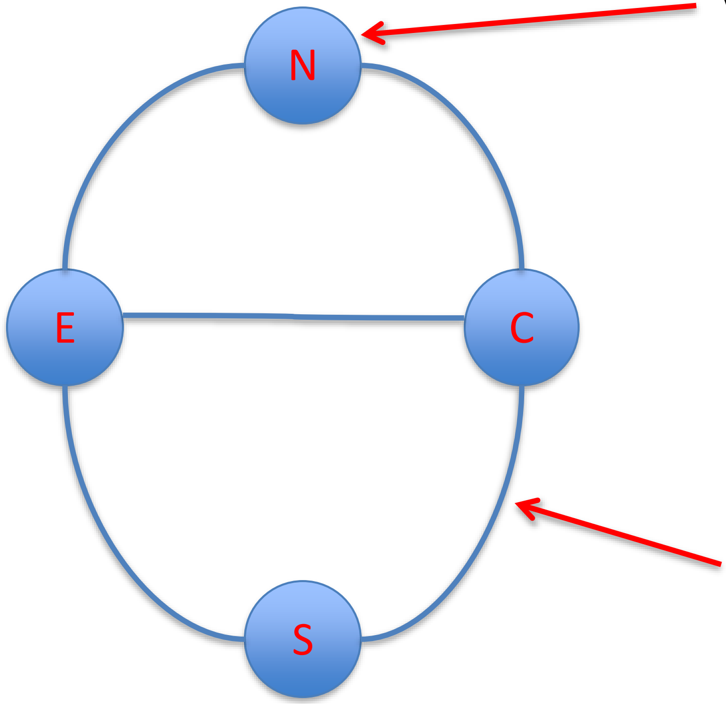


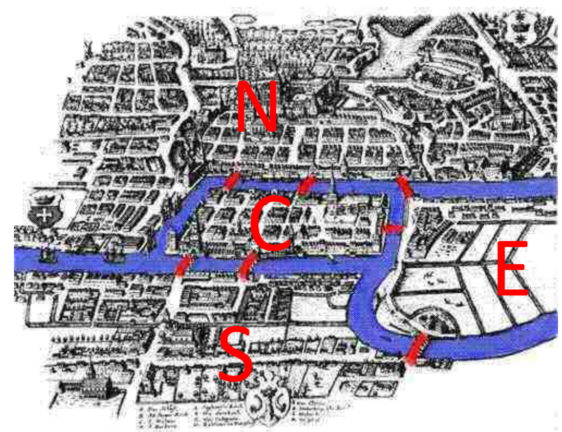
A degree of a node is the number of edges which END at the node

What are the degrees of N, C, S and E?

