Contact of Rough Spheres: Greenwood-Tripp model

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Model description

To make a truly predictive model linking contact area and force in contact of rough spheres, Greenwood and Tripp constructed a model taking into account averaged deformations induced by the contact pressure due to asperity contact [1]. This model, initially constructed for spherical contact, could be extended to arbitrary contact shape (its implementation for the axisymmetric contact is provided in github.com/vyastreb/Greenwood-Tripp-Model).

The main feature of the model is that it solves the problem in an iterative way by computing pressures from the separation field using multi-asperity model, then it computes induced displacements using integral convolution, recomputing separations and so on before the convergence. The pressure $p^i(x,y)$ induced by asperity contact based on a statistical multi-asperity model for the separation $z_0^i(x,y)$ between two surfaces is given by the following equation

$$p^i(x,y) = \tfrac{4}{3}\eta E^* r^{1/2} \int\limits_{z_0^i(x,y)}^{\infty} (z - z_0^i(x,y))^{3/2} P(z)\,dz, \tag{1}$$

where η is the asperity density (m^{-2}), r (m) their average geometrically mean curvature radius and $P(z)$ (m^{-1}) is the probability density of asperity heights. This average pressure is used to induce macroscopic deformation following Boussinesq approach, the total induced displacement (summing both sides) is given by

$$u^i(x,y) = \frac{1}{\pi E^*}\ \text{p.v.}\int\limits_{\mathbb{R}^2} \frac{p^i(x',y')\,dx'dy'}{\sqrt{(x-x')^2+(y-y')^2}}, \tag{2}$$

where p.v. denotes principal value of the integral. Then the separation between the surfaces is updated $z_0^{i+1} = z_0^i + u^i$, new pressure p^{i+1} is recomputed (1) as well as the induced displacement u^{i+1} from (2). For an axisymmetric problem, a simpler form is available [2][see Eq. (3.96a)]:

$$u_z(r) = \frac{4}{\pi E^*}\int_0^{\infty} \frac{\rho}{\rho + r}\, p(\rho)\, K(k(r,\rho))\, d\rho \tag{3}$$

where r and ρ are the radial coordinate and the modulus k of the complete elliptic integral of the first kind $K(k)$ is given by:

$$k(r,\rho) = \frac{4\rho r}{(r+\rho)^2}, \quad K(k) = \int_0^{\pi/2} [1 - k\sin^2(t)]^{-1/2}\, dt. \tag{4}$$

At the convergence separation $z^*(x,y)$, we have the information about contact area fraction, which can be computed as

$$A^*(x,y) = \pi\eta r \int_{z^*}^{\infty} (z^* - z) P(z)\, dz. \tag{5}$$

Example

For the set of parameters provided in Table 1, the roughness parameter [3] can be evaluated

$$\alpha = \frac{\sigma R}{a^2} \approx 0.361,$$

placing this problem in the class of problems where the deviation from the Hertz theory is significant.

We use a relaxation technique by weighting the obtained displacement as

$$u'_{k+1} = \kappa u_{k+1} + (1-\kappa) u_k,$$

Therefore, the change in displacement $u'_{k+1} - u'_k = \kappa(u_{k+1} - u_k)$ and for the convergence we will require that

$$\frac{\|u_{k+1} - u_k\|_\infty}{\|u_k\|_\infty + \varepsilon_0} \le \epsilon \quad \Leftrightarrow \quad \frac{\|u'_{k+1} - u_k\|_\infty}{\|u_k\|_\infty + \varepsilon_0} \le \kappa\epsilon,$$

where ε_0 is a small parameter, set by default to 10^{-20}, and used to avoid division by zero. The iterative algorithm converges in 17 iterations for the tolerance of $\epsilon = 10^{-3}$ defined by The obtained pressure, contact area distribution, deformed configuration and the resulting surface displacement are shown in Figure 1 and compared with the reference Hertz solution for the same load.

