DeepLearning.AI

Math for Machine Learning

Linear algebra - Week 3

Vectors

Matrices

Dot product

Matrix multiplication

Linear transformations

DeepLearning.AI

Vectors and Linear Transformations

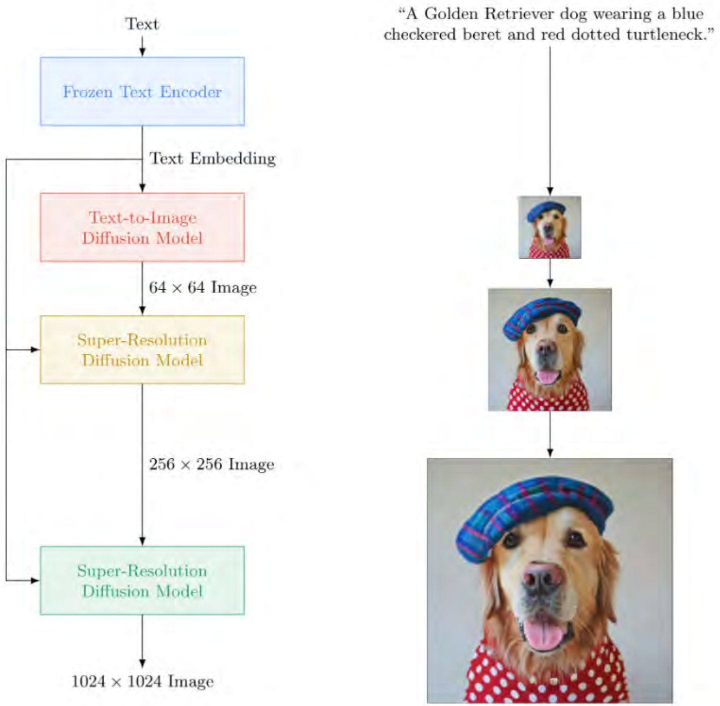
Machine Learning motivation

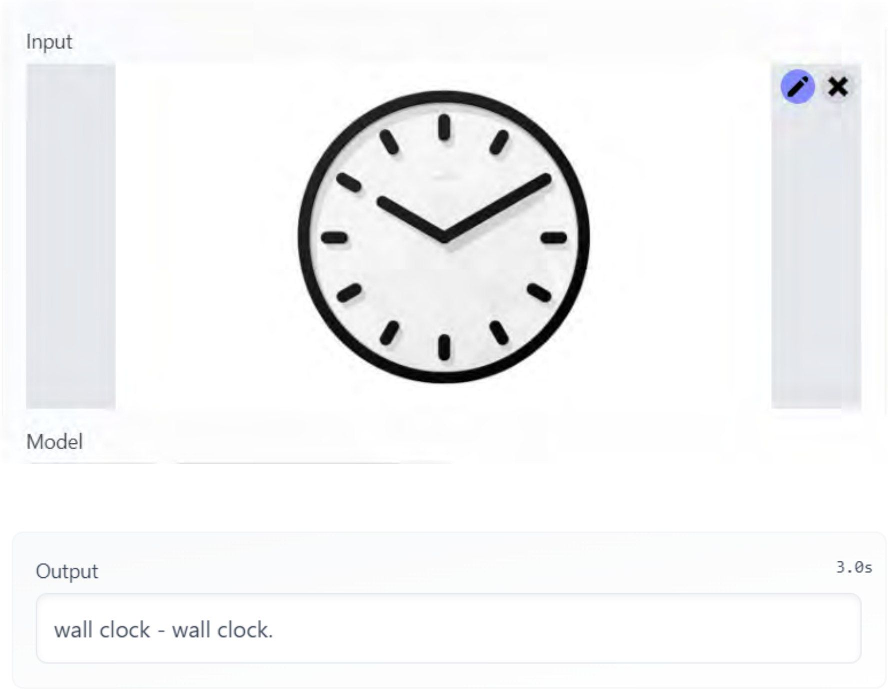
Neural Networks - AI generated images

AI-generated human faces.

- Generative learning: Generating realistic looking images.

Text-to-image and image-to-text generation



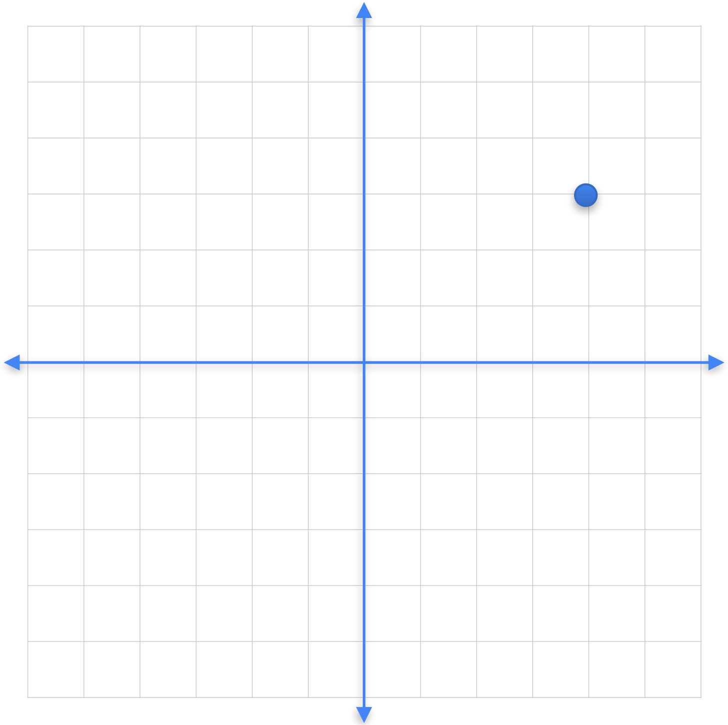


DeepLearning.AI

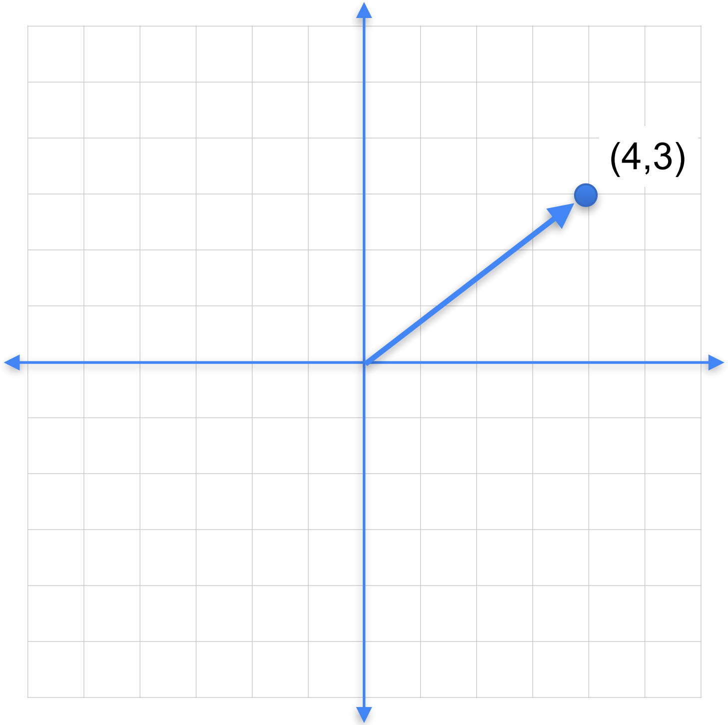
Vectors and Linear Transformations

Vectors and their properties

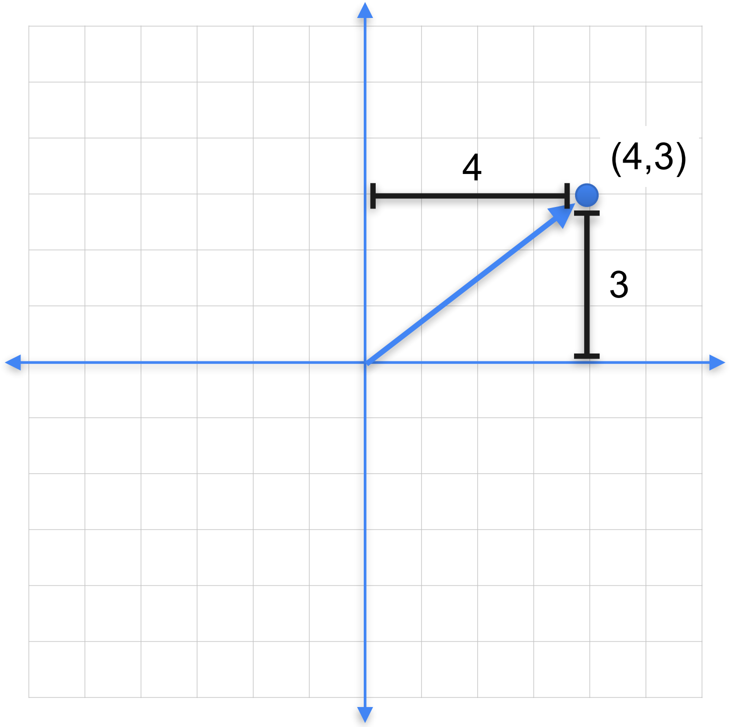
Vectors



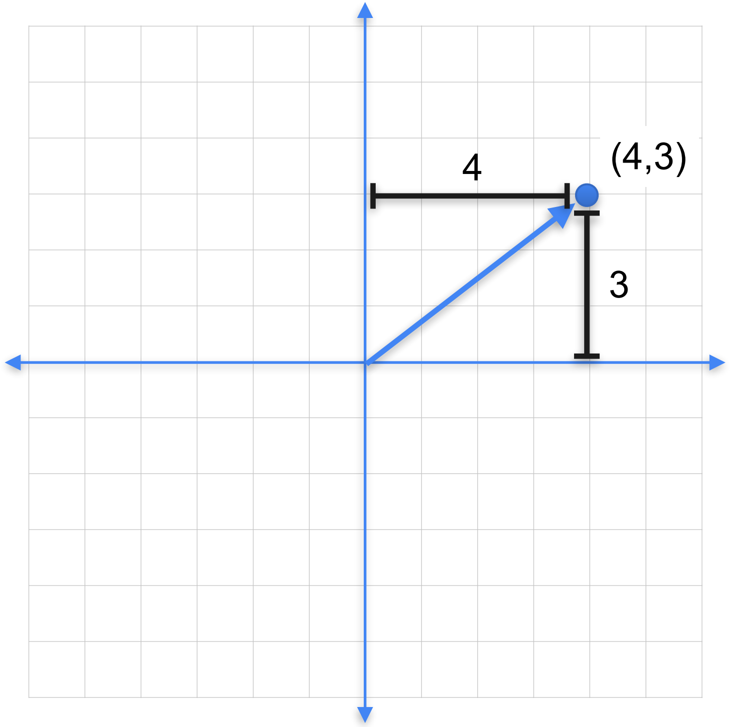
Vectors

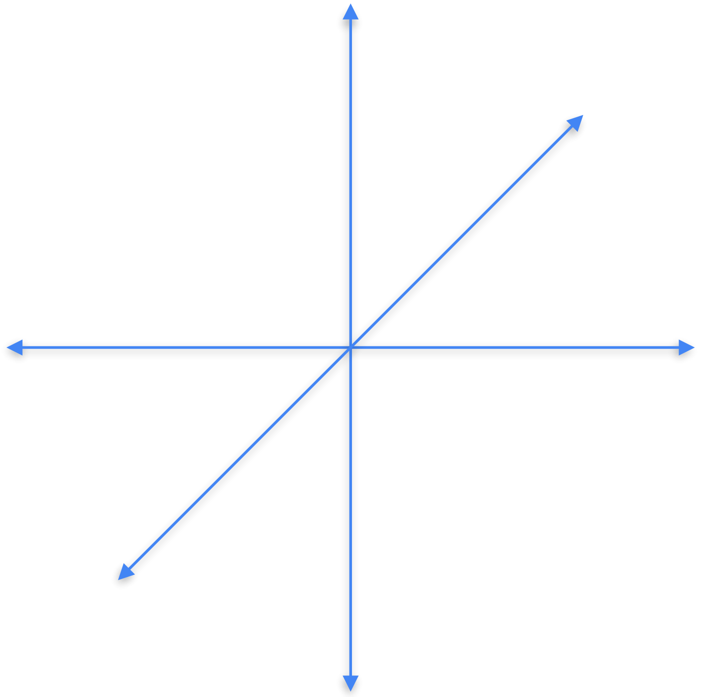


Vectors

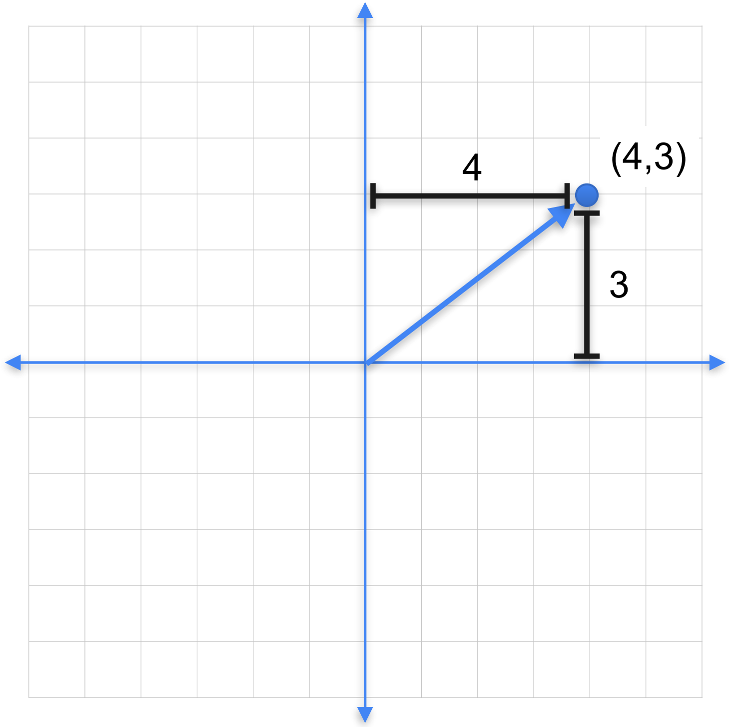


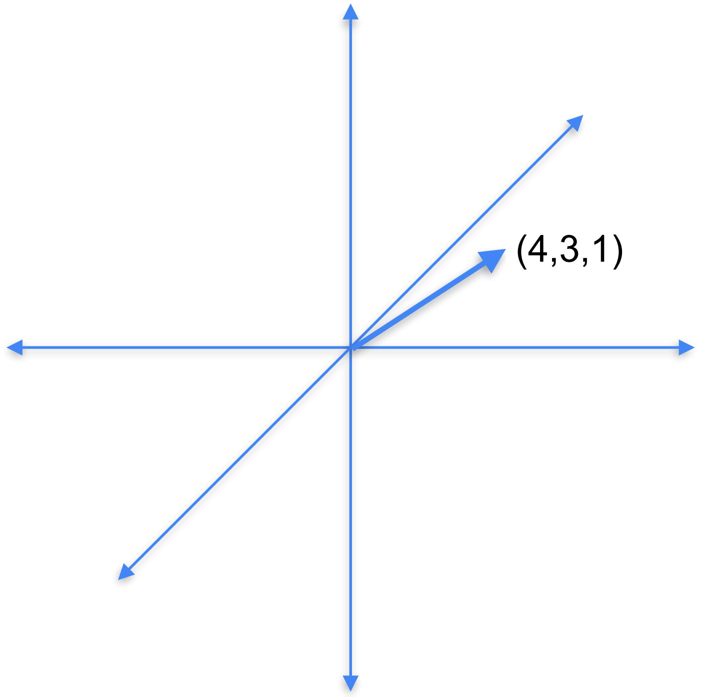
Vectors



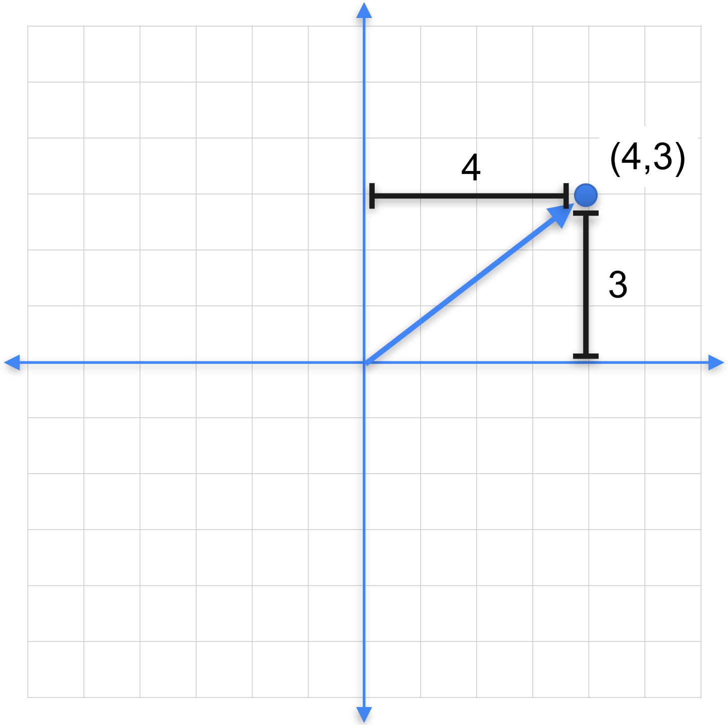


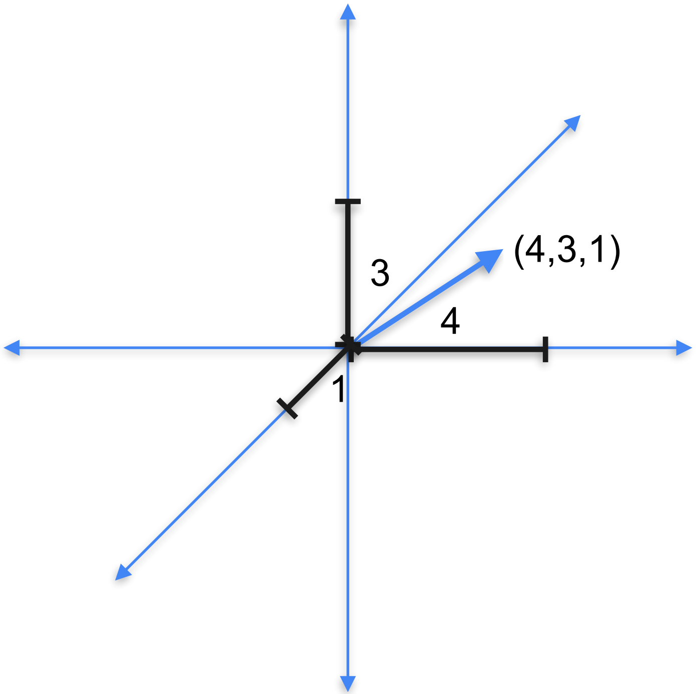
Vectors





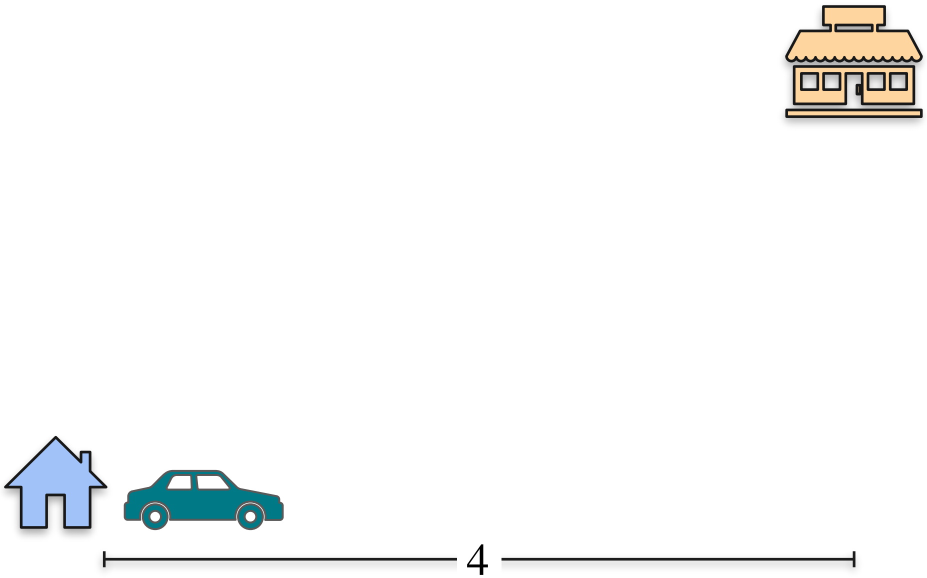
Vectors



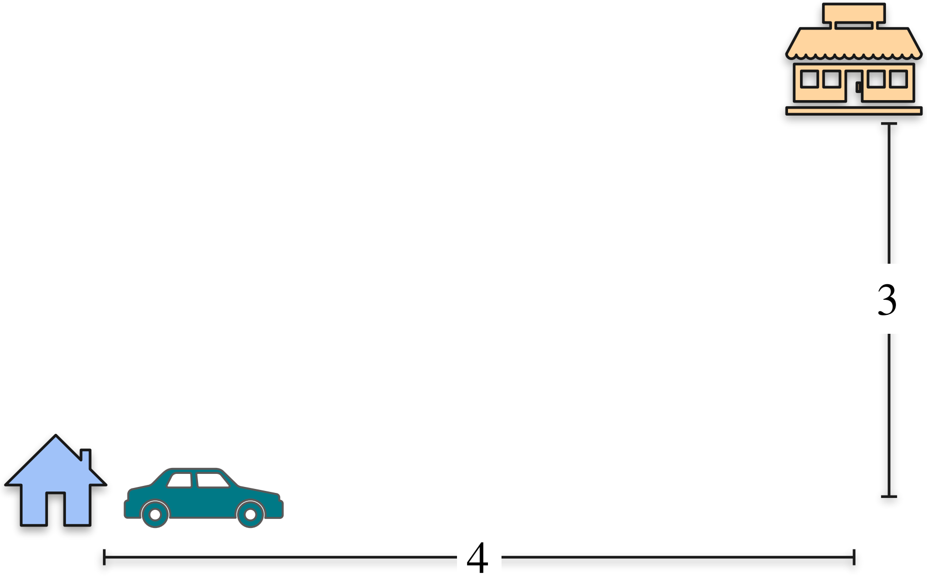


How to get from point A to point B?

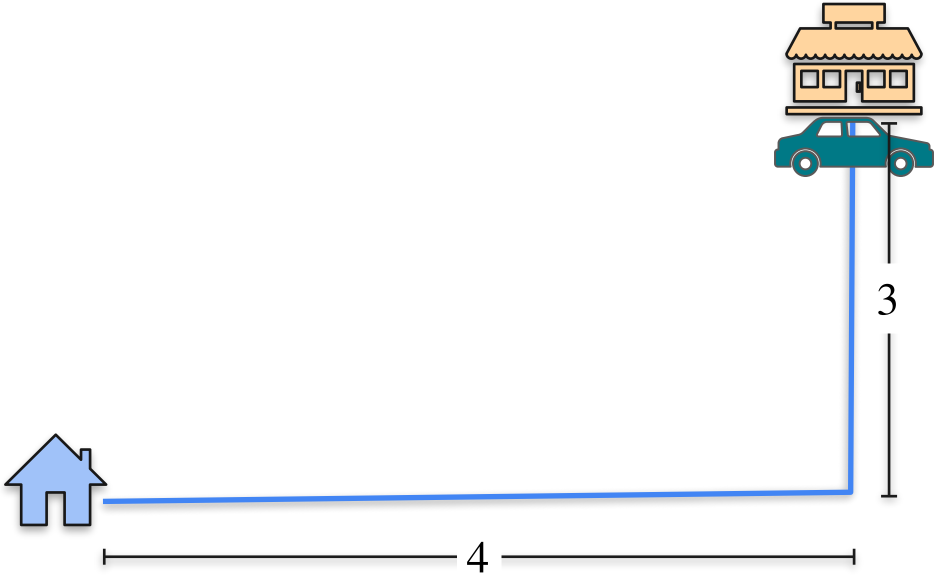
How to get from point A to point B?



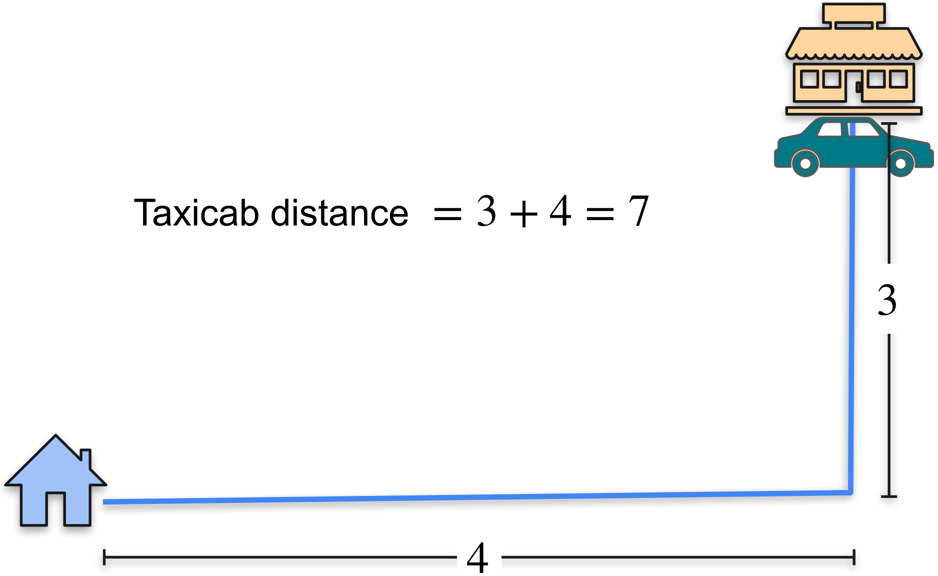
How to get from point A to point B?



How to get from point A to point B?

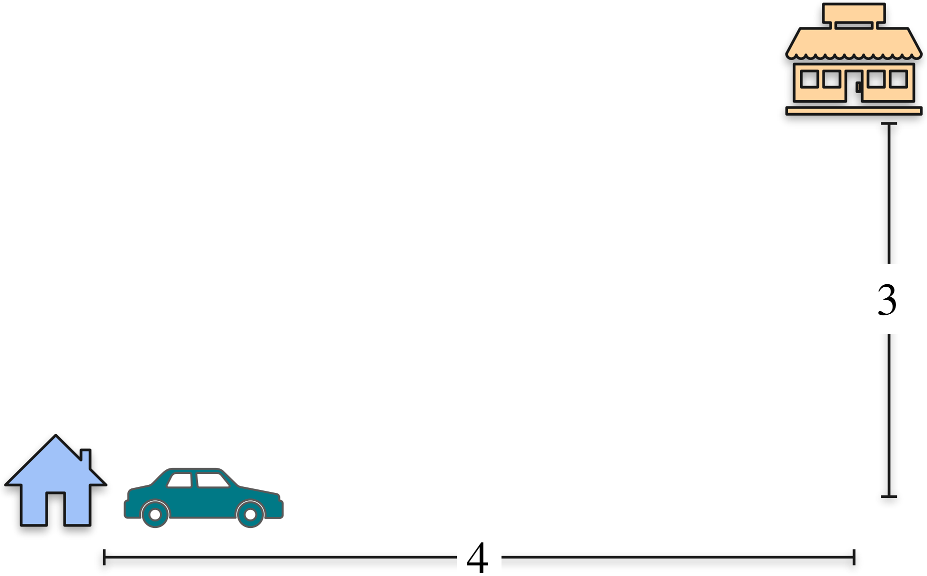


How to get from point A to point B?

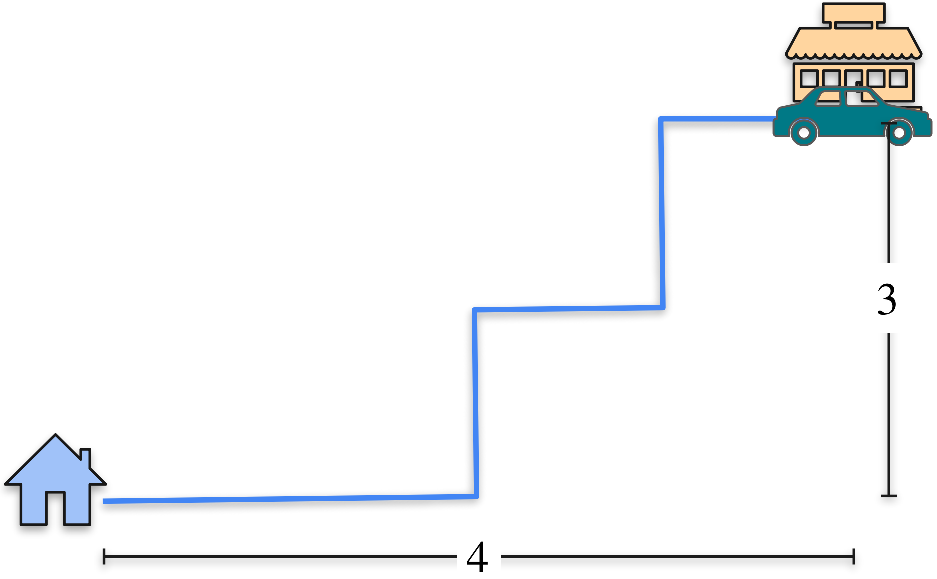


Taxicab distance $= 3 + 4 = 7$

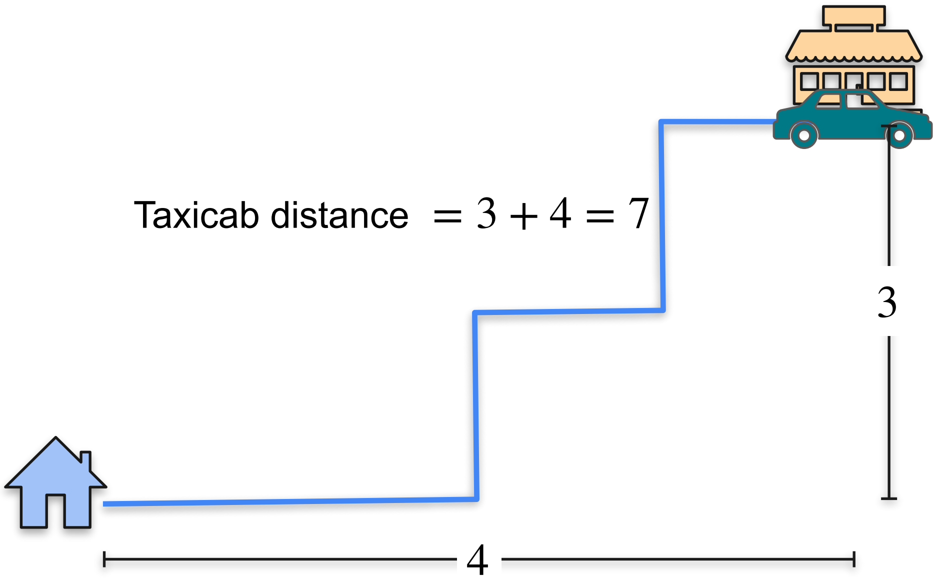
How to get from point A to point B?



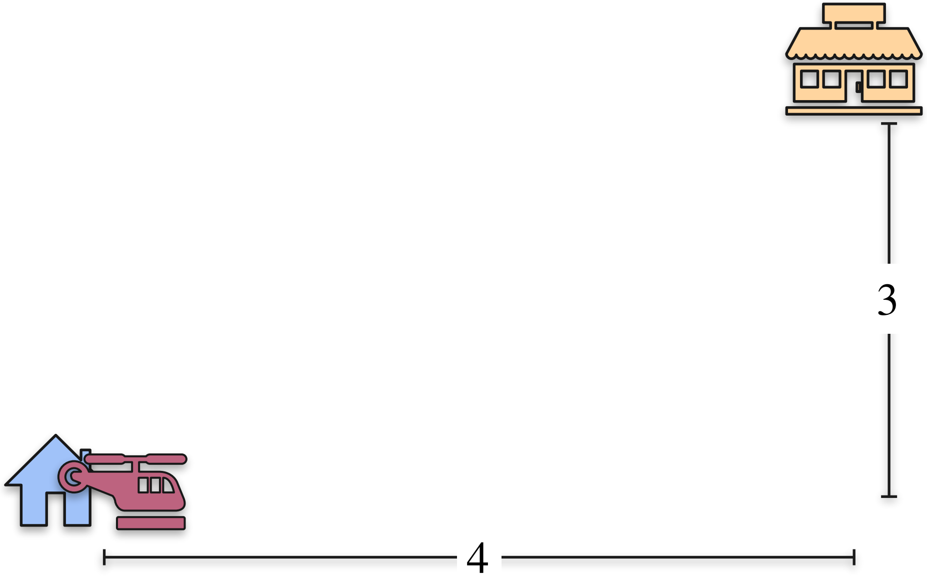
How to get from point A to point B?



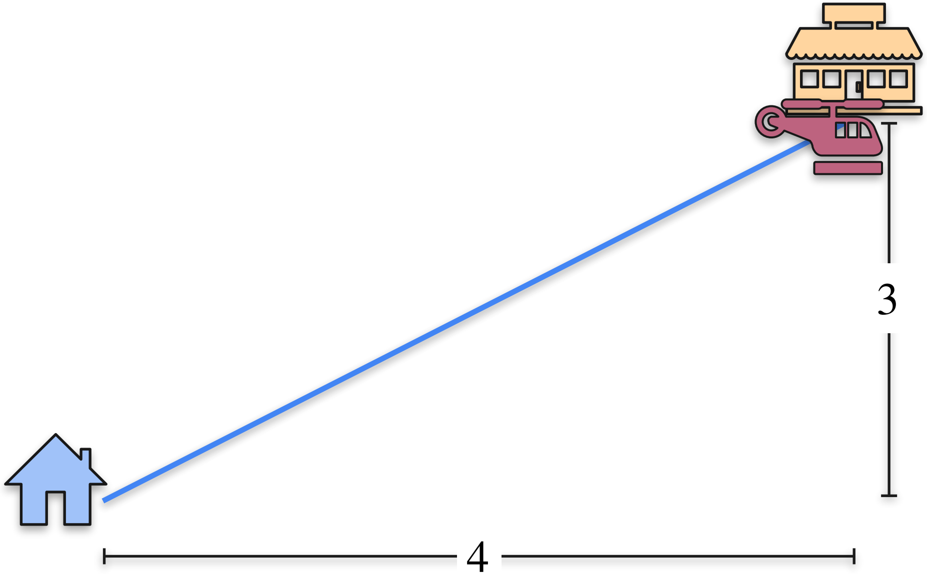
How to get from point A to point B?



How to get from point A to point B?



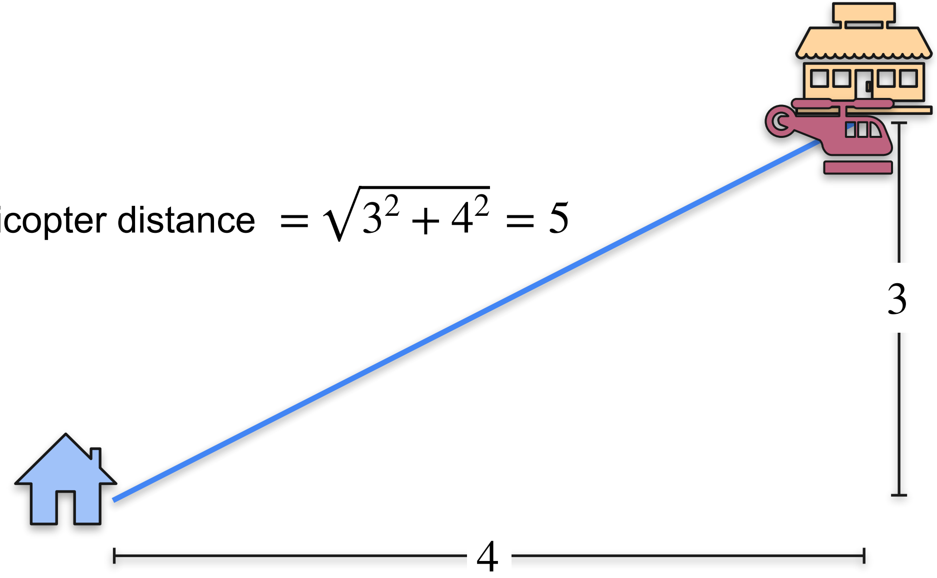
How to get from point A to point B?



DeepLearning.AI

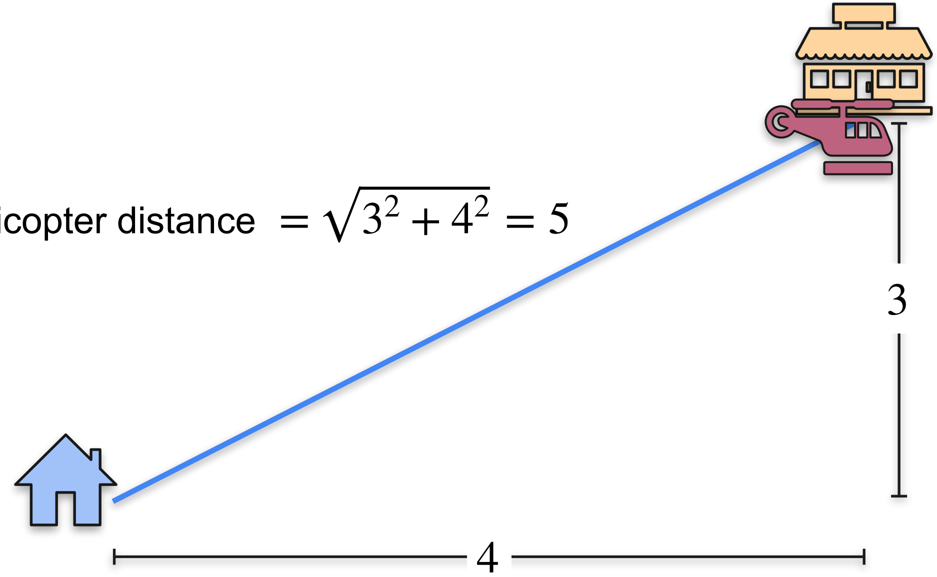
How to get from point A to point B?

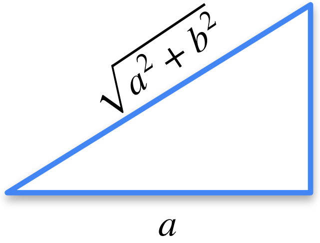
Helicopter distance $= \sqrt{3^2 + 4^2} = 5$



How to get from point A to point B?

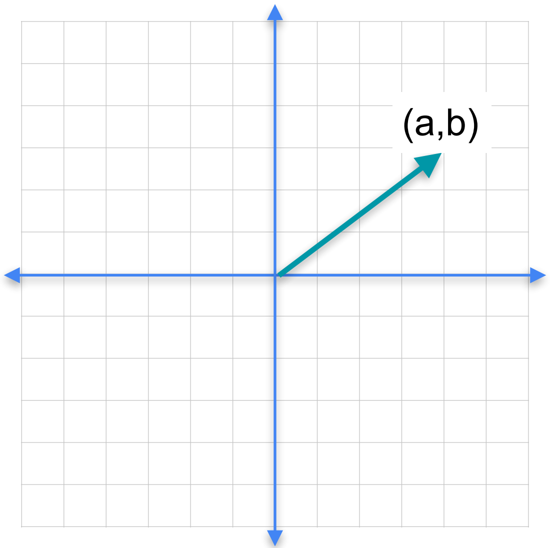
Helicopter distance $= \sqrt{3^2 + 4^2} = 5$



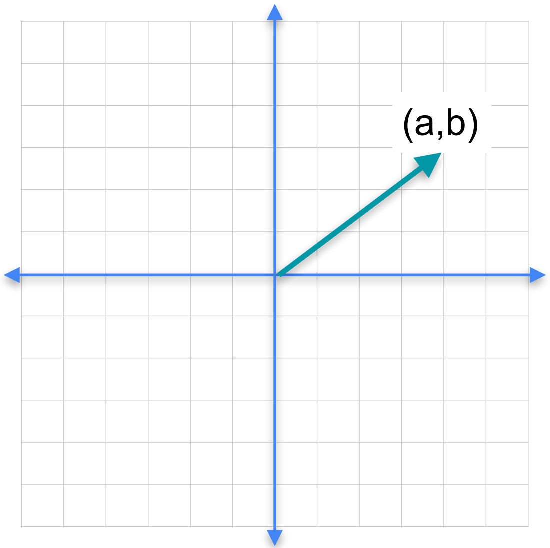


Pythagorean Theorem

Norms

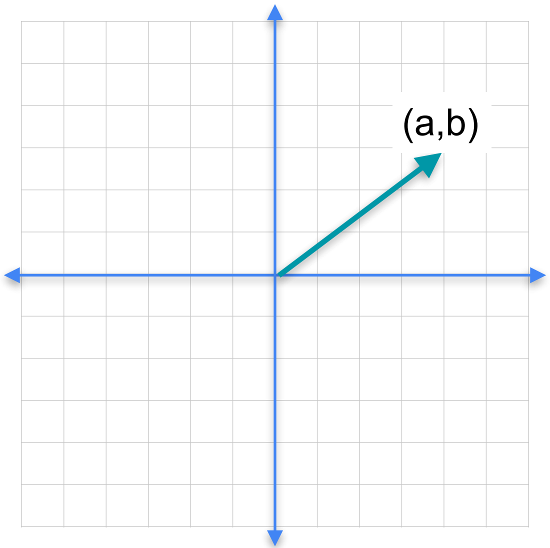


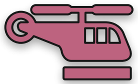
Norms



L1-norm $= |(a,b)|_1 = |a| + |b|$

Norms

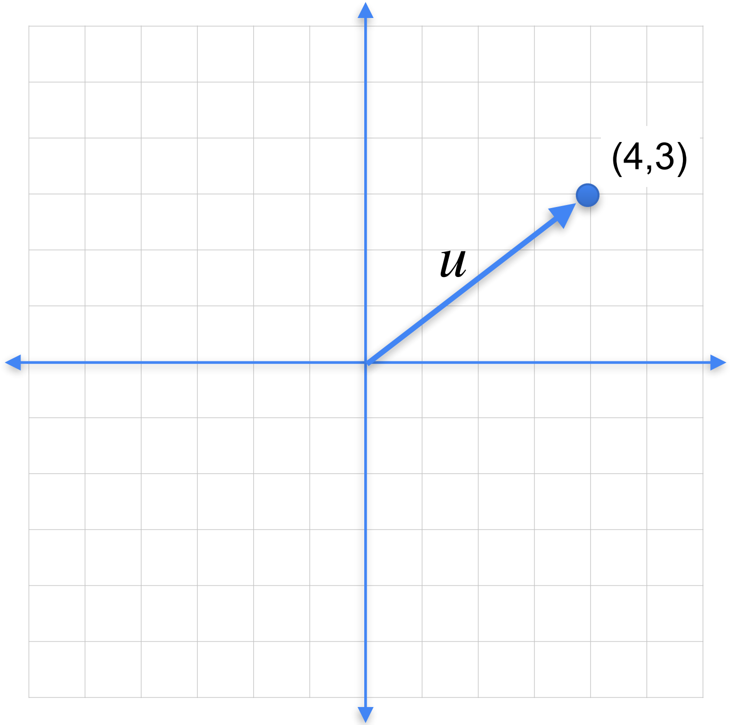




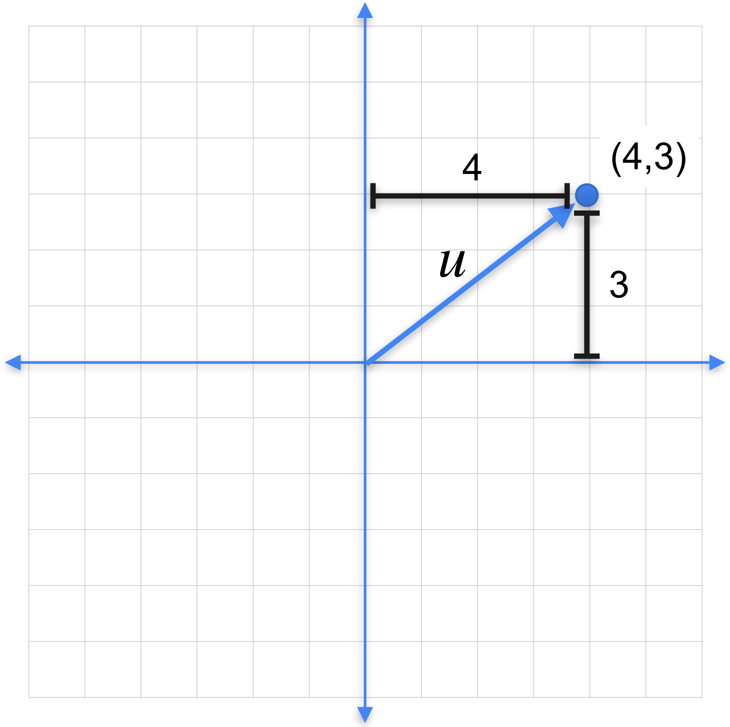
L1-norm $= |(a, b)|_1 = |a| + |b|$

L2-norm $= |(a, b)|_2 = \sqrt{a^2 + b^2}$

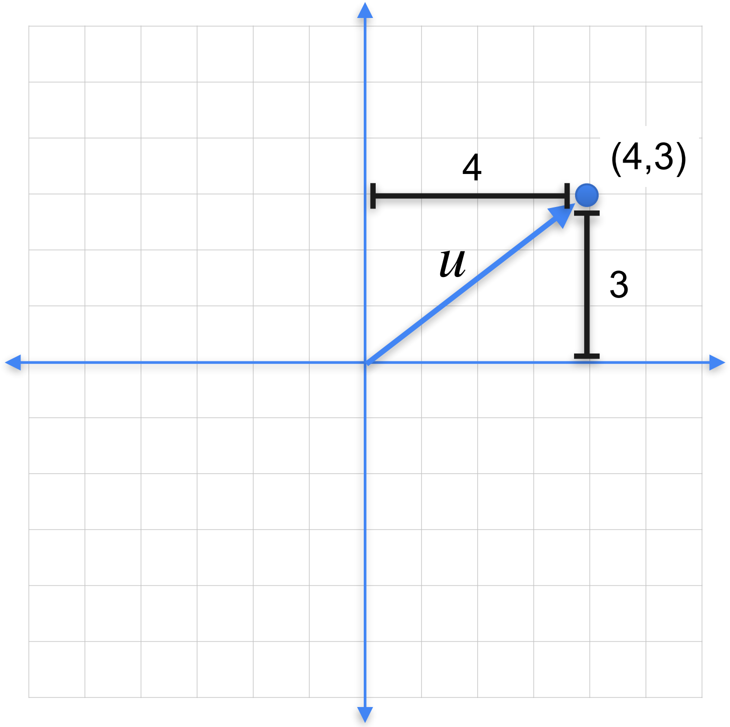
Norm of a vector



Norm of a vector

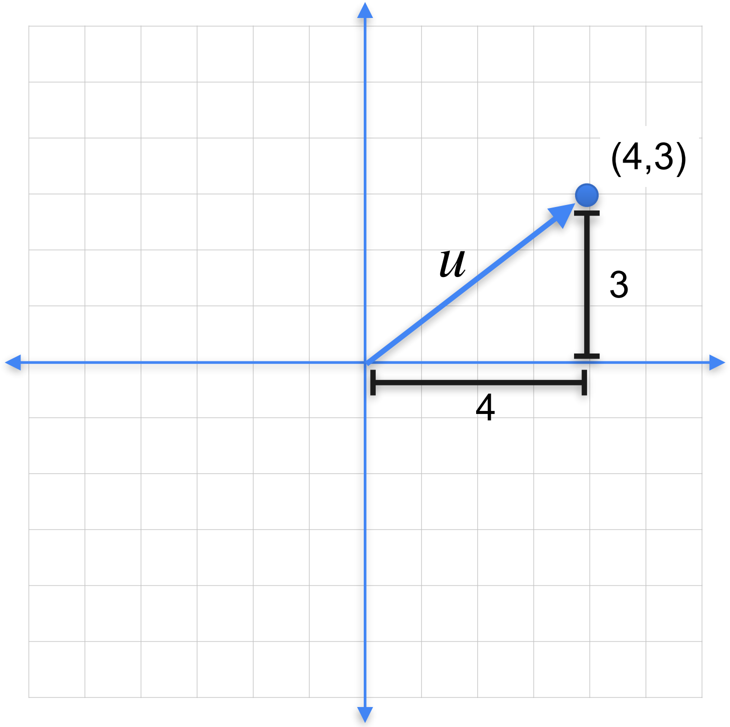


Norm of a vector

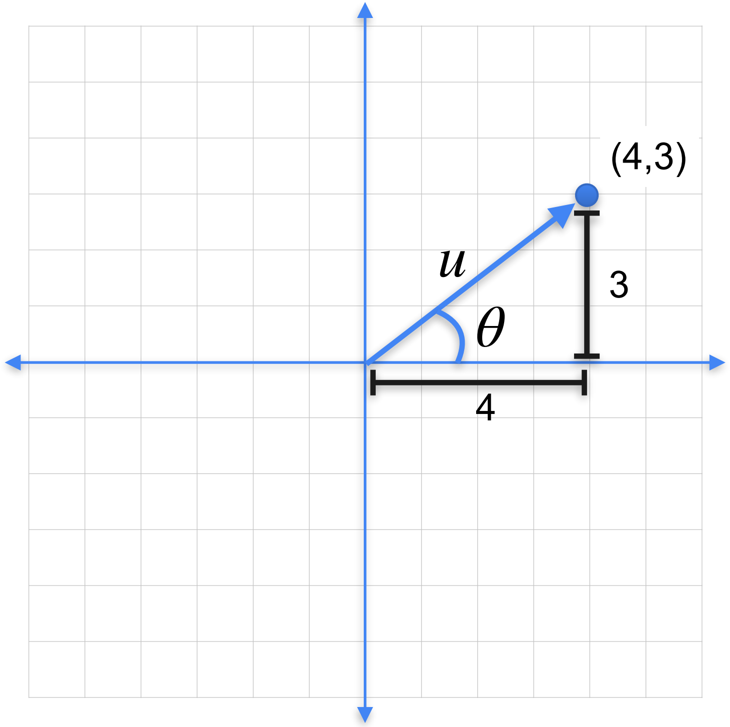


$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

Direction of a vector

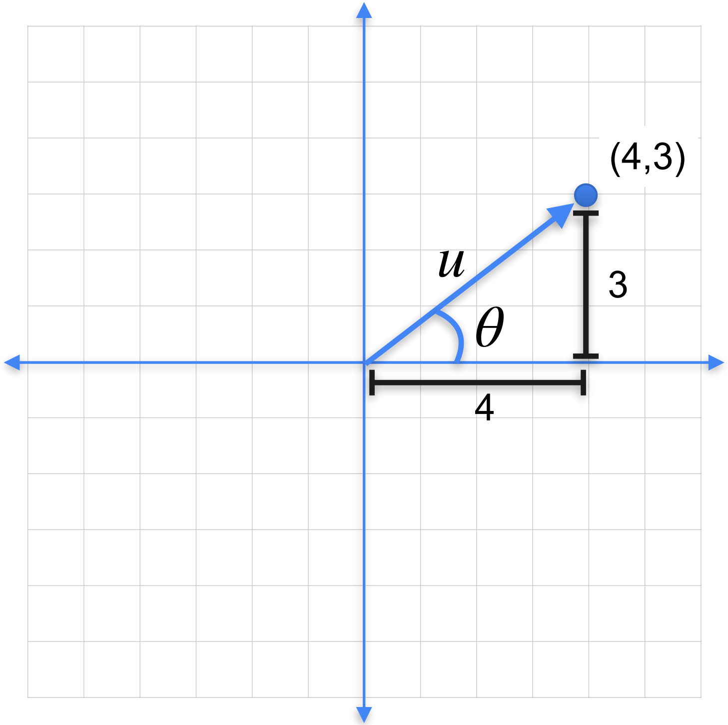


Direction of a vector



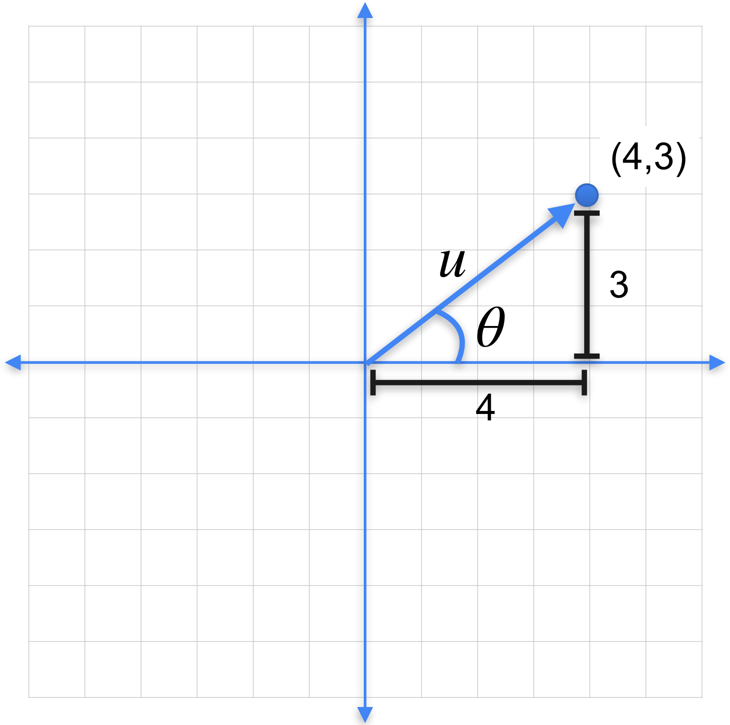
DeepLearning.AI

Direction of a vector



$$\tan(\theta) = \frac{3}{4}$$

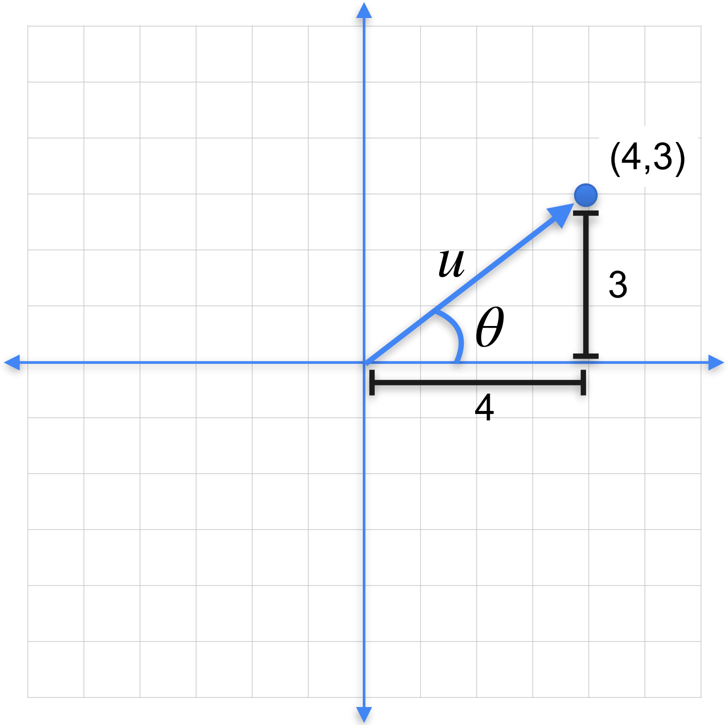
Direction of a vector



$$\tan(\theta) = \frac{3}{4}$$

$$\theta = \arctan(3/4) = 0.64$$

Direction of a vector



$$\tan(\theta) = \frac{3}{4}$$

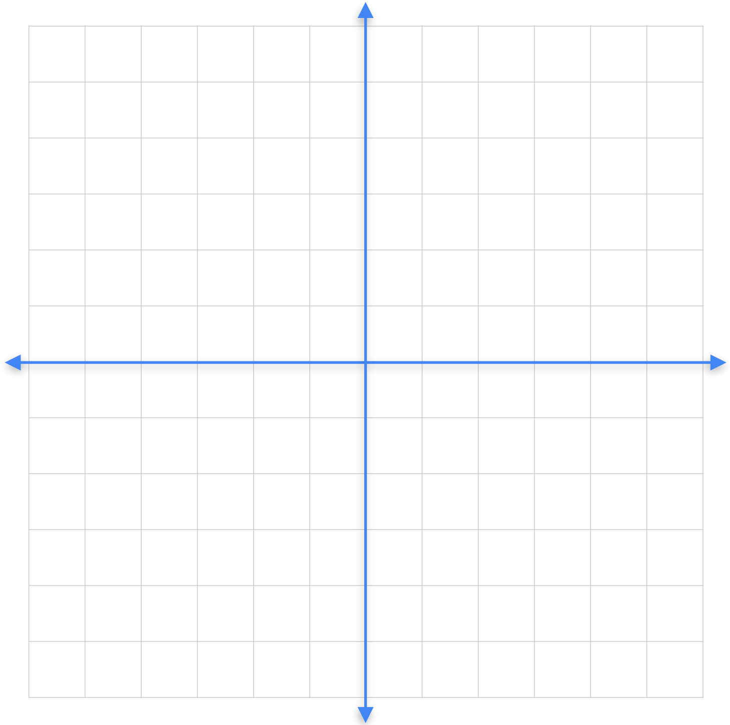
$$\theta = \arctan(3/4) = 0.64 \ = 36.87°$$