Königsberg Bridges: a Graph

Vertex or Node





Edge

Degree (N) = 2
Degree (C) = 3
Degree (S) = 2
Degree (E) = 3

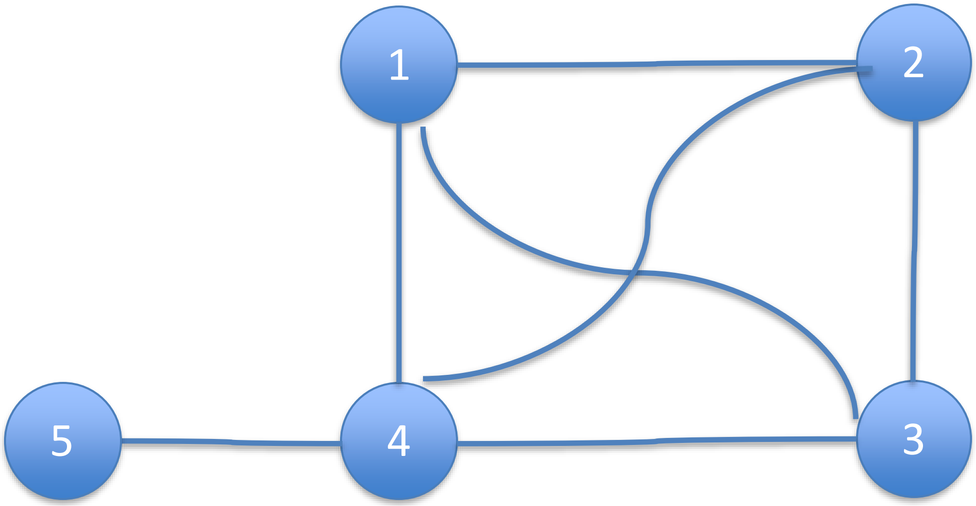
Simple Graph Representations

- A simple graph has a set of edges
- A graph may be directed or undirected
- Directed if edges are ordered pairs
  - { (1,2), (3,4) }
- Undirected could be represented as sets
  - {   {1,2}, {2,3} … }
  - Or a symmetric relation of course!! (as in the previous example)

Exercise

- Draw the following undirected graph, G
- $V(G) = \{1, 2, 3, 4, 5\}$
- $E(G) = \{ \{1,2\}, \{2,3\}, \{3,4\}, \{1,4\}, \{3,4\},$
  $\{4,1\}, \{4,5\} \}$

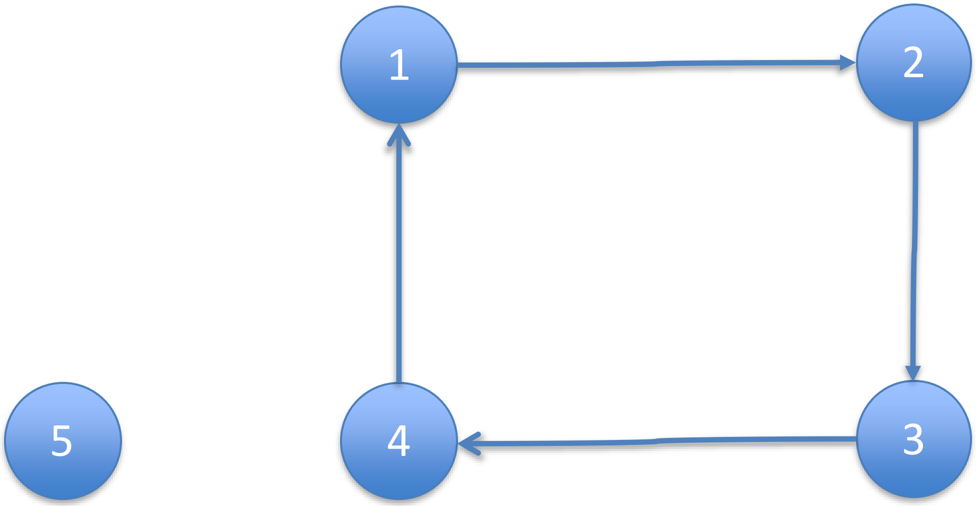
Exercise Solution



Exercise

- Draw the following directed graph G
- V(G) = {1,2,3,4,5}
- E(G) = { (1,2), (2,3), (3,4), (4,1) }

Exercise Solution



More definitions about Graphs

- Two vertices connected by an edge are said to be **adjacent**
- **Degree**: number of edges ending at a vertex
- **Path**: a list of vertices linked by edges
- **Simple path**: a path with no duplicate vertices
- **Cycle**: a path with the same start/end vertex
- Graphs may be **connected** or **disconnected**

