Economic Design of Variables-Based Acceptance Sampling Plan with Rectifying Inspection Based on Advanced Capability Index

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Abstract

Acceptance sampling plans are used in the industry for the inspection of the quality of incoming and outgoing lots, primarily due to their economic feasibility over 100% inspection. It is an established fact that variables-based sampling plans require smaller sample size for same process capability requirements. However, most of the available sampling plans are attributes-based and do not explicitly consider the economic aspects. In this paper, we propose an economic model of a variables-based single sampling plan based on the advanced process capability index, C_{pkm} which combines process yield and quality loss. The optimisation problem is modelled as a constrained non-linear multivariate problem using the cumulative distribution function of $\hat{C}_{pkm}$. The objective function is the total quality cost which considers the inspection, internal failure and external failure cost, and constraints are defined using producer's and consumer's risks. Comparison with the current state of the art economic sampling plans based on C_{pk} show that our quality cost is lower by approximately 8%, while also providing a smaller confidence interval for the process mean due to inherent superiority of C_{pkm} over C_{pk}.

1 Introduction

Acceptance sampling has been a cost and time-effective inspection tool adopted by the vendor and the buyer for product quality or reliability assurance (Montgomery 2009; Schilling and Neubauer 2009). In an acceptance sampling plan, a sample of items is randomly selected from a lot and based on the inspection results, the disposition of the lot is determined. The acceptance/rejection decision can be based on attributes (e.g., percentage of defective items) or variables (e.g., using various process capability indices, PCIs).

With the ever increasing demands of higher quality products by the consumers, it has become absolutely necessary for the industries to produce products of very high quality. Thus, there is a growing interest in the adoption of variables-based sampling plans by the industry as they provide better statistical control and require less sampling than attributes-based sampling plans. However, the precise measurements required by a variables-based plan would probably cost more than the simple classification of items required by an attributes-based plan, but the reduction in sample size, may more than offset this exact expense. Such saving may be especially marked if inspection is destructive and the item is expensive (see Schilling, 1982; Duncan, 1986; Montgomery, 2001).

Constructing economic models of acceptance sampling plans has been a topic of interest to researchers for years (Wetherrill and Chiu 1975). However, designing economically optimal acceptance sampling plans has not been widely addressed even though sampling remains a commonly used technique in certain quality engineering systems (Ferrell and Chhoker, 2002). Hsu and Hsu (2012) developed an attributes-based economic model of acceptance sampling plans in a two-stage supply chain. Yen et al. (2015) proposed a variables-based acceptance sampling plan with rectifying inspection based on C_{pk} index, which is based on process yield. However, high-tech industrial processes have more strict requirements for product quality and reliability than other industries. Therefore, it would be more suitable for high-tech industry to use C_{pkm} rather than C_p, C_{pk} and C_{pm} (Wu and Wang 2017).

The remainder of this paper is organized as follows. Section 1.1 lists all acronyms and notation used in this paper. Section 1.2 states the problem definition. Section 2 provides a brief review of various PCIs, the estimation of the estimated C_{pkm}, and analysis associated with the producer's and consumer's risks. We design a variable economic sampling plan in Section 3. The performance comparison and analysis is given in Section 4. Section 5 presents the sensitivity analysis of optimum sample size, critical acceptance value and total quality cost. We end with concluding remarks and the scope for future research in section 6.