DeepLearning.AI

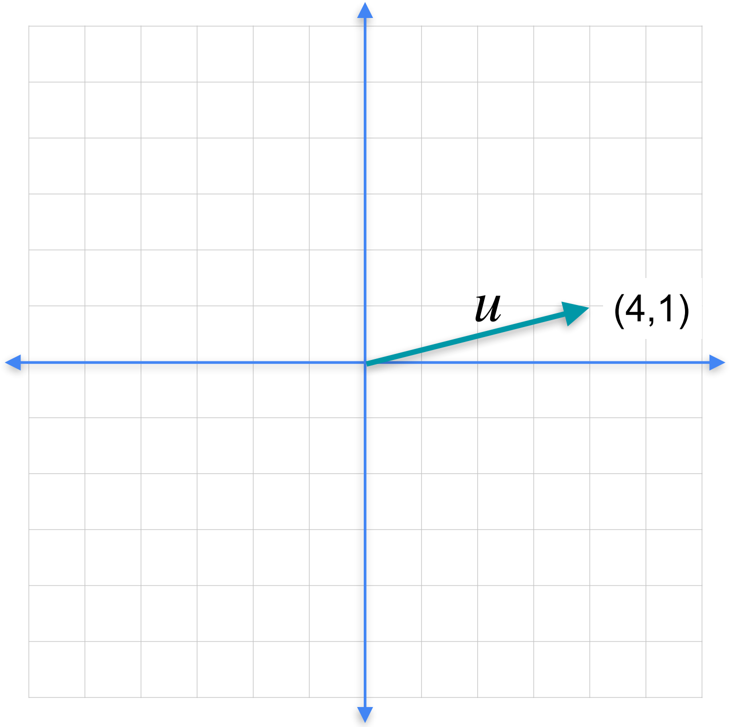
Vectors and Linear Transformations

Sum and difference of vectors

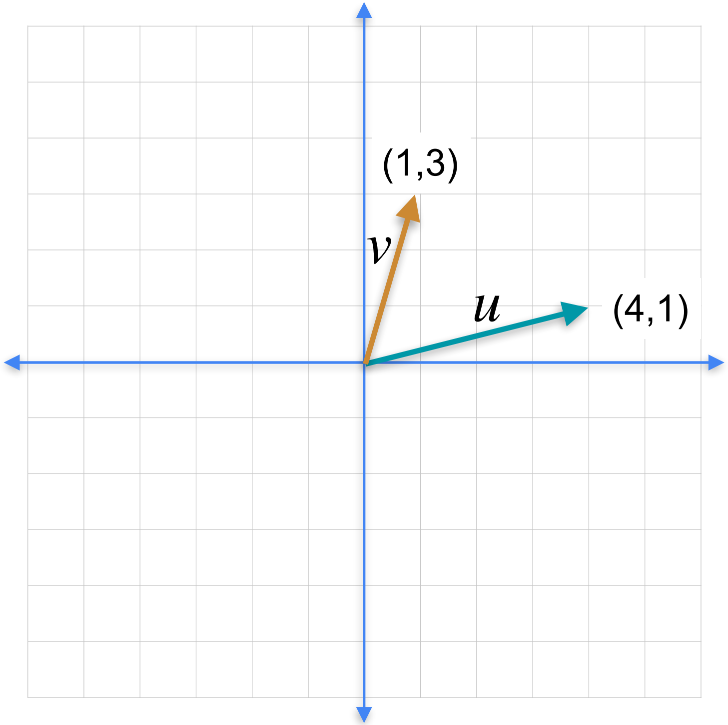
Sum of vectors



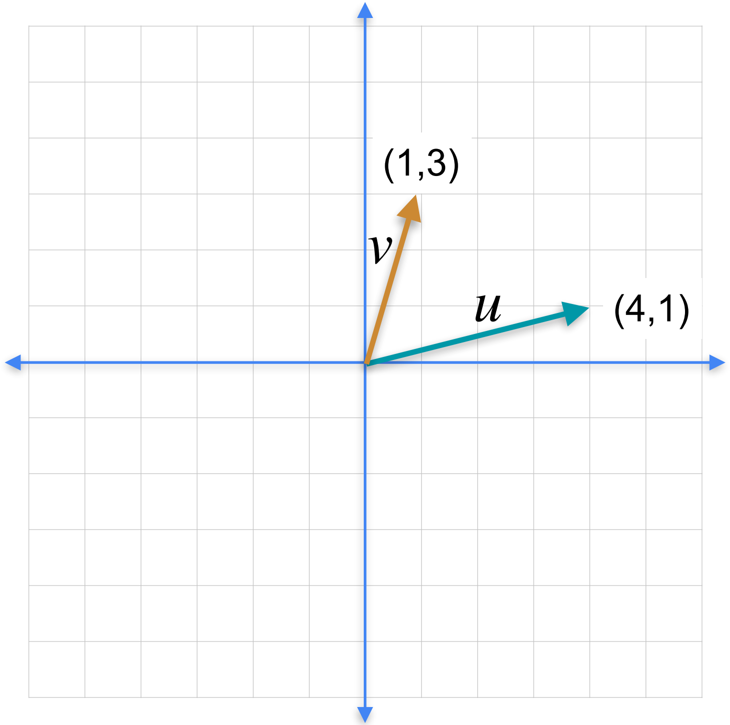
Sum of vectors



Sum of vectors

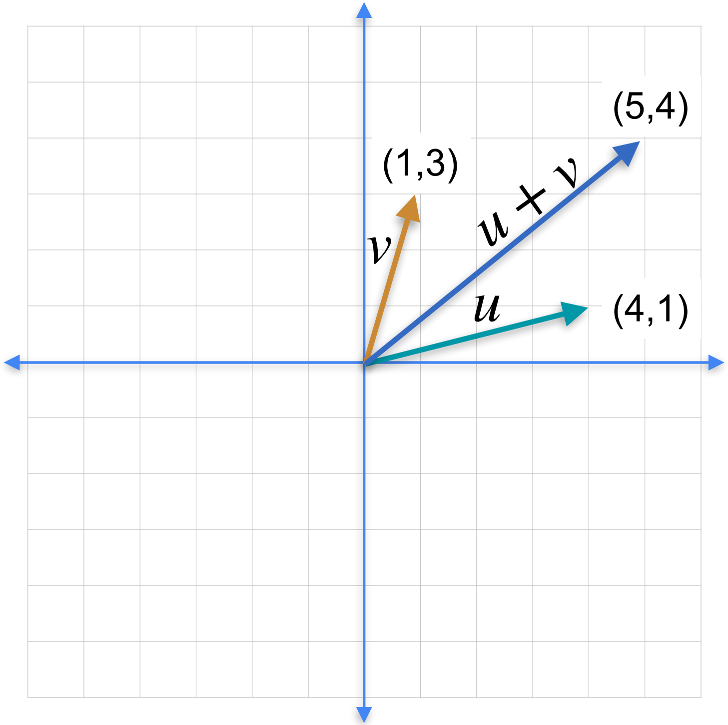


Sum of vectors



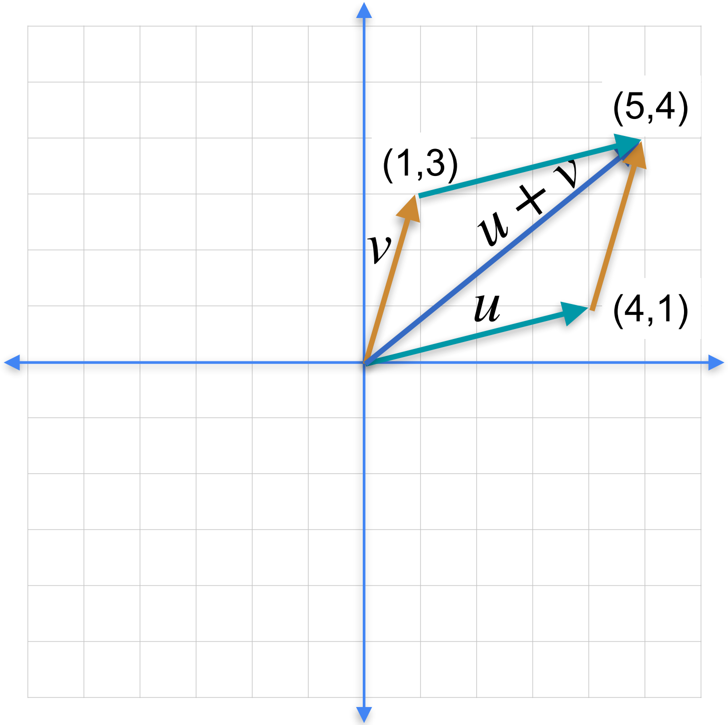
$$u + v = (4 + 1, 1 + 3) = (5, 4)$$

Sum of vectors



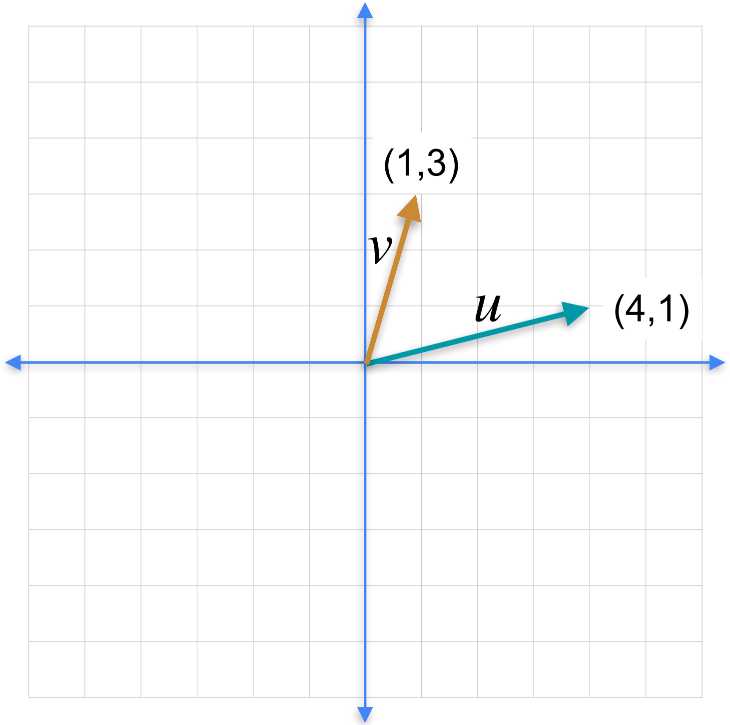
$$u + v = (4 + 1, 1 + 3) = (5,4)$$

Sum of vectors

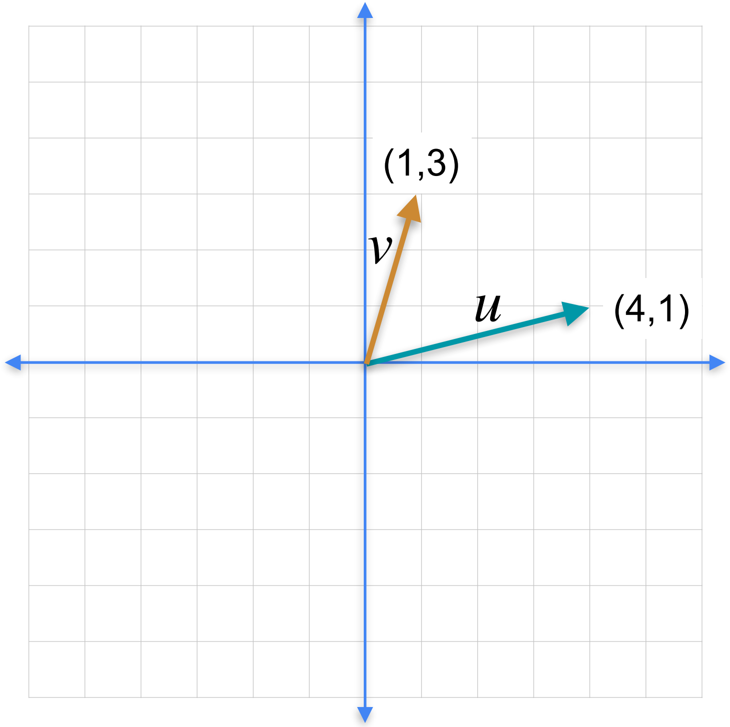


$$u + v = (4 + 1, 1 + 3) = (5,4)$$

Difference of vectors

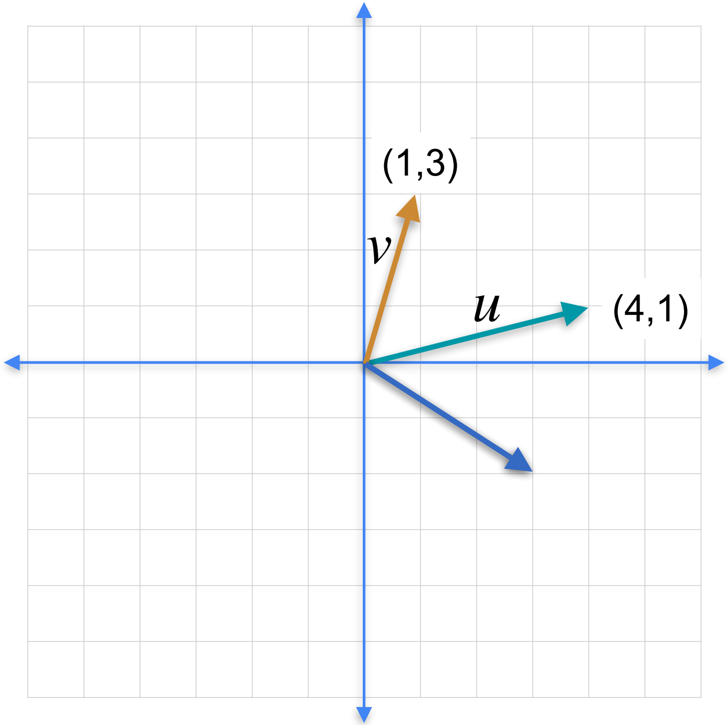


Difference of vectors



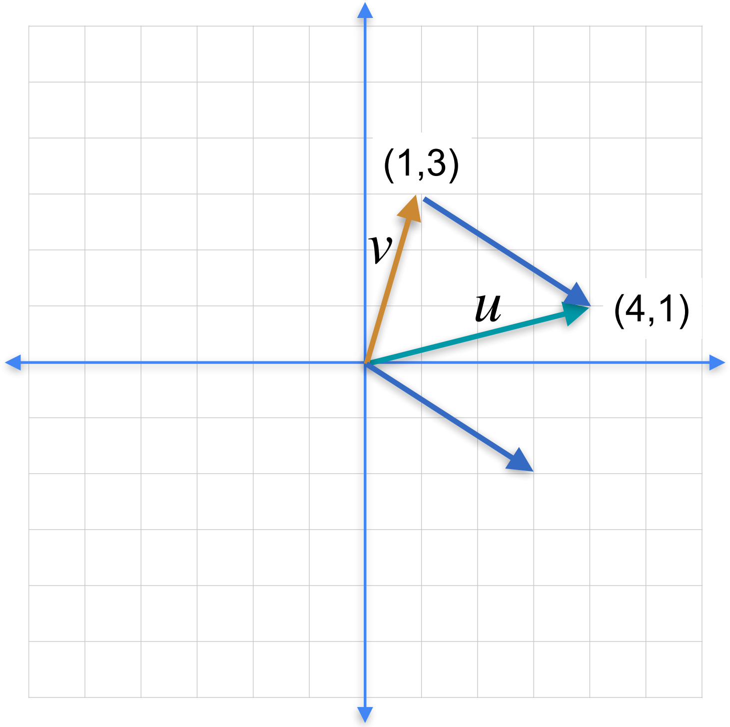
$$u - v = (4 - 1, 1 - 3) = (3, -2)$$

Difference of vectors



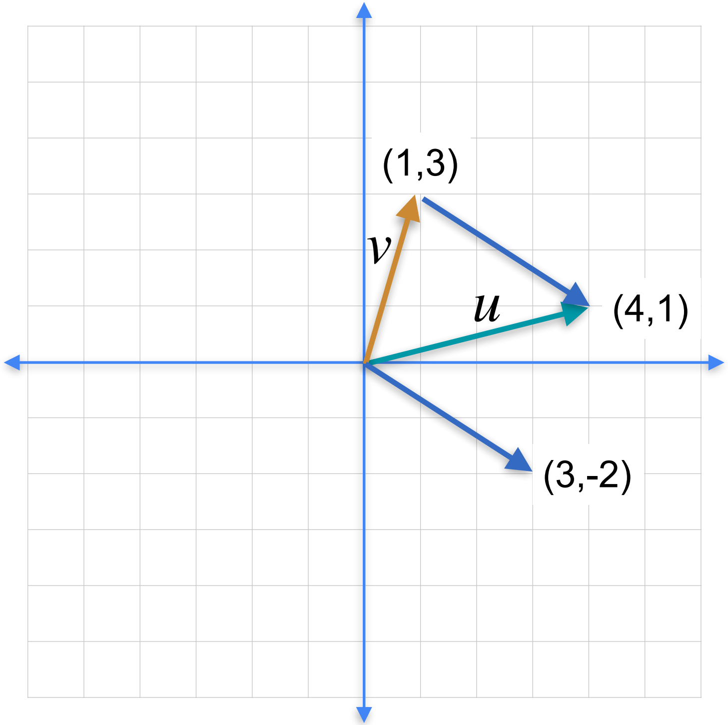
$$u - v = (4 - 1, 1 - 3) = (3, -2)$$

Difference of vectors



$$u - v = (4 - 1, 1 - 3) = (3, -2)$$

Difference of vectors



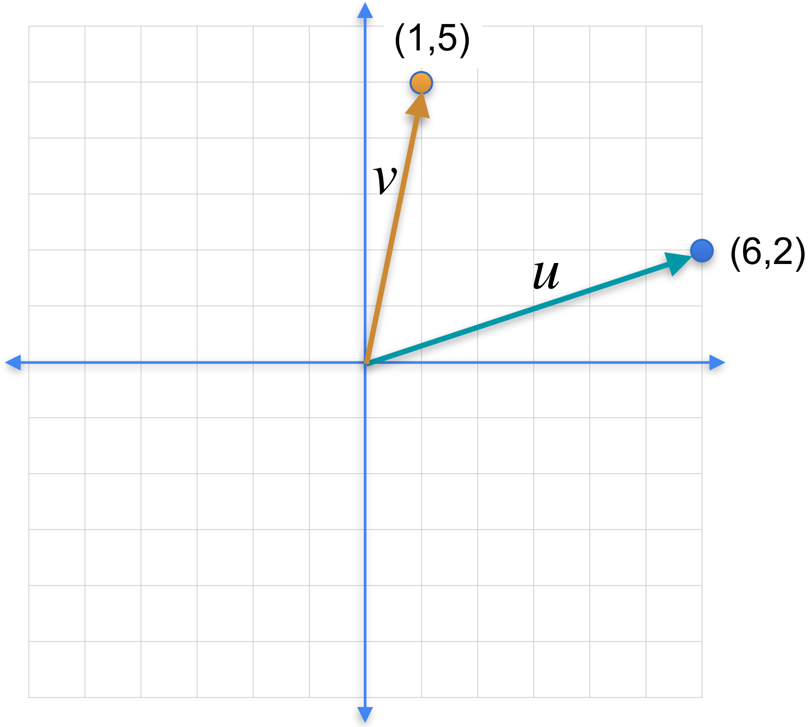
$$u - v = (4 - 1, 1 - 3) = (3, -2)$$

DeepLearning.AI

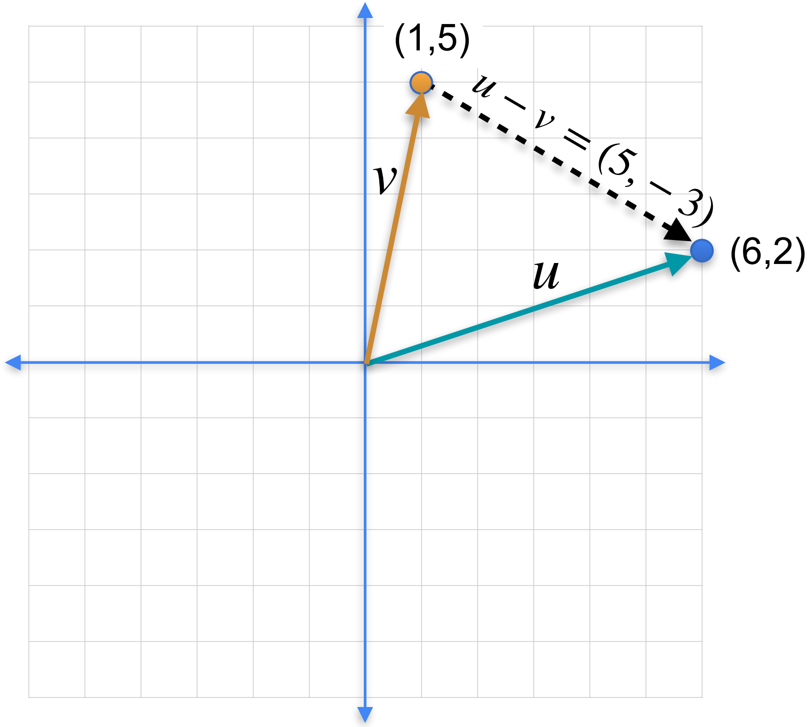
Vectors and Linear Transformations

Distance between vectors

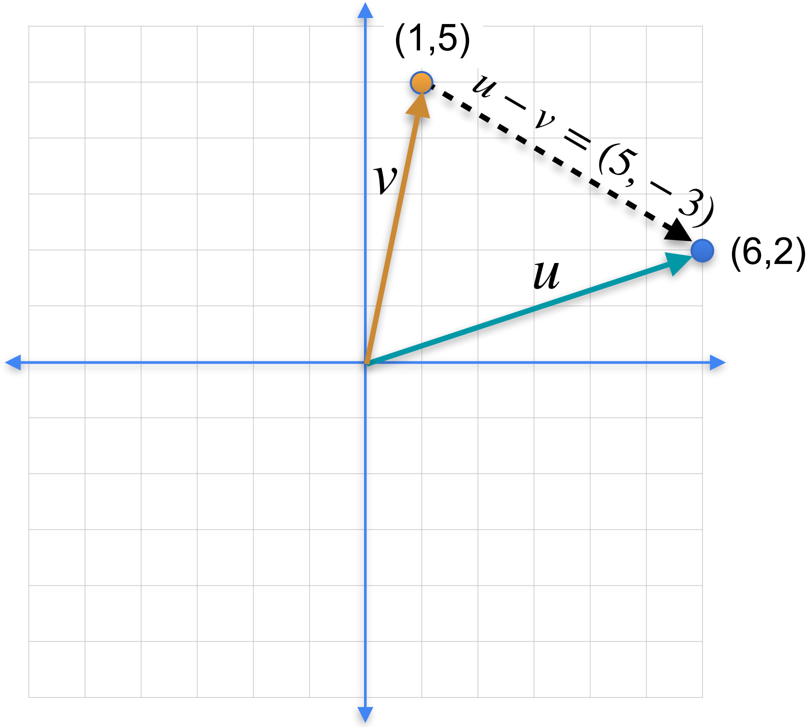
Distances



Distances



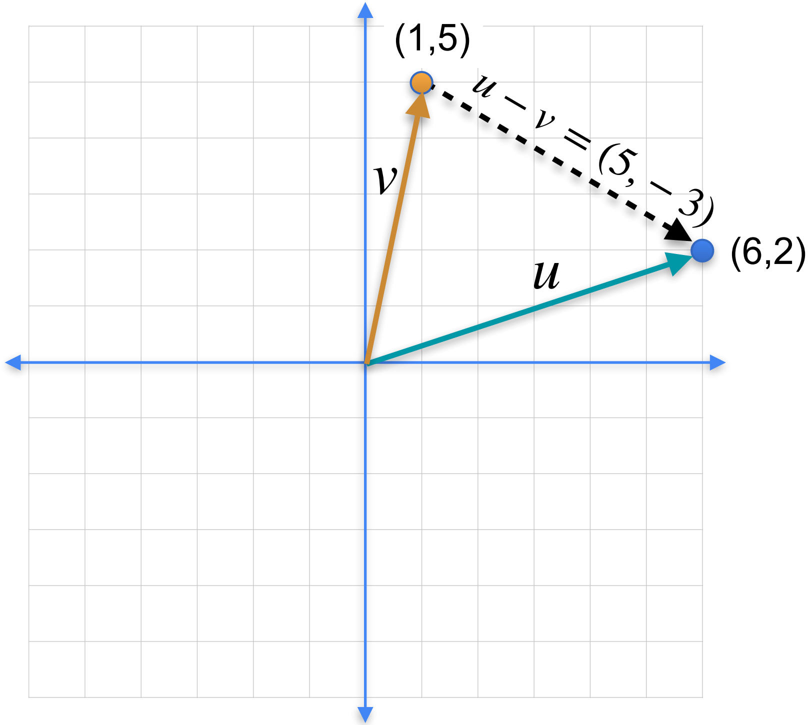
Distances



L1-distance

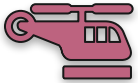
$$|u - v|_1 = |5| + |-3| = 8$$

Distances



L1-distance

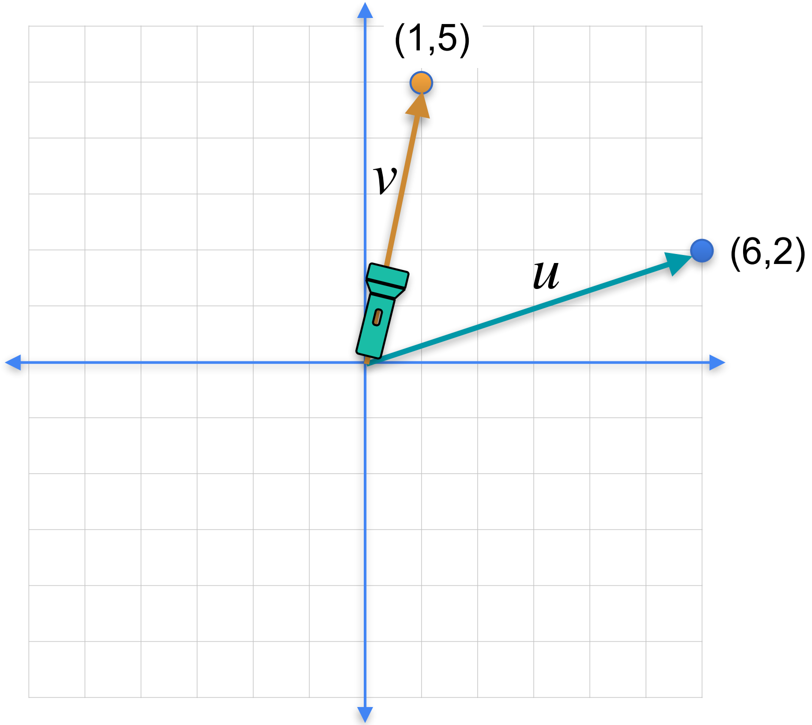
$$|u - v|_1 = |5| + |-3| = 8$$



L2-distance

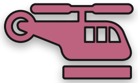
$$|u - v|_2 = \sqrt{5^2 + 3^2} = 5.83$$

Distances



L1-distance

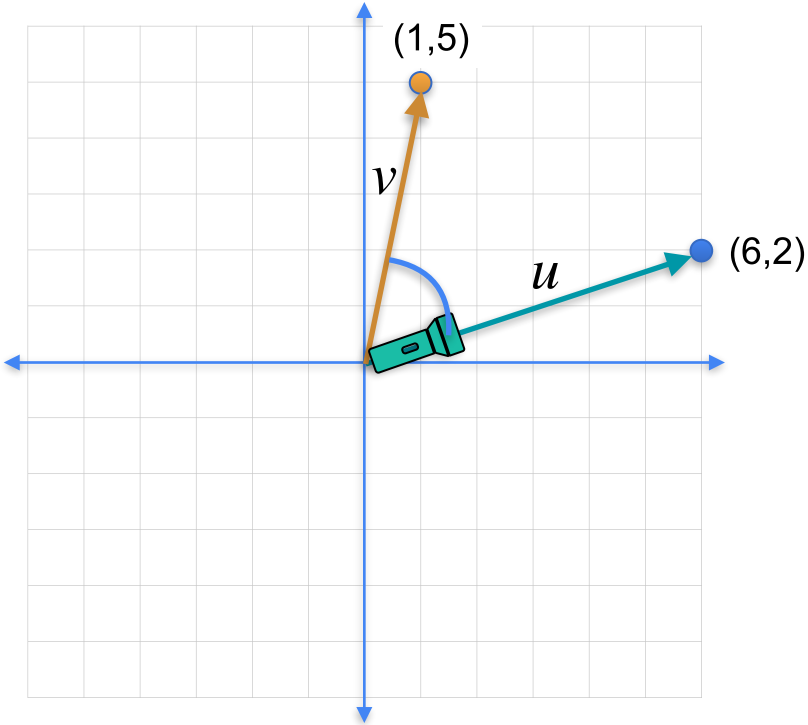
$$|u-v|_1 = |5| + |-3| = 8$$



L2-distance

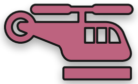
$$|u-v|_2 = \sqrt{5^2+3^2} = 5.83$$

Distances



L1-distance

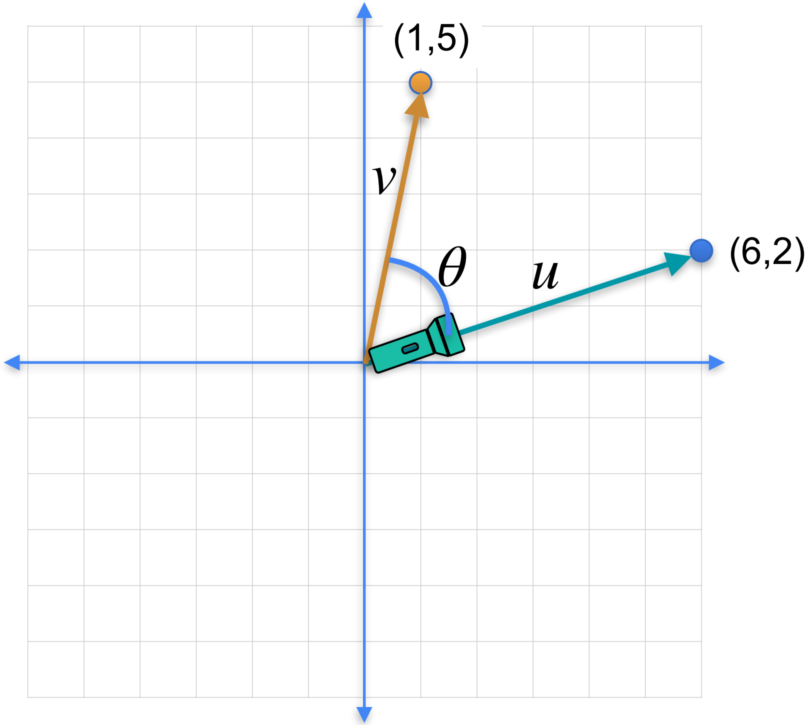
$$|u-v|_1 = |5| + |-3| = 8$$



L2-distance

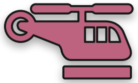
$$|u-v|_2 = \sqrt{5^2 + 3^2} = 5.83$$

Distances



L1-distance

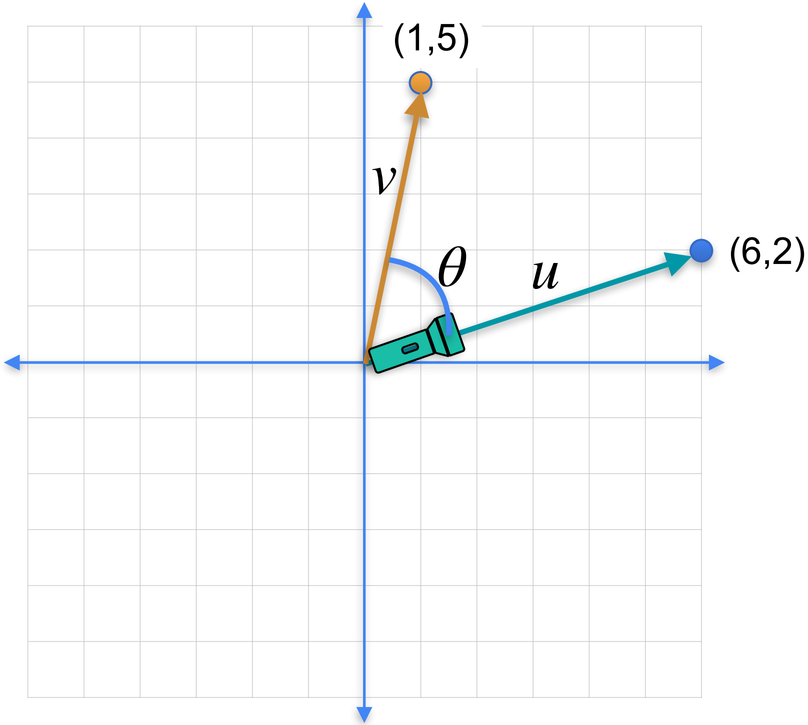
$$|u - v|_1 = |5| + |-3| = 8$$



L2-distance

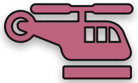
$$|u - v|_2 = \sqrt{5^2 + 3^2} = 5.83$$

Distances



L1-distance

$$|u - v|_1 = |5| + |-3| = 8$$



L2-distance

$$|u - v|_2 = \sqrt{5^2 + 3^2} = 5.83$$

Cosine distance

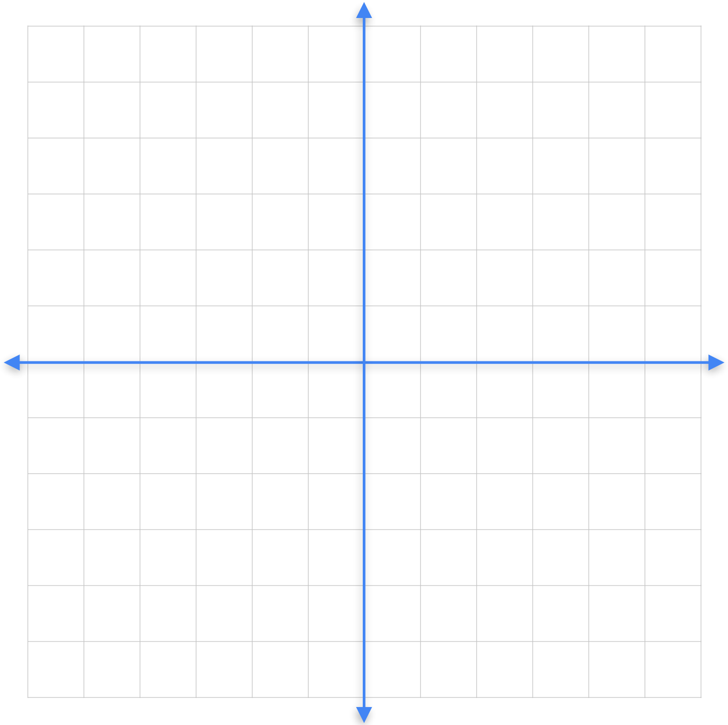
$$\cos(\theta)$$

DeepLearning.AI

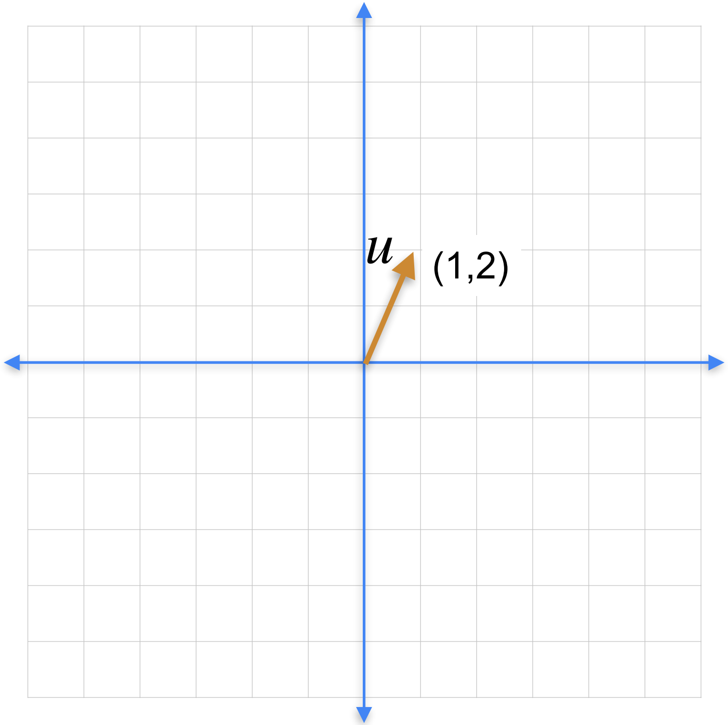
Vectors and Linear Transformations

Multiplying a vector by a scalar

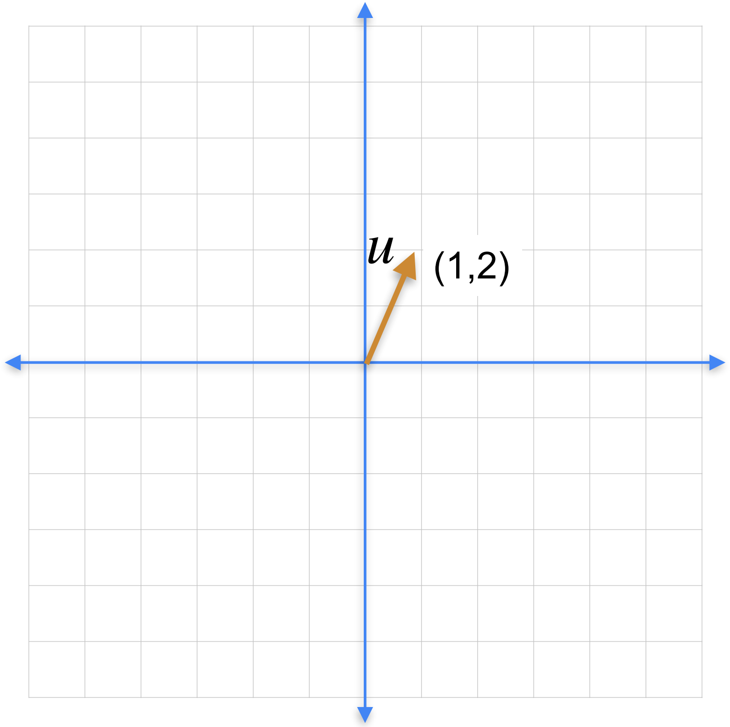
Multiplying a vector by a scalar



Multiplying a vector by a scalar

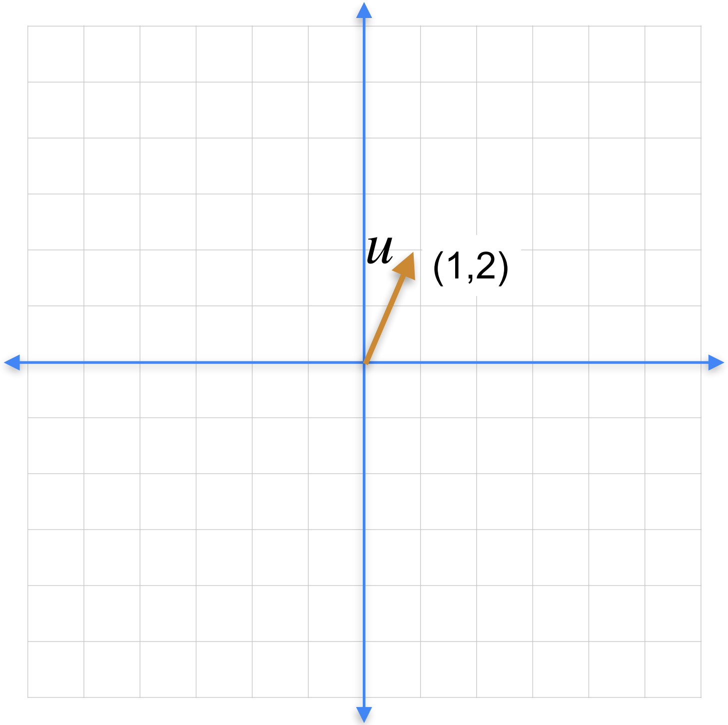


Multiplying a vector by a scalar



$$u = (1,2)$$

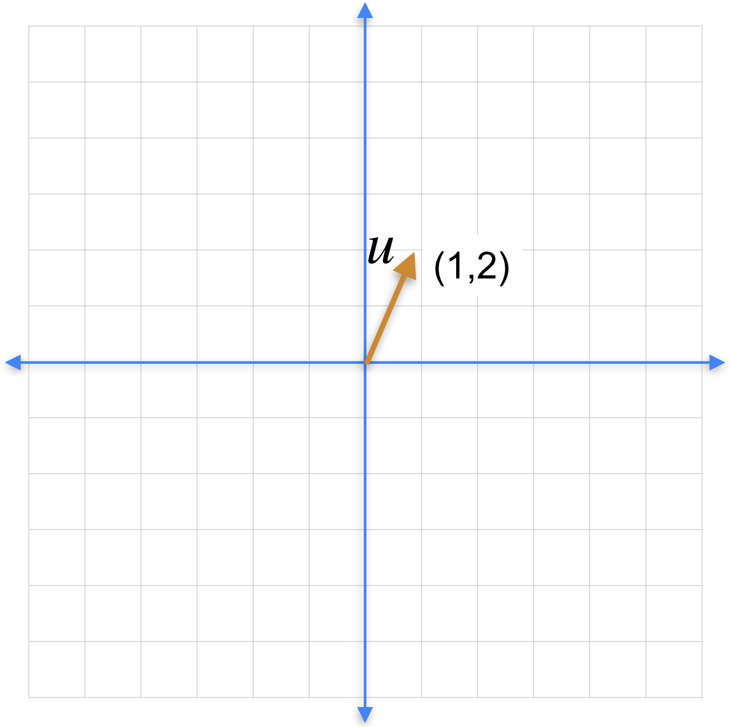
Multiplying a vector by a scalar



$u = (1,2)$

$\lambda = 3$

Multiplying a vector by a scalar

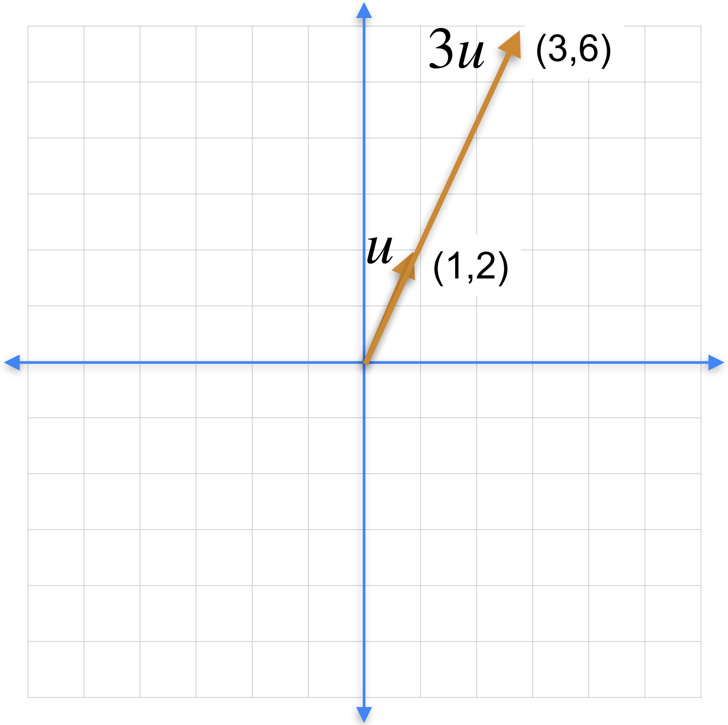


$$u = (1,2)$$

$$\lambda = 3$$

$$\lambda u = (3,6)$$

Multiplying a vector by a scalar

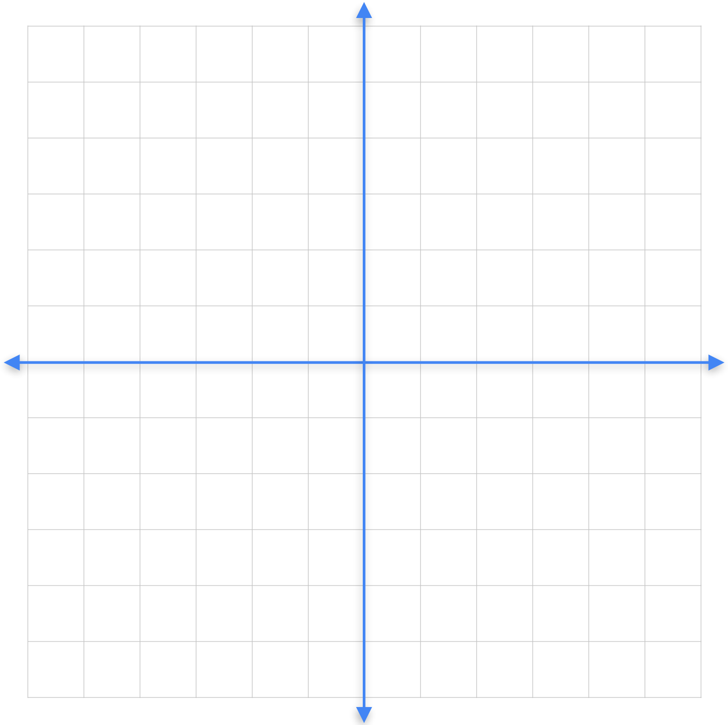


$$u = (1,2)$$

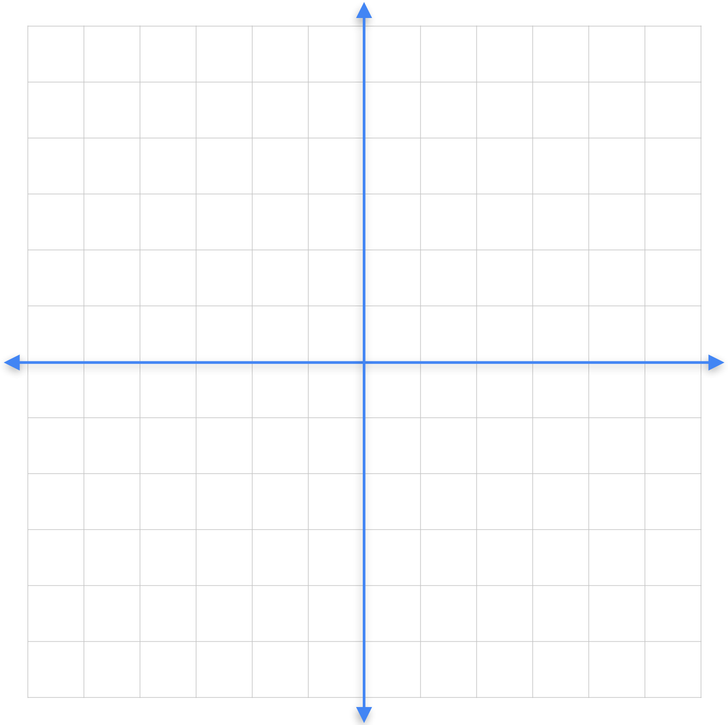
$$\lambda = 3$$

$$\lambda u = (3,6)$$

If the scalar is negative

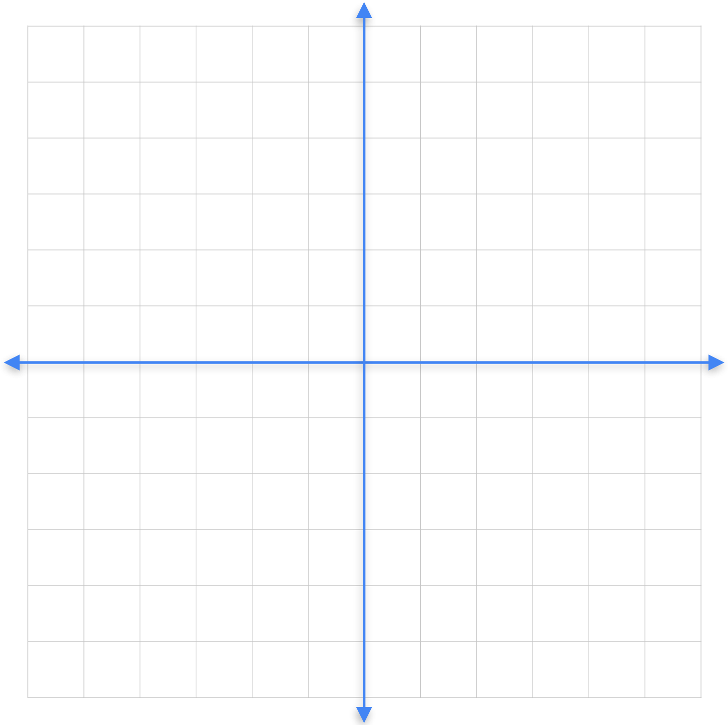


If the scalar is negative



$$u = (1,2)$$

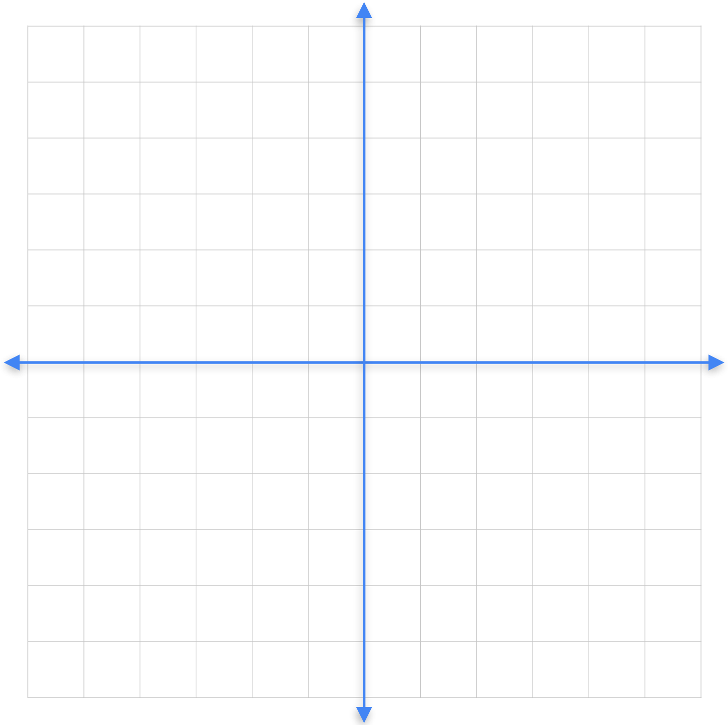
If the scalar is negative



$$u = (1,2)$$

$$\lambda = -2$$

If the scalar is negative

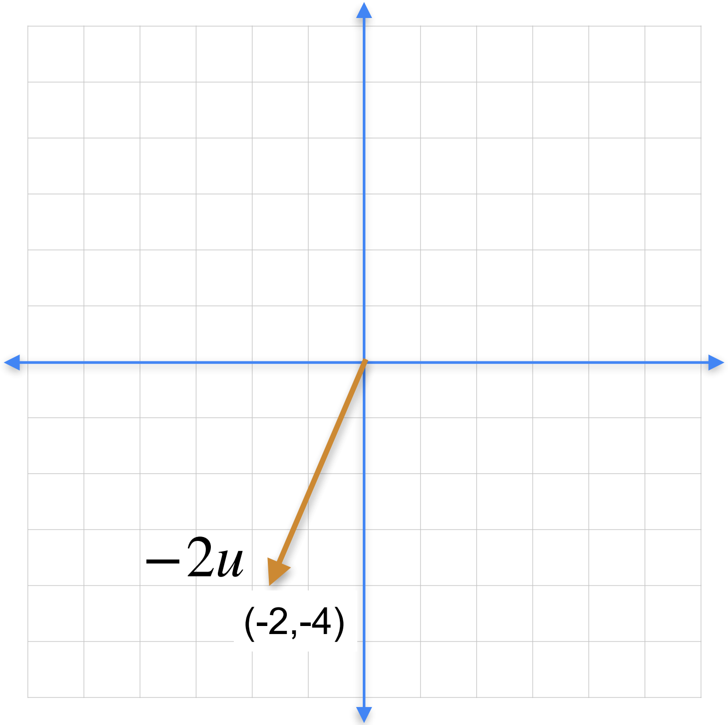


$$u = (1,2)$$

$$\lambda = -2$$

$$\lambda u = (-2, -4)$$

If the scalar is negative

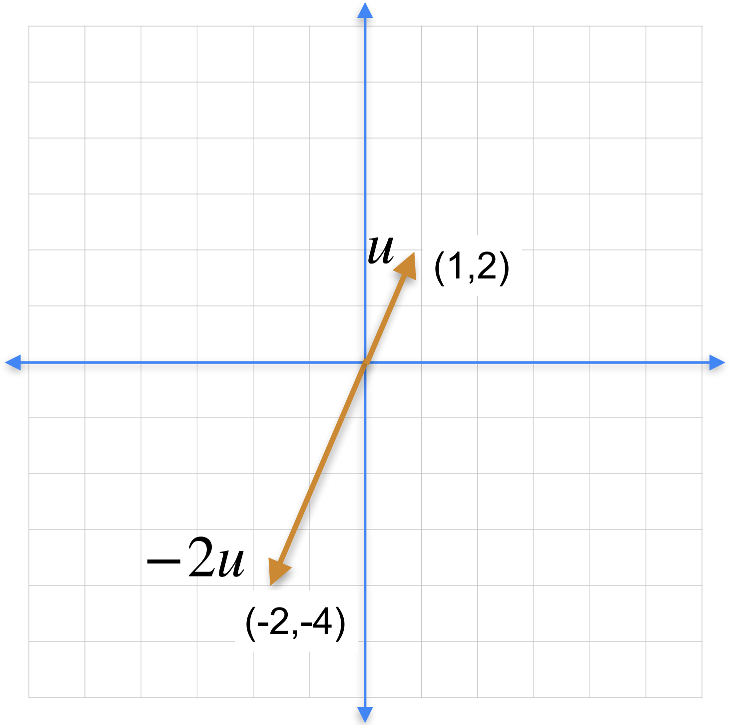


$$u = (1,2)$$

$$\lambda = -2$$

$$\lambda u = (-2, -4)$$

If the scalar is negative



$$u = (1,2)$$

$$\lambda = -2$$

$$\lambda u = (-2, -4)$$

Vectors and Linear Transformations

The dot product

A shortcut for linear operations

A shortcut for linear operations

Quantities
2 apples
4 bananas
1 cherry

A shortcut for linear operations

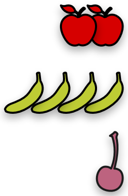
Quantities	**Prices**
2 apples	apples: $3
4 bananas	bananas: $5
1 cherry	cherries: $2

A shortcut for linear operations

Quantities	**Prices**	**Total price**
2 apples	apples: $3	
4 bananas	bananas: $5	
1 cherry	cherries: $2	

A shortcut for linear operations

Quantities
2 apples
4 bananas
1 cherry



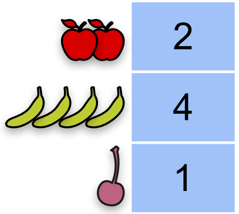
$$\begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}$$

Prices
apples: $3
bananas: $5
cherries: $2

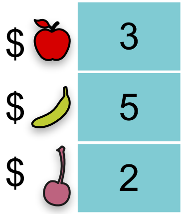
Total price

A shortcut for linear operations

Quantities
2 apples
4 bananas
1 cherry



Prices
apples: $3
bananas: $5
cherries: $2



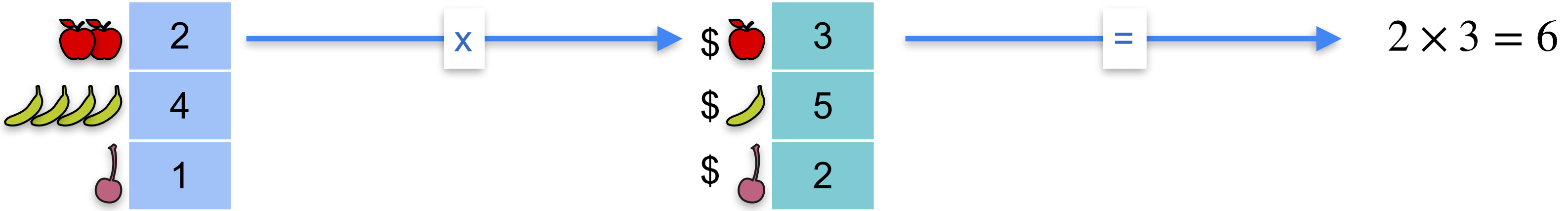
Total price

A shortcut for linear operations

Quantities
2 apples
4 bananas
1 cherry

Prices
apples: $3
bananas: $5
cherries: $2

Total price

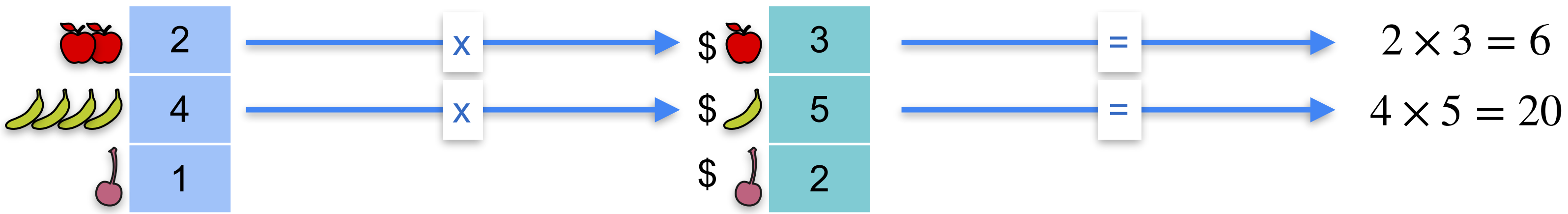


A shortcut for linear operations

Quantities
2 apples
4 bananas
1 cherry

Prices
apples: $3
bananas: $5
cherries: $2

Total price

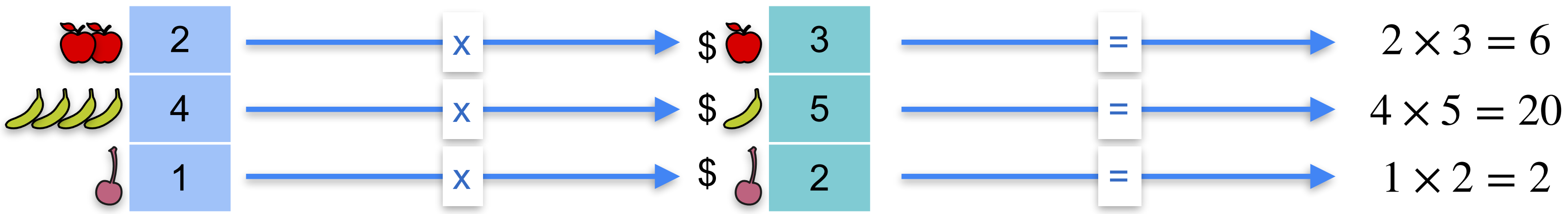


A shortcut for linear operations

Quantities
2 apples
4 bananas
1 cherry

Prices
apples: $3
bananas: $5
cherries: $2

Total price

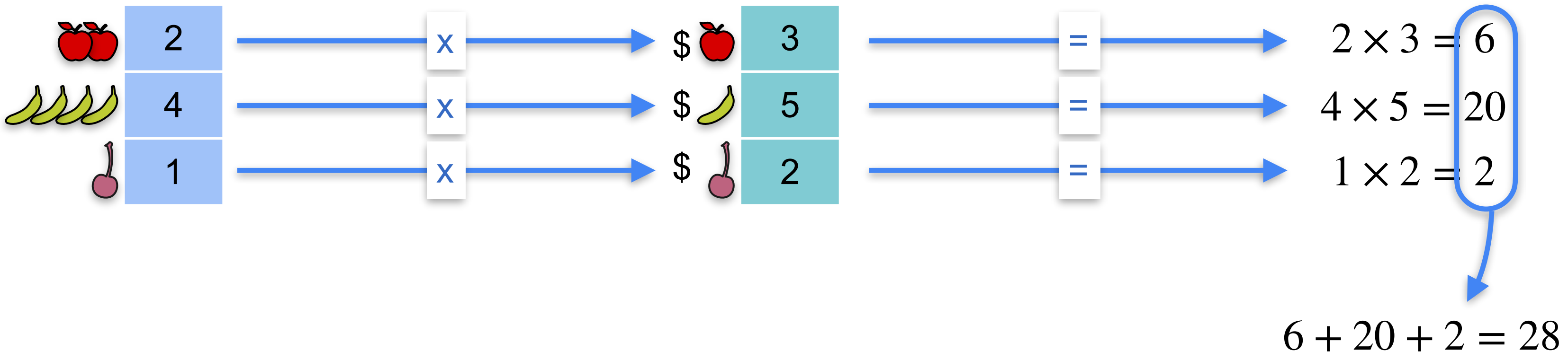


A shortcut for linear operations

Quantities
2 apples
4 bananas
1 cherry

Prices
apples: $3
bananas: $5
cherries: $2

Total price

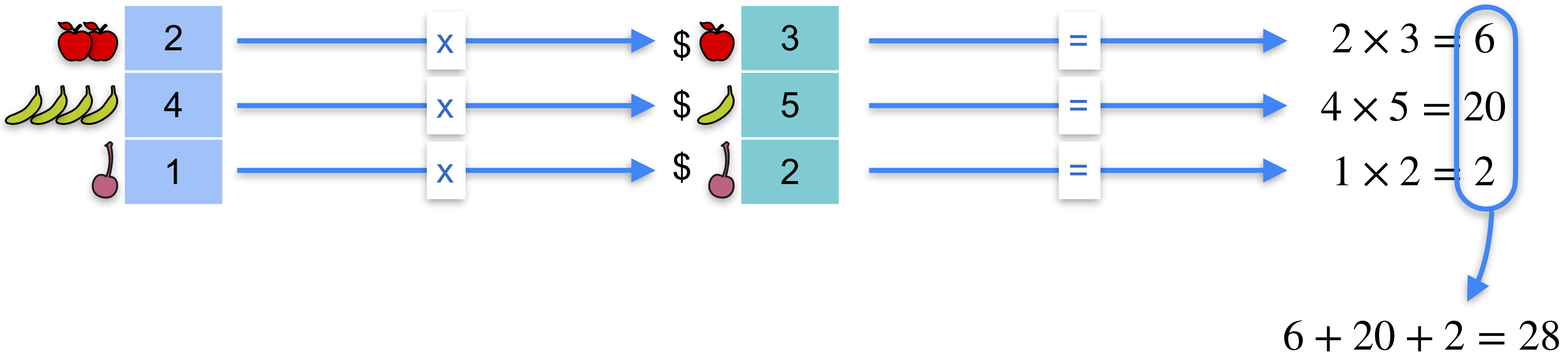


A shortcut for linear operations

Quantities
2 apples
4 bananas
1 cherry

Prices
apples: $3
bananas: $5
cherries: $2

Total price
$28



The dot product

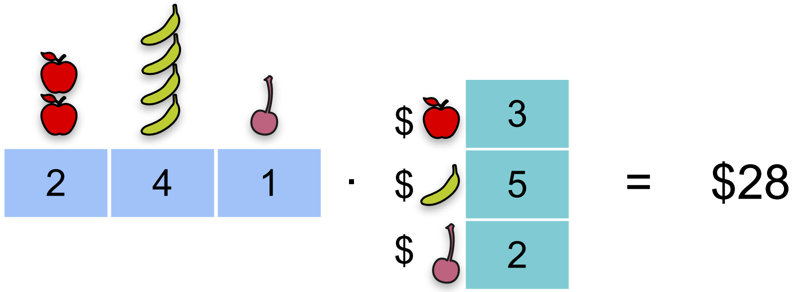
$$\begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix} \cdot \begin{array}{c} \$ \\ \$ \\ \$ \end{array}\begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} = \$28$$

