Mopping Up

(start of week 7)

The plan

- Remembering Your Matrices
- Power Sets
- Countability

Summary of transformation matrices that you should learn or be able to deduce quickly

Reflection in the x-axis: $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

Reflection in the y-axis: $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

Reflection in the $y = x$: $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Reflection in the $y = -x$: $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

Enlargement by scale factor k, centre at $(0,0)$: $\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$

Rotation 90° anticlockwise about $(0,0)$: $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

Rotation 180° $(0,0)$: $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

Rotation 270° anticlockwise about $(0,0)$: $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

Rotation θ° anticlockwise about $(0,0)$: $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

Shear in the x-direction, shear factor k: $\begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$

Shear in the y-direction, shear factor k: $\begin{pmatrix} 1 & 0 \\ k & 1 \end{pmatrix}$

Also enlargement of m along x-axis and n along y-axis

m 0
0 n

- Power Set of A is the set of all subsets of A
- **P** {a, b} = { {}, {a}, {b}, {a,b} }
- **P** {a, b, c } = { {}, {a}, {b}, {c}, {a,b},
  {a,c}, {b,c}, {a,b,c } }

Countability

- An infinite set A is countable is there is a 1 to 1 correspondence between A and the Natural Numbers (looking for an injective and surjective function f from **A** to **N** or **N** to **A**

  (can we make a countable list?)
- Examples

  A = Even natural numbers. F : n -> n/2

  A = { n : N | n MOD 3 = 1}. F : n -> (n -1) / 3

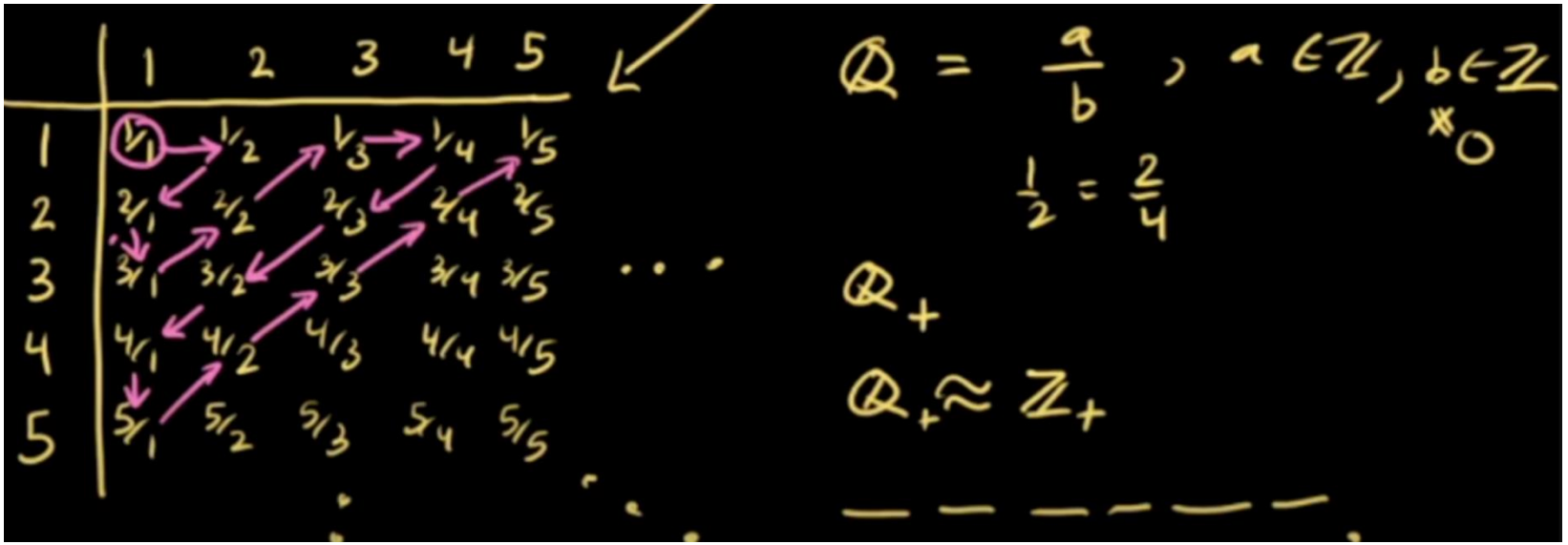
  A = Integers

$$f: \mathbb{N} \to \mathbb{Z}$$

$$n \mapsto f(n) = \begin{cases} \frac{n+1}{2} & \text{if } n \text{ is odd} \\ -\frac{n}{2} & \text{if } n \text{ is even} \end{cases}$$

Countability

– The Rational Numbers are countable



– The Real Numbers are not countable