1.1 Nomenclature, Acronyms, and Notation

- USL and LSL: the upper and lower specification limits, respectively. They are the targets set for the process/product by the customer.
- d= (USL-LSL)/2 is the half specification width
- T: the target value for the quality characteristic
- μ_X, σ_X, the mean and standard deviation of the random variable X, which represents the quality characteristic. Let X follows a normal distribution.
- process capability index, PCIs
  - Basic PCIs, $C_p = \frac{\text{USL}-\text{LSL}}{6\sigma}$, where $\sigma = E(X-\mu_X)^2$;
  - Yield-based PCIs, C_{pk}=min(C_{pu}, C_{pl}), where $C_{pu} = \frac{\text{USL}-\mu}{3\sigma}$ and $C_{pl} = \frac{\mu-\text{LSL}}{3\sigma}$;
  - Loss-based PCIs, $C_{pm} = \frac{\text{USL}-\text{LSL}}{6\tau} = \frac{C_p}{\sqrt{(1+\xi^2)}}$, where $\tau^2 = E(X-T)^2$, $\xi^2 = [(\mu - T)/\sigma]^2$ and $T = (\text{USL}+\text{LSL})/2$ is the target;
  - Yield-loss based PCIs, $C_{pkm} = \frac{C_{pk}}{\sqrt{(1+\xi^2)}}$, where ξ is defined in C_{pm}
- C_{pkm}^{AQL}: Associated C_{pkm} value at Acceptable Quality Limit (AQL),
  i.e, P(accept the lot | $C_{pkm} = C_{pkm}^{\text{AQL}}) \geq 1-\alpha$
- C_{pkm}^{RQL} or C_{pkm}^{LTPD}: Associated C_{pkm} value at Rejected Quality Limit (RQL) or Lot Tolerance Percent Defective (LTPD),
  i.e, P(accept the lot | $C_{pkm}) = C_{pkm}^{\text{RQL}} \leq \beta$
- N: Size of the lot
- n: Size of the sample randomly taken from the lot of size N

- p, the probability of defective item in the lot
- TQC: Total quality cost per lot (with rectifying inspection)
- Y: $\hat{C}_{pkm}$ value calculated from the sample
- c^*: threshold of $\hat{C}_{pkm}$ value, i.e., accept the lot if $Y \geq c^*$
- c_{ip}: Cost of inspection per unit
- c_{if}: Cost of internal failure per unit
- c_{ef}: Cost of external failure per unit
- ATI: Average total inspection per lot
- D_d: Average no. of defects detected per lot
- D_n: Average no. of defects that go undetected per lot
- α: producer's risk, the probability of rejecting a good lot, the probability of Type I error under the testing H_0: the lot is good, H_1: the lot is poor
- β: consumer's risk, the probability of accepting a poor lot, the probability of Type II error under the testing H_0: the lot is good, H_1: the lot is poor
- $\hat{C}_{pkm}$: the estimated C_{pkm}, which is a random variable
- N(0,1), standard normal distribution.

1.2 Problem Definition

For the sake of completeness, we start with a general structure of a sampling plan and develop the proposed economic model of a single variables-based acceptance sampling plan based on C_{pkm}.

A general single sampling plan broadly consists of two steps:

(1) Finding the criterion for acceptance or rejection of lots.

   (i) Determining the cost function (or objective function) of the sampling plan to be minimized, e.g. sample size, average total inspection, total quality cost, etc.

   (ii) Determining the sample statistic to be used for making acceptance and rejection decisions, e.g. number of poor items in the sample or PCIs value for the sample.

   (iii) Defining constraint equations for pre-specified producer's risk α and consumer's risk β using the sample statistic.

   (iv) Minimising the cost function as per the constraints to obtain the optimum sample size and critical value of sample statistic.

(2) Comparing the sample statistic calculated by picking a random sample of optimum size from a lot with the critical value of the statistic to make the acceptance/rejection decision for that lot.

We now consider one attributes-based and one variables-based model which have different sample statistic. The cost function may or may not be the same.

(Model 1) An attributes-based plan (traditional approach) with the critical acceptance number c^* as the decision making criterion

Minimize Cost

such that

$$P(\hat{T} \leq c^* | p = p^{\text{AQL}}) \geq 1 - \alpha \tag{1}$$

$$P(\hat{T} \leq c^* | p = p^{\text{RQL}}) \leq \beta, \tag{2}$$

Here, $\hat{T}$ is the number of defective items in the sample of size n, and p denotes the probability of defective items in the lot.