The dot product

$$\begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix} \cdot \begin{array}{c} \$ \\ \$ \\ \$ \end{array}\begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} = \$28$$

$$2 \cdot 3 + 4 \cdot 5 + 1 \cdot 2 = 28$$

The dot product



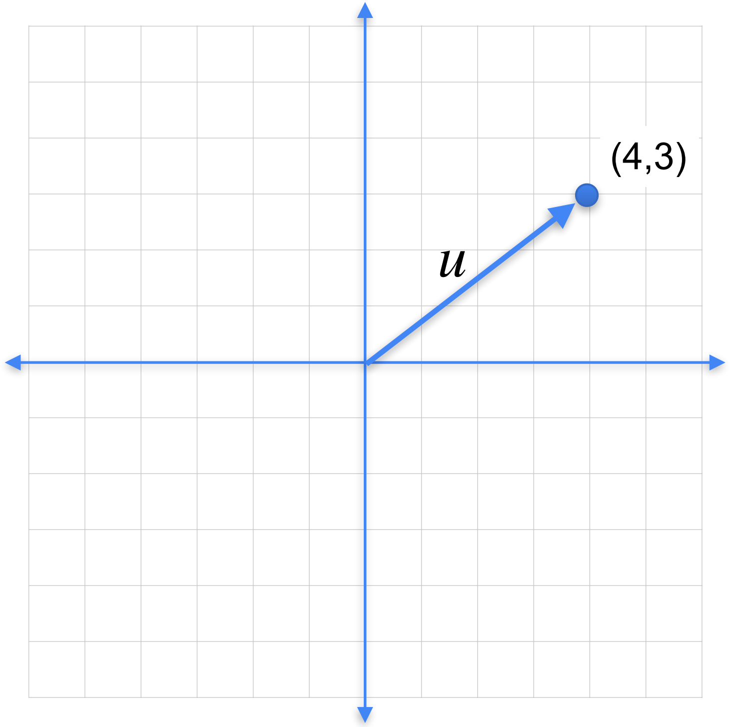
$$2 \cdot 3 + 4 \cdot 5 + 1 \cdot 2 = 28$$

The dot product

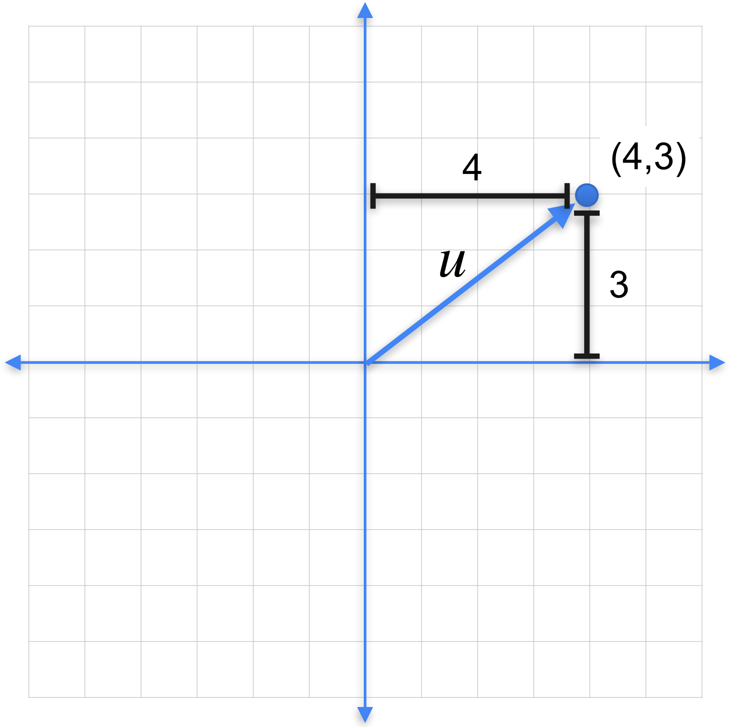
$$\begin{bmatrix} 2 & 4 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} = 28$$

$$2 \cdot 3 + 4 \cdot 5 + 1 \cdot 2 = 28$$

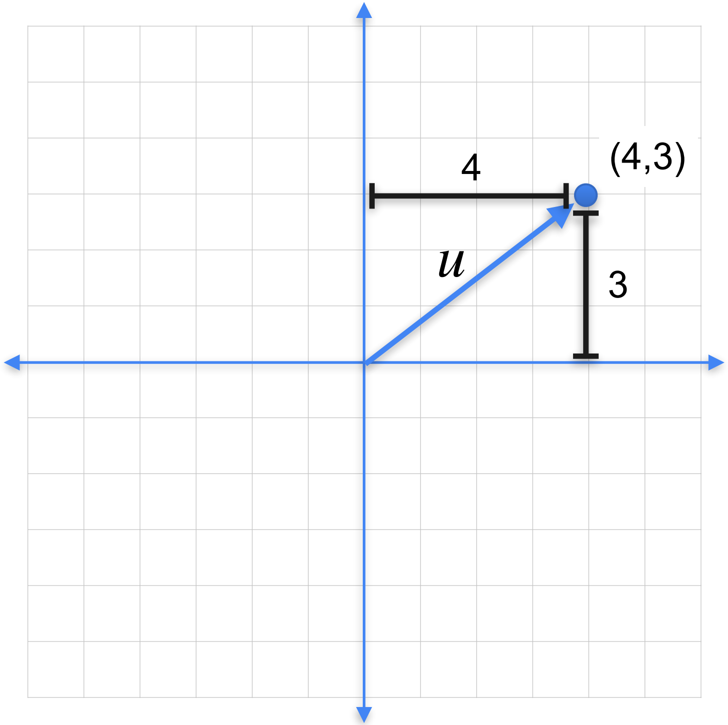
Norm of a vector using dot product



Norm of a vector using dot product

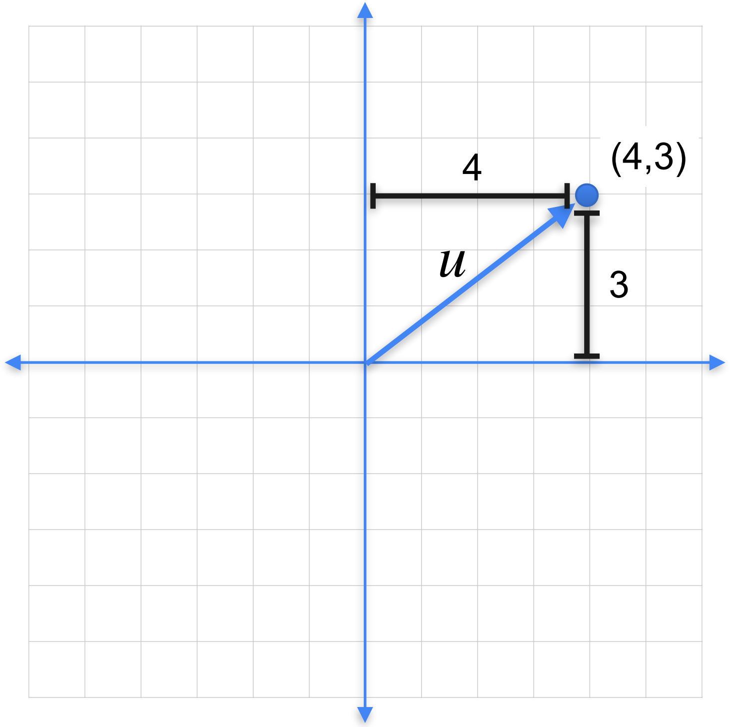


Norm of a vector using dot product



$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

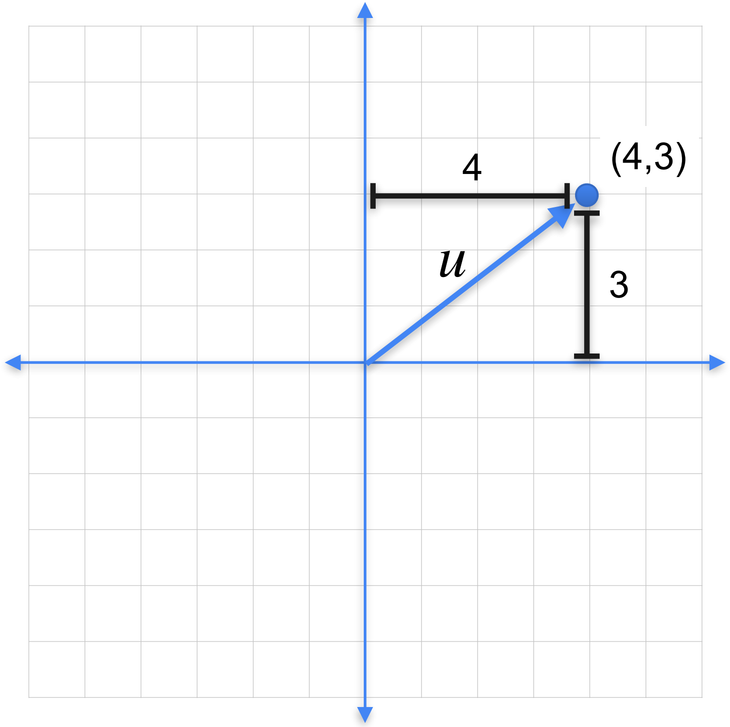
Norm of a vector using dot product



$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

$$\begin{bmatrix} 4 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = 25$$

Norm of a vector using dot product

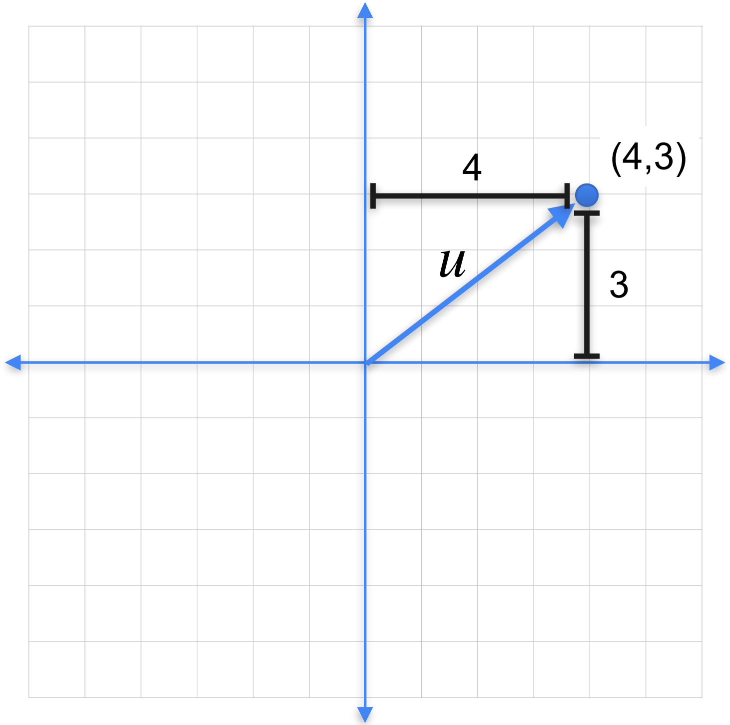


$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

$$\begin{bmatrix} 4 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = 25$$

$$L2-norm = \sqrt{dot\ \ product(u,u)}$$

Norm of a vector using dot product



$$\sqrt{4^2+3^2}=\sqrt{25}=5$$

$$\begin{bmatrix}4 & 3\end{bmatrix}\begin{bmatrix}4\\3\end{bmatrix}=25$$

$$L2-norm=\sqrt{dot\ \ product(u,u)}$$

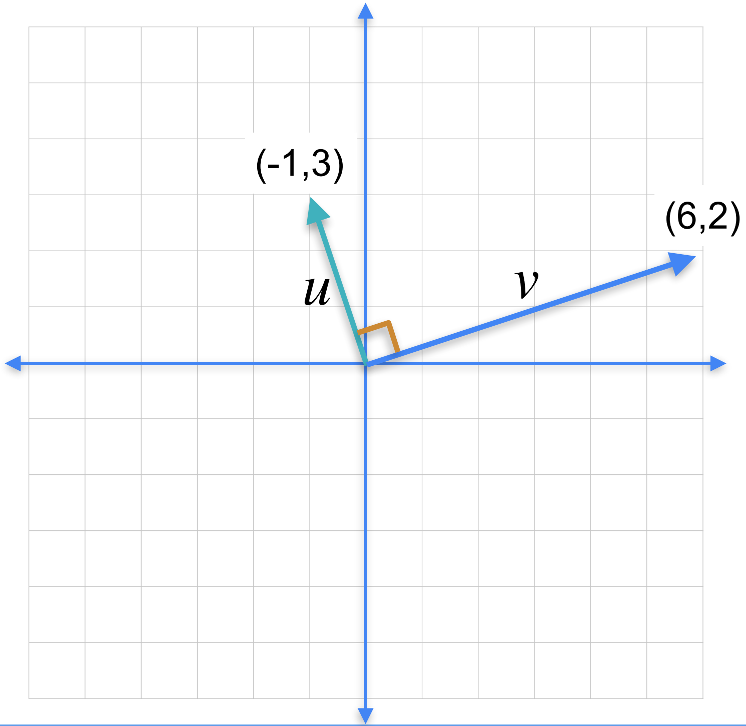
$$|u|_2=\sqrt{\langle u,u\rangle}$$

DeepLearning.AI

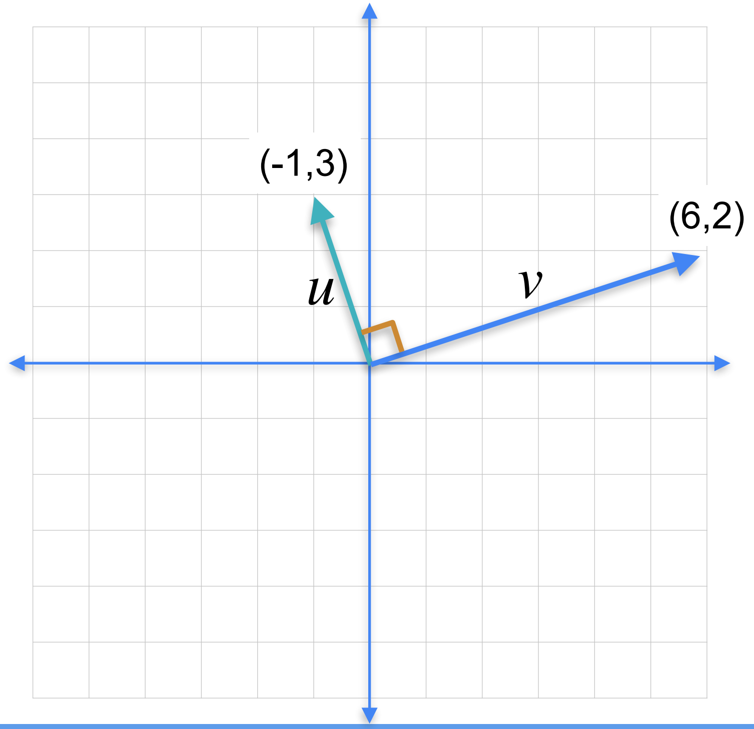
Vectors and Linear Transformations

Geometric dot product

Orthogonal vectors have dot product 0

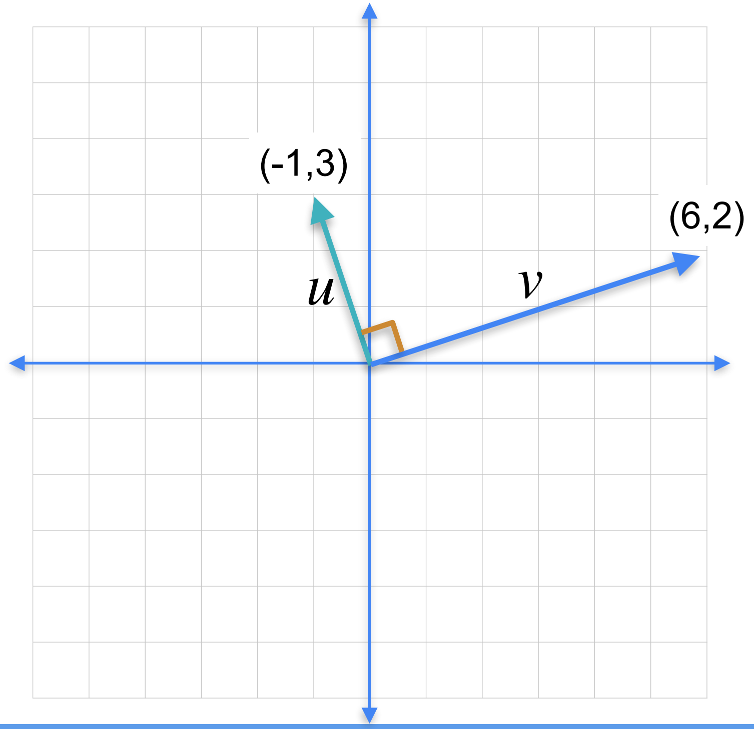


Orthogonal vectors have dot product 0



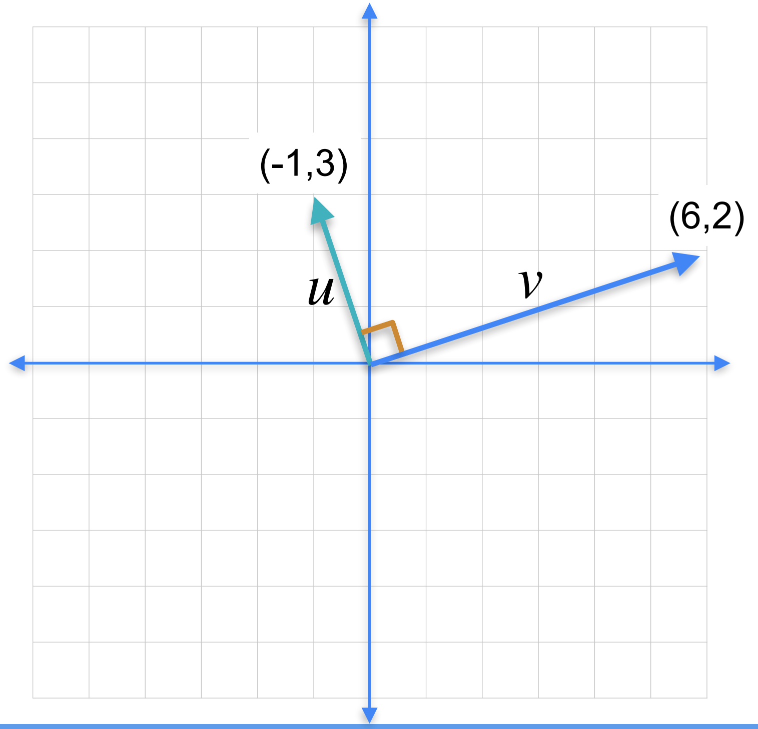
6	2

Orthogonal vectors have dot product 0



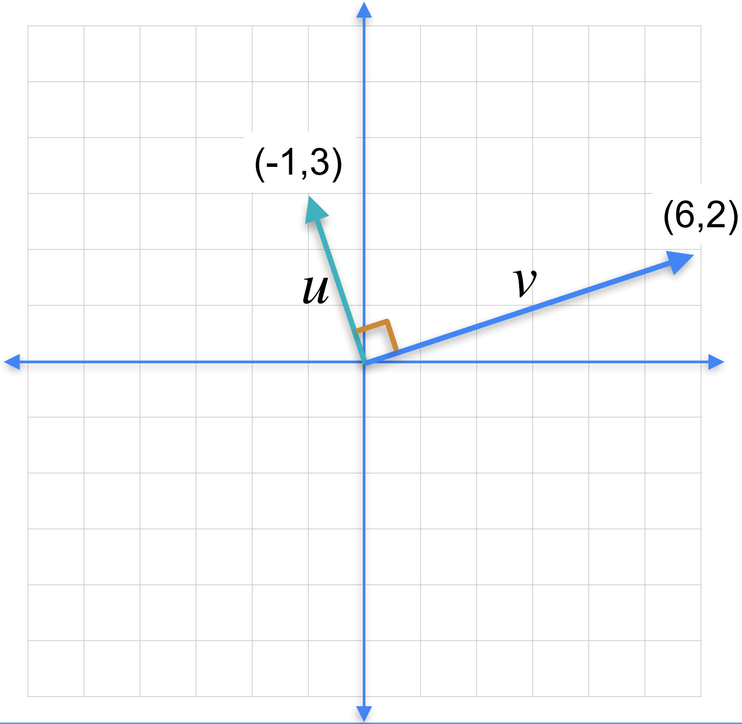
$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

Orthogonal vectors have dot product 0



$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix} = 0$$

Orthogonal vectors have dot product 0



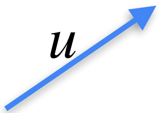
$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

$$\langle u, v \rangle = 0$$

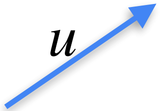
DeepLearning.AI

The dot product

The dot product

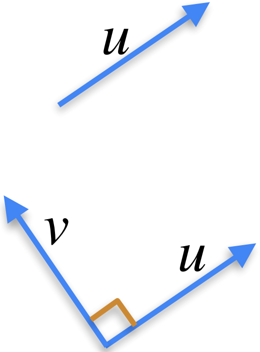


The dot product



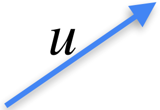
$$\langle u, u \rangle = |u|^2$$

The dot product

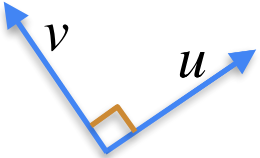


$$\langle u, u \rangle = |u|^2$$

The dot product

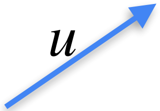


$$\langle u, u \rangle = |u|^2$$

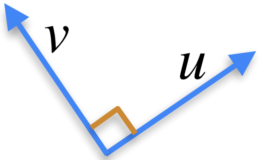


$$\langle u, v \rangle = 0$$

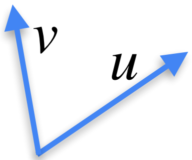
The dot product



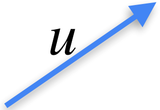
$$\langle u, u \rangle = |u|^2$$



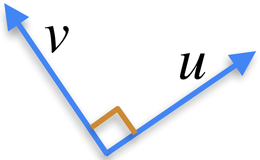
$$\langle u, v \rangle = 0$$



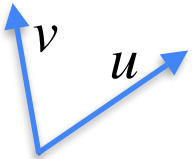
The dot product



$$\langle u, u \rangle = |u|^2$$



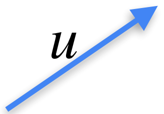
$$\langle u, v \rangle = 0$$



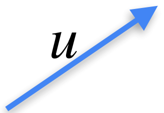
$$\langle u, v \rangle = ?$$

The dot product

The dot product

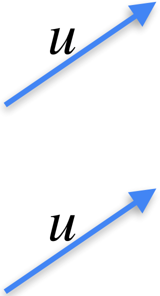


The dot product



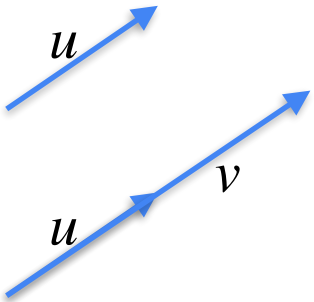
$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

The dot product



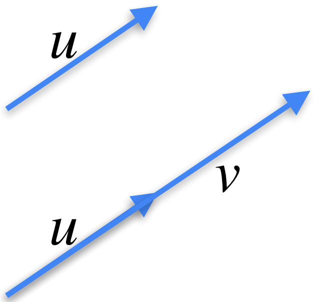
$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

The dot product



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

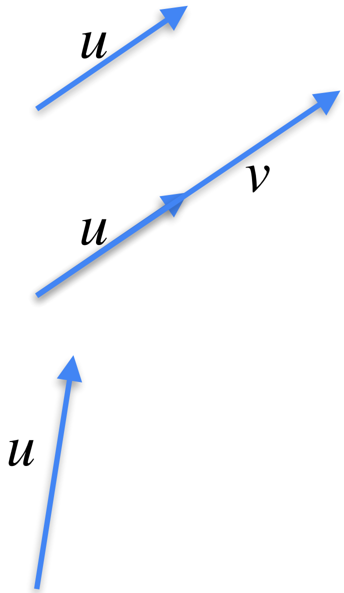
The dot product



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v \rangle = |u| \cdot |v|$$

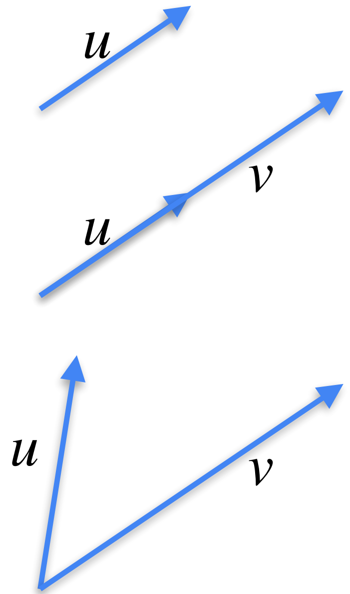
The dot product



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v \rangle = |u| \cdot |v|$$

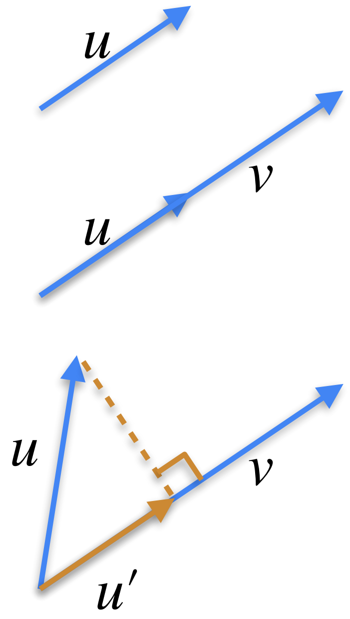
The dot product



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v \rangle = |u| \cdot |v|$$

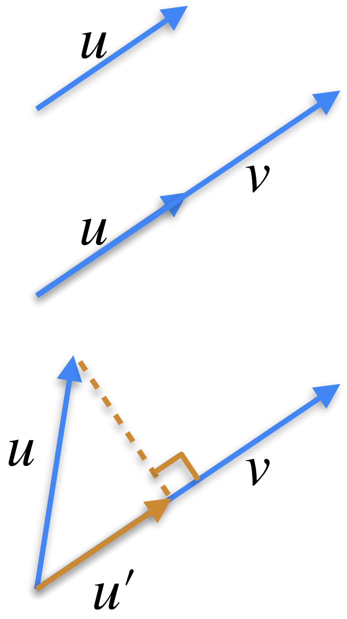
The dot product



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v \rangle = |u| \cdot |v|$$

The dot product

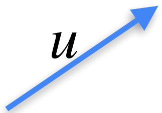


$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

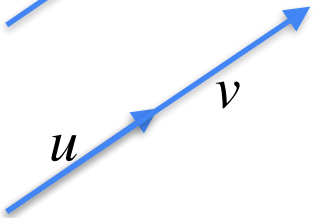
$$\langle u, v \rangle = |u| \cdot |v|$$

$$\langle u, v \rangle = |u'| \cdot |v|$$

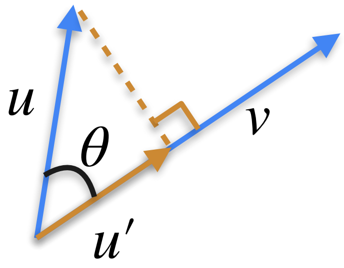
The dot product



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

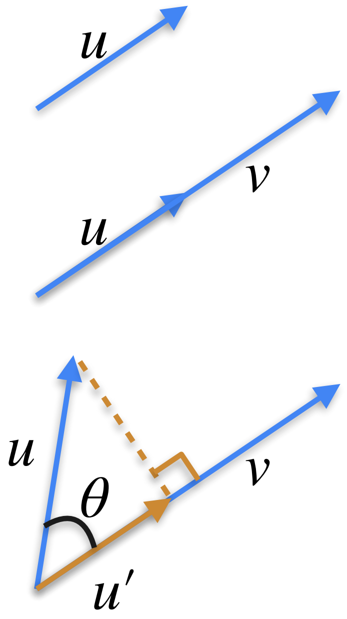


$$\langle u, v \rangle = |u| \cdot |v|$$



$$\langle u, v \rangle = |u'| \cdot |v|$$

The dot product



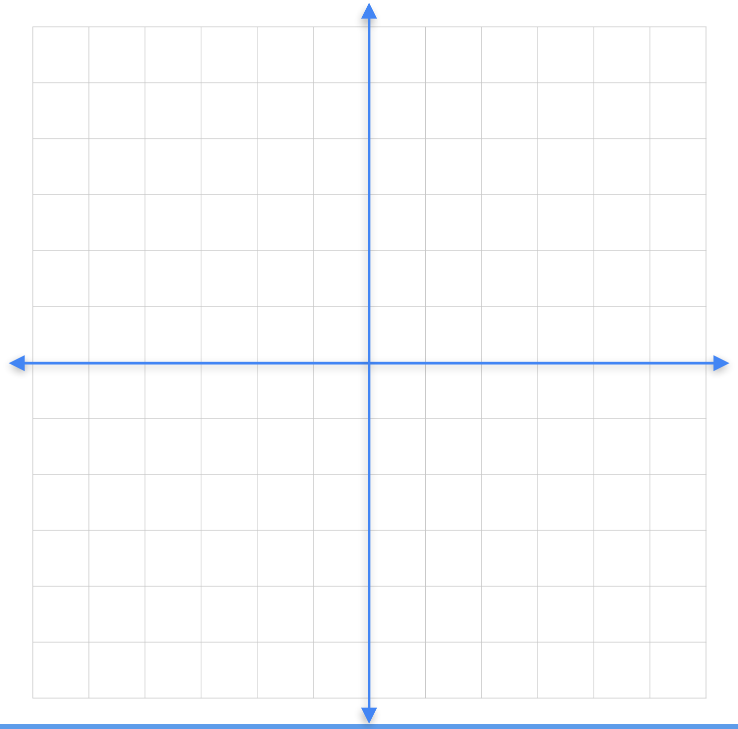
$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v \rangle = |u| \cdot |v|$$

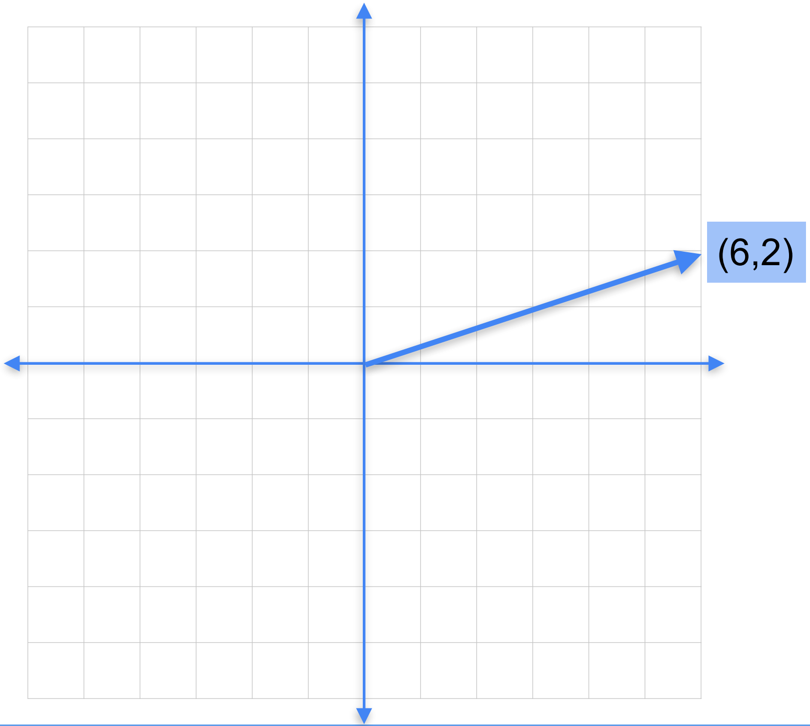
$$\langle u, v \rangle = |u'| \cdot |v|$$

$$= |u||v|\cos(\theta)$$

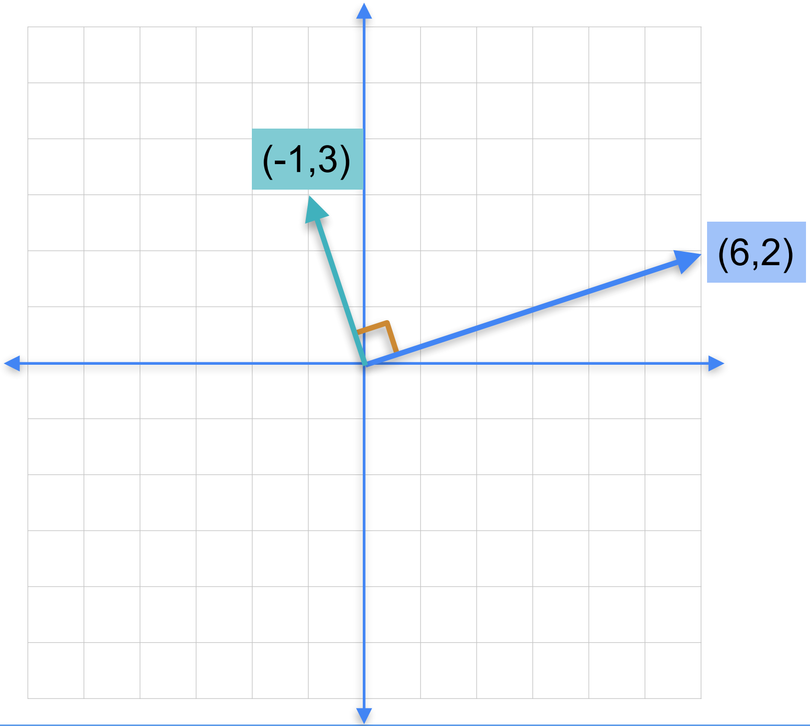
Geometric dot product



Geometric dot product

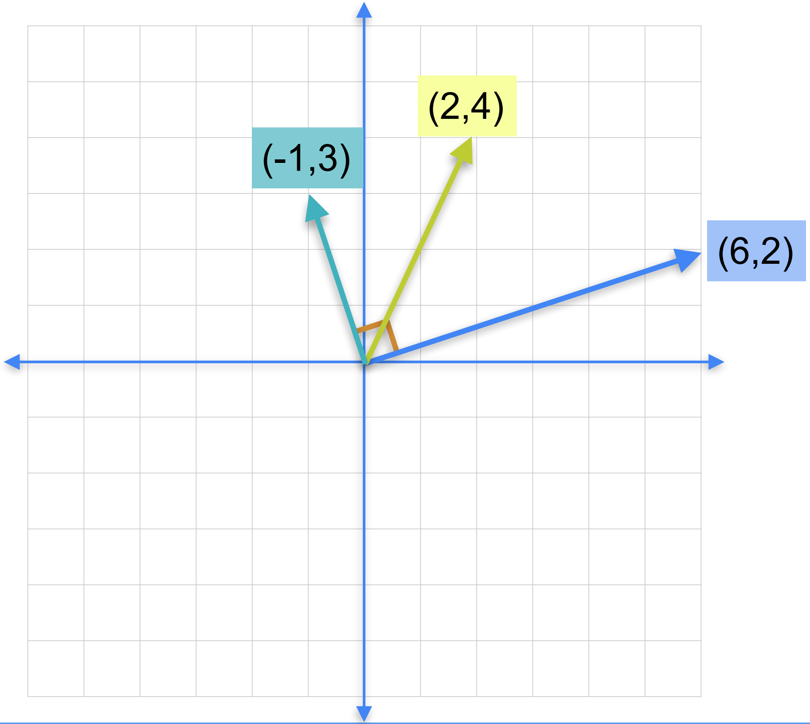


Geometric dot product



$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix} = 0$$

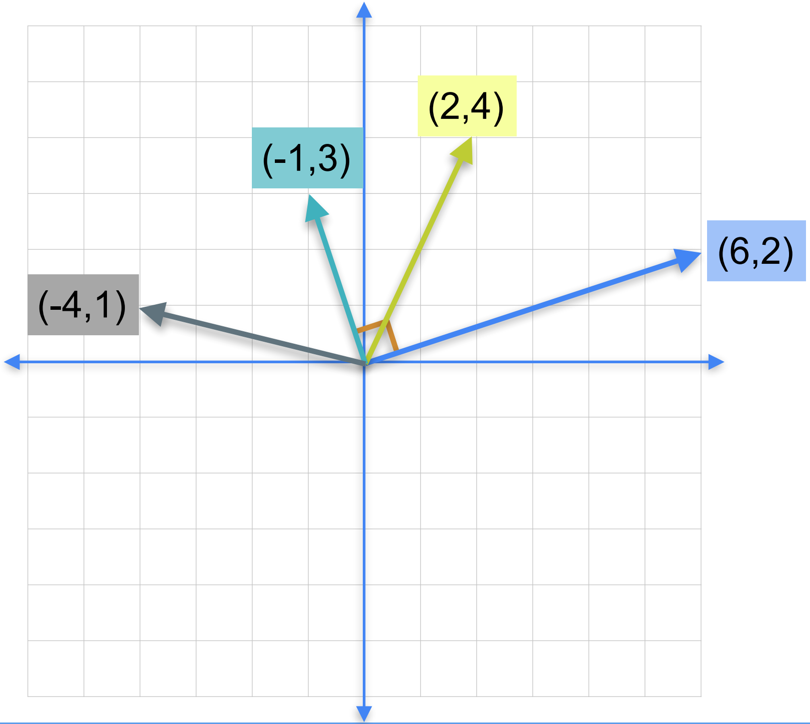
Geometric dot product



$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \end{bmatrix} = 20$$

$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix} = 0$$

Geometric dot product

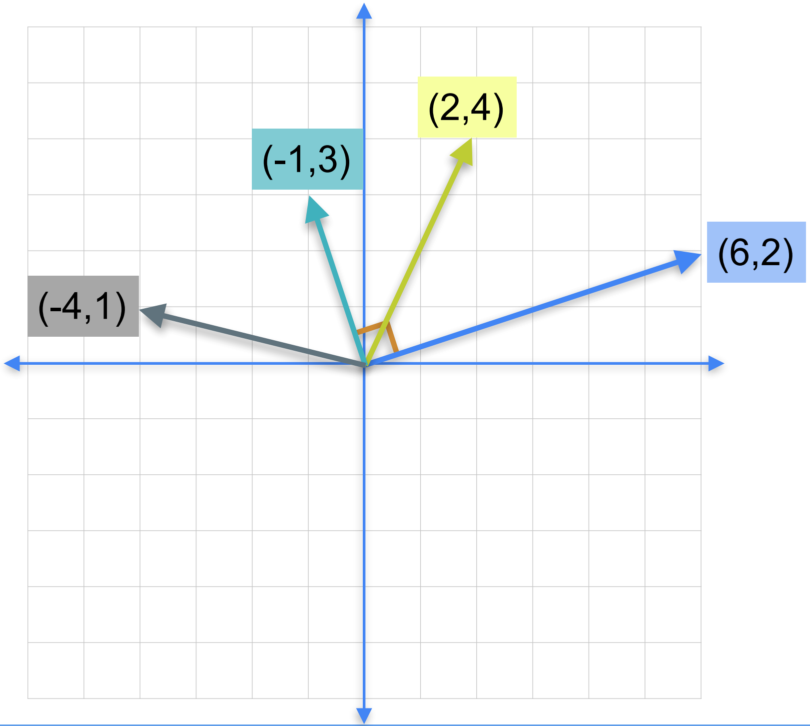


$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \end{bmatrix} = 20$$

$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} -4 \\ 1 \end{bmatrix} = -22$$

Geometric dot product



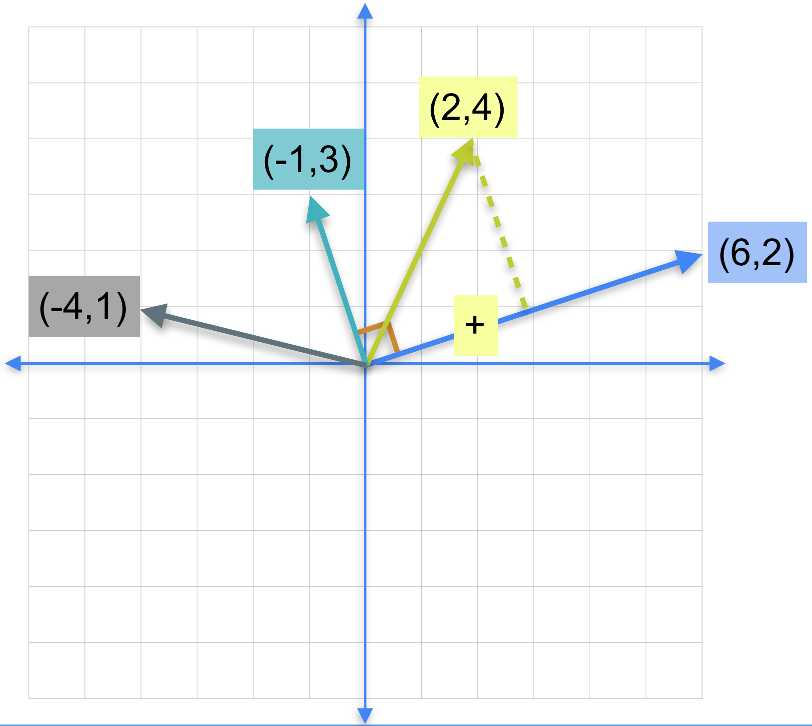
$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \end{bmatrix} = 20$$

Positive

$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} -4 \\ 1 \end{bmatrix} = -22$$

Geometric dot product



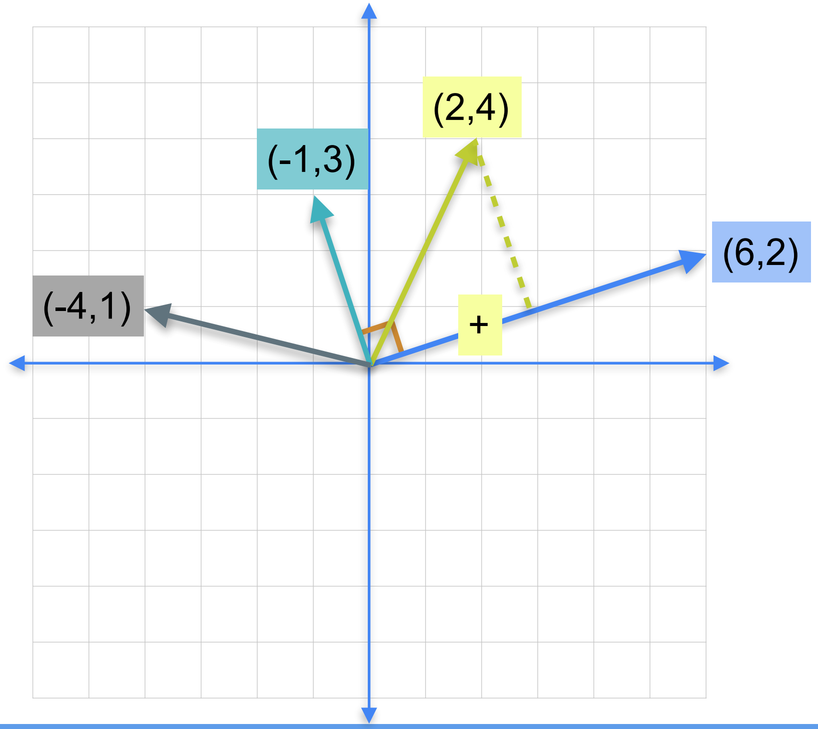
$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \end{bmatrix} = 20$$

Positive

$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} -4 \\ 1 \end{bmatrix} = -22$$

Geometric dot product

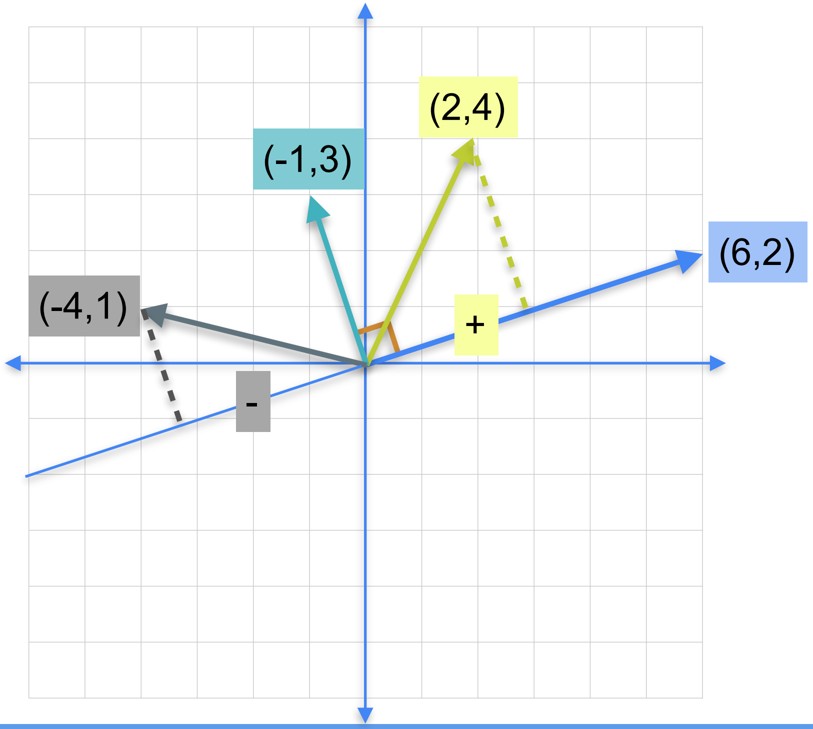


$$\begin{bmatrix}6 & 2\end{bmatrix}\begin{bmatrix}2\\4\end{bmatrix} = 20$$ Positive

$$\begin{bmatrix}6 & 2\end{bmatrix}\begin{bmatrix}-1\\3\end{bmatrix} = 0$$

$$\begin{bmatrix}6 & 2\end{bmatrix}\begin{bmatrix}-4\\1\end{bmatrix} = -22$$ Negative

Geometric dot product

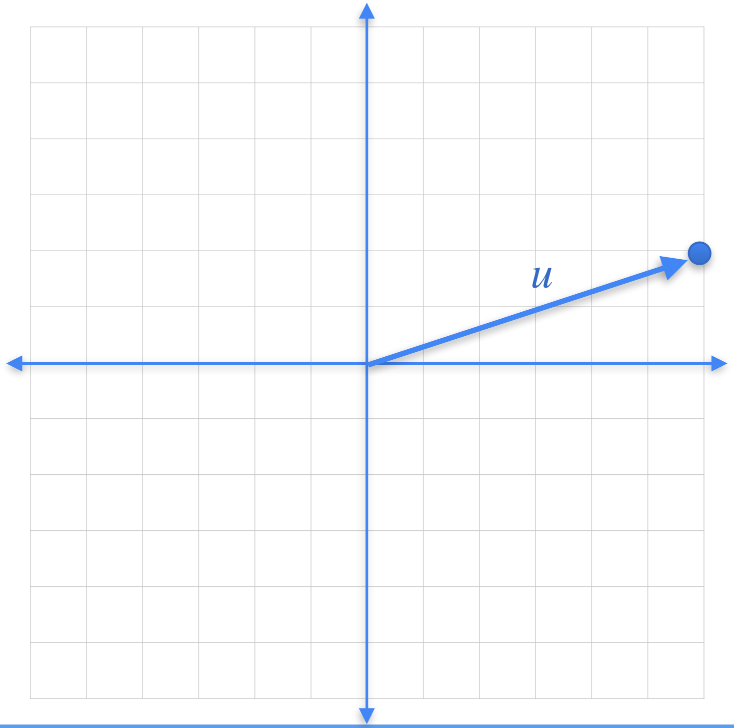


$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \end{bmatrix} = 20$$ Positive

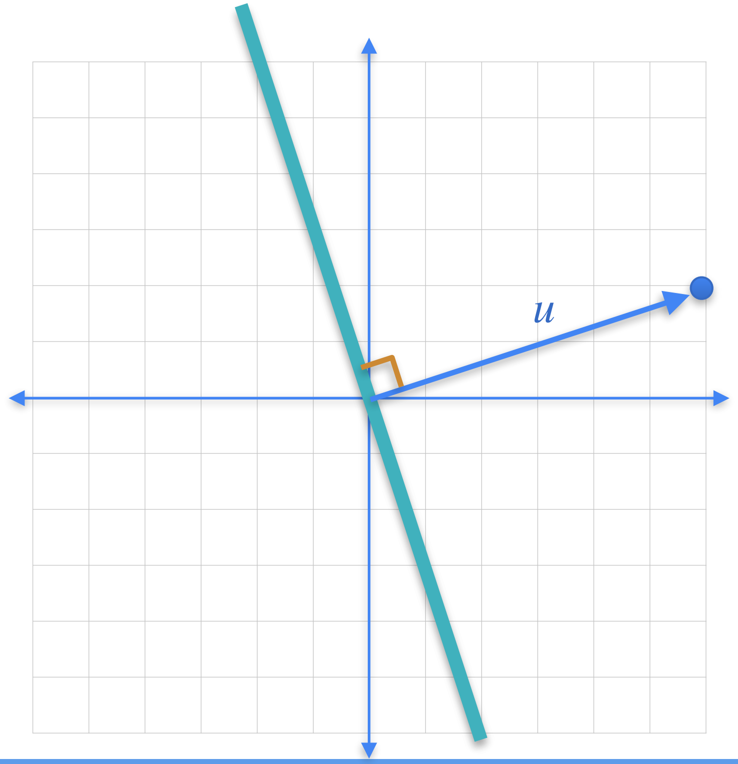
$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} -4 \\ 1 \end{bmatrix} = -22$$ Negative

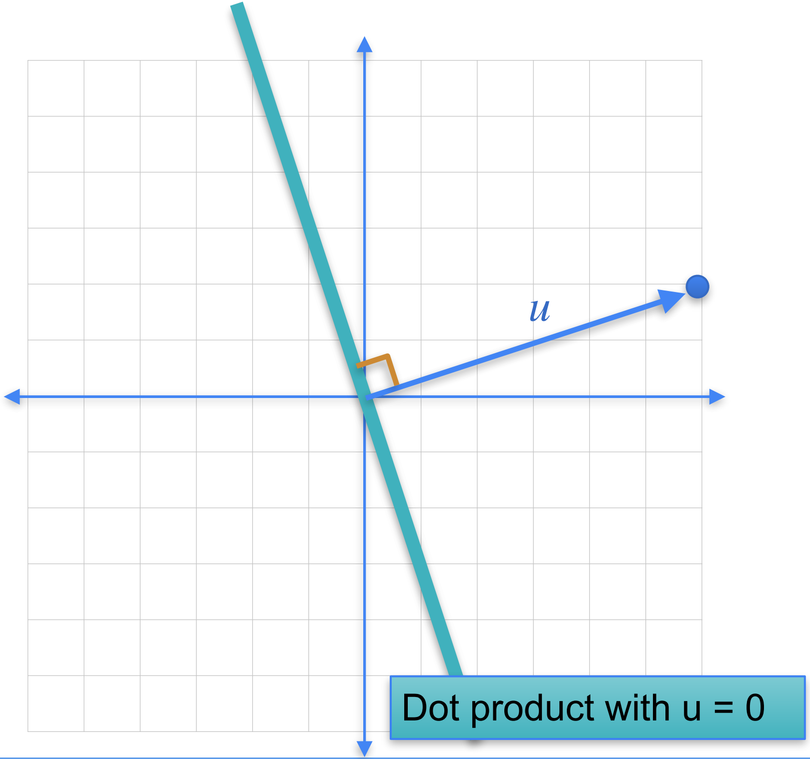
Geometric dot product



Geometric dot product

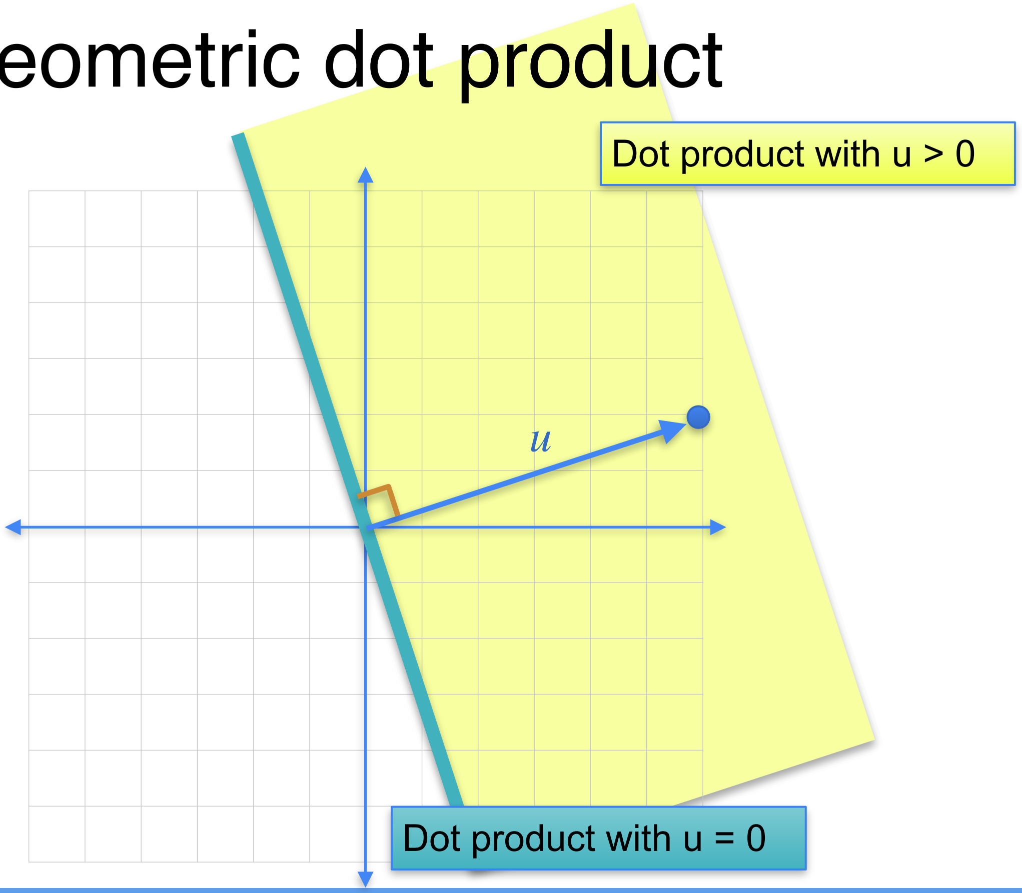


Geometric dot product



$$\langle u, v \rangle = 0$$

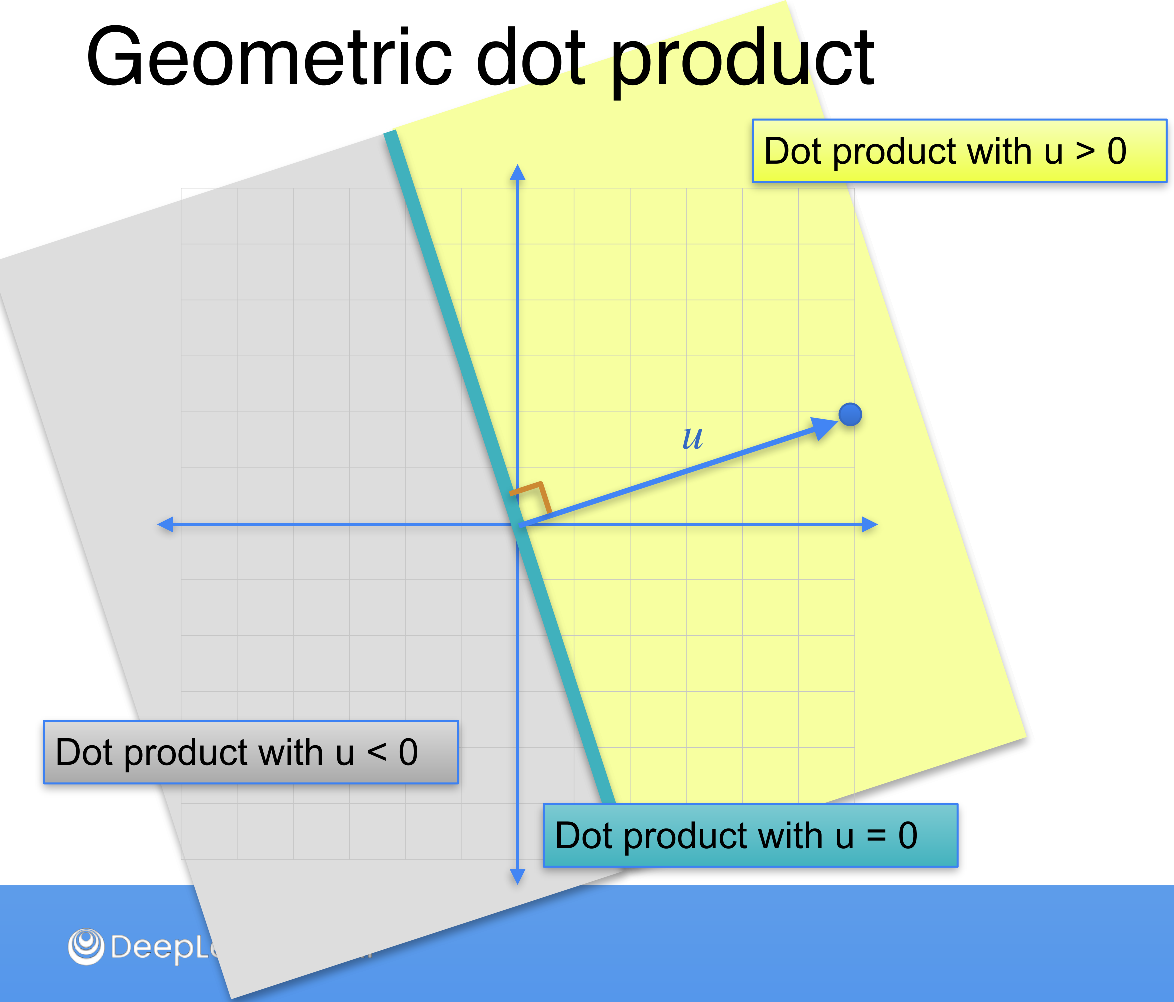
Geometric dot product



$$\langle u, v \rangle > 0$$

$$\langle u, v \rangle = 0$$

Geometric dot product



$$\langle u, v \rangle > 0$$

$$\langle u, v \rangle = 0$$

$$\langle u, v \rangle < 0$$

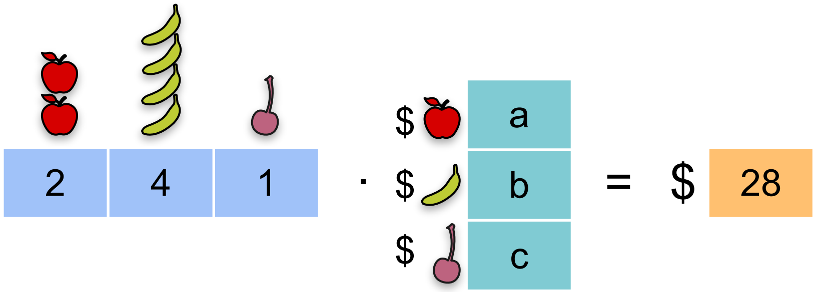
DeepLearning.AI

Vectors and Linear Transformations

Multiplying a matrix by a vector

Equations as dot product

$$2a + 4b + c = 28$$



Equations as dot product

$a + b + c = 10$

$a + 2b + c = 15$

$a + b + 2c = 12$

Equations as dot product

$a + b + c = 10$

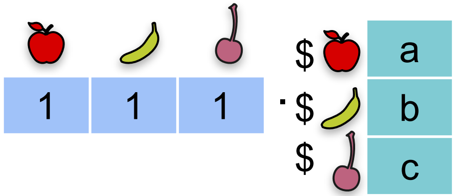
$a + 2b + c = 15$

$a + b + 2c = 12$

1	1	1

Equations as dot product

$a + b + c = 10$



$a + 2b + c = 15$

$a + b + 2c = 12$

Equations as dot product

$a + b + c = 10$

$a + 2b + c = 15$

$a + b + 2c = 12$

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} \$\ a \\ \$\ b \\ \$\ c \end{bmatrix} = \$\ \boxed{10}$$

Equations as dot product

$$a + b + c = 10$$

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \cdot \begin{array}{c} \$ \\ \$ \\ \$ \end{array} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \$ \begin{bmatrix} 10 \end{bmatrix}$$

$$a + 2b + c = 15$$

$$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$$

$$a + b + 2c = 12$$

Equations as dot product

$$a + b + c = 10$$

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \cdot \begin{array}{c} \$ \\ \$ \\ \$ \end{array}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \$\ \boxed{10}$$

$$a + 2b + c = 15$$

$$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \cdot \begin{array}{c} \$ \\ \$ \\ \$ \end{array}\begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$a + b + 2c = 12$$