References

1. J. A. Greenwood and J. H. Tripp (1967). *The elastic contact of rough spheres.* Journal of Applied Mechanics, **34**(1), 153–159. doi:10.1115/1.3607616

2. K. L. Johnson (1985). *Contact Mechanics.* Cambridge University Press, Cambridge. Ninth printing, 2003.

3. J. A. Greenwood, K. L. Johnson, and E. Matsubara (1984). *A surface roughness parameter in Hertz contact.* Wear, **100**(1–3), 47–57. doi:10.1016/0043-1648(84)90005-X

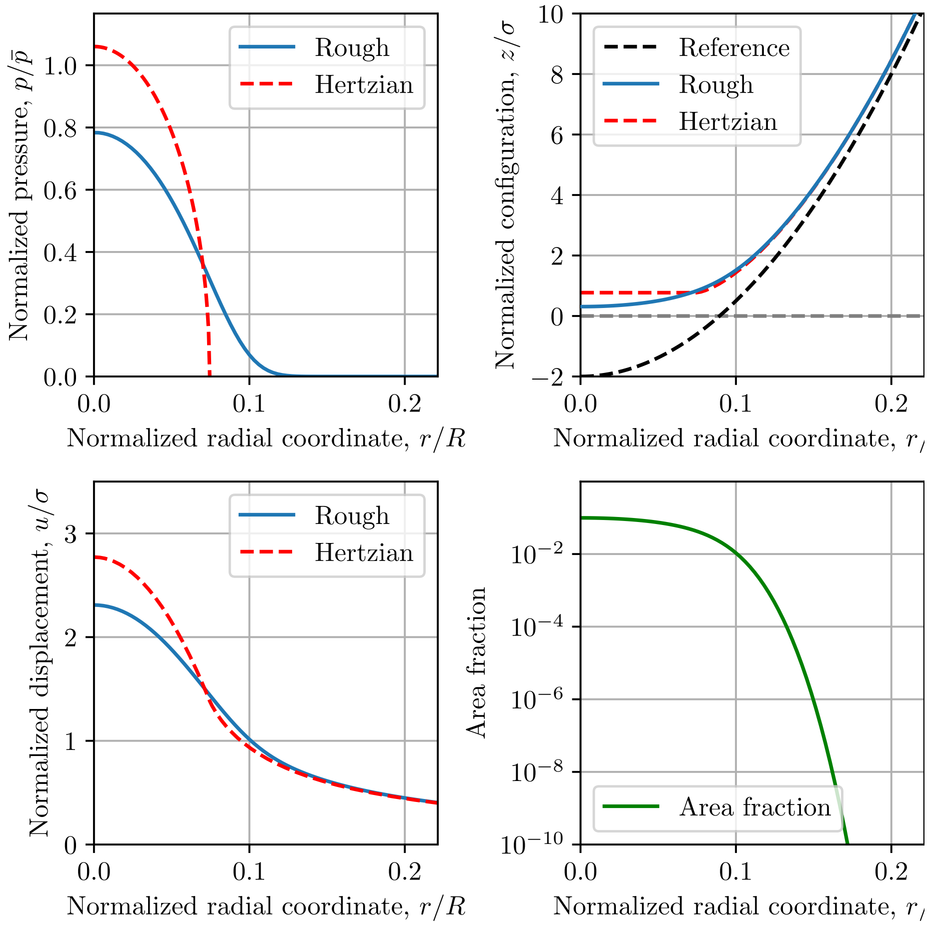

Figure 1: Converged pressure distribution and the corresponding Hertzian pressure (left column, upper pane); initial penetration (dashed line) and the resulting indenter's configuration for Greenwood-Tripp and Hertzian configuration (right column, upper panel); vertical displacement of Greenwood-Tripp and Hertzian displacement (left column, lower panel); contact area fraction for Greenwood-Tripp model (right column, lower panel).

Parameter	Symbol	Value	Units	Description
Material Properties				
Young's modulus	E	2.1×10^{11}	Pa	Steel elastic modulus
Poisson's ratio	ν	0.3	-	Poisson's ratio
Combined modulus	E^*	$\approx 1.15 \times 10^{11}$	Pa	$E/(2(1-\nu^2))$
Roughness Parameters				
RMS roughness	σ	20	µm	Root mean square height
Asperity density	η	2×10^8	m^{-2}	Number of asperities per unit area
Asperity tip radius	β	30	µm	Mean radius of curvature
Indenter Geometry				
Type	-	Sphere	-	Indenter shape
Radius	R	10	mm	Indenter radius
Loading Parameters				
Initial separation	d	-40	µm	-2σ
Numerical Parameters				
Convergence tolerance	ϵ	10^{-3}	(-)	
Relaxation parameter	κ	0.2	(-)	

Table 1: Model parameters