(Model 2) A variables-based plan (contemporary approach) with the critical acceptance value c^* (in terms of a PCIs, say C_{pkm}) as the decision making criterion.

Minimize Cost,

such that

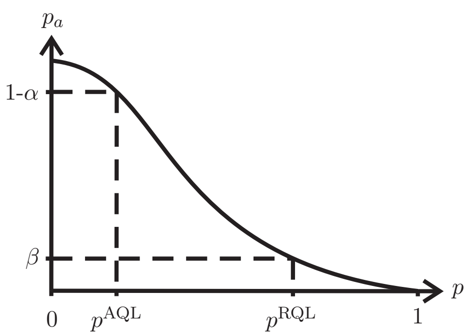
$$P(\hat{T} \geq c^* | C_{pkm} = C_{pkm}^{\text{AQL}}) \geq 1 - \alpha \tag{3}$$

$$P(\hat{T} \geq c^* | C_{pkm} = C_{pkm}^{\text{RQL}}) \leq \beta, \tag{4}$$

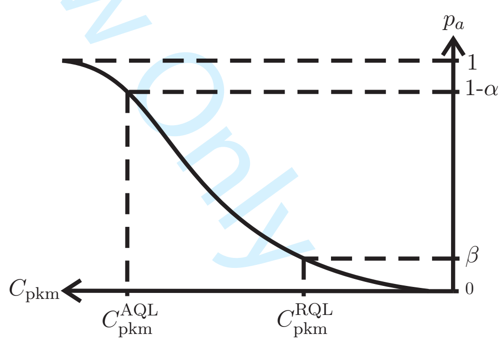
Here, $\hat{T}$ is the $\hat{C}_{pkm}$ value of the sample.

For our sampling plan, the cost function is a linear function of three types of quality costs:

- (i) inspection cost, which is the fixed cost incurred when inspecting one item from the sample
- (ii) internal failure cost, such as scrap and rework costs for the materials, labor, and overhead associated with production, and
- (iii) external failure cost, including costs due to customer complaints, which include the costs of investigation and adjustments, and those associated with receipt, handling, repair, and replacement of nonconforming products.



(a) A Traditional Sampling Plan



(b) The Proposed Sampling Plan

Figure 1: Comparison Between (a) a Traditional and (b) the proposed sampling plans

Figure 1 illustrates the operating characteristic curve of an attributes-based sampling plan (i.e., the probability of acceptance of the lot as a function of the proportion of defective items in the lot) and a variables-based sampling plan (i.e., the probability of acceptance of the lot as a function of the C_{pmk} value of the process).

To summarize, the objective of this paper is to solve Model 2 with total quality cost as the cost function to obtain the associated n and c^* and then evaluate the performance of the corresponding sampling plan.

2 Literature Review

2.1 A Review of Various Process Capability Indices (PCIs)

Process capability indices (PCIs) are useful management tools, particularly in the manufacturing industry. PCIs provide common quantitative measures on manufacturing capability and production quality. Most supplier certification manuals describe the recommended procedure for computing a process capability index and use them for making acceptance/rejection decisions. (Wu and Pearn 2007). Below we list four commonly used PCIs and the associated explanation follows.

(a) $C_p = \dfrac{\text{USL-LSL}}{6\sigma}$, where $\sigma = \mathrm{E}(X - \mu_X)^2$;

(b) $C_{pk} = \min\left[\dfrac{\text{USL} - \mu}{3\sigma}, \dfrac{\mu - \text{LSL}}{3\sigma}\right]$;

(c) $C_{pm} = \dfrac{\text{USL-LSL}}{6\tau} = \dfrac{C_p}{\sqrt{1+\xi^2}}$, where $\tau^2 = \mathrm{E}(X-T)^2$; $\xi = (\mu - T)/\sigma$, $T = (\text{USL+LSL})/2$, the quality target;

(d) $C_{pkm} = C_{pmk} = \dfrac{C_{pk}}{\sqrt{1+\xi^2}}$. Through out this paper, we use C_{pkm}.