Equations as dot product

$a + b + c = 10$

$$\begin{bmatrix}1 & 1 & 1\end{bmatrix} \cdot \begin{bmatrix}\$\ a\\ \$\ b\\ \$\ c\end{bmatrix} = \$\ 10$$

$a + 2b + c = 15$

$$\begin{bmatrix}1 & 2 & 1\end{bmatrix} \cdot \begin{bmatrix}\$\ a\\ \$\ b\\ \$\ c\end{bmatrix} = \$\ 15$$

$a + b + 2c = 12$

Equations as dot product

$a + b + c = 10$

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \cdot \begin{array}{c} \$ \\ \$ \\ \$ \end{array} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \$ \ \boxed{10}$$

$a + 2b + c = 15$

$$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \cdot \begin{array}{c} \$ \\ \$ \\ \$ \end{array} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \$ \ \boxed{15}$$

$a + b + 2c = 12$

$$\begin{bmatrix} 1 & 1 & 2 \end{bmatrix}$$

Equations as dot product

$$a + b + c = 10$$

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} \$\ a \\ \$\ b \\ \$\ c \end{bmatrix} = \$\ 10$$

$$a + 2b + c = 15$$

$$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} \$\ a \\ \$\ b \\ \$\ c \end{bmatrix} = \$\ 15$$

$$a + b + 2c = 12$$

$$\begin{bmatrix} 1 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} \$\ a \\ \$\ b \\ \$\ c \end{bmatrix}$$

Equations as dot product

$a + b + c = 10$

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \cdot \begin{array}{c} \$ \\ \$ \\ \$ \end{array}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \$\ 10$$

$a + 2b + c = 15$

$$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \cdot \begin{array}{c} \$ \\ \$ \\ \$ \end{array}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \$\ 15$$

$a + b + 2c = 12$

$$\begin{bmatrix} 1 & 1 & 2 \end{bmatrix} \cdot \begin{array}{c} \$ \\ \$ \\ \$ \end{array}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \$\ 12$$

Equations as dot product

$a + b + c = 10$

$$\begin{bmatrix}1 & 1 & 1\end{bmatrix} \cdot \begin{array}{c}\$\\ \$\\ \$\end{array}\begin{bmatrix}a\\ b\\ c\end{bmatrix} = \$\ \boxed{10}$$

$a + 2b + c = 15$

$$\begin{bmatrix}1 & 2 & 1\end{bmatrix} \cdot \begin{array}{c}\$\\ \$\\ \$\end{array}\begin{bmatrix}a\\ b\\ c\end{bmatrix} = \$\ \boxed{15}$$

$a + b + 2c = 12$

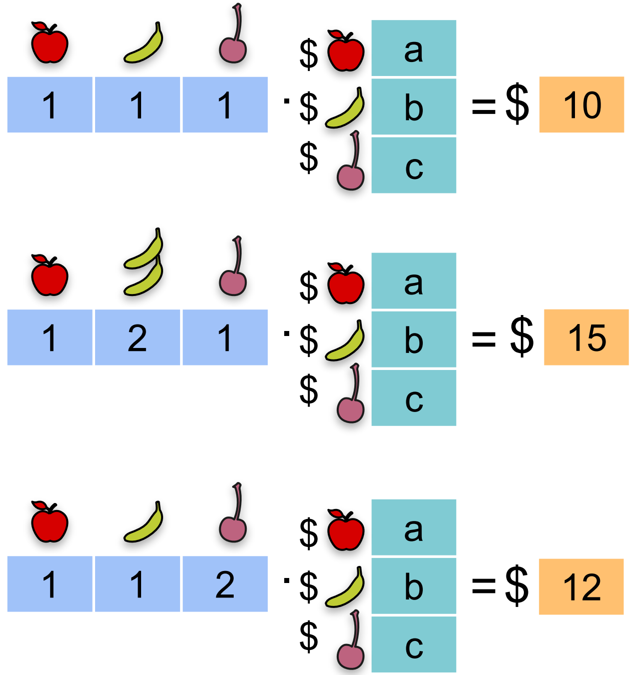
$$\begin{bmatrix}1 & 1 & 2\end{bmatrix} \cdot \begin{array}{c}\$\\ \$\\ \$\end{array}\begin{bmatrix}a\\ b\\ c\end{bmatrix} = \$\ \boxed{12}$$

Equations as dot product

$a + b + c = 10$

$a + 2b + c = 15$

$a + b + 2c = 12$

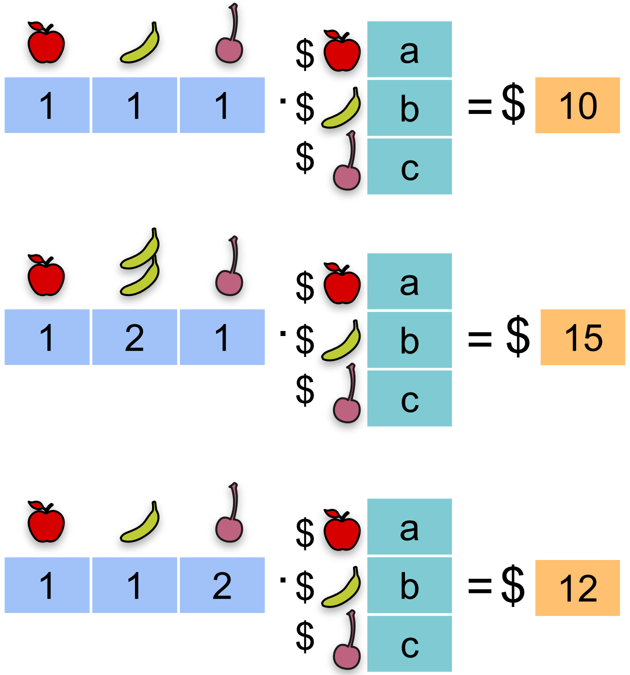


Equations as dot product

$a + b + c = 10$

$a + 2b + c = 15$

$a + b + 2c = 12$



Equations as dot product

System of equations

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Matrix product

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \cdot \begin{array}{c} \$ \\ \$ \\ \$ \end{array} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \$ \begin{bmatrix} 10 \\ 15 \\ 12 \end{bmatrix}$$

Equations as dot product

System of equations

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Matrix product

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 10 \\ 15 \\ 12 \end{bmatrix}$$

DeepLearning.AI

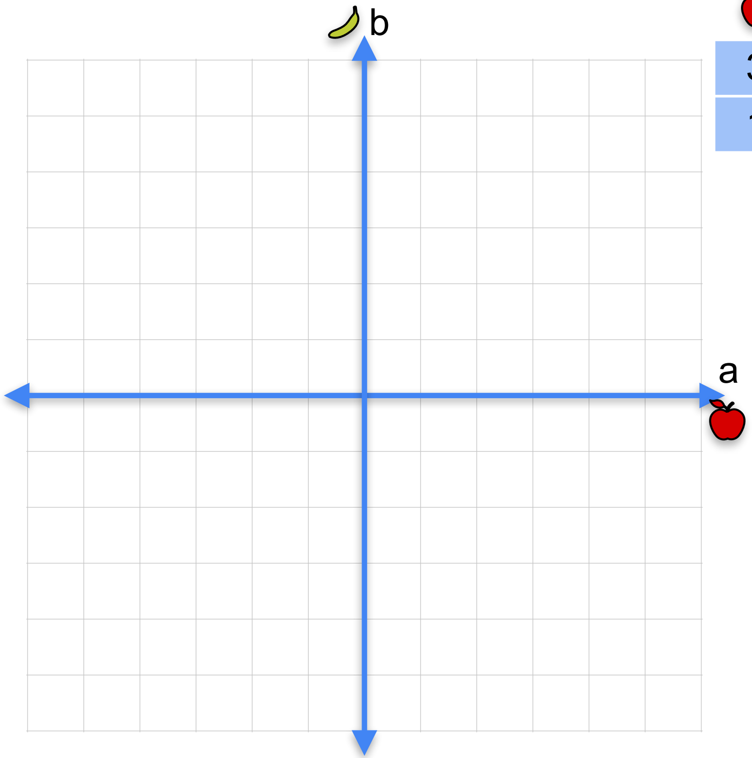
Vectors and Linear Transformations

Matrices as linear transformations

Matrices as linear transformations

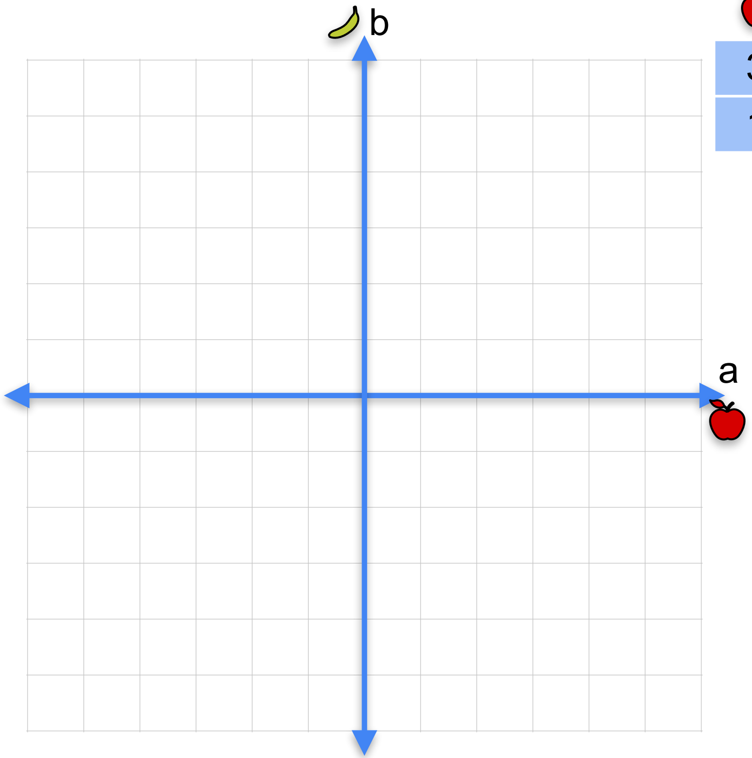
3	1
1	2

Matrices as linear transformations

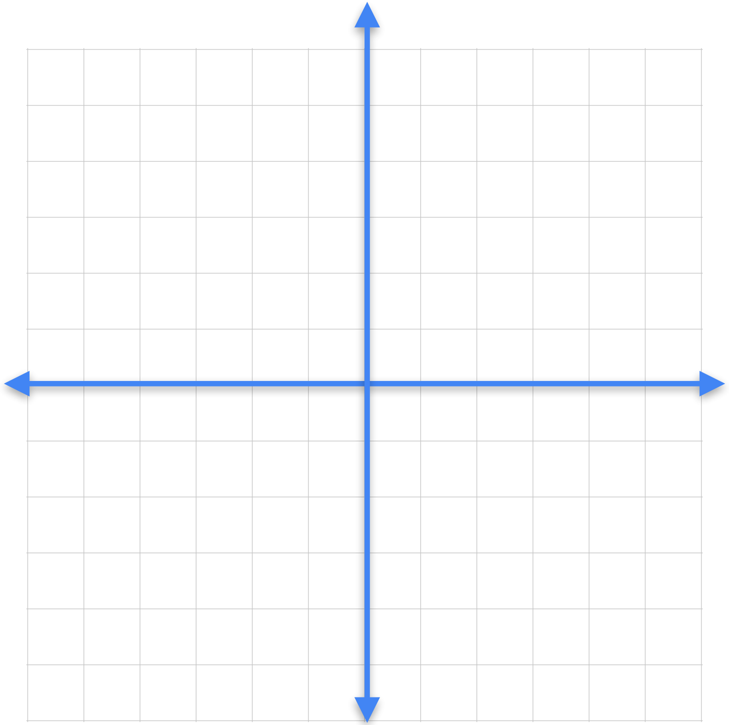


🍎	🍌
3	1
1	2

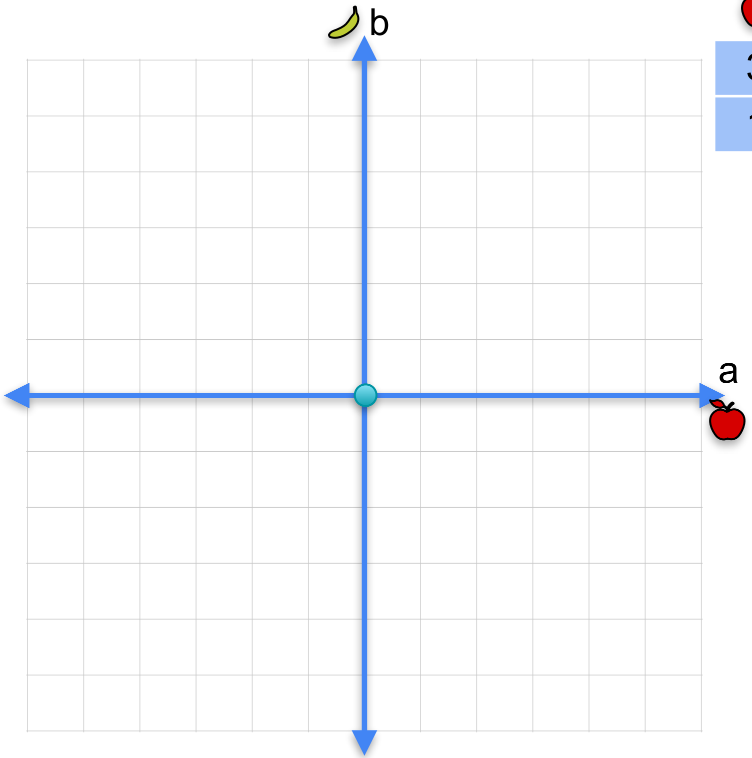
Matrices as linear transformations



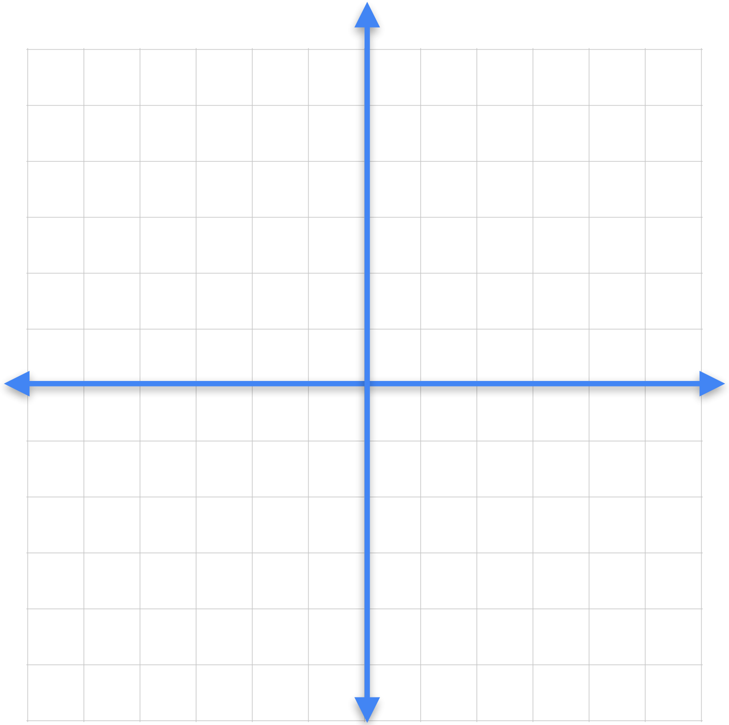
3	1
1	2



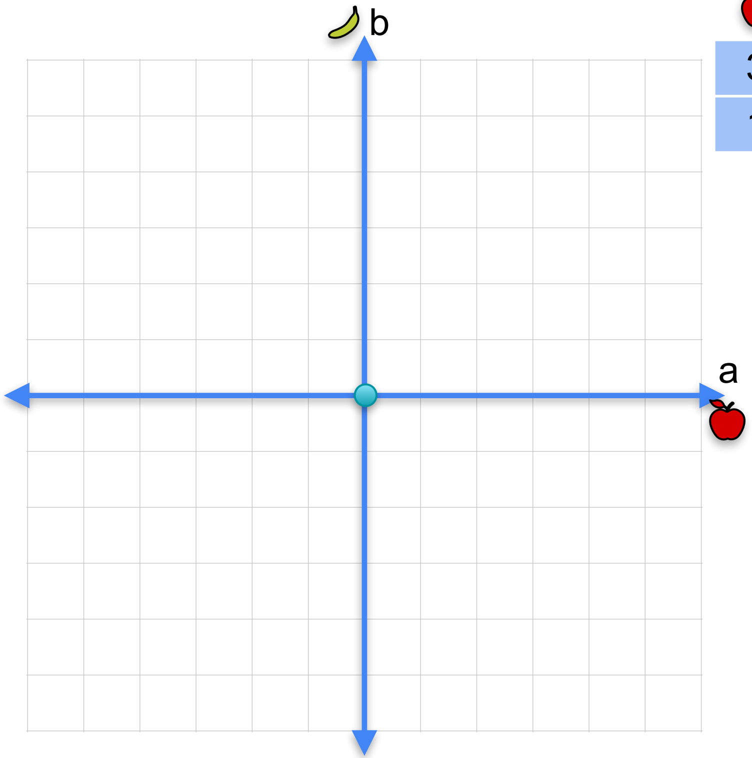
Matrices as linear transformations



3	1
1	2

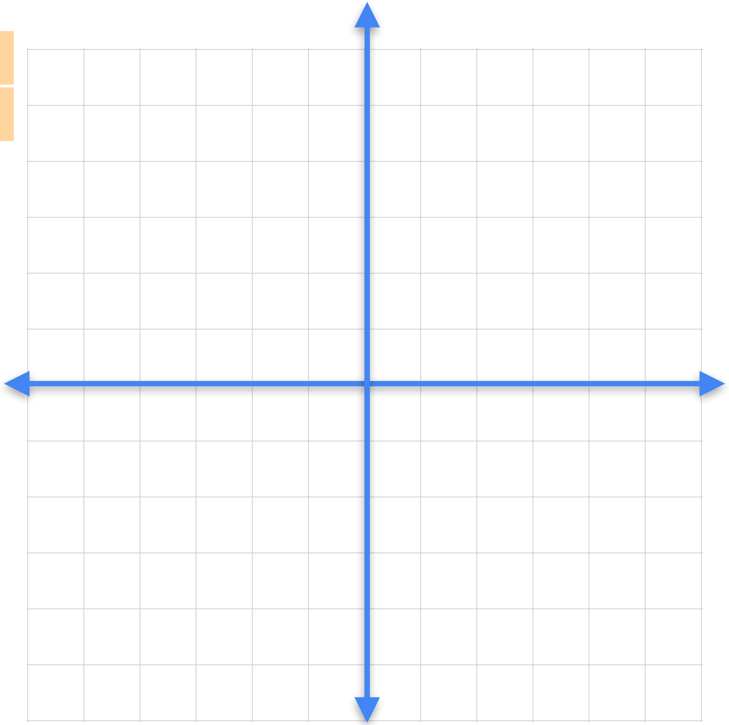


Matrices as linear transformations

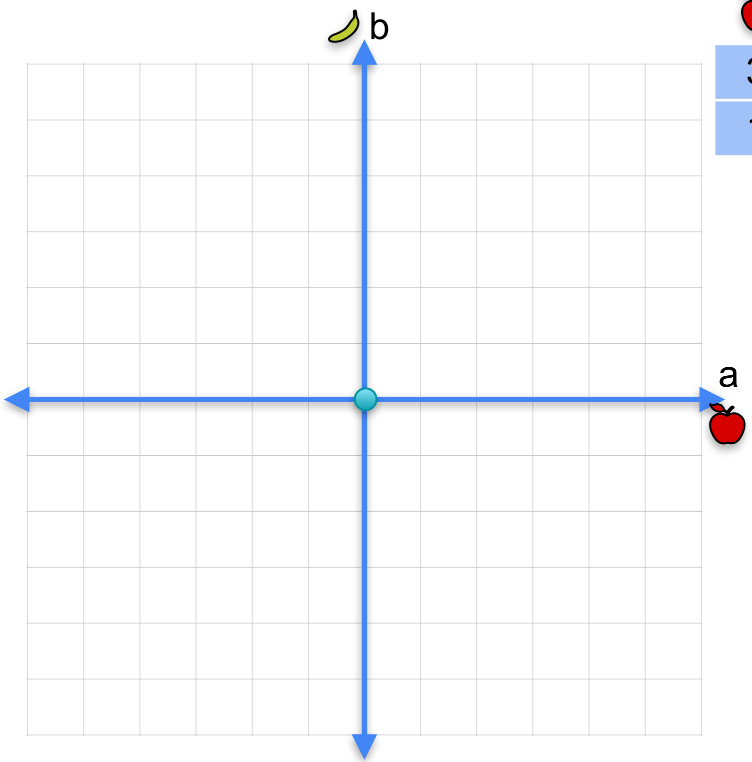


$$\begin{bmatrix}3 & 1\\ 1 & 2\end{bmatrix}\begin{bmatrix}0\\ 0\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$(0,0) \rightarrow (0,0)$



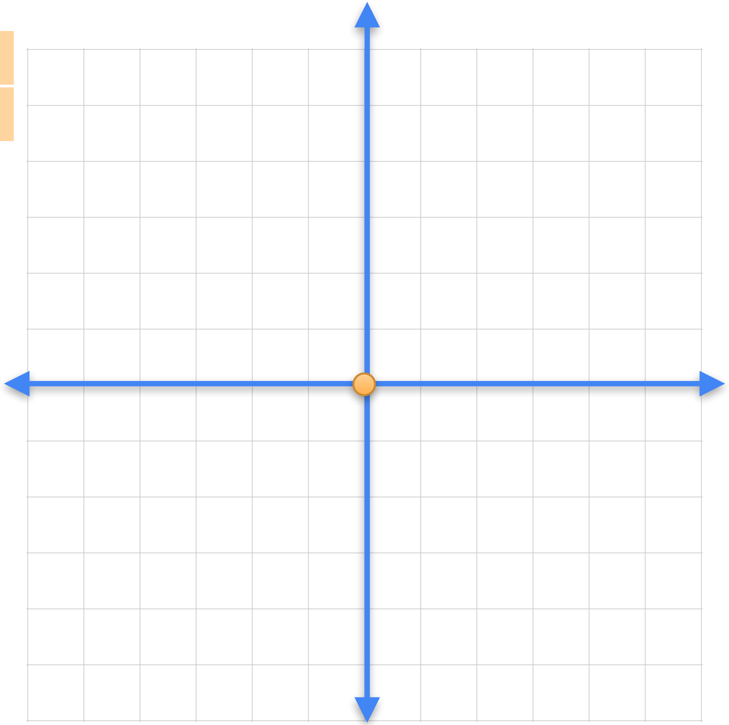
Matrices as linear transformations



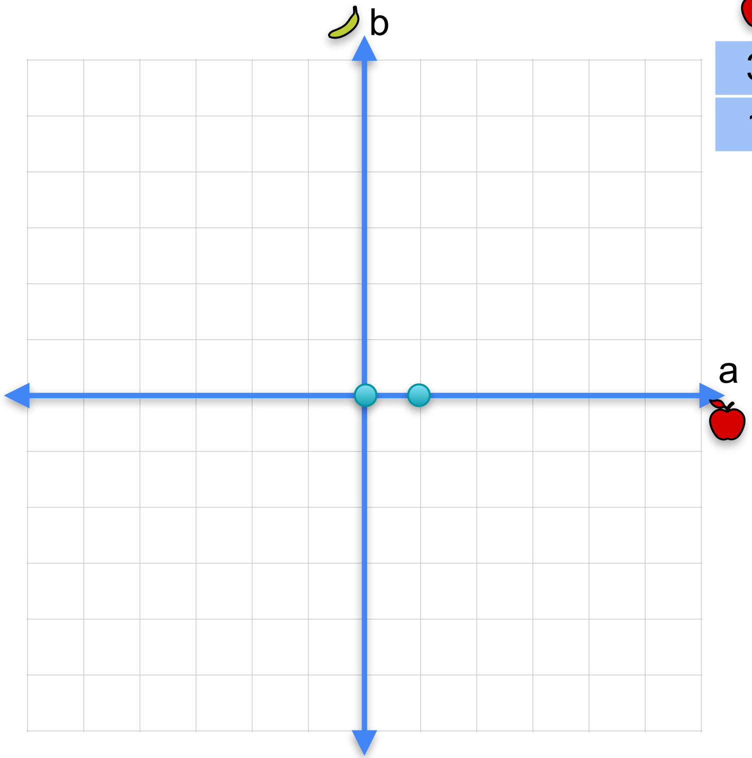
$$\begin{array}{cc} 🍎 & 🍌 \\ \end{array}$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$(0,0) \rightarrow (0,0)$

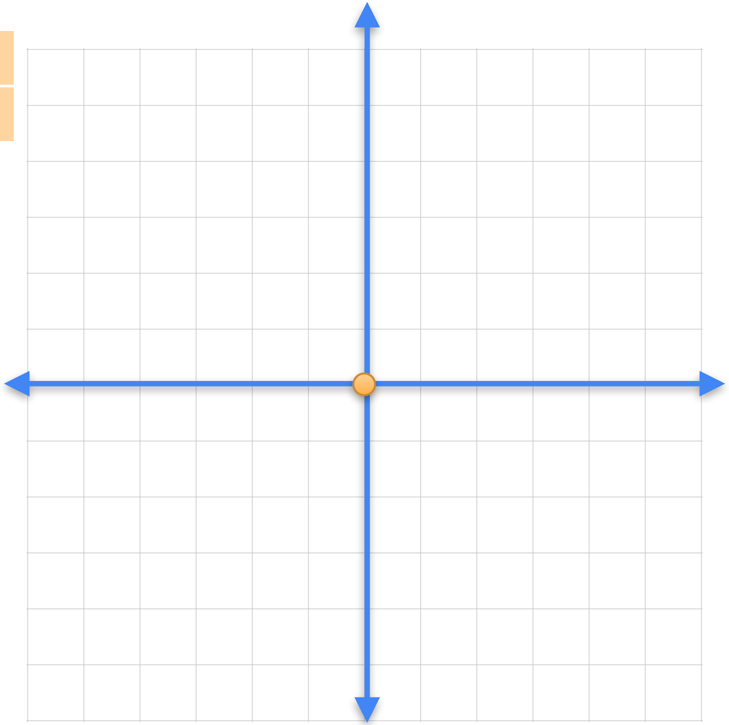


Matrices as linear transformations

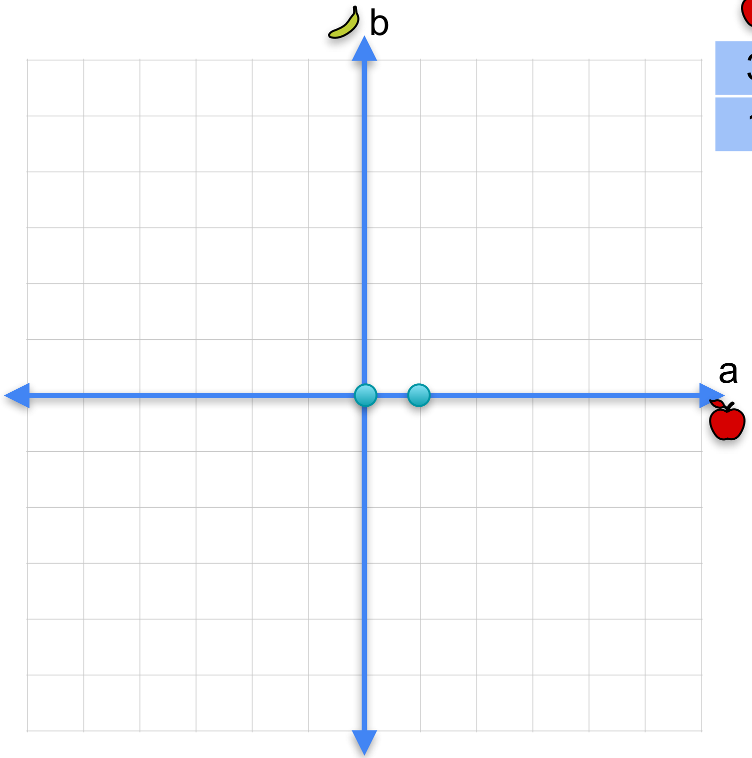


$$\begin{bmatrix}3&1\\1&2\end{bmatrix}\begin{bmatrix}0\\0\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

$(0,0) \rightarrow (0,0)$



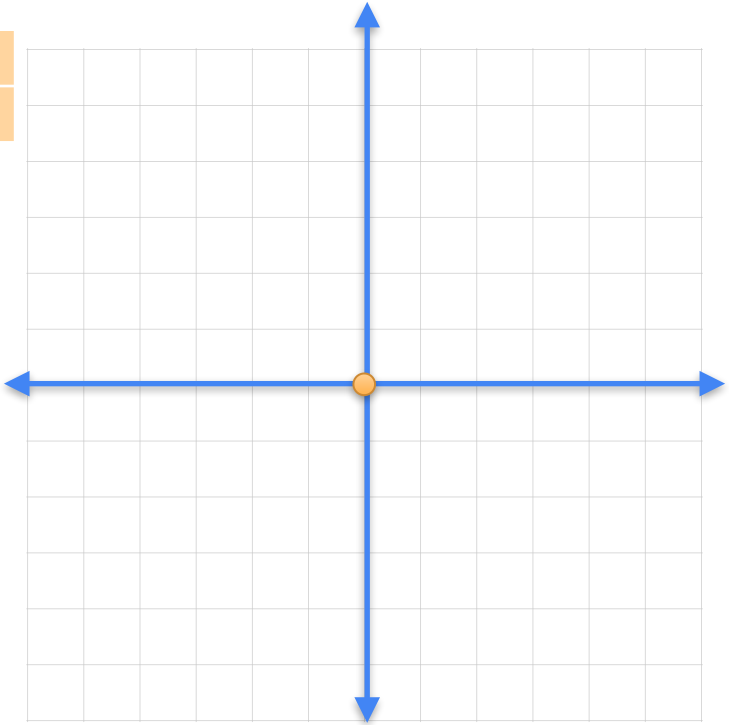
Matrices as linear transformations



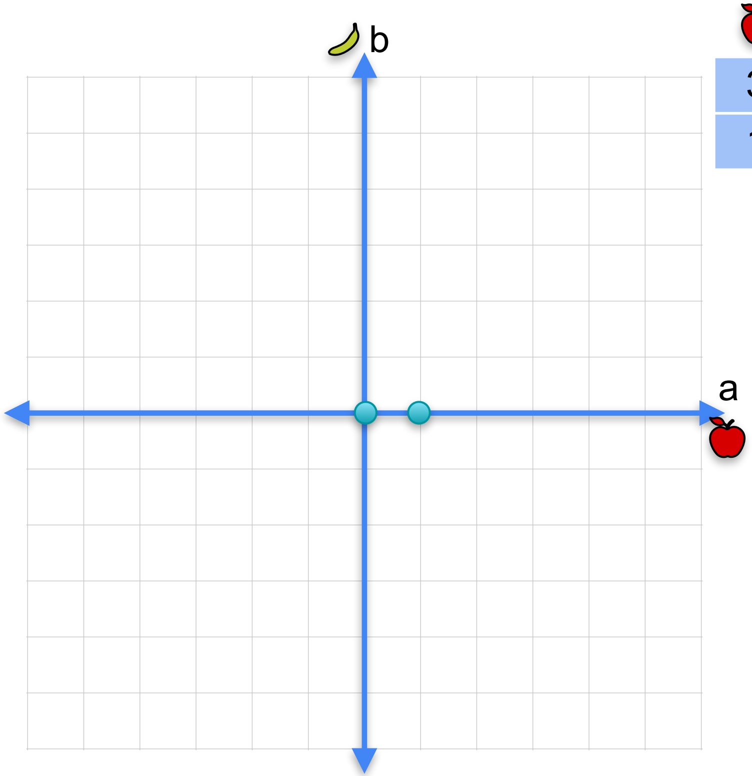
$$\begin{bmatrix}3 & 1\\1 & 2\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}3\\1\end{bmatrix}$$

$(0,0) \rightarrow (0,0)$

$(1,0) \rightarrow (3,1)$



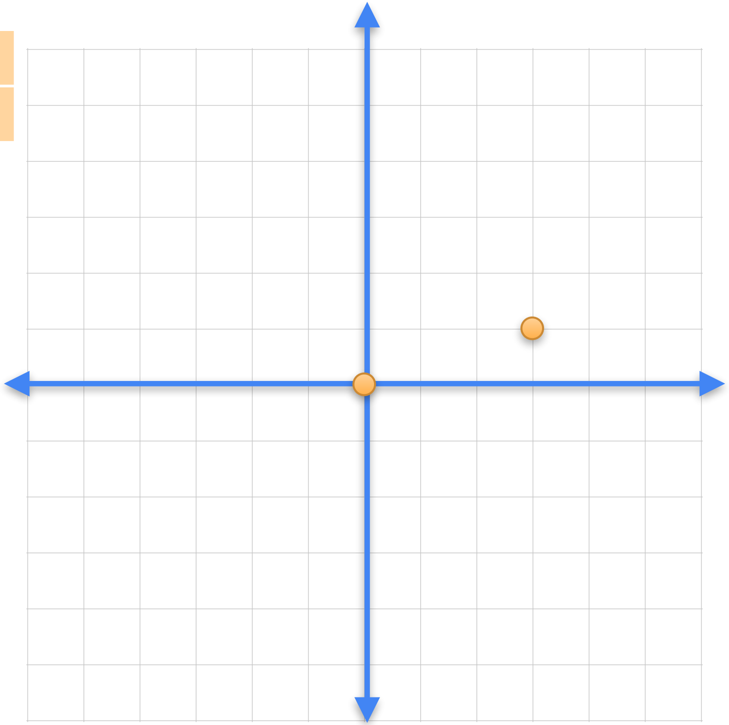
Matrices as linear transformations



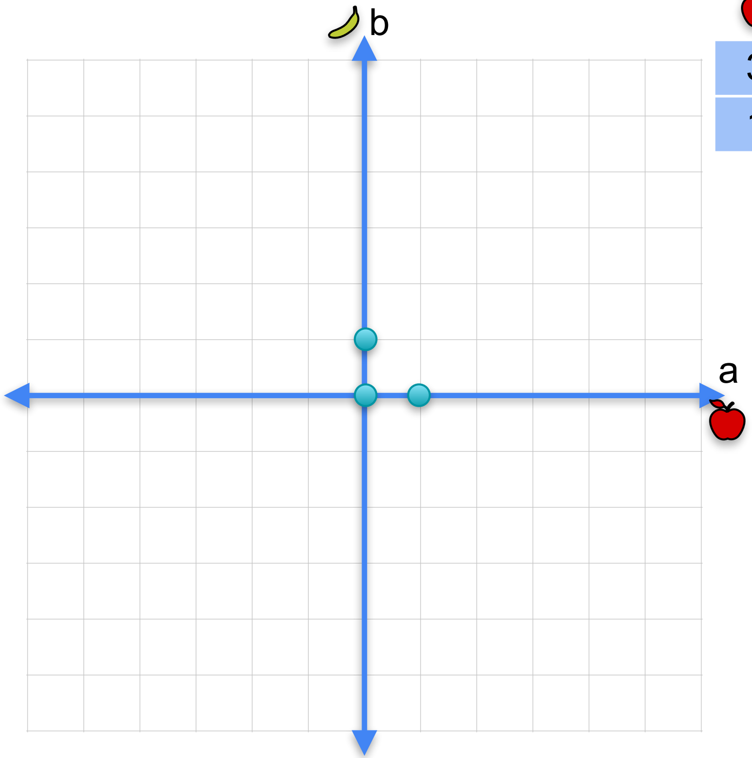
$$\begin{array}{cc} \text{🍎} & \text{🍌} \end{array}$$
$$\begin{bmatrix}3 & 1\\1 & 2\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}3\\1\end{bmatrix}$$

$(0,0) \rightarrow (0,0)$

$(1,0) \rightarrow (3,1)$



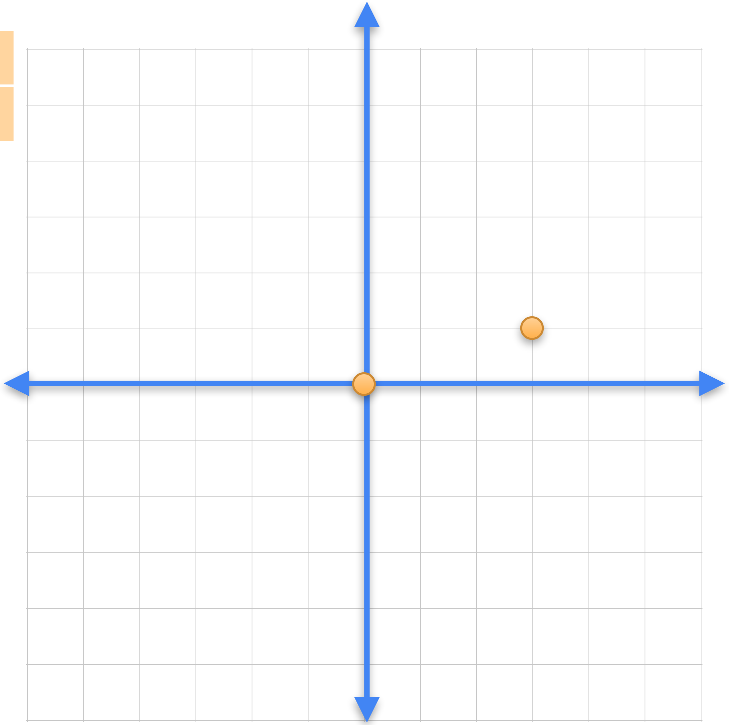
Matrices as linear transformations



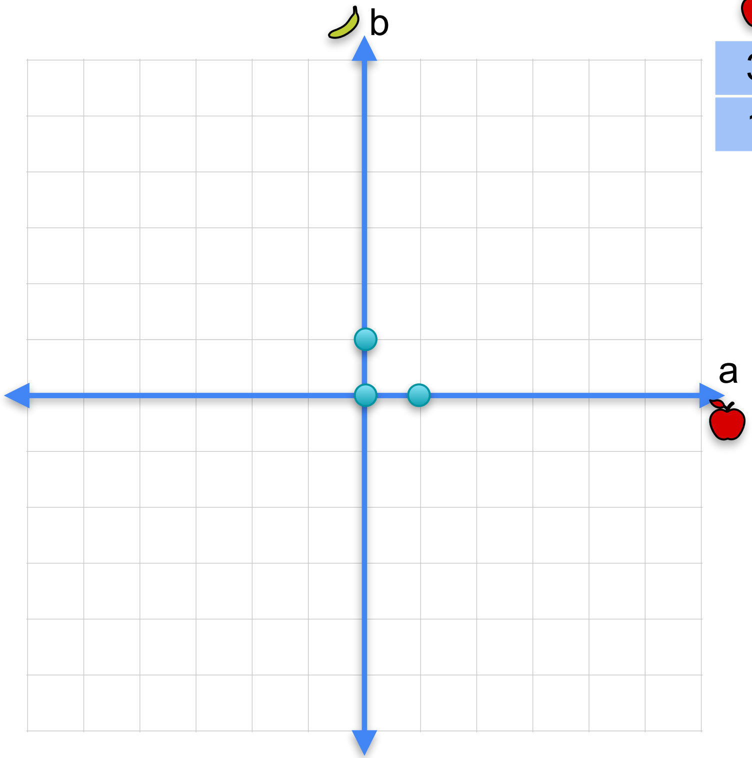
$$\begin{bmatrix}3 & 1\\ 1 & 2\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix}=\begin{bmatrix}3\\ 1\end{bmatrix}$$

(0,0) → (0,0)

(1,0) → (3,1)



Matrices as linear transformations

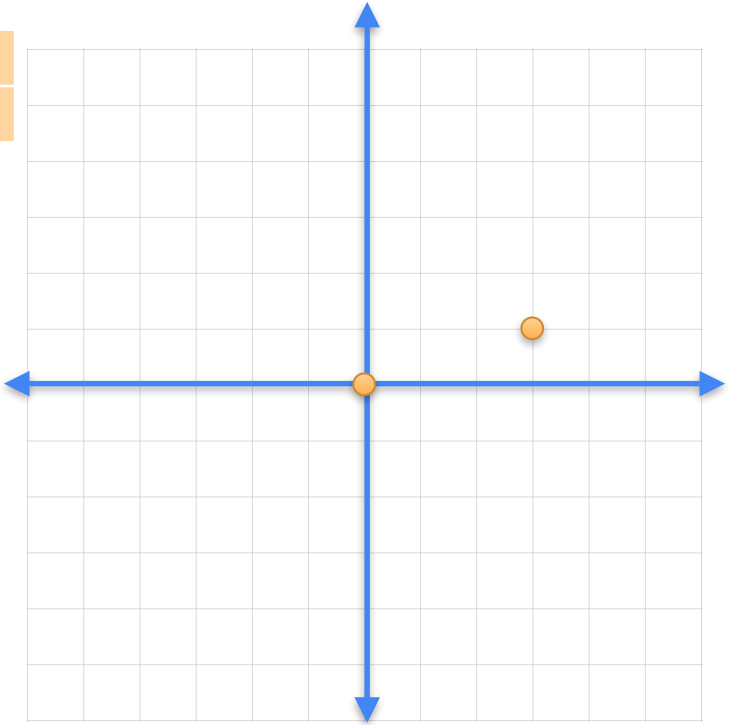


$$\begin{bmatrix}3 & 1\\ 1 & 2\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix}=\begin{bmatrix}1\\ 2\end{bmatrix}$$

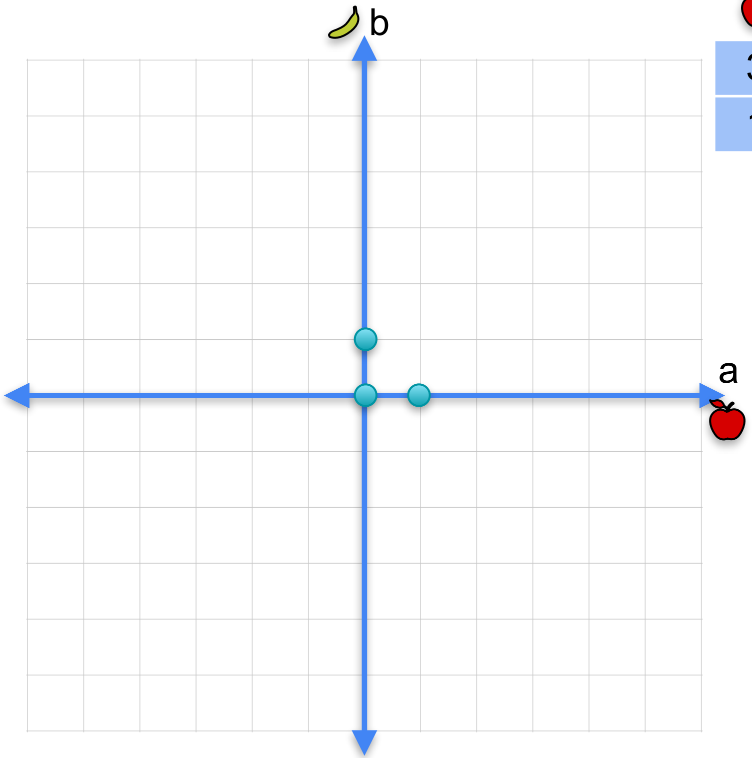
$(0,0) \rightarrow (0,0)$

$(1,0) \rightarrow (3,1)$

$(0,1) \rightarrow (1,2)$



Matrices as linear transformations

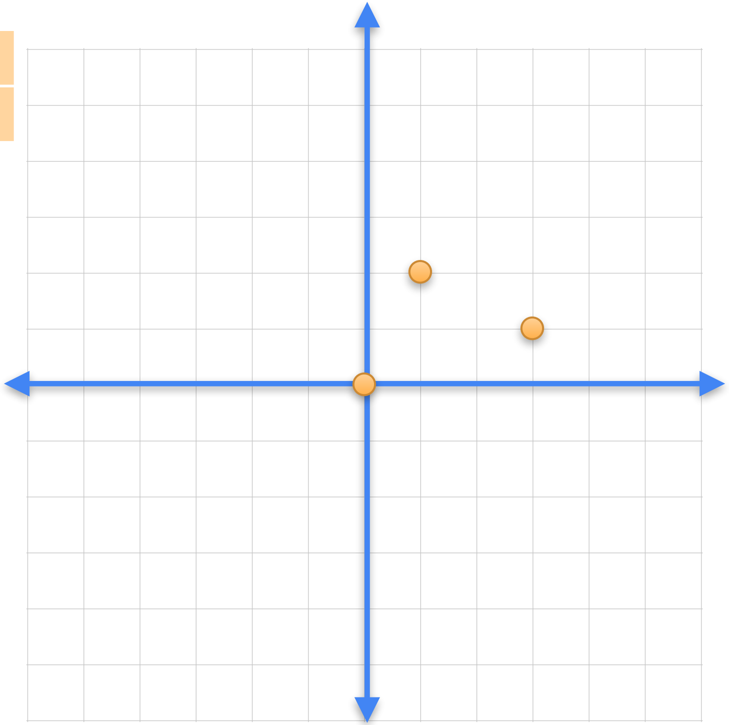


$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

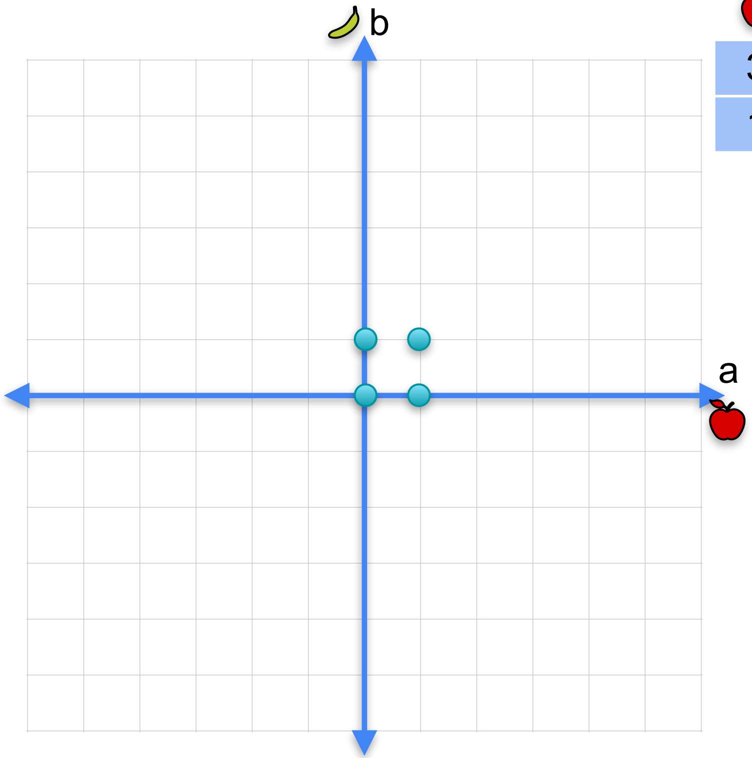
(0,0) → (0,0)

(1,0) → (3,1)

(0,1) → (1,2)

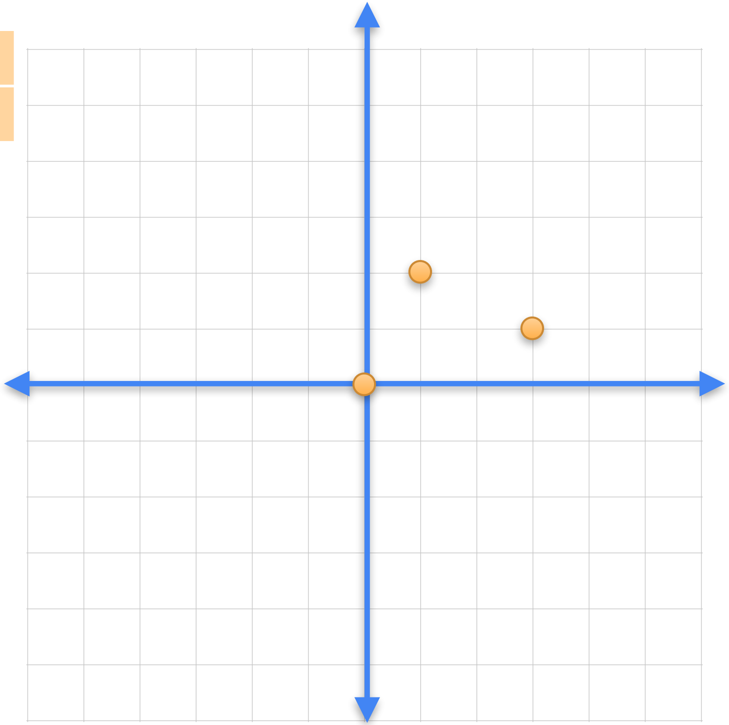


Matrices as linear transformations

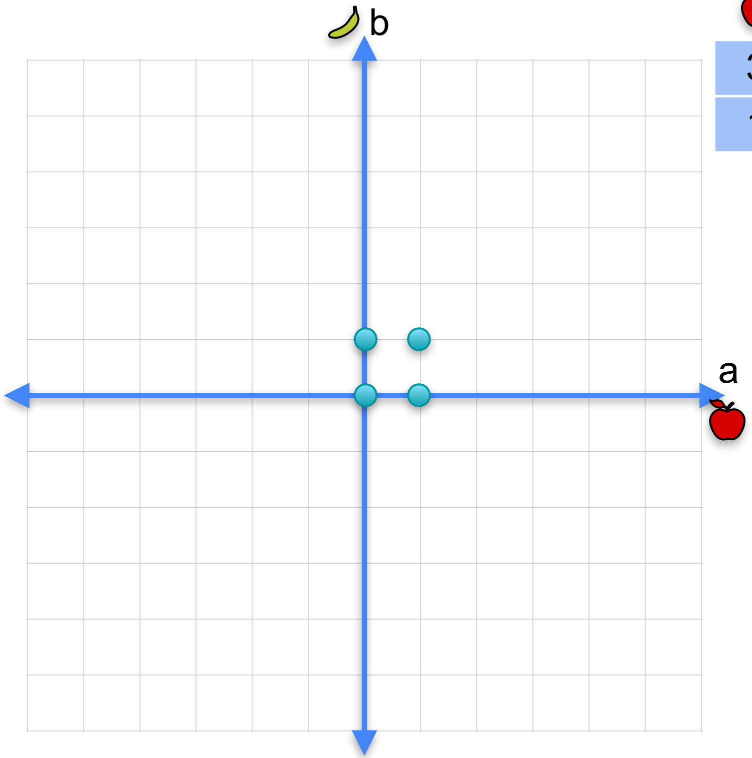


$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

(0,0) → (0,0)
(1,0) → (3,1)
(0,1) → (1,2)



Matrices as linear transformations



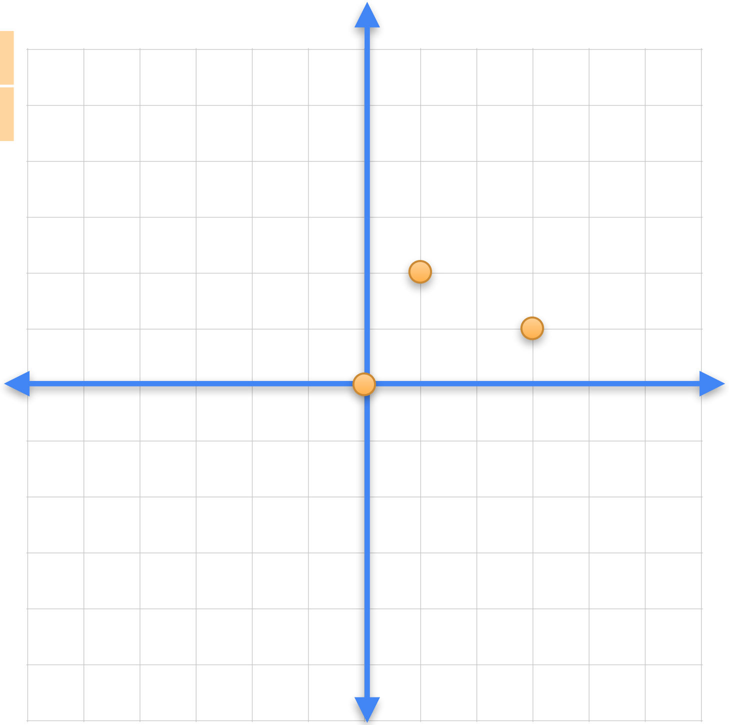
$$\begin{bmatrix}3 & 1\\ 1 & 2\end{bmatrix}\begin{bmatrix}1\\ 1\end{bmatrix}=\begin{bmatrix}4\\ 3\end{bmatrix}$$

(0,0) → (0,0)

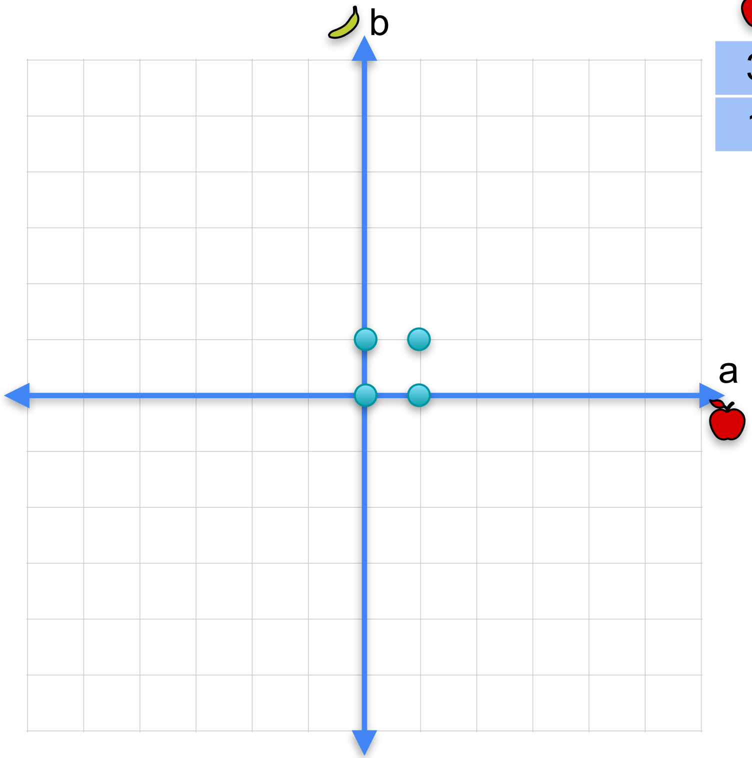
(1,0) → (3,1)

(0,1) → (1,2)

(1,1) → (4,3)



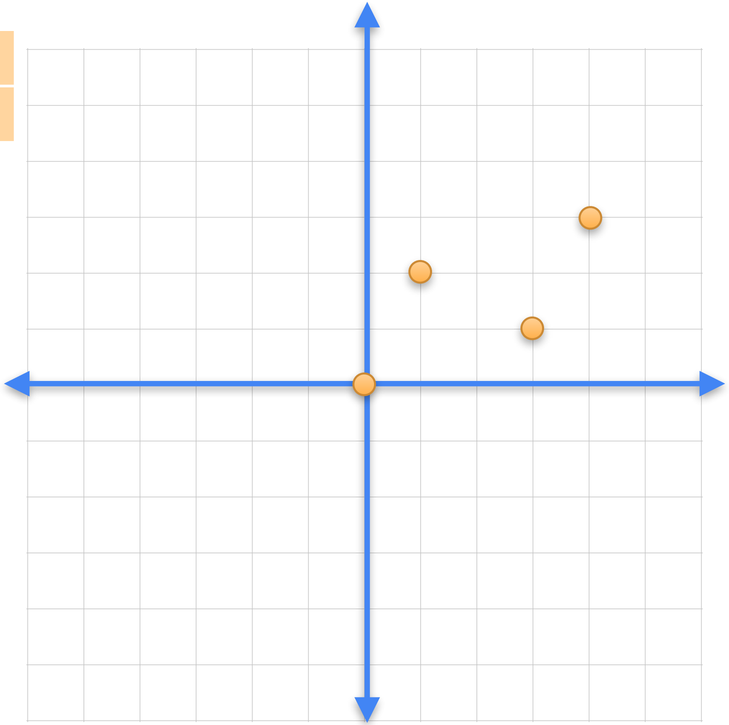
Matrices as linear transformations



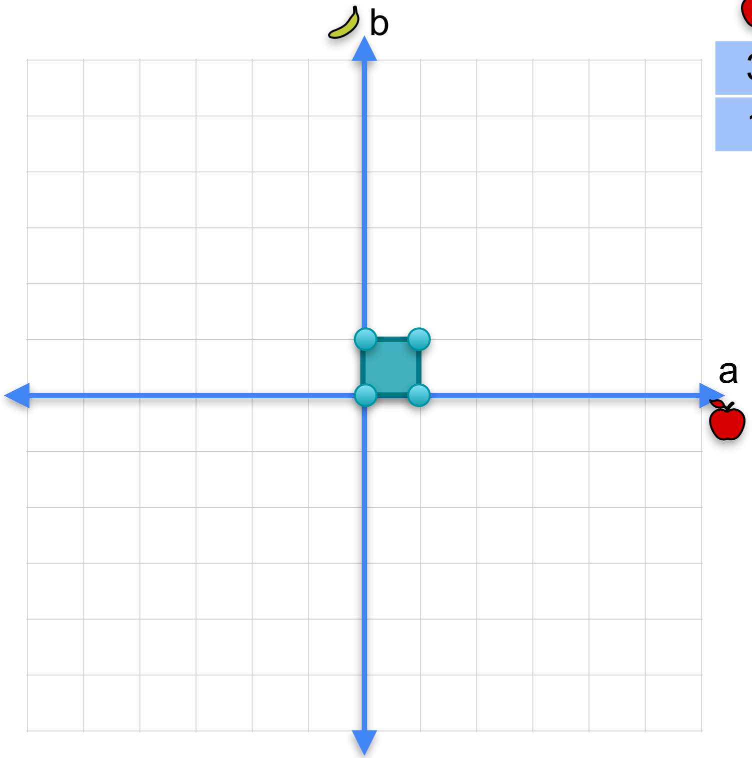
$$\begin{array}{cc} 🍎 & 🍌 \end{array}$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

(0,0) → (0,0)
(1,0) → (3,1)
(0,1) → (1,2)
(1,1) → (4,3)



Matrices as linear transformations



$$\begin{array}{cc} 🍎 & 🍌 \\ \end{array}$$

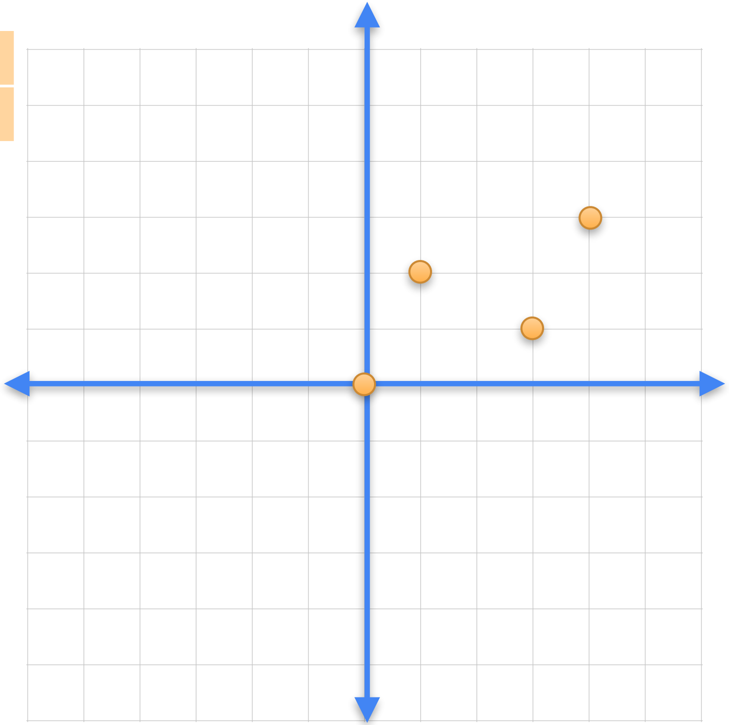
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

(0,0) → (0,0)