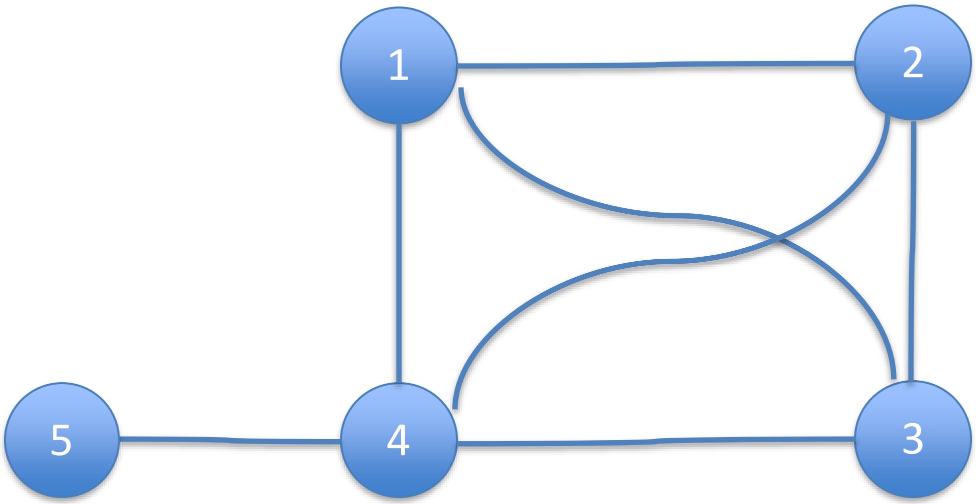
Module Evaluation Survey

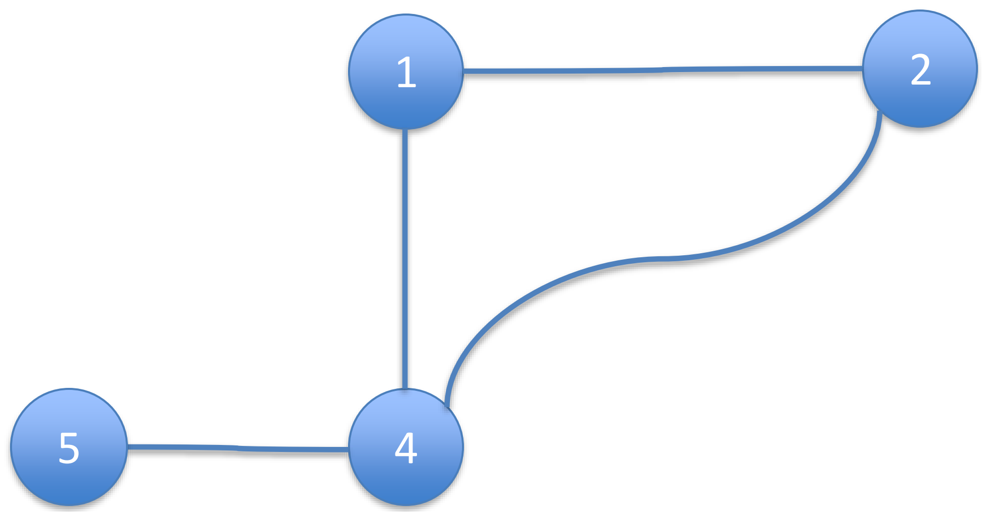
- A formal way to gather your feedback to improve the student experience
- More information on the Student Hub
- Access all open surveys on the Student Survey Portal or via the QR code (you will also be emailed a link)
- https://city.surveys.evasysplus.co.uk

Exam

- January 7^{th} (Tuesday)
- I'll do a video of last years exam
- It lasts 1 hour
- It covers the whole course content
- Seven questions, choose five.
- Or if you do more, we will mark them and give the result based on the top 5

