



DeepLearning.AI

# Math for Machine Learning

---

## **Linear algebra - Week 3**

Vectors

Matrices

Dot product

Matrix multiplication

Linear transformations



DeepLearning.AI

# Vectors and Linear Transformations

---

## **Machine Learning motivation**

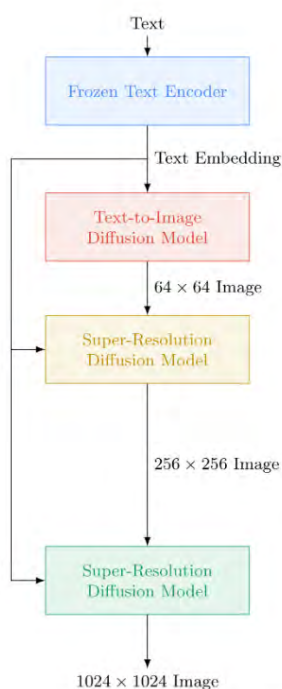
# Neural Networks - AI generated images



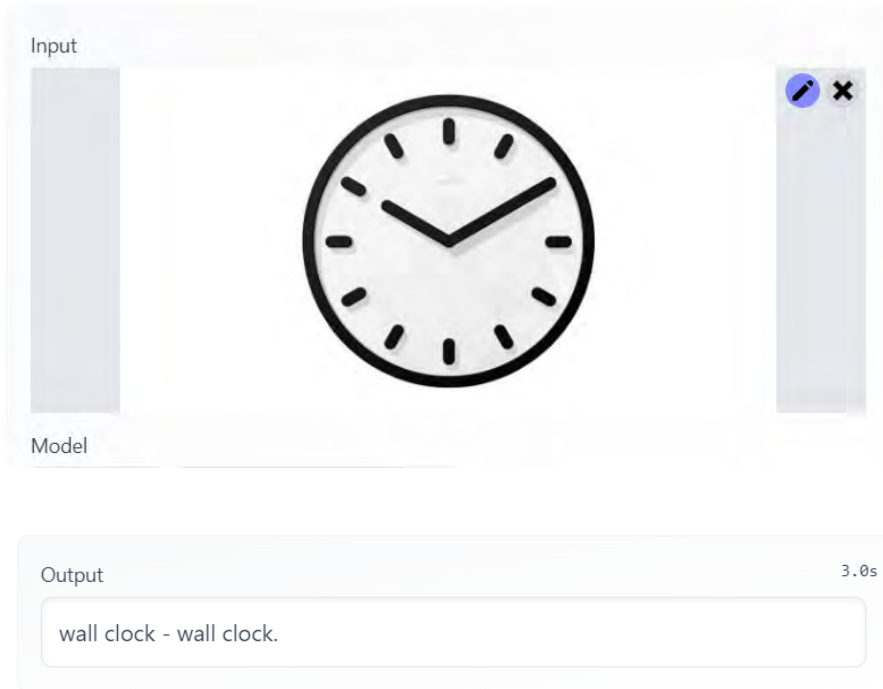
AI-generated human faces.

- Generative learning: Generating realistic looking images.

# Text-to-image and image-to-text generation



"A Golden Retriever dog wearing a blue checkered beret and red dotted turtleneck."





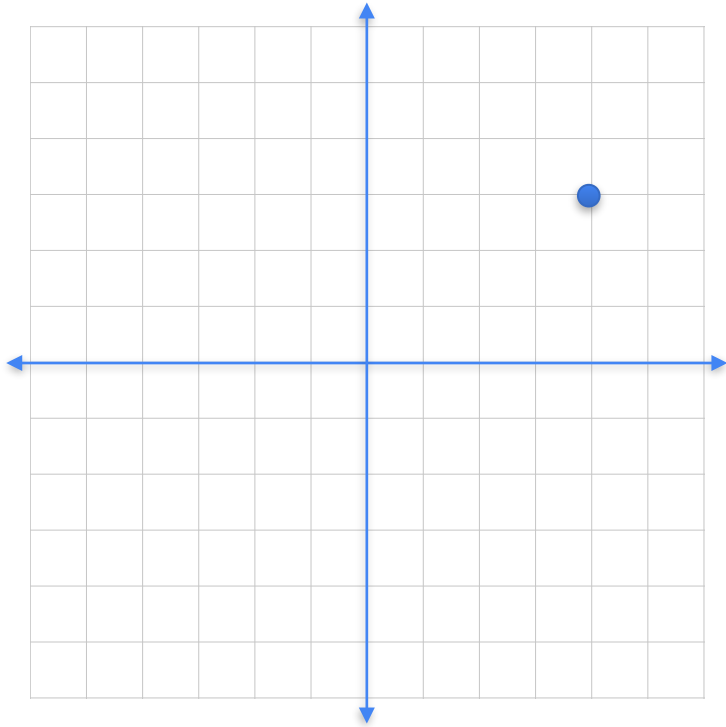
DeepLearning.AI

# Vectors and Linear Transformations

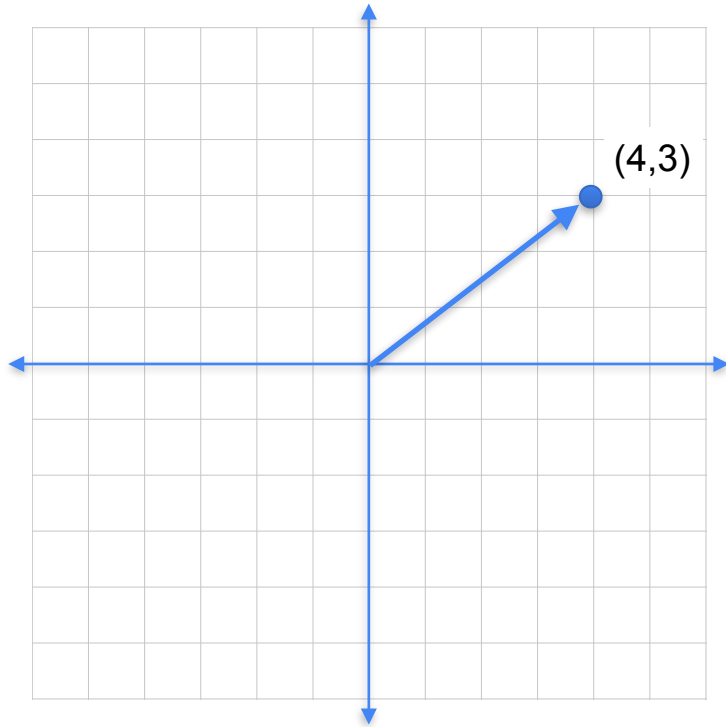
---

## **Vectors and their properties**

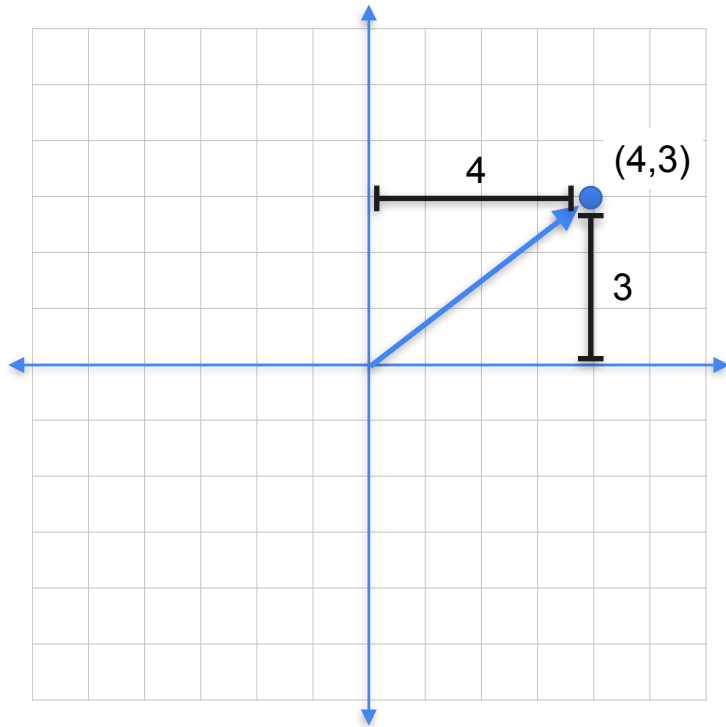
# Vectors



# Vectors

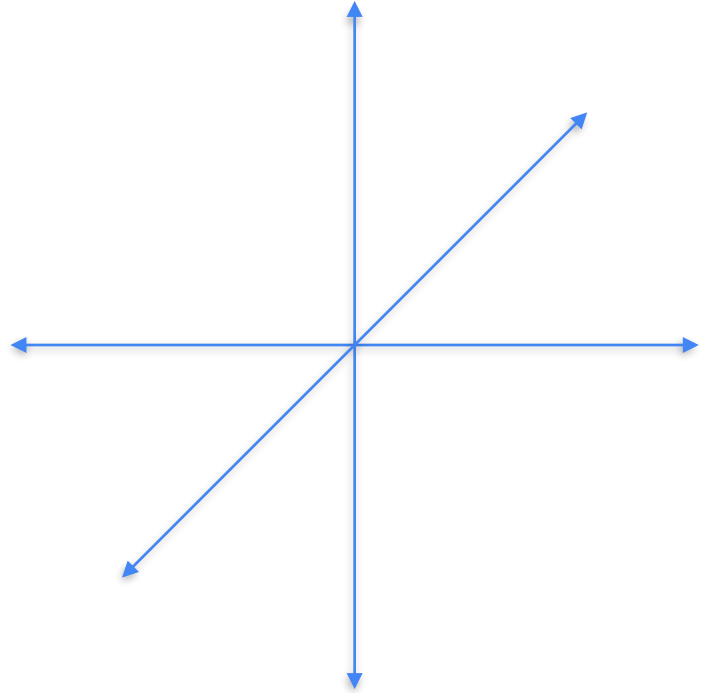
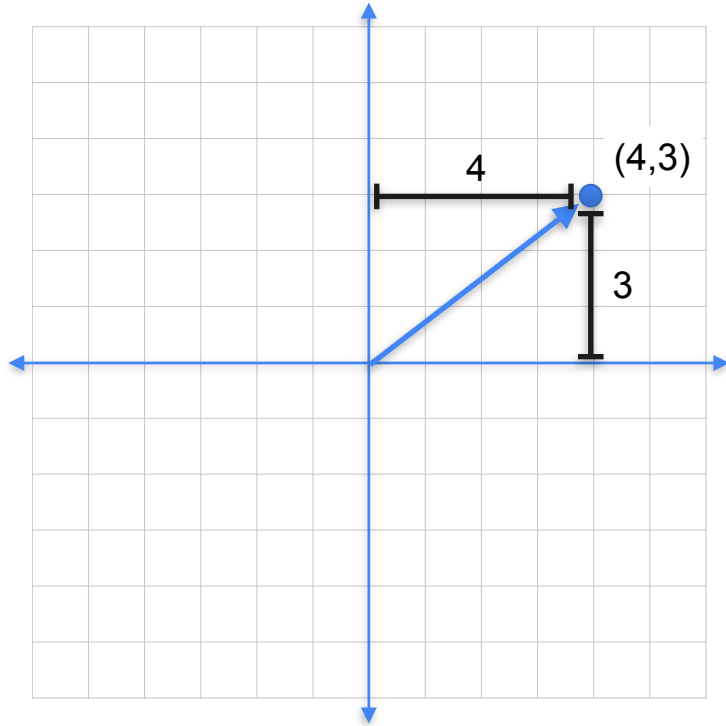


# Vectors

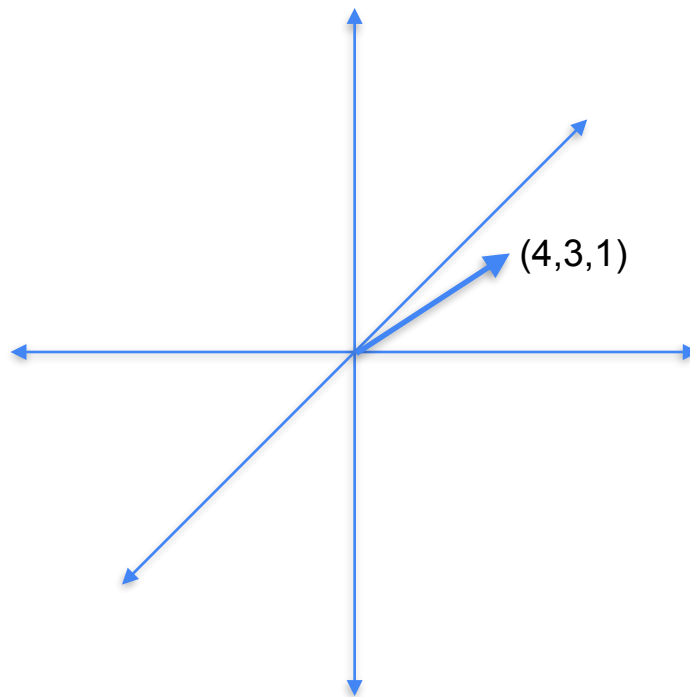
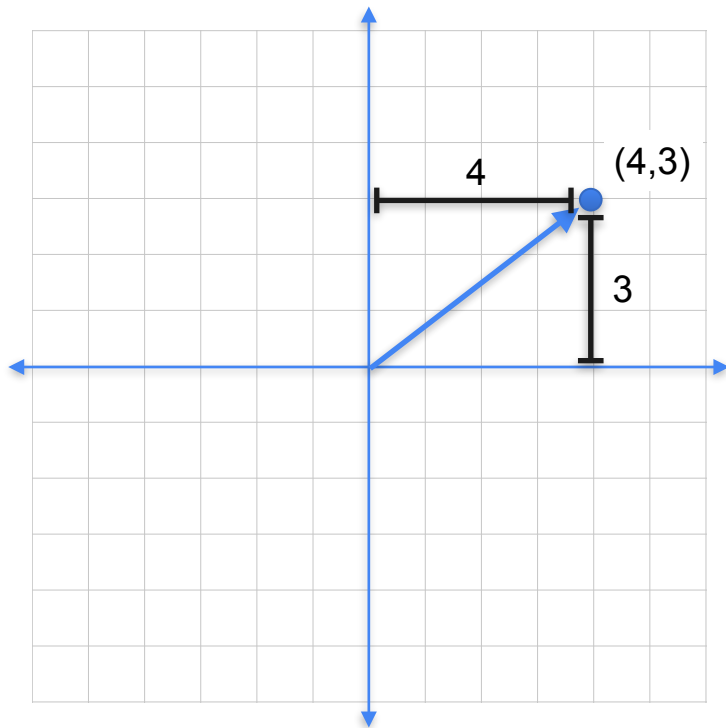




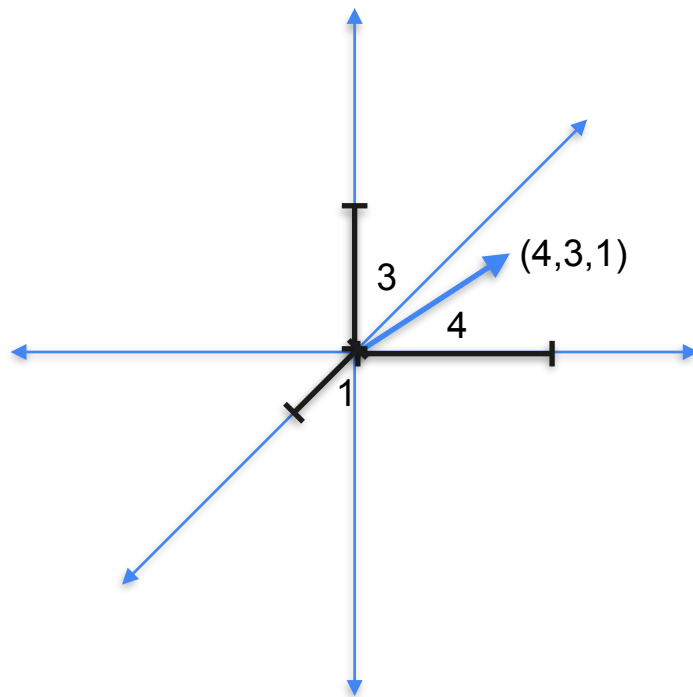
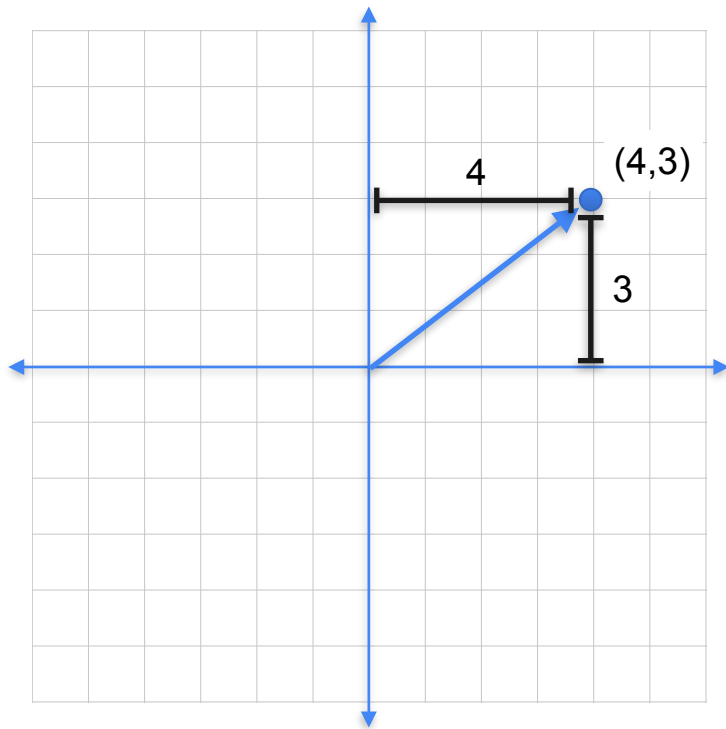
# Vectors



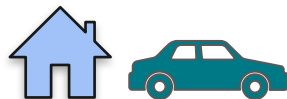
# Vectors



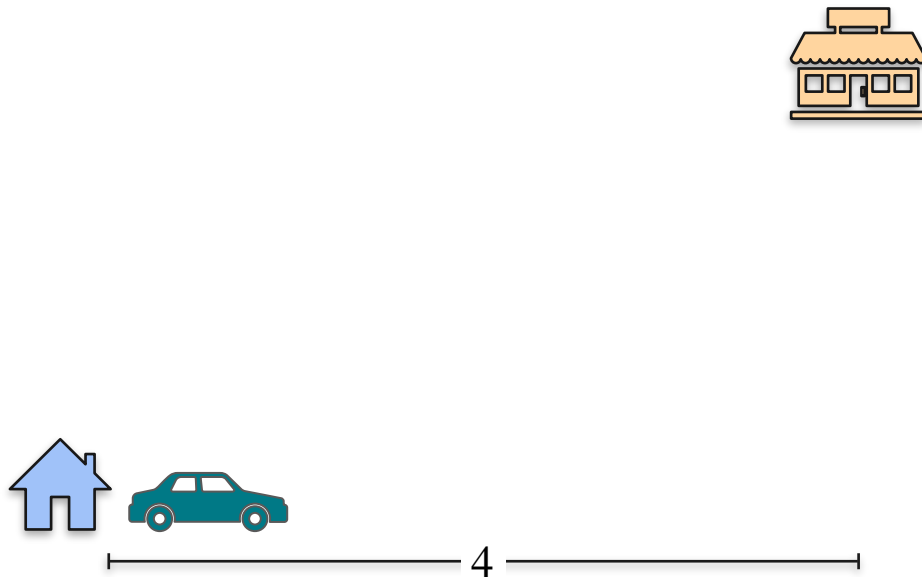
# Vectors



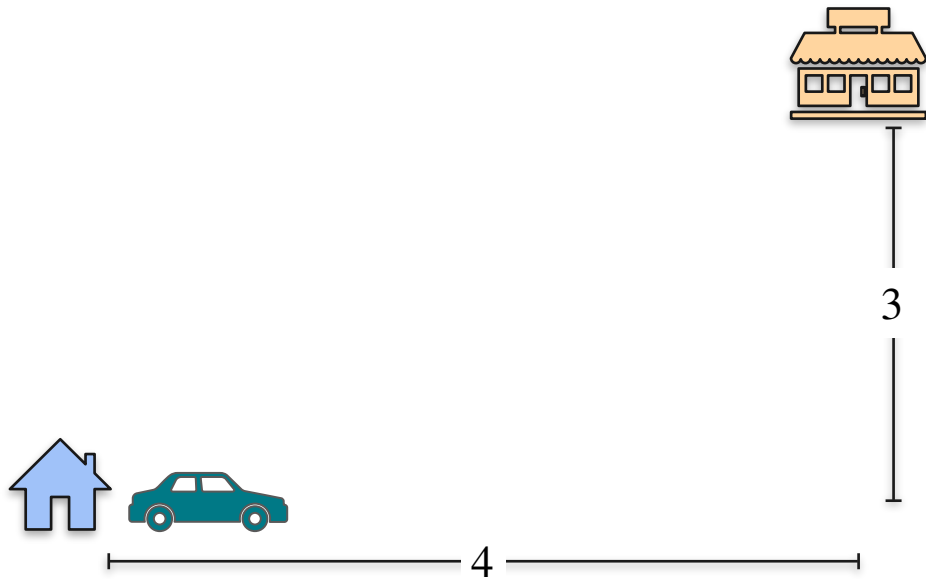
# How to get from point A to point B?



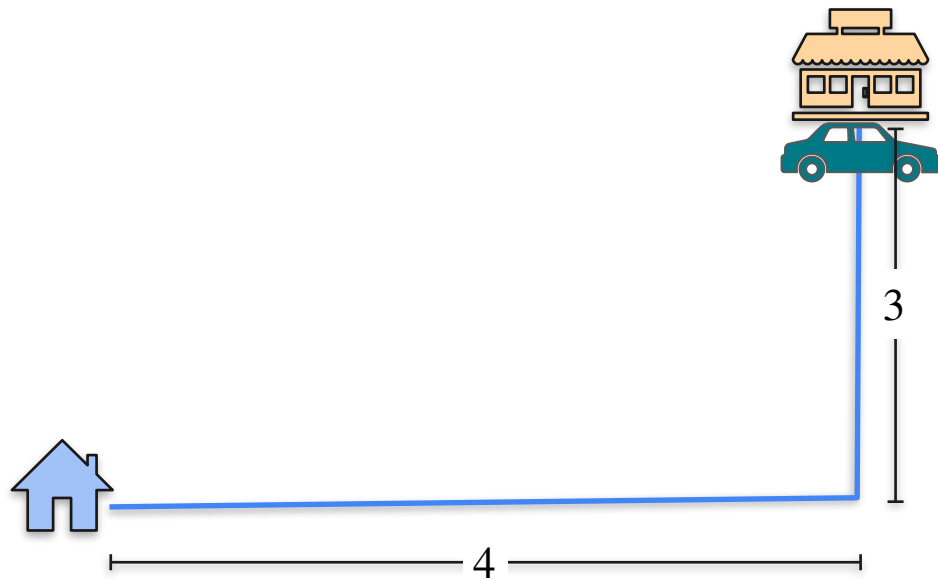
# How to get from point A to point B?



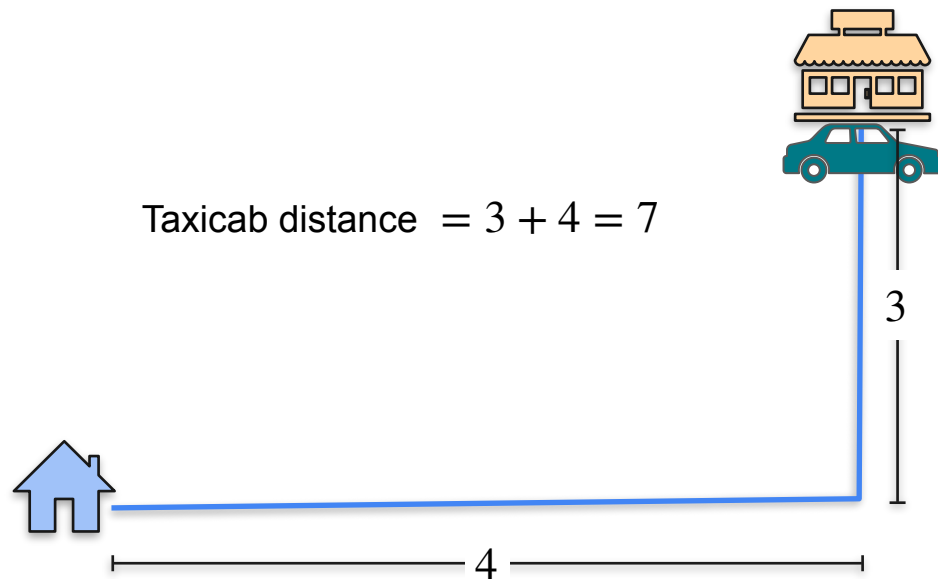
# How to get from point A to point B?



# How to get from point A to point B?

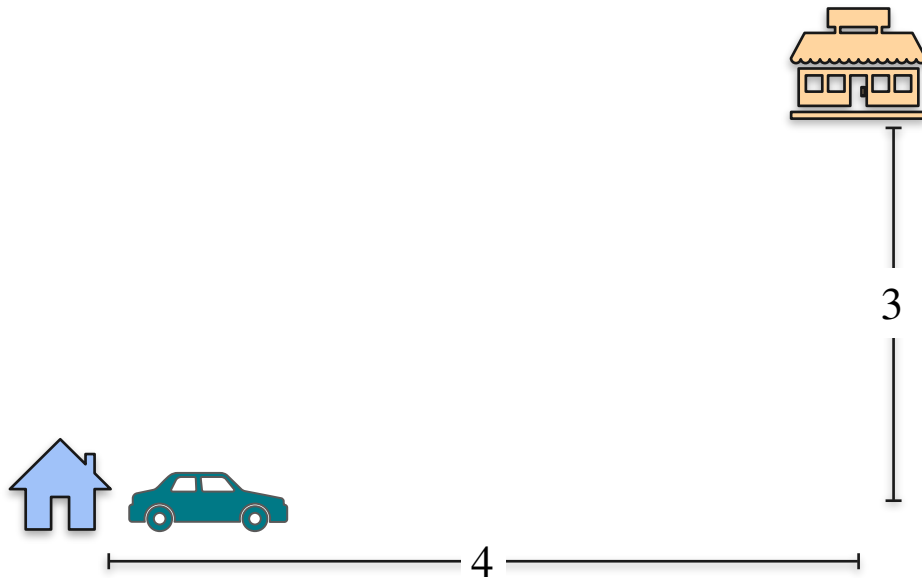


# How to get from point A to point B?

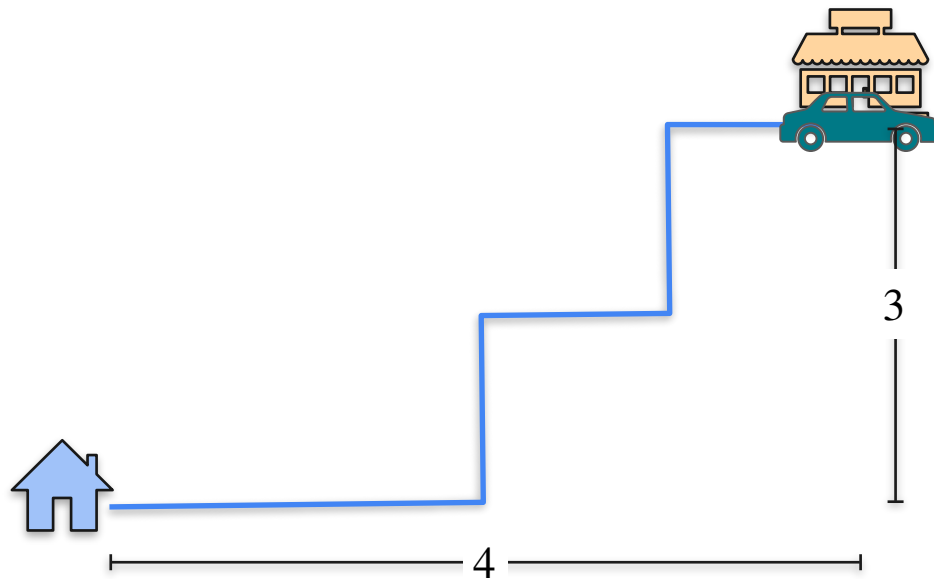




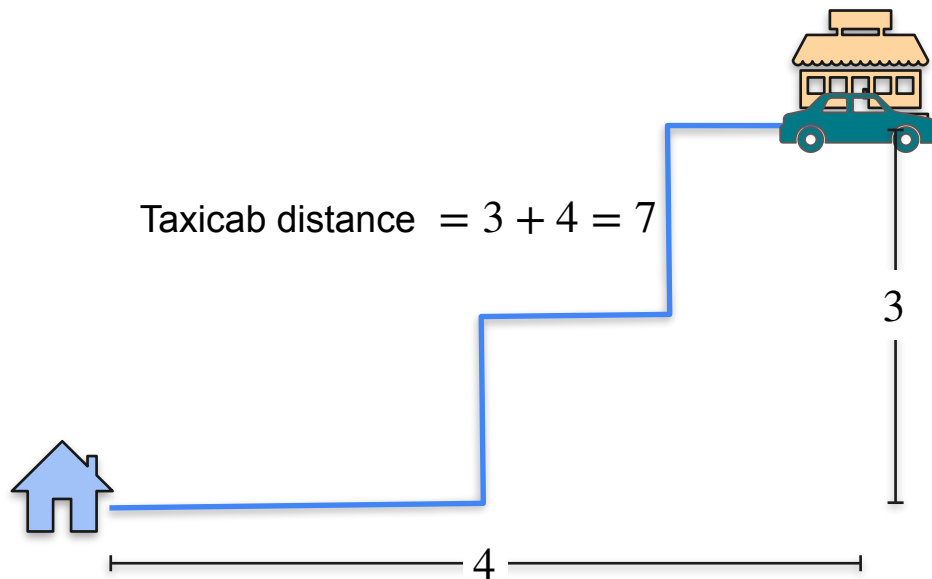
# How to get from point A to point B?



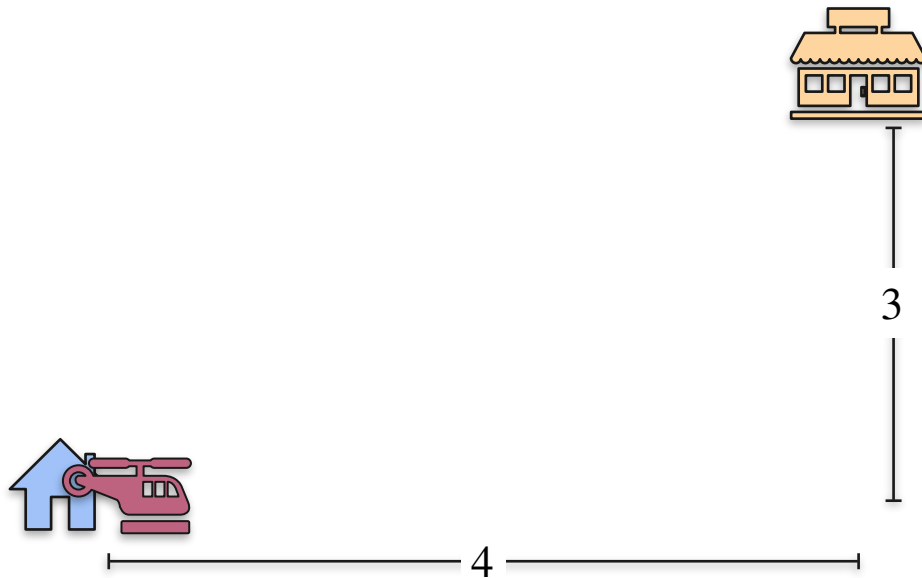
# How to get from point A to point B?



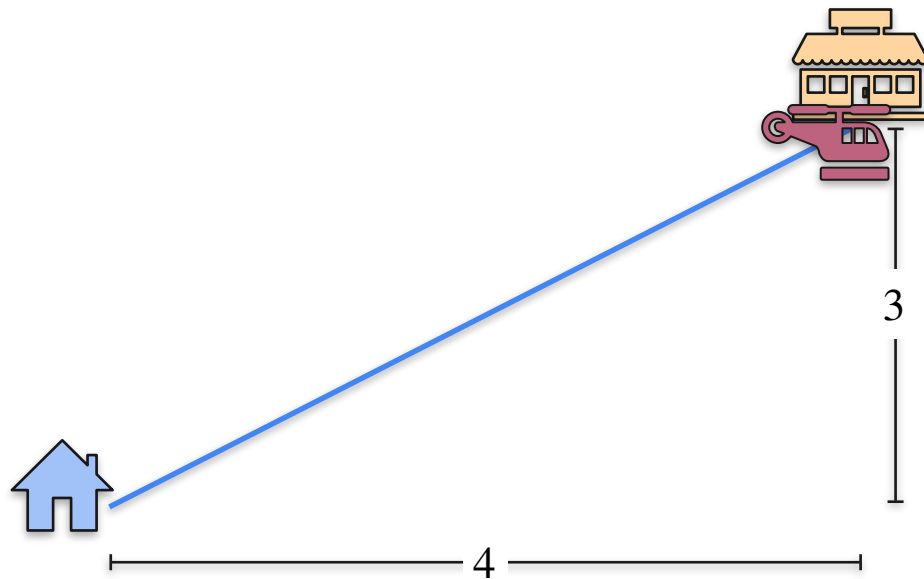
# How to get from point A to point B?



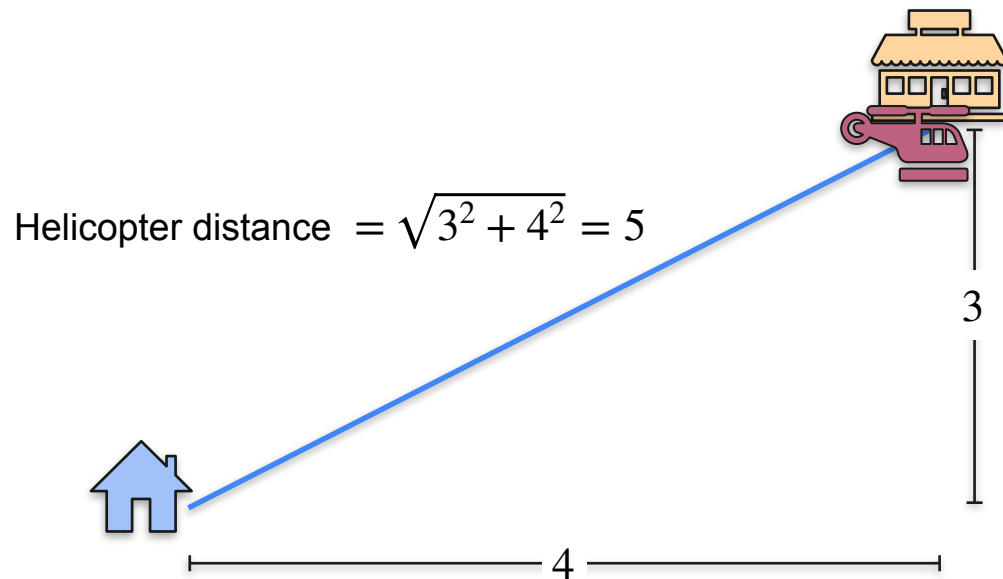
# How to get from point A to point B?



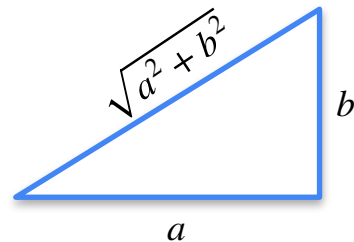
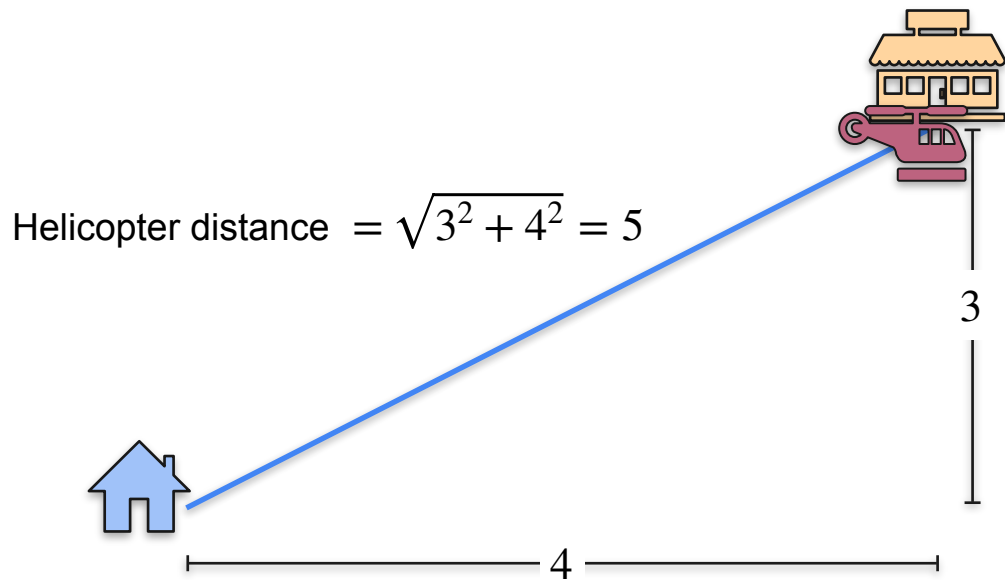
# How to get from point A to point B?