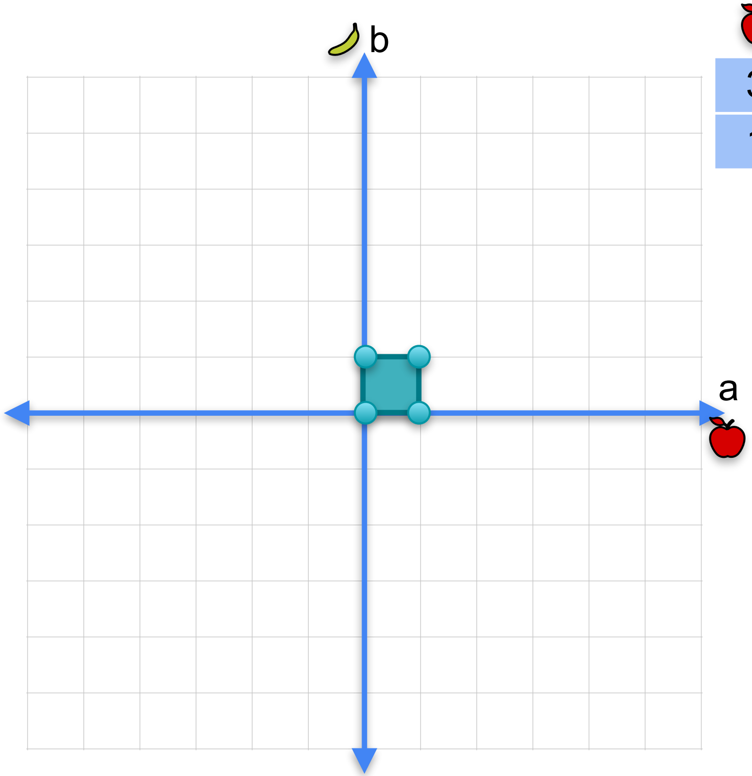
(1,0) → (3,1)

(0,1) → (1,2)

(1,1) → (4,3)



Matrices as linear transformations



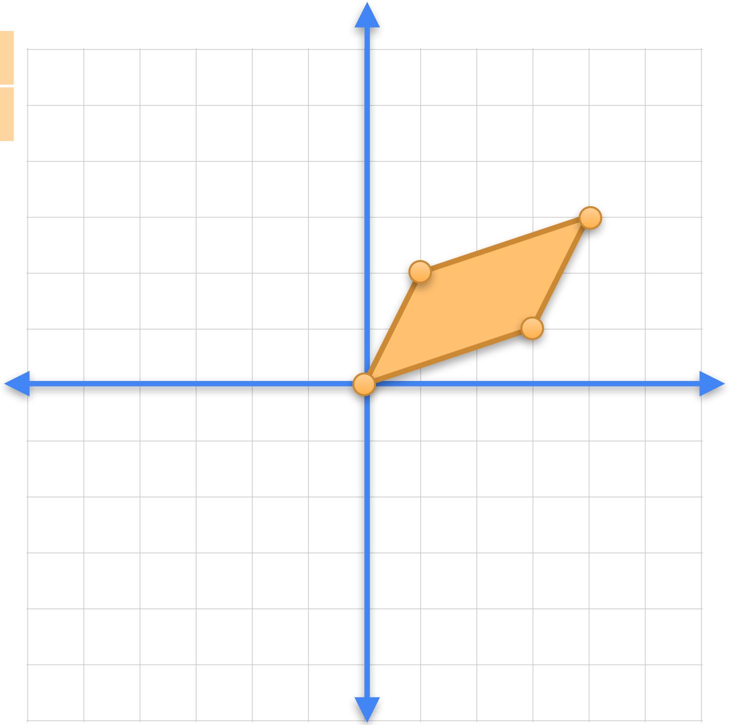
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

(0,0) → (0,0)

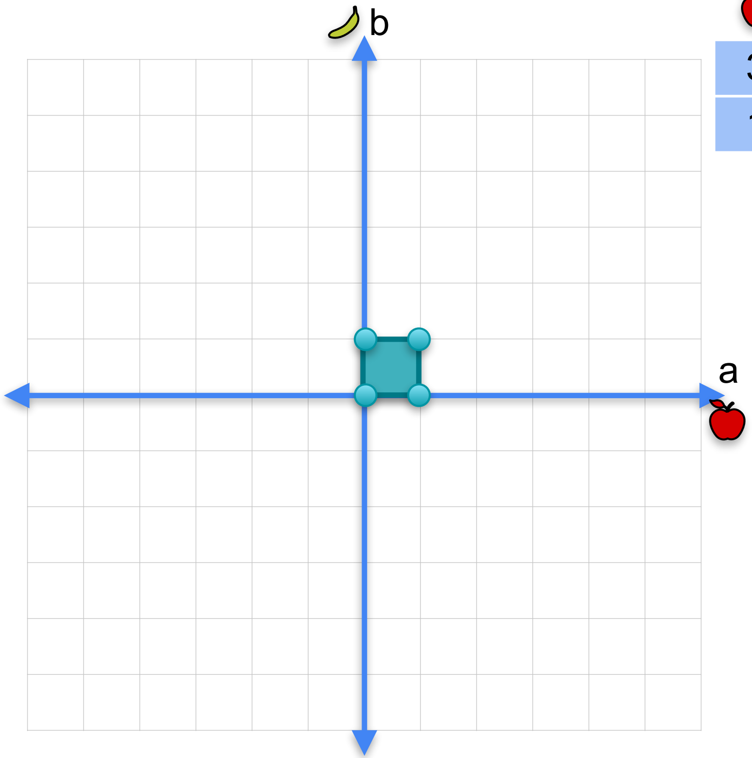
(1,0) → (3,1)

(0,1) → (1,2)

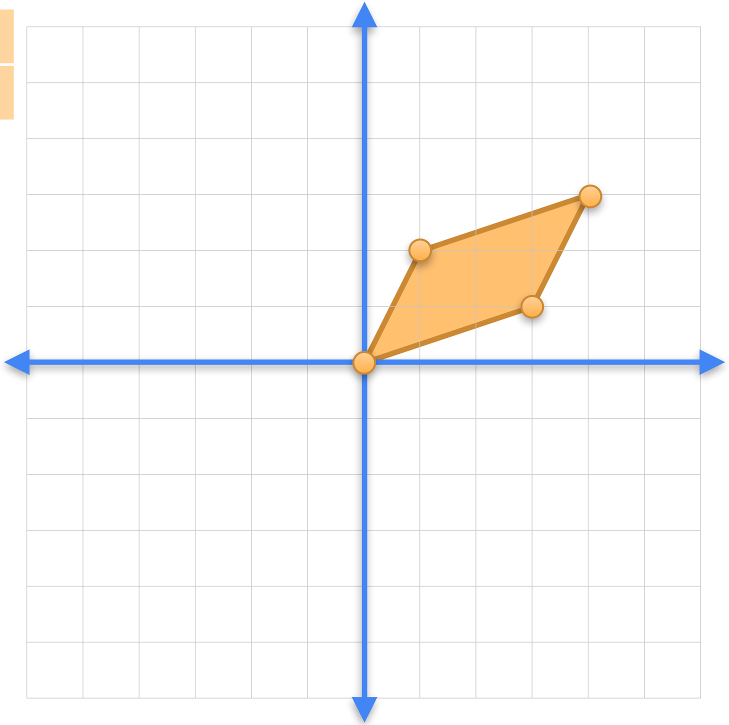
(1,1) → (4,3)



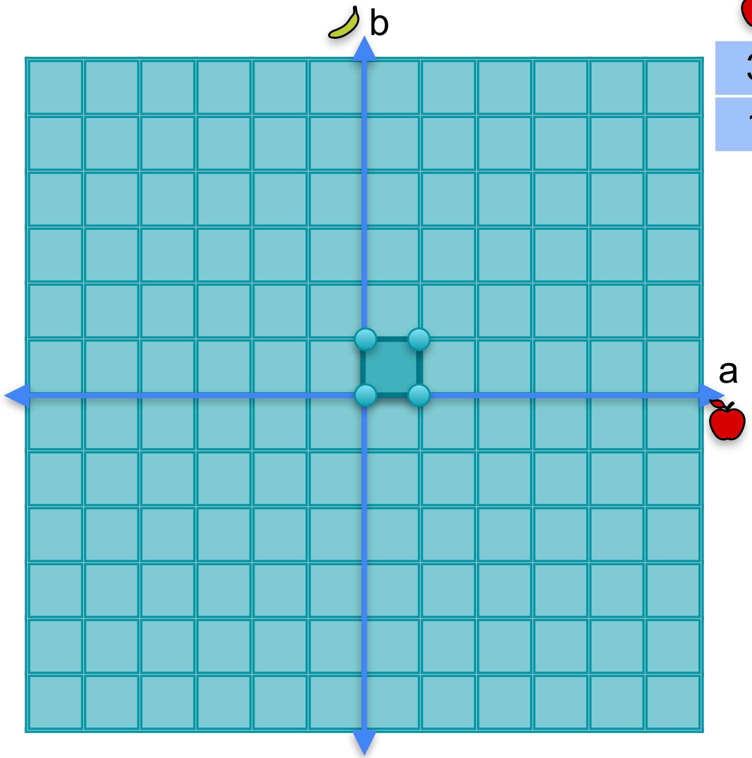
Matrices as linear transformations



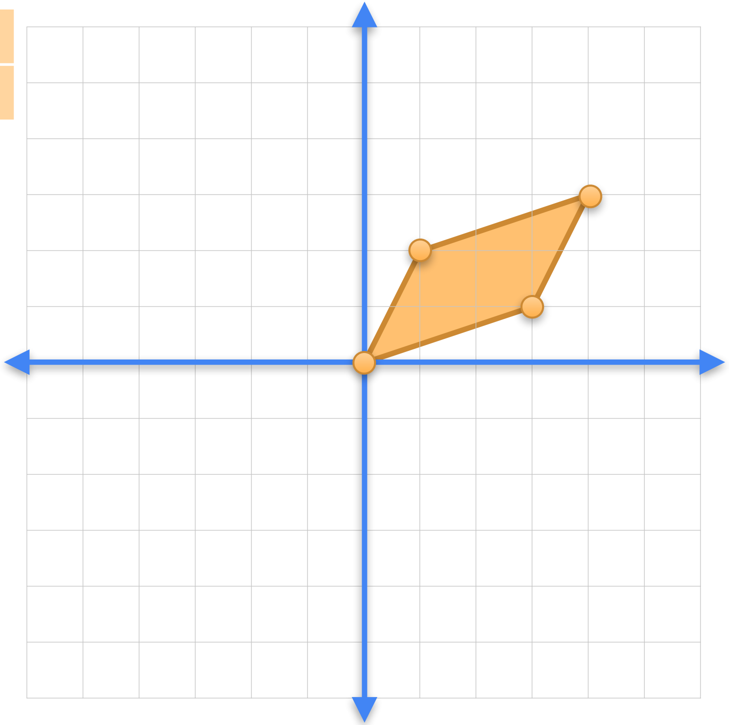
$$\begin{array}{cc} 🍎 & 🍌 \end{array}$$
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$



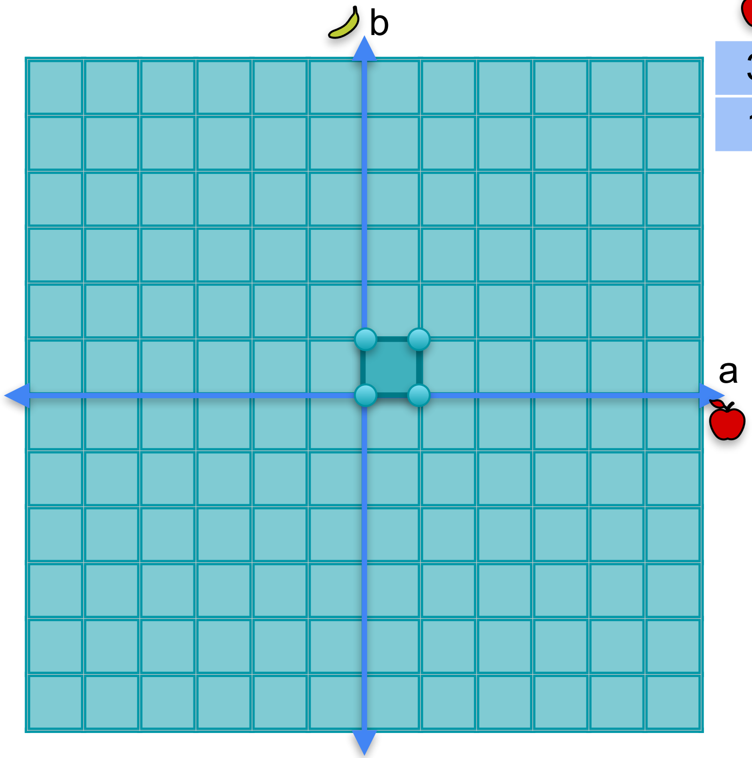
Matrices as linear transformations



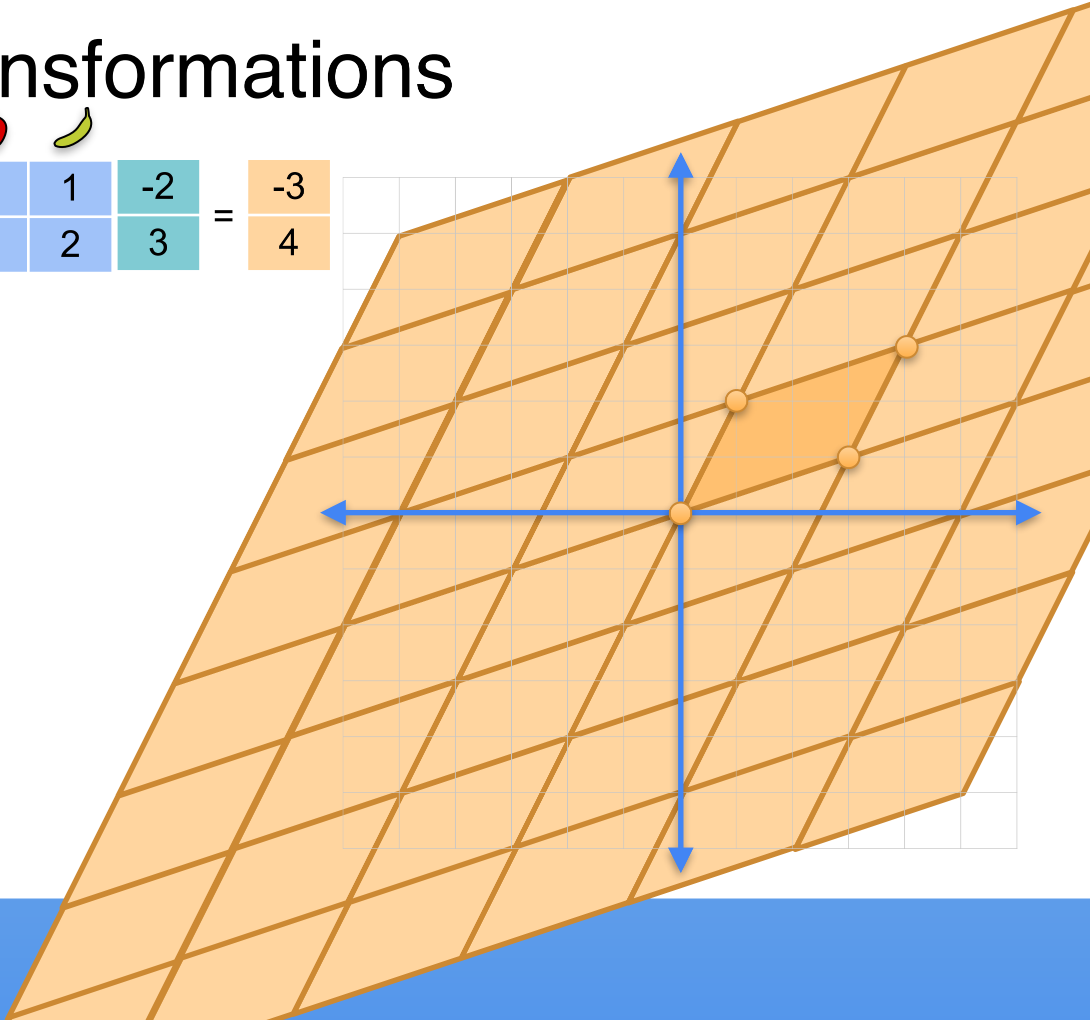
$$\begin{bmatrix}3 & 1\\1 & 2\end{bmatrix}\begin{bmatrix}-2\\3\end{bmatrix}=\begin{bmatrix}-3\\4\end{bmatrix}$$



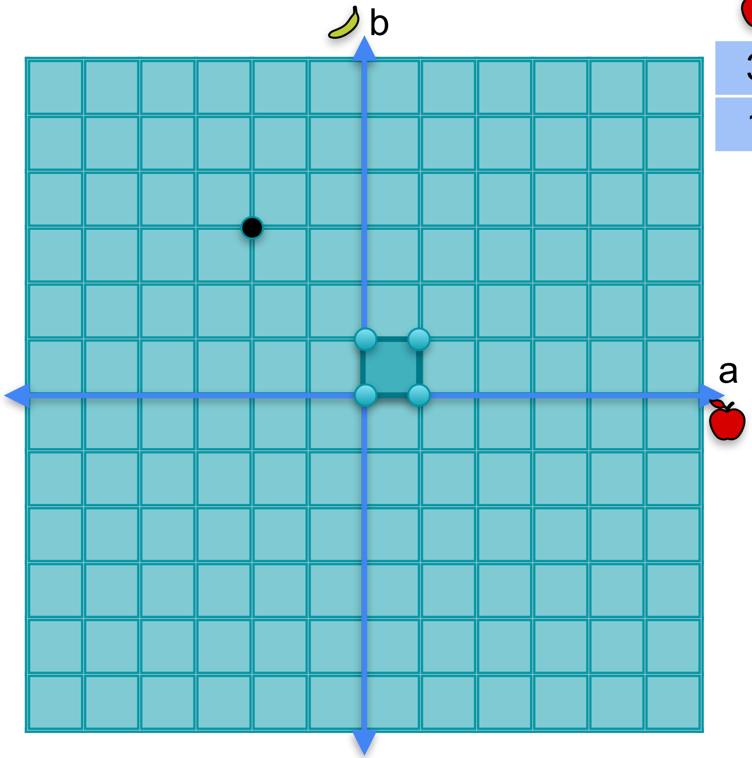
Matrices as linear transformations



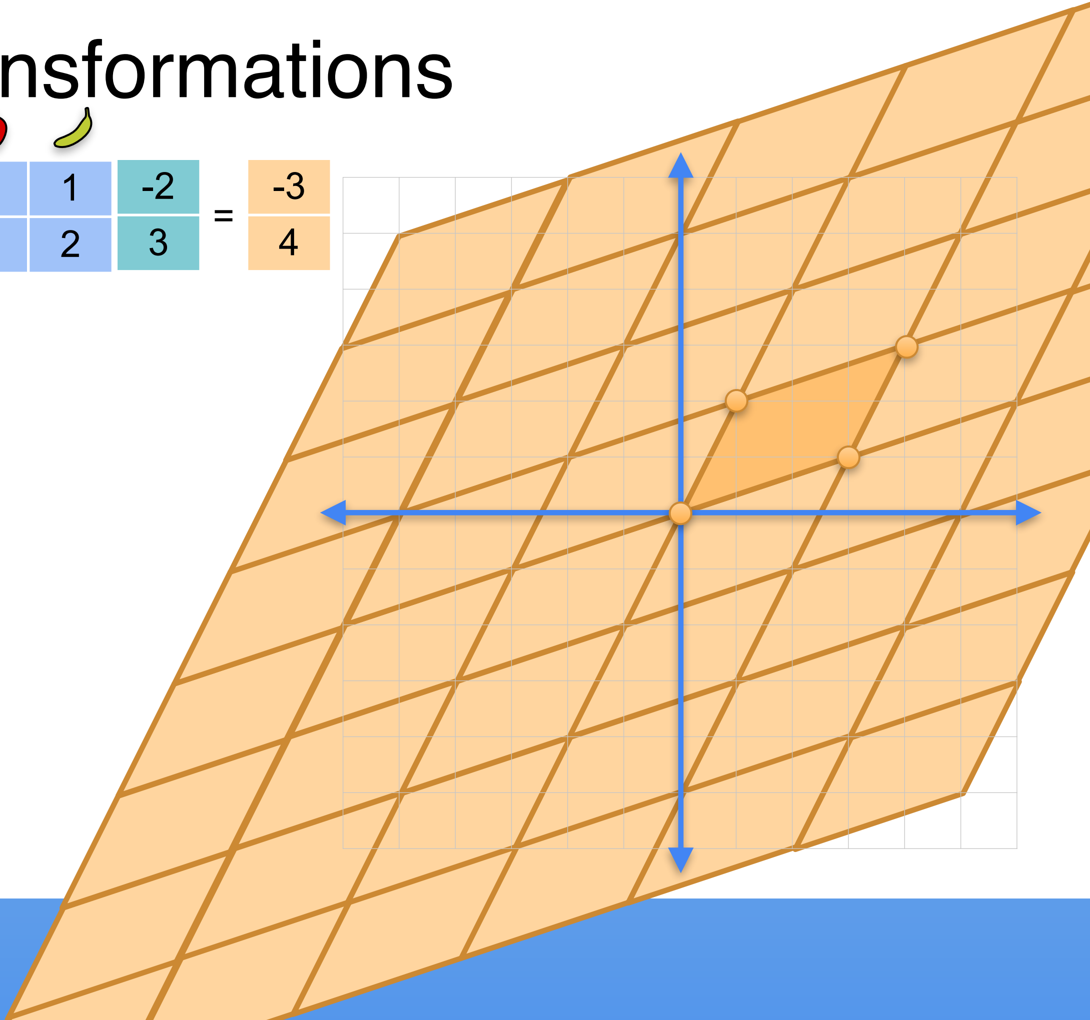
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$



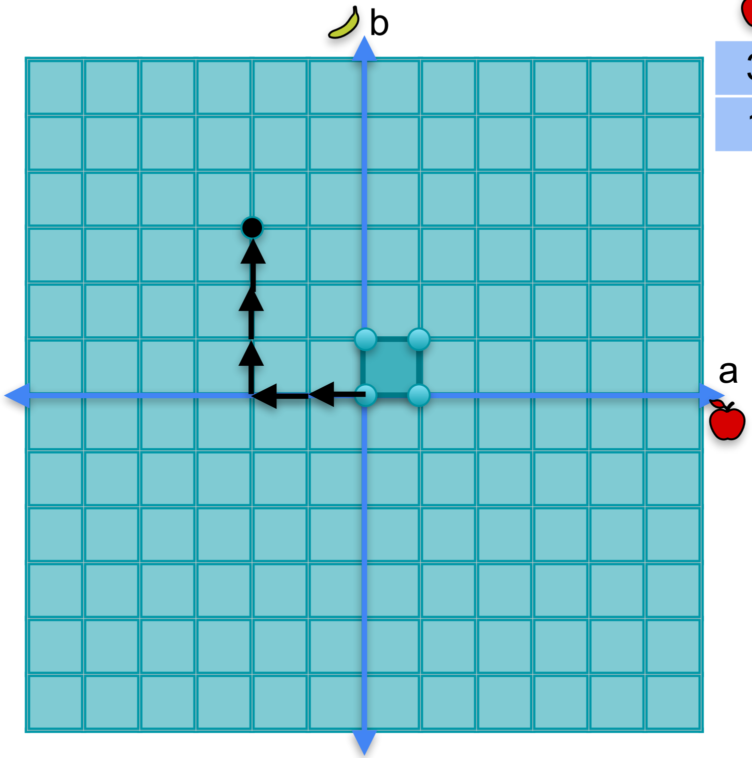
Matrices as linear transformations



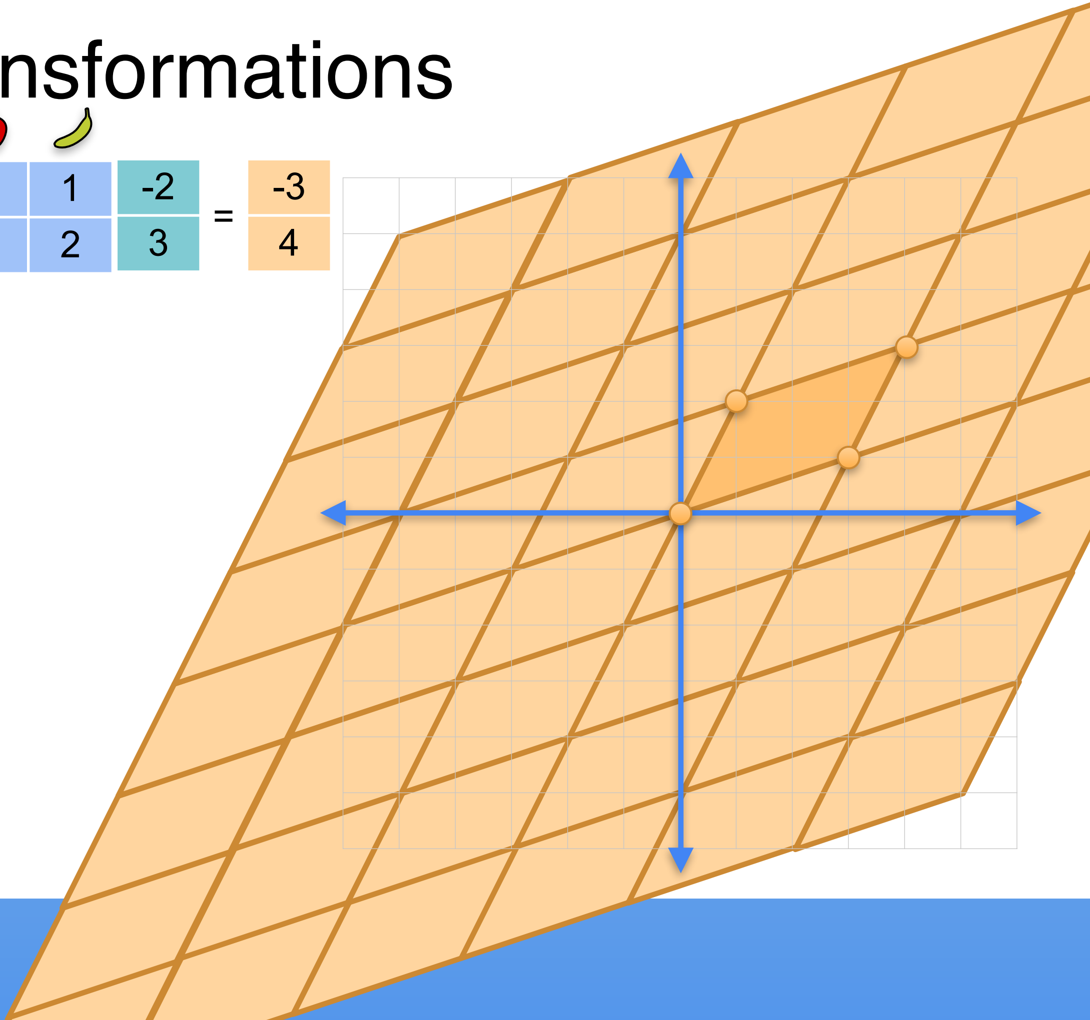
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$



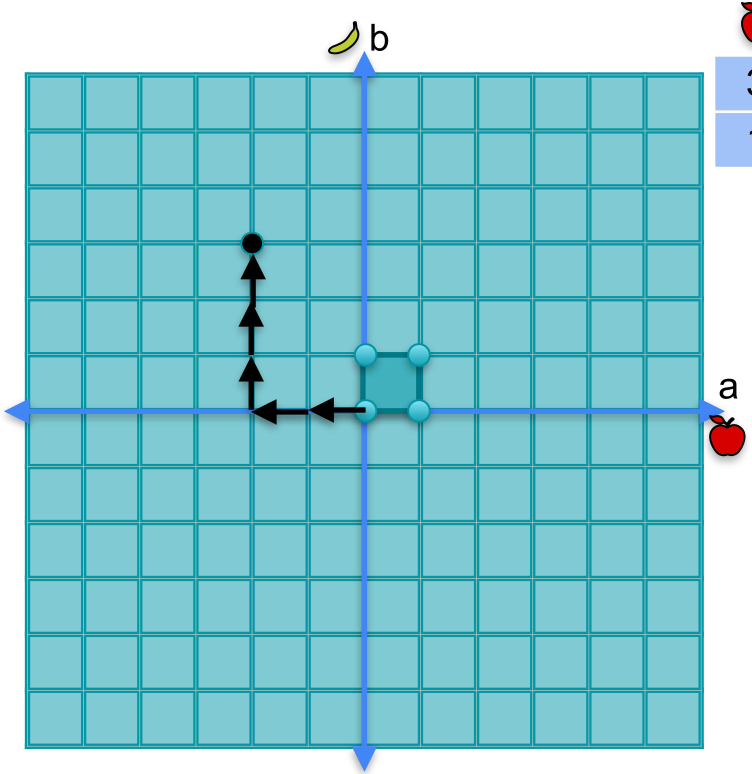
Matrices as linear transformations



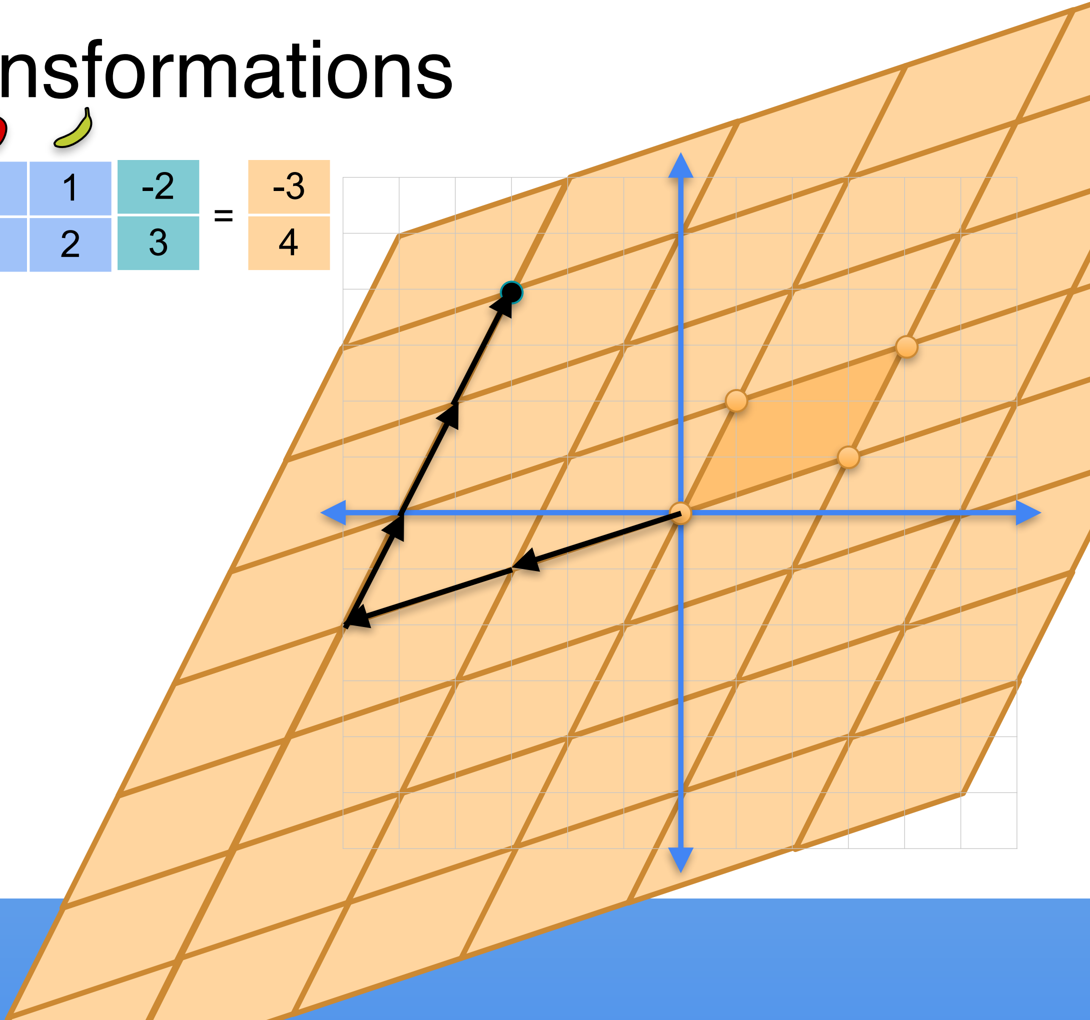
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$



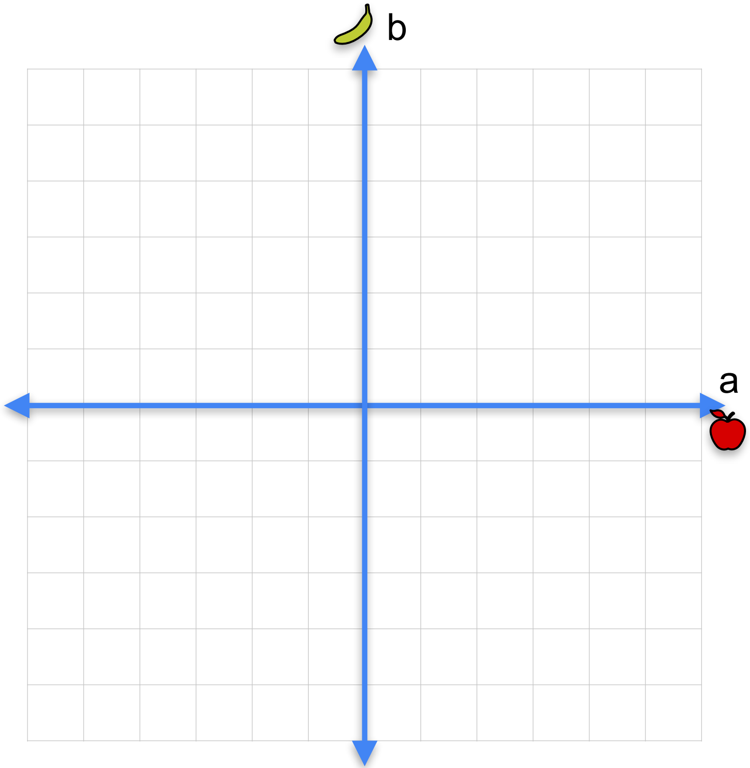
Matrices as linear transformations

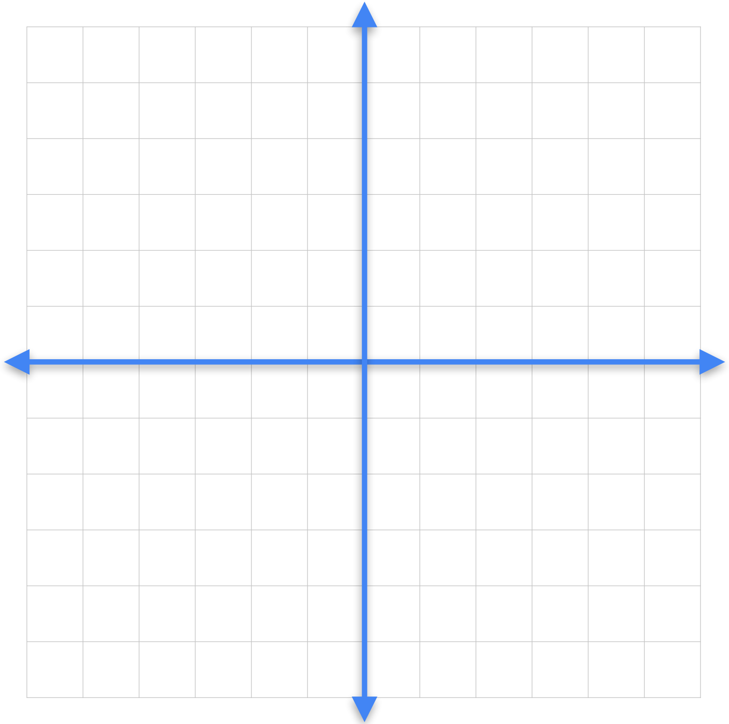


$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$

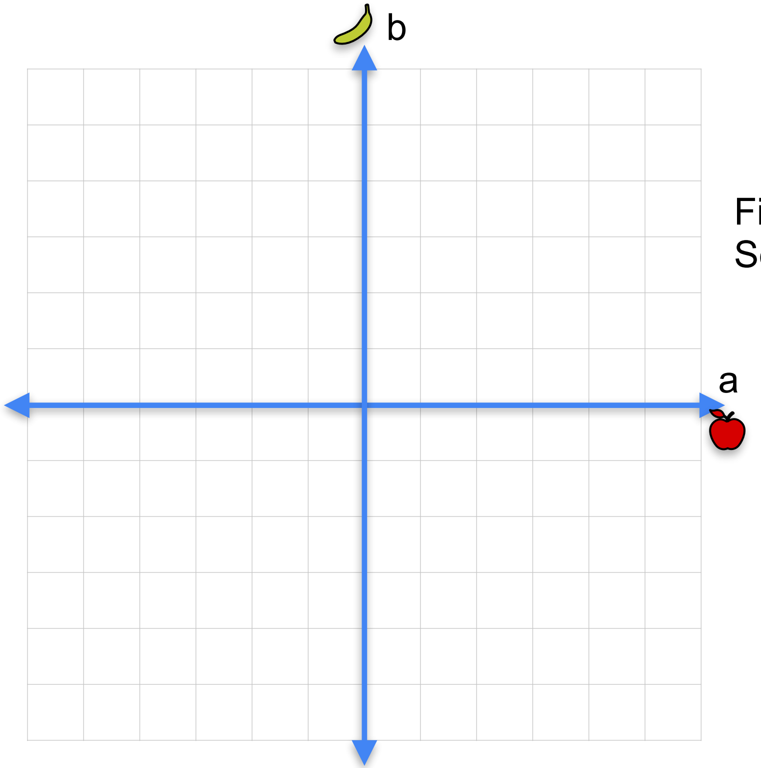


Systems of equations as linear transformations

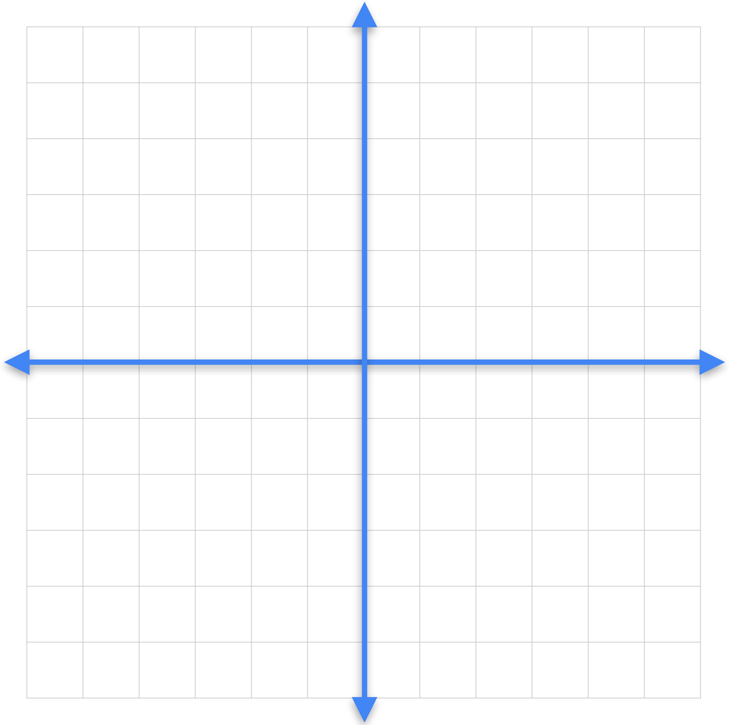




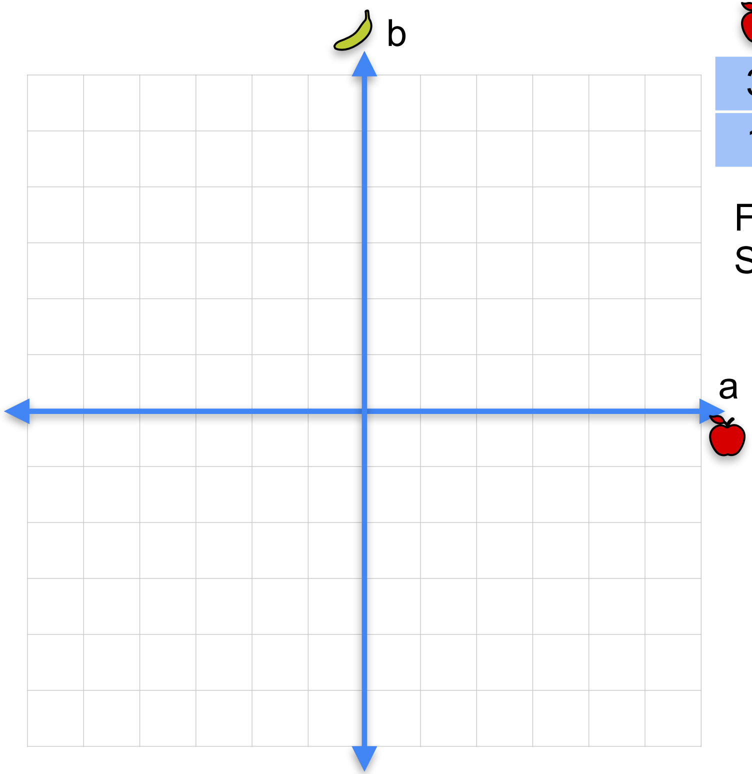
Systems of equations as linear transformations



First day: 3a + b
Second day: a + 2b

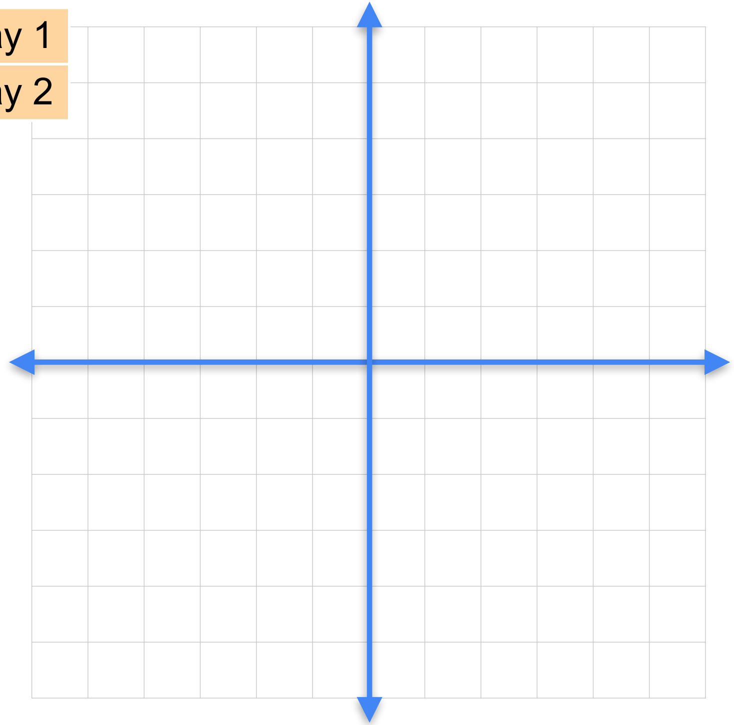


Systems of equations as linear transformations

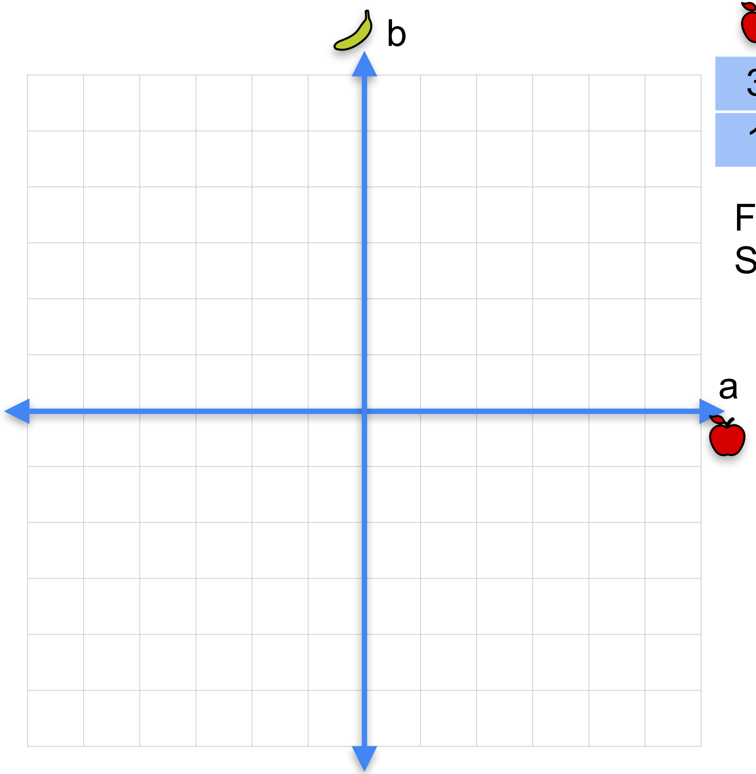


$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \text{Day 1} \\ \text{Day 2} \end{bmatrix}$$

First day: $3a + b$
Second day: $a + 2b$

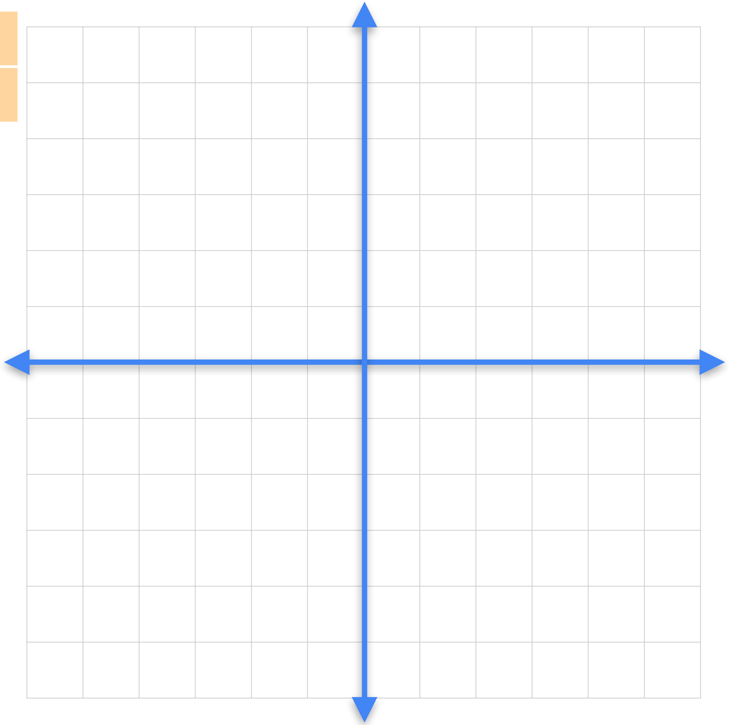


Systems of equations as linear transformations

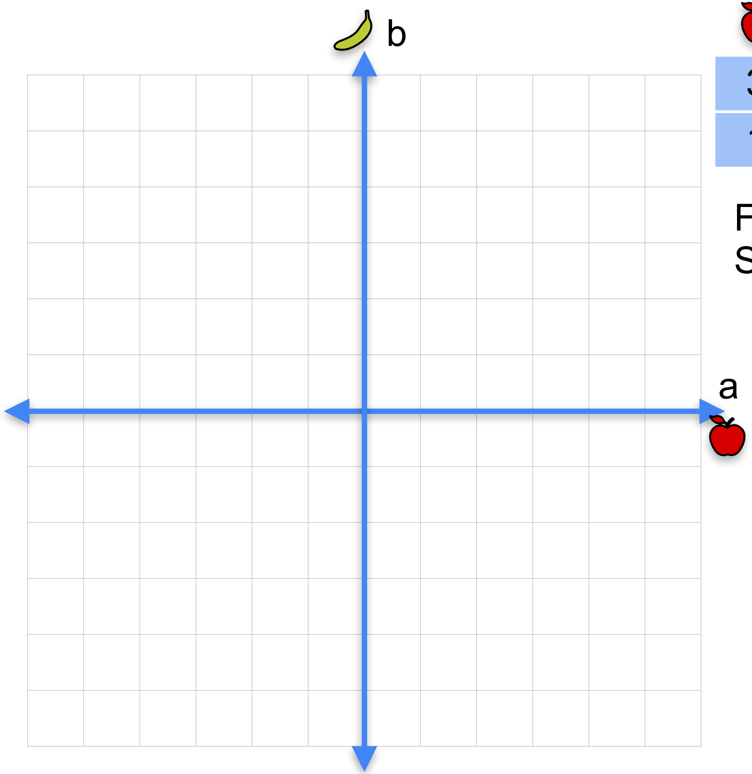


$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

First day: $3a + b$
Second day: $a + 2b$



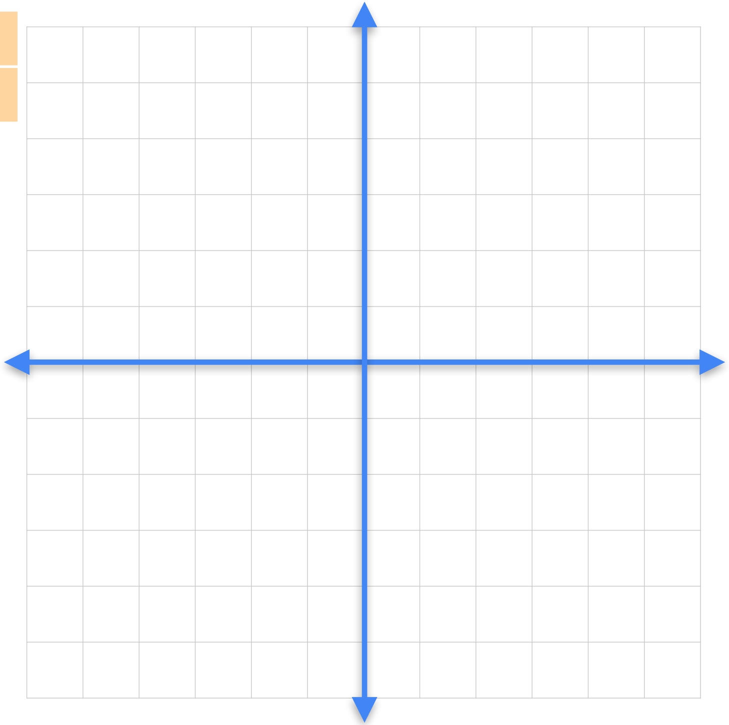
Systems of equations as linear transformations



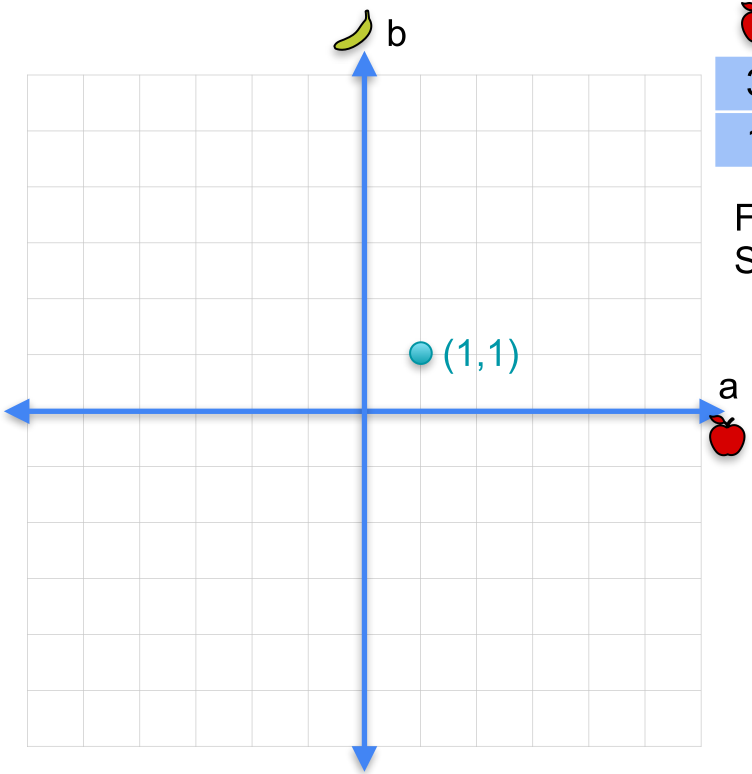
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

First day:	$3a + b$
Second day:	$a + 2b$

$(1,1) \rightarrow (4,3)$



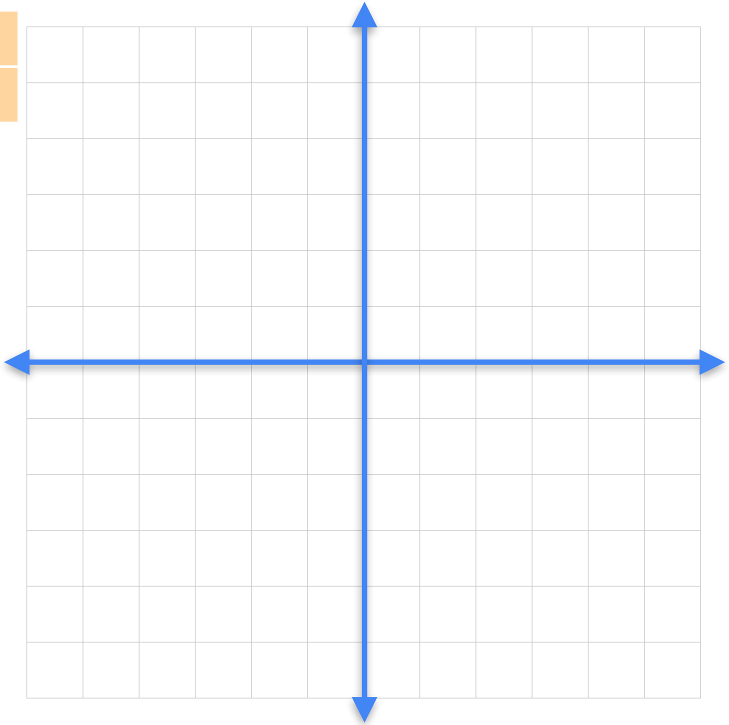
Systems of equations as linear transformations



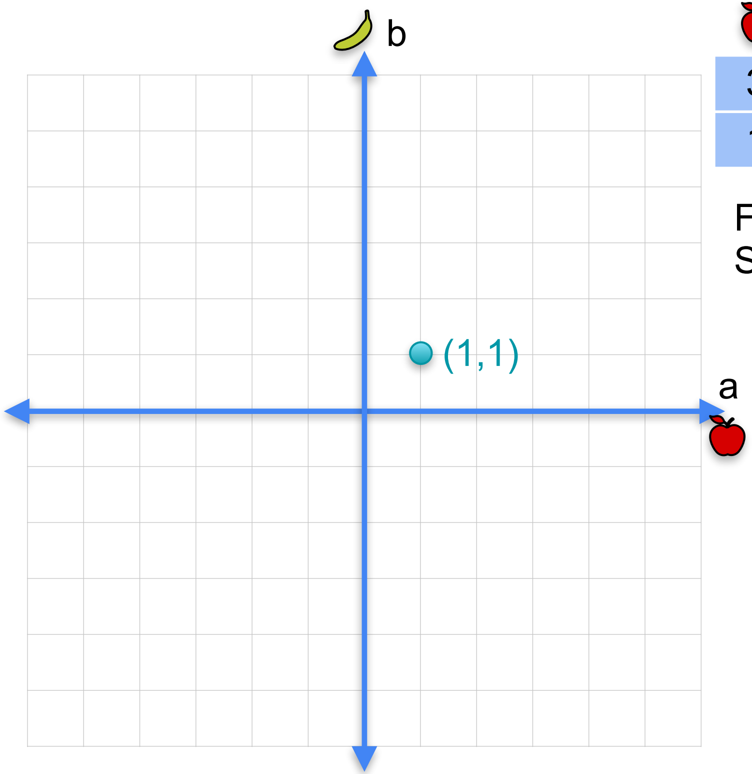
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

First day: $3a + b$
Second day: $a + 2b$

$(1,1) \rightarrow (4,3)$



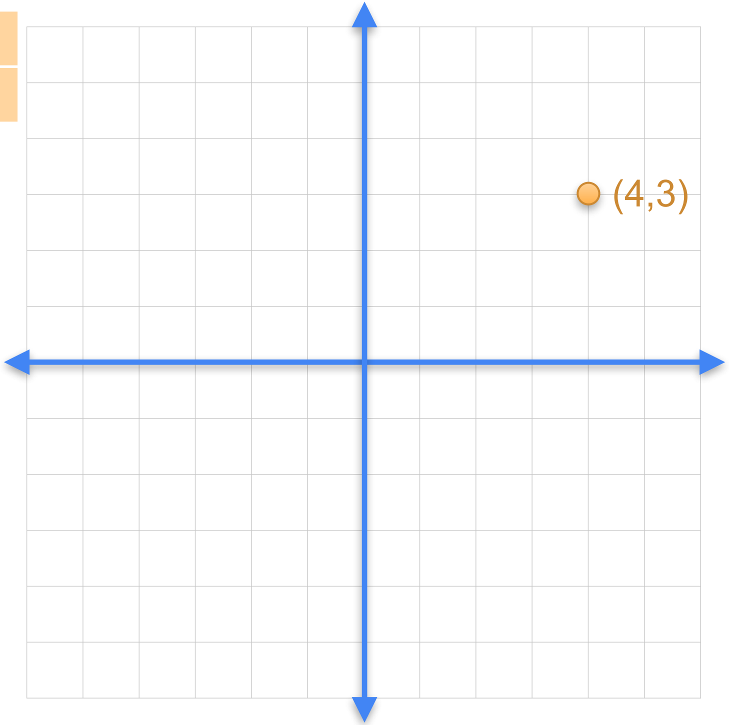
Systems of equations as linear transformations



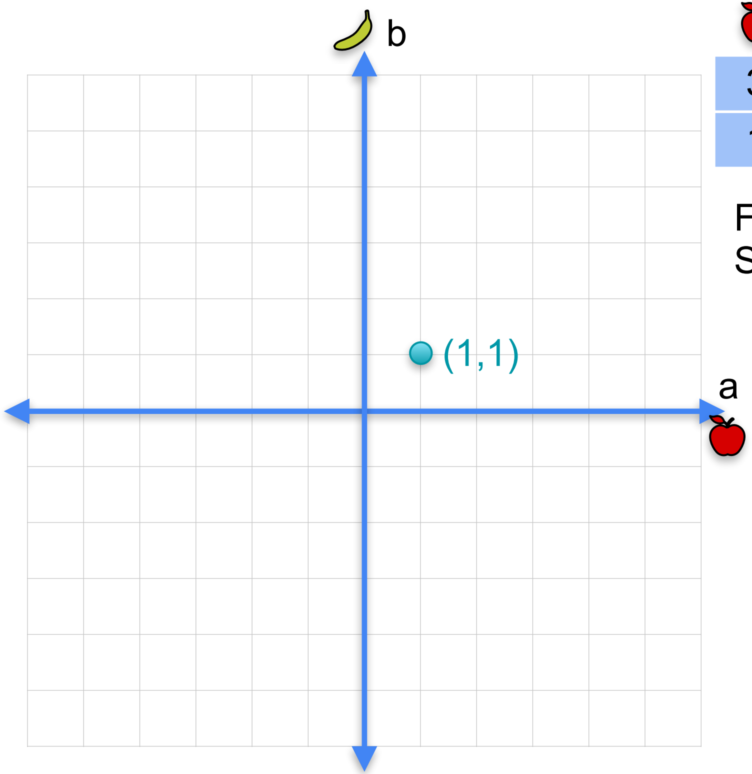
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

First day: 3a + b
Second day: a + 2b

(1,1) → (4,3)



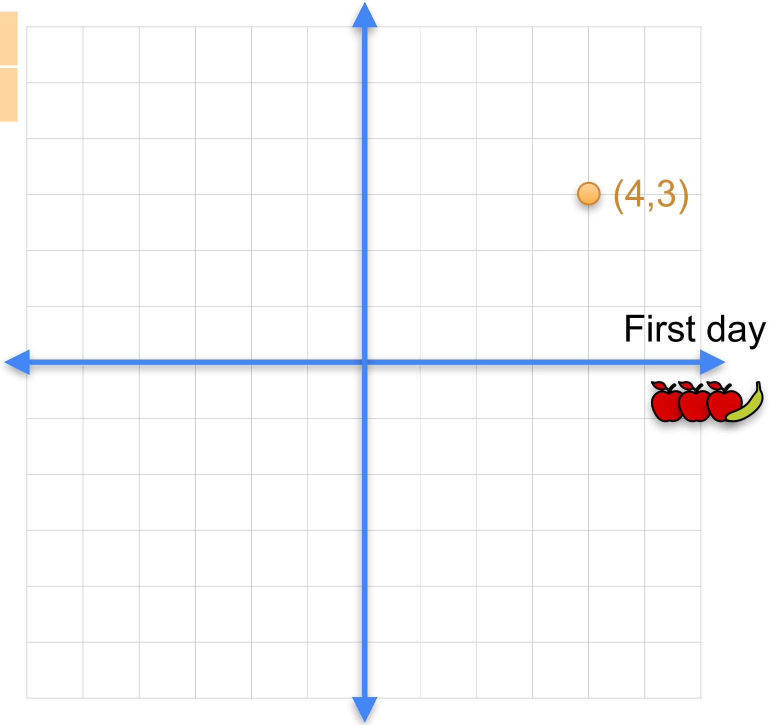
Systems of equations as linear transformations