The first PCIs appearing in the engineering literature was presumably the simple precision index, C_p (Juran, 1974; Sullivan, 1984, 1985; Kane 1986). C_p is defined as the ratio of the process specification spread (also known as the allowable process spread) to actual process spread.

C_p does not take into account the location of the process mean and hence turns out to be a suboptimal index. The index C_{pk}, on the other hand, takes both the magnitude of the process variation and the process departure from the midpoint m into consideration (Wu et al. 2009). Both the C_p and C_{pk} indices are aimed at reducing the variability in a process and do not consider the loss incurred to the producer or the consumer.

To incorporate these losses into the process capability indices, Hsiang and Taguchi (1985) introduced a new and improved loss based process capability index, C_{pm} also known as the Taguchi index, which was also later proposed independently by Chan et al. (1988).

Pearn et al. (1992) proposed the process capability index C_{pkm}, which combines the features of the yield based index C_{pk} and the loss based index C_{pm}. It alerts the user whenever the process variance increases and/or the process mean deviates from its target value (Wu et al. 2009). C_{pkm} provides many advantages over C_p, C_{pk}, and C_{pm}:

i) When the process mean departs from its target value, the amount of variation in C_p, C_{pk}, C_{pm}, and C_{pkm} is in the order: $C_{pkm} \geq C_{pm} \geq C_{pk} \geq C_p$ (Pearn and Kotz, 1995). Thus, C_{pkm} is most sensitive index among all 4 indices.

ii) For some positive value s, if $C_{pk} \geq s$, it does not imply that $C_{pm} \geq s$ and vice versa. But if $C_{pkm} \geq s$, then $C_{pk} \geq s$ and $C_{pm} \geq s$ (Wu and Pearn, 2008).

iii) Assuming T, the target value of the process mean, to be the mid point of specification limits (i.e, $T = (\text{USL} + \text{LSL})/2$),
$|\mu - T| < d$, if $C_p = 1$,
$|\mu - T| < d/3$, if $C_{pk} = 1$,
$|\mu - T| < d/4$, if $C_{pkm} = 1$,
where $d = (\text{USL} - \text{LSL})/2$, and μ is the process mean (Wu and Pearn 2008). Thus, C_{pkm} gives the smallest length of the confidence interval containing the process mean μ. That is, C_{pkm} leads to the greatest reduction in process-loss among all capability indices.

iv) As C_{pkm} is derived by combining C_{pk} and C_{pm}, it behaves more like C_{pk} when σ^2 is large and more like C_{pm} if σ^2 is small (Jessenberger and Weihs, 2000).

2.2 A Review of the Estimated C_{pkm} and its Sampling Distribution

Since its first introduction by Pearn et al. (1992), many researchers have described the statistical properties of C_{pkm} index. As the process mean μ and process standard deviation σ are generally unknown, Pearn et al. (1992) suggested using the natural estimator of C_{pkm} denoted by $\hat{C}_{pkm}$:

$$\hat{C}_{pkm} = \min\left\{\frac{\text{USL} - \overline{X}}{3\sqrt{S_n^2 + (\overline{X} - T)^2}}, \frac{\overline{X} - \text{LSL}}{3\sqrt{S_n^2 + (\overline{X} - T)^2}}\right\} = \frac{d - |\overline{X} - m|}{3\sqrt{S_n^2 + (\overline{X} - T)^2}} \tag{5}$$

where $\overline{X} = \sum_{i=1}^{n} X_i/n$ and $S_n^2 = \sum_{i=1}^{n} (X_i - \overline{X})^2/(n-1)$ are the maximum likelihood estimators (MLEs) of μ and σ^2, respectively. Using these estimators, Chen and Hsu (1995) derived the asymptotic distribution of the $\hat{C}_{pkm}$ index. Wright (1998) proposed a rather complicated expression for the probability density function of $\hat{C}_{pkm}$. Under the normality assumption for the quality characteristic, Vännman (1997) derived the explicit expression for the cumulative distribution function (CDF) of the random variable $\hat{C}_{pkm}$ (which we'll denote by Y from now on to simplify notation):