# How to get from point A to point B?

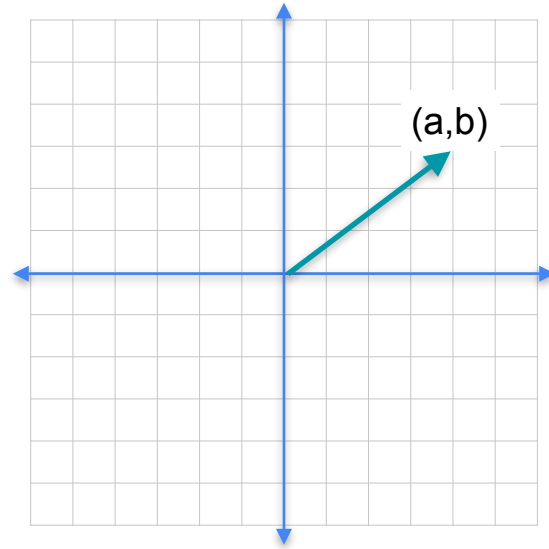


# How to get from point A to point B?



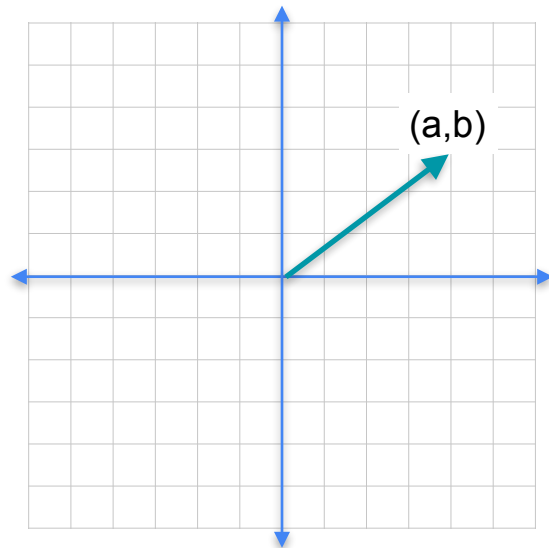
Pythagorean Theorem

# Norms



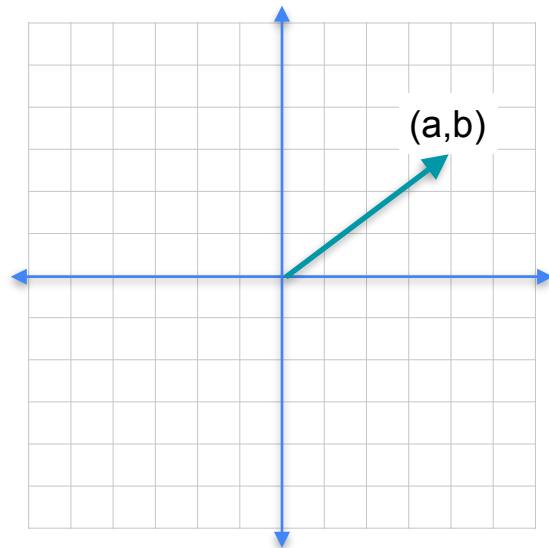


# Norms



$$\text{L1-norm} = |(a,b)|_1 = |a| + |b|$$

# Norms

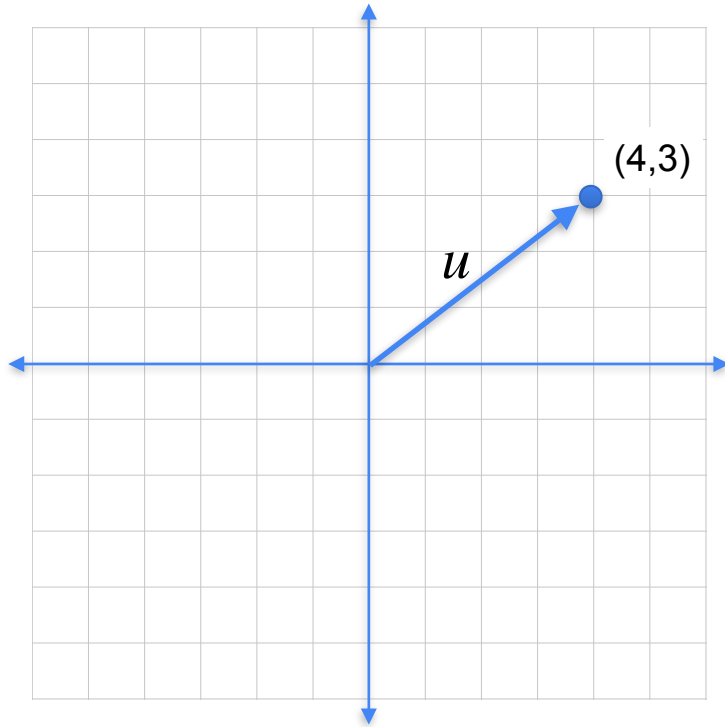


$$\text{L1-norm} = |(a,b)|_1 = |a| + |b|$$

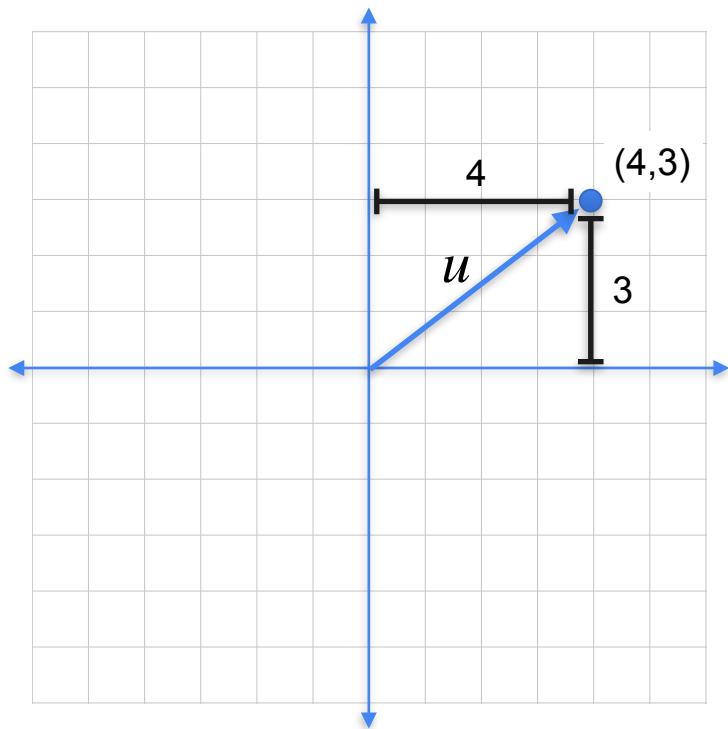


$$\text{L2-norm} = |(a,b)|_2 = \sqrt{a^2 + b^2}$$

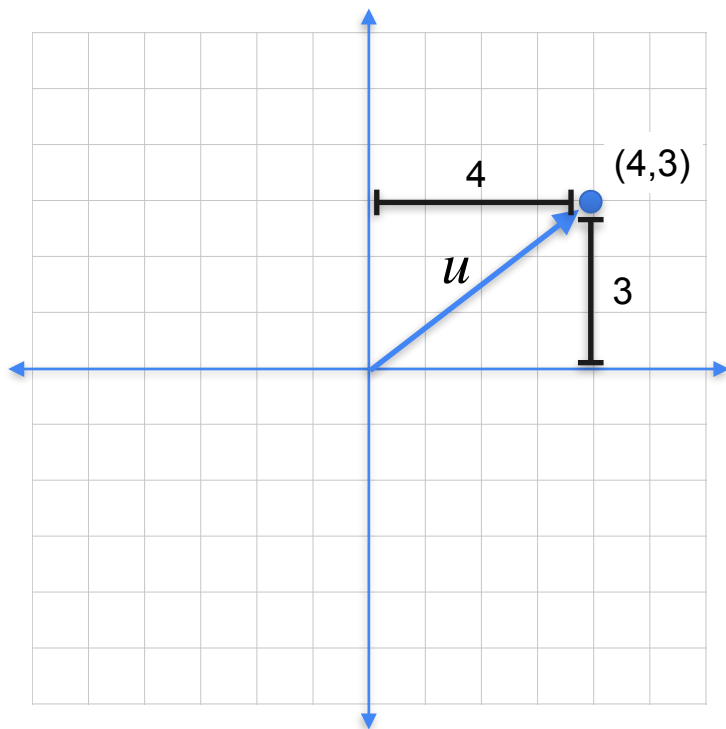
# Norm of a vector



# Norm of a vector

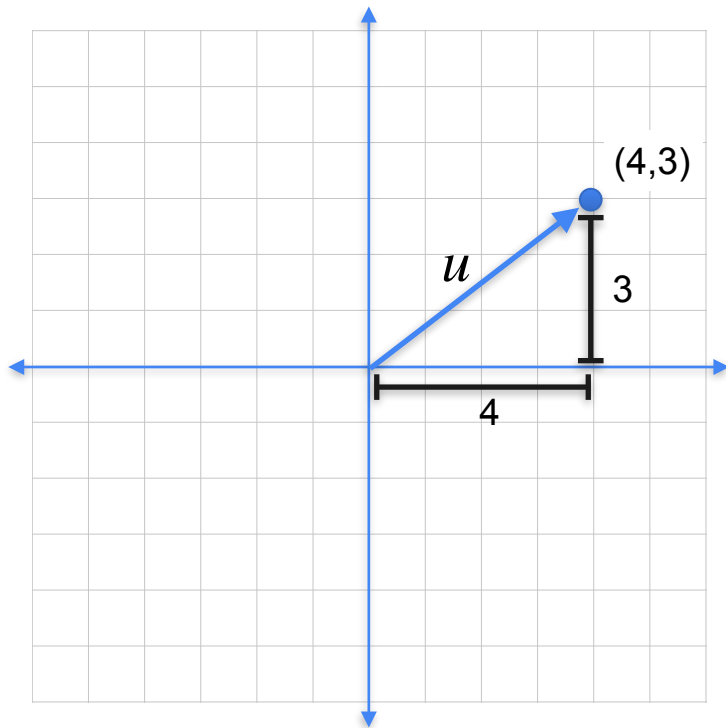


# Norm of a vector

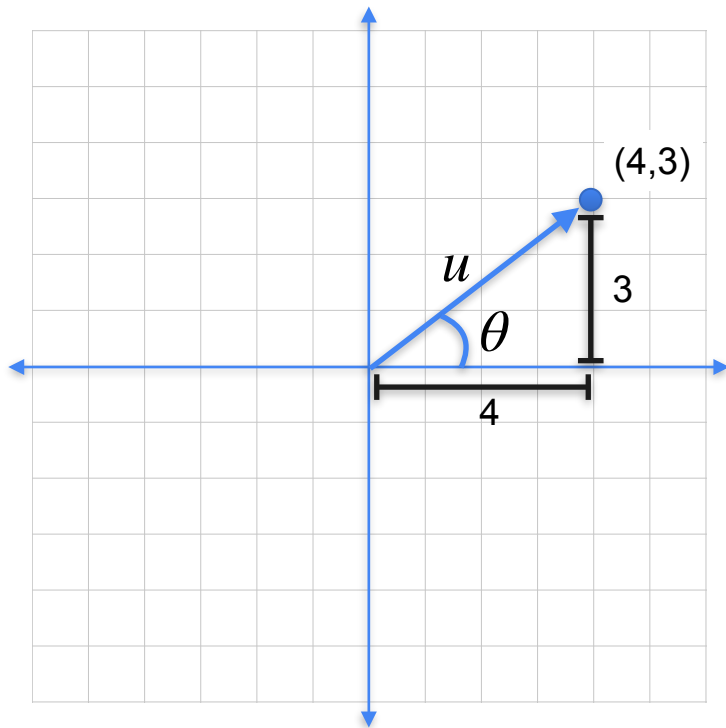


$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

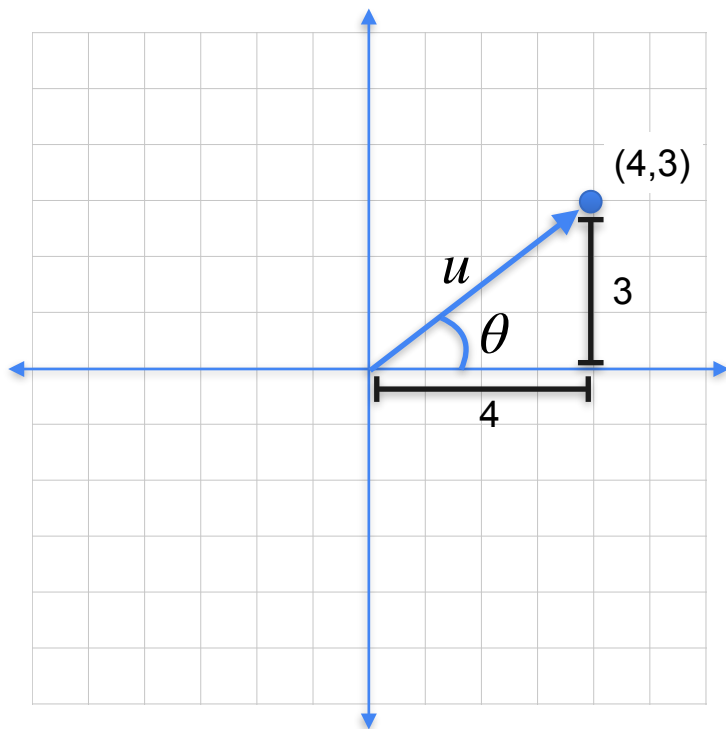
# Direction of a vector



# Direction of a vector



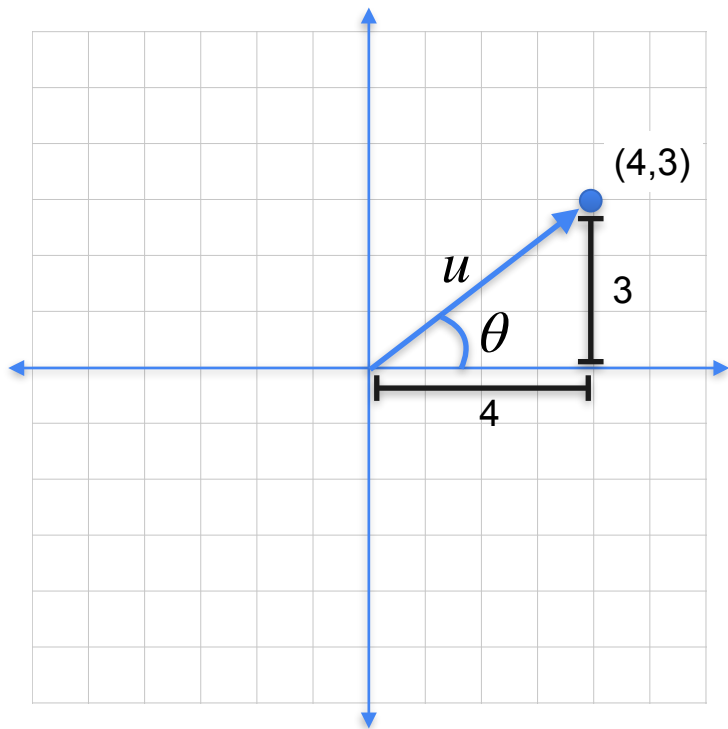
# Direction of a vector



$$\tan(\theta) = \frac{3}{4}$$



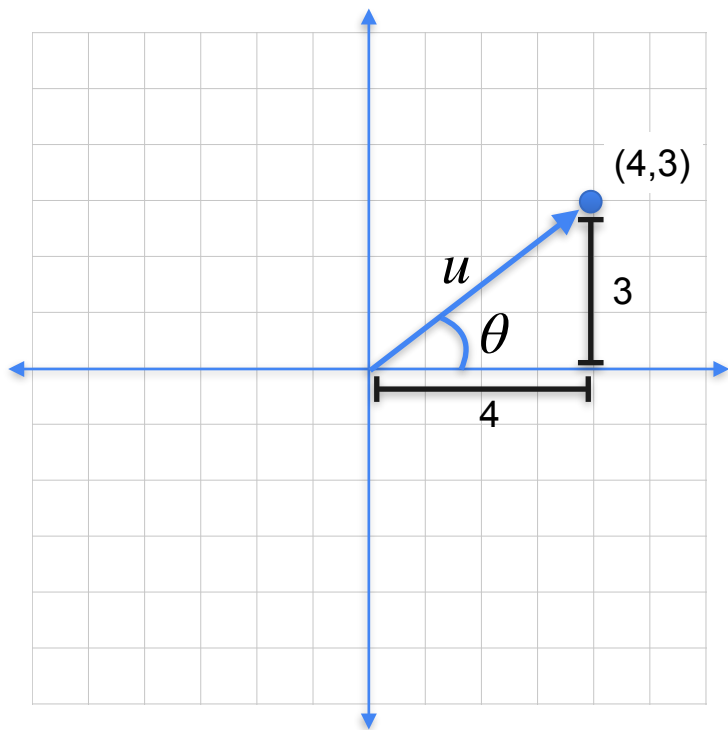
# Direction of a vector



$$\tan(\theta) = \frac{3}{4}$$

$$\theta = \arctan(3/4) = 0.64$$

# Direction of a vector



$$\tan(\theta) = \frac{3}{4}$$

$$\theta = \arctan(3/4) = 0.64 = 36.87^\circ$$



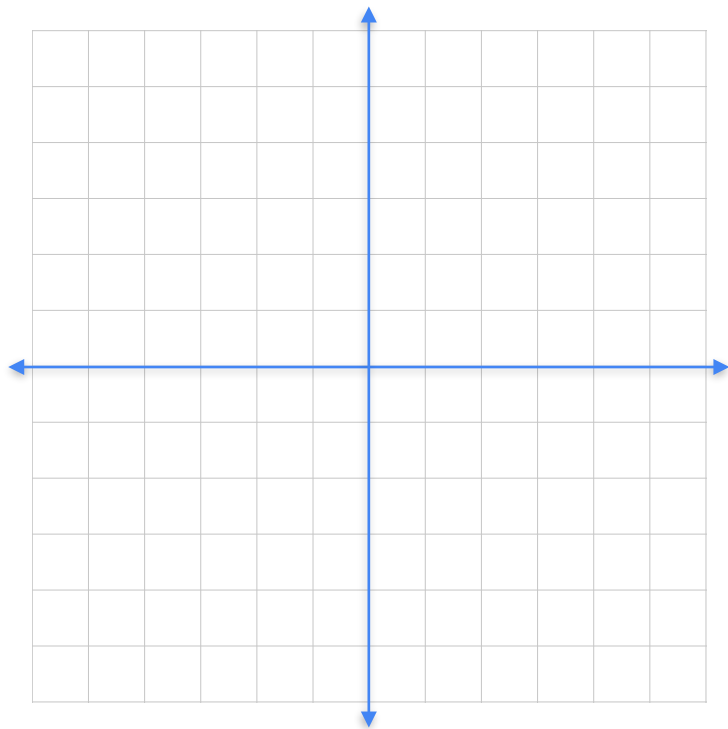
DeepLearning.AI

# Vectors and Linear Transformations

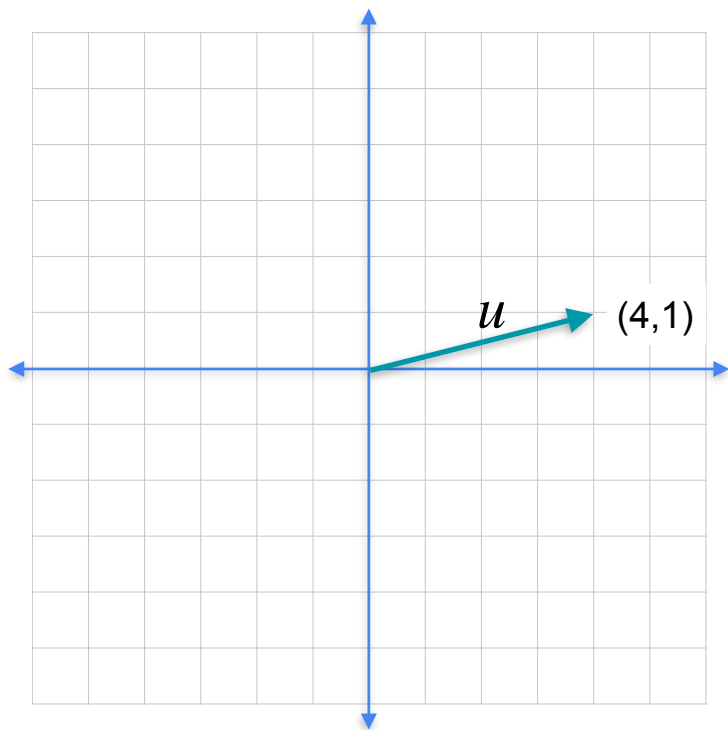
---

## **Sum and difference of vectors**

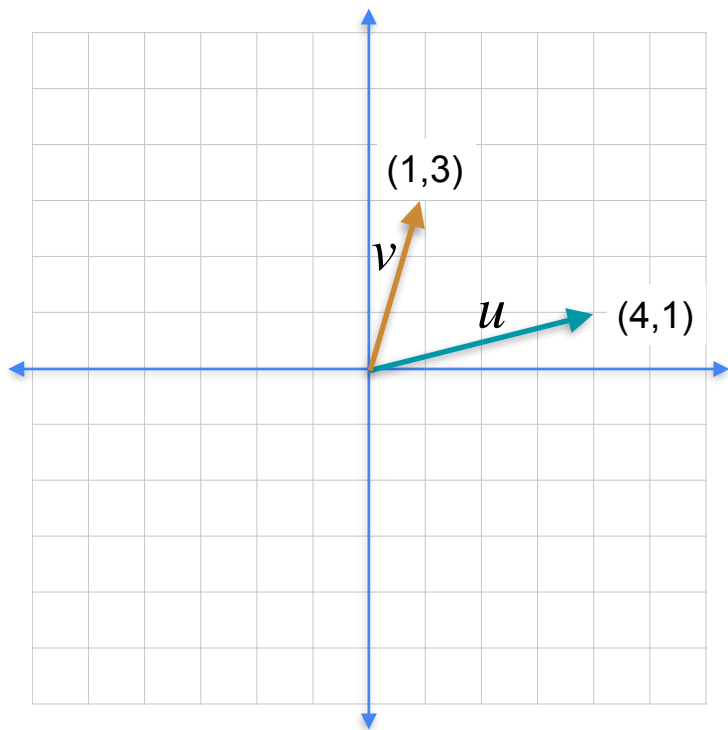
# Sum of vectors



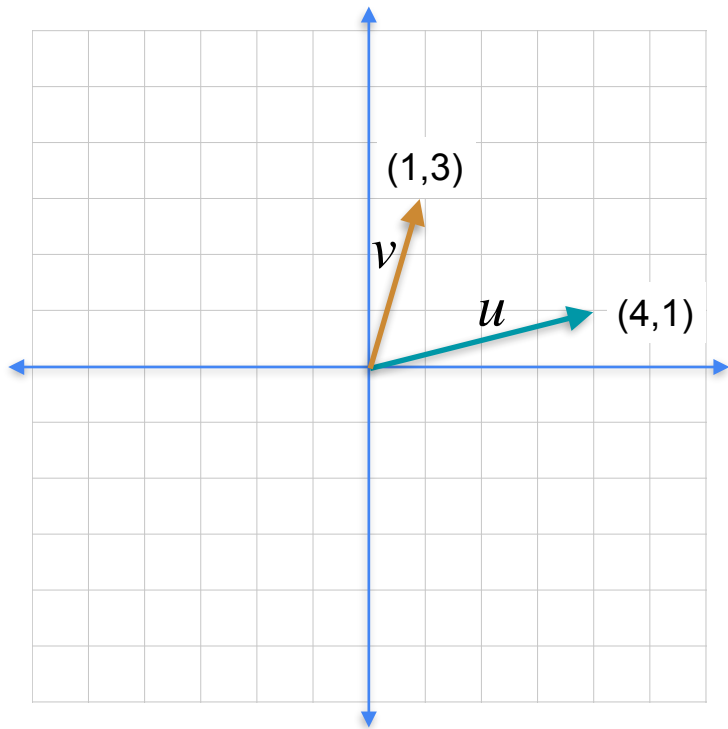
# Sum of vectors



# Sum of vectors

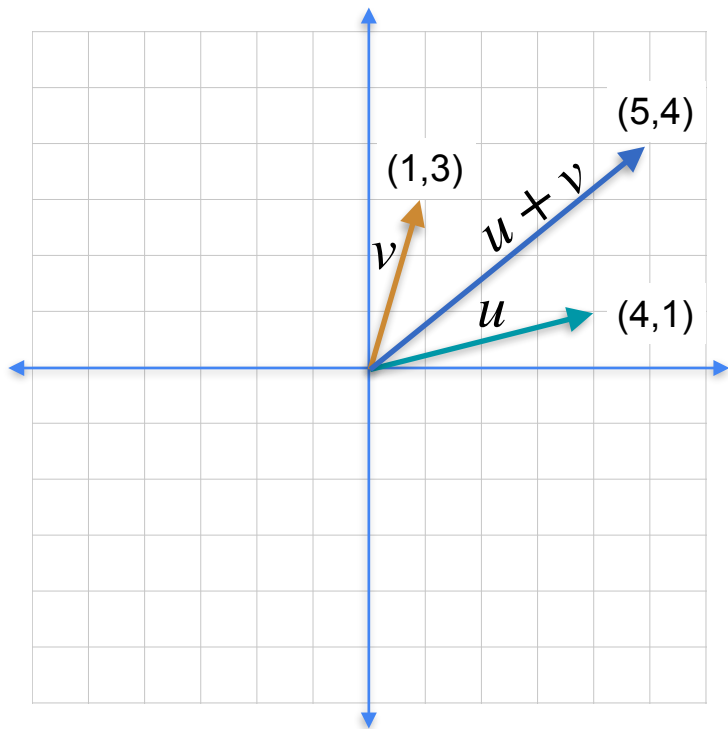


# Sum of vectors



$$u + v = (4 + 1, 1 + 3) = (5, 4)$$

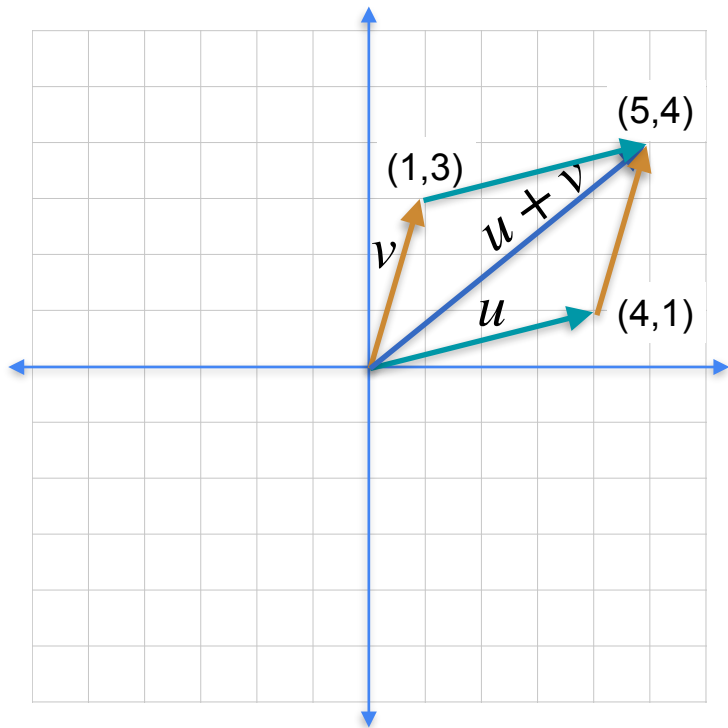
# Sum of vectors



$$u + v = (4 + 1, 1 + 3) = (5, 4)$$

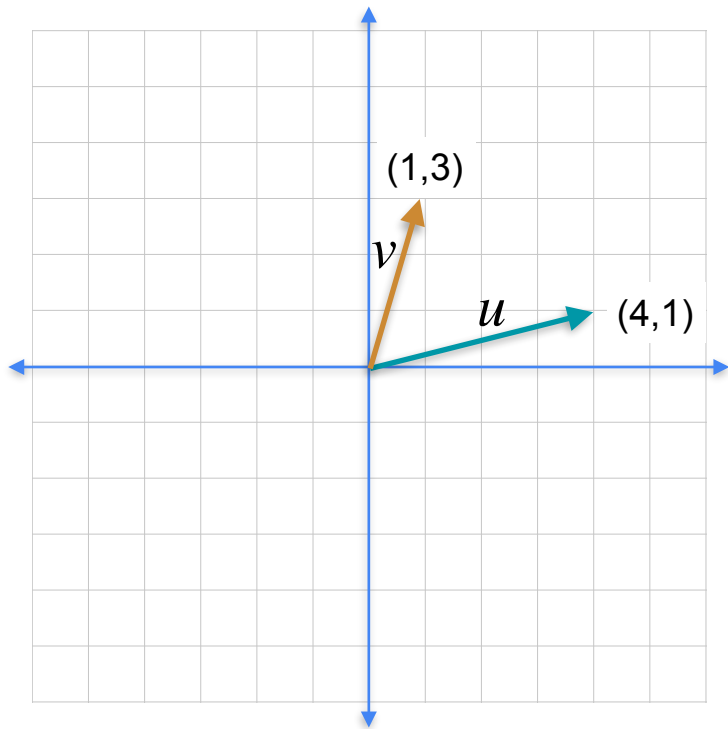


# Sum of vectors

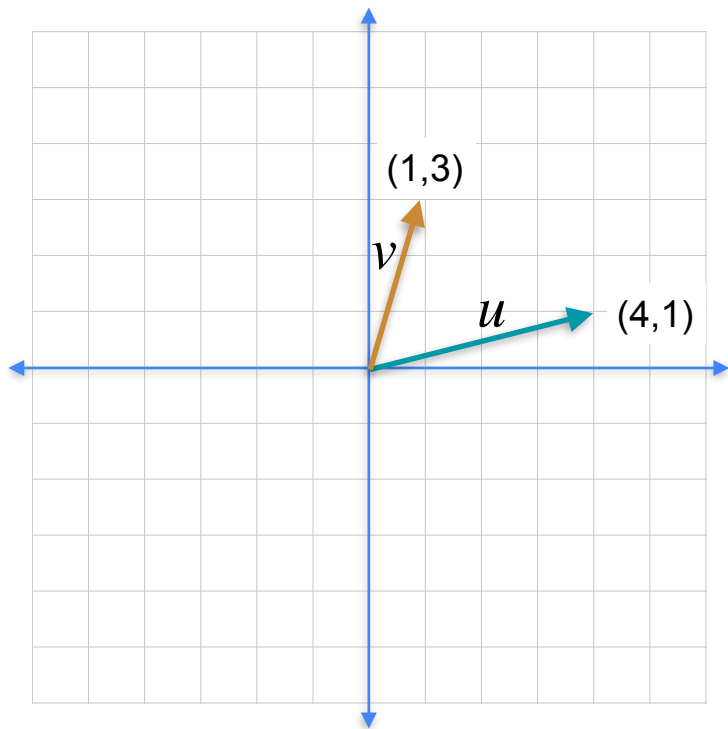


$$u + v = (4 + 1, 1 + 3) = (5, 4)$$

# Difference of vectors

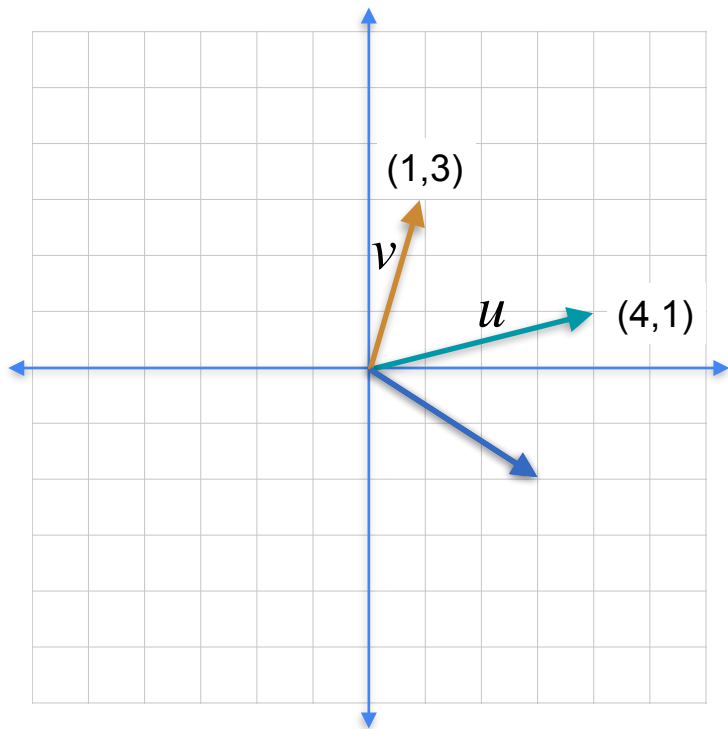


# Difference of vectors



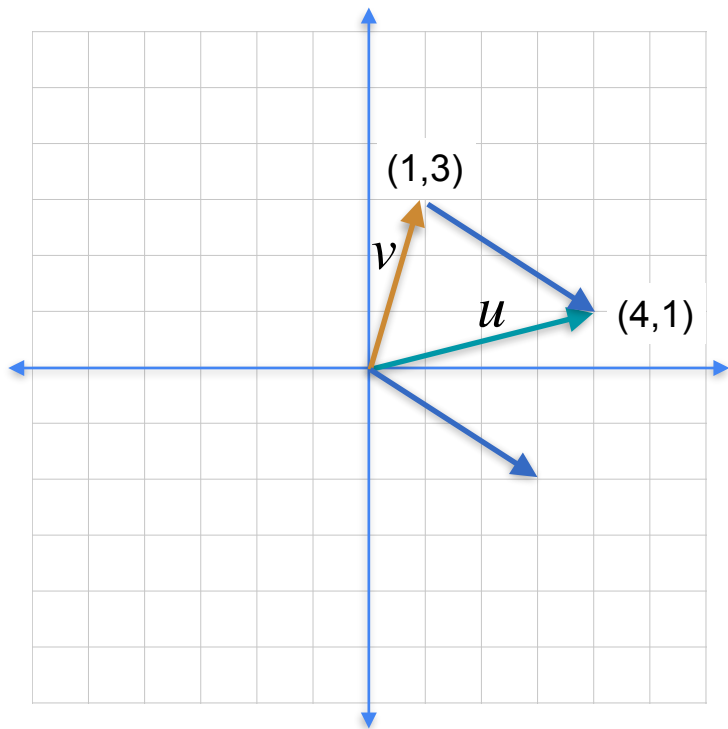
$$u - v = (4 - 1, 1 - 3) = (3, -2)$$

# Difference of vectors



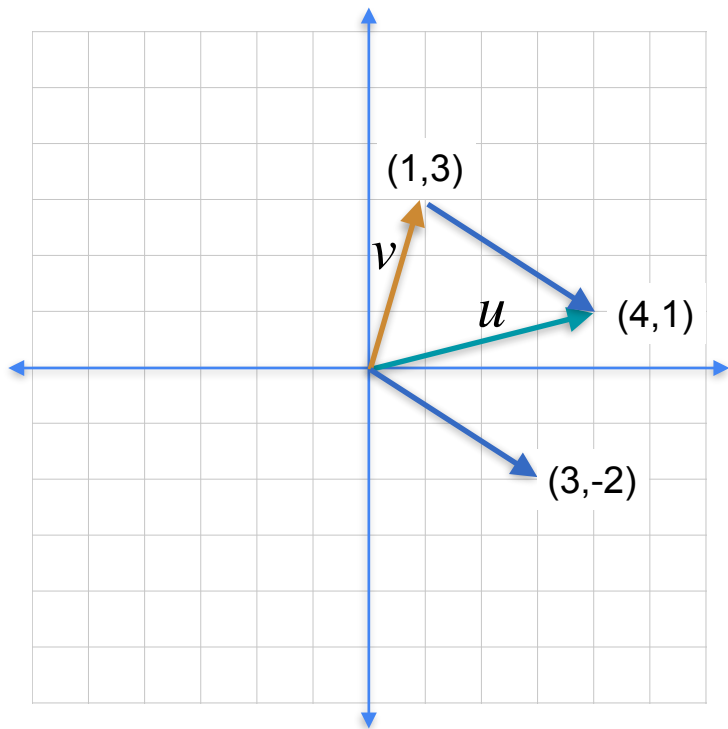
$$u - v = (4 - 1, 1 - 3) = (3, -2)$$

# Difference of vectors



$$u - v = (4 - 1, 1 - 3) = (3, -2)$$

# Difference of vectors



$$u - v = (4 - 1, 1 - 3) = (3, -2)$$



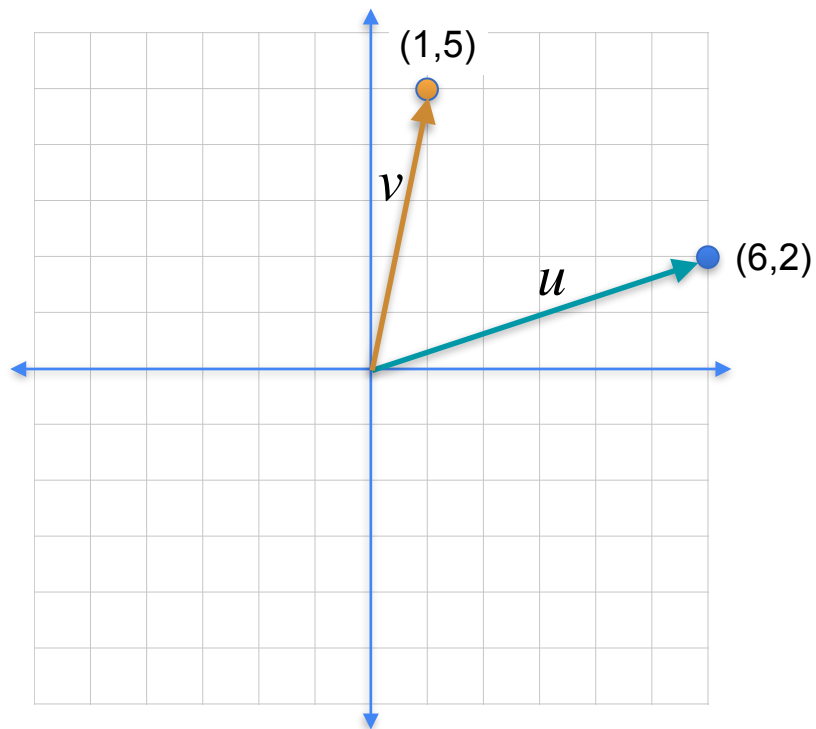
DeepLearning.AI

# Vectors and Linear Transformations

---

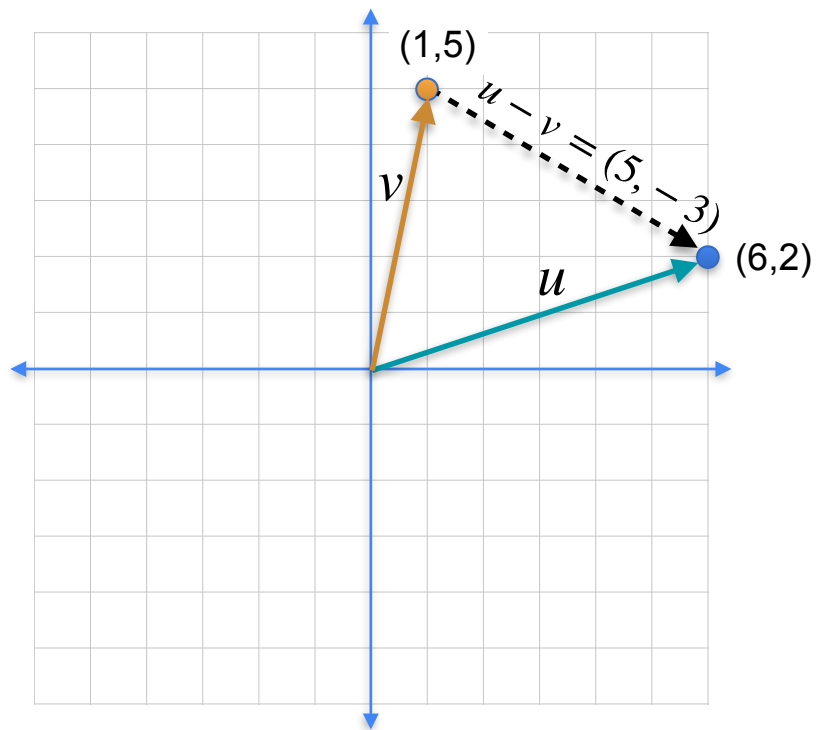
## **Distance between vectors**

# Distances

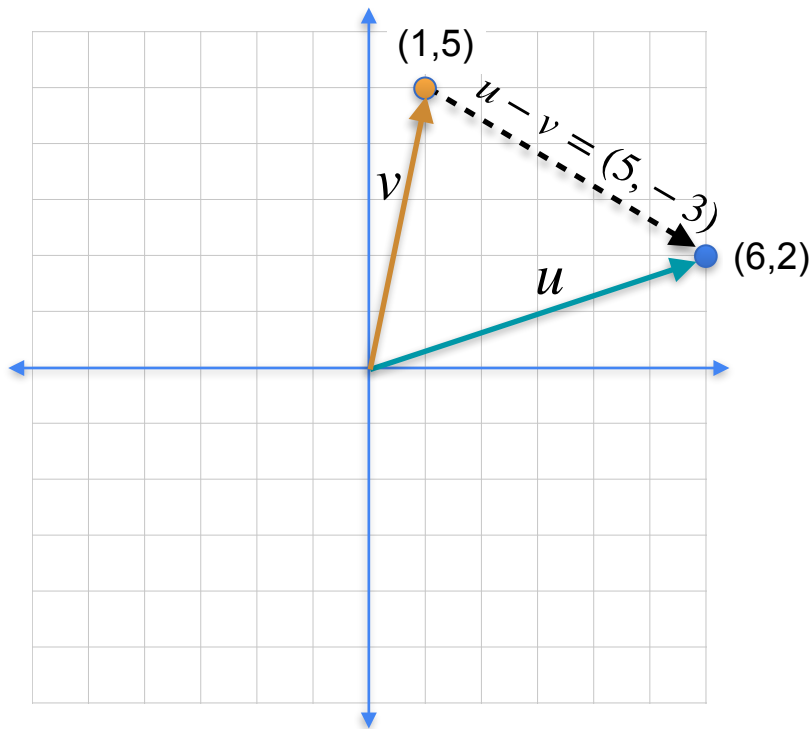




# Distances



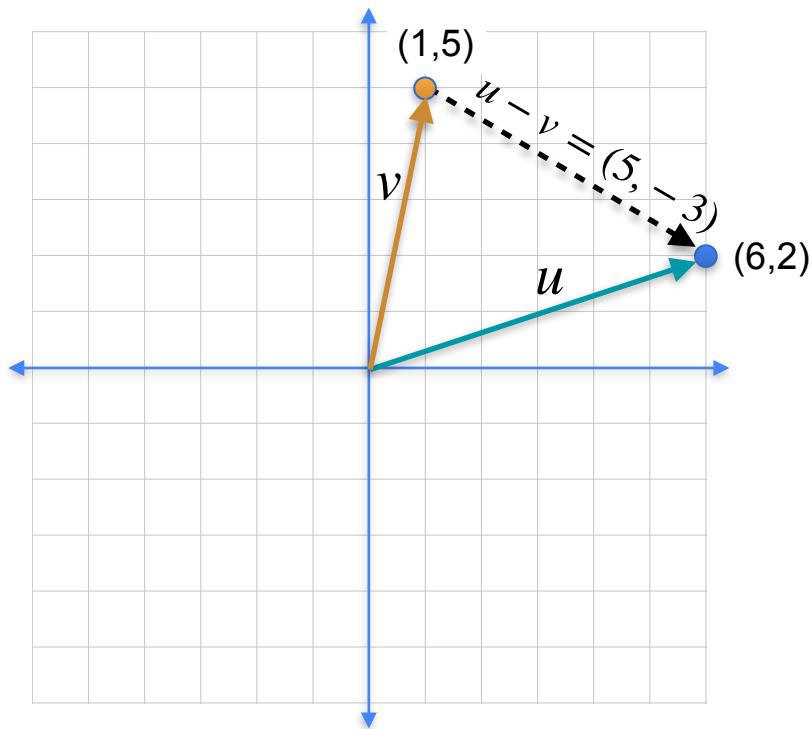
# Distances



L1-distance

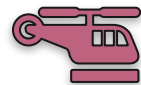
$$|u - v|_1 = |5| + |-3| = 8$$

# Distances



L1-distance

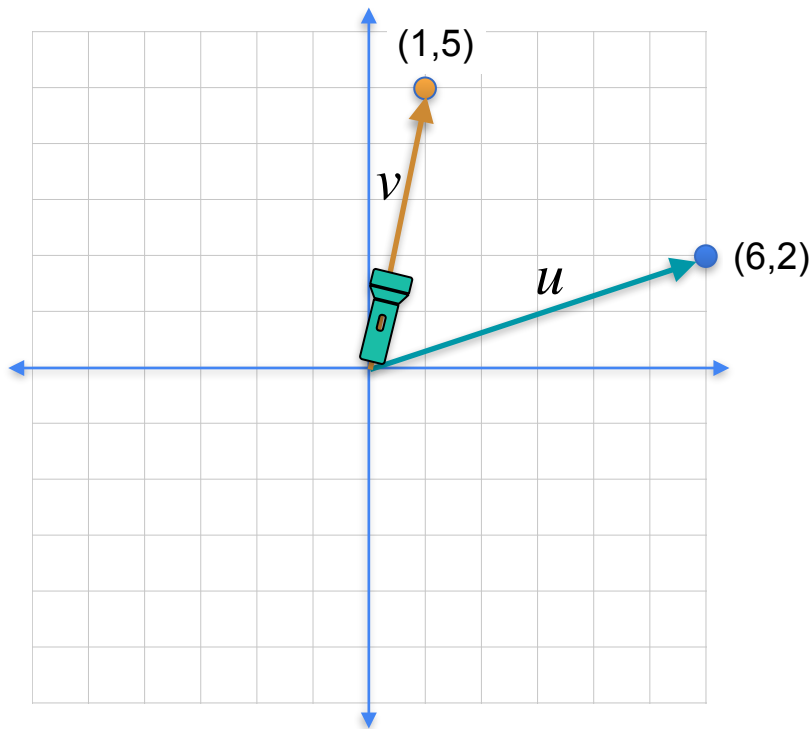
$$|u - v|_1 = |5| + |-3| = 8$$



L2-distance

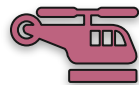
$$|u - v|_2 = \sqrt{5^2 + 3^2} = 5.83$$

# Distances



L1-distance

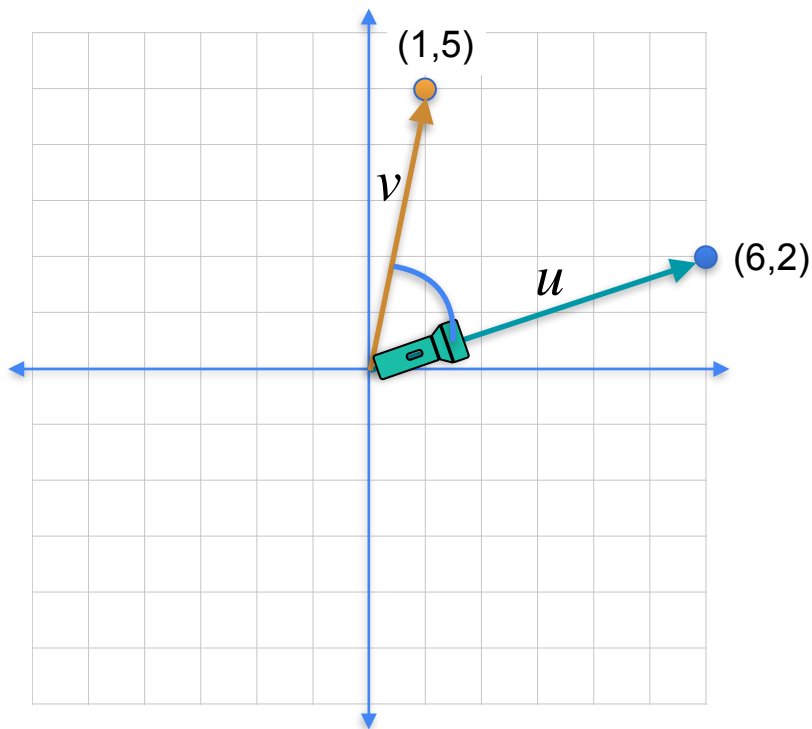
$$|u - v|_1 = |5| + |-3| = 8$$



L2-distance

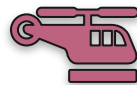