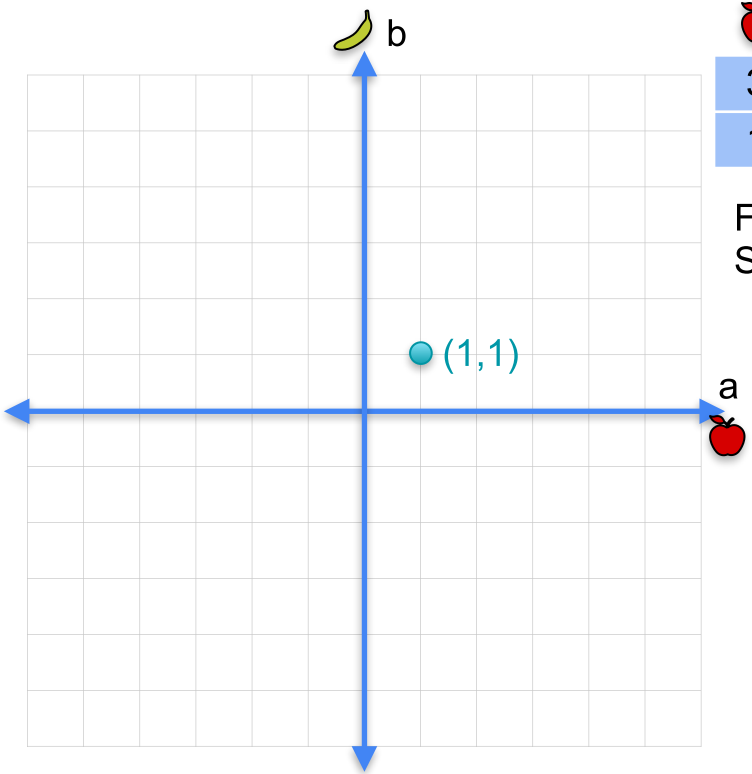
$$\begin{bmatrix}3 & 1\\1 & 2\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}=\begin{bmatrix}4\\3\end{bmatrix}$$

First day: $3a + b$
Second day: $a + 2b$

$(1,1) \rightarrow (4,3)$



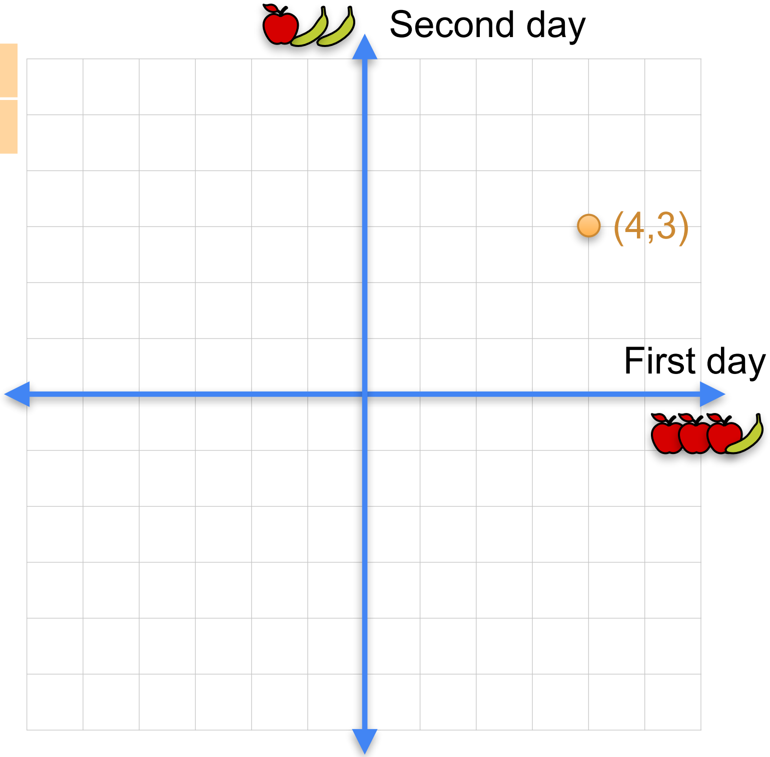
Systems of equations as linear transformations



$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

First day: 3a + b
Second day: a + 2b

(1,1) → (4,3)

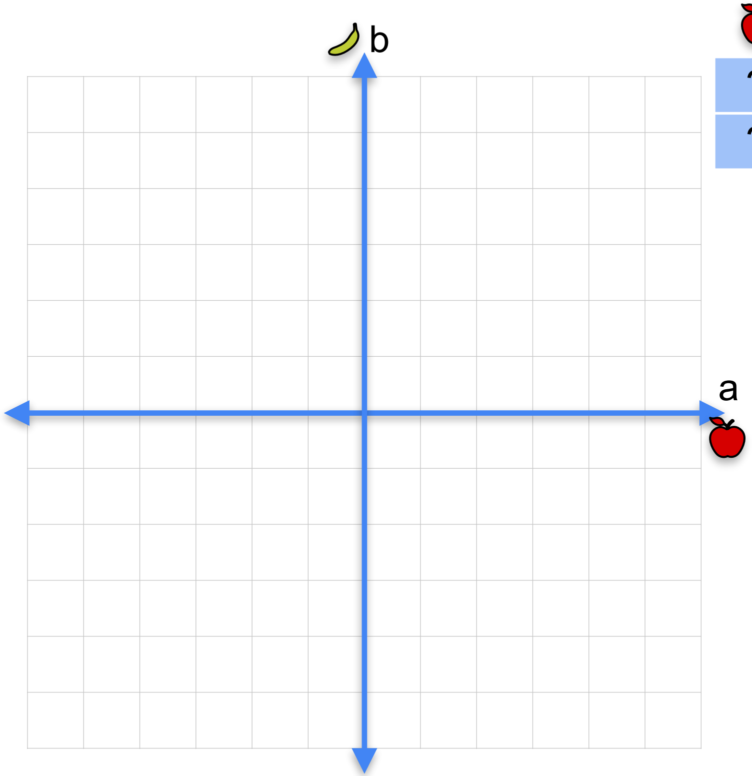


DeepLearning.AI

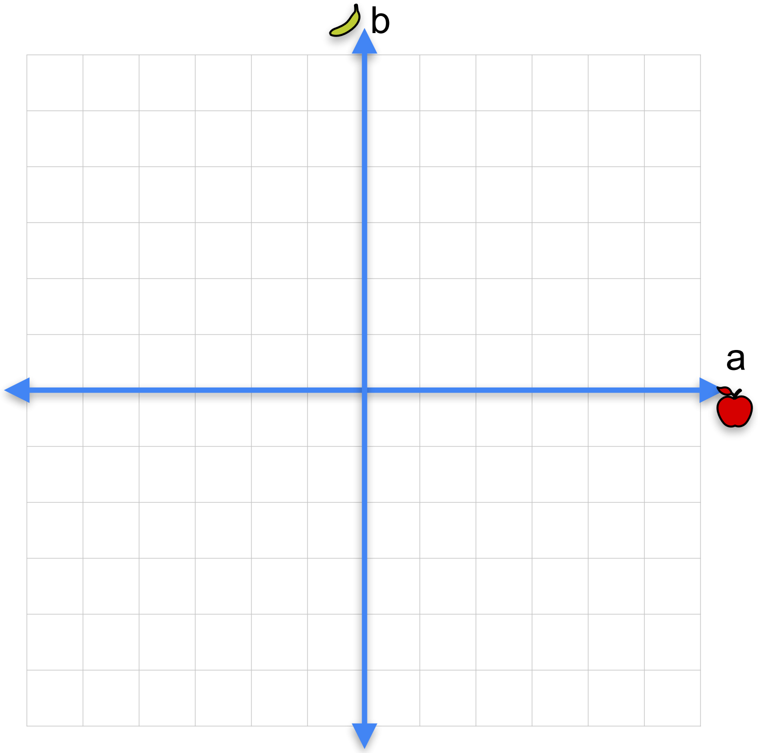
Vectors and Linear Transformations

Linear transformations as matrices

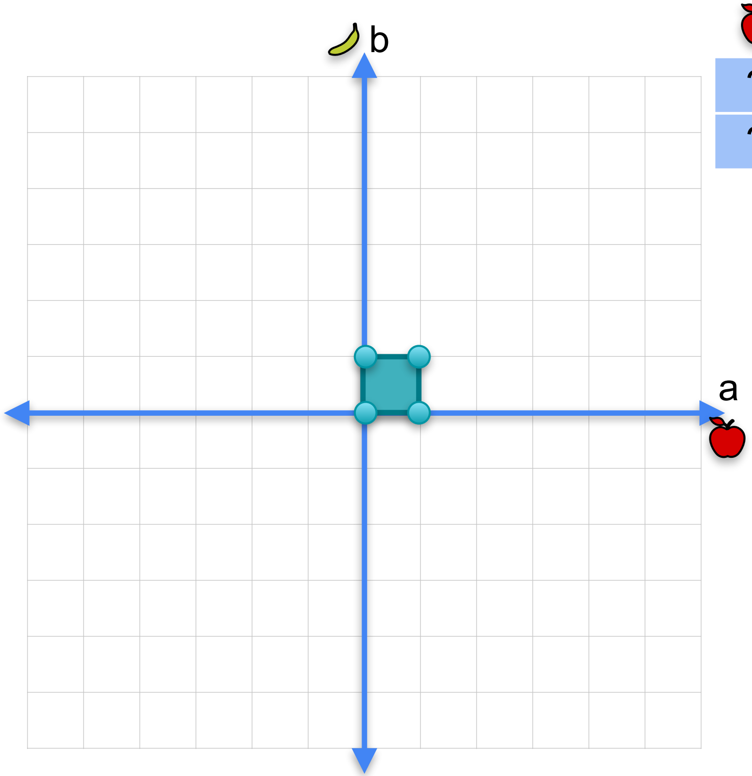
Linear transformations as matrices



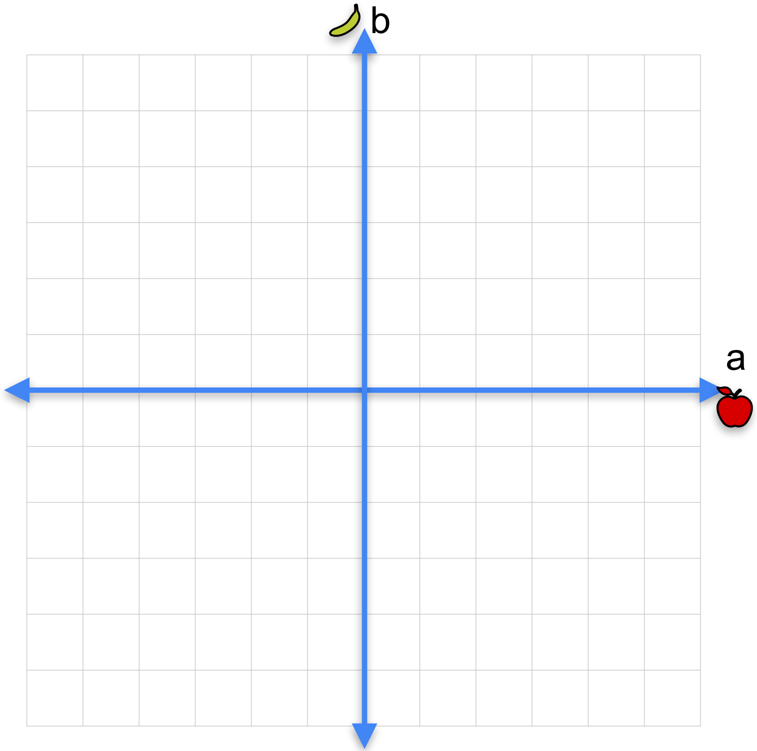
$$\begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$



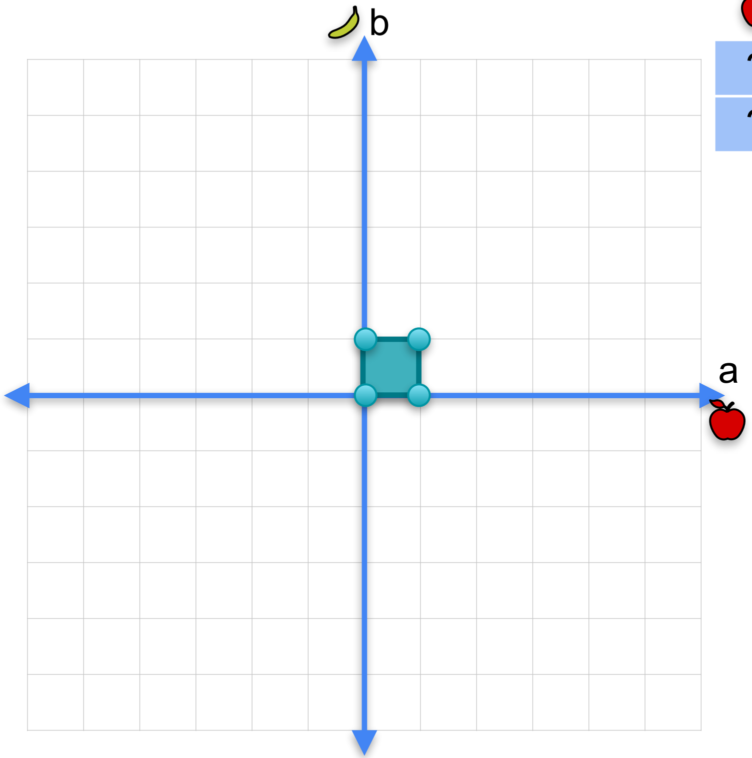
Linear transformations as matrices



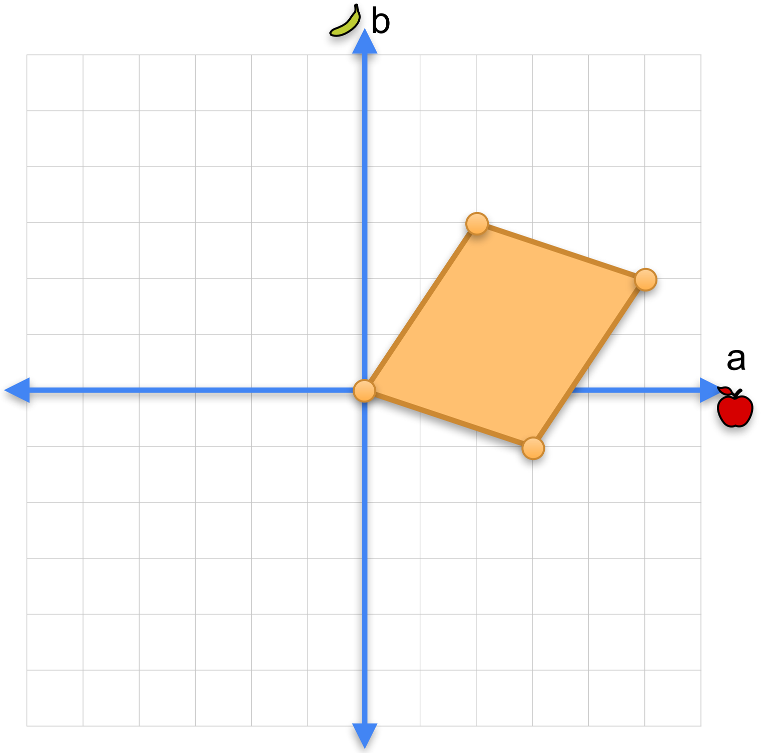
🍎	🍌
?	?
?	?



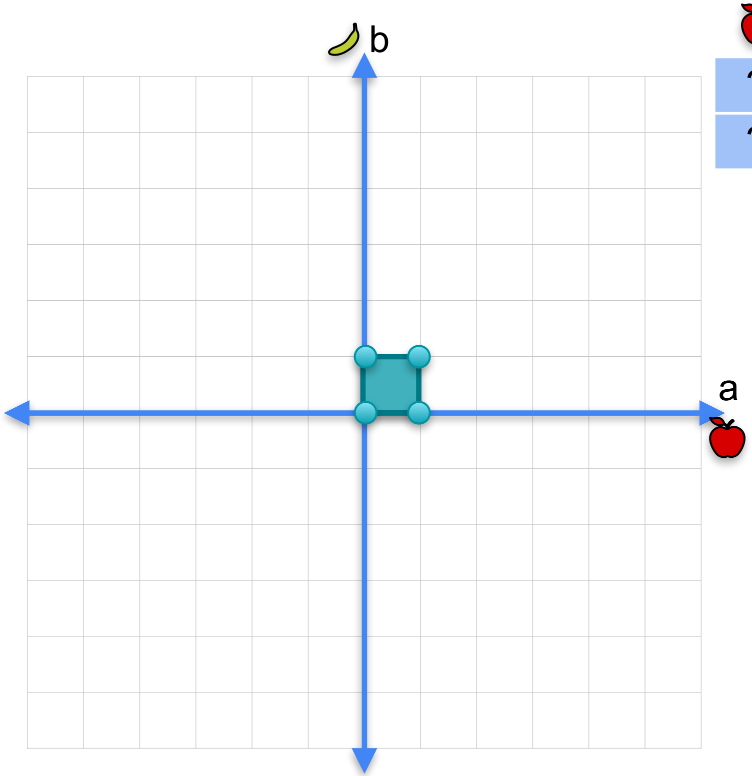
Linear transformations as matrices



?	?
?	?

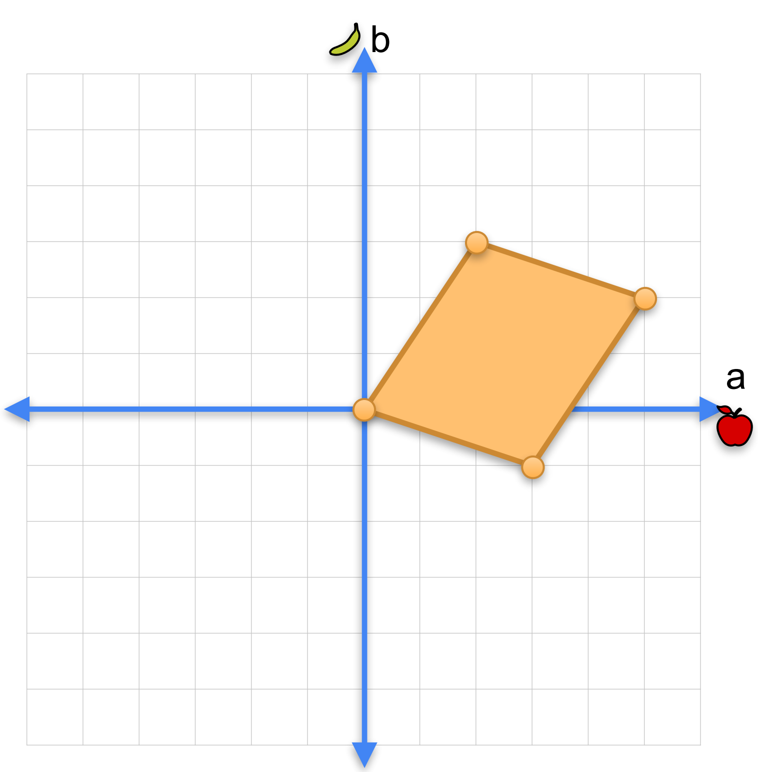


Linear transformations as matrices

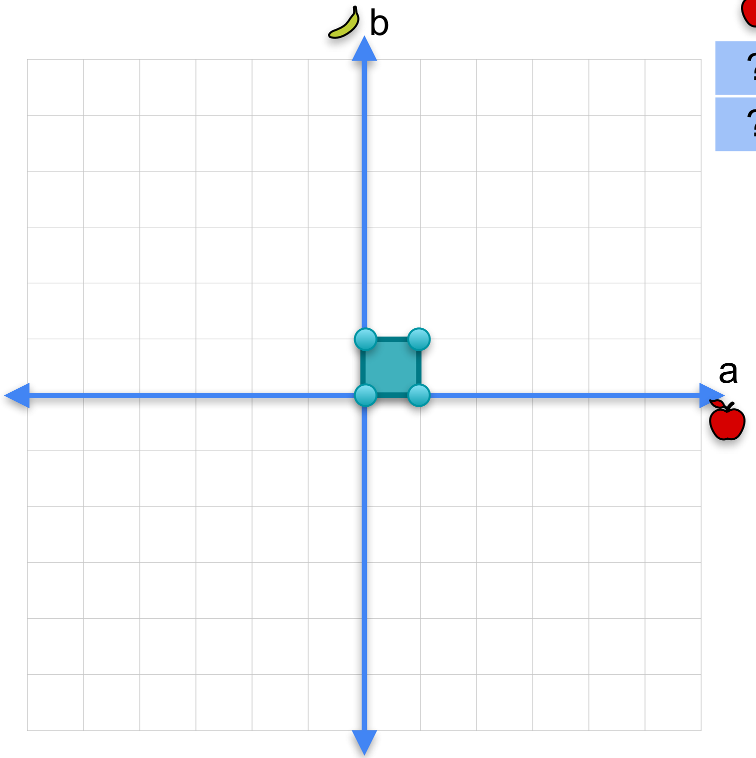


🍎	🍌
?	?
?	?

(0,0) → (0,0)



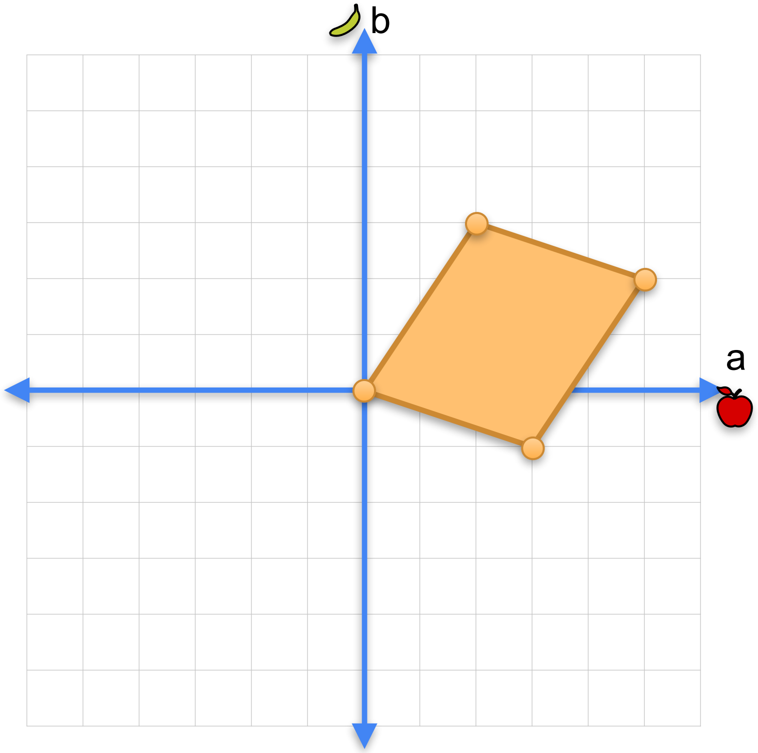
Linear transformations as matrices



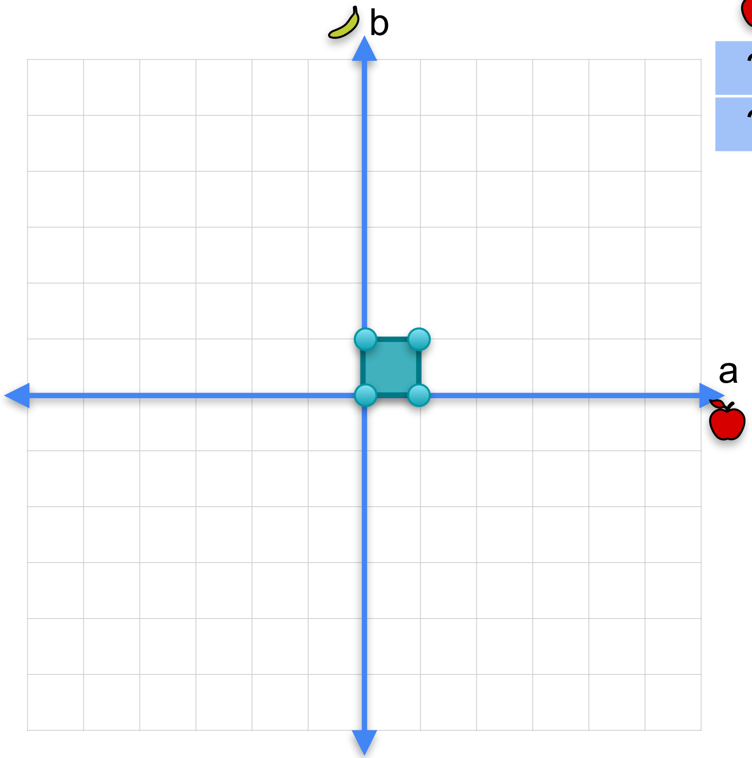
?	?
?	?

(0,0) → (0,0)

(1,0) → (3,-1)

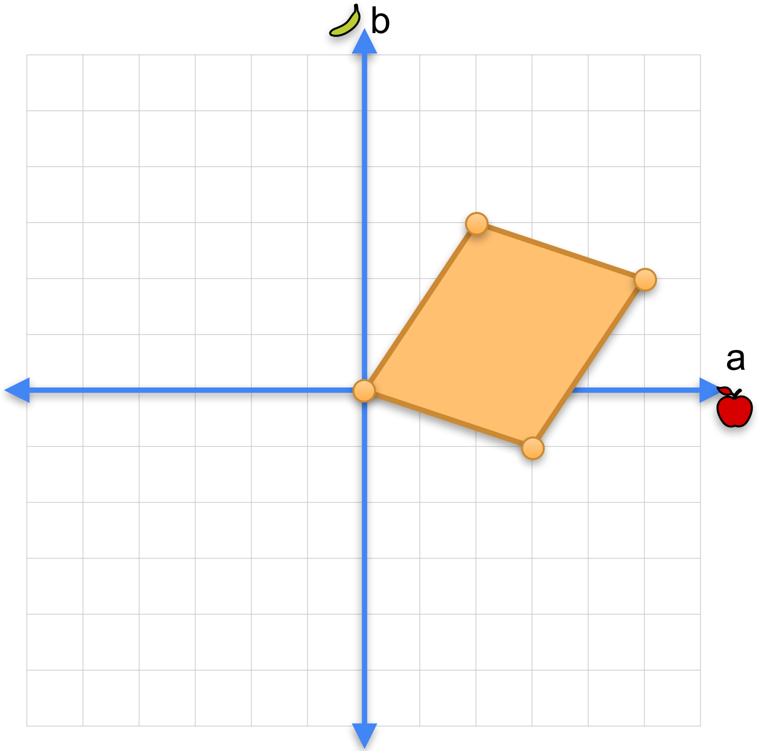


Linear transformations as matrices

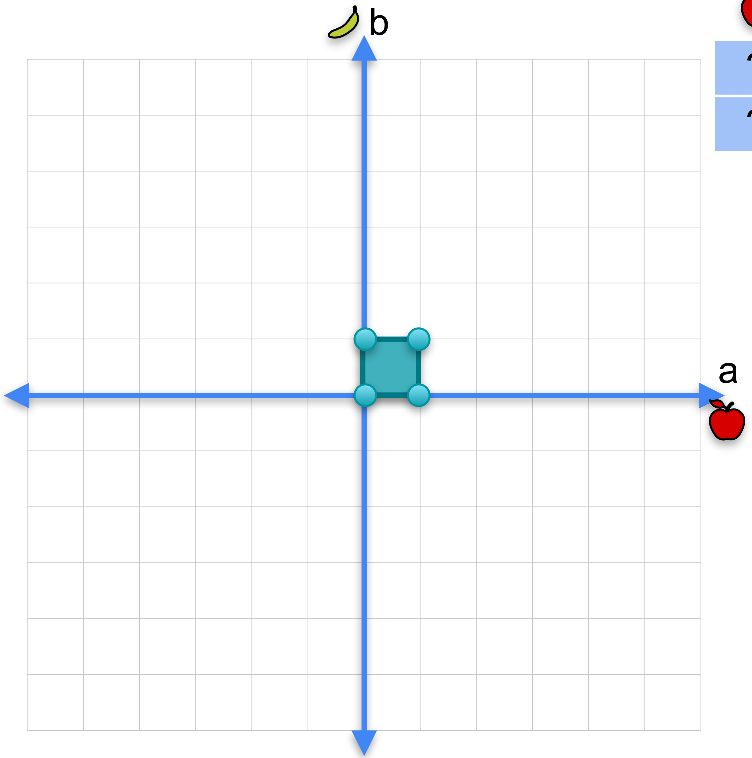


?	?
?	?

(0,0) → (0,0)
(1,0) → (3,-1)
(0,1) → (2,3)



Linear transformations as matrices



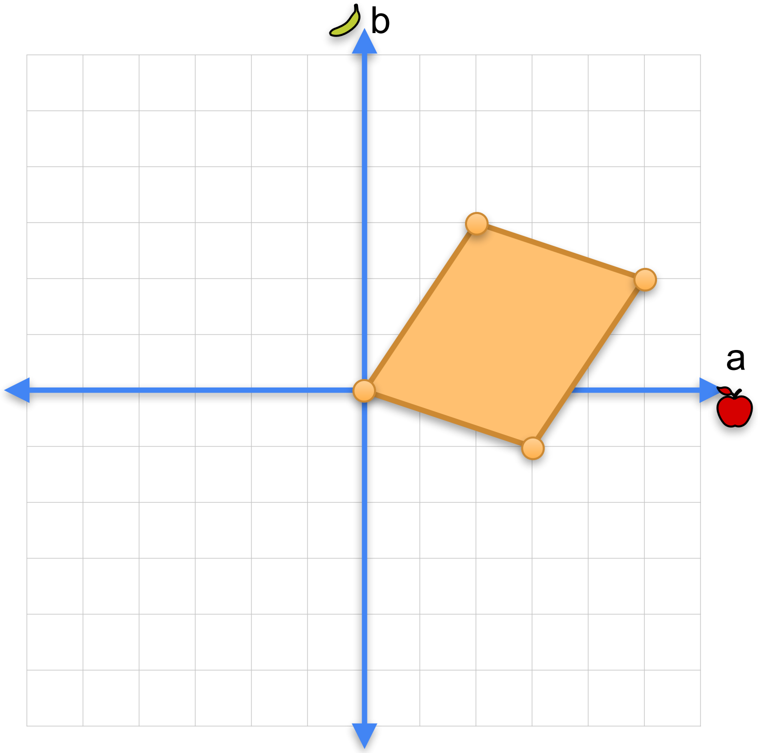
?	?
?	?

(0,0) → (0,0)

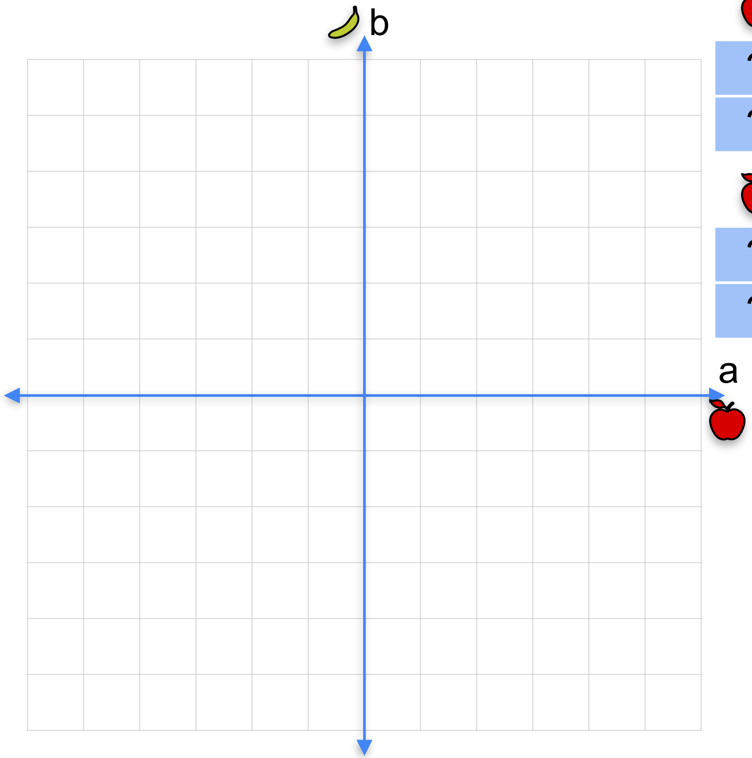
(1,0) → (3,-1)

(0,1) → (2,3)

(1,1) → (5,2)



Linear transformations as matrices

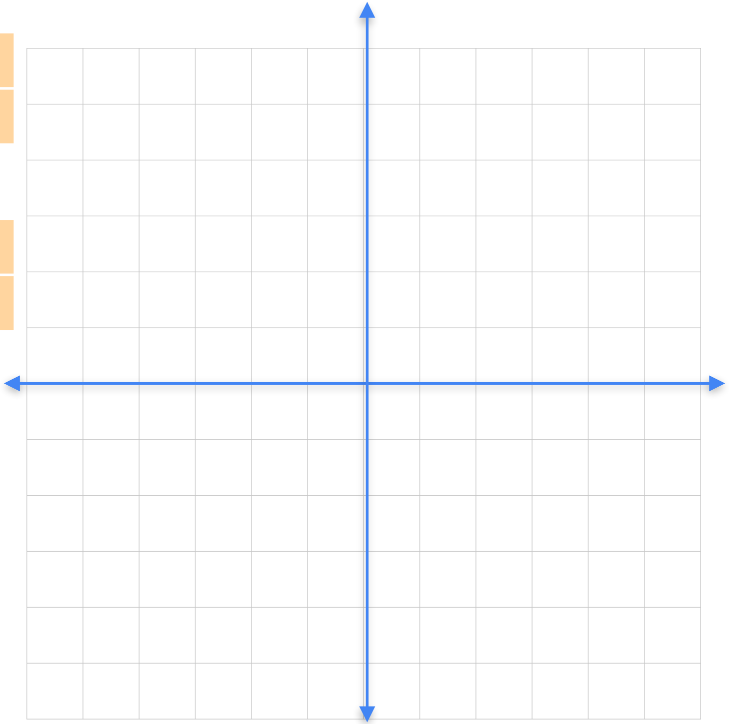


$$\begin{array}{cc} 🍎 & 🍌 \\ \end{array}$$
$$\begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

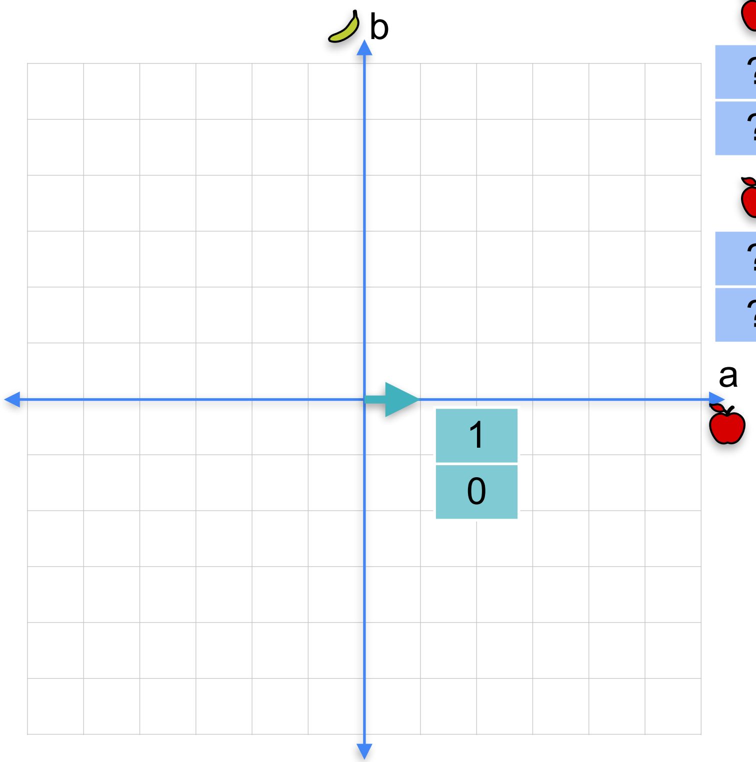
$$\begin{array}{cc} 🍎 & 🍌 \\ \end{array}$$
$$\begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

$(1,0) \rightarrow (3,-1)$

$(0,1) \rightarrow (2,3)$



Linear transformations as matrices

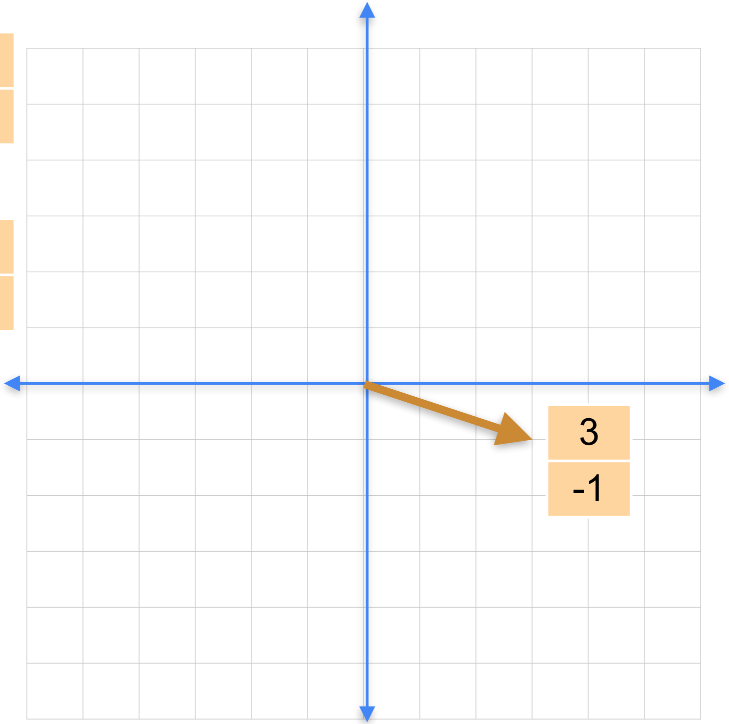


$$\begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

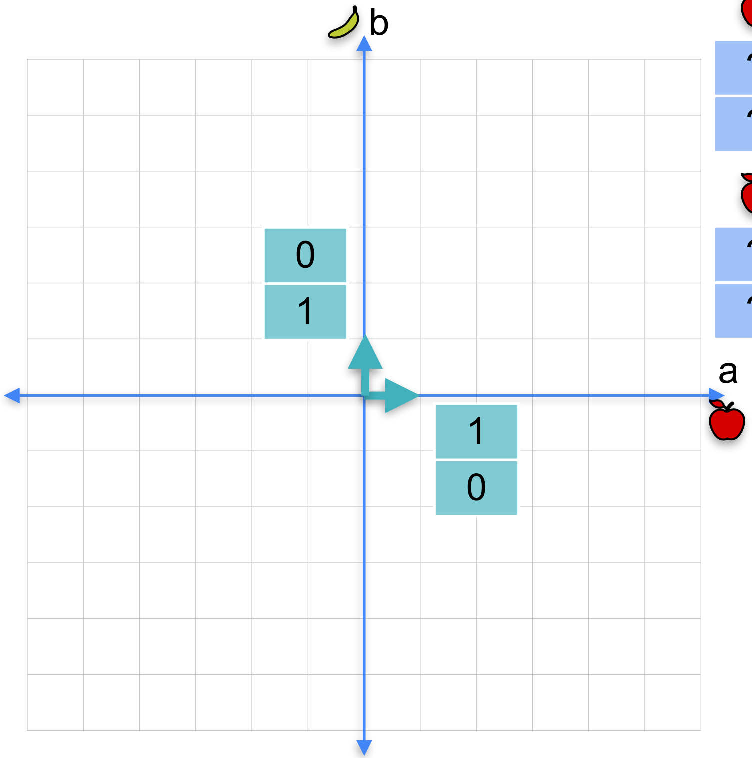
$$\begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

$(1,0) \rightarrow (3,-1)$

$(0,1) \rightarrow (2,3)$



Linear transformations as matrices

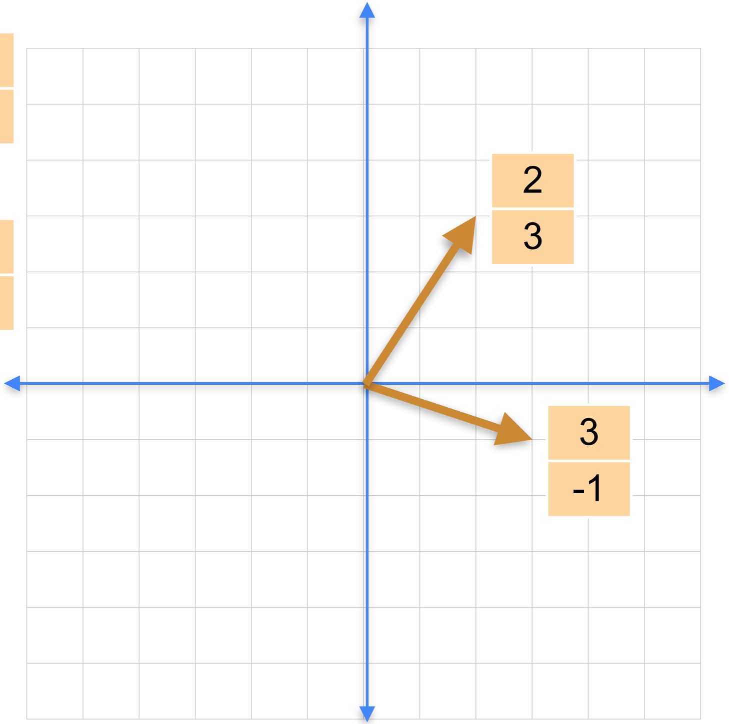


$$\begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

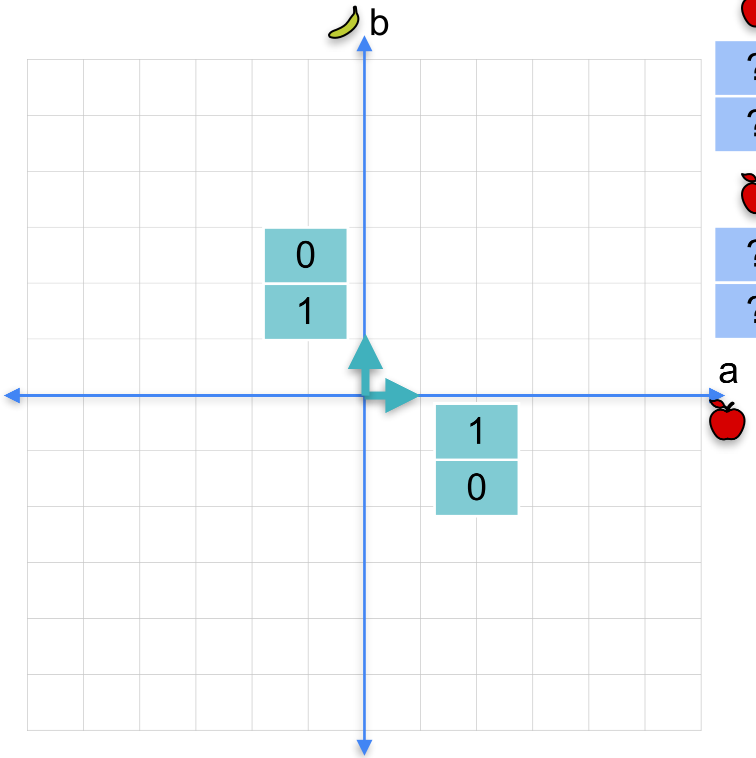
$$\begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

$(1,0) \rightarrow (3,-1)$

$(0,1) \rightarrow (2,3)$



Linear transformations as matrices

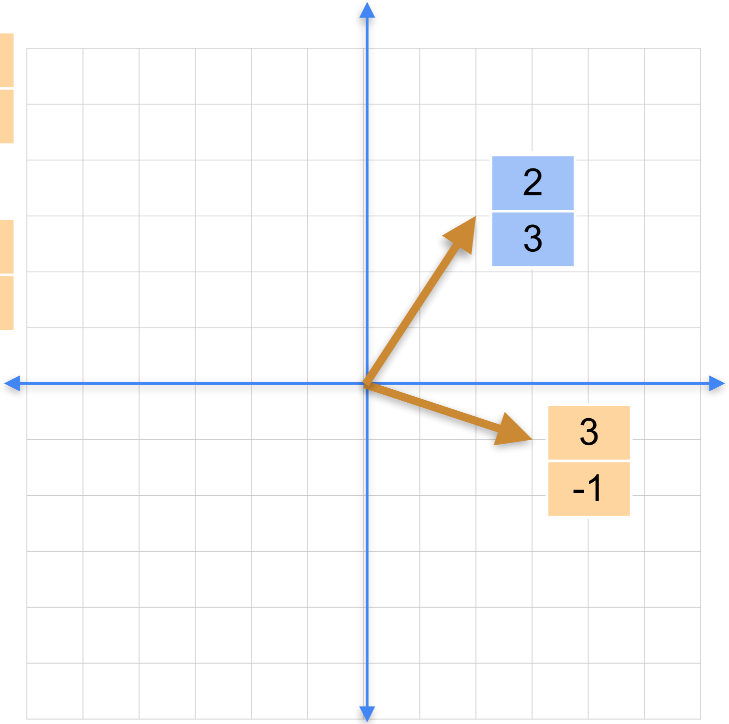


$$\begin{array}{cc} 🍎 & 🍌 \end{array}$$
$$\begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

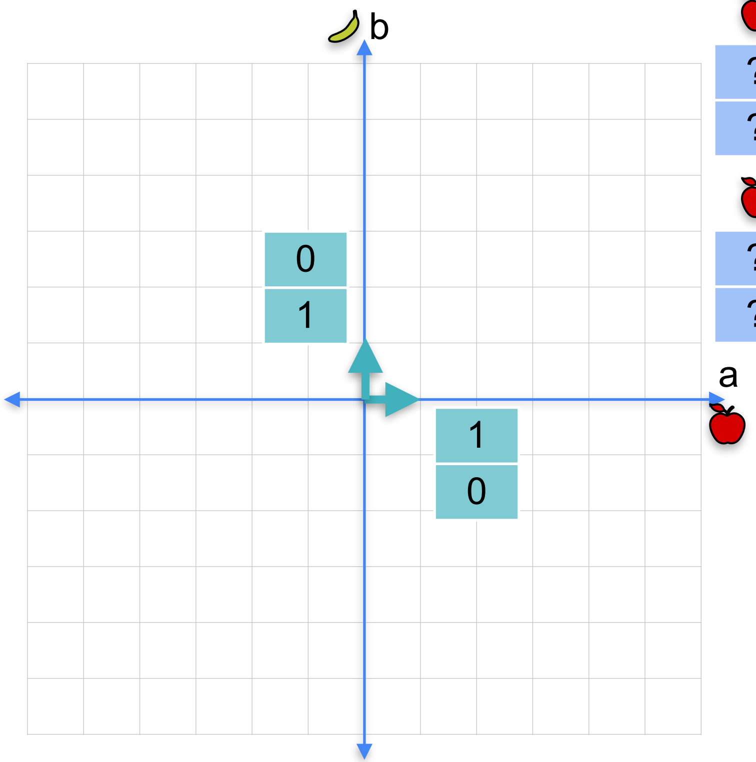
$$\begin{array}{cc} 🍎 & 🍌 \end{array}$$
$$\begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

$(1,0) \rightarrow (3,-1)$

$(0,1) \rightarrow (2,3)$



Linear transformations as matrices

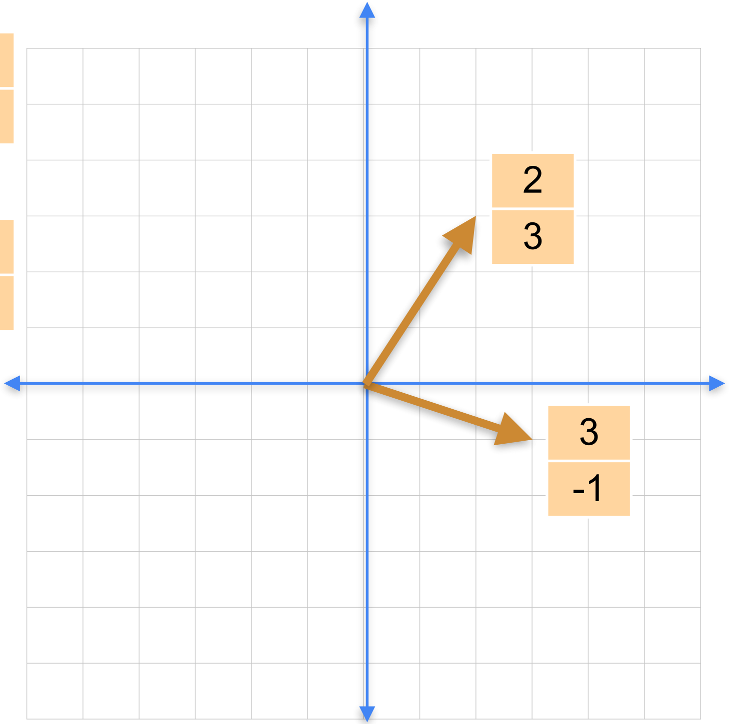


$$\begin{bmatrix} ? & 2 \\ ? & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

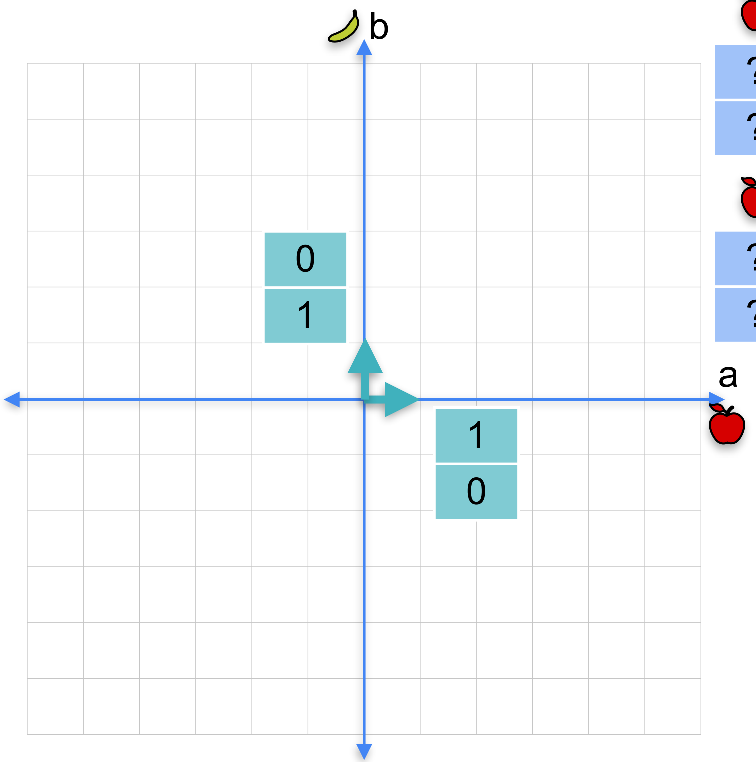
$$\begin{bmatrix} ? & 2 \\ ? & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

$(1,0) \rightarrow (3,-1)$

$(0,1) \rightarrow (2,3)$



Linear transformations as matrices

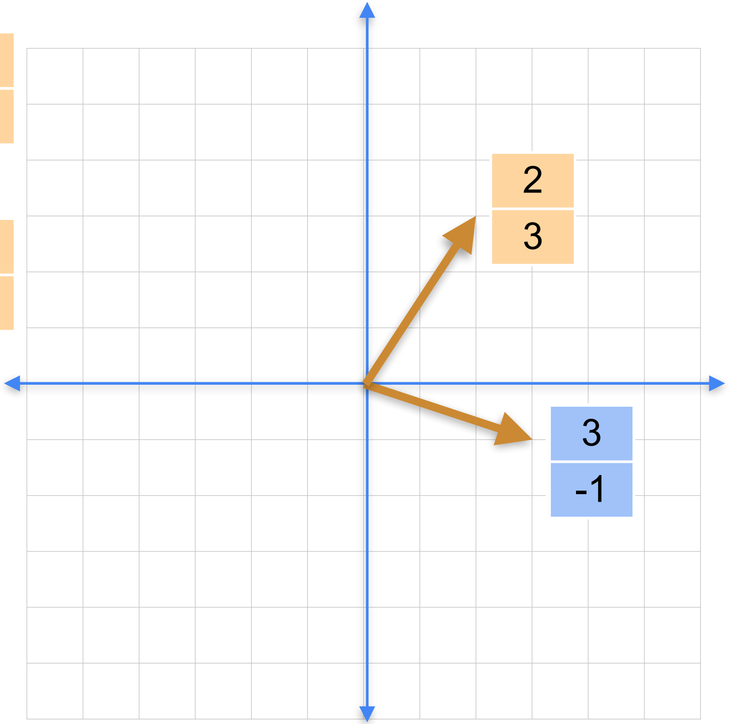


$$\begin{bmatrix} ? & 2 \\ ? & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

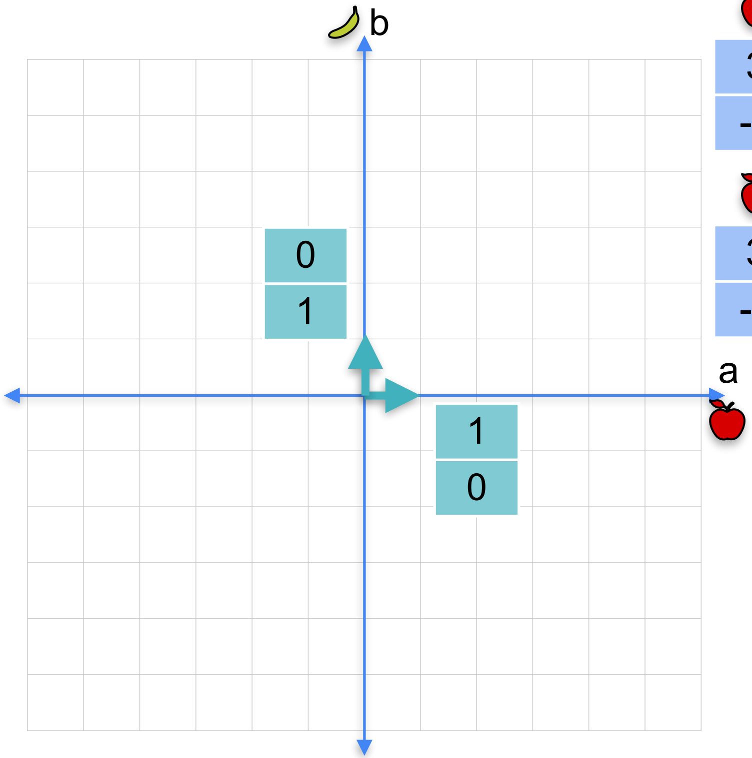
$$\begin{bmatrix} ? & 2 \\ ? & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

$(1,0) \rightarrow (3,-1)$

$(0,1) \rightarrow (2,3)$



Linear transformations as matrices

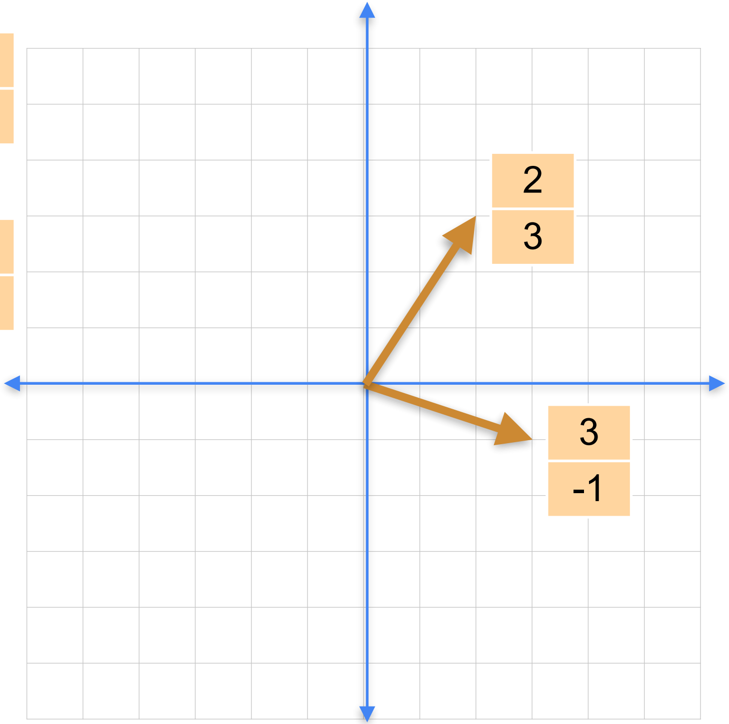


$$\begin{bmatrix}3 & 2\\-1 & 3\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}2\\3\end{bmatrix}$$

$$\begin{bmatrix}3 & 2\\-1 & 3\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}3\\-1\end{bmatrix}$$

$(1,0) \rightarrow (3,-1)$

$(0,1) \rightarrow (2,3)$

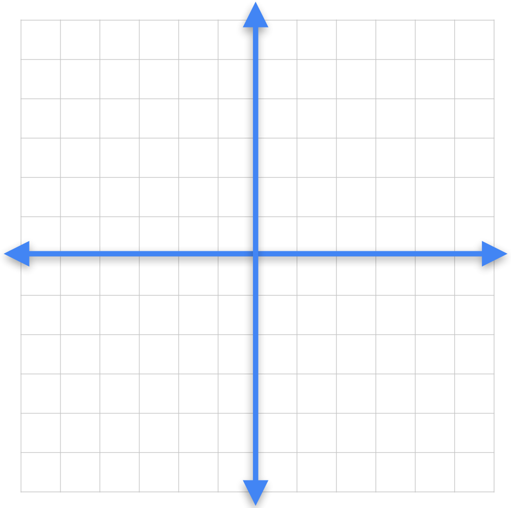


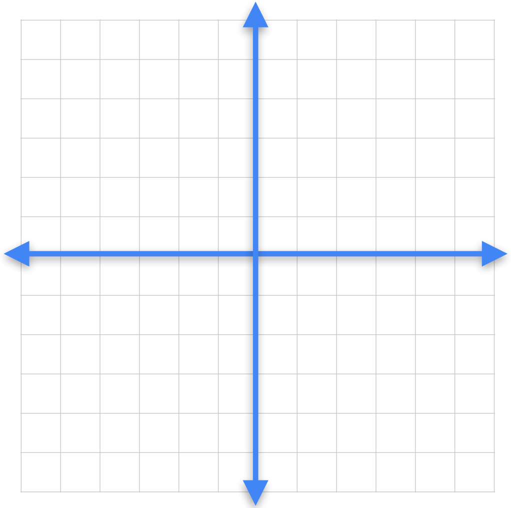
DeepLearning.AI

Vectors and Linear Transformations

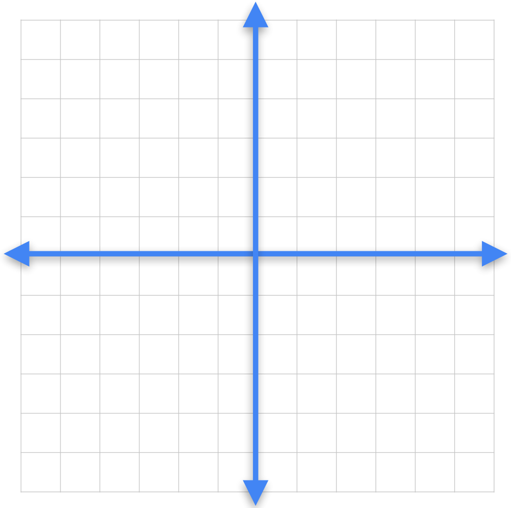
Matrix multiplication

Combining linear transformations

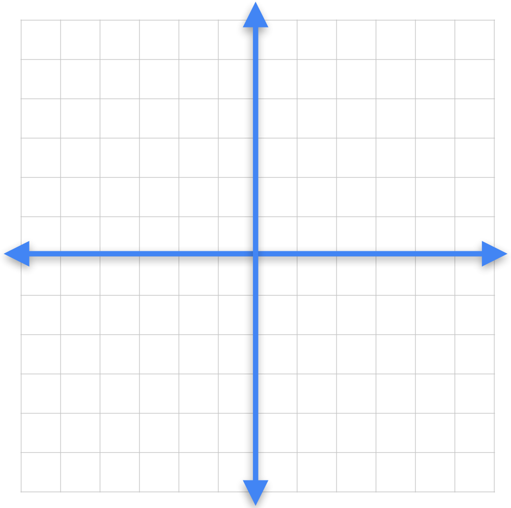




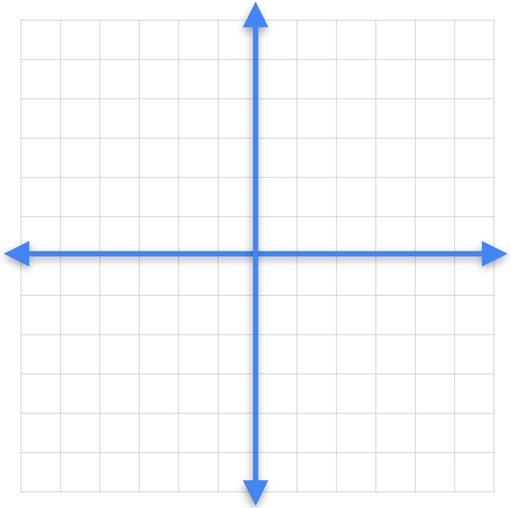
Combining linear transformations



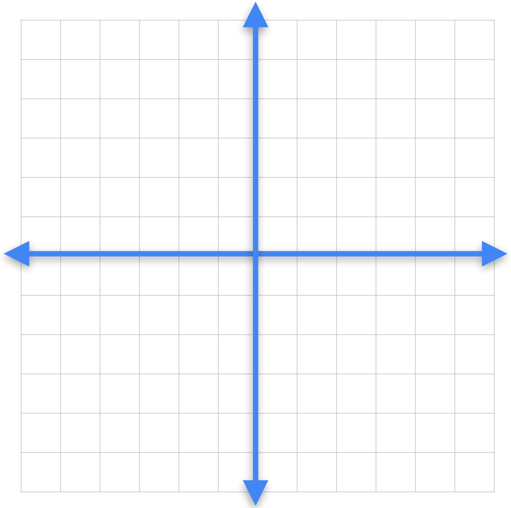
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$$