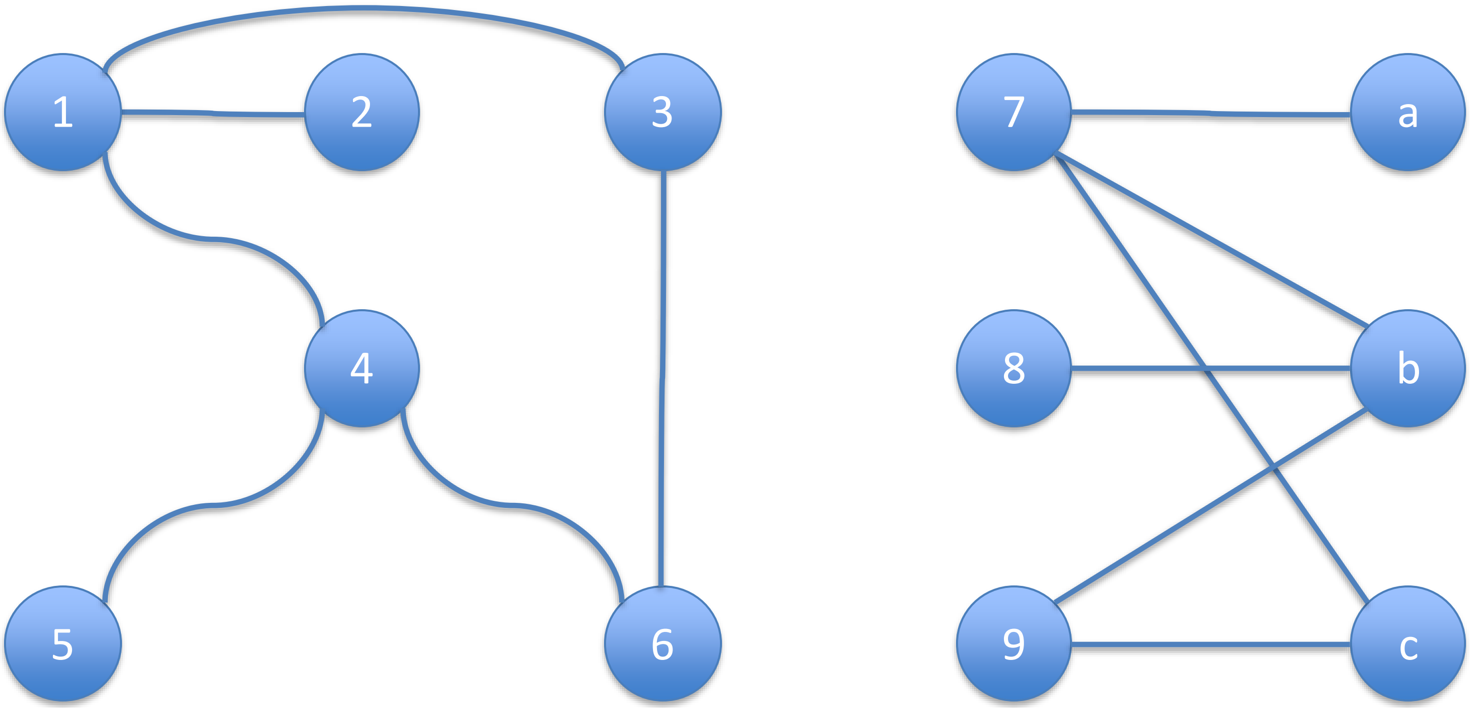
Subgraphs





Exercise

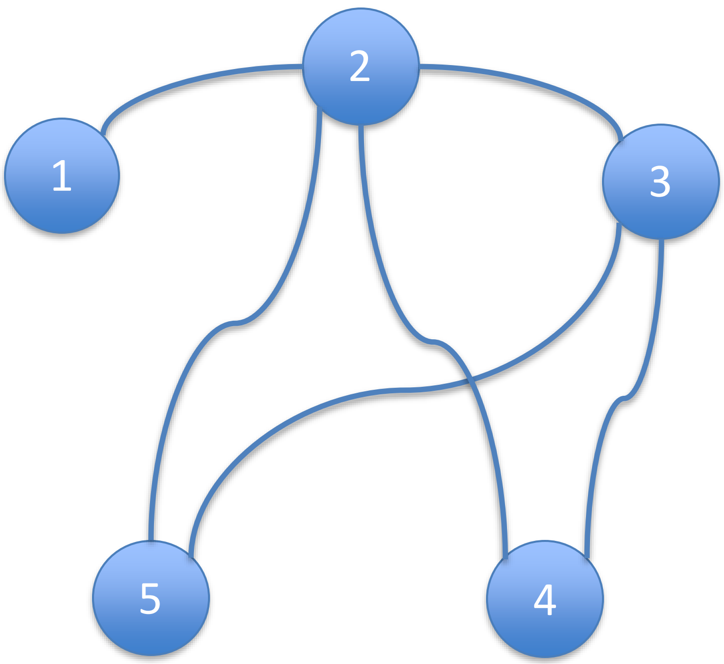
- Are the following isomorphic? (if and only if we can find a bijective function from nodes to nodes which keeps all the relations in tact)