$$|u - v|_2 = \sqrt{5^2 + 3^2} = 5.83$$

# Distances



L1-distance

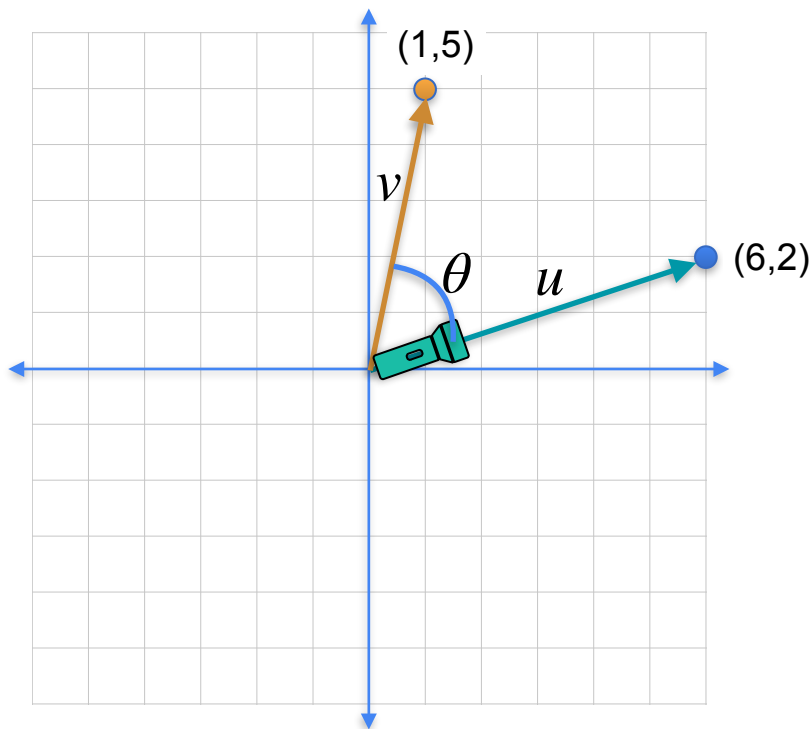
$$|u - v|_1 = |5| + |-3| = 8$$



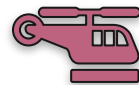
L2-distance

$$|u - v|_2 = \sqrt{5^2 + 3^2} = 5.83$$

# Distances

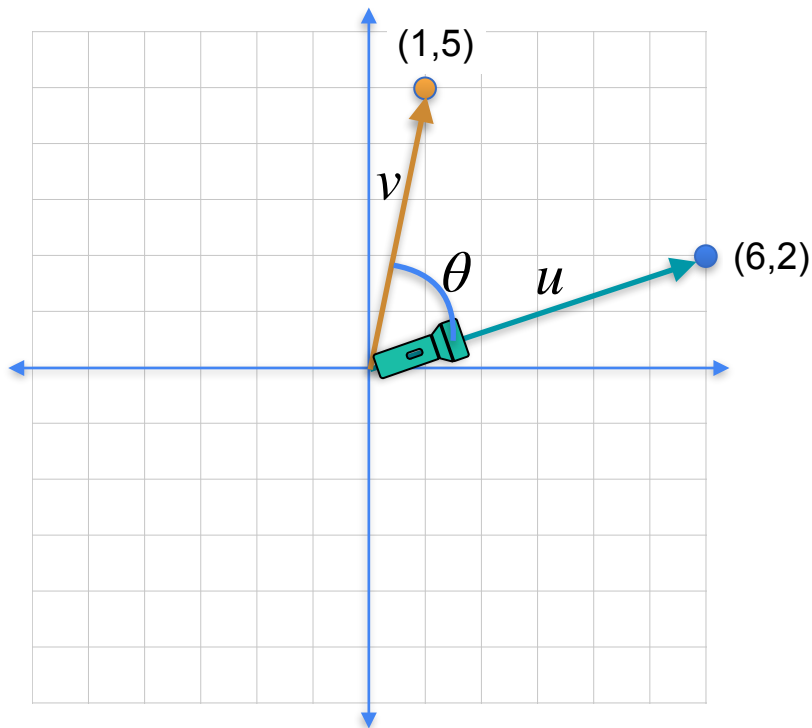



L1-distance  $|u - v|_1 = |5| + |-3| = 8$




L2-distance  $|u - v|_2 = \sqrt{5^2 + 3^2} = 5.83$

# Distances



  $|u - v|_1 = |5| + |-3| = 8$   
L1-distance

  $|u - v|_2 = \sqrt{5^2 + 3^2} = 5.83$   
L2-distance

  $\cos(\theta)$   
Cosine distance



DeepLearning.AI

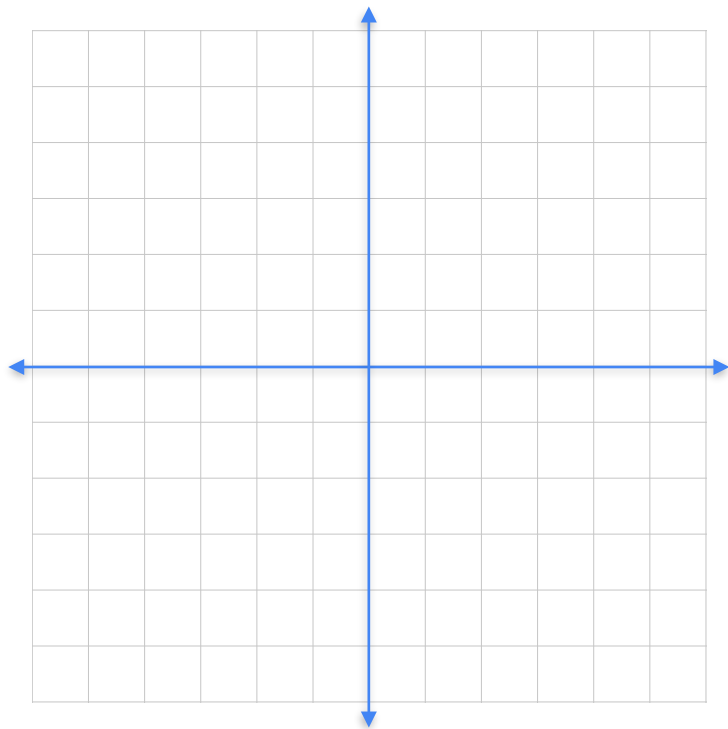
# Vectors and Linear Transformations

---

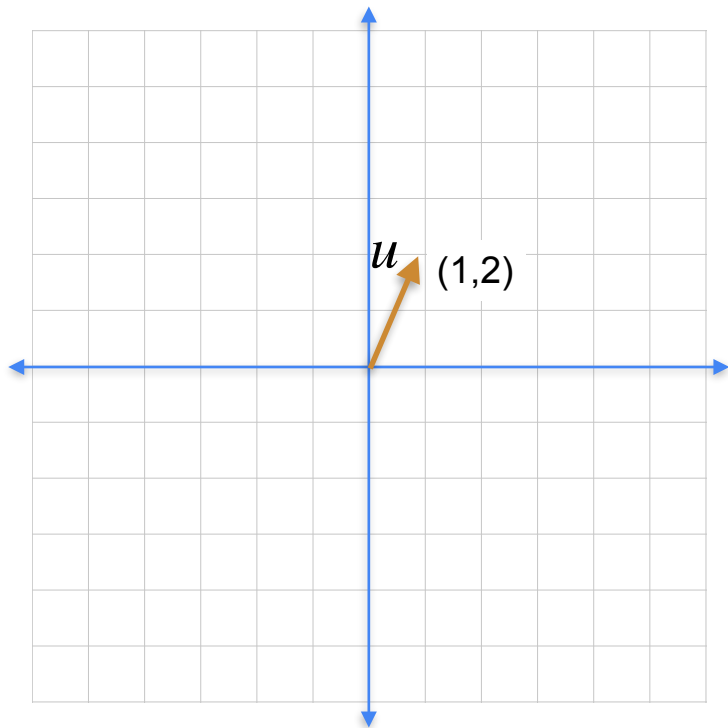
**Multiplying a vector by a scalar**



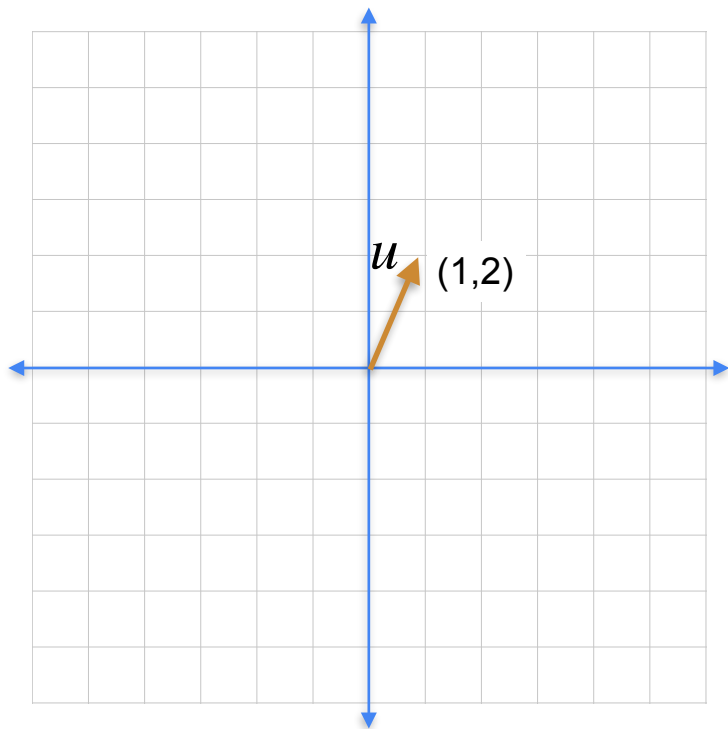
# Multiplying a vector by a scalar



# Multiplying a vector by a scalar

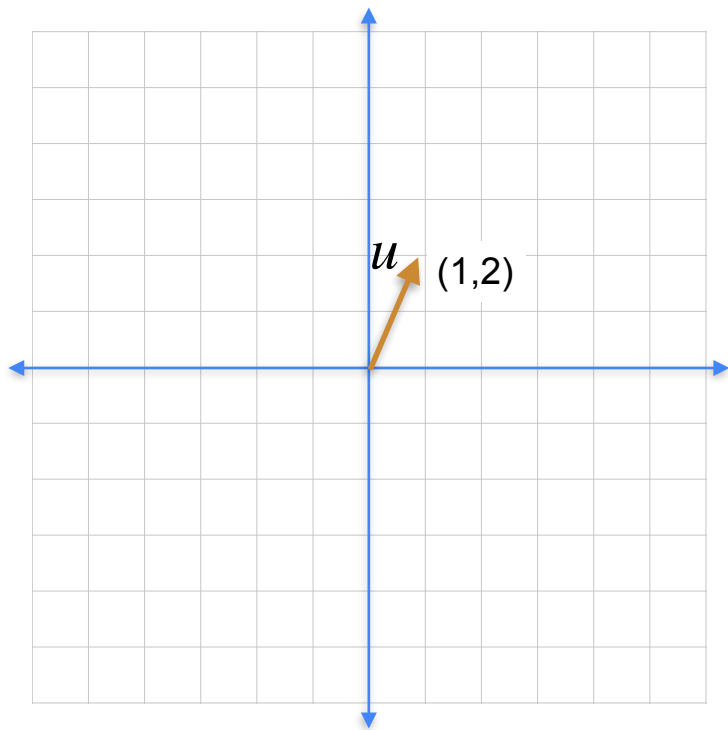


# Multiplying a vector by a scalar



$$u = (1,2)$$

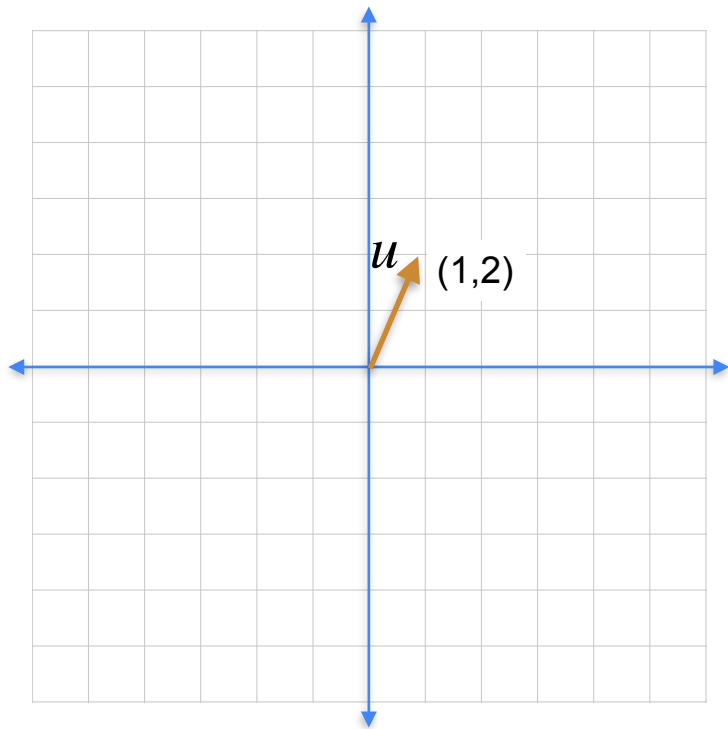
# Multiplying a vector by a scalar



$$u = (1,2)$$

$$\lambda = 3$$

# Multiplying a vector by a scalar

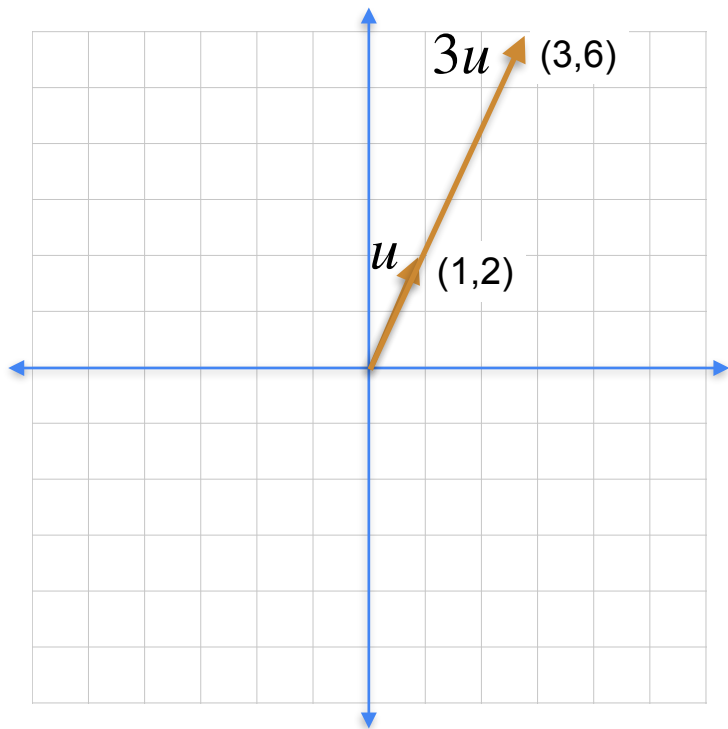


$$u = (1,2)$$

$$\lambda = 3$$

$$\lambda u = (3,6)$$

# Multiplying a vector by a scalar

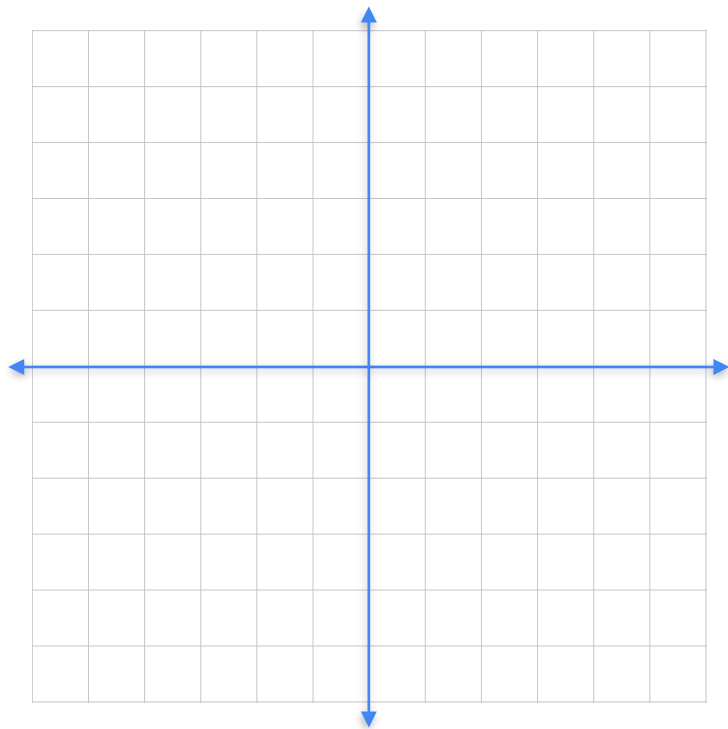


$$u = (1,2)$$

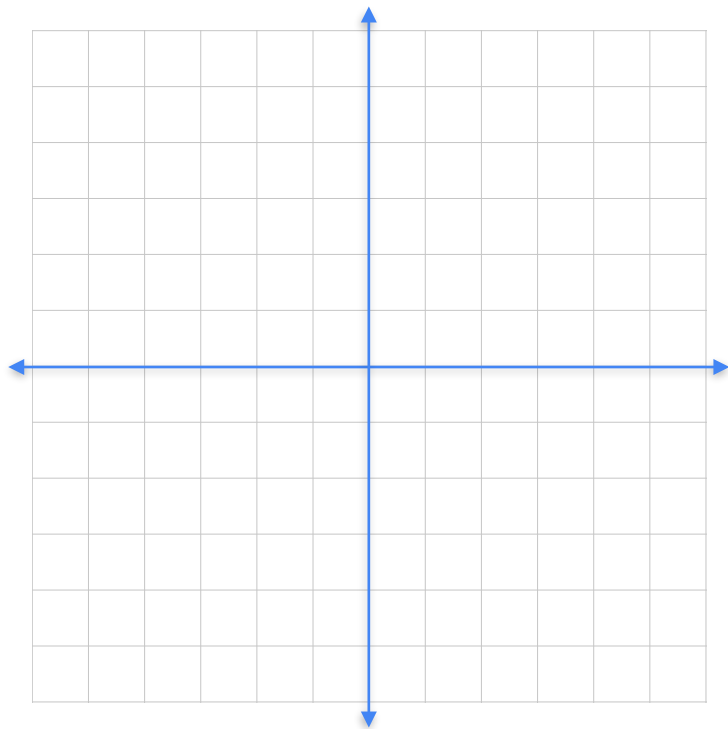
$$\lambda = 3$$

$$\lambda u = (3,6)$$

# If the scalar is negative



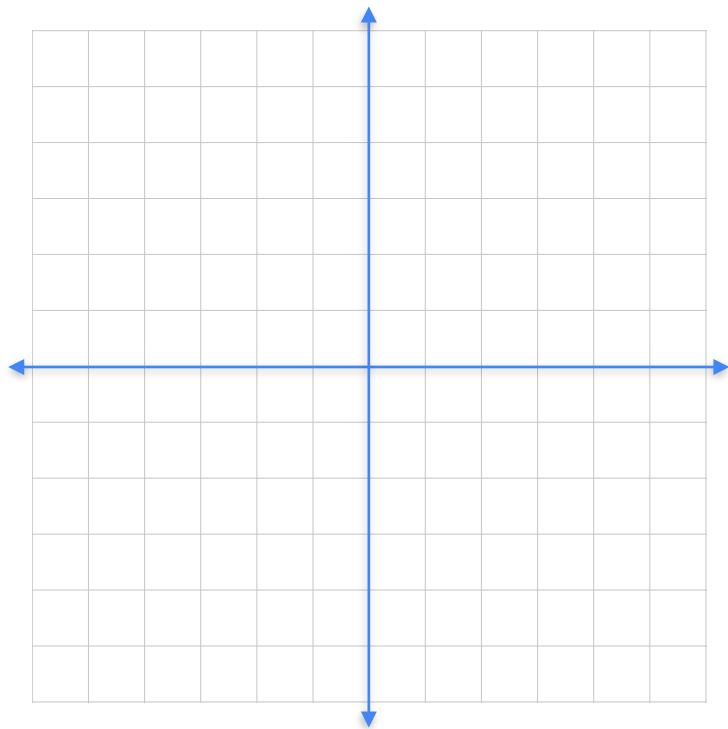
# If the scalar is negative



$$u = (1,2)$$



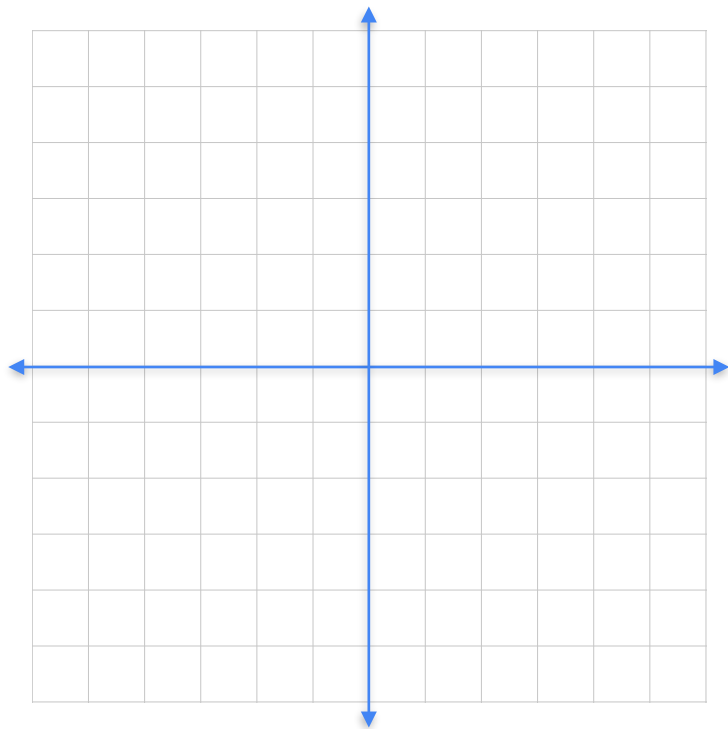
# If the scalar is negative



$$u = (1, 2)$$

$$\lambda = -2$$

# If the scalar is negative

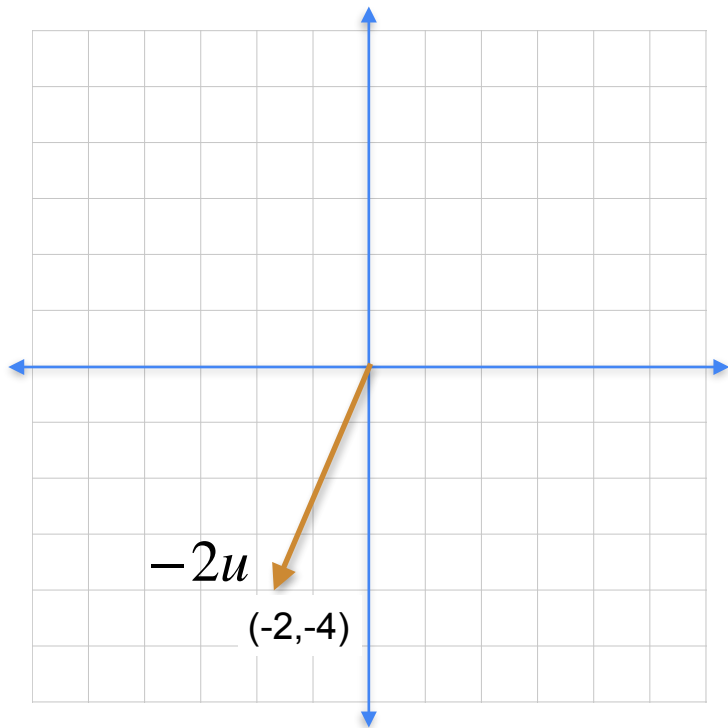


$$u = (1, 2)$$

$$\lambda = -2$$

$$\lambda u = (-2, -4)$$

# If the scalar is negative

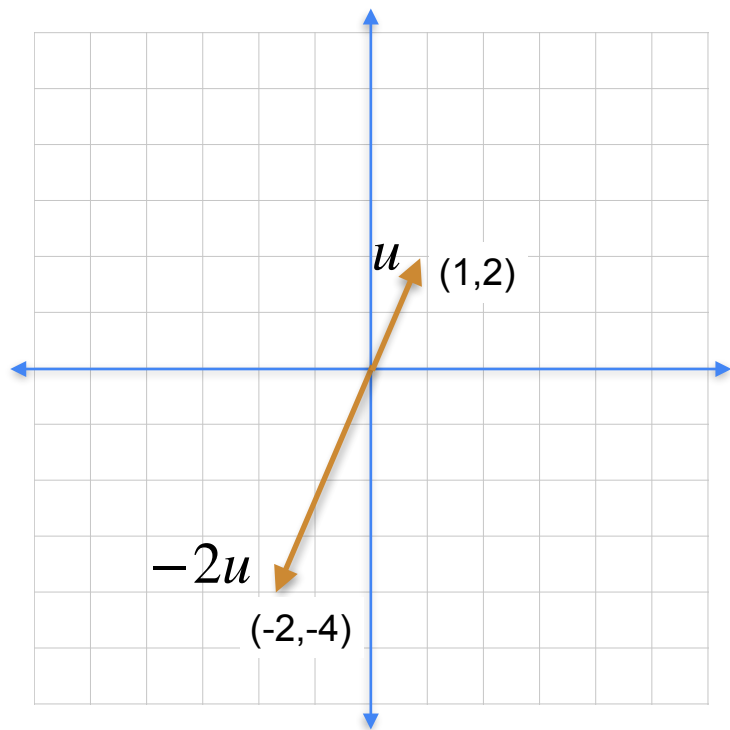


$$u = (1, 2)$$

$$\lambda = -2$$

$$\lambda u = (-2, -4)$$

# If the scalar is negative



$$u = (1,2)$$

$$\lambda = -2$$

$$\lambda u = (-2, -4)$$



DeepLearning.AI

# Vectors and Linear Transformations

---

## **The dot product**

# A shortcut for linear operations

# A shortcut for linear operations

## **Quantities**

2 apples

4 bananas

1 cherry

# A shortcut for linear operations

## **Quantities**

2 apples  
4 bananas  
1 cherry

## **Prices**

apples: \$3  
bananas: \$5  
cherries: \$2



# A shortcut for linear operations

## **Quantities**

2 apples  
4 bananas  
1 cherry

## **Prices**

apples: \$3  
bananas: \$5  
cherries: \$2

## **Total price**




# A shortcut for linear operations

## Quantities

2 apples

4 bananas

1 cherry

|   |   |
|---|---|
|  | 2 |
|  | 4 |
|  | 1 |

## Prices

apples: \$3

bananas: \$5

cherries: \$2

## Total price




# A shortcut for linear operations

## Quantities

2 apples

4 bananas

1 cherry

|   |   |
|---|---|
|  | 2 |
|  | 4 |
|  | 1 |

## Prices

apples: \$3

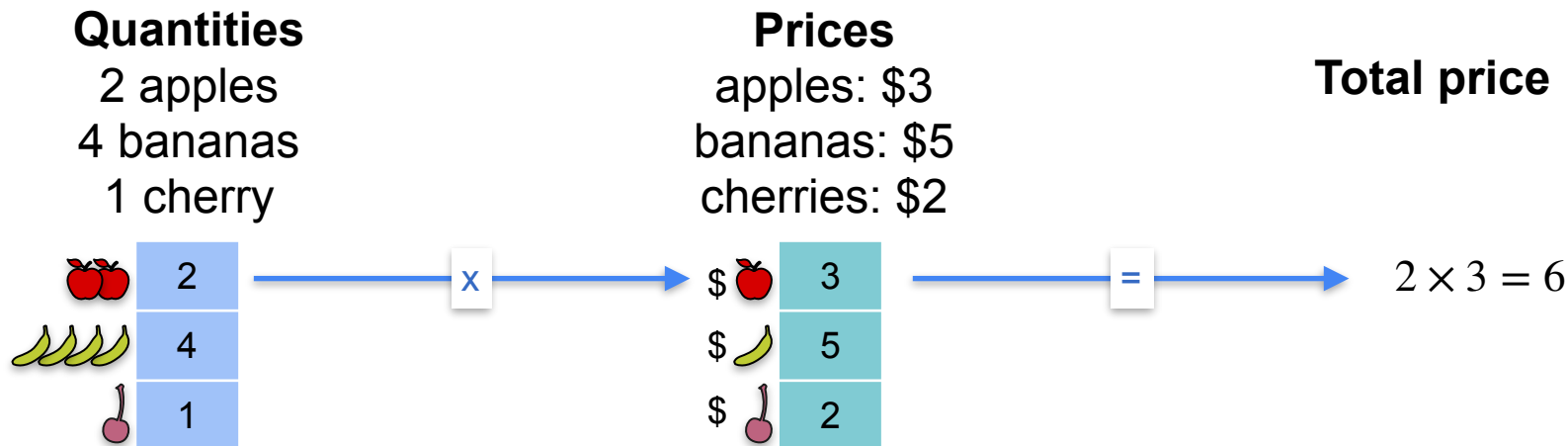
bananas: \$5

cherries: \$2

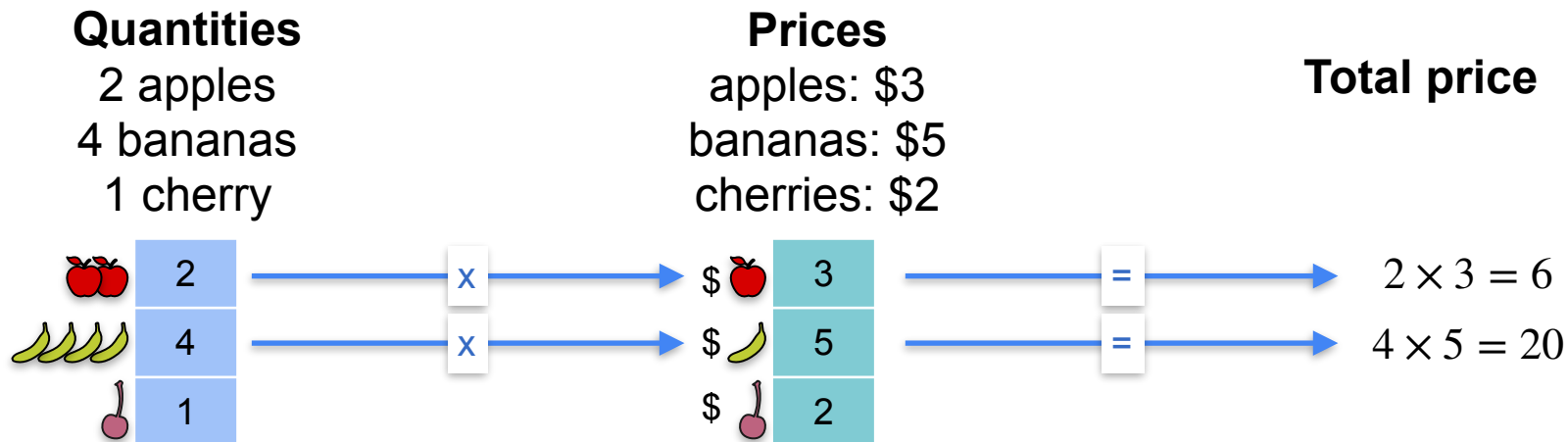
|  |   |
|--|---|
| \$  | 3 |
| \$  | 5 |
| \$  | 2 |