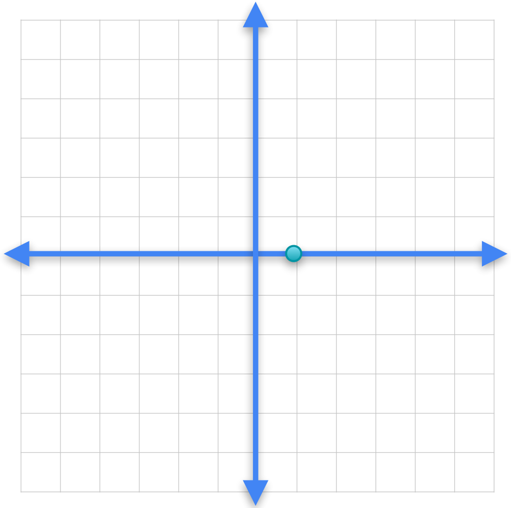
Combining linear transformations



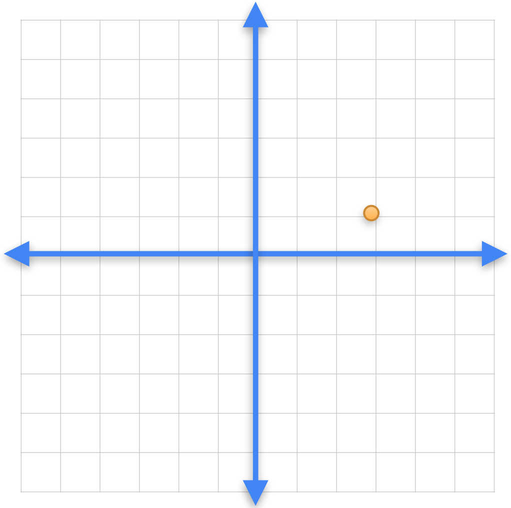
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$



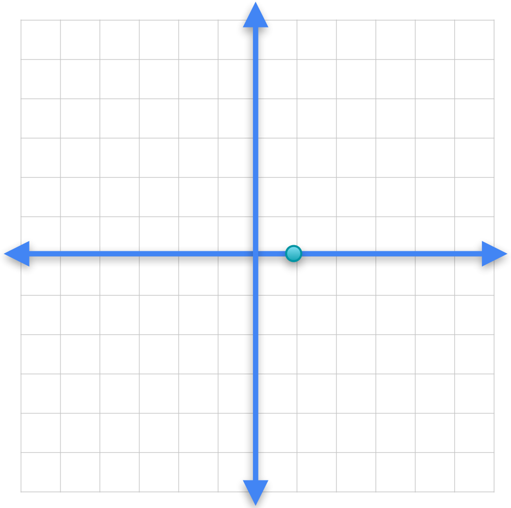
Combining linear transformations



$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

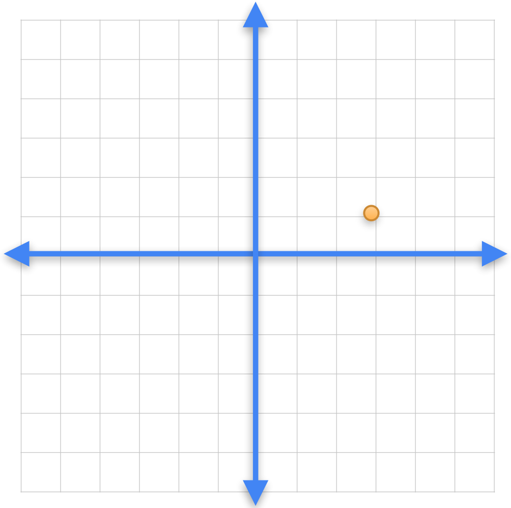


Combining linear transformations

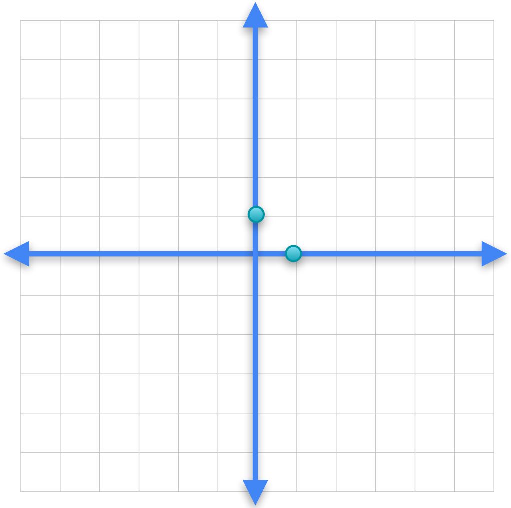


$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

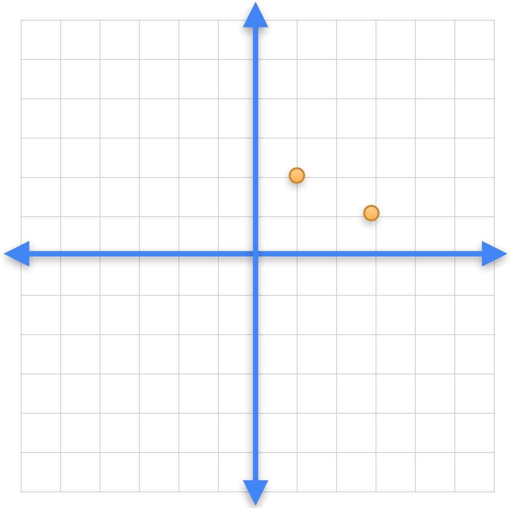


Combining linear transformations

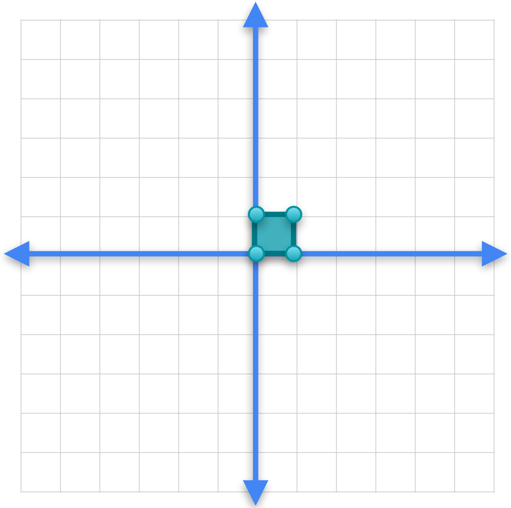


$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

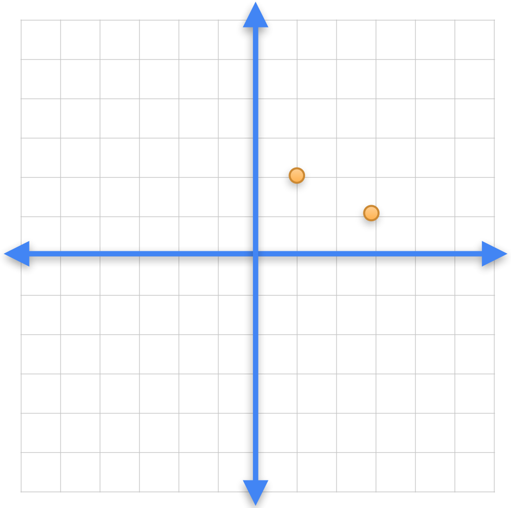


Combining linear transformations

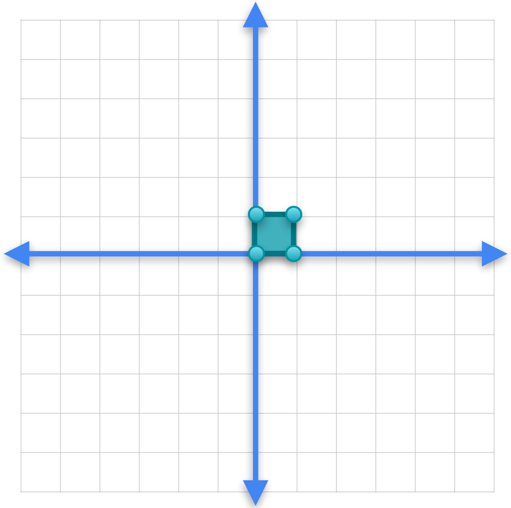


$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

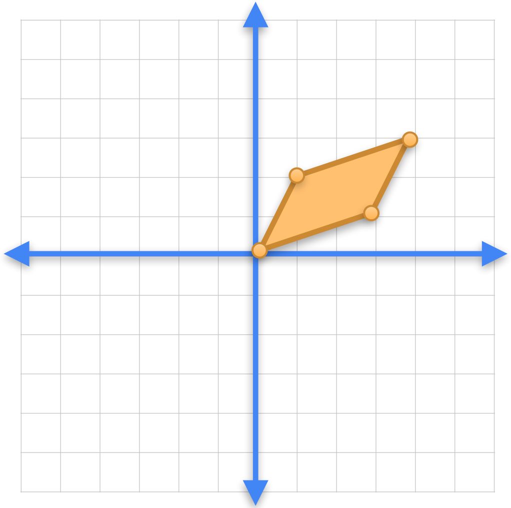


Combining linear transformations

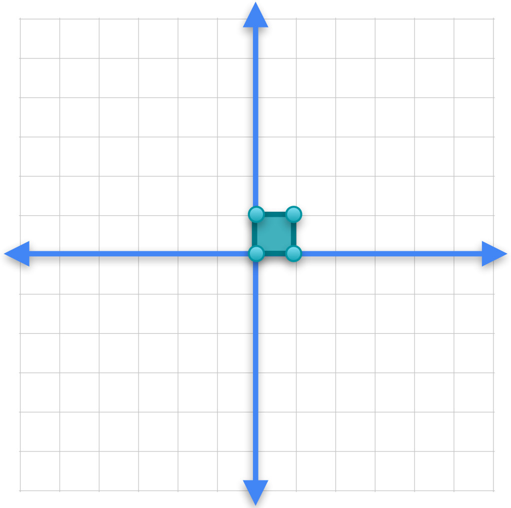


$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

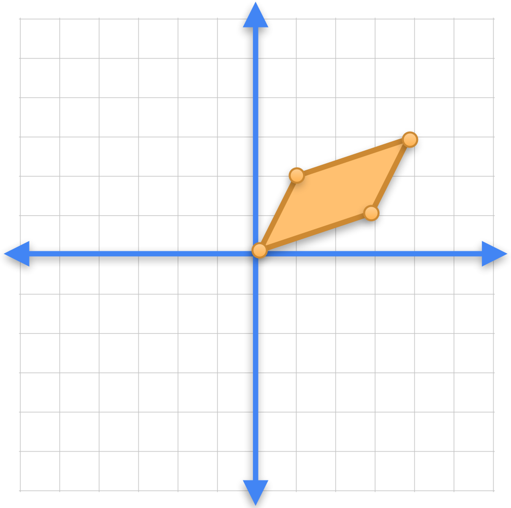


Combining linear transformations

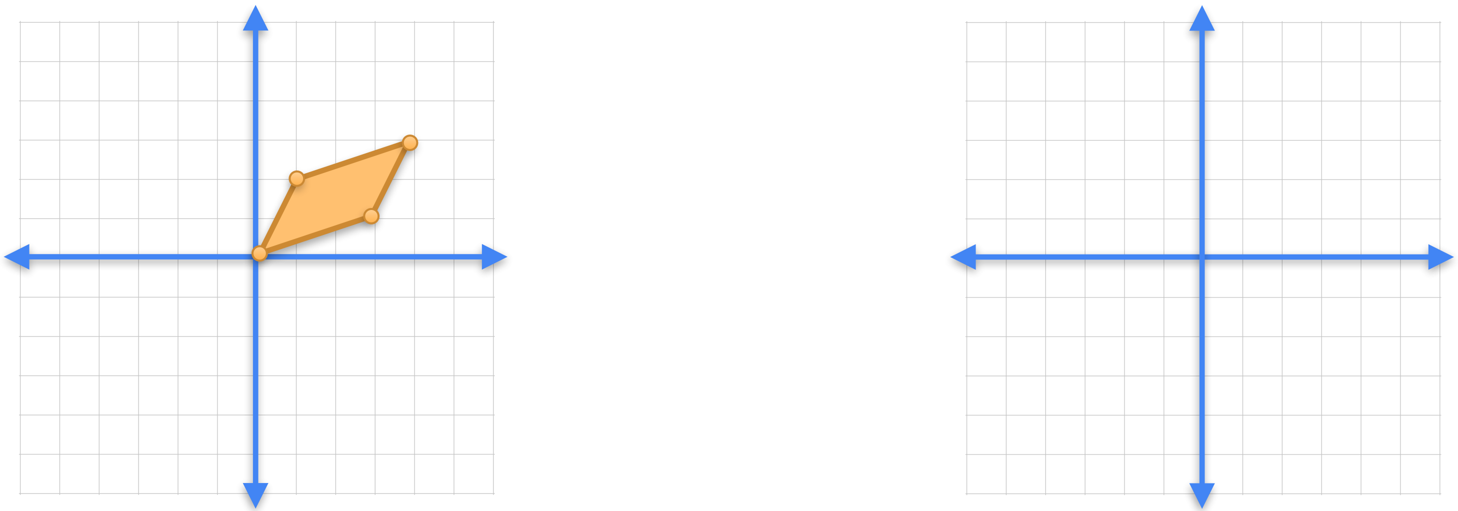


$$\begin{bmatrix}3 & 1\\ 1 & 2\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix} = \begin{bmatrix}3\\ 1\end{bmatrix}$$

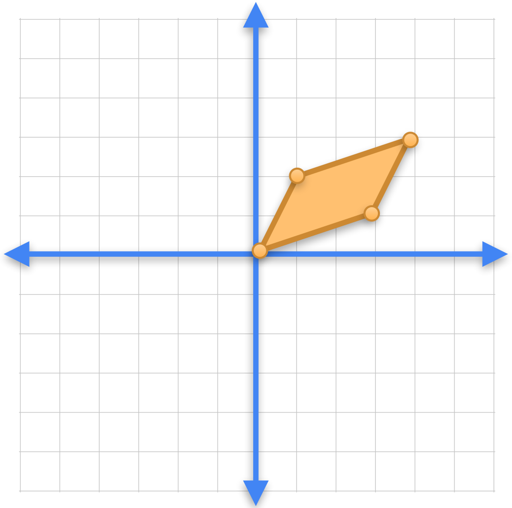
$$\begin{bmatrix}3 & 1\\ 1 & 2\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix} = \begin{bmatrix}1\\ 2\end{bmatrix}$$



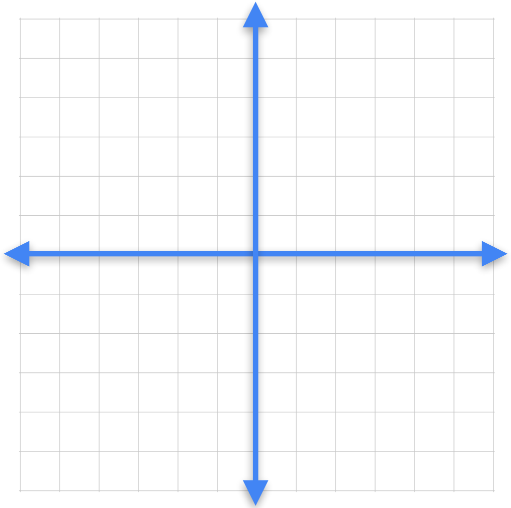
Combining linear transformations



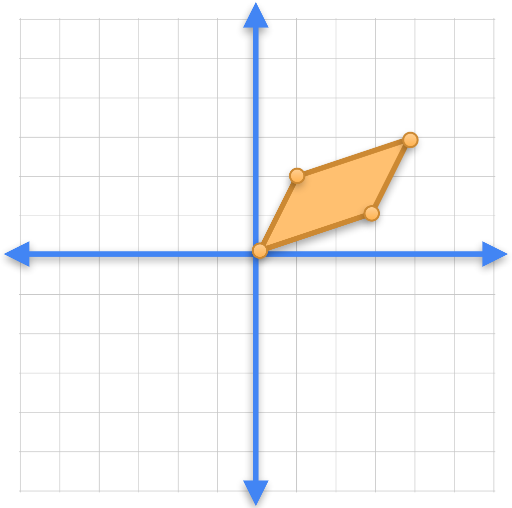
Combining linear transformations



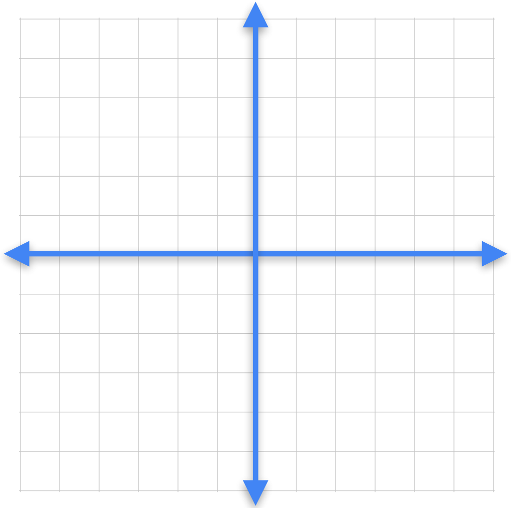
2	-1
0	2



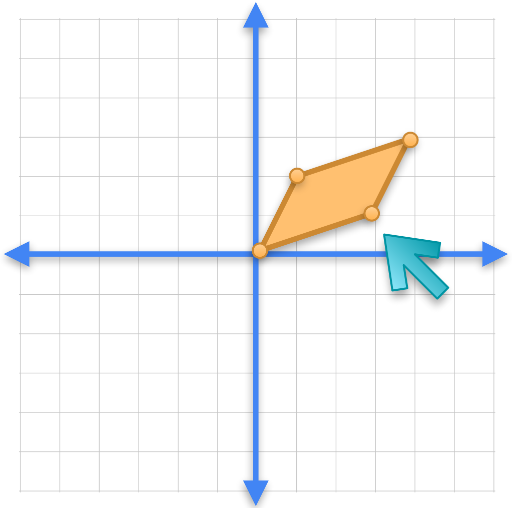
Combining linear transformations



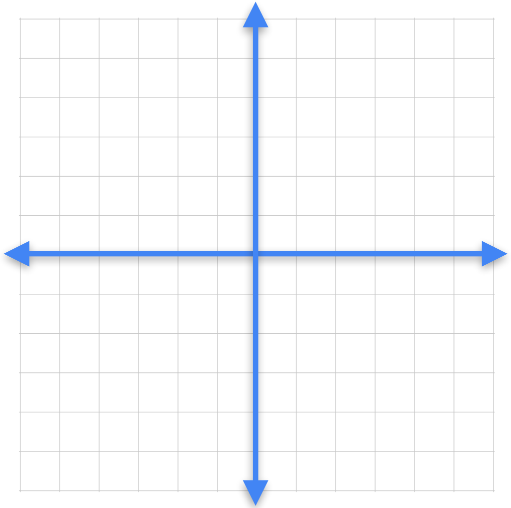
$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$



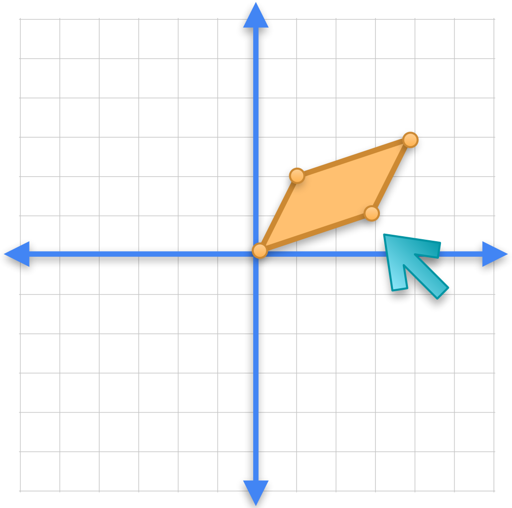
Combining linear transformations



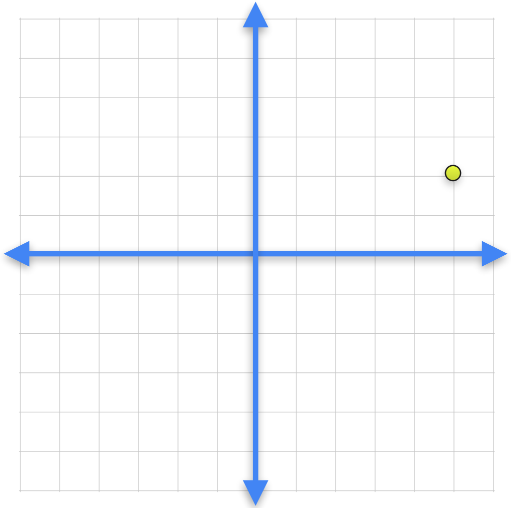
$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$



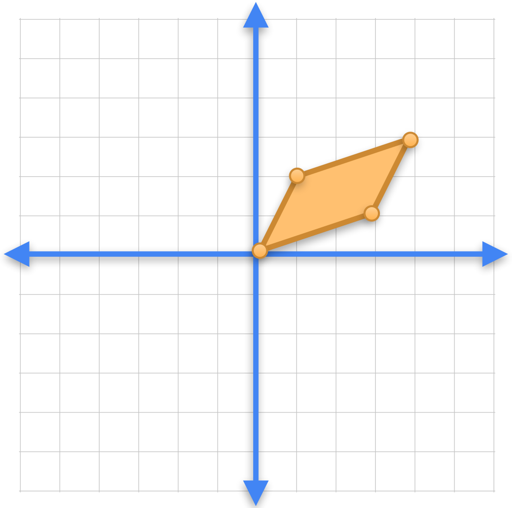
Combining linear transformations



$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

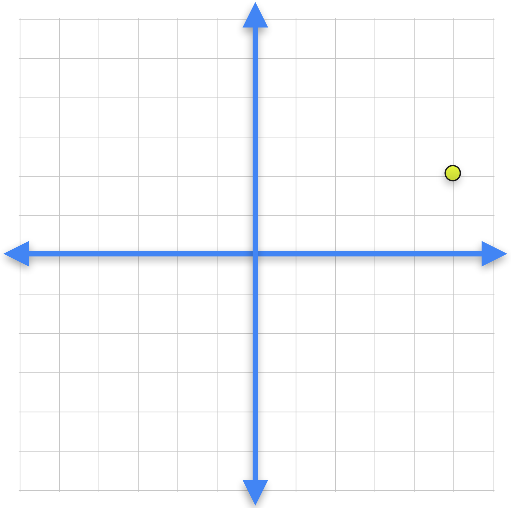


Combining linear transformations

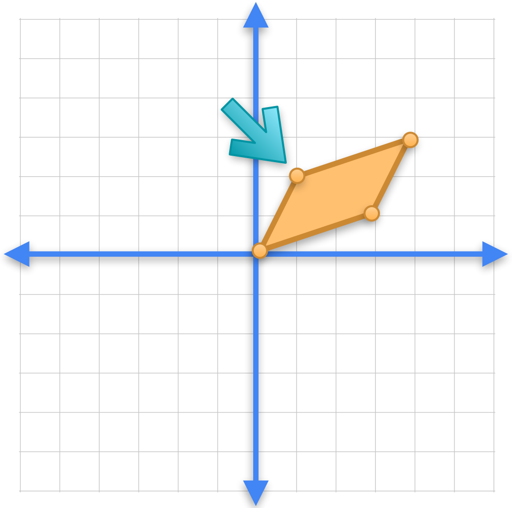


$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

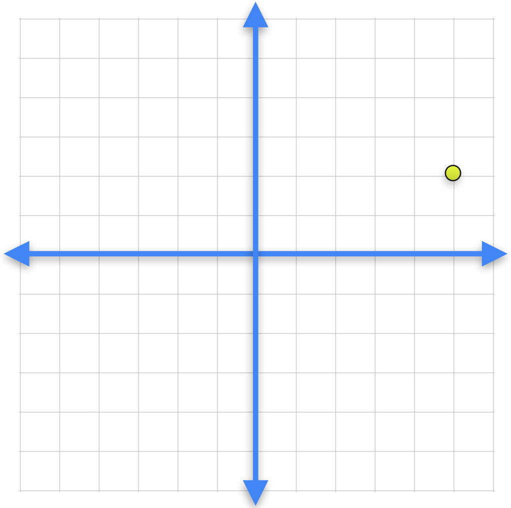


Combining linear transformations

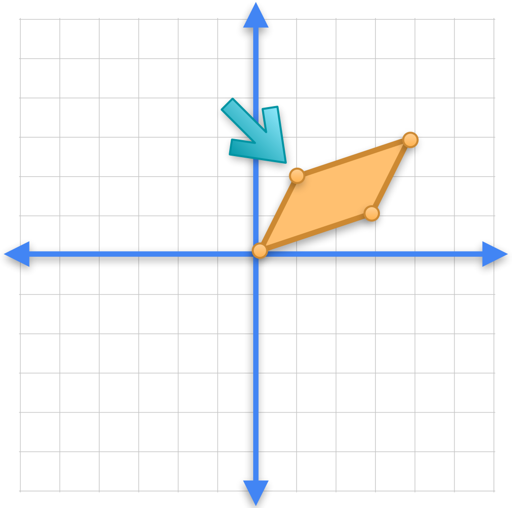


$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

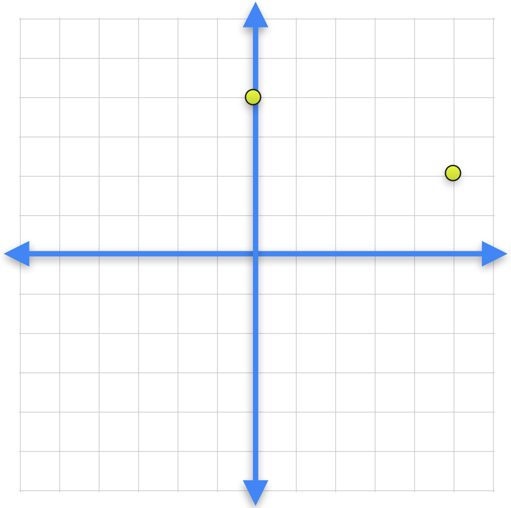


Combining linear transformations

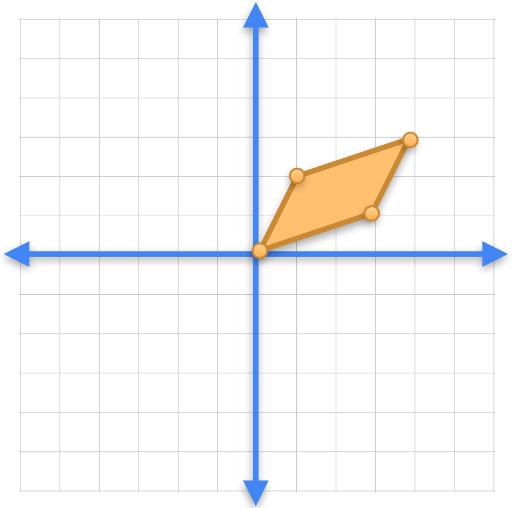


$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

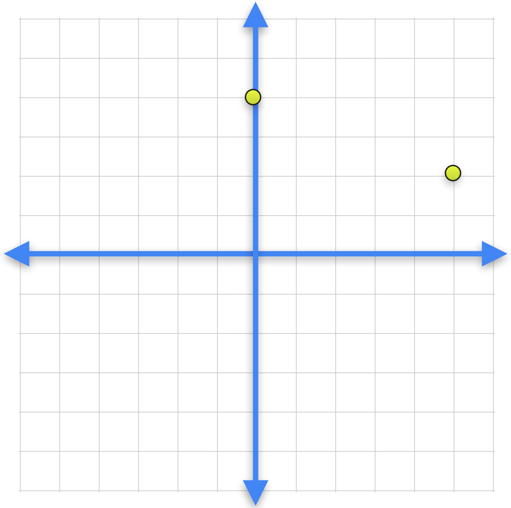


Combining linear transformations

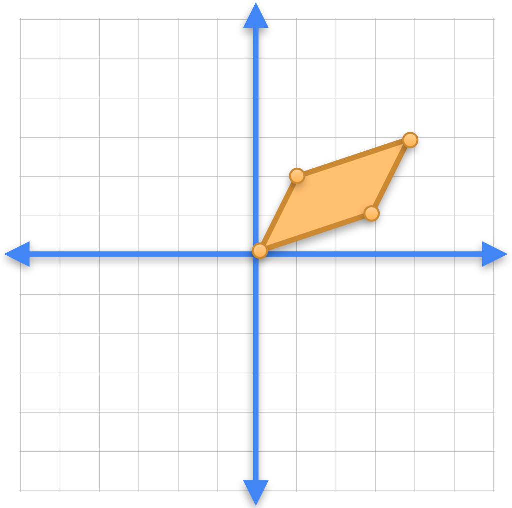


$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

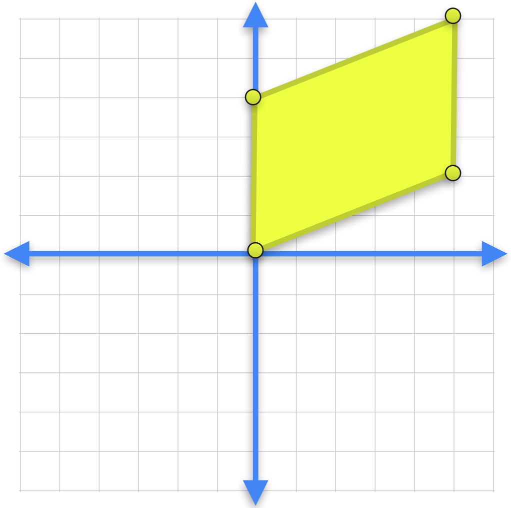


Combining linear transformations

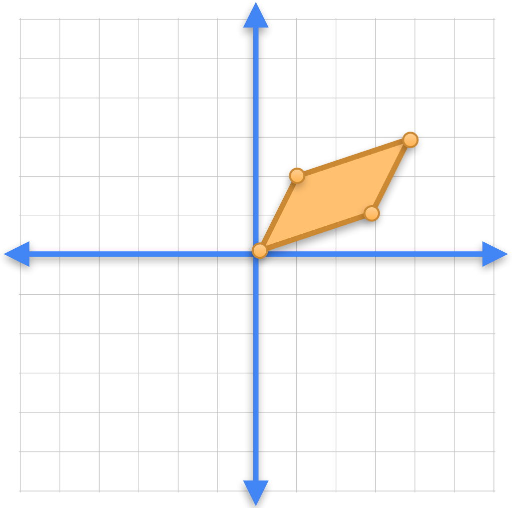


$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

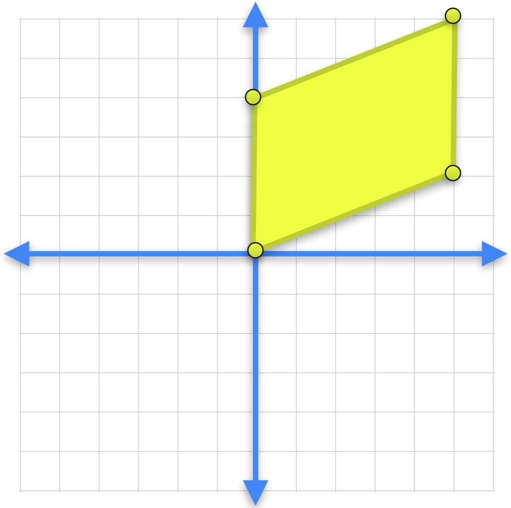


Combining linear transformations

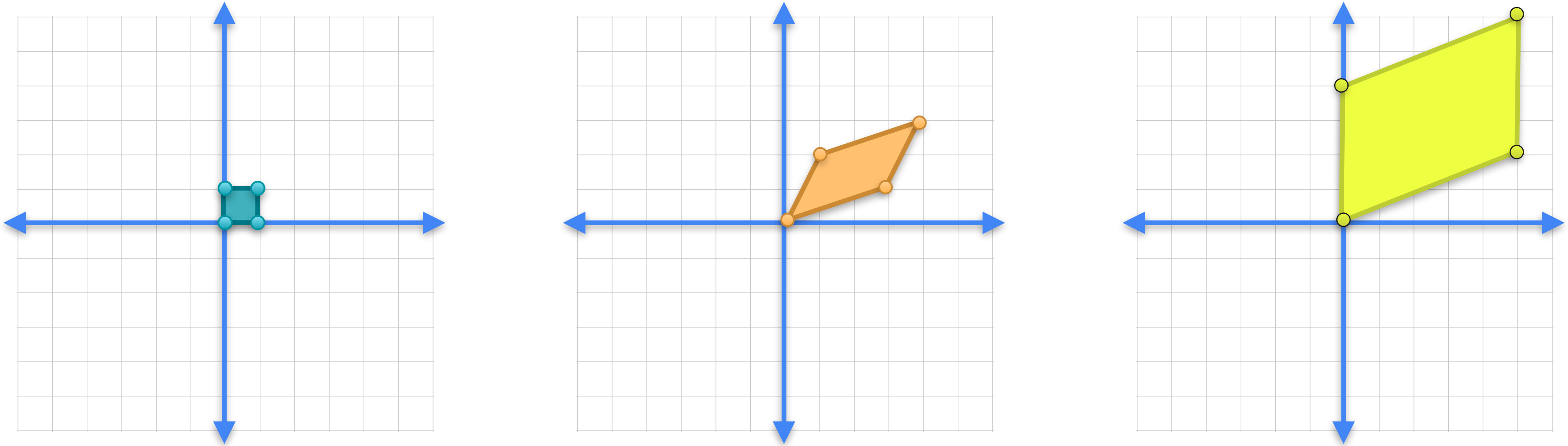


$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

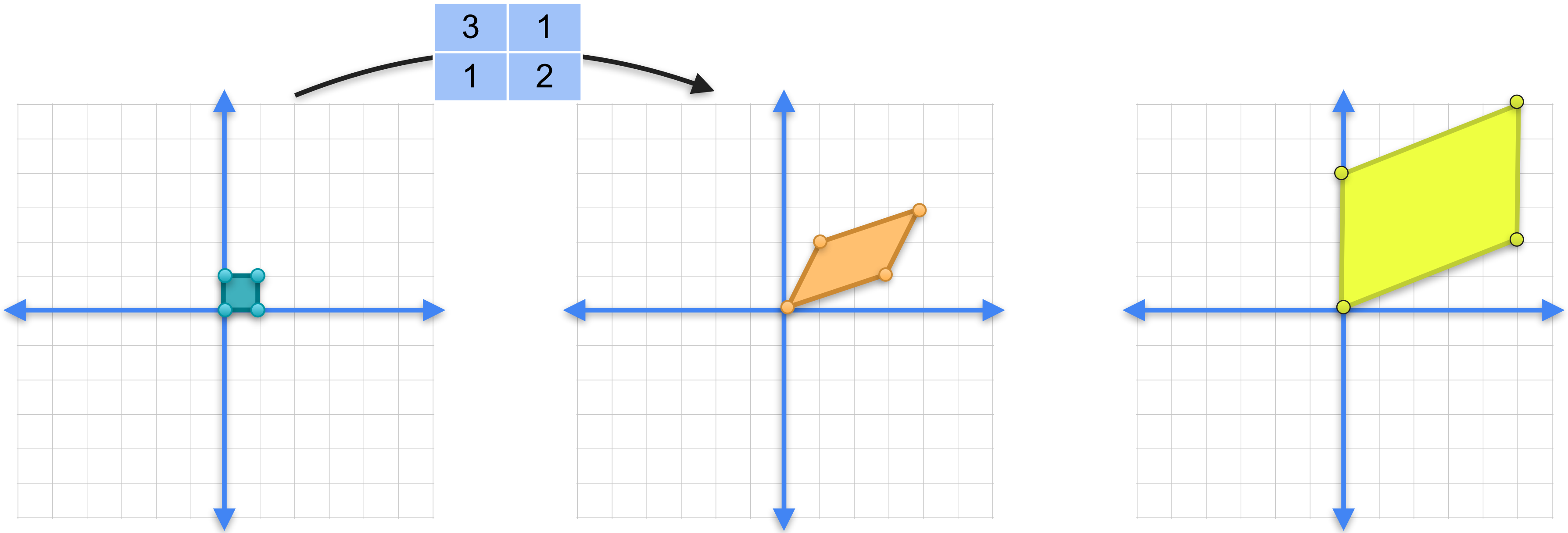
$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$



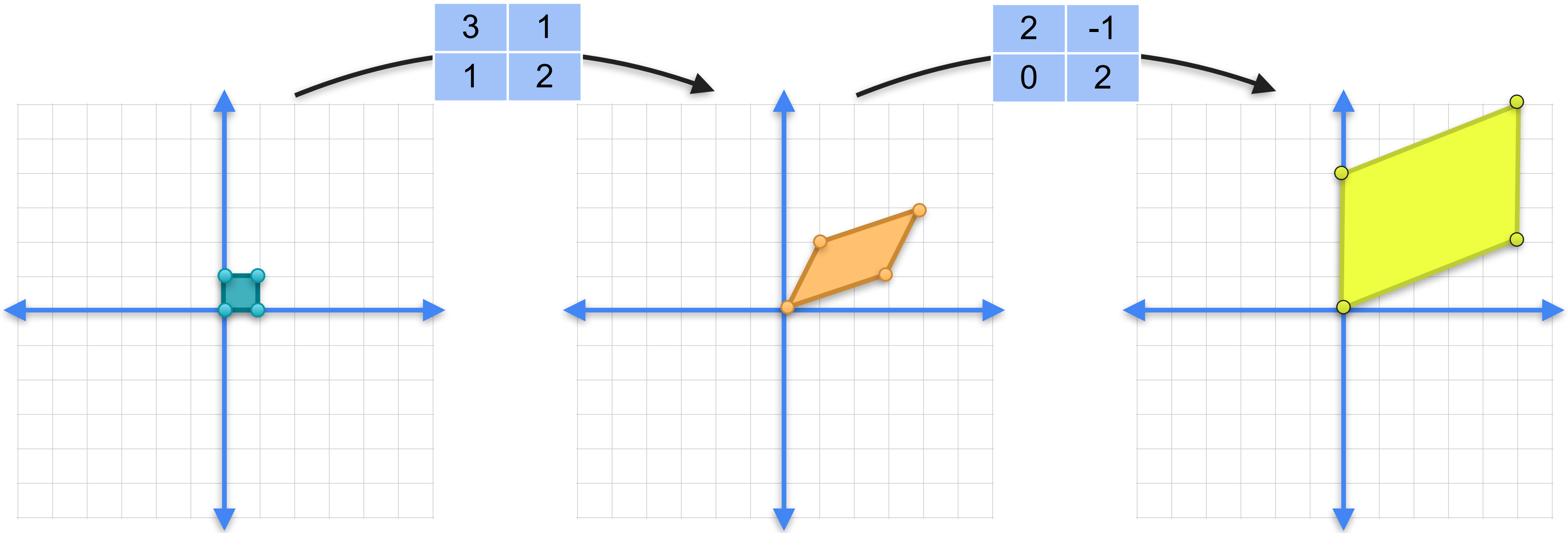
Combining linear transformations



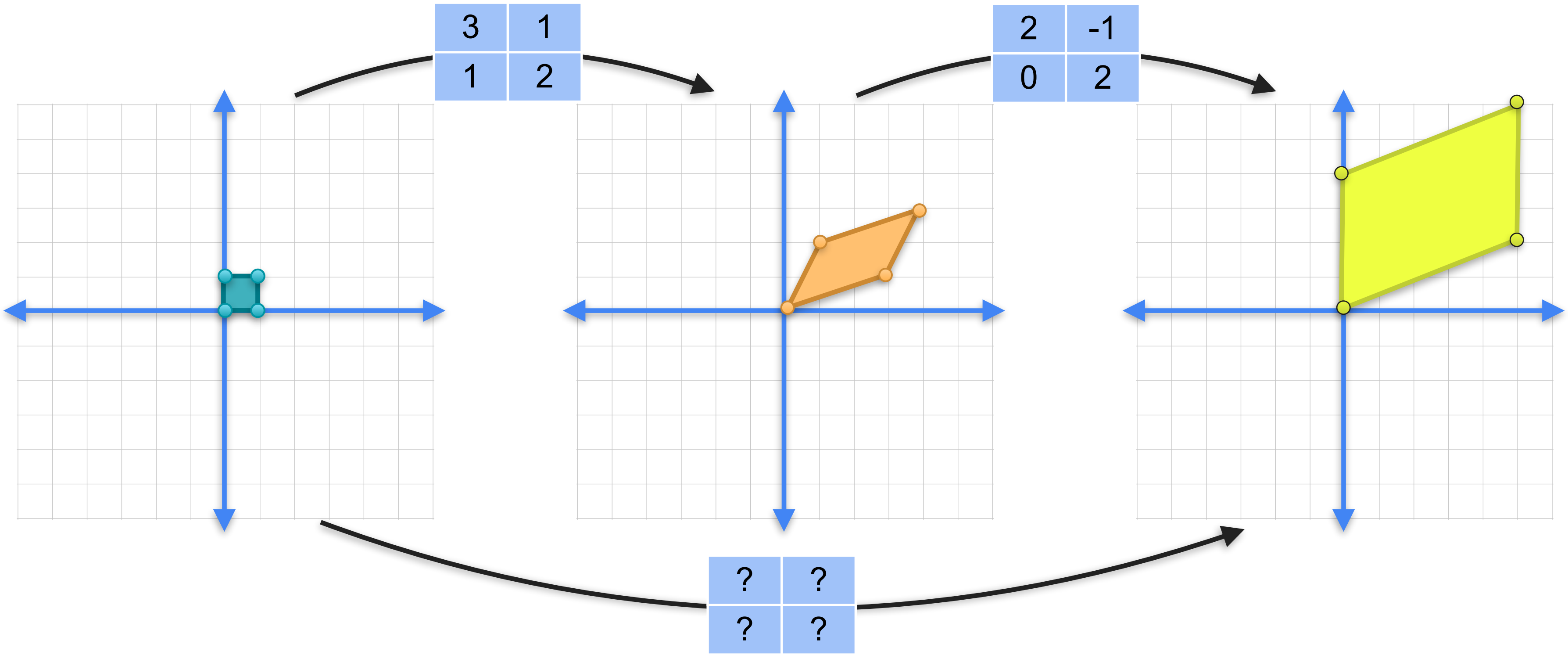
Combining linear transformations



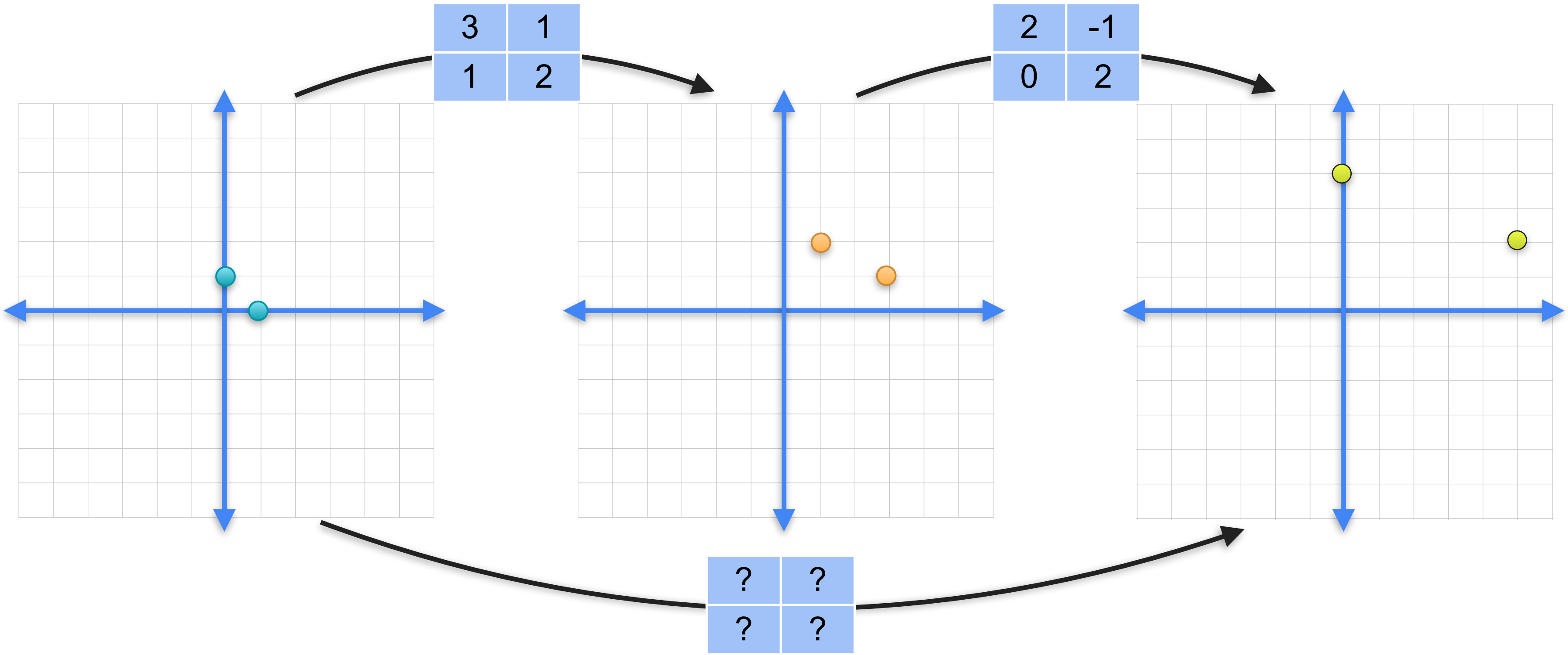
Combining linear transformations



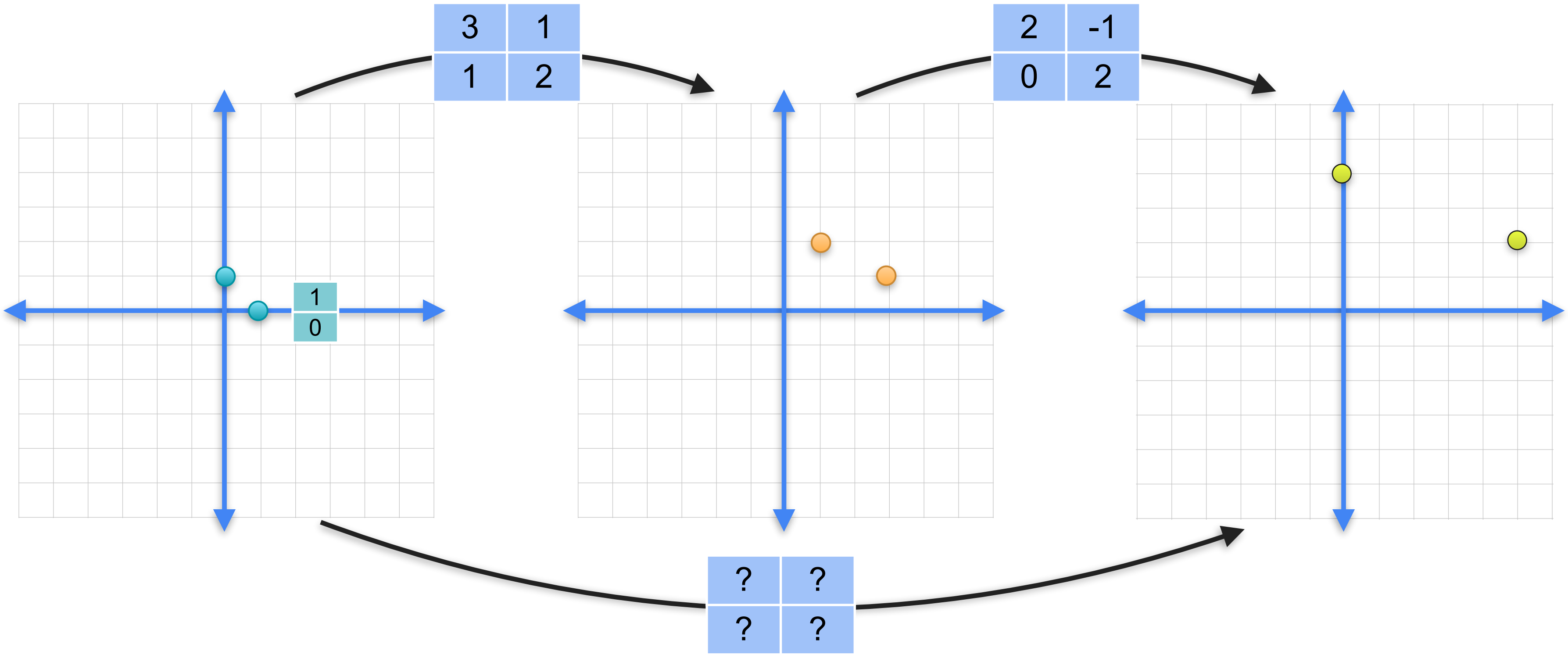
Combining linear transformations



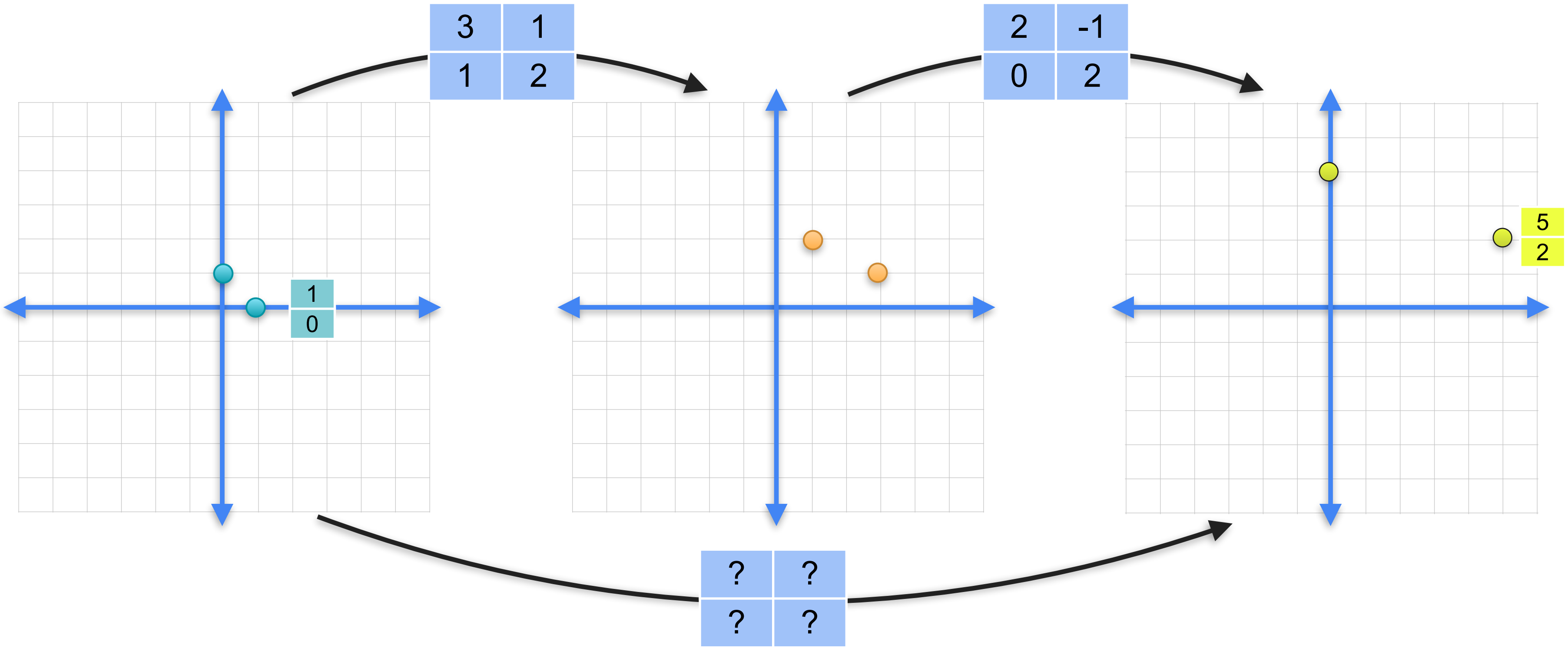
Combining linear transformations



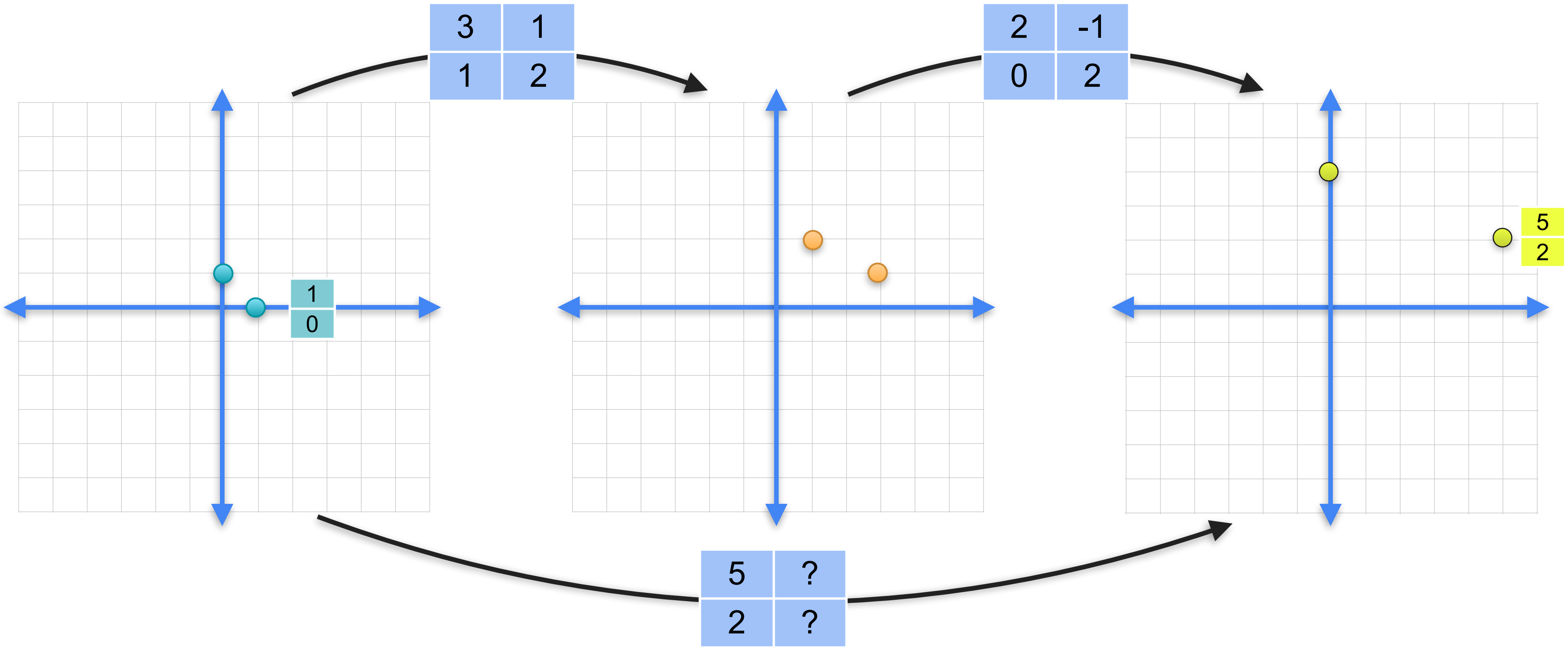
Combining linear transformations



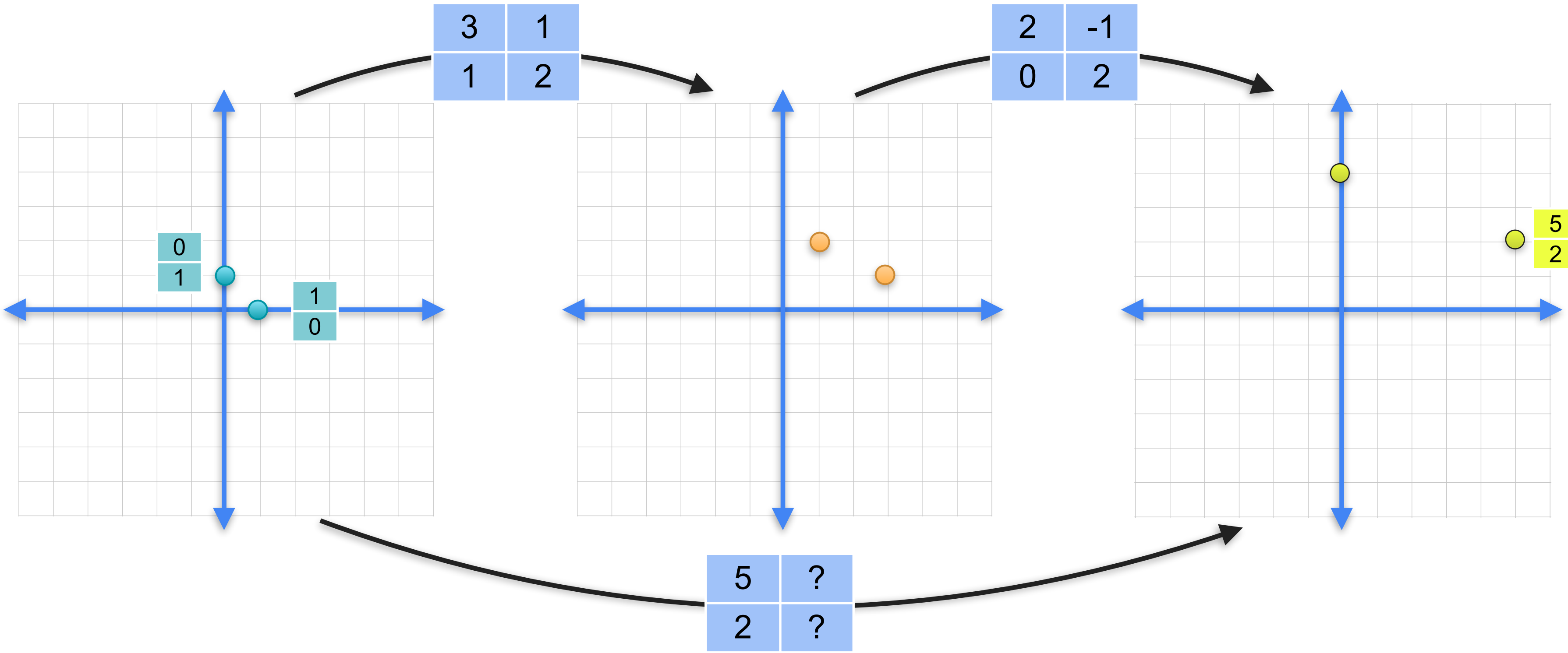
Combining linear transformations



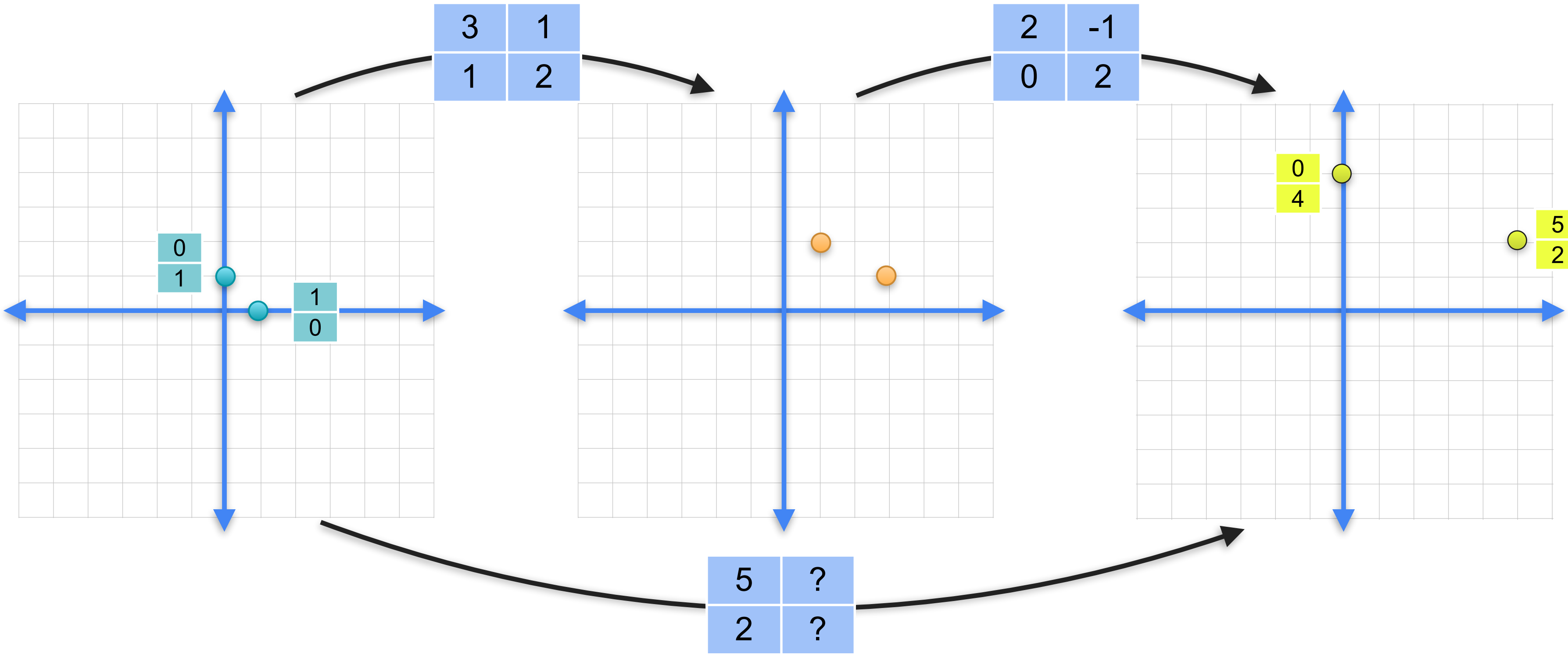
Combining linear transformations



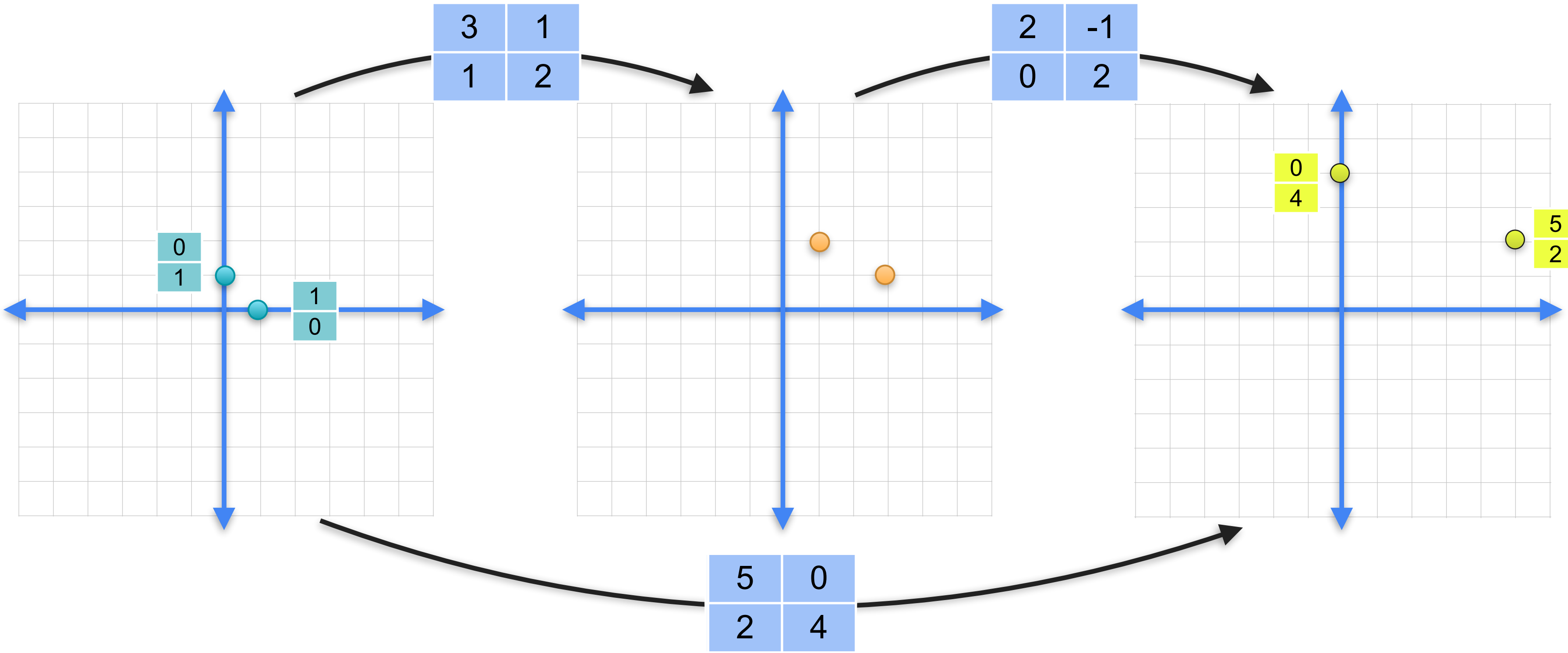
Combining linear transformations



Combining linear transformations



Combining linear transformations



Combining linear transformations

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 2 & 4 \end{bmatrix}$$

Combining linear transformations

First

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 2 & 4 \end{bmatrix}$$

Combining linear transformations

Second $\downarrow$ First $\downarrow$

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 2 & 4 \end{bmatrix}$$

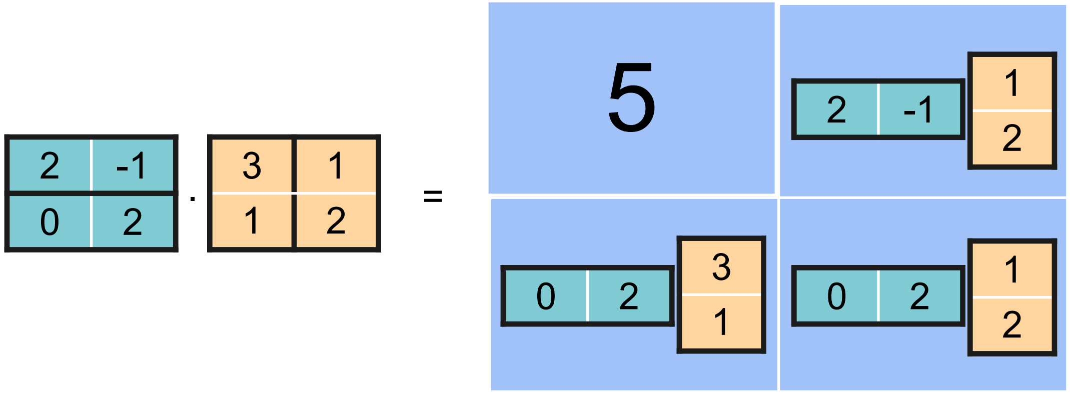
Multiplying matrices

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 5 & 0 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} & \\ & \end{bmatrix}$$

