ENEL 649 Fall 2022 Project

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Introduction

The purpose of this project is to give you the opportunity to compare the theoretical principles discussed in the lectures with simulated realizations of random variables and stochastic processes.

Requirements

- These projects are to be done individually.
- For this project, you can use either Matlab/Octave or Python with the NumPy, SciPy and PyPlot libraries.
- **Important:** You are allowed to use library functions to generate uniform random numbers and calculate histograms. No other library functions may be used beyond basic math operations and matrix manipulation functions. For example, if library functions exist to calculate a Chi-squre goodness of fit test or generate an exponentially distributed vector of random samples, you cannot use them. You must code these operations from scratch.

Problems

1. Generate 100,000 samples of an exponentially distributed random variable with a mean of 8. Generate a histogram of these samples, normalize to have the same area as a PDF. Plot your histogram and the theoretical PDF function together on the same figure. They should match.
2. Generate 100,000 samples of the sum of 2, 6 and 50 exponentially distributed random variables, each with a mean of 6. Create histograms of each sum, normalize to have the same area as a PDF and plot. For each distribution, choose the number of histogram bins that produce plots that clearly show the shape of the distribution.
3. Apply a Chi-squared goodness-of-fit test to see if the random vector you generated in Problem 1 matches a theoretical exponential distribution. Your test should calculate and display a confidence value that should reveal your vector of random numbers does match an exponential distribution.

4. Apply a Chi-squared goodness-of-fit test to the sum of 50 exponentially distributed random vectors from Problem 2 and see if it matches a Gaussian theoretical distribution. In your PDF file, comment on what these results say about the utility of the central limit theorem in this particular case.

5. Create a 10,000 waveform ensemble of a stochastic process where the waveform is $X(t) = exp(-Yt)$, where Y is uniformly distributed between 0 and 3. Your time vector should go from 0 to 4 seconds with a sampling interval of 1 ms.

6. Use your 10,000 waveform ensemble from Problem 5 to numerically calculate a histogram that represents the first order PDF of this stochastic process. Normalize your histogram to have the same area as a PDF and plot your histogram on the same figure as the theoretical expression for the first order PDF for this stochastic process. They should match. You can generate your plot for a single time sample that does a good job of illustrating the zero and non-zero regions of the PDF.

7. Use your 10,000 waveform ensemble from Problem 5 to numerically calculate the mean of the stochastic process. Plot the numerical mean along with the theoretical mean expression on the same figure. They should match.

What to Hand In

In a single PDF file, submit your source code and any required plots for each problem.

Marking

Each problem will be marked as follows:

- Good (4 marks): Code is well written, easy to understand and produces the correct results. All required plots and other problem components are present and correct.
- Acceptable (3 marks): Some minor errors/omissions and/or results are correct but the code is poorly written and difficult to understand.
- Marginal (2 marks): A major error and/or a required component or plot is missing.
- Unacceptable (1 mark): Several major errors.
- Missing (0 marks): Nothing submitted.