$$F_Y(y) = 1 - \int_0^{b\sqrt{n}/(1+3y)} G\left(\frac{(b\sqrt{n} - t)^2}{9y^2} - t^2\right)\left[\phi(t + \xi\sqrt{n}) + \phi(t - \xi\sqrt{n})\right] dt = P(Y \leq y) \tag{6}$$

where:

- $b = 3C_{pkm}\sqrt{1 + \xi^2} + |\xi|$, $\xi = (\mu - T)/\sigma$
- $G(\cdot)$ is the CDF of the χ^2_{n-1}
- $\phi(\cdot)$ is the probability distribution function of standard normal distribution

2.3 Producer's and Consumer's risk and Critical Acceptance value

The critical acceptance value, c^*, is the minimum value of $\hat{C}_{pkm}$ (as calculated from the sample) for which the lot will be accepted. If $\hat{C}_{pkm} \geq c^*$, the lot is accepted, but if $\hat{C}_{pkm} < c^*$, the lot undergoes 100% inspection and then accepted.

For a given C_{pkm} for the process, the operating characteristic equation, i.e., the probability of acceptance of the lot as a function of C_{pkm} is given by:

$$P_a(C_{pkm}) = P(Y \geq c^*) = 1 - P(Y \leq c^*) = 1 - F_Y(c^*) \tag{7}$$

where $F_Y(y)$ is defined in Equation (6). More explicitly, using Equation (6), we can write

$$P_a(C_{pkm}) = \int_0^{b\sqrt{n}/(1+3c^*)} G\left(\frac{(b\sqrt{n} - t)^2}{9c^{*2}} - t^2\right)\left[\phi(t + \xi\sqrt{n}) + \phi(t - \xi\sqrt{n})\right] dt \tag{8}$$

where: $b = 3C_{pkm}(1 + \xi^2) + |\xi|$.

Equation (7) can be used to calculate $P_a(C_{pkm}^{AQL})$ at the producer's risk point $(C_{pkm}^{AQL}, 1 - \alpha)$ and $P_a(C_{pkm}^{RQL})$ at the consumer's risk point (C_{pkm}^{RQL}, β).

3 Designing a Variables-Based Economic Sampling Plan

In Subsection 3.1, we construct a mathematical model to determine the optimum sample size n and critical acceptance value, c^*, of the acceptance sampling plan. The associated variables-based economic sampling plan is proposed in Subsection 3.2.

3.1 Determine the Sample Size n and Critical Acceptance Value c^*

We now develop a variables-based economic sampling model to solve for the two decision variables - the optimum sample size n and critical acceptance value, c^*.

$$\text{Minimize} \quad \text{TQC} = (c_{ip} \times \text{ATI}) + (c_{if} \times D_d) + (c_{ef} \times D_n) \tag{9}$$

such that

$$\text{P}(\hat{C}_{pkm} \geq c^* | C_{pkm} = C_{pkm}^{\text{AQL}}) \geq 1 - \alpha, \tag{10}$$

$$\text{P}(\hat{C}_{pkm} \geq c^* | C_{pkm} = C_{pkm}^{\text{RQL}}) \leq \beta, \tag{11}$$

where the two constraints (Equations (10) and (11)), are used to satisfy producer's and consumer's risks. Specifically, Equations (10) and (11) can be further expressed as

$$\int_0^{b_1\sqrt{n}/(1+3c^*)} G\left(\frac{(b_1\sqrt{n}-t)^2}{9c^{*2}} - t^2\right)\left[\phi(t+\xi\sqrt{n}) + \phi(t-\xi\sqrt{n})\right] dt \geq 1-\alpha, \tag{12}$$