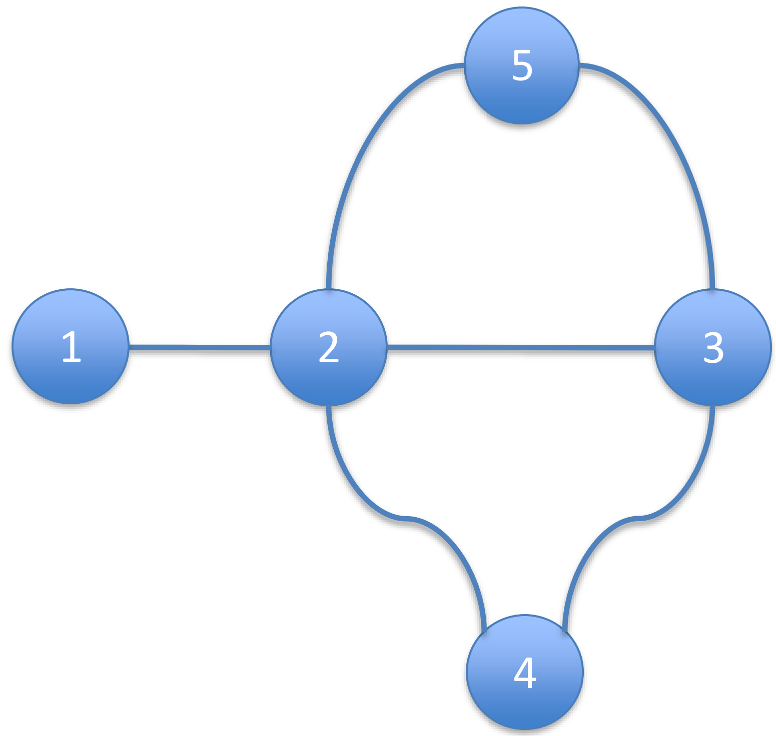


- F = { (1,7), (5,8), (6,9), (2,a), (4,b), (3,c) }

Exercise

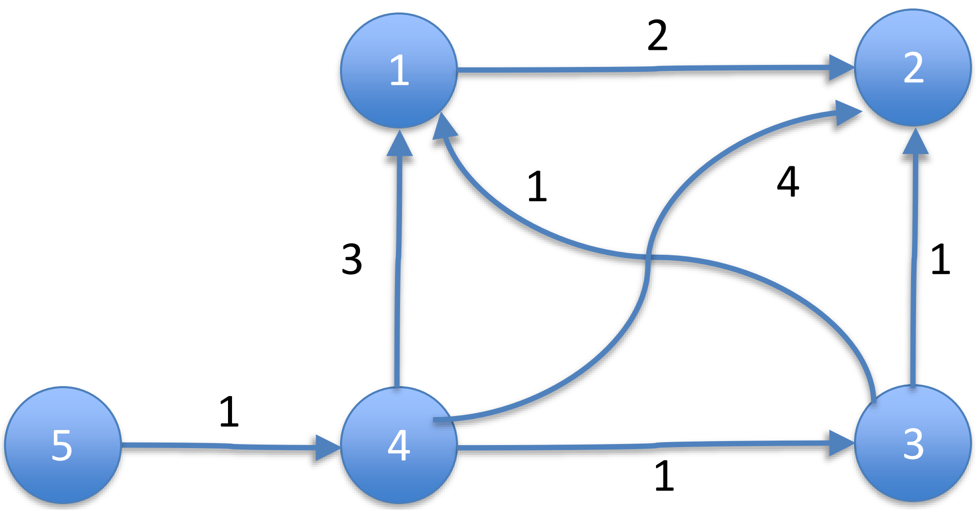
- What edge to you need to add to the second graph to make it isomorphic to the first?





- (2,3)

Weighted directed graph



- What is the shortest path in this graph?

Euler circuit

- A Euler circuit is a cycle with each vertex visited and each edge used once.
- **Theorem:** An undirected graph has a Euler circuit if and only if every vertex has even degree
- Hence, there is no solution to the Königsberg Bridges problem
  - Just cannot be done!