**Total price**

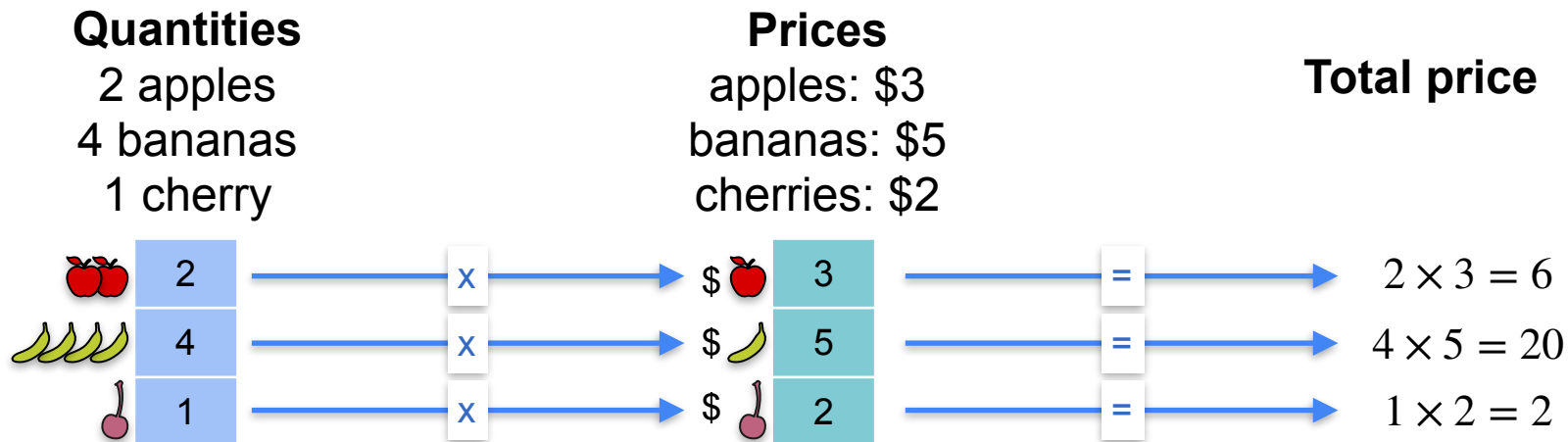
# A shortcut for linear operations



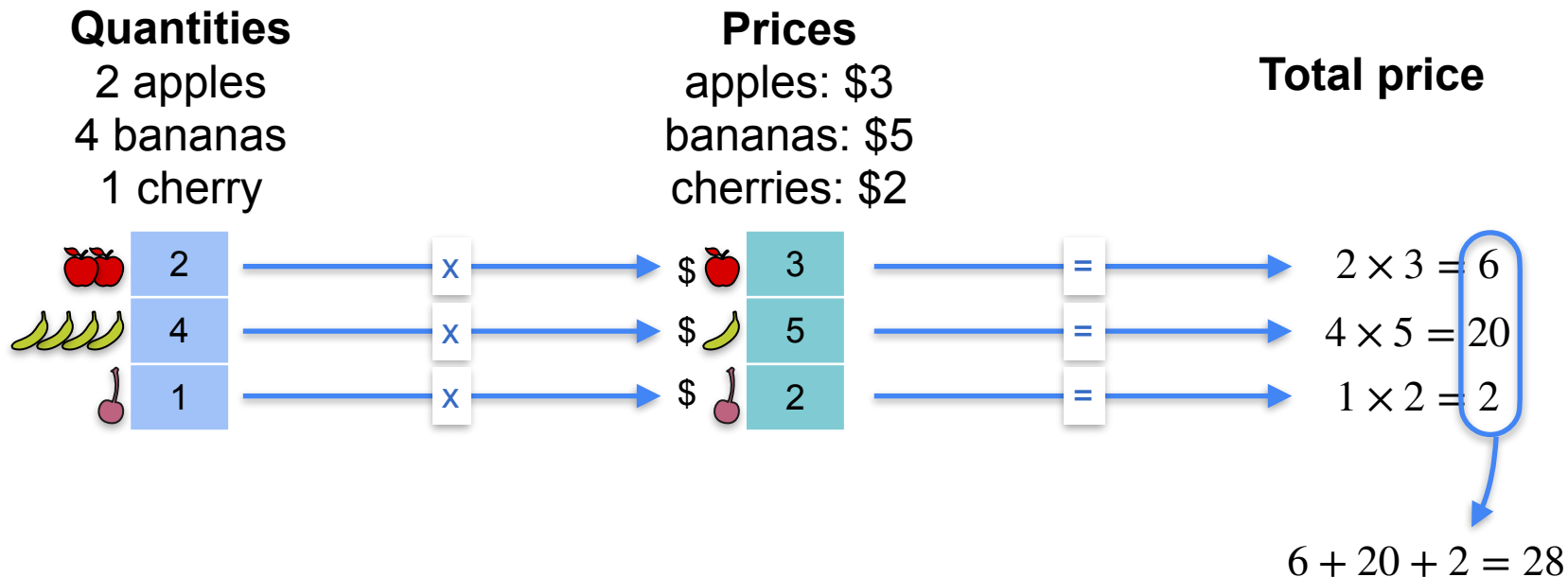
# A shortcut for linear operations



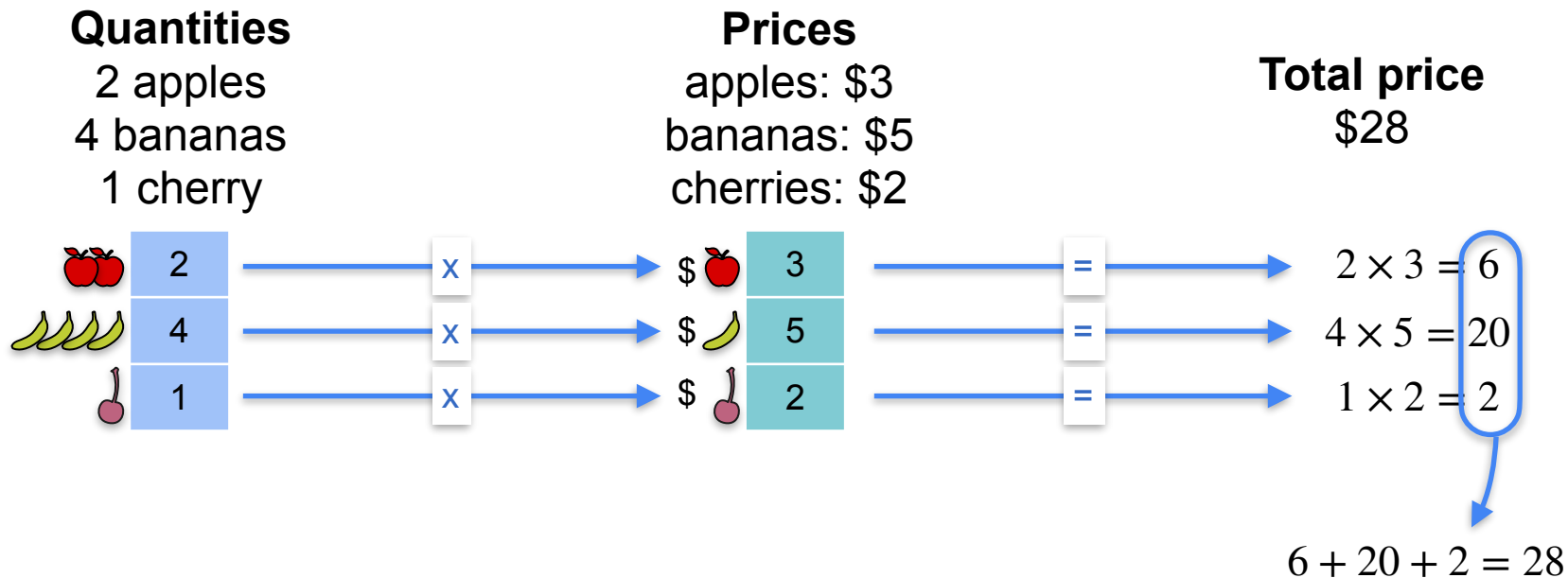
# A shortcut for linear operations



# A shortcut for linear operations






# A shortcut for linear operations








# The dot product

The diagram illustrates the dot product of two vectors. The first vector, represented by a blue column, contains the values 2, 4, and 1, corresponding to 2 apples, 4 bananas, and 1 cherry. The second vector, represented by a teal column, contains the values 3, 5, and 2, corresponding to \$3 for an apple, \$5 for a banana, and \$2 for a cherry. The dot product is calculated as  $2 \times 3 + 4 \times 5 + 1 \times 2 = 28$ , resulting in a total cost of \$28.

|   |   |
|---|---|
|  | 2 |
|  | 4 |
|  | 1 |


·




|  |   |
|--|---|
| \$  | 3 |
| \$  | 5 |
| \$  | 2 |

=




\$28

# The dot product



|   |   |
|---|---|
|  | 2 |
|  | 4 |
|  | 1 |

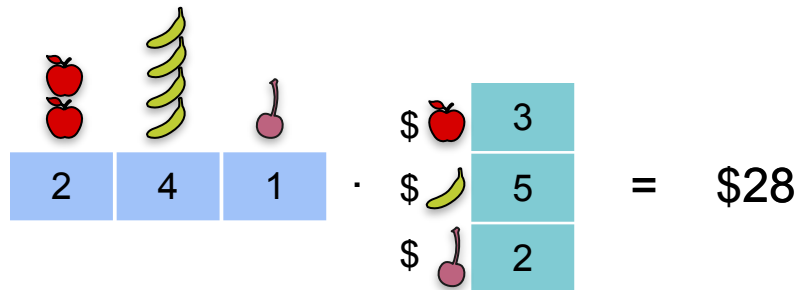
 $\cdot$ 

|  |   |
|--|---|
| \$  | 3 |
| \$  | 5 |
| \$  | 2 |







 $= \$28$

$$2 \cdot 3 + 4 \cdot 5 + 1 \cdot 2 = 28$$

# The dot product



The diagram illustrates a dot product calculation for fruit prices. On the left, a vector of quantities is shown in blue boxes: 2 apples, 4 bananas, and 1 cherry. This is multiplied (indicated by a dot) by a vector of prices in teal boxes: \$3 for an apple, \$5 for a banana, and \$2 for a cherry. The result is \$28.

|   |   |   |   |
|---|---|---|---|
|  |  |  |   |
| 2   | 4   | 1   | ·   |
|   |   |   |  3 |
|   |   |   |  5 |
|   |   |   |  2 |
|   |   |   | = \$28  |

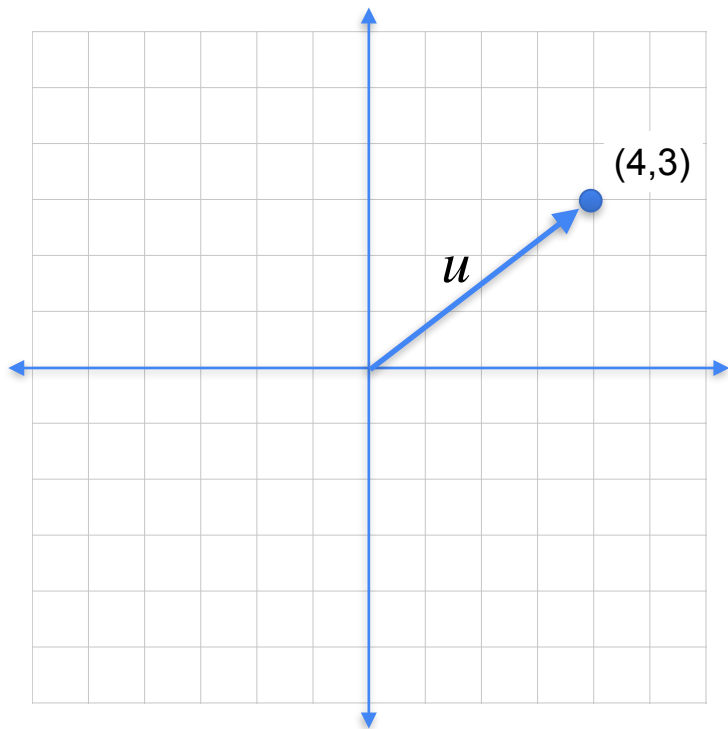
$$2 \cdot 3 + 4 \cdot 5 + 1 \cdot 2 = 28$$

# The dot product

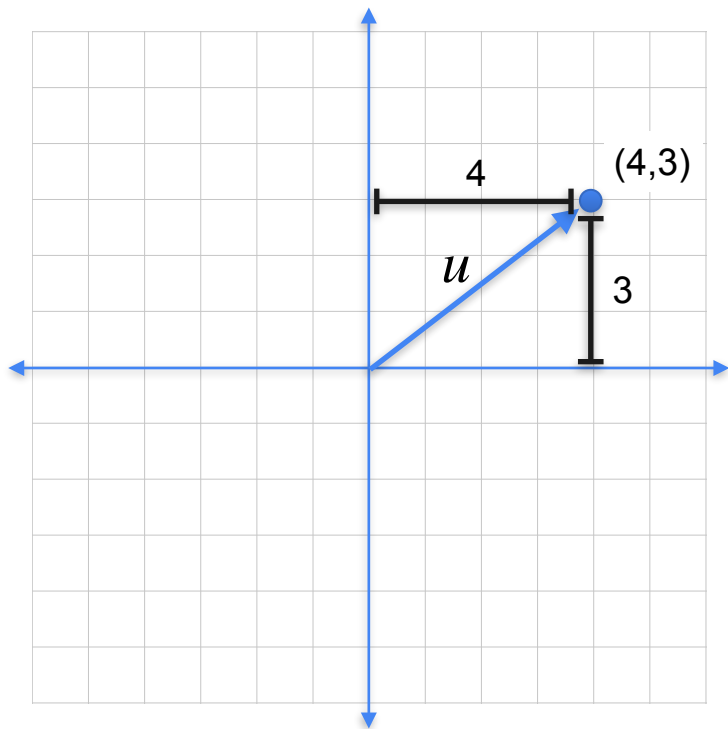
$$\begin{bmatrix} 2 & 4 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} = 28$$

$$2 \cdot 3 + 4 \cdot 5 + 1 \cdot 2 = 28$$

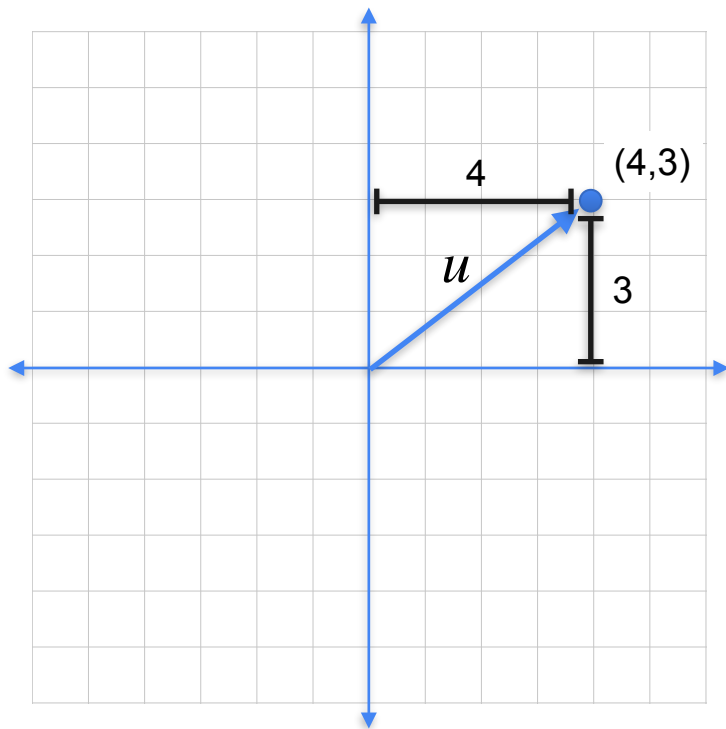
# Norm of a vector using dot product



# Norm of a vector using dot product

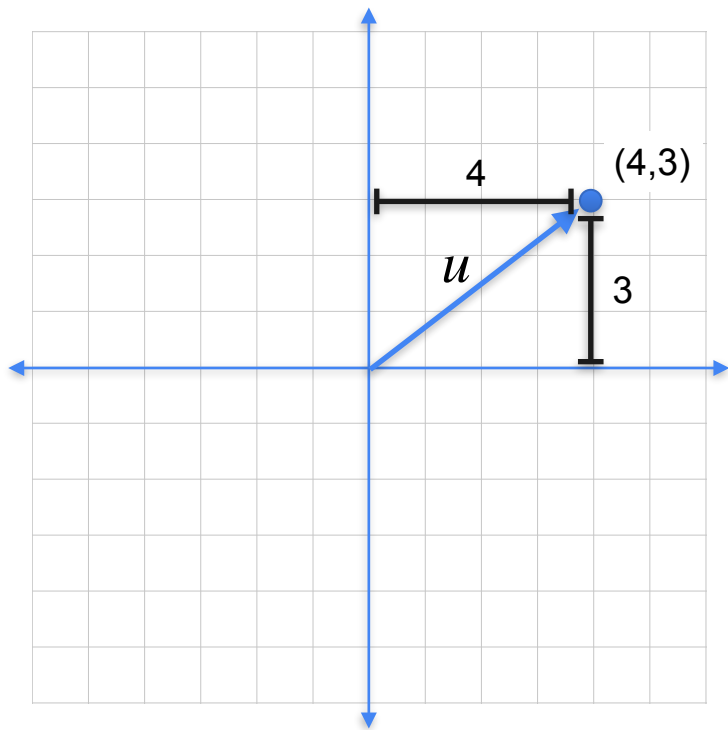


# Norm of a vector using dot product



$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

# Norm of a vector using dot product

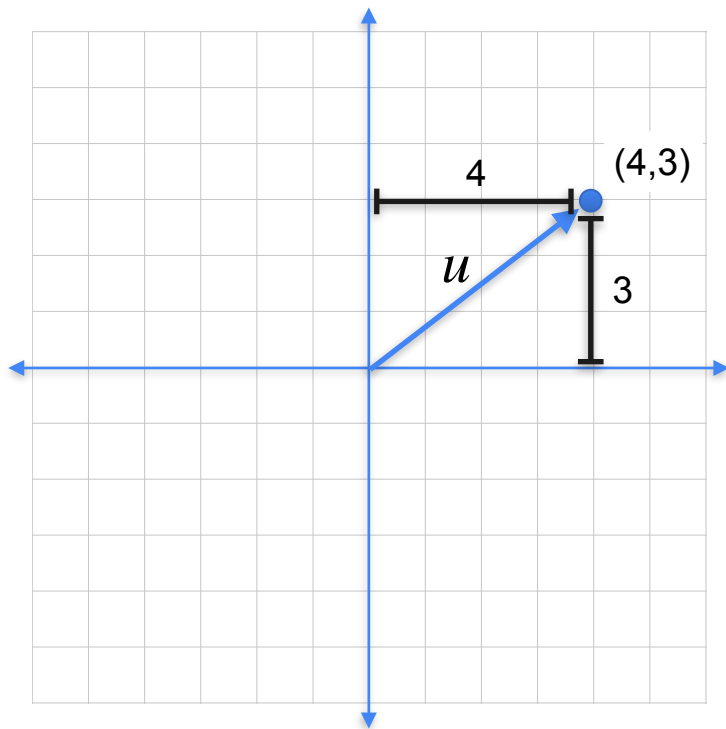


$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

$$\begin{bmatrix} 4 & 3 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 3 \end{bmatrix} = 25$$



# Norm of a vector using dot product

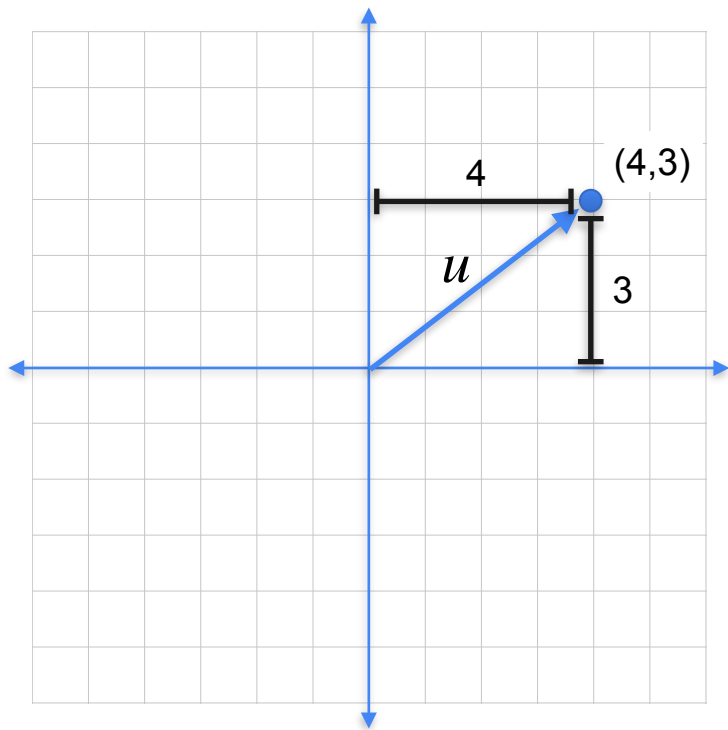


$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

$$\begin{bmatrix} 4 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = 25$$

$$L2 - norm = \sqrt{\text{dot product}(u, u)}$$

# Norm of a vector using dot product



$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

$$\begin{bmatrix} 4 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = 25$$

$$L2 - norm = \sqrt{\text{dot product}(u, u)}$$

$$|u|_2 = \sqrt{\langle u, u \rangle}$$



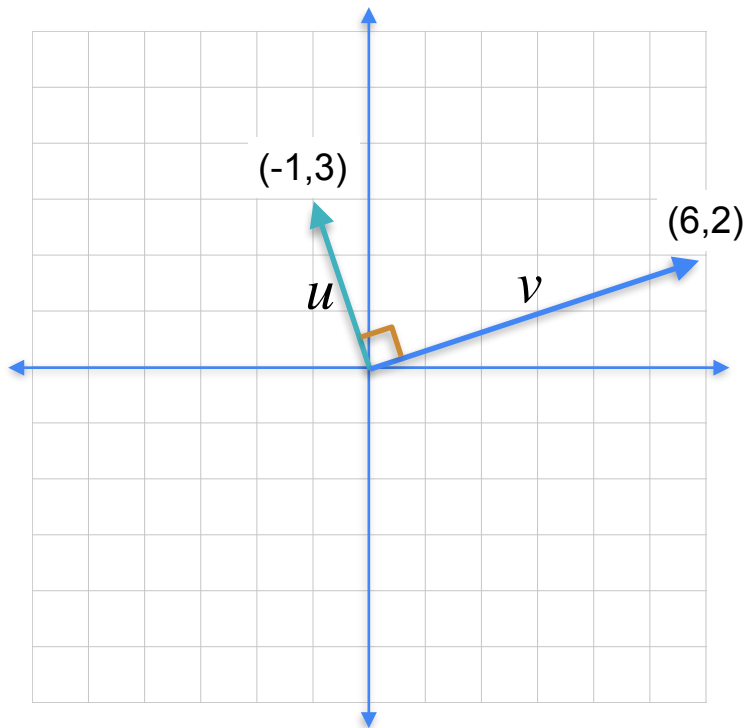
DeepLearning.AI

# Vectors and Linear Transformations

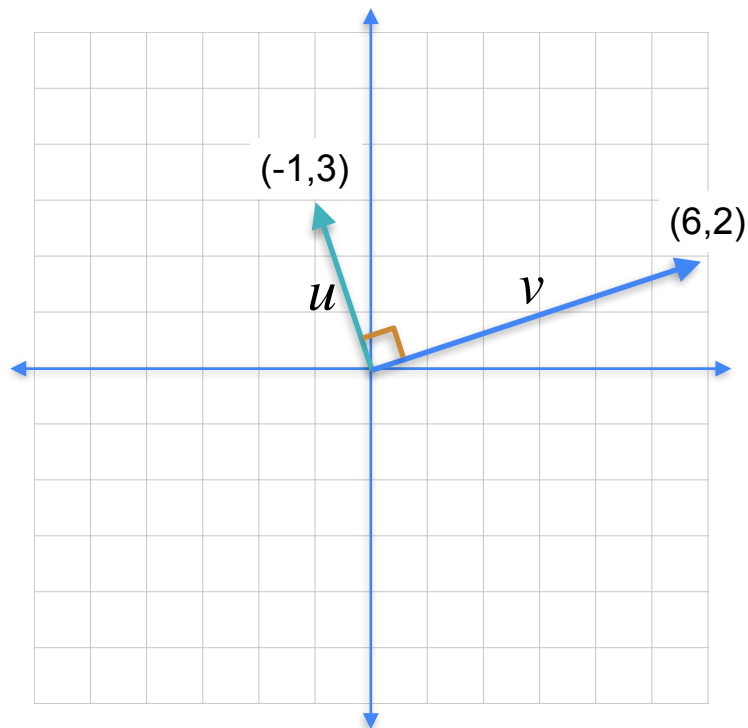
---

## **Geometric dot product**

# Orthogonal vectors have dot product 0



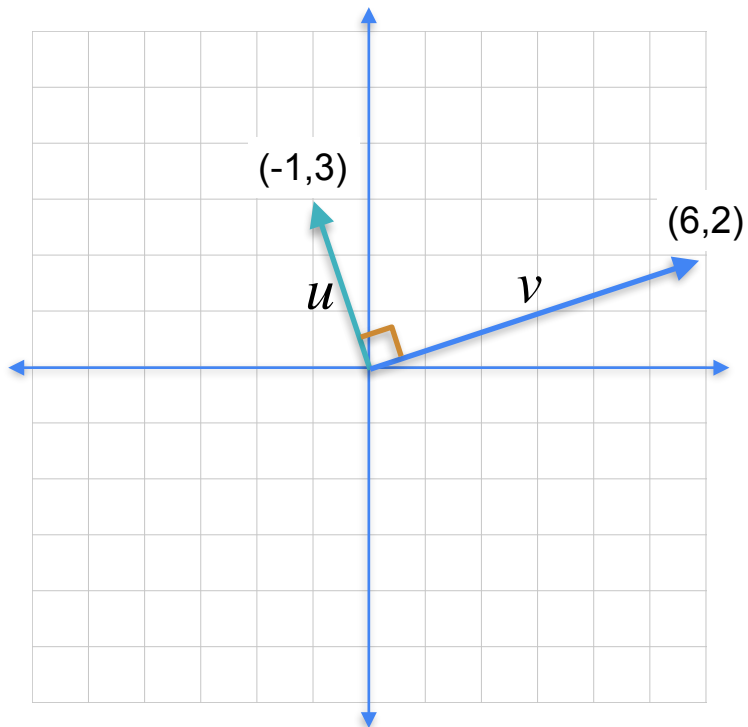
# Orthogonal vectors have dot product 0



6

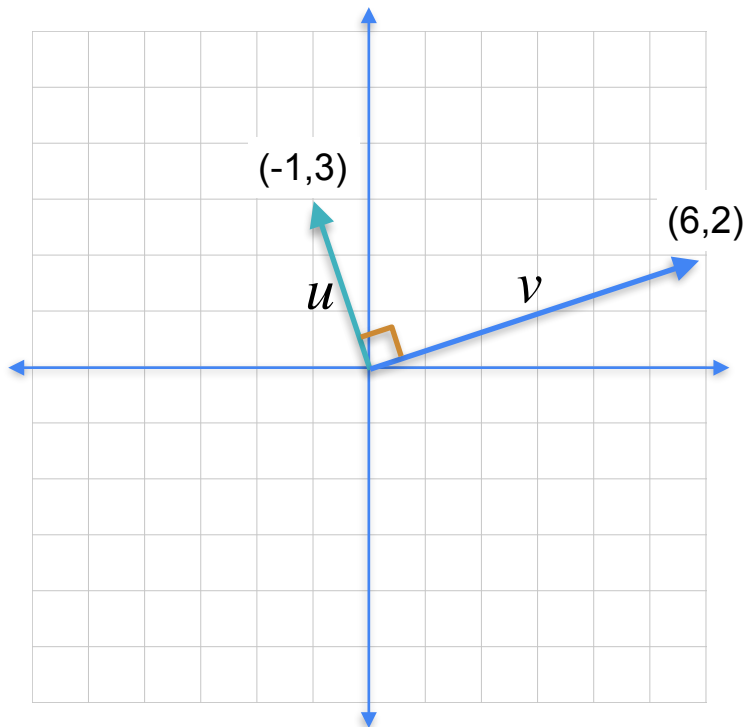
2

# Orthogonal vectors have dot product 0



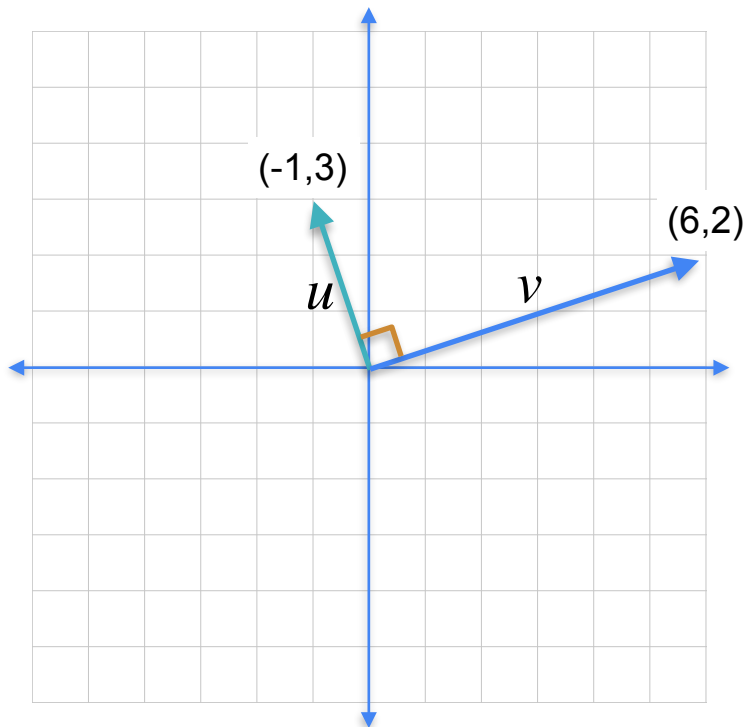
|   |   |    |
|---|---|----|
| 6 | 2 | -1 |
|   |   | 3  |

# Orthogonal vectors have dot product 0



$$\begin{bmatrix} 6 & 2 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 3 \end{bmatrix} = 0$$

# Orthogonal vectors have dot product 0



$$\begin{bmatrix} 6 & 2 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 3 \end{bmatrix} = 0$$

$$\langle u, v \rangle = 0$$

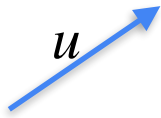


# The dot product

# The dot product

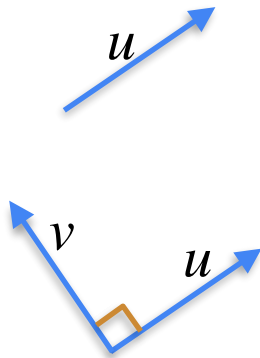


# The dot product



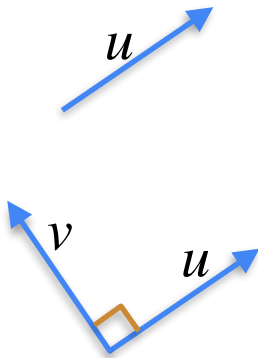
$$\langle u, u \rangle = |u|^2$$

# The dot product



$$\langle u, u \rangle = |u|^2$$

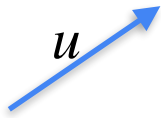
# The dot product



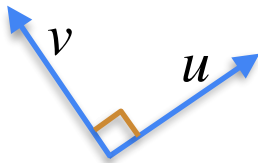
$$\langle u, u \rangle = |u|^2$$

$$\langle u, v \rangle = 0$$

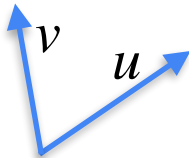
# The dot product



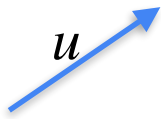
$$\langle u, u \rangle = |u|^2$$



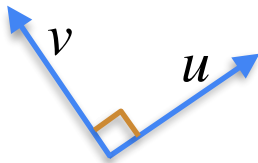
$$\langle u, v \rangle = 0$$



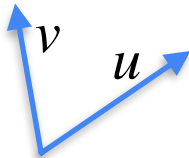
# The dot product



$$\langle u, u \rangle = |u|^2$$



$$\langle u, v \rangle = 0$$



$$\langle u, v \rangle = ?$$

# The dot product



# The dot product

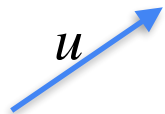


# The dot product



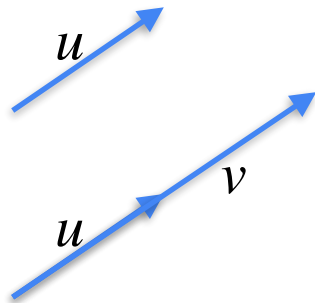
$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

# The dot product



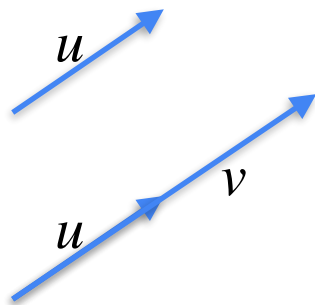
$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

# The dot product



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

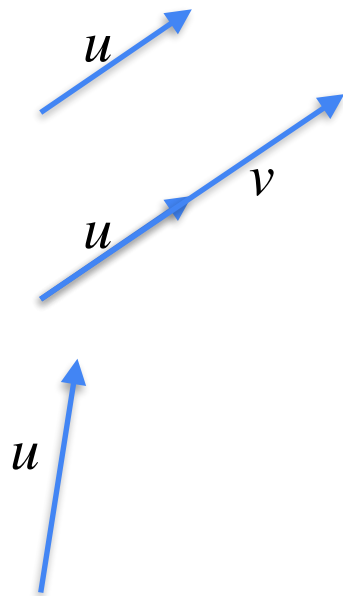
# The dot product



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v \rangle = |u| \cdot |v|$$

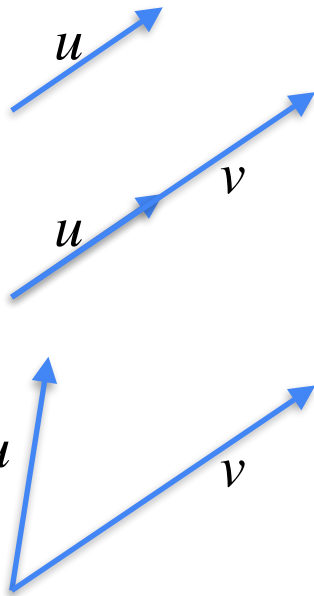
# The dot product



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v \rangle = |u| \cdot |v|$$

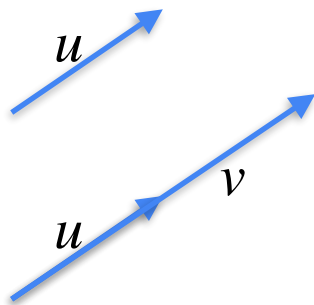
# The dot product



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

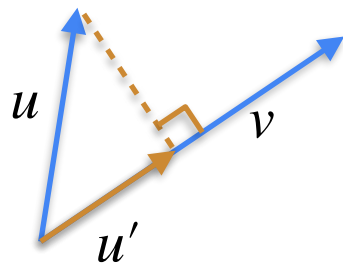
$$\langle u, v \rangle = |u| \cdot |v|$$

# The dot product



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v \rangle = |u| \cdot |v|$$

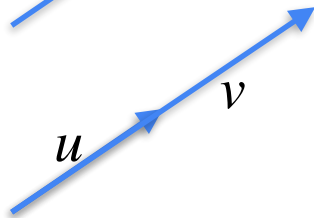




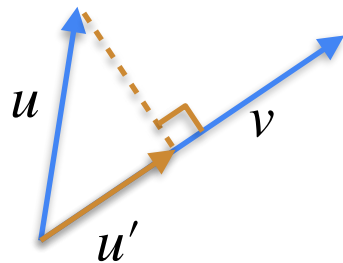
# The dot product



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

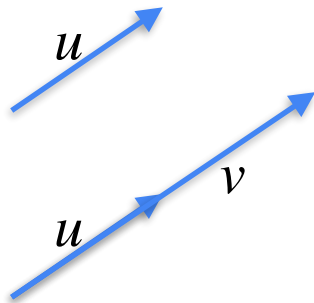


$$\langle u, v \rangle = |u| \cdot |v|$$



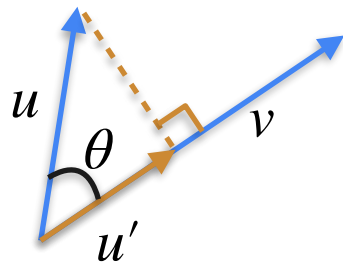
$$\langle u, v \rangle = |u'| \cdot |v|$$

# The dot product



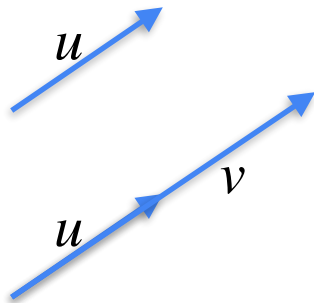
$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v \rangle = |u| \cdot |v|$$



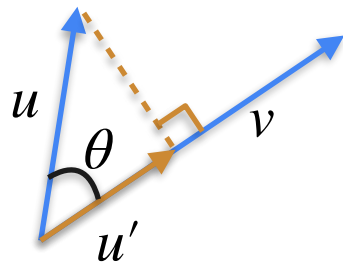
$$\langle u, v \rangle = |u'| \cdot |v|$$

# The dot product



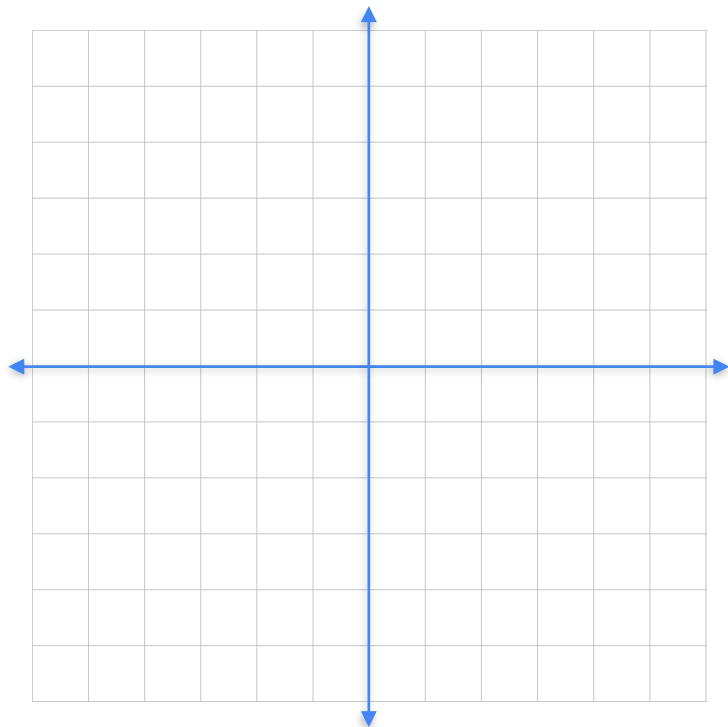
$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v \rangle = |u| \cdot |v|$$

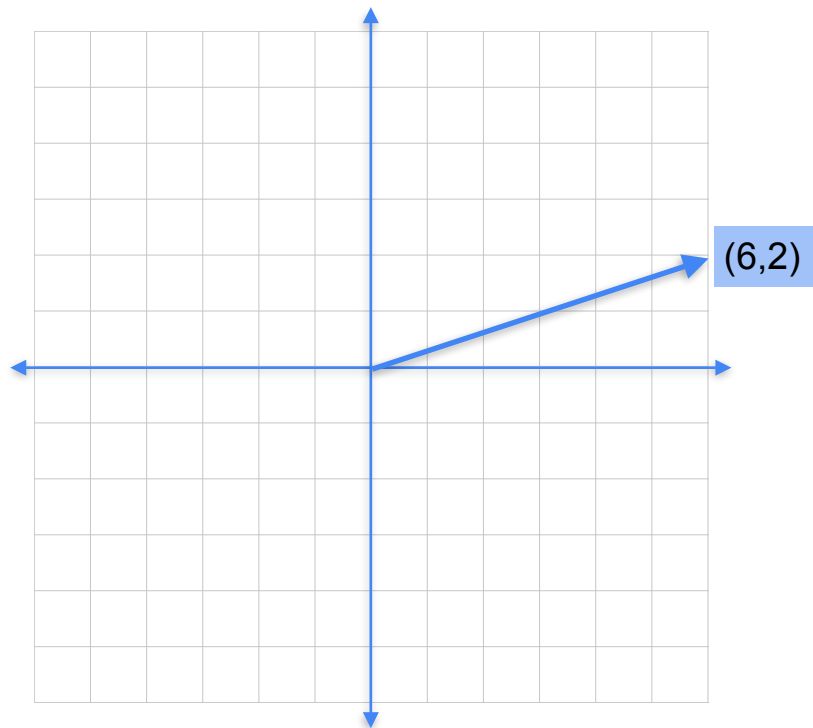


$$\begin{aligned}\langle u, v \rangle &= |u'| \cdot |v| \\ &= |u| |v| \cos(\theta)\end{aligned}$$

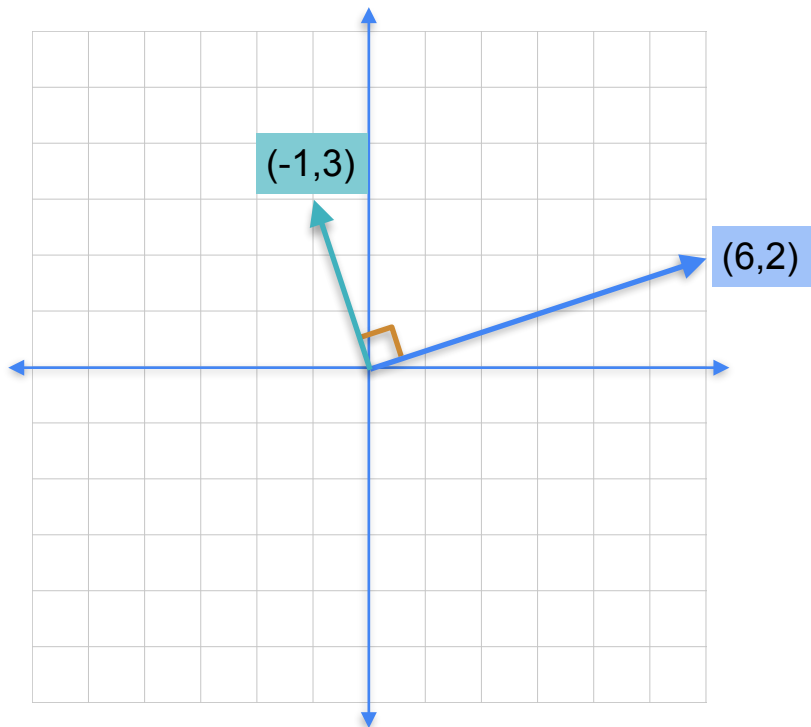
# Geometric dot product



# Geometric dot product

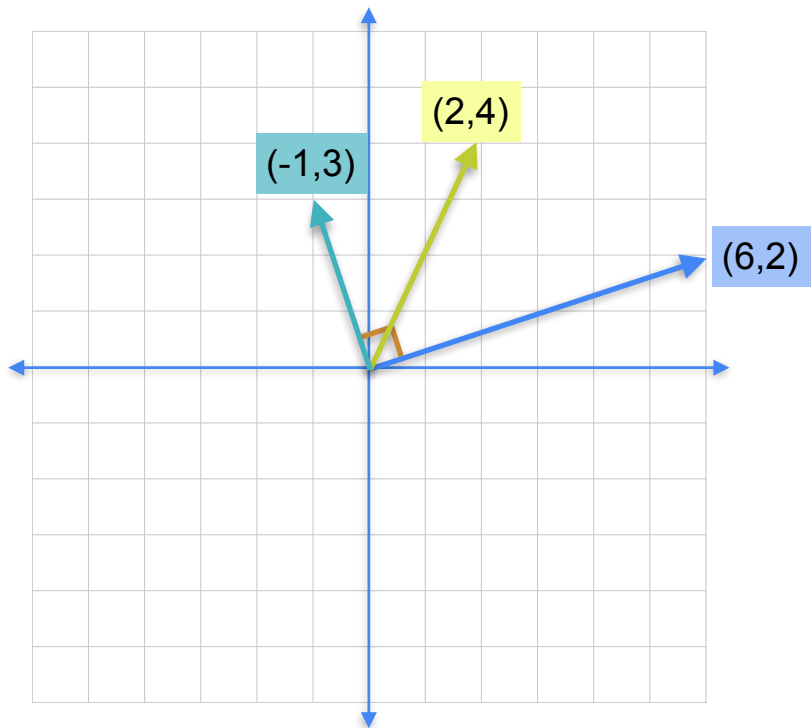


# Geometric dot product



$$\begin{bmatrix} 6 & 2 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 3 \end{bmatrix} = 0$$