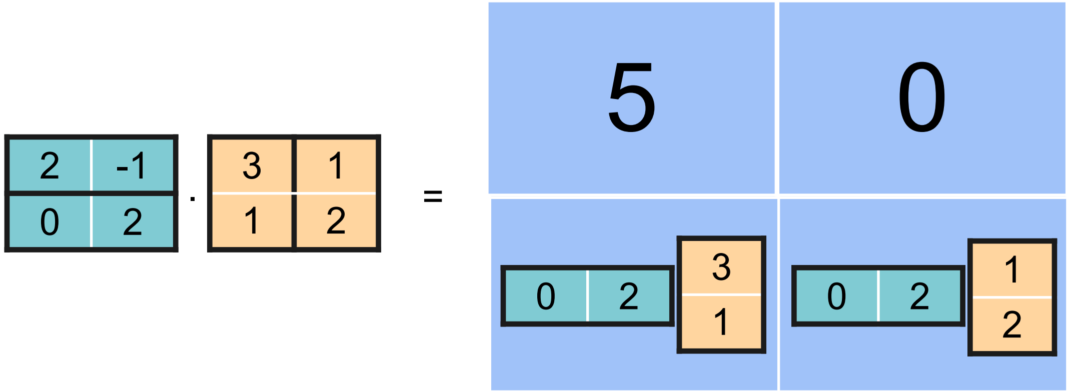
Multiplying matrices

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} 2 & -1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} & \begin{bmatrix} 2 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} \\ \begin{bmatrix} 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} & \begin{bmatrix} 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} \end{bmatrix}$$

Multiplying matrices



Multiplying matrices



Multiplying matrices

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 2 & \begin{bmatrix} 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} \end{bmatrix}$$

Multiplying matrices

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 2 & 4 \end{bmatrix}$$

Vectors and Linear Transformations

The identity matrix

The identity matrix

1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

The identity matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix}$$

The identity matrix

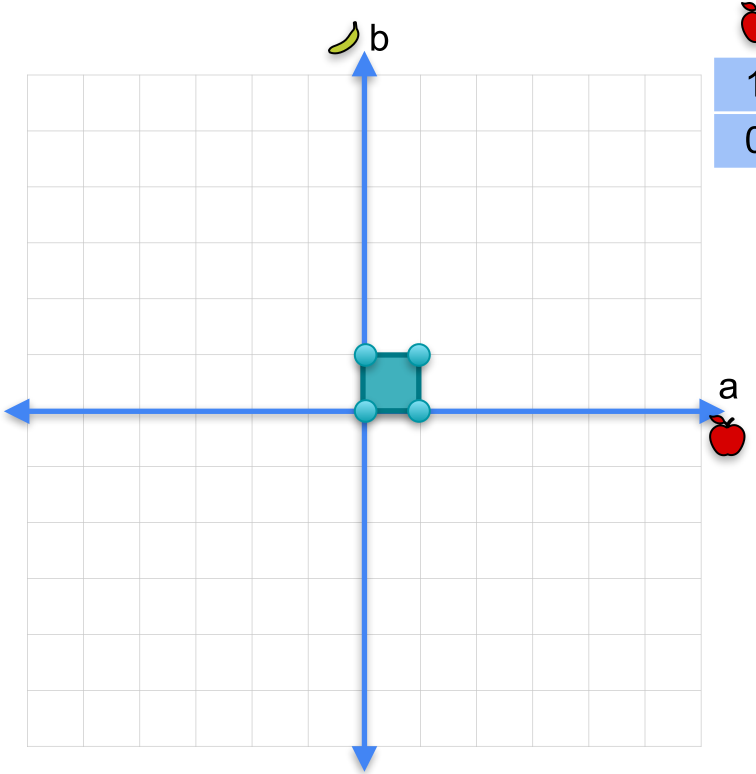
$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix}$$

The identity matrix

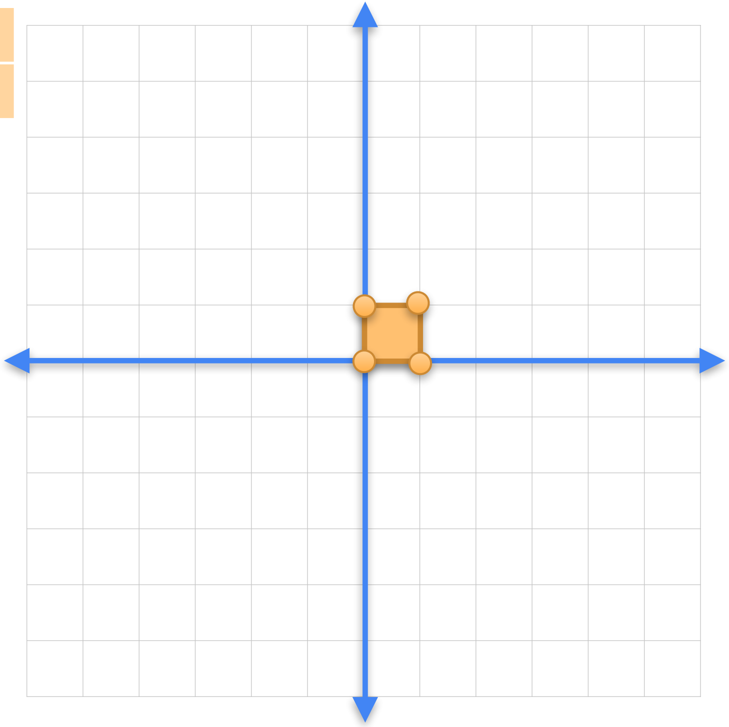
$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix} = a$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix}$$

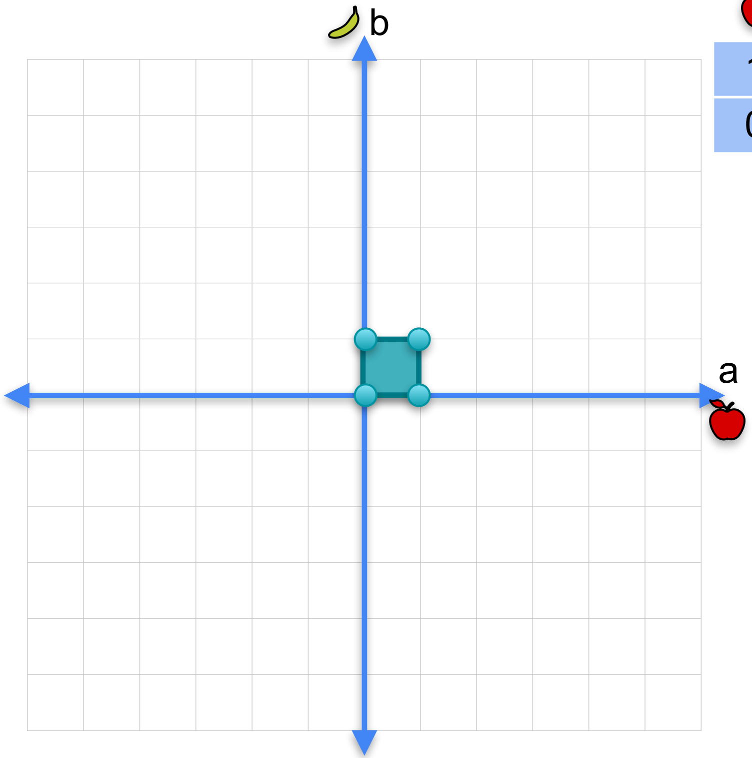
The identity matrix



$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$



The identity matrix



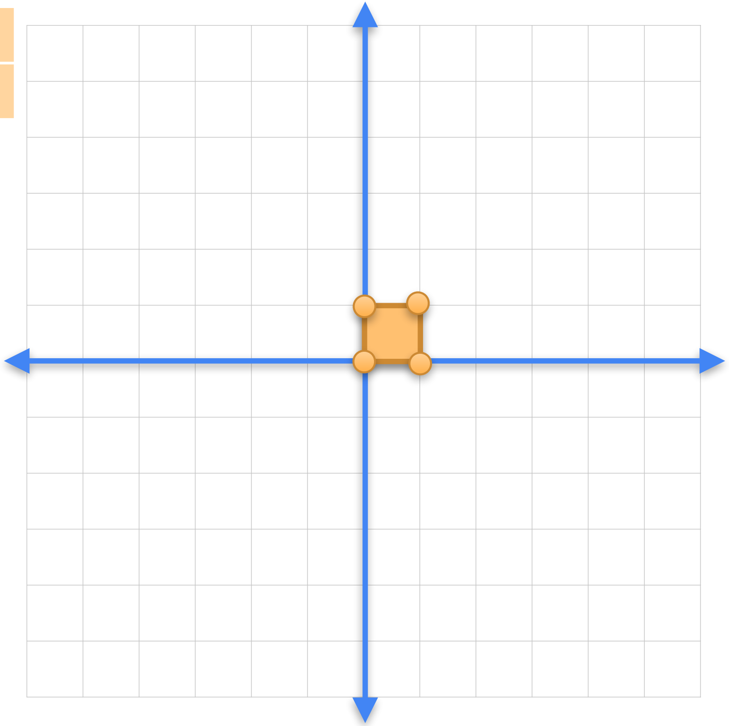
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

(0,0) → (0,0)

(1,0) → (1,0)

(0,1) → (0,1)

(1,1) → (1,1)



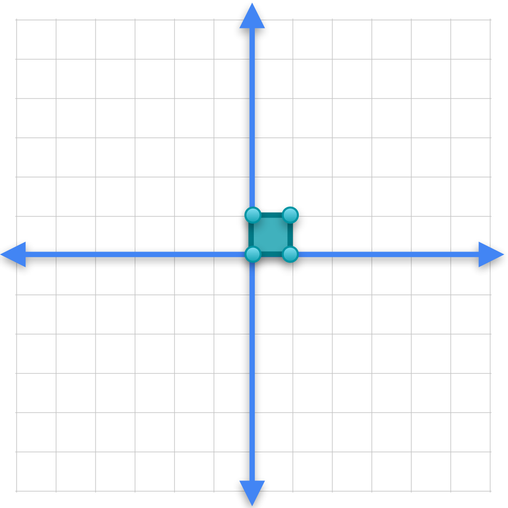
DeepLearning.AI

Vectors and Linear Transformations

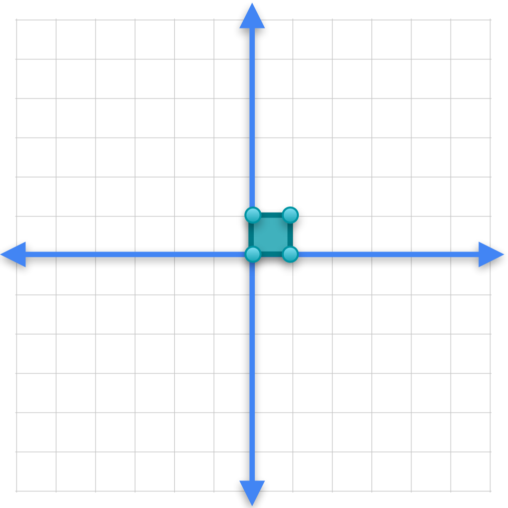
Matrix inverse

Matrix inverses

Matrix inverses



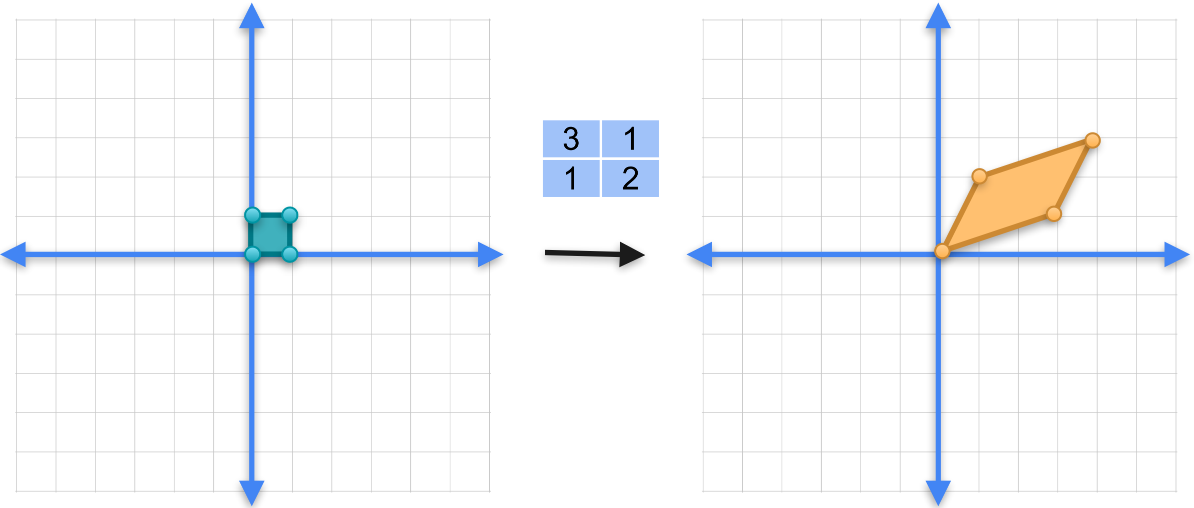
Matrix inverses



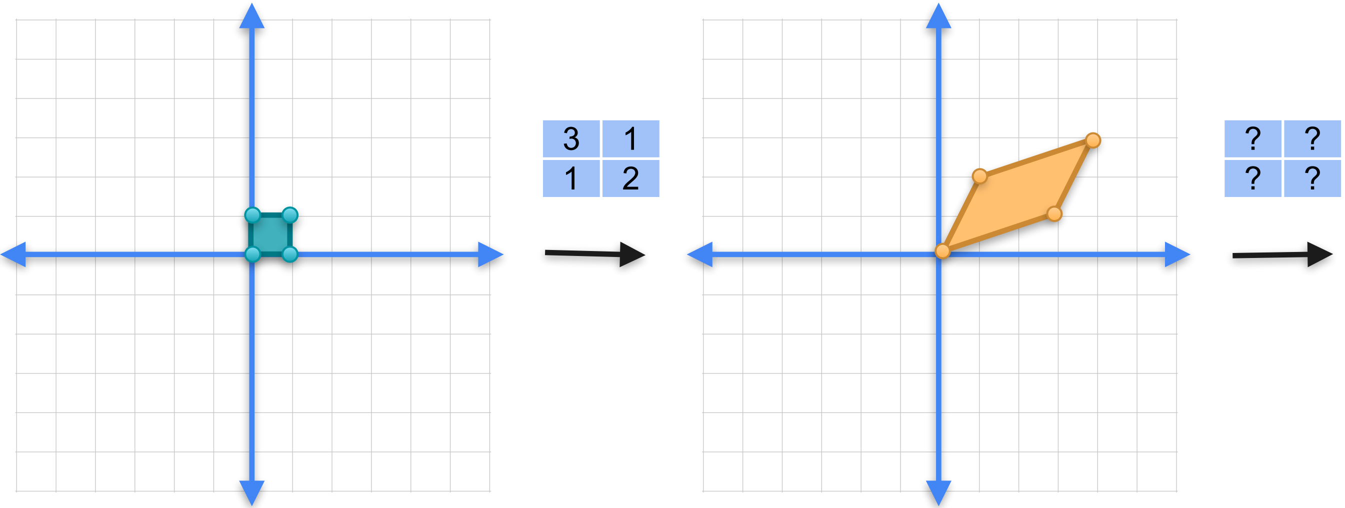
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$$

$\longrightarrow$

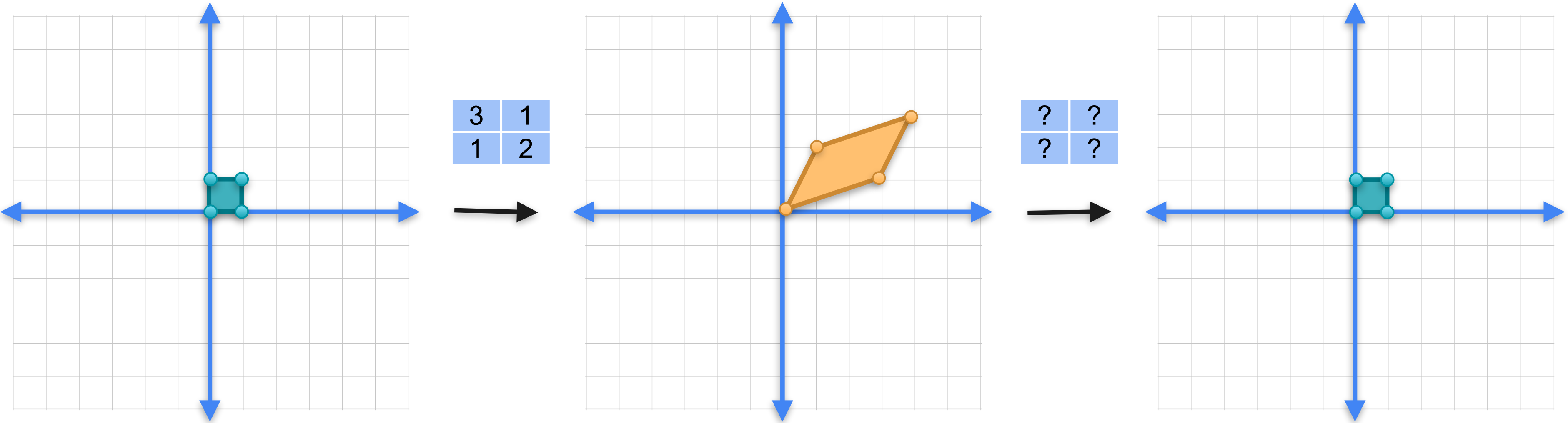
Matrix inverses



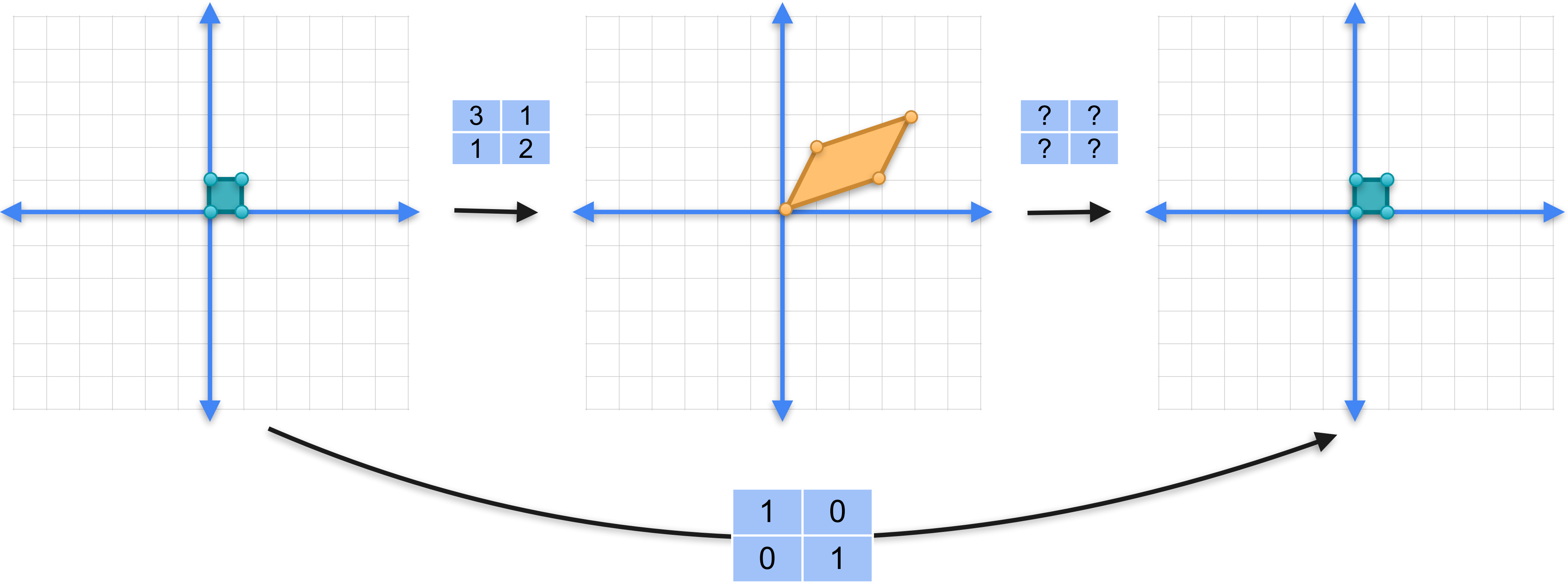
Matrix inverses



Matrix inverses



Matrix inverses



Multiplying matrices

Multiplying matrices

a	b
c	d

Multiplying matrices

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$$

Multiplying matrices

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Multiplying matrices

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1}$$

Multiplying matrices

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 2/5 & -1/5 \\ -1/5 & 3/5 \end{bmatrix}$$

How to find an inverse?

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

How to find an inverse?

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = 1$$

$$\begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 0$$

$$\begin{bmatrix} c & d \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} c & d \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 1$$

How to find an inverse?

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = 1 \qquad 3a + 1b = 1$$

$$\begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 0 \qquad 1a + 2b = 0$$

$$\begin{bmatrix} c & d \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = 0 \qquad 3c + 1d = 0$$

$$\begin{bmatrix} c & d \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 1 \qquad 1c + 2d = 1$$

How to find an inverse?

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix} \qquad 3a + 1b = 1 \qquad a = \frac{2}{5}$$

$$\begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix} \qquad 1a + 2b = 0 \qquad b = -\frac{1}{5}$$

$$\begin{bmatrix} c & d \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix} \qquad 3c + 1d = 0 \qquad c = -\frac{1}{5}$$

$$\begin{bmatrix} c & d \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix} \qquad 1c + 2d = 1 \qquad d = \frac{3}{5}$$

Quiz

- Find the inverse of the following matrix. If you find that the task is impossible, feel free to click on “I couldn’t find it”

5	2
1	2

Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 1$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 1$$

Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix}$$

- **$5a + 2c = 1$**
- **$5b + 2d = 0$**
- **$a + 2c = 0$**
- **$b + 2d = 1$**

Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix}$$

- **$5a + 2c = 1$**
- **$5b + 2d = 0$**
- **$a + 2c = 0$**
- **$b + 2d = 1$**

- **$a = 1/4$**

Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix}$$

- **5a + 2c = 1**
- **5b + 2d = 0**
- **a + 2c = 0**
- **b + 2d = 1**

- **a = 1/4**
- **b = -1/4**

Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix}$$

- **$5a + 2c = 1$**
- **$5b + 2d = 0$**
- **$a + 2c = 0$**
- **$b + 2d = 1$**

- **$a = 1/4$**
- **$b = -1/4$**
- **$c = -1/8$**

Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 1$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 1$$

- **$5a + 2c = 1$**
- **$5b + 2d = 0$**
- **$a + 2c = 0$**
- **$b + 2d = 1$**

- **$a = 1/4$**
- **$b = -1/4$**
- **$c = -1/8$**
- **$d = 5/8$**

Quiz

- Find the inverse of the following matrix. If you find that the task is impossible, feel free to click on “I’m reaching a dead end”

1	1
2	2

Solutions

- The inverse doesn't exist!

We need to solve the following system of linear equations:

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$a + c = 1$

$2b + 2d = 1$

$2a + 2c = 0$

$b + d = 0$

This is clearly a contradiction, since equation 1 says $a+c=1$, and equation 3 says $2a+2c=0$.

DeepLearning.AI

Vectors and Linear Transformations

Which matrices have an inverse?

Which matrices have inverses?

Which matrices have inverses?

$$5^{-1} = 0.2$$

Which matrices have inverses?

$$5^{-1} = 0.2 \qquad 8^{-1} = 0.125$$

Which matrices have inverses?

$5^{-1} = 0.2$ $\quad$ $8^{-1} = 0.125$ $\quad$ $0^{-1} = ???$

Which matrices have inverses?

$$5^{-1} = 0.2 \qquad 8^{-1} = 0.125 \qquad 0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Which matrices have inverses?

$$5^{-1} = 0.2 \qquad 8^{-1} = 0.125 \qquad 0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix} \qquad \begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Which matrices have inverses?

$$5^{-1} = 0.2 \qquad 8^{-1} = 0.125 \qquad 0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix} \qquad \begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Which matrices have inverses?

$$5^{-1} = 0.2 \qquad 8^{-1} = 0.125 \qquad 0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix} \qquad \begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Non-singular matrix

Which matrices have inverses?

$$5^{-1} = 0.2 \qquad 8^{-1} = 0.125 \qquad 0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Non-singular matrix

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Non-singular matrix

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Which matrices have inverses?

$$5^{-1} = 0.2 \qquad 8^{-1} = 0.125 \qquad 0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Non-singular matrix

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Non-singular matrix

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Singular matrix

Which matrices have inverses?

$$5^{-1} = 0.2 \qquad 8^{-1} = 0.125 \qquad 0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Non-singular matrix
Invertible

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Non-singular matrix

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Singular matrix

Which matrices have inverses?

$$5^{-1} = 0.2 \qquad 8^{-1} = 0.125 \qquad 0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Non-singular matrix
Invertible

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Non-singular matrix
Invertible

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Singular matrix

Which matrices have inverses?

$$5^{-1} = 0.2 \qquad 8^{-1} = 0.125 \qquad 0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Non-singular matrix
Invertible

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Non-singular matrix
Invertible

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Singular matrix
Non-invertible

Which matrices have inverses?

$$5^{-1} = 0.2 \qquad 8^{-1} = 0.125 \qquad 0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Non-singular matrix

Invertible

Det = 5

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Non-singular matrix

Invertible

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Singular matrix

Non-invertible

Which matrices have inverses?

$5^{-1} = 0.2$

$8^{-1} = 0.125$

$0^{-1} = ???$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Non-singular matrix

Invertible

Det = 5

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Non-singular matrix

Invertible

Det = 8

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Singular matrix

Non-invertible

Which matrices have inverses?

$$5^{-1} = 0.2 \qquad 8^{-1} = 0.125 \qquad 0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Non-singular matrix

Invertible

Det = 5

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Non-singular matrix

Invertible

Det = 8

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Singular matrix

Non-invertible

Det = 0

Which matrices have inverses?

$5^{-1} = 0.2$ $\quad$ $8^{-1} = 0.125$ $\quad$ $0^{-1} = ???$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix} \qquad \begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Non-singular matrix
Invertible

Det = 5

Non-singular matrix
Invertible

Det = 8

Singular matrix
Non-invertible

Det = 0

Non-zero determinants

Which matrices have inverses?

$$5^{-1} = 0.2 \qquad 8^{-1} = 0.125 \qquad 0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Non-singular matrix

Invertible

Det = 5

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Non-singular matrix

Invertible

Det = 8

Non-zero determinants

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Singular matrix

Non-invertible

Det = 0

Zero determinant

DeepLearning.AI

Vectors and Linear Transformations

Neural networks and matrices


AI , ML , DL , RL
MATRIX
MULTIPLICATION

Quiz: Natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Quiz: Natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Scores:

Lottery: ___ points

Win: ___ points

Quiz: Natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Scores:

Lottery: ___ points

Win: ___ points

Examples

Lottery: 3 point
Win: 2 points
“Win, win the lottery!” : 7points

Quiz: Natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Scores:

Lottery: ___ points

Win: ___ points

Examples

Lottery: 3 point
Win: 2 points
"Win, win the lottery!" : 7points

Rule:
If the number of points of the sentence is bigger than ____, then the email is spam.

Quiz: Natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Scores:

Lottery: ___ points

Win: ___ points

Examples

Lottery: 3 point
Win: 2 points
"Win, win the lottery!" : 7points

Rule:
If the number of points of the sentence is bigger than ____, then the email is spam.

Goal: Find the best points and threshold
Lottery: ___ point
Win: ___ point
Threshold: ___ points

Quiz: Natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Score	> 1.5?
2	Yes
3	Yes
0	No
2	Yes
1	No
1	No
4	Yes
2	Yes
3	Yes

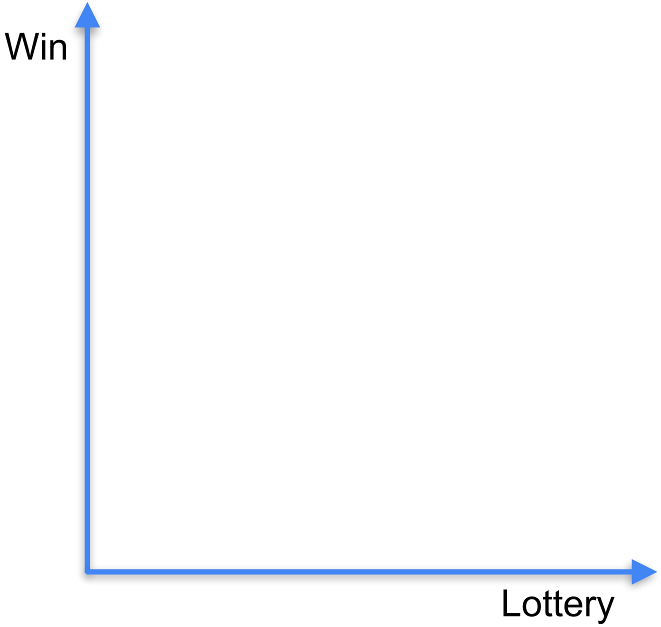
Solution:
Lottery: 1 point
Win: 1 point
Threshold: 1.5 points

Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

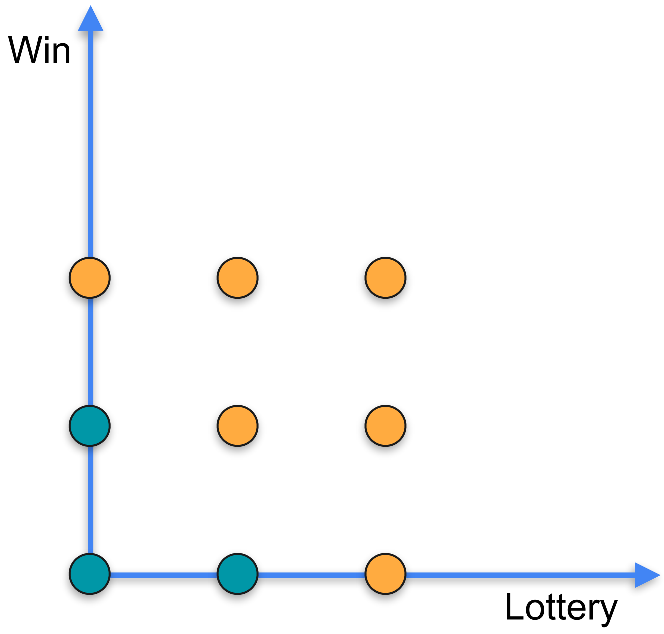
Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2



Graphical natural language processing

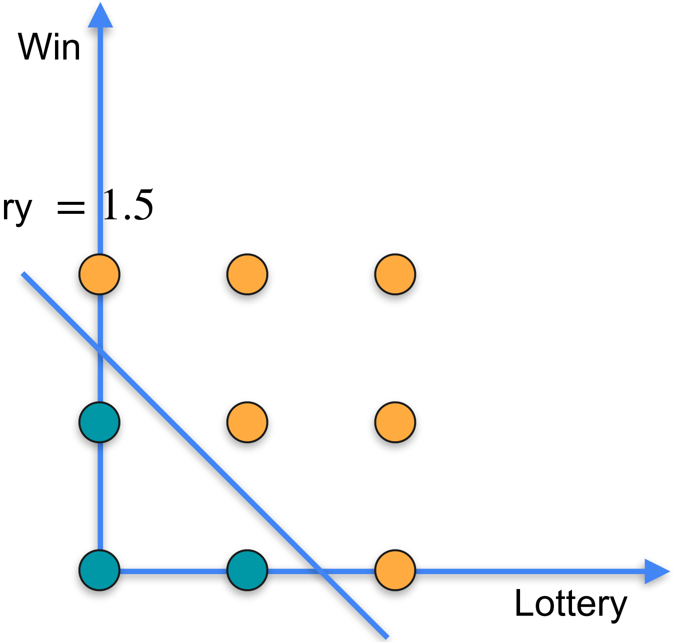
Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2



Graphical natural language processing

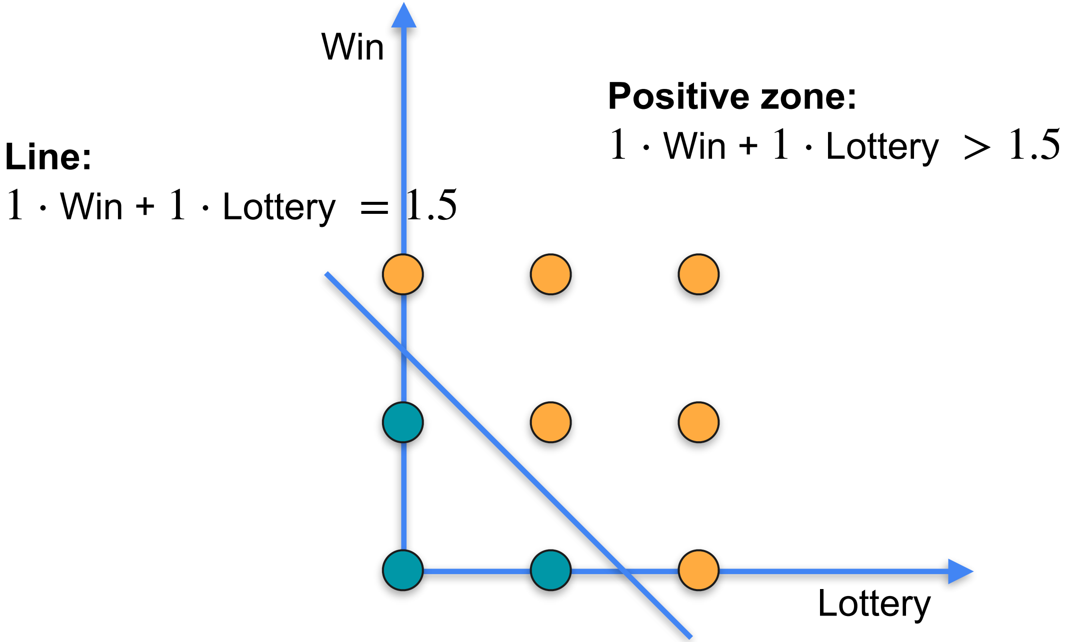
Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Line:
$1 \cdot \text{Win} + 1 \cdot \text{Lottery} = 1.5$



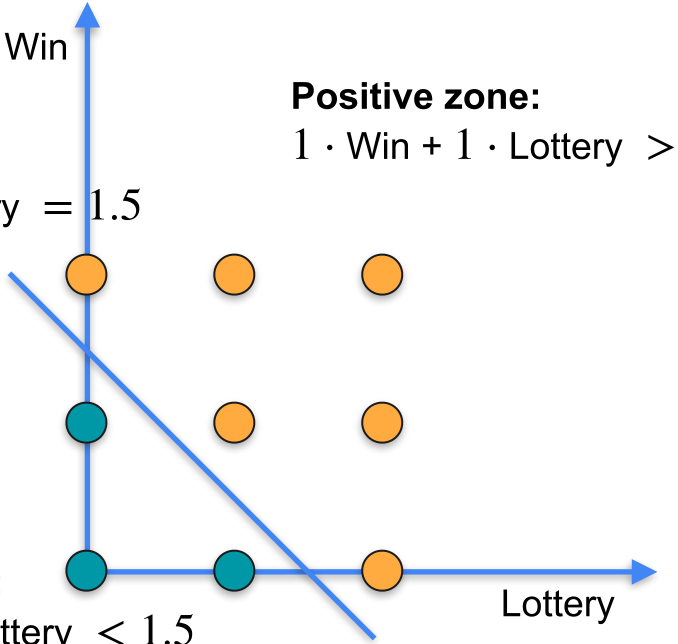
Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2



Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2



Line:
$1 \cdot \text{Win} + 1 \cdot \text{Lottery} = 1.5$

Positive zone:
$1 \cdot \text{Win} + 1 \cdot \text{Lottery} > 1.5$

Negative zone:
$1 \cdot \text{Win} + 1 \cdot \text{Lottery} < 1.5$

Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Model
1
1

Check: > 1.5?

Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

2	1

Model
1
1

Check: > 1.5?

Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

2	1

Model
1
1

= 3

Check: > 1.5?

Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

2	1

Model
1
1

= 3

Check: > 1.5?

Spam

Dot product between vectors

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Model
1
1

Check: > 1.5?

Dot product between vectors

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

0	1

Model
1
1

Check: > 1.5?

Dot product between vectors

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

0	1

Model
1
1

= 1

Check: > 1.5?

Dot product between vectors

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

$$\begin{bmatrix} 0 & 1 \end{bmatrix} \begin{array}{c} \textbf{Model} \\ \begin{bmatrix} 1 \\ 1 \end{bmatrix} \end{array} = 1$$

Check: > 1.5?

Not spam

Matrix multiplication

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Model
1
1

Matrix multiplication

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Model
1
1

=

Prod
2
3
0
2
1
1
4
2
3

Matrix multiplication

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Model
1
1

=

Prod
2
3
0
2
1
1
4
2
3

Check: >1.5?

Matrix multiplication

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Model
1
1

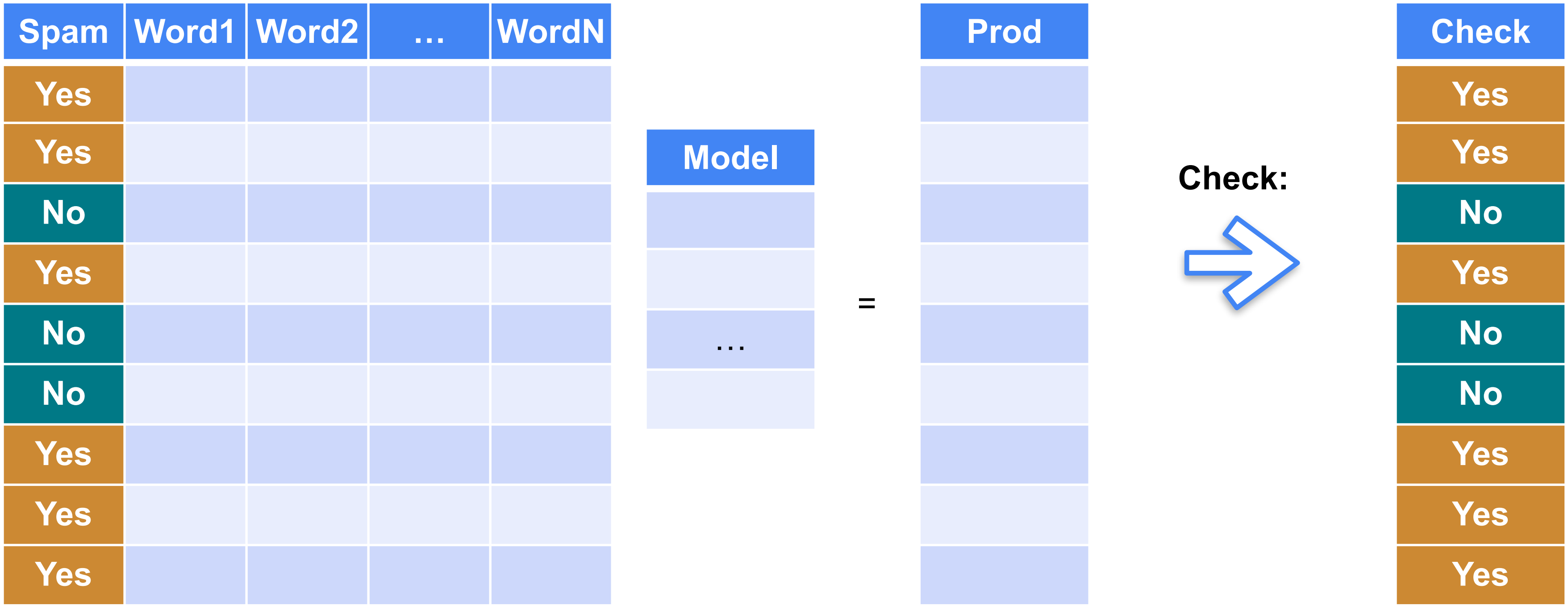
=

Prod
2
3
0
2
1
1
4
2
3

Check: >1.5?

Check
Yes
Yes
No
Yes
No
No
Yes
Yes
Yes

Perceptrons



Threshold and bias

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Model
1
1

Check: > 1.5?

Threshold and bias

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Threshold

Check

$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$

Model
1
1

Check: > 1.5?

Threshold and bias

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Check

Threshold

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

Bias

Model
1
1

Check: > 1.5?

Threshold and bias

Spam	Lottery	Win	Bias
Yes	1	1	1
Yes	2	1	1
No	0	0	1
Yes	0	2	1
No	0	1	1
No	1	0	1
Yes	2	2	1
Yes	2	0	1
Yes	1	2	1

Threshold

Check

$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$

$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$

Bias

Model
1
1

Check: > 1.5?

Threshold and bias

Spam	Lottery	Win	Bias
Yes	1	1	1
Yes	2	1	1
No	0	0	1
Yes	0	2	1
No	0	1	1
No	1	0	1
Yes	2	2	1
Yes	2	0	1
Yes	1	2	1

Threshold

Check

$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$

$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$

Bias

Check: > 1.5?

Model
1
1
-1.5

Bias

Threshold and bias

Spam	Lottery	Win	Bias
Yes	1	1	1
Yes	2	1	1
No	0	0	1
Yes	0	2	1
No	0	1	1
No	1	0	1
Yes	2	2	1
Yes	2	0	1
Yes	1	2	1

Threshold

Check

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

Bias

Check: > 0?

Model
1
1
-1.5

Bias

The AND operator

AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1

The AND operator

AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1

Model
1
1

The AND operator

AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1

Model
1
1

=

Dot prod
0
1
1
2

The AND operator

AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1

Model
1
1

=

Dot prod
0
1
1
2

Check: >1.5?

The AND operator

AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1

Model
1
1

=

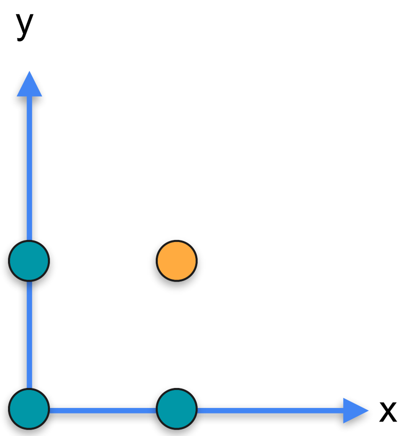
Dot prod
0
1
1
2

Check: >1.5?

Check
No
No
No
Yes

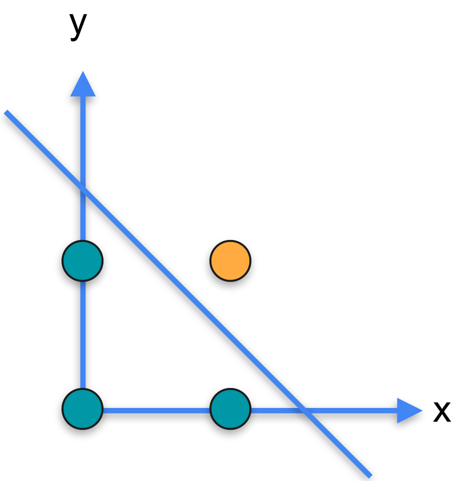
The AND operator

AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1



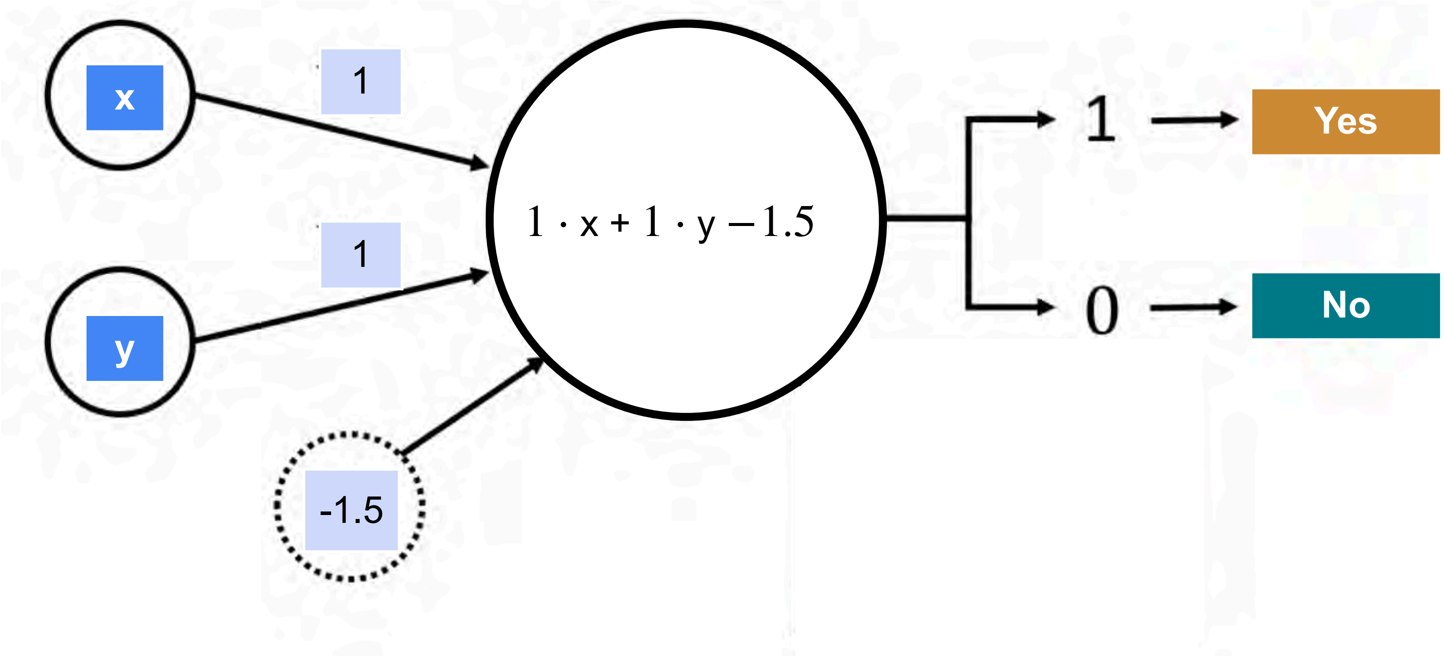
The AND operator

AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1

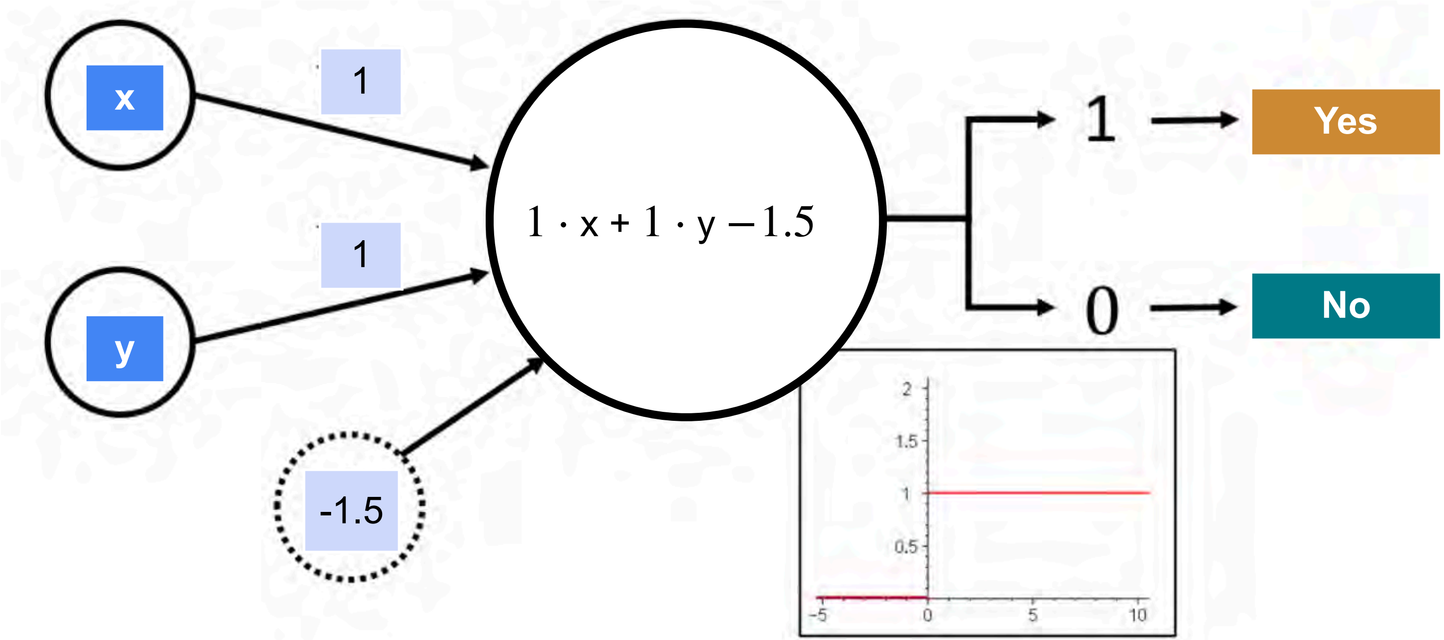


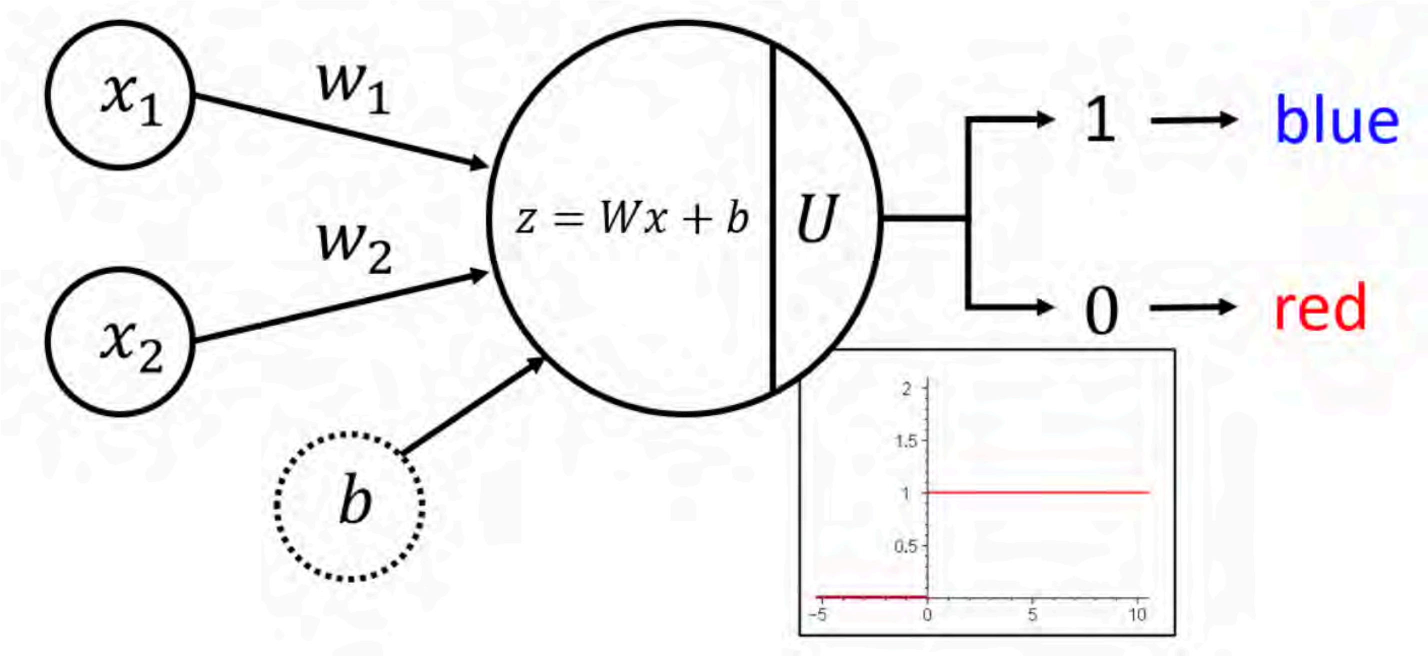
$$1 \cdot x + 1 \cdot y = 1.5$$

The perceptron



The perceptron




x_1
x_2
w_1
w_2
b
$z = Wx + b$
U
1
0
blue
red
2
1.5
1
0.5
-5
0
5
10

DeepLearning.AI

Vectors and Linear Transformations

Conclusion