$$\int_0^{b_2\sqrt{n}/(1+3c^*)} G\left(\frac{(b_2\sqrt{n}-t)^2}{9c^{*2}} - t^2\right)\left[\phi(t+\xi\sqrt{n}) + \phi(t-\xi\sqrt{n})\right] dt \leq \beta, \tag{13}$$

where

$$b_1 = 3C_{pkm}^{\text{AQL}}(\sqrt{1+\xi^2}) + |\xi| \tag{14}$$

$$b_2 = 3C_{pkm}^{\text{RQL}}(\sqrt{1+\xi^2}) + |\xi|. \tag{15}$$

The objective function defined in Equation (9) is the total quality cost, which is the sum of three types of costs:

$$\text{(i) Total inspection cost} = c_{ip} \times \text{ATI}, \tag{16}$$

$$\text{where ATI} = n + (1 - \text{P}_a(C_{pkm}))(N-n).$$

$$\text{(ii) Total internal failure cost} = c_{if} \times D_d, \tag{17}$$

$$\text{where } D_d = (1 - \Phi(3C_{pkm}\sqrt{1+\xi^2}))(n + (1 - \text{P}_a(C_{pkm})))(N-n)$$

and $\Phi(\cdot)$ denotes the CDF of the N(0,1).

$$\text{(iii) Total external failure cost} = c_{ef} \times D_n \tag{18}$$

$$\text{where } D_n = \text{P}_a(C_{pkm})(1 - \Phi(3C_{pkm}\sqrt{1+\xi^2}))(N-n).$$

We used an algorithm known as sequential quadratic programming (SQP) to solve for the plan parameters n and c^*. SQP is an iterative method for solving constrained non-linear optimisation problems which is a generalisation of the Newton's method for unconstrained optimisation. It replaces the objective function with a quadratic approximation and the non-linear constraints with linear approximations. It has been implemented in almost all popular scientific computing languages. For our purpose, we utilised "fmincon" function in MATLAB by setting the *Algorithm* option to *sqp*.

3.2 The Proposed Variables-Based Economic Sampling Plan

We now present the step by step procedure for the application of the proposed variables-based economic sampling plan.

i) Given the pre-determined values:

- process capability requirements $(C_{pkm}^{\text{AQL}}, 1-\alpha)$ and $(C_{pkm}^{\text{RQL}}, \beta)$
- C_{pkm} of the process
- cost values: c_{ip}, c_{if} and c_{ef}
- lot size N

ii) Find the optimum sample size n and critical acceptance value c^* using the model described in Subsection 3.1.

iii) Randomly pick a sample of size n from the lot of size N and calculate the associated value of $\hat{C}_{pkm}$.

iv) The decision criterion are given as follows:

$$\text{If} \begin{cases} \hat{C}_{pkm} \geq c^* & \text{accept the lot and replace defective items in the sample with good ones} \\ \hat{C}_{pkm} < c^* & \text{perform 100\% inspection and replace all the defective items with good ones,} \\ & \text{and then accept the lot.} \end{cases}$$

4 Performance Comparison and Analysis

For illustration, we use a numerical example to demonstrate the results of our sampling plan. The proposed sampling plan is compared with the sampling plan proposed by Yen et al. (2015), who used C_{pk} index for developing a variables-based acceptance sampling plan. To ensure that the various parameters in their plan and corresponding parameters in our plan are consistent, a ξ value of 0 is used. The values for various parameters are as follows:

- $N = 1000$
- $C_{pkm}^{\text{AQL}} = 1.33$ and $C_{pkm}^{RQL} = 1.00$
- $C_{pkm} = 1.2$
- $c_{ip} = 10$, $c_{if} = 20$ and $c_{ef} = 50$
- $\alpha = \{0.01, 0.025, 0.05, 0.075, 0.1\}$
- $\beta = \{0.01, 0.025, 0.05, 0.075, 0.1\}$

The optimal combination of n and c^* for both the plans are summarized in Table 1. The values in columns 3, 4 and 5 are the n, c^* and TQC values, respectively, for our plan. Similarly, the values in columns 6, 7 and 8 are the n, c^* and TQC values, respectively, for Yen et al. plan.