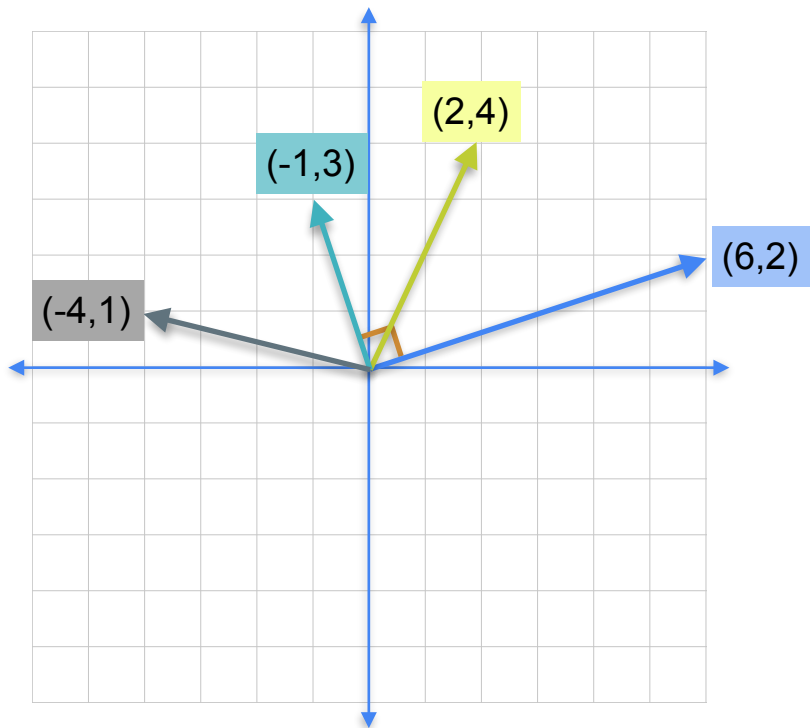
# Geometric dot product



$$\begin{bmatrix} 6 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 4 \end{bmatrix} = 20$$

$$\begin{bmatrix} 6 & 2 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 3 \end{bmatrix} = 0$$

# Geometric dot product



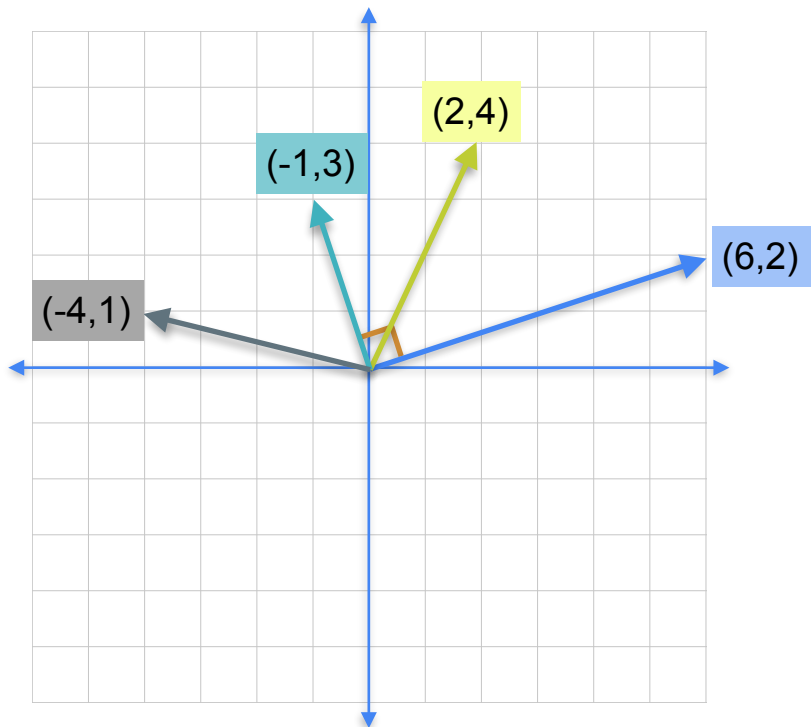
$$\begin{bmatrix} 6 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 4 \end{bmatrix} = 20$$

$$\begin{bmatrix} 6 & 2 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 6 & 2 \end{bmatrix} \cdot \begin{bmatrix} -4 \\ 1 \end{bmatrix} = -22$$



# Geometric dot product

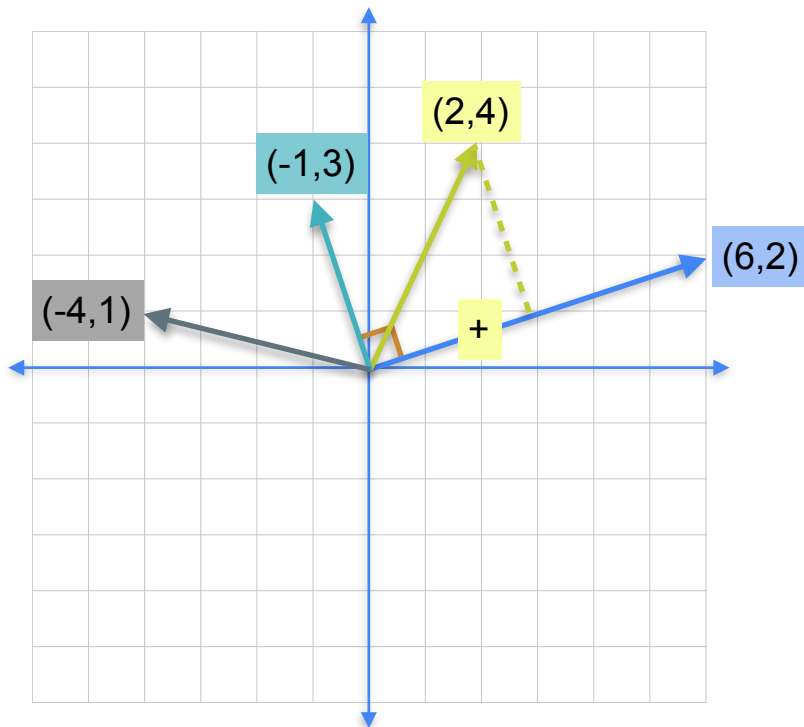


$$\begin{bmatrix} 6 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 4 \end{bmatrix} = 20 \quad \text{Positive}$$

$$\begin{bmatrix} 6 & 2 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 6 & 2 \end{bmatrix} \cdot \begin{bmatrix} -4 \\ 1 \end{bmatrix} = -22$$

# Geometric dot product

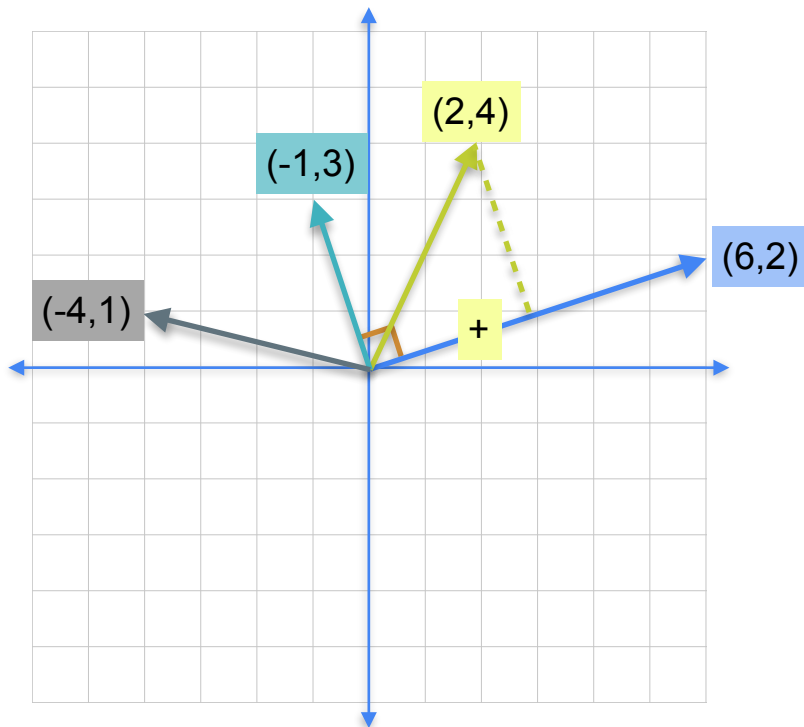


$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \end{bmatrix} = 20 \quad \text{Positive}$$

$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} -4 \\ 1 \end{bmatrix} = -22$$

# Geometric dot product

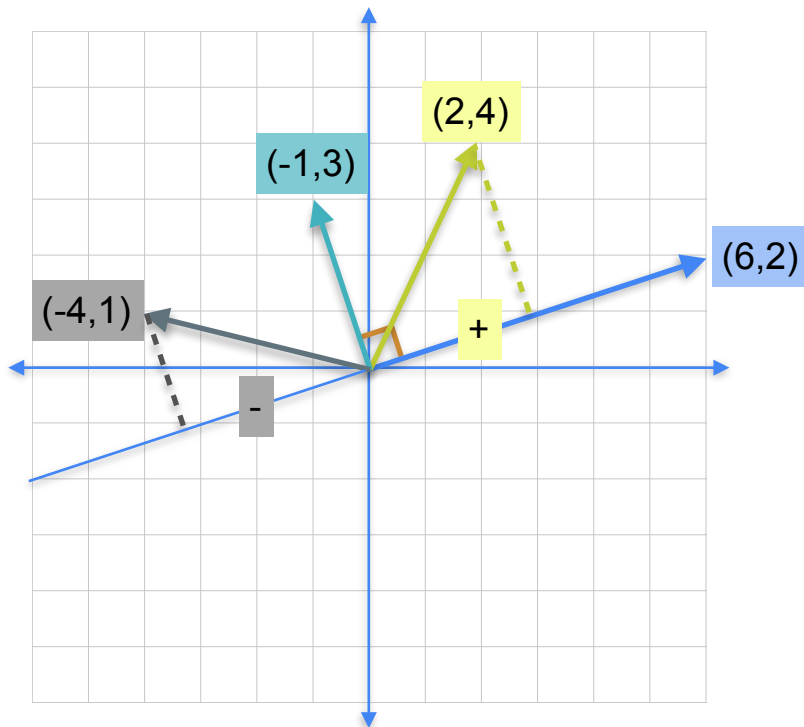


$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \end{bmatrix} = 20 \quad \text{Positive}$$

$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} -4 \\ 1 \end{bmatrix} = -22 \quad \text{Negative}$$

# Geometric dot product

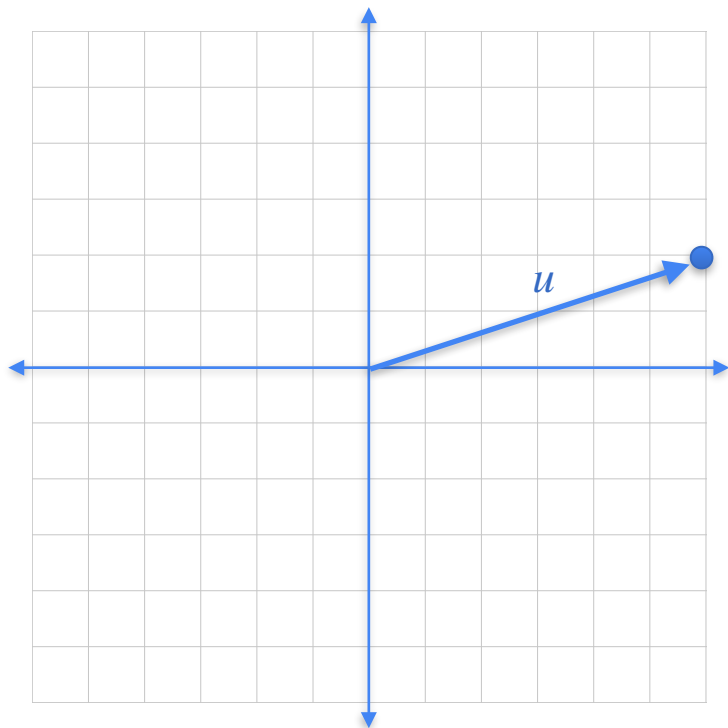


$$\begin{bmatrix} 6 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 4 \end{bmatrix} = 20 \quad \text{Positive}$$

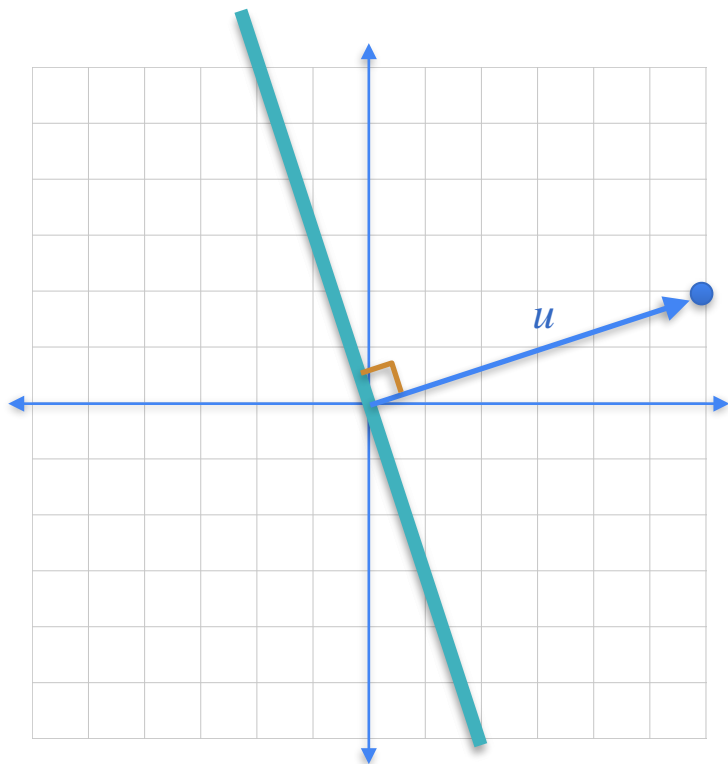
$$\begin{bmatrix} 6 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 6 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} -4 \\ 1 \end{bmatrix} = -22 \quad \text{Negative}$$

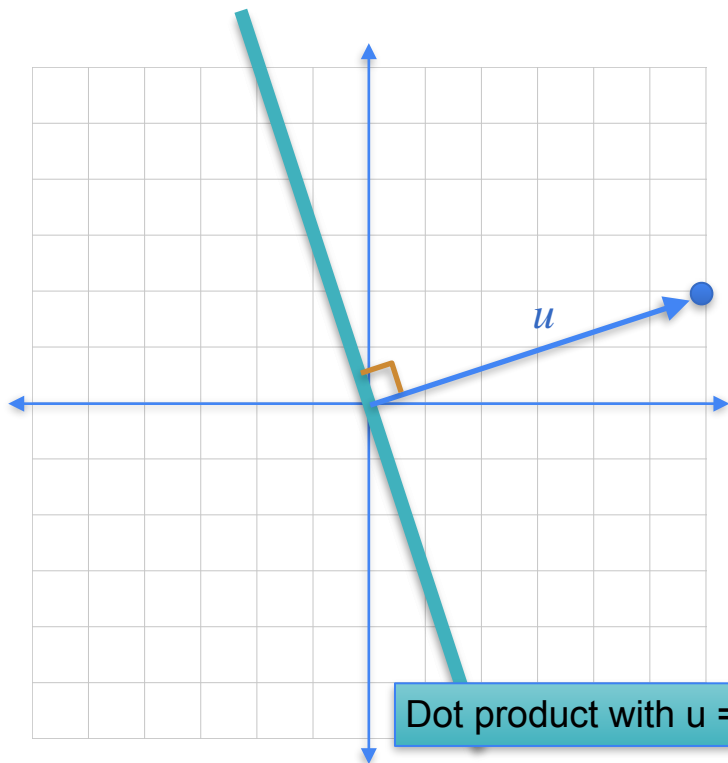
# Geometric dot product



# Geometric dot product



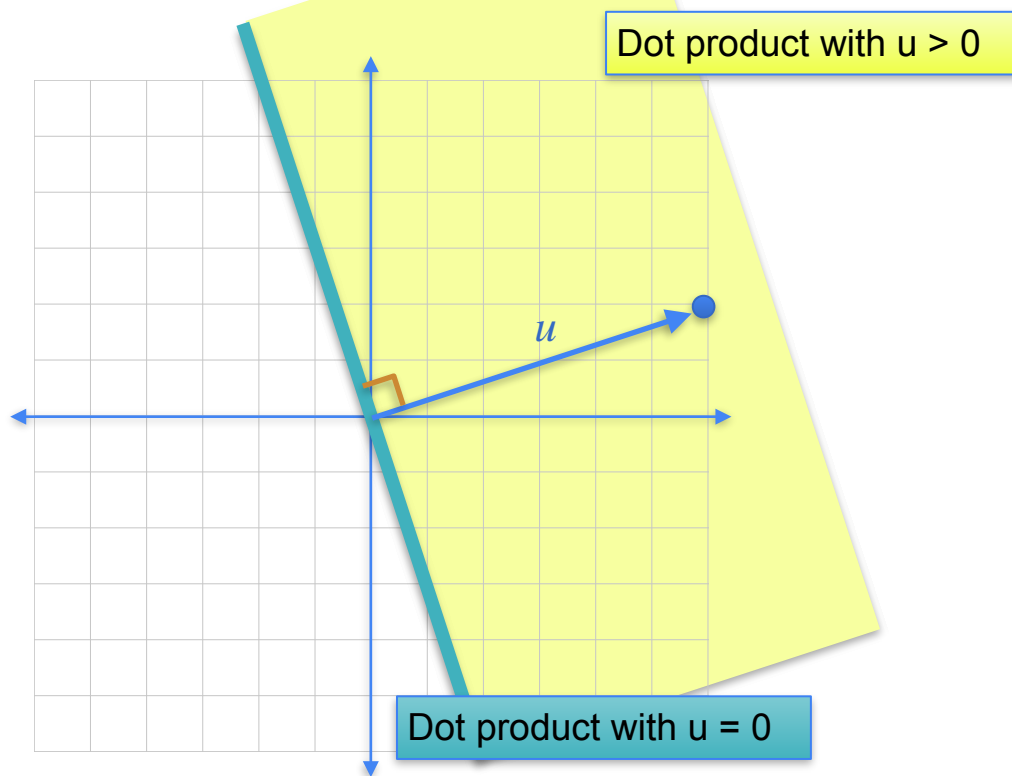
# Geometric dot product



$$\langle u, v \rangle = 0$$

Dot product with  $u = 0$

# Geometric dot product



$$\langle u, v \rangle > 0$$

$$\langle u, v \rangle = 0$$



# Geometric dot product

Dot product with  $u > 0$

$$\langle u, v \rangle > 0$$

$$\langle u, v \rangle = 0$$

$$\langle u, v \rangle < 0$$

Dot product with  $u < 0$

Dot product with  $u = 0$



DeepLearning.AI

# Vectors and Linear Transformations

---

**Multiplying a matrix by a  
vector**

# Equations as dot product

$$2a + 4b + c = 28$$

The diagram illustrates the equation  $2a + 4b + c = 28$  using fruit icons and boxes. On the left, there are three blue boxes containing the numbers 2, 4, and 1. Above the box with 2 are two red apples. Above the box with 4 are four yellow bananas. Above the box with 1 is one red cherry. To the right of these boxes is a dot operator. Further right is a vertical stack of three light blue boxes labeled 'a', 'b', and 'c'. To the left of the 'a' box is a red apple icon with a dollar sign. To the left of the 'b' box is a yellow banana icon with a dollar sign. To the left of the 'c' box is a red cherry icon with a dollar sign. To the right of this stack is an equals sign, followed by a dollar sign and an orange box containing the number 28.

# Equations as dot product

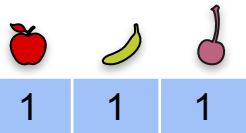
$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

# Equations as dot product

$$a + b + c = 10$$

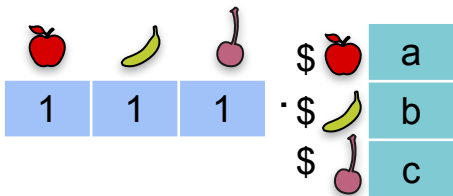


$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

# Equations as dot product

$$a + b + c = 10$$



$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

# Equations as dot product

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

The diagram illustrates the dot product for the equation  $a + b + c = 10$ . It shows a row vector  $[1, 1, 1]$  (represented by blue boxes) multiplied by a column vector  $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$  (represented by teal boxes). The column vector is preceded by a dollar sign (\$) and each element is accompanied by a fruit icon: an apple for  $a$ , a banana for  $b$ , and a cherry for  $c$ . The result of the dot product is shown as  $= \$ 10$  (represented by an orange box).

# Equations as dot product

$$a + b + c = 10$$

Diagram illustrating the dot product for the equation  $a + b + c = 10$ . The row vector  $[1, 1, 1]$  is multiplied by the column vector  $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$  to equal 10. The fruit icons (apple, banana, cherry) are used to represent the variables  $a$ ,  $b$ , and  $c$  respectively.

$$a + 2b + c = 15$$

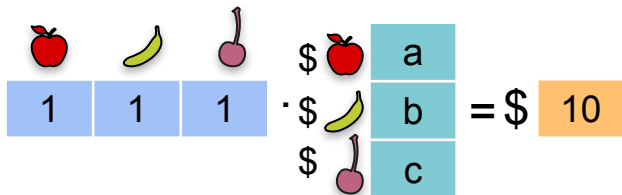
Diagram illustrating the dot product for the equation  $a + 2b + c = 15$ . The row vector  $[1, 2, 1]$  is multiplied by the column vector  $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$  to equal 15. The fruit icons (apple, banana, cherry) are used to represent the variables  $a$ ,  $b$ , and  $c$  respectively.

$$a + b + 2c = 12$$

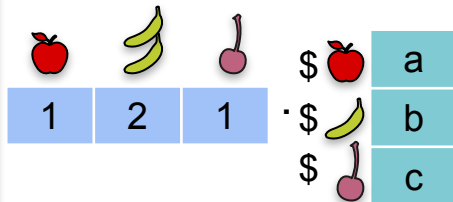


# Equations as dot product

$$a + b + c = 10$$



$$a + 2b + c = 15$$



$$a + b + 2c = 12$$

# Equations as dot product

$$a + b + c = 10$$

Diagram illustrating the dot product for the equation  $a + b + c = 10$ . The row vector  $[1, 1, 1]$  is multiplied by the column vector  $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$  to equal 10.

$$a + 2b + c = 15$$

Diagram illustrating the dot product for the equation  $a + 2b + c = 15$ . The row vector  $[1, 2, 1]$  is multiplied by the column vector  $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$  to equal 15.

$$a + b + 2c = 12$$

# Equations as dot product

$$a + b + c = 10$$

Diagram illustrating the dot product for the equation  $a + b + c = 10$ . The input vector is  $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$  (represented by 1 apple, 1 banana, 1 cherry). The weight vector is  $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$  (represented by 1 apple, 1 banana, 1 cherry). The dot product is calculated as  $1 \cdot a + 1 \cdot b + 1 \cdot c = 10$ .

$$a + 2b + c = 15$$

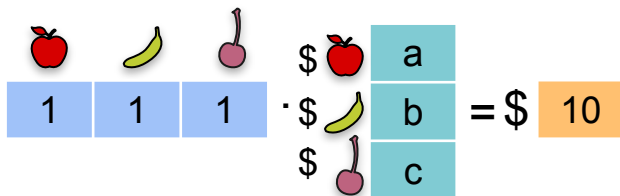
Diagram illustrating the dot product for the equation  $a + 2b + c = 15$ . The input vector is  $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$  (represented by 1 apple, 2 bananas, 1 cherry). The weight vector is  $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$  (represented by 1 apple, 1 banana, 1 cherry). The dot product is calculated as  $1 \cdot a + 2 \cdot b + 1 \cdot c = 15$ .

$$a + b + 2c = 12$$

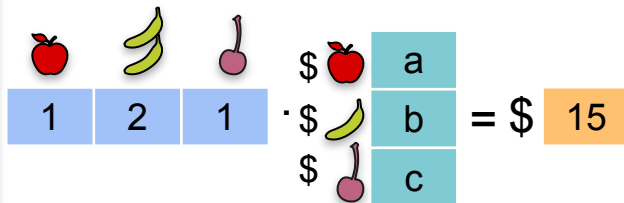
Diagram illustrating the dot product for the equation  $a + b + 2c = 12$ . The input vector is  $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$  (represented by 1 apple, 1 banana, 2 cherries). The weight vector is  $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$  (represented by 1 apple, 1 banana, 1 cherry). The dot product is calculated as  $1 \cdot a + 1 \cdot b + 2 \cdot c = 12$ .

# Equations as dot product

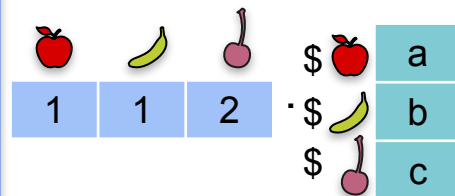
$$a + b + c = 10$$



$$a + 2b + c = 15$$



$$a + b + 2c = 12$$



# Equations as dot product

$$a + b + c = 10$$

$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} \$ & \text{apple} & \text{banana} \\ \$ & \text{cherry} & \text{cherry} \end{bmatrix} = \$ 10$

$$a + 2b + c = 15$$

$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} \$ & \text{apple} & \text{banana} \\ \$ & \text{cherry} & \text{cherry} \end{bmatrix} = \$ 15$

$$a + b + 2c = 12$$

$\begin{bmatrix} 1 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} \$ & \text{apple} & \text{banana} \\ \$ & \text{cherry} & \text{cherry} \end{bmatrix} = \$ 12$

# Equations as dot product

$$a + b + c = 10$$

Diagram illustrating the dot product for the equation  $a + b + c = 10$ . The row vector  $[1, 1, 1]$  is multiplied by the column vector  $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$  to equal 10.

$$a + 2b + c = 15$$

Diagram illustrating the dot product for the equation  $a + 2b + c = 15$ . The row vector  $[1, 2, 1]$  is multiplied by the column vector  $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$  to equal 15.

$$a + b + 2c = 12$$

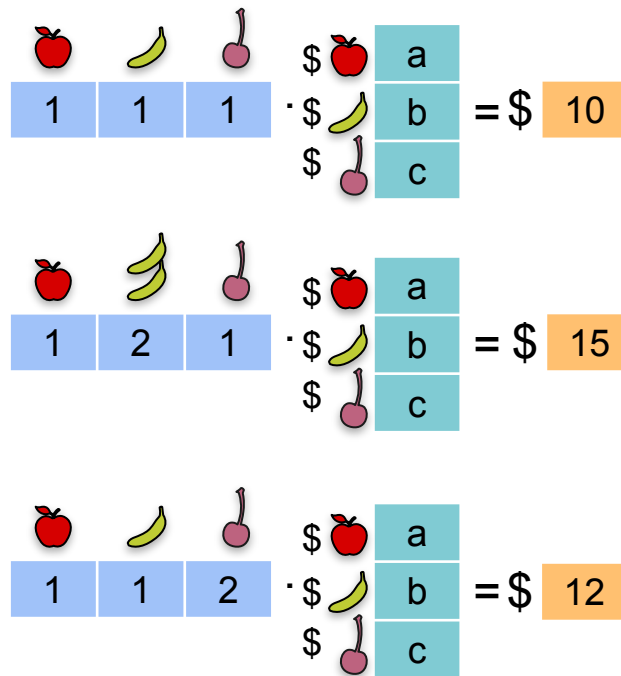
Diagram illustrating the dot product for the equation  $a + b + 2c = 12$ . The row vector  $[1, 1, 2]$  is multiplied by the column vector  $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$  to equal 12.

# Equations as dot product

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

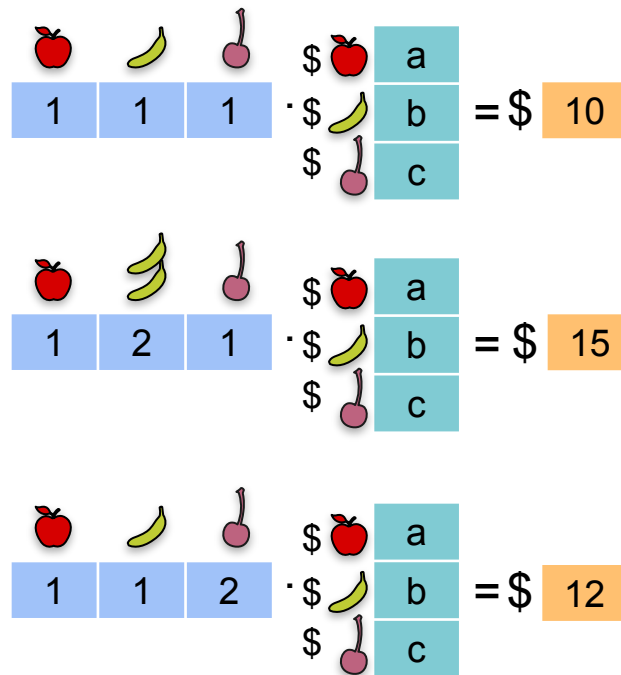


# Equations as dot product

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$





# Equations as dot product







## System of equations

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

## Matrix product

|   |   |   |  |   |    |
|---|---|---|--|---|----|
|  |  |  |  |   |    |
| 1   | 1   | 1   | \$  | a | 10 |
| 1   | 2   | 1   | \$  | b | 15 |
| 1   | 1   | 2   | \$  | c | 12 |

The matrix product is represented as:

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} \$ \text{apple} & a \\ \$ \text{banana} & b \\ \$ \text{cherry} & c \end{bmatrix} = \$ \begin{bmatrix} 10 \\ 15 \\ 12 \end{bmatrix}$$

# Equations as dot product

## System of equations

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

## Matrix product

|   |   |   |   |   |    |
|---|---|---|---|---|----|
| 1 | 1 | 1 | a | = | 10 |
| 1 | 2 | 1 | b | = | 15 |
| 1 | 1 | 2 | c | = | 12 |





DeepLearning.AI

# Vectors and Linear Transformations

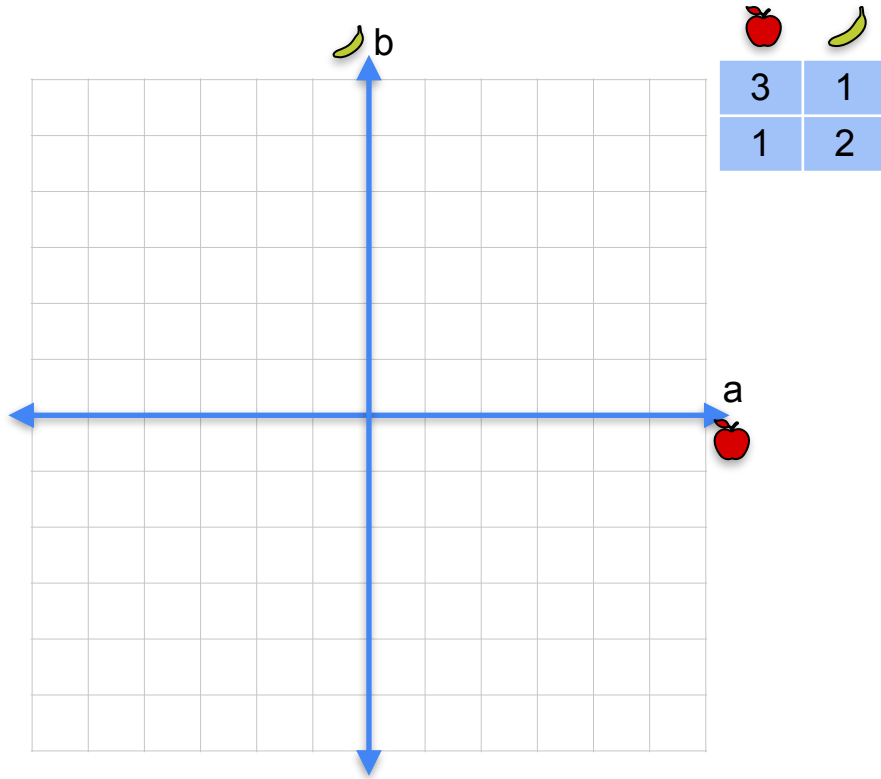
---

**Matrices as linear  
transformations**

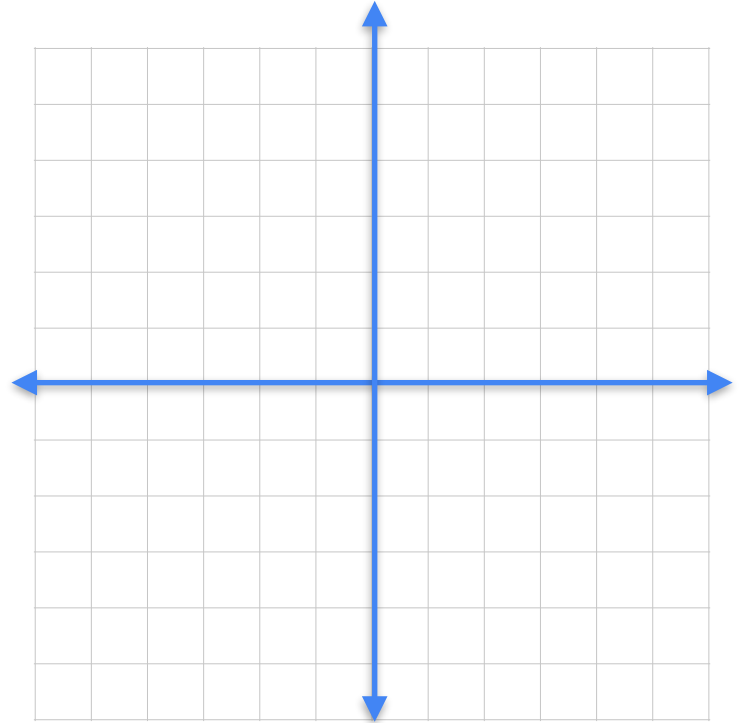
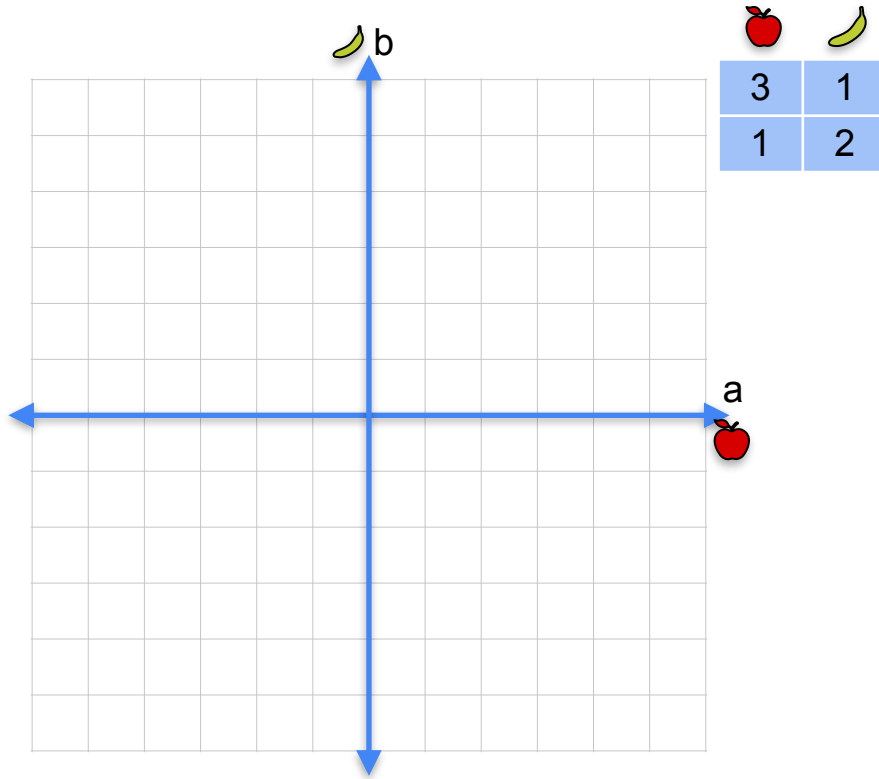
# Matrices as linear transformations

|   |   |
|---|---|
|  |  |
| 3   | 1   |
| 1   | 2   |

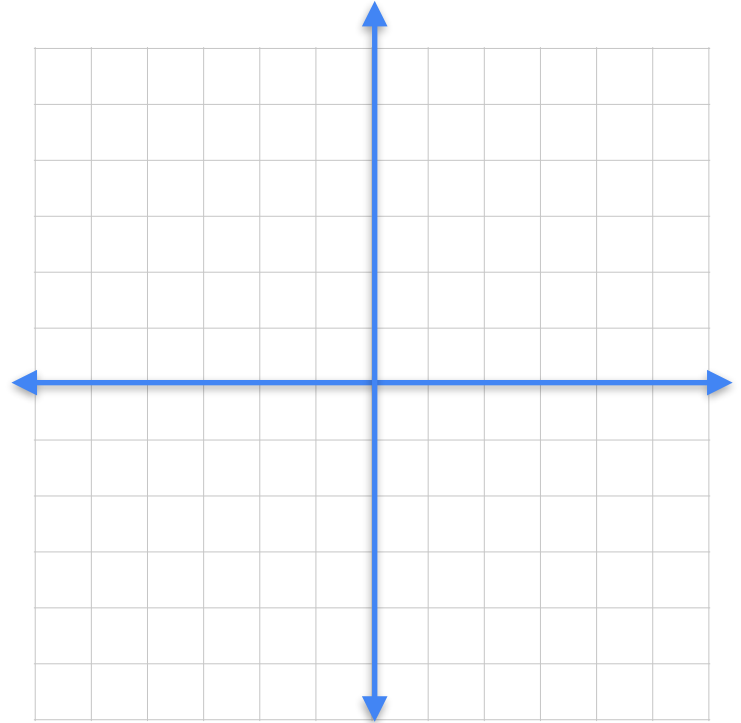
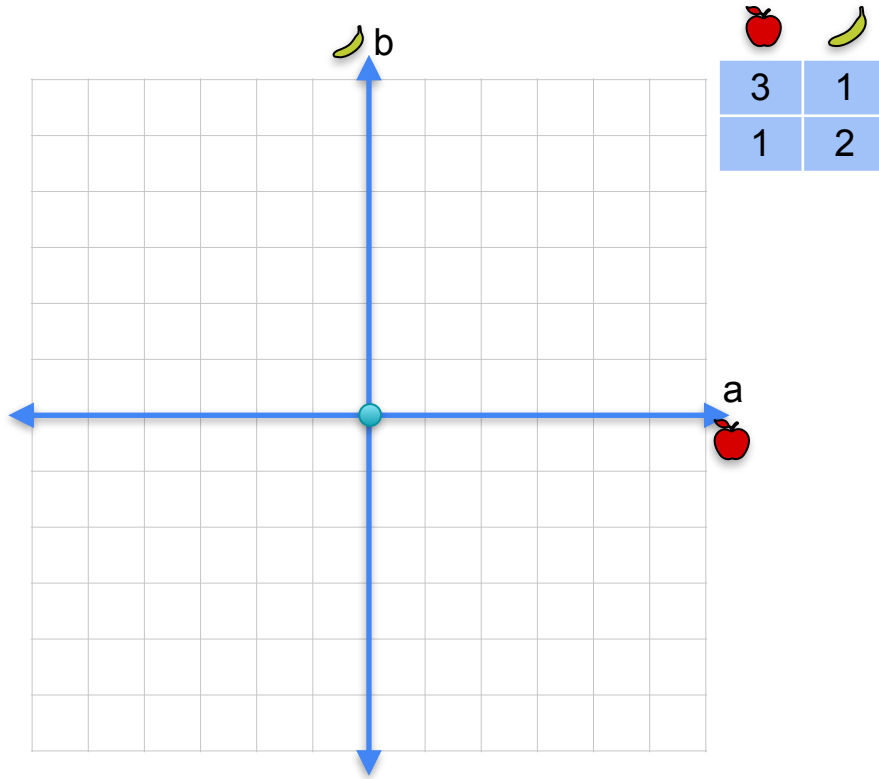
# Matrices as linear transformations



