7. Probability

The plan

- What is probability?
- Sample space and events
- Independent events
- Mutually exclusive events
- Conditional probability
- Bayes's Theorem
- Worked Exampls

Probability

- Likelihood of an event occurring on a scale of **0** (won't occur) to **1** (will occur).
- Eg coin toss heads has probability 0.5
- The range of possibilities is drawn from the **sample space**.
- Sample space for a coin toss is {H,T}
- Two events that cannot occur simultaneously are called **mutually exclusive events**.
- Events that don't influence each other are called **independent events**.

Sample Spaces - exercises

- What is the sample space for two coin tosses is {(H,H) …. }. ?
- What is the sample space for throwing a dice?
- What is the sample space for throwing two dice?

Exercise

1. What is the probability that an integer drawn from 1,…,10 is a square number?
2. If a coin is tossed 3 times what is the probability that the result is **HTH** ?
3. If a pair of dice is cast, what is the probability that the result does not sum to 11 ?

Exercise

1. $P(\text{square from } [1..10]) = |\{1,4,9\}| \,/\, |\{1,\ldots,10\}|$
   $= 3/10 = 0.3$

2. $P(HTH) = ½ * ½ * ½ = 1/8 = 0.125$

3. $P(\text{not } 11) = 1 – 2 * (1/6) * (1/6) = 17/18$

Exercise

- If I have a bag with 4 red balls and 4 blue balls in it, and I draw two balls, what is the probability that I draw one ball of each colour:
  - with replacement
  - without replacement

Exercise

- With replacement:
  - $2 * (1/2) * (1/2) = 1/2$
- Without replacement:
  - $(1/2 * 4/7) + (1/2 * 4/7) = 8/14 = 4/7$

Exercise

- If I have a drawer with 5 red socks, 4 blue socks and 1 green sock, and I draw two socks without replacement. What is the probability of
  - Picking two red socks
  - Picking two green socks
  - Picking a matching pair of socks
  - Picking an Odd pair of socks
- Will I increase my probability of picking a matching pair if I take out 3 socks?
- If I take out 4 socks, am I guaranteed a matching pair?

Assessed MCQ 3

- The third assessed MCQ is this week
- It’s about vectors & matrices and probability, covering chapters 6 and 7
- As before, 40mins, open book in your own time.
- +4 for correct, -1 for incorrect, 0 for leave blank
- The quiz opens at 5pm Thursday 5th December and closes 8pm Sunday 8th Dec (so start by 7:20pm).

Module Evaluation Survey

- A formal way to gather your feedback to improve the student experience
- More information on the Student Hub
- Access all open surveys on the Student Survey Portal or via the QR code (you will also be emailed a link)
    - https://city.surveys.evasysplus.co.uk

Exam

- January 7th (Tuesday)
- I'll do a video of last years exam
- It lasts 1 hour
- It covers the whole course content
- Seven questions, choose five.
- Or if you do more, we will mark them and give the result based on the top 5

Probability distribution

- A probability distribution is a table summarising all possible (hence sums to 1)

	T	¬T
A	0.04	0.01
¬A	0.06	0.89

Conditional probability

- Conditional probability is given by:

$$P(A|B) = \frac{P(A \land B)}{P(B)}$$

- This read as Probabilty that “A occurs given B has occurred”

Exercise

- Consider the following probability distribution (L for left, R for right, H for handed, F for footed, assuming non-ambidexterousness):

	LH	RH
LF	0.15	0.10
RF	0.05	0.70

- Please calculate the following probabilties
  - P(LH)
  - P(RF)
  - $P(RH \wedge LF)$
  - $P(LF \mid LH)$

Exercise

- $P(LH) = 0.15 + 0.05 = 0.2$
- $P(RF) = 0.05 + 0.70 = 0.75$
- $P(RH \wedge LF) = 0.10$
- $P(LF \mid LH) = 0.15/0.20 = 0.75$

Bayes's Theorem

$$\boxed{P(A|B) = \frac{P(B|A)P(A)}{P(B)}}$$

Proof [edit]

For events [edit]

Bayes' theorem may be derived from the definition of conditional probability:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \text{ if } P(B) \neq 0,$$

where $P(A \cap B)$ is the probability of both A and B being true. Similarly,

$$P(B|A) = \frac{P(A \cap B)}{P(A)}, \text{ if } P(A) \neq 0.$$

Solving for $P(A \cap B)$ and substituting into the above expression for $P(A|B)$ yields Bayes' theorem:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}, \text{ if } P(B) \neq 0.$$

Bayes's Theorem