Table 1 also presents the comparative analysis between plan proposed by Yen et. al (2015) and our plan. Observations and inferences from Table 1:

- The required sample size decreases as the consumer's risk (β) increases but remains constant with an increase in producer's risk (α).
- The total quality cost decreases as consumer's risk (β) increases but remains constant with an increase in producer's risk (α).

- When C_{pkm} is used instead of C_{pk} as the PCIs, then on an average both n and TQC decrease approximately by 8%. This implies that our sampling plan not only achieves a lower total quality cost for same producer's and consumer's risk but also achieves it with a smaller sample size. Thus, we are able to attain a win-win situation for both the producer and the consumer.

Table 1: Comparison between plan proposed by our plan and Yen et al.

		Our Plan			Yen et. al (2015)			Reduction	
α	β	n	c^*	TQC	n	c^*	TQC	% ↓ n	% ↓ TQC
0.01	0.01	233	1.106	2909	250	1.127	3170	6.80	8.23
0.01	0.025	197	1.094	2493	218	1.114	2712	9.63	8.08
0.01	0.05	169	1.082	2159	183	1.104	2343	7.65	7.85
0.01	0.075	151	1.073	1955	165	1.096	2119	8.48	7.74
0.01	0.1	139	1.065	1804	149	1.090	1954	6.71	7.68
0.01	0.01	233	1.106	2909	250	1.127	3170	6.80	8.23
0.01	0.025	197	1.094	2493	218	1.114	2712	9.63	8.08
0.01	0.05	169	1.082	2159	183	1.104	2343	7.65	7.85
0.01	0.075	151	1.073	1955	165	1.096	2119	8.48	7.74
0.01	0.1	139	1.065	1804	149	1.090	1954	6.71	7.68
0.01	0.01	233	1.106	2909	250	1.127	3170	6.80	8.23
0.01	0.025	197	1.094	2493	218	1.114	2712	9.63	8.08
0.01	0.05	169	1.082	2159	183	1.104	2343	7.65	7.85
0.01	0.075	151	1.073	1955	165	1.096	2119	8.48	7.74
0.01	0.1	139	1.065	1804	149	1.090	1954	6.71	7.68
0.01	0.01	233	1.106	2909	250	1.127	3170	6.80	8.23
0.01	0.025	197	1.094	2493	218	1.114	2712	9.63	8.08
0.01	0.05	169	1.082	2159	183	1.104	2343	7.65	7.85
0.01	0.075	151	1.073	1955	165	1.096	2119	8.48	7.74
0.01	0.1	139	1.065	1804	149	1.090	1954	6.71	7.68
0.01	0.01	233	1.106	2909	250	1.127	3170	6.80	8.23
0.01	0.025	197	1.094	2493	218	1.114	2712	9.63	8.08
0.01	0.05	169	1.082	2159	183	1.104	2343	7.65	7.85
0.01	0.075	151	1.073	1955	165	1.096	2119	8.48	7.74
0.01	0.1	139	1.065	1804	149	1.090	1954	6.71	7.68

5 Sensitivity Analysis of n, c^* and TQC

To perform a sensitivity analysis of n, c^* and TQC w.r.t. c_{ip}, c_{if} and c_{ef}.

- $N = 1000$
- $C_{pkm}^{\text{AQL}} = 1.33$ and $C_{pkm}^{\text{RQL}} = 1.00$
- $C_{pkm} = 1.2$
- $\alpha = 0.05$
- $\beta = 0.05$

Table 2 shows how TQC varies as c_{ip} varies from 10 to 100, while $c_{if} = 20$ and $c_{ef} = 50$. For Yen et al. plan, $n = 183$ while for our plan, $n = 169$ which remain constant for all values of c_{ip}.