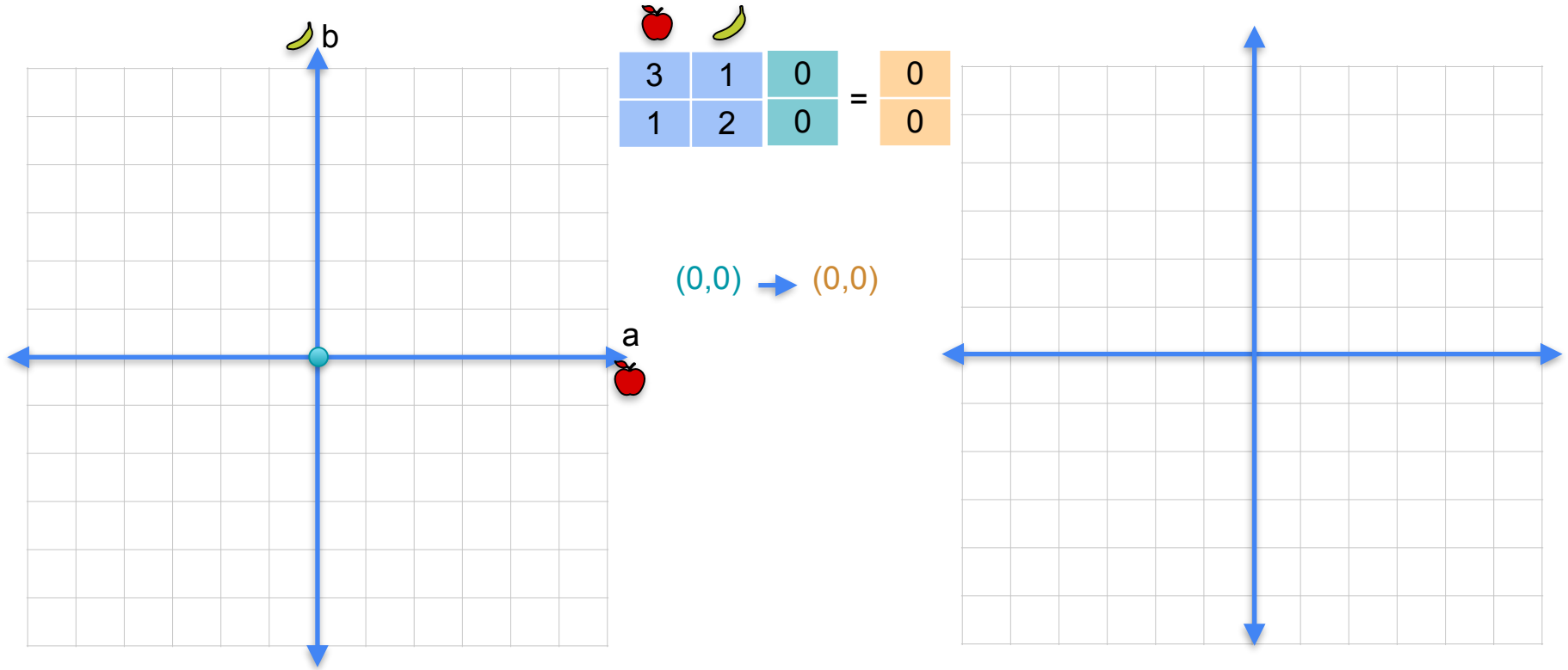
# Matrices as linear transformations



# Matrices as linear transformations

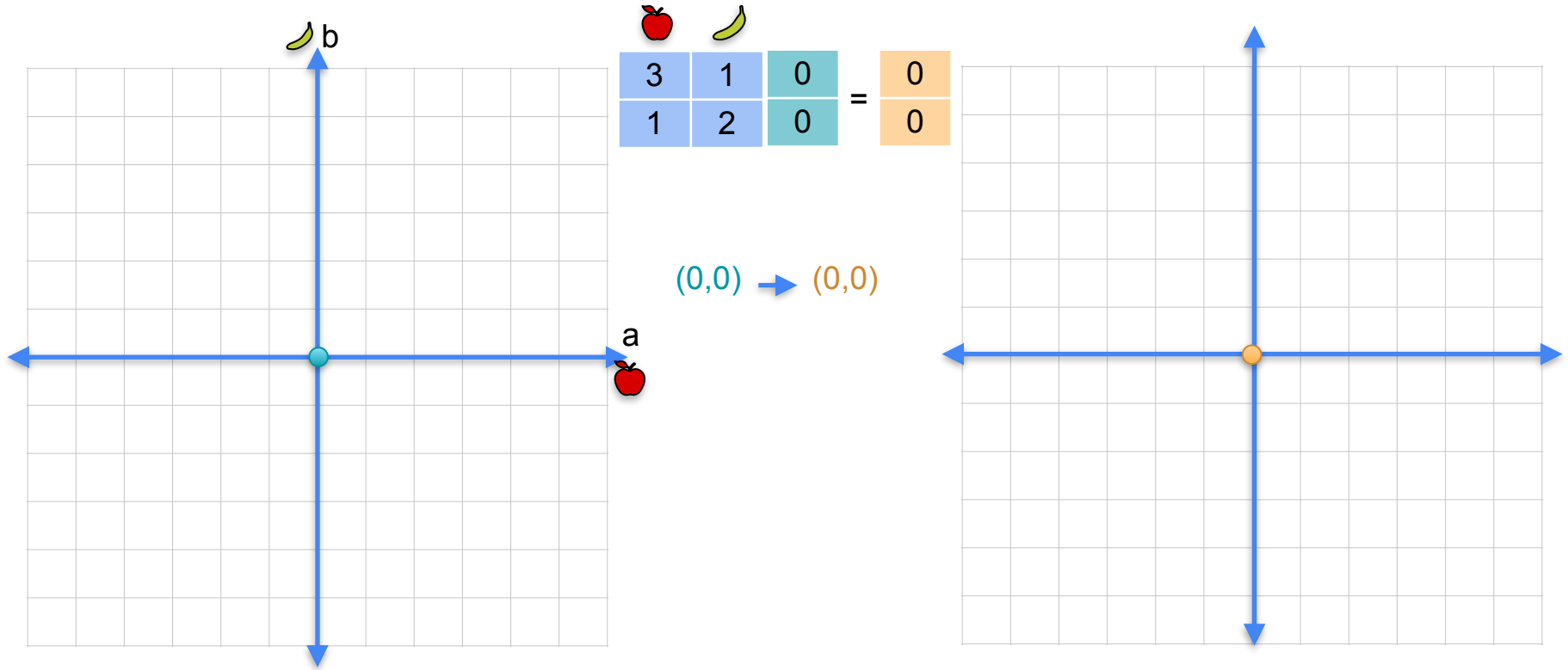


# Matrices as linear transformations

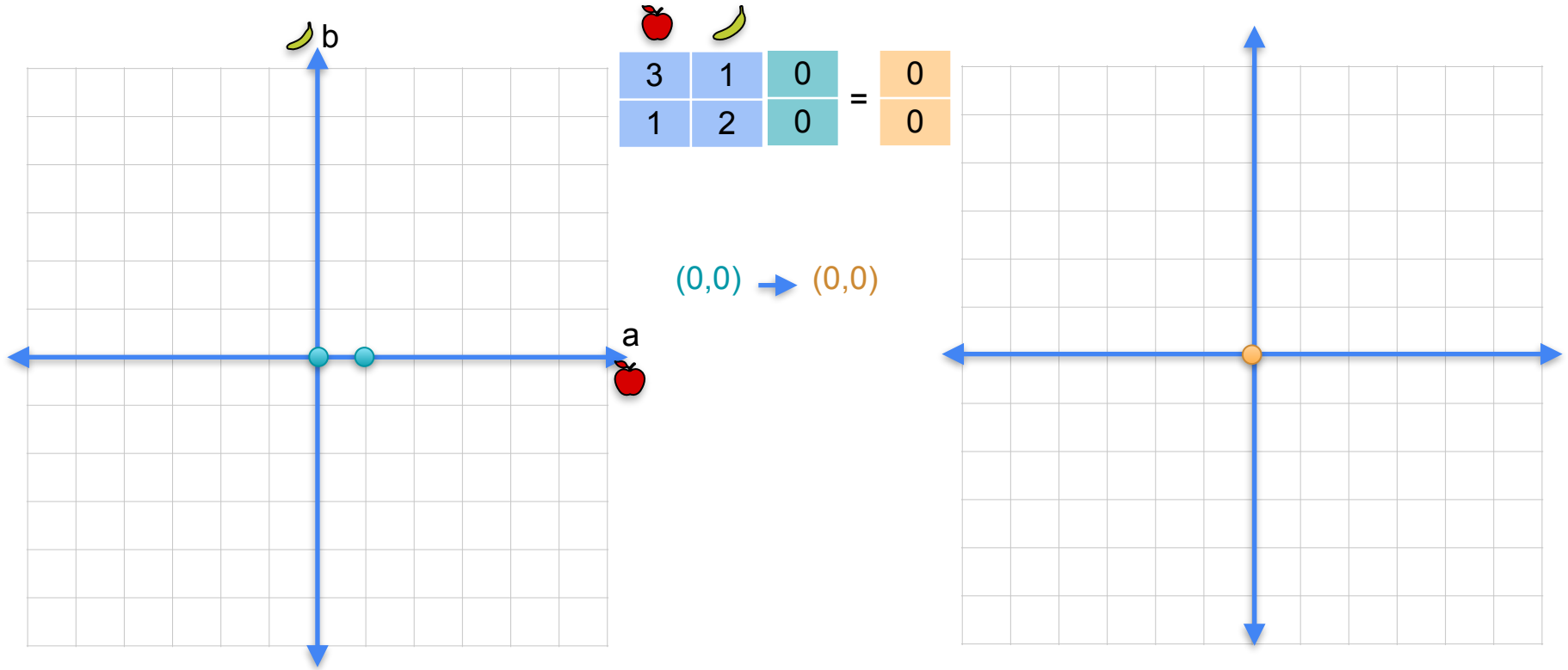




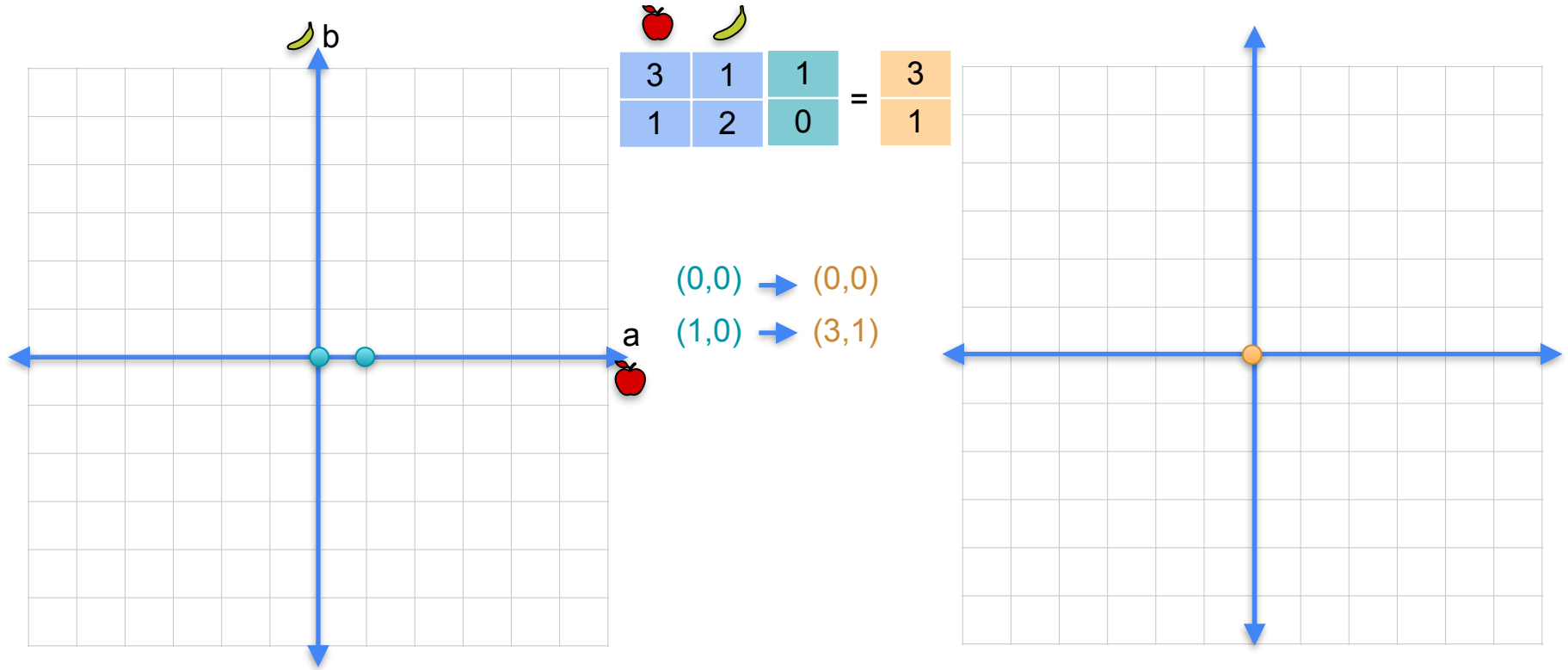
# Matrices as linear transformations



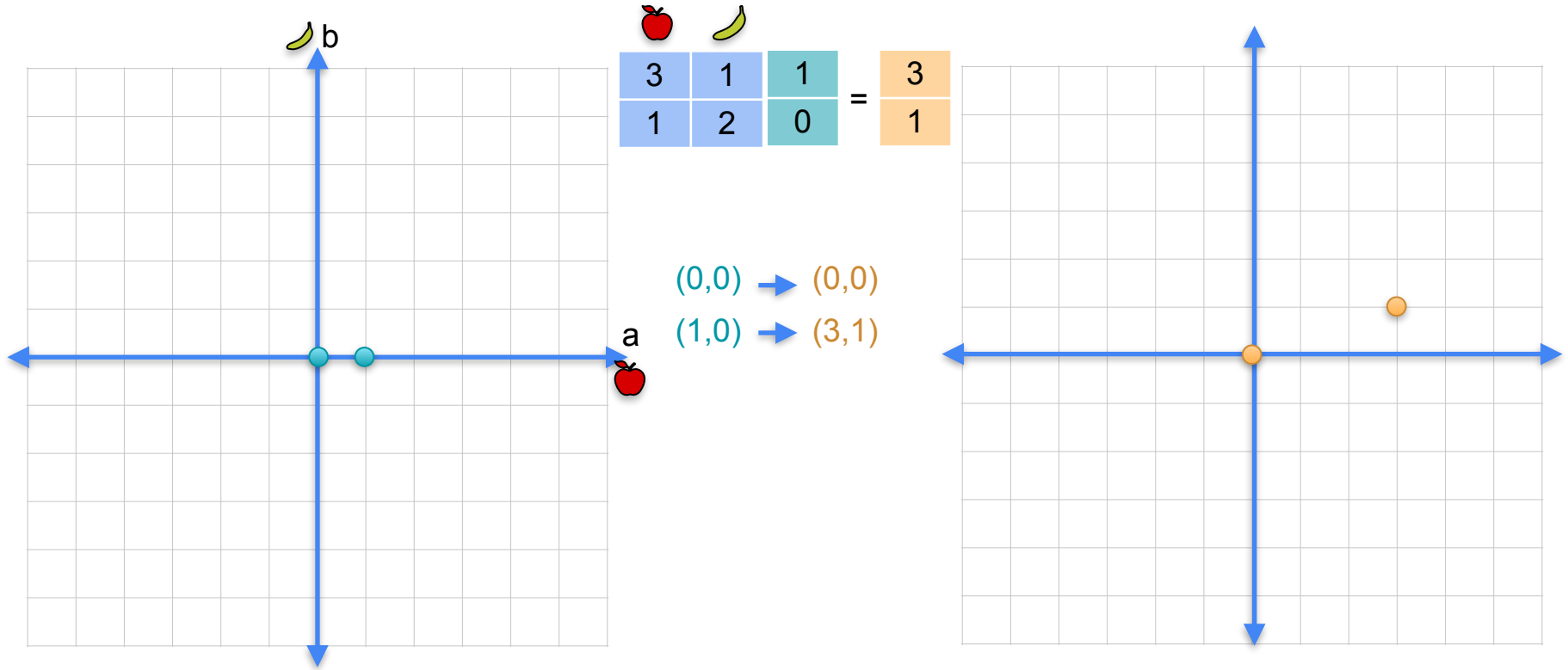
# Matrices as linear transformations



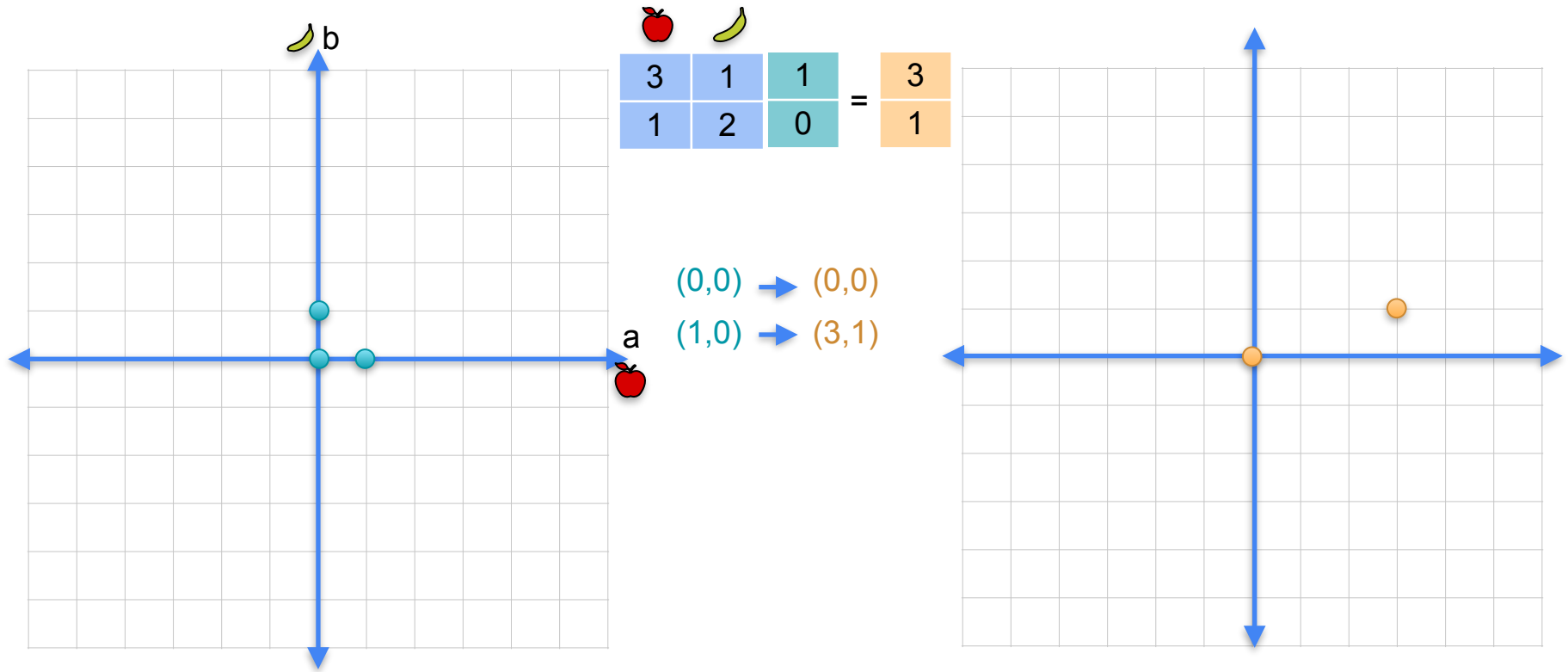
# Matrices as linear transformations



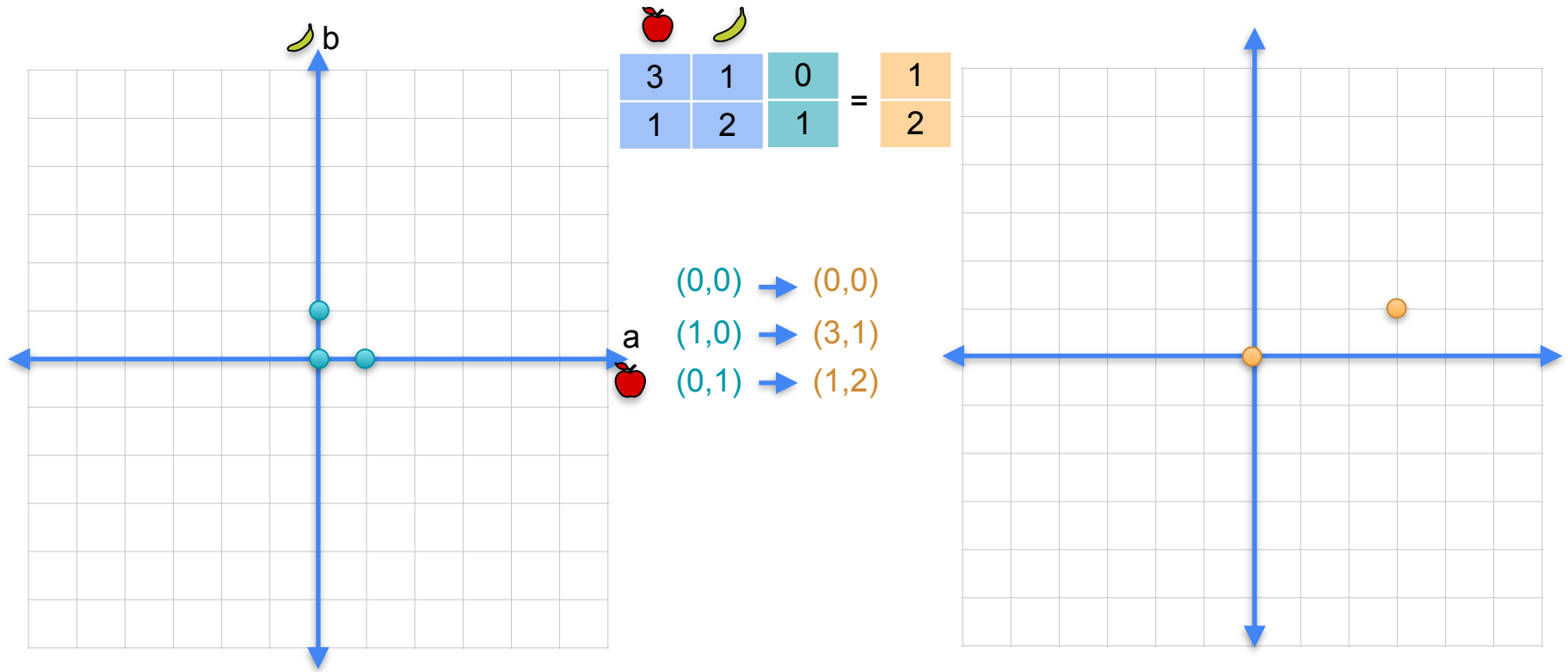
# Matrices as linear transformations



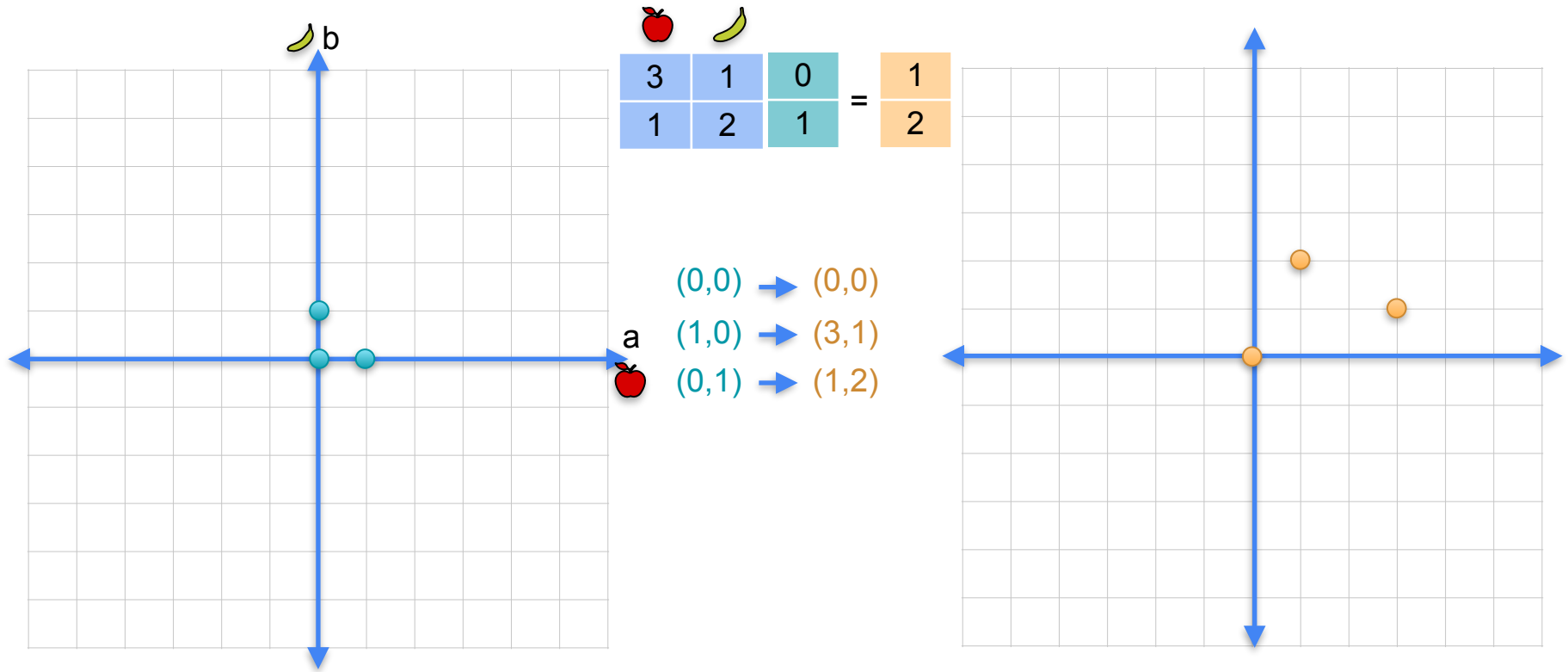
# Matrices as linear transformations



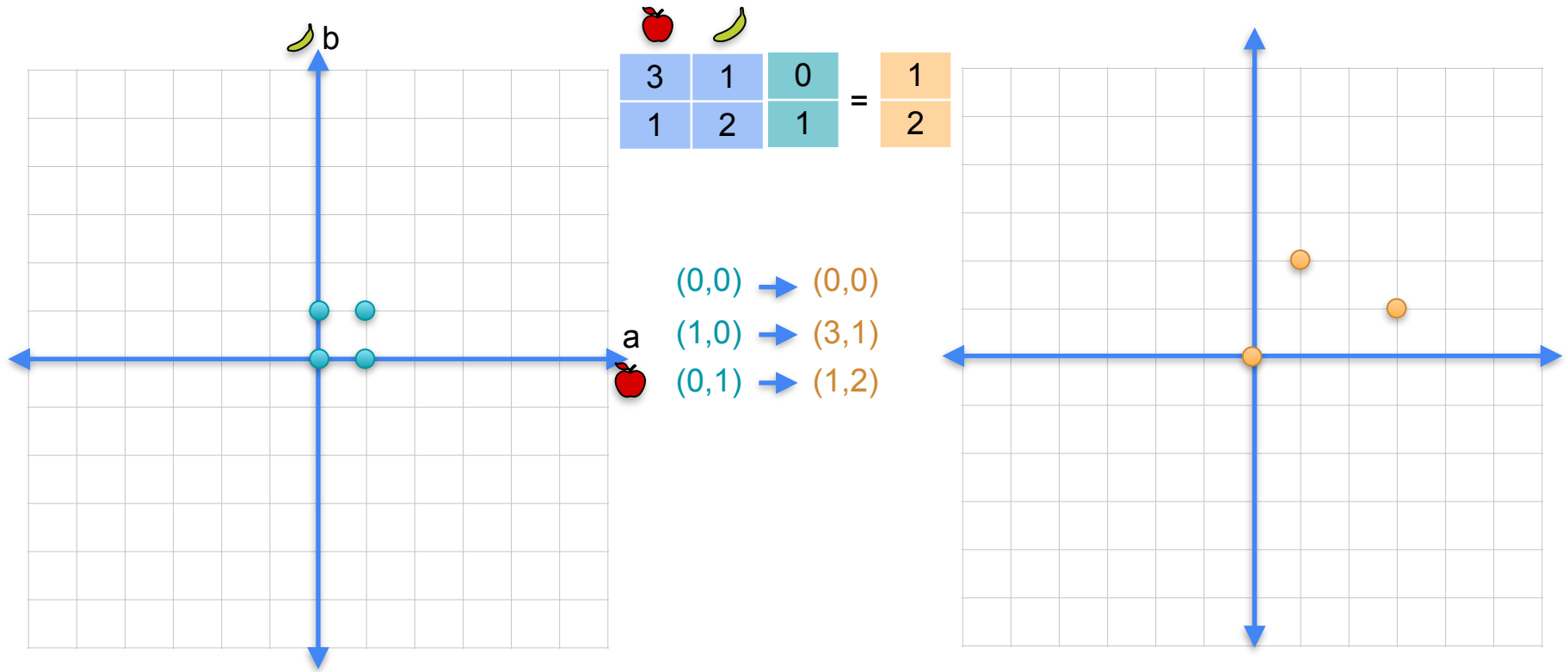
# Matrices as linear transformations



# Matrices as linear transformations

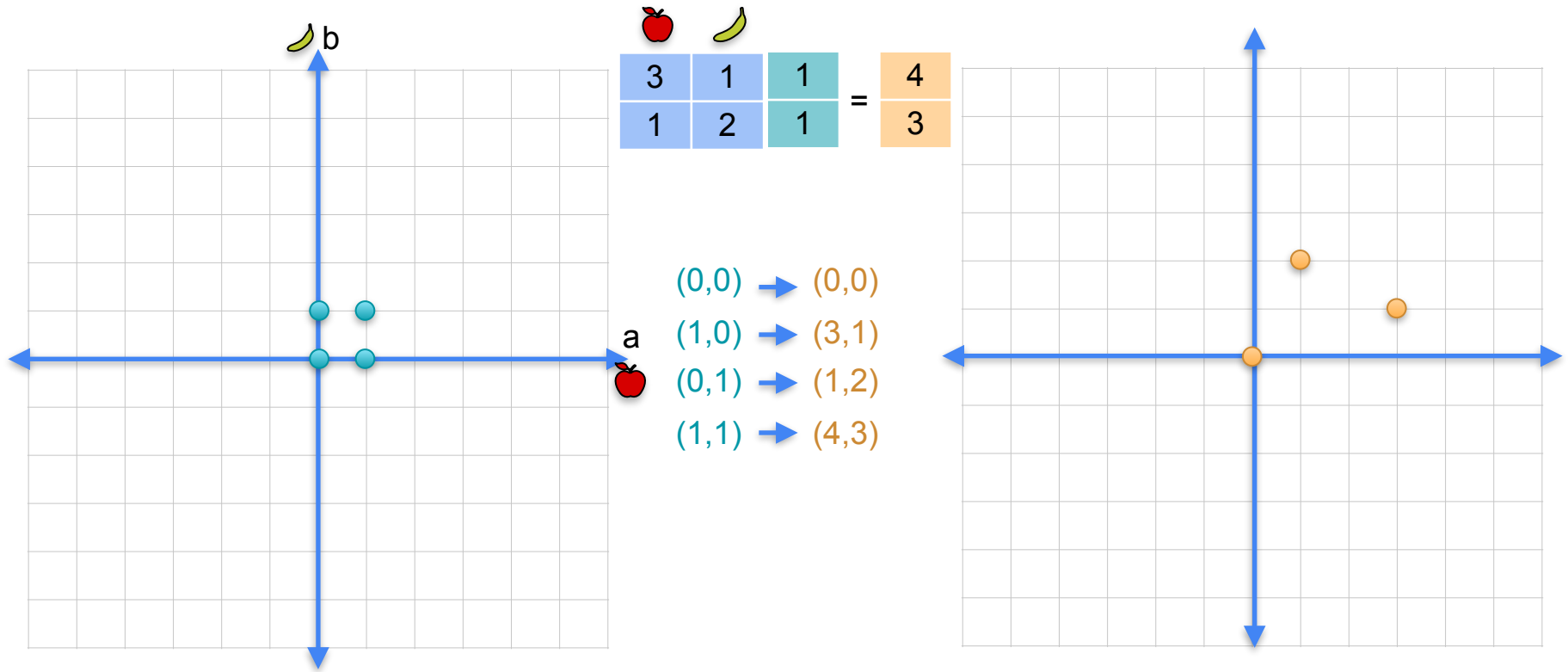


# Matrices as linear transformations

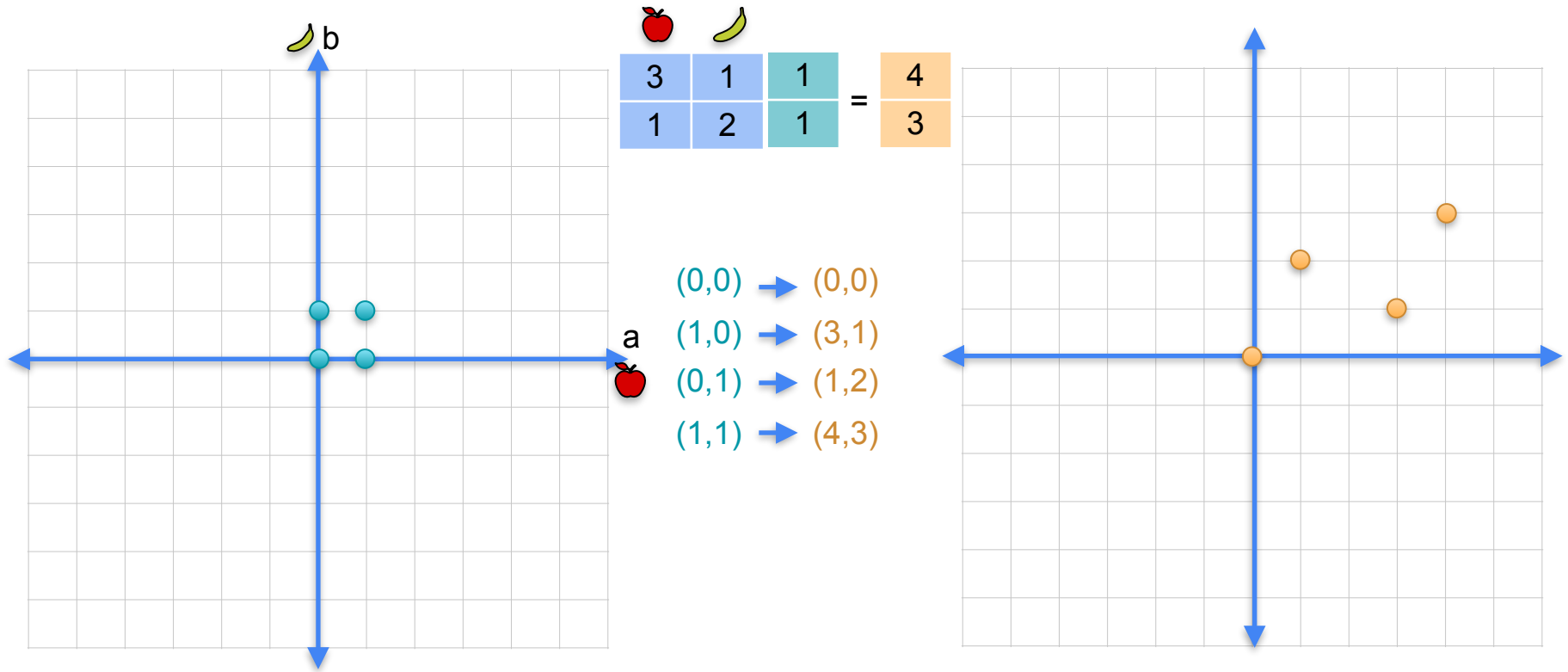




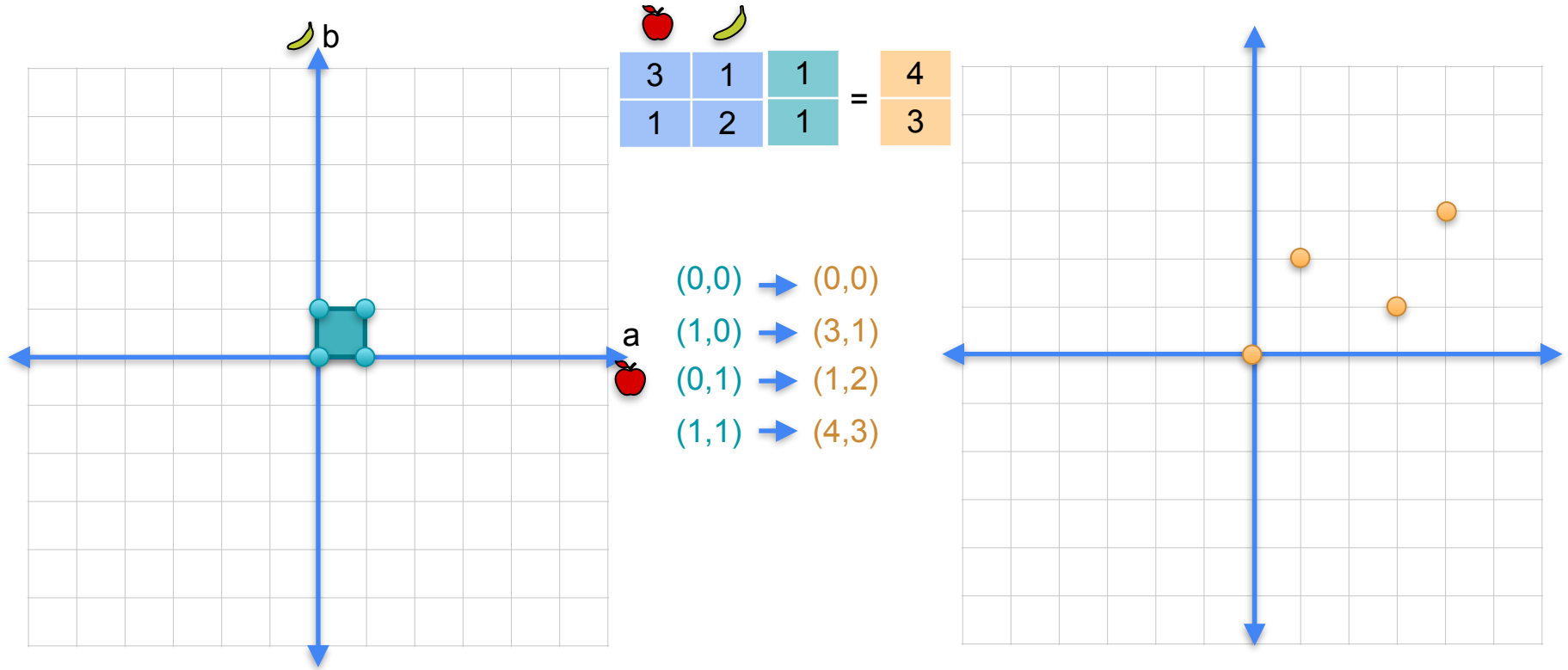
# Matrices as linear transformations



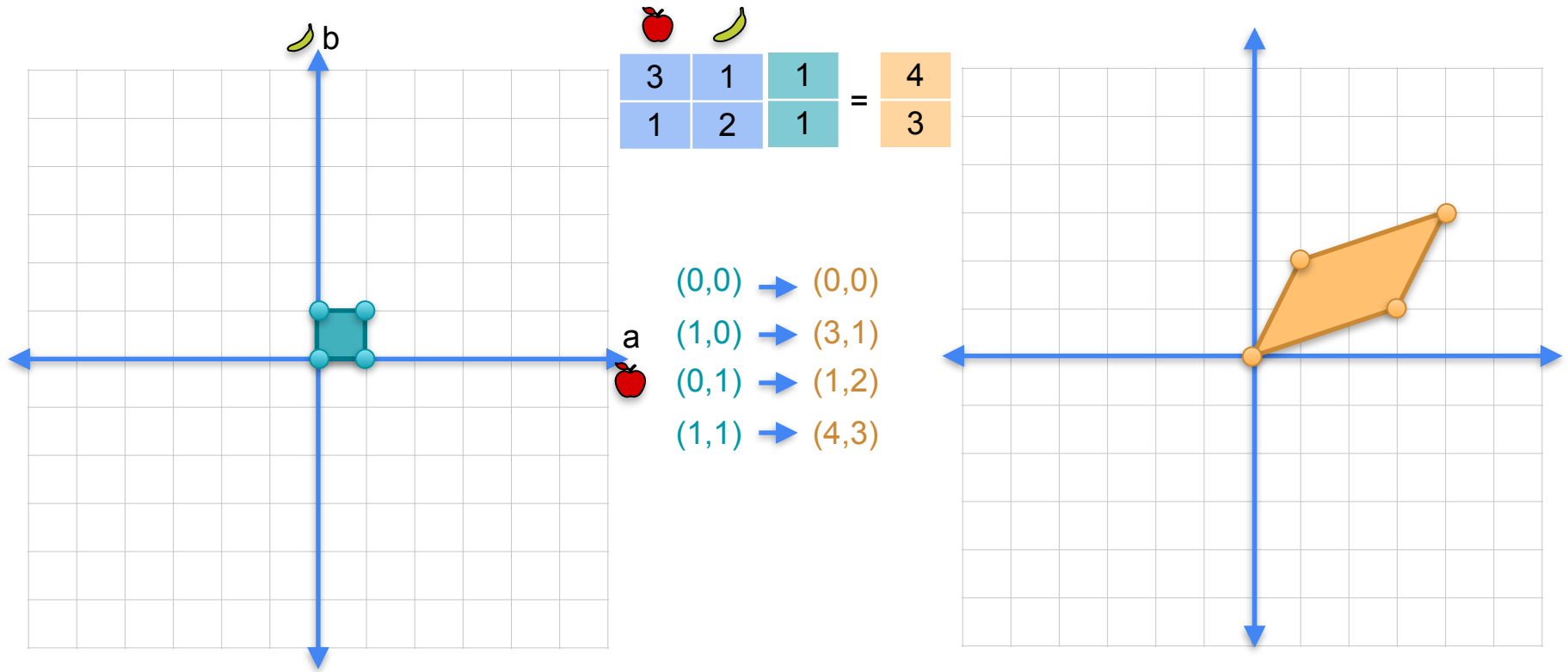
# Matrices as linear transformations



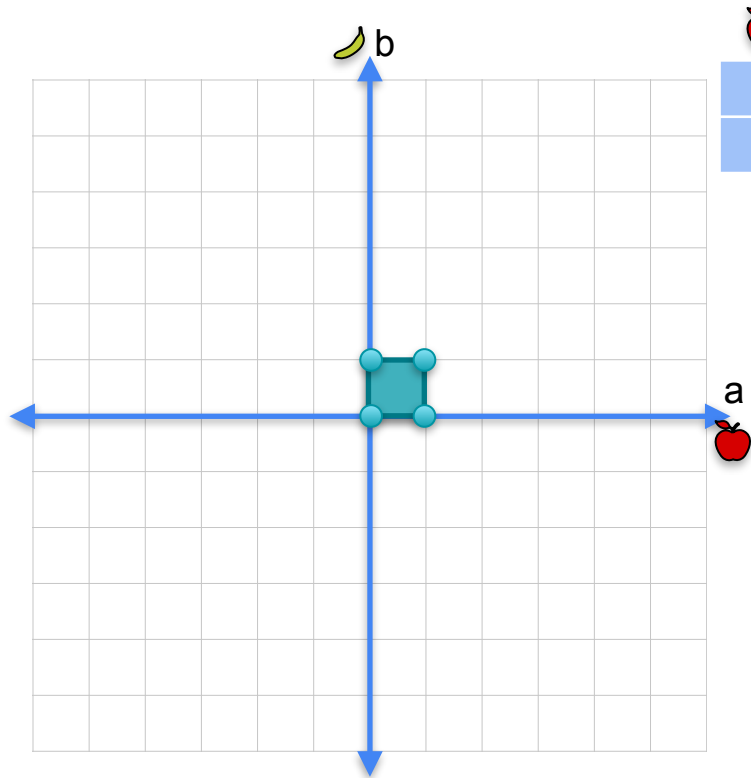
# Matrices as linear transformations



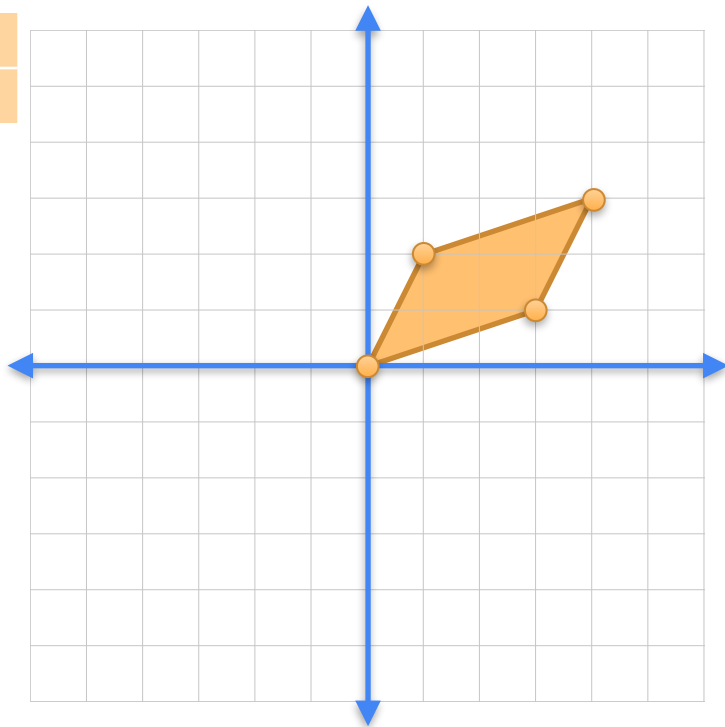
# Matrices as linear transformations



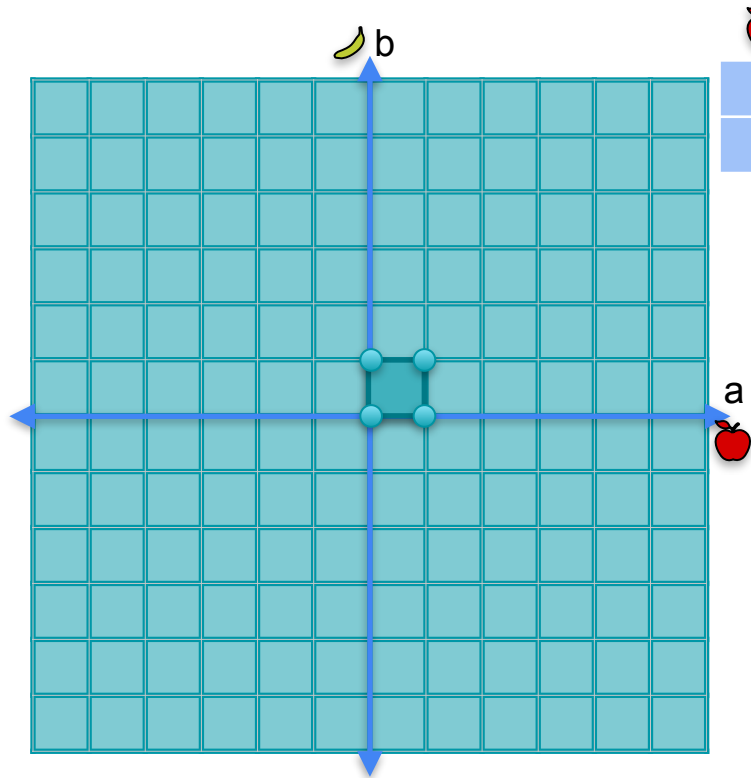
# Matrices as linear transformations



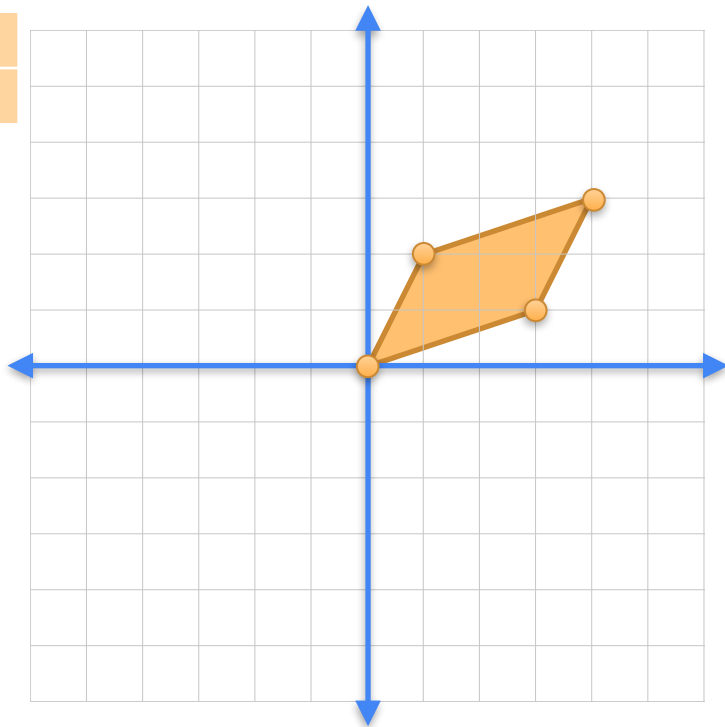
$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} & \begin{bmatrix} -2 \\ 3 \end{bmatrix} \end{matrix} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$



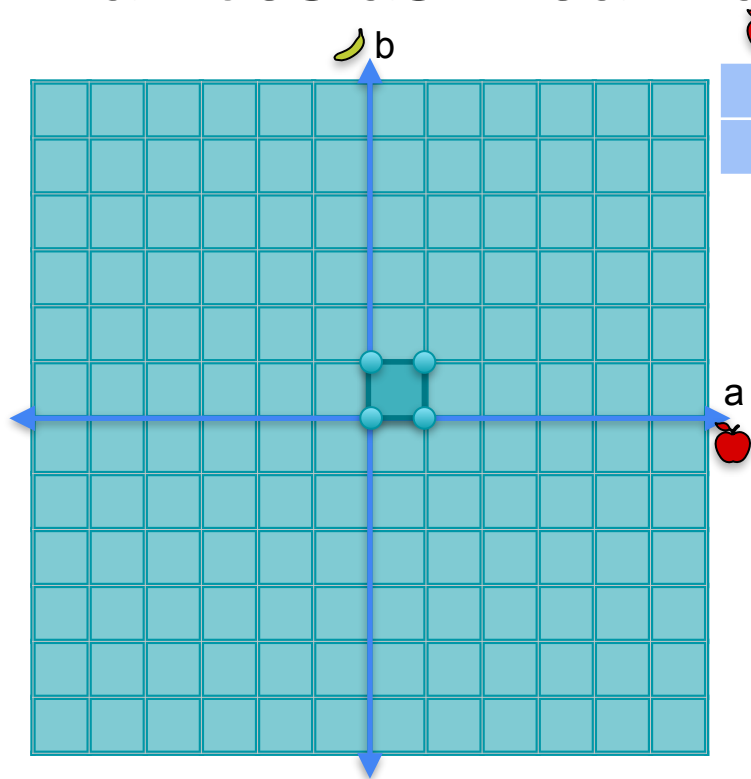
# Matrices as linear transformations



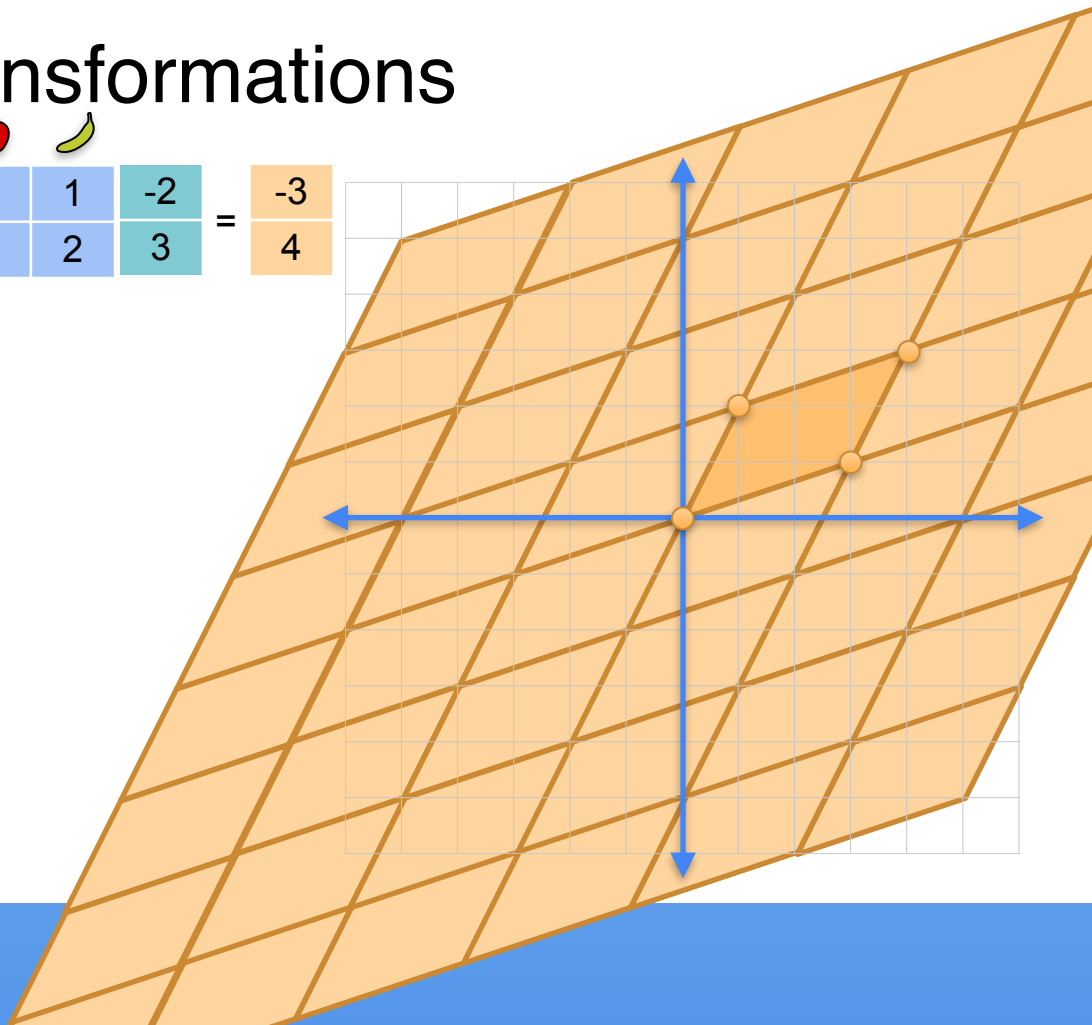
$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} & \begin{bmatrix} -2 \\ 3 \end{bmatrix} \end{matrix} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$



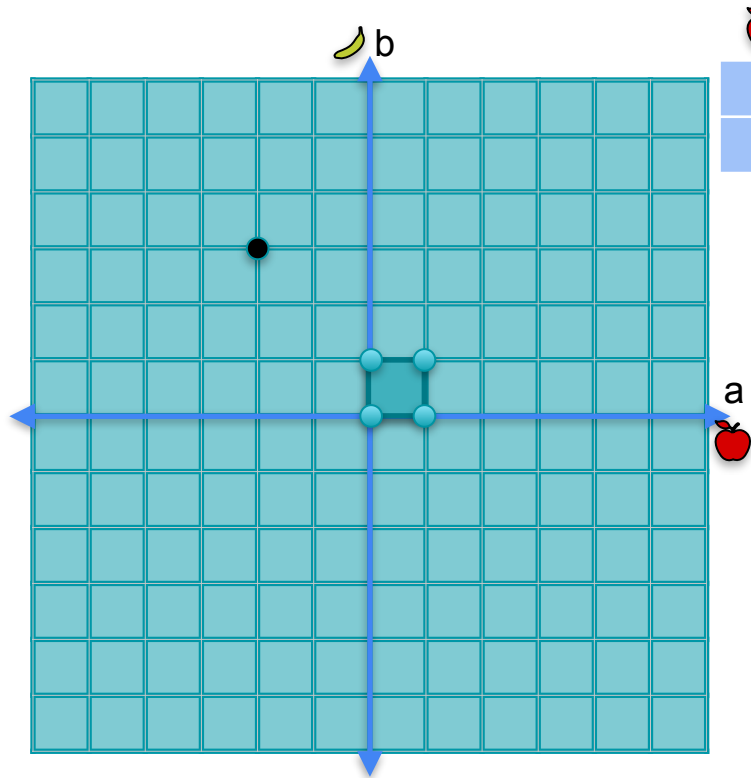
# Matrices as linear transformations



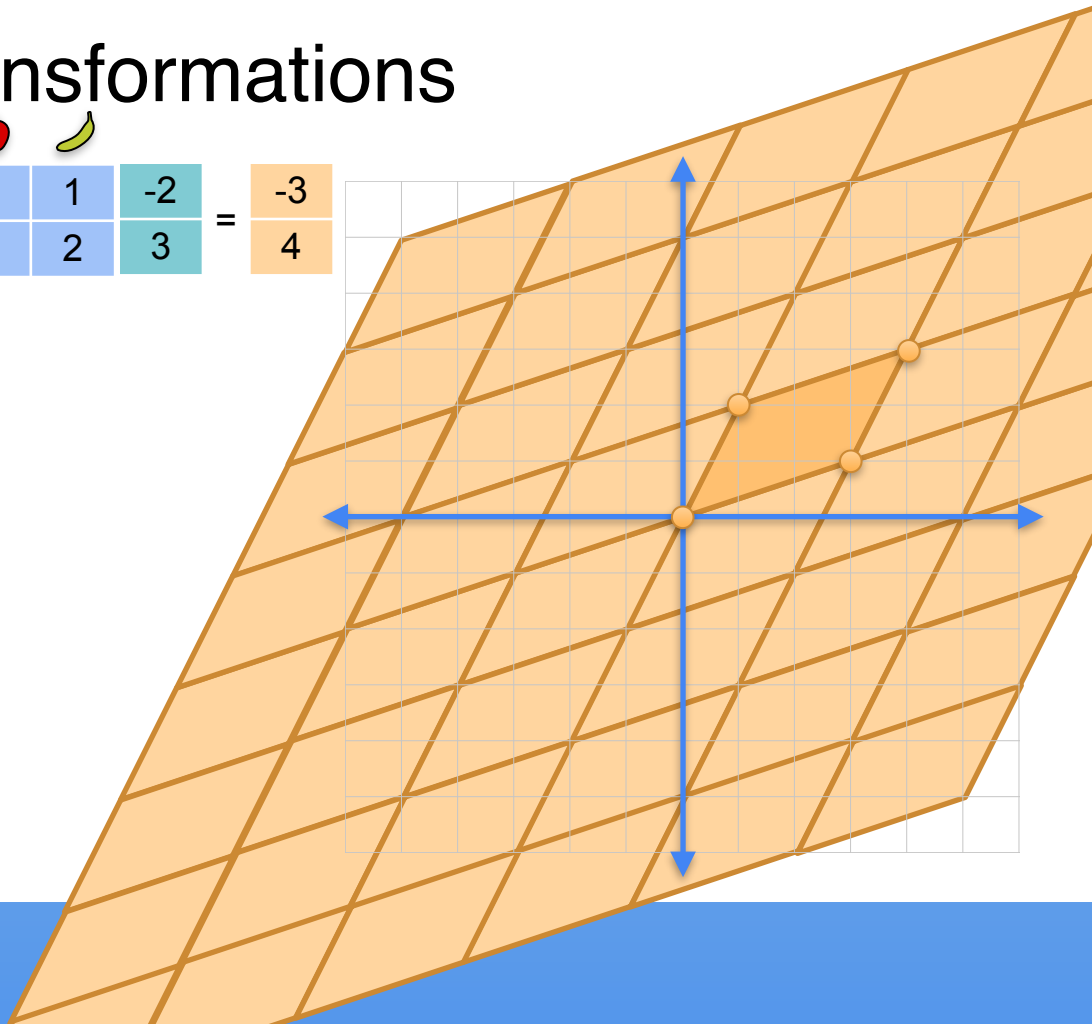
$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} & \begin{bmatrix} -2 \\ 3 \end{bmatrix} \end{matrix} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$



# Matrices as linear transformations

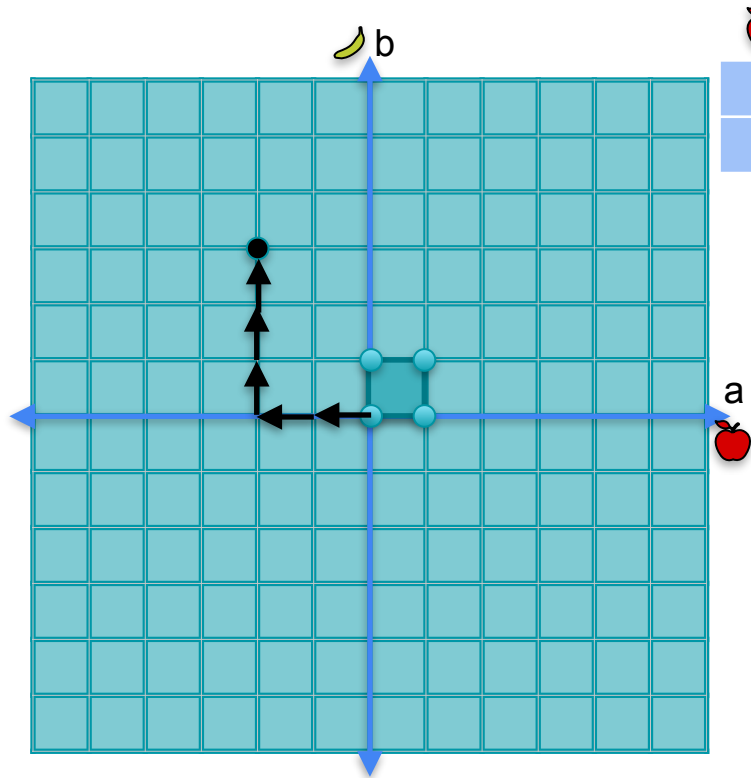


$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} & \begin{bmatrix} -2 \\ 3 \end{bmatrix} \end{matrix} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$

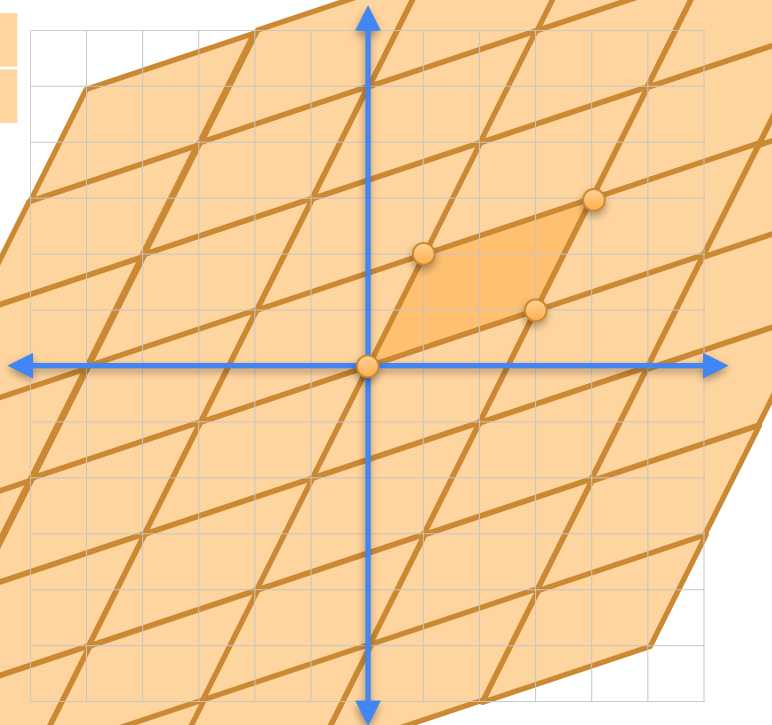




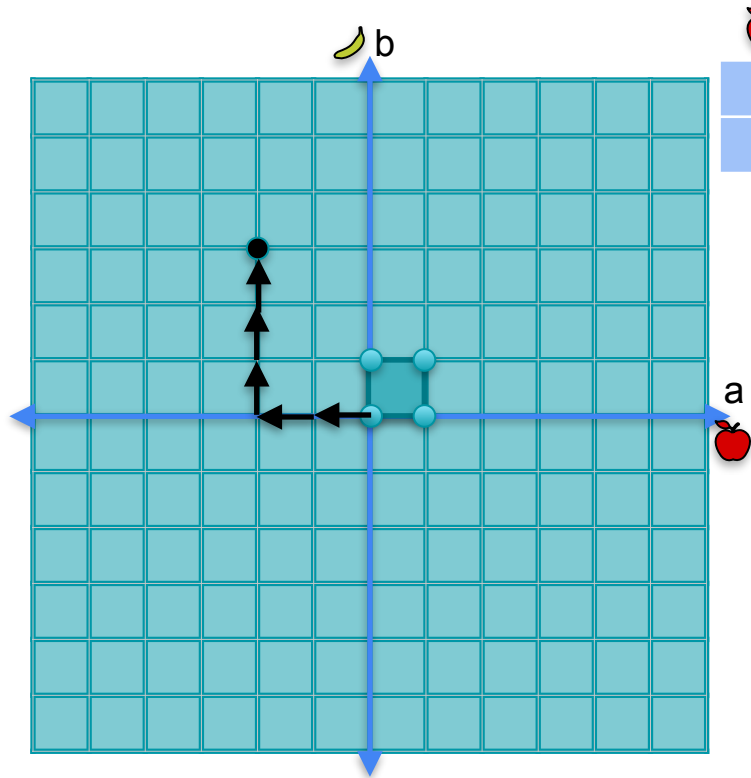
# Matrices as linear transformations



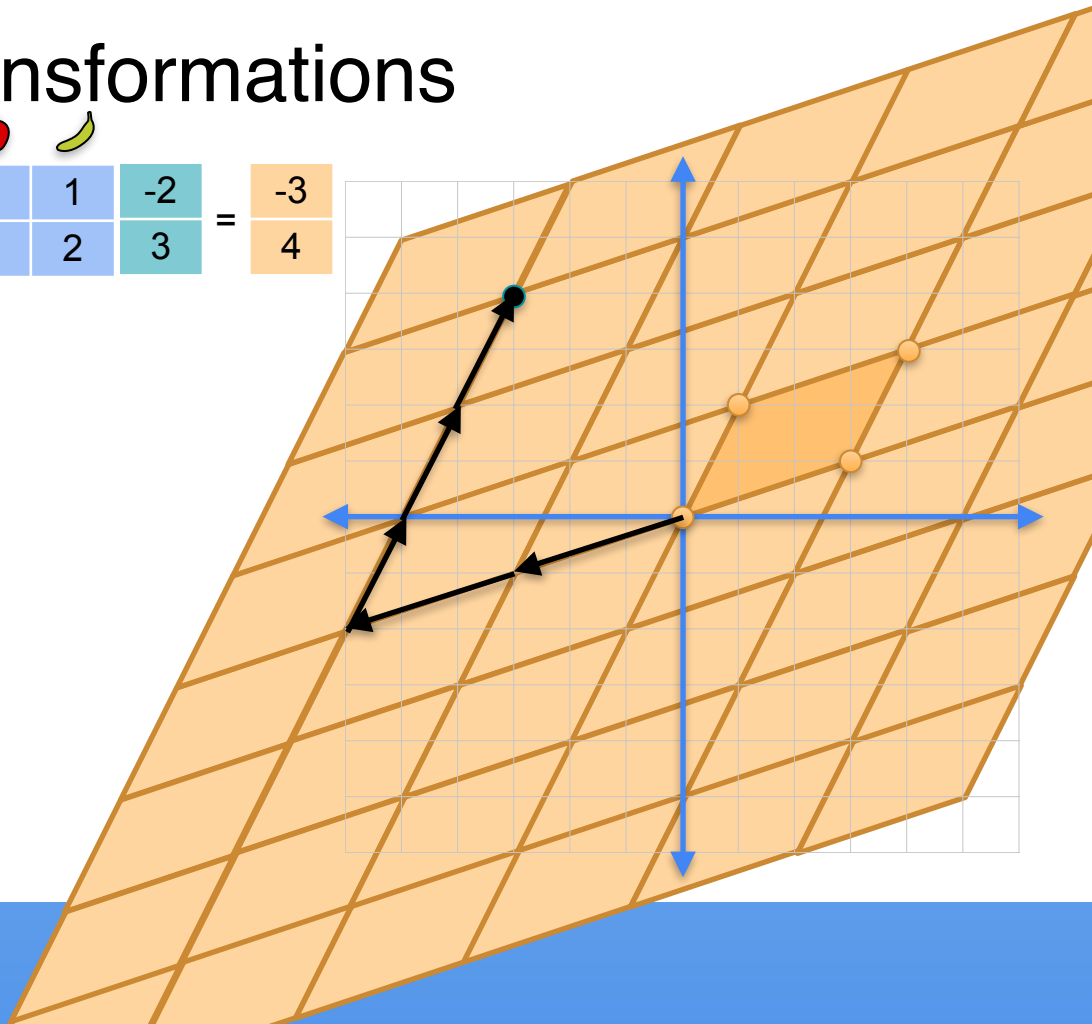
$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} & \begin{bmatrix} -2 \\ 3 \end{bmatrix} \end{matrix} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$



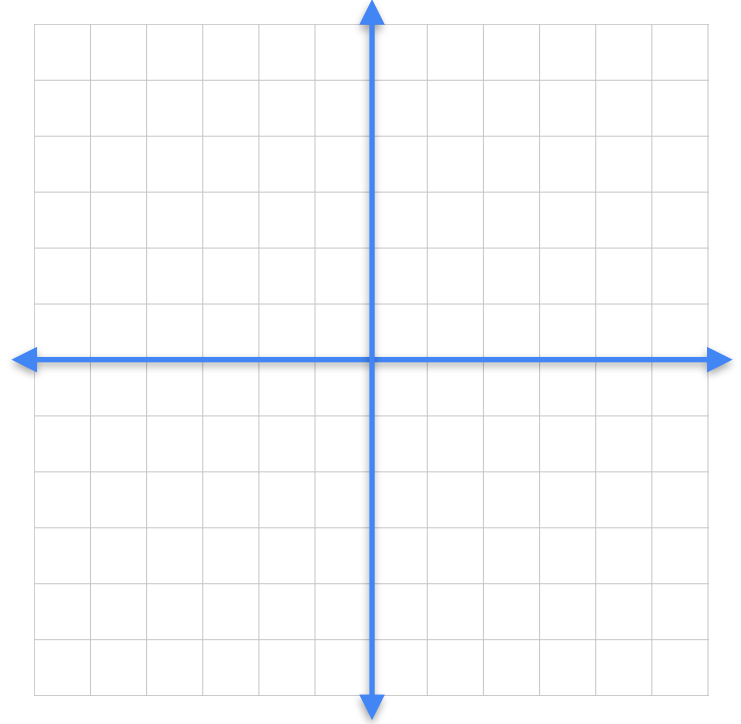
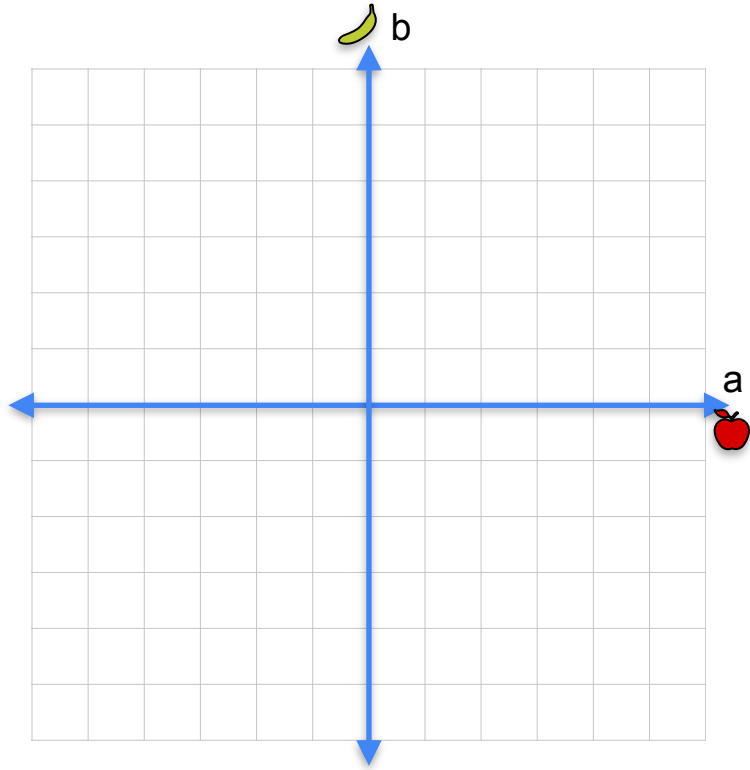
# Matrices as linear transformations



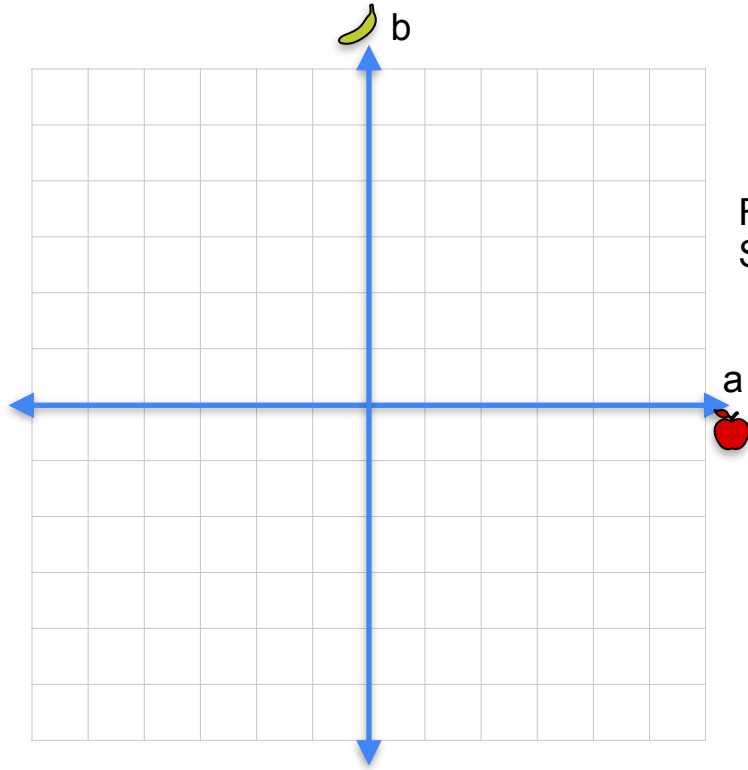
$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} & \begin{bmatrix} -2 \\ 3 \end{bmatrix} \end{matrix} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$



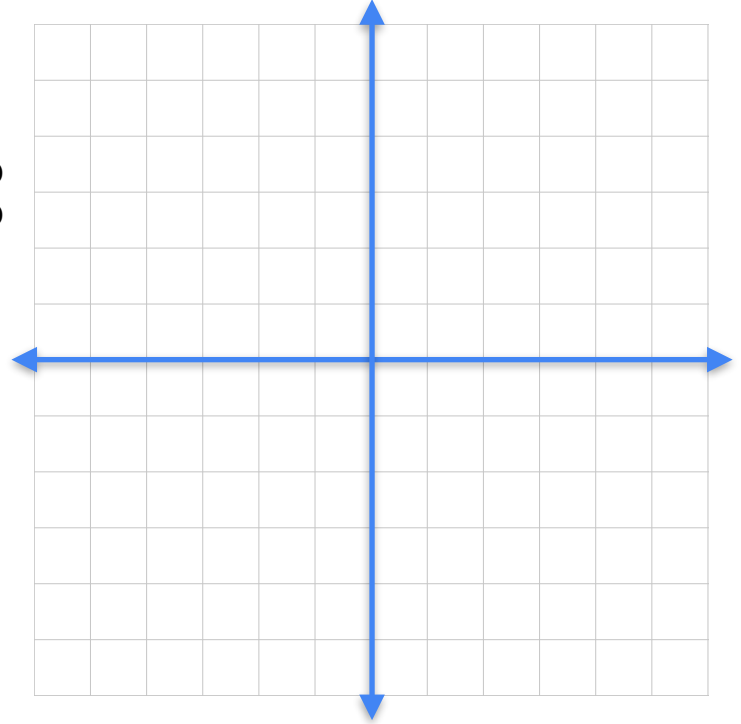
# Systems of equations as linear transformations



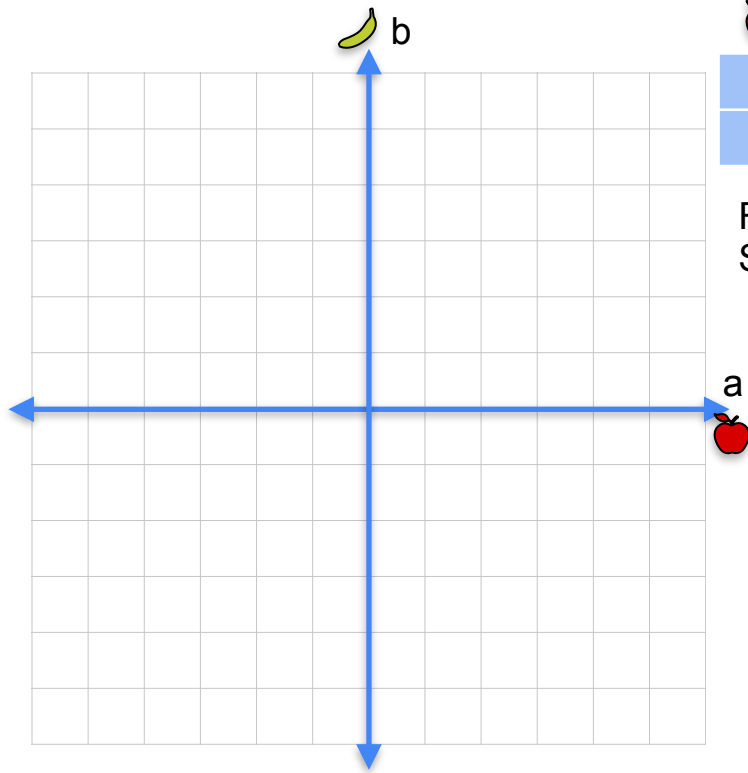
# Systems of equations as linear transformations



First day:  $3a + b$   
Second day:  $a + 2b$



# Systems of equations as linear transformations

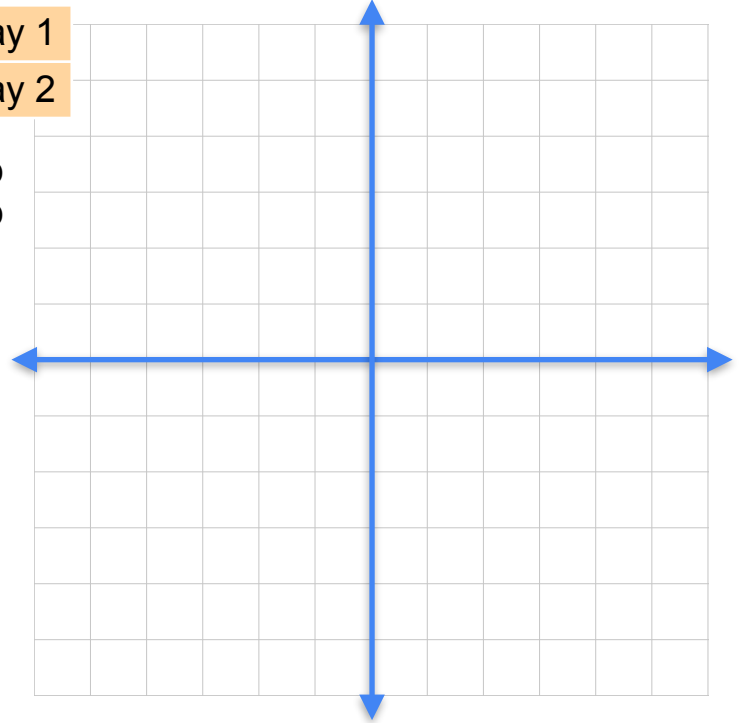


|   |   |   |
|---|---|---|
| 3 | 1 | a |
| 1 | 2 | b |

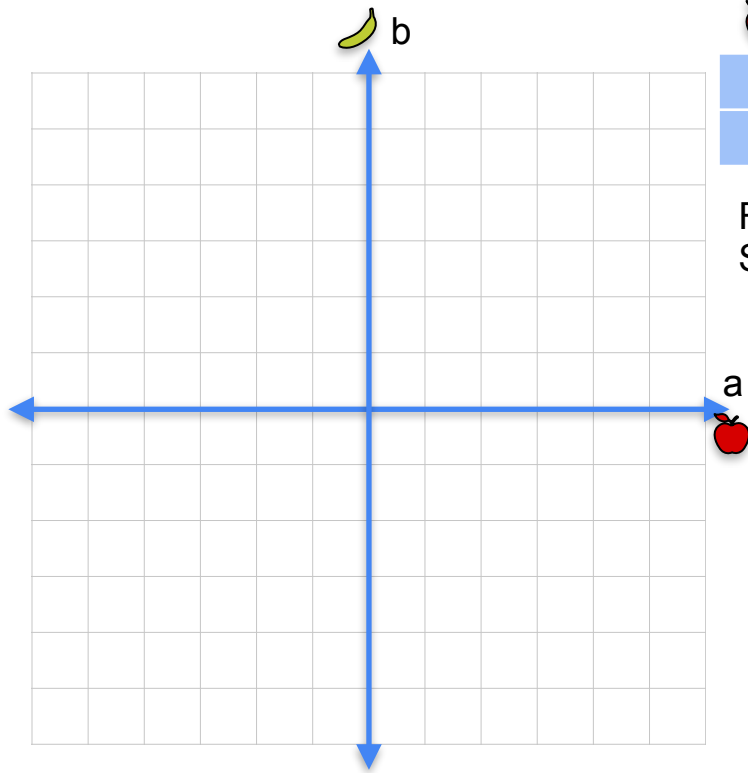
 = 

|       |
|-------|
| Day 1 |
| Day 2 |

First day:  $3a + b$   
Second day:  $a + 2b$

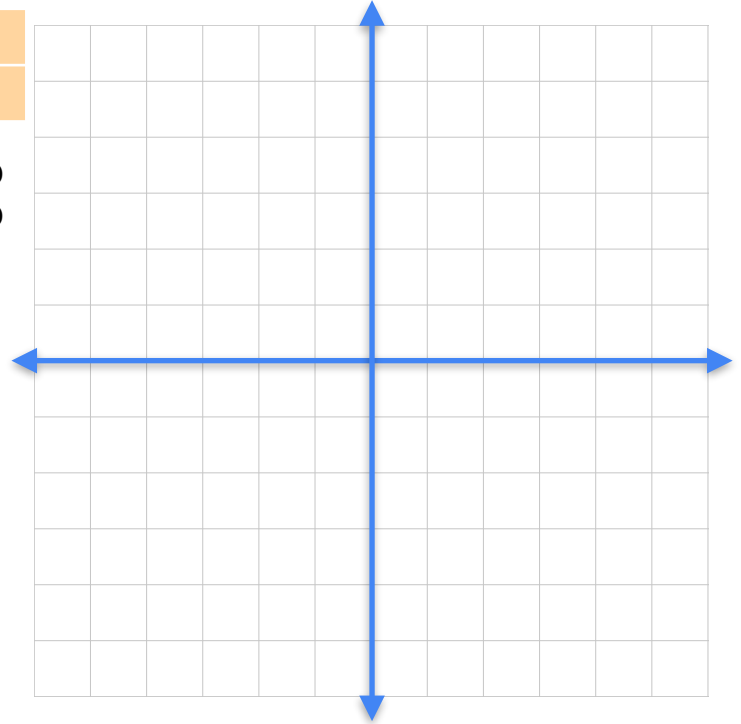


# Systems of equations as linear transformations



|   |   |   |   |   |
|---|---|---|---|---|
| 3 | 1 | 1 | = | 4 |
| 1 | 2 | 1 | = | 3 |

First day:  $3a + b$   
Second day:  $a + 2b$



# Systems of equations as linear transformations

 b

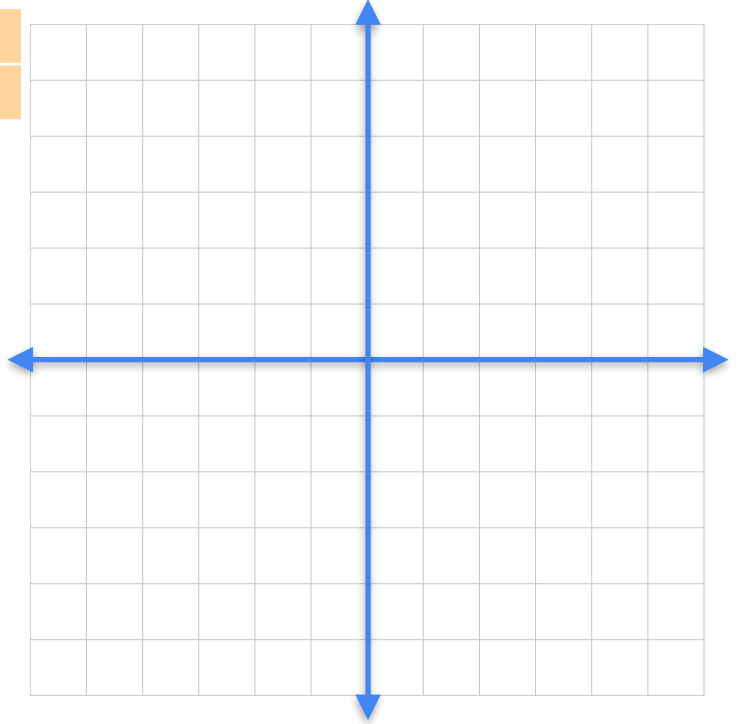
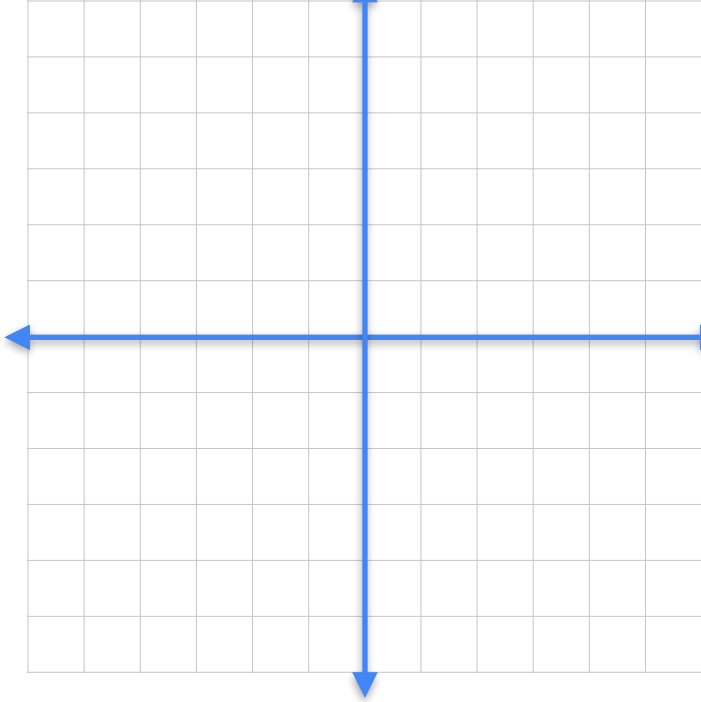


|   |   |   |   |   |
|---|---|---|---|---|
| 3 | 1 | 1 | = | 4 |
| 1 | 2 | 1 | = | 3 |

First day:  $3a + b$   
Second day:  $a + 2b$

a

$(1,1) \rightarrow (4,3)$



# Systems of equations as linear transformations

 b



|   |   |   |   |   |
|---|---|---|---|---|
| 3 | 1 | 1 | = | 4 |
| 1 | 2 | 1 | = | 3 |

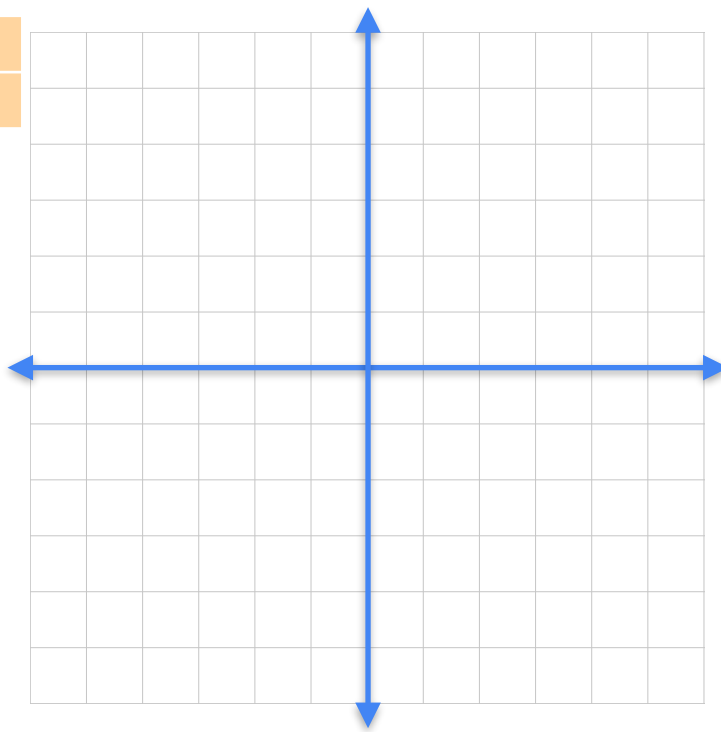
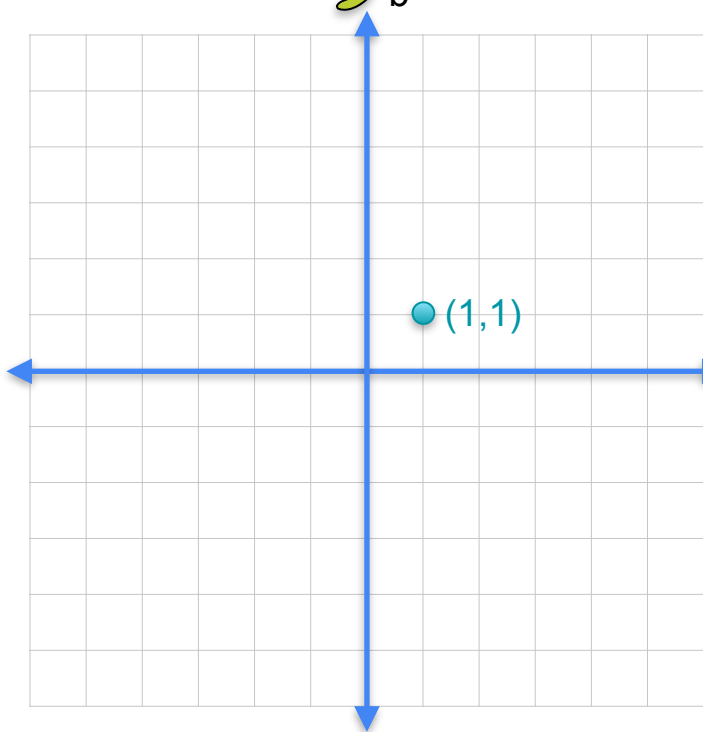
First day:  $3a + b$

Second day:  $a + 2b$

 (1,1)

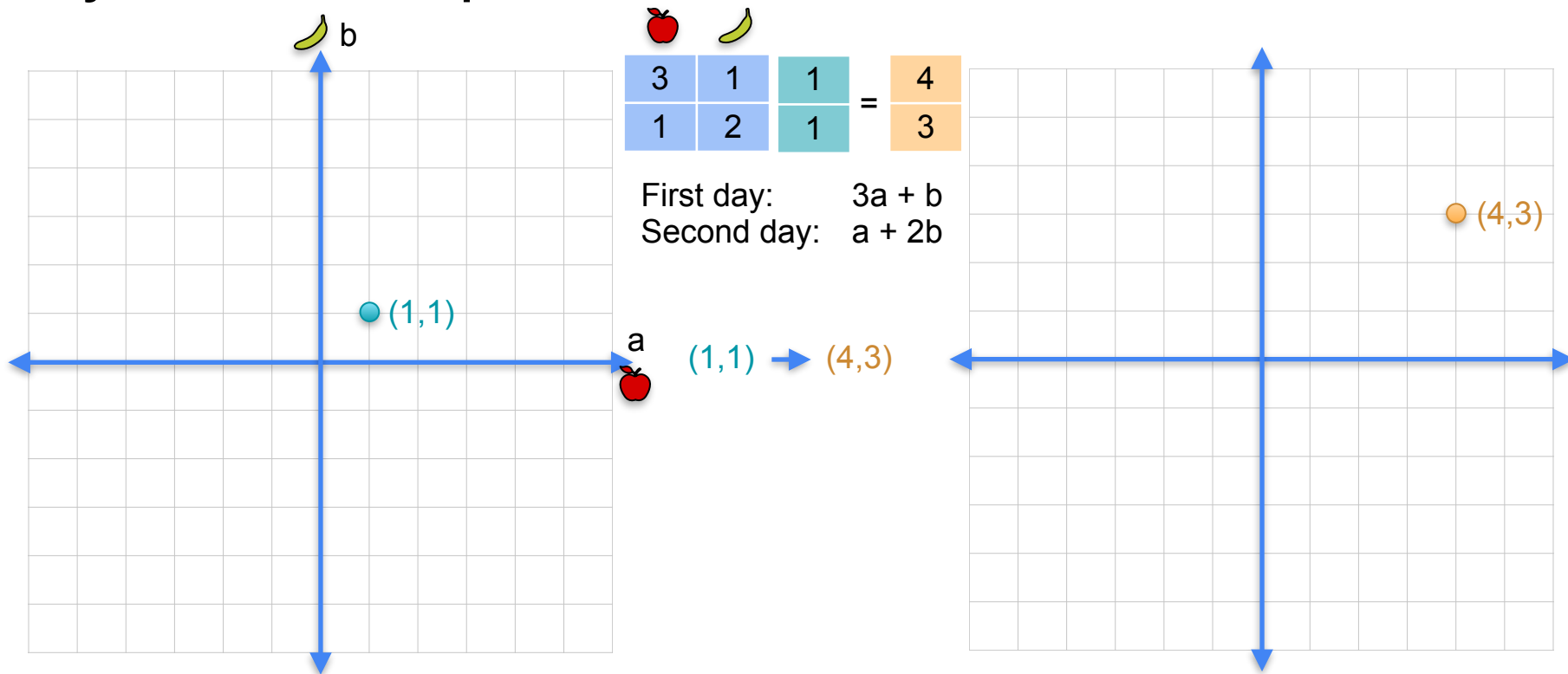
a

(1,1) → (4,3)

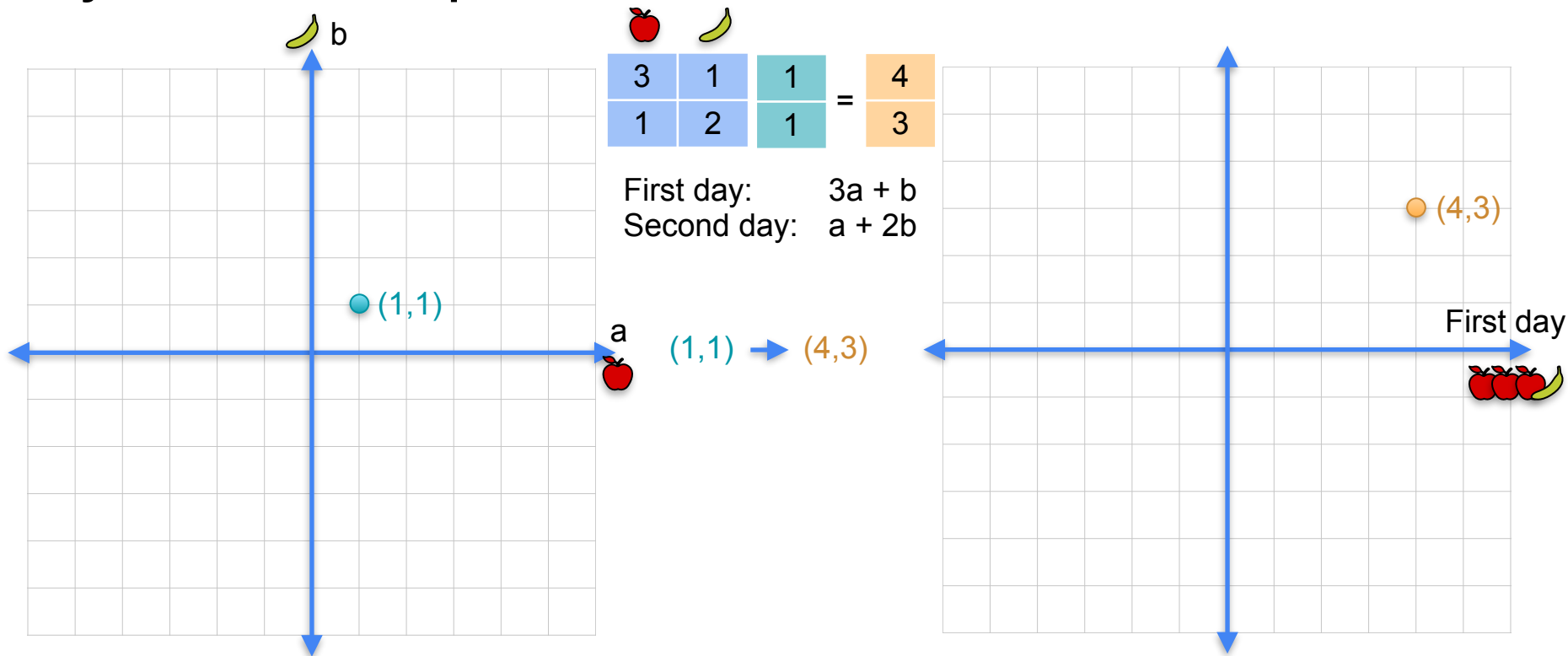




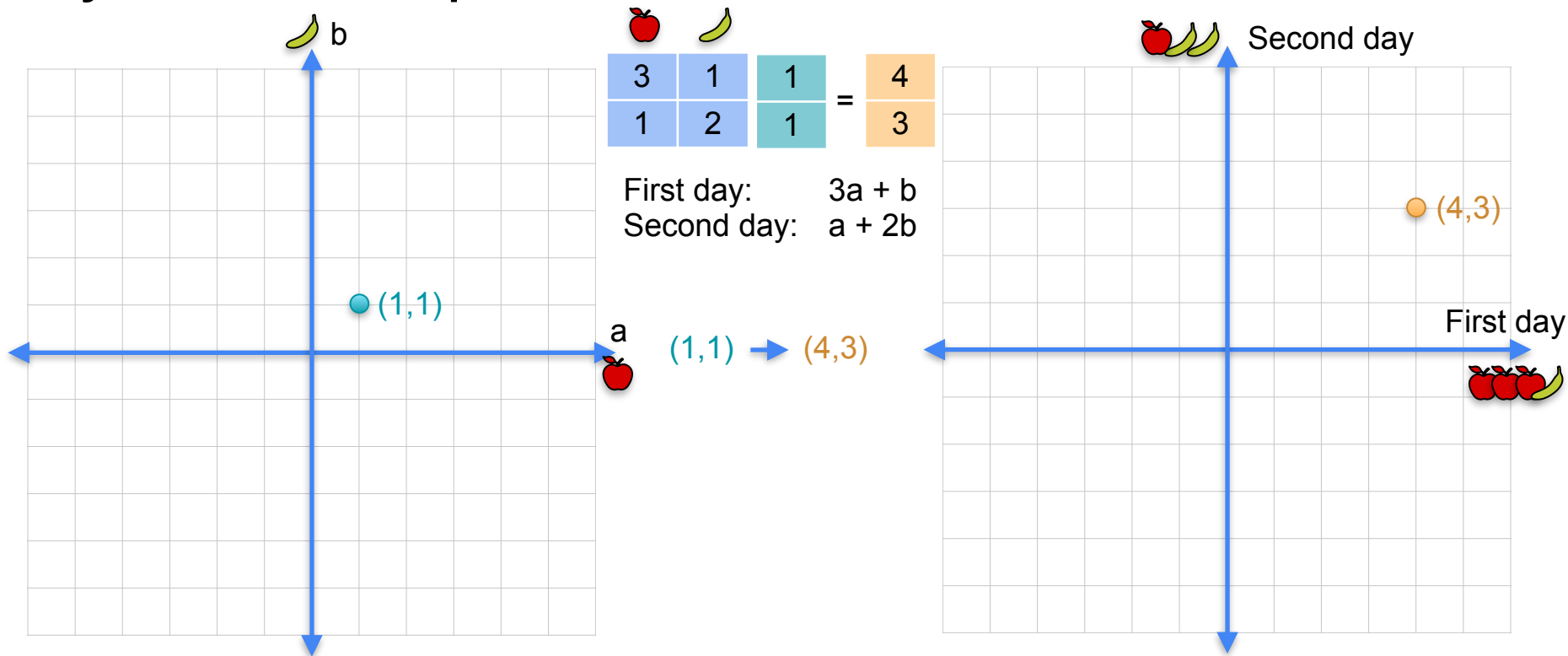
# Systems of equations as linear transformations



# Systems of equations as linear transformations



# Systems of equations as linear transformations





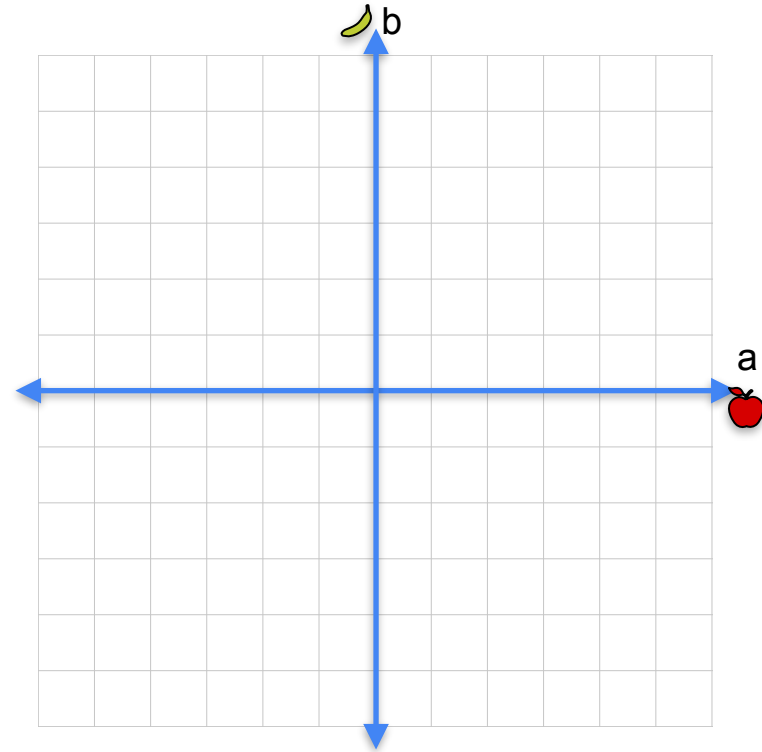
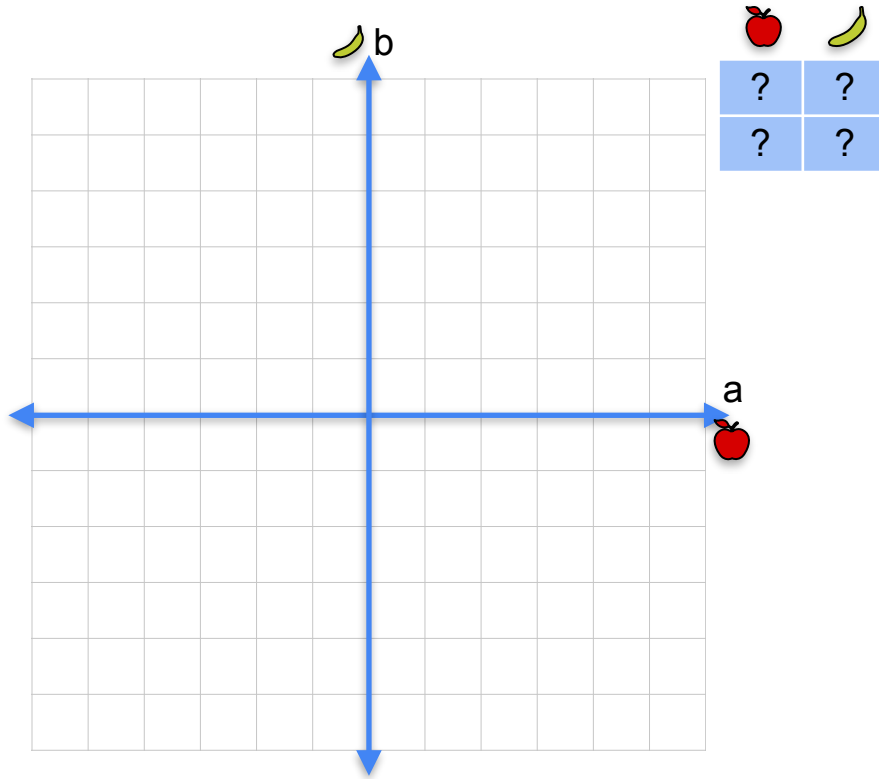
DeepLearning.AI

# Vectors and Linear Transformations

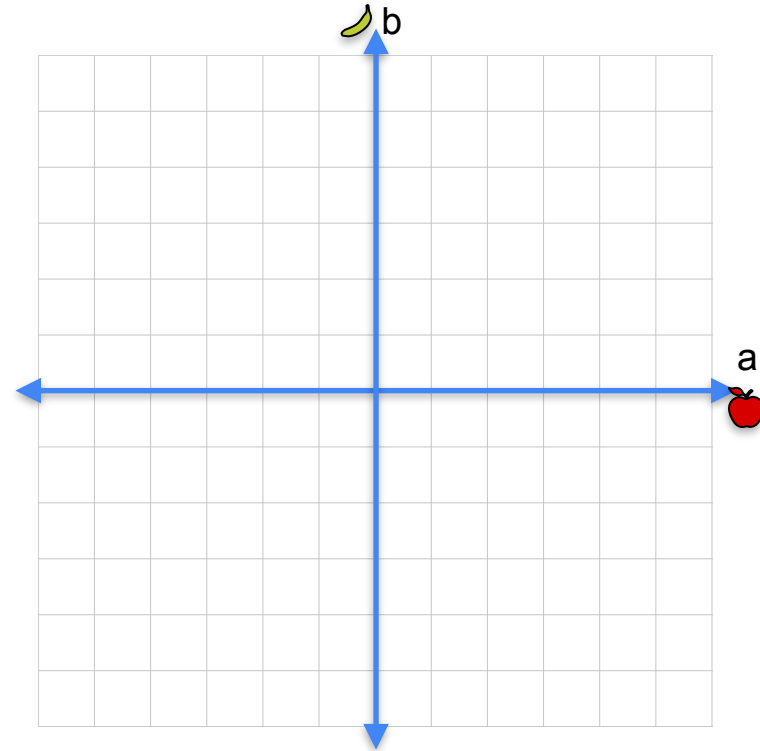
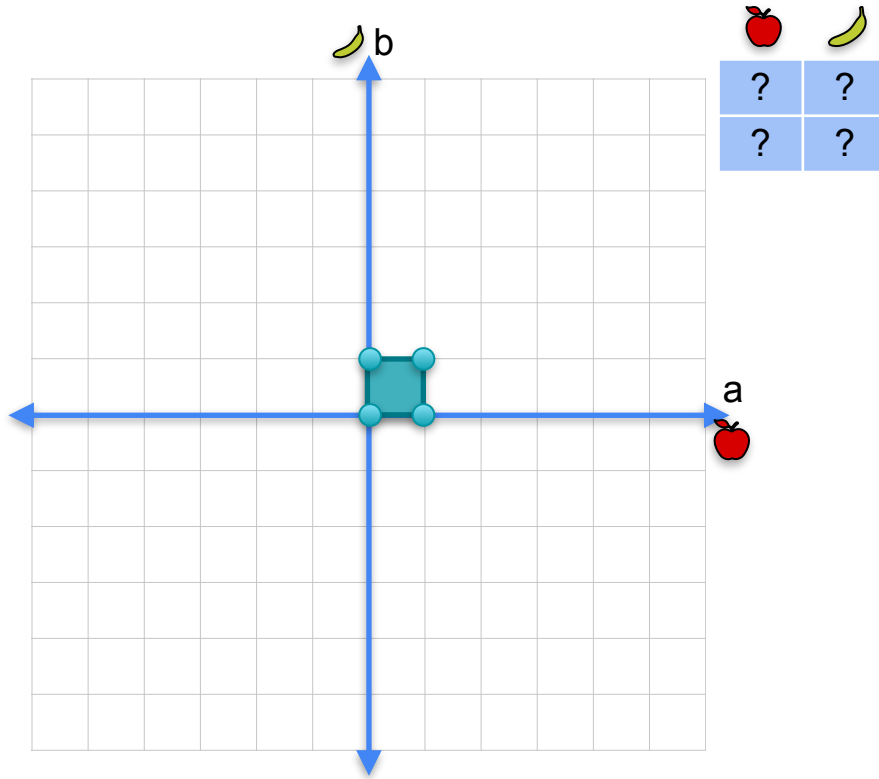
---

**Linear transformations as  
matrices**

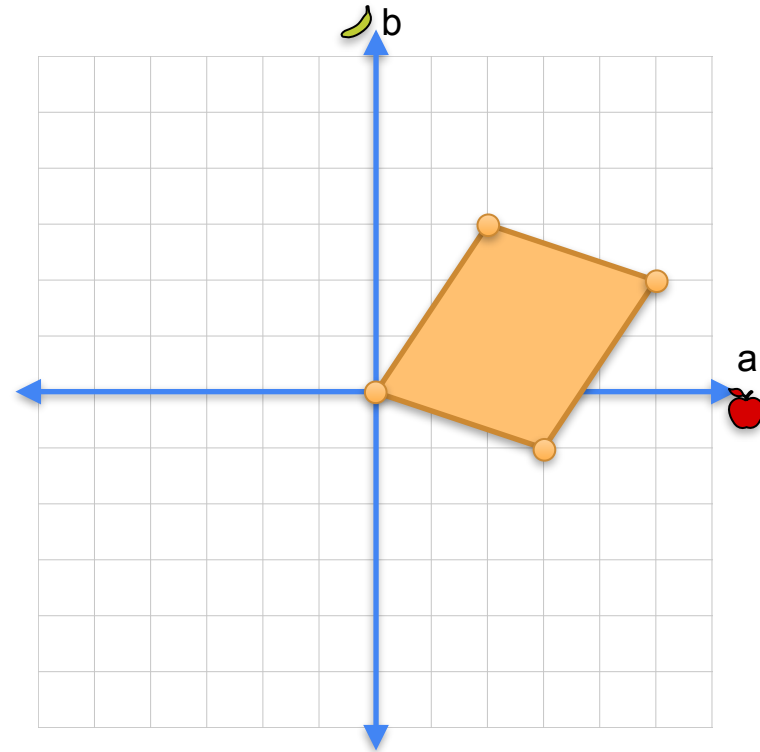
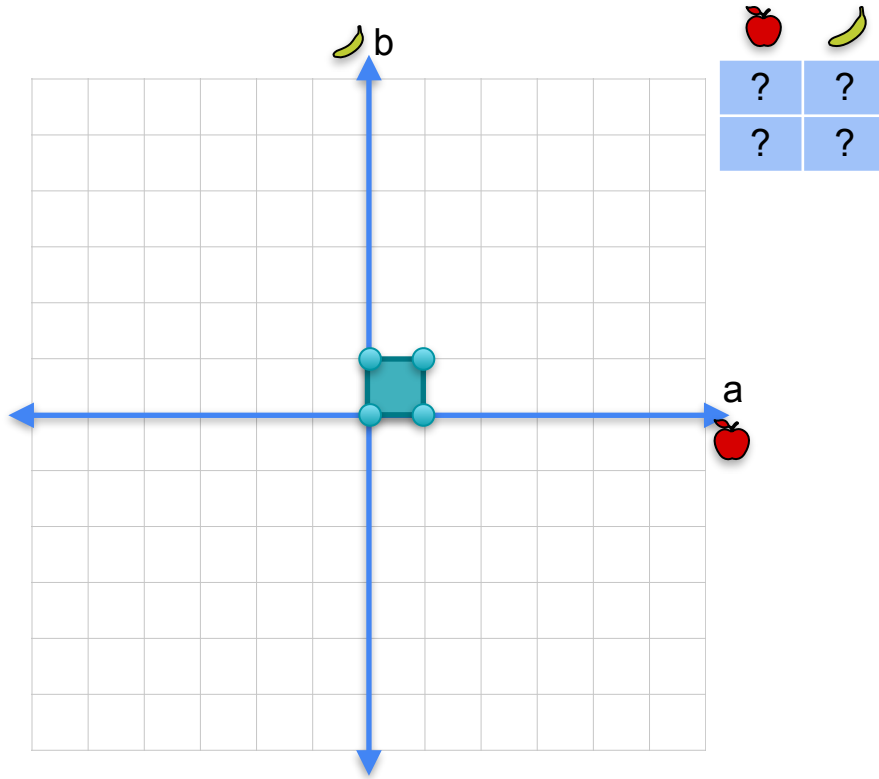
# Linear transformations as matrices



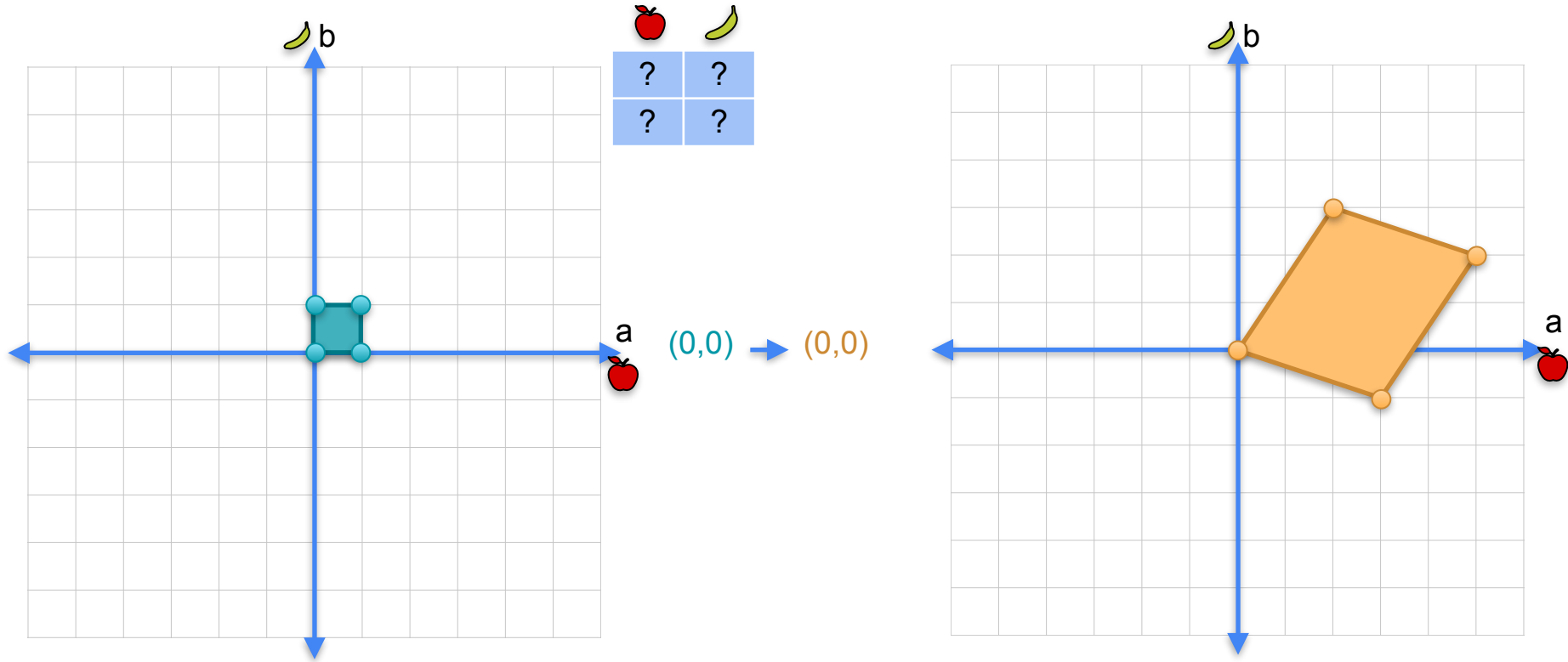
# Linear transformations as matrices



# Linear transformations as matrices

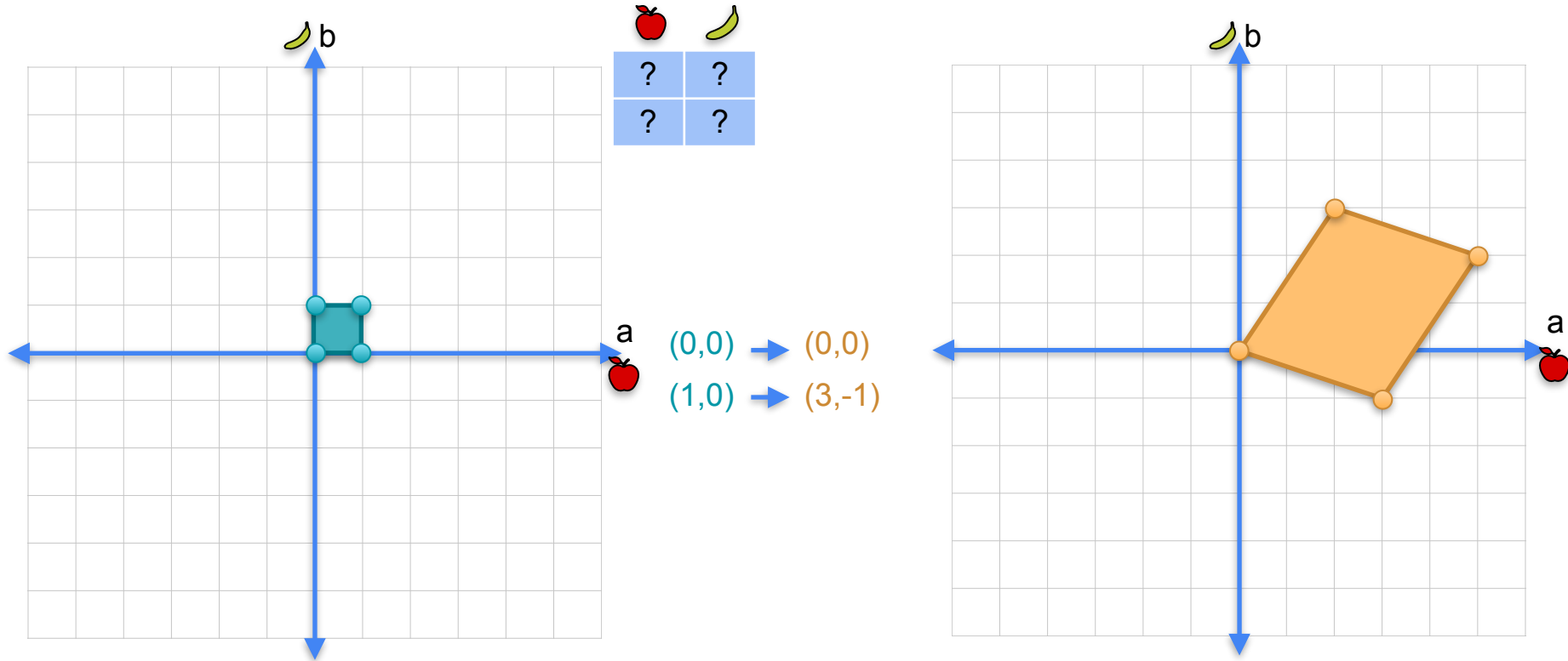


# Linear transformations as matrices