Table 2: Comparison of TQC: c_{ip} varies from 10 to 100, while $c_{if} = 20$ and $c_{ef} = 50$

c_{ip}	10	20	30	40	50	60	70	80	90	100
TQC: our plan	2159	4311	6463	8616	10768	12920	15072	17224	19376	21529
TQC: Yen et al.	2343	4679	7015	9351	11687	14024	16360	18696	21032	23368
% ↓ in TQC	7.85	7.86	7.87	7.86	7.86	7.87	7.87	7.87	7.87	7.87

Table 3 shows how TQC varies as c_{if} varies from 10 to 100, while $c_{ip} = 10$ and $c_{ef} = 50$. For Yen et al. plan, $n = 183$ and for our plan, $n = 169$ which remain constant for all values of c_{if}.

Table 3: Comparison of TQC: c_{if} varies from 10 to 100, while $c_{ip} = 10$ and $c_{ef} = 50$

c_{if}	10	20	30	40	50	60	70	80	90	100
TQC: our plan	2159	2159	2159	2159	2160	2160	2160	2161	2161	2161
TQC: Yen et al.	2343	2343	2343	2344	2344	2344	2345	2345	2345	2345
% ↓ in TQC	7.85	7.85	7.85	7.89	7.85	7.85	7.89	7.85	7.85	7.85

Table 4 shows how TQC varies as c_{ef} varies from 10 to 100, while $c_{ip} = 10$ and $c_{if} = 20$. For Yen et al. plan, $n = 183$ and for our plan, $n = 169$ which remain constant for all values of c_{if}.

Table 4: Comparison of TQC: c_{ef} varies from 10 to 100, while $c_{ip} = 10$ and $c_{if} = 20$

c_{ef}	10	20	30	40	50	60	70	80	90	100
TQC: our plan	2338	2339	2341	2342	2343	2344	2345	2347	2348	2349
TQC: Yen et al.	2154	2155	2156	2158	2159	2161	2162	2163	2164	2165
% ↓ in TQC	7.87	7.87	7.90	7.86	7.85	7.81	7.80	7.84	7.84	7.83

From Tables (2), (3) and (4), we conclude that the total quality cost is primarily dependent on the inspection cost and almost independent of the internal and external failure costs. Thus, industries should focus on minimising inspection costs to lower their total quality costs. Moreover, we observe that the optimum sample size is independent of all cost values.

6 Conclusion

Acceptance sampling plans are used for minimising the total expenditure before and after inspection while ensuring that both the producer and consumer achieve the required satisfactory level of product quality. In this paper, we developed an economic model of a variables-based single acceptance sampling plan using the advanced process capability index, C_{pkm}. All the 3 types of costs that are incurred during and after inspection are considered and the results are compared with the C_{pk} based plans proposed by Yen et al. (2015). The results indicate that the proposed plan incurs approximately 8% less quality cost while also providing better statistical properties due to inherent superiority of C_{pkm} over C_{pk}. The results of sensitivity analysis indicate that the total quality cost depends primarily on inspection cost only while the optimum sample size and critical acceptance value are independent of all types of costs . This result implies that industries should focus on decreasing their inspection cost to achieve lower quality costs.

Our model is developed under the assumption that the quality characteristic X is normally distributed. This assumption is not always true and some methodologies can be developed for processes for which the normality assumption is violated. The various cost values are assumed to be constant which is not always the case. Taguchi proposed the quadratic cost of the quality model in which the economic loss due to poor quality is given by a quadratic function which is more appropriate for estimating quality costs and should be taken up as a future research work. Also, as Wu and Wang (2017) pointed out, multiple dependent state sampling plans provide more efficiency than the single sampling plans under the same conditions, which can be a future extension to the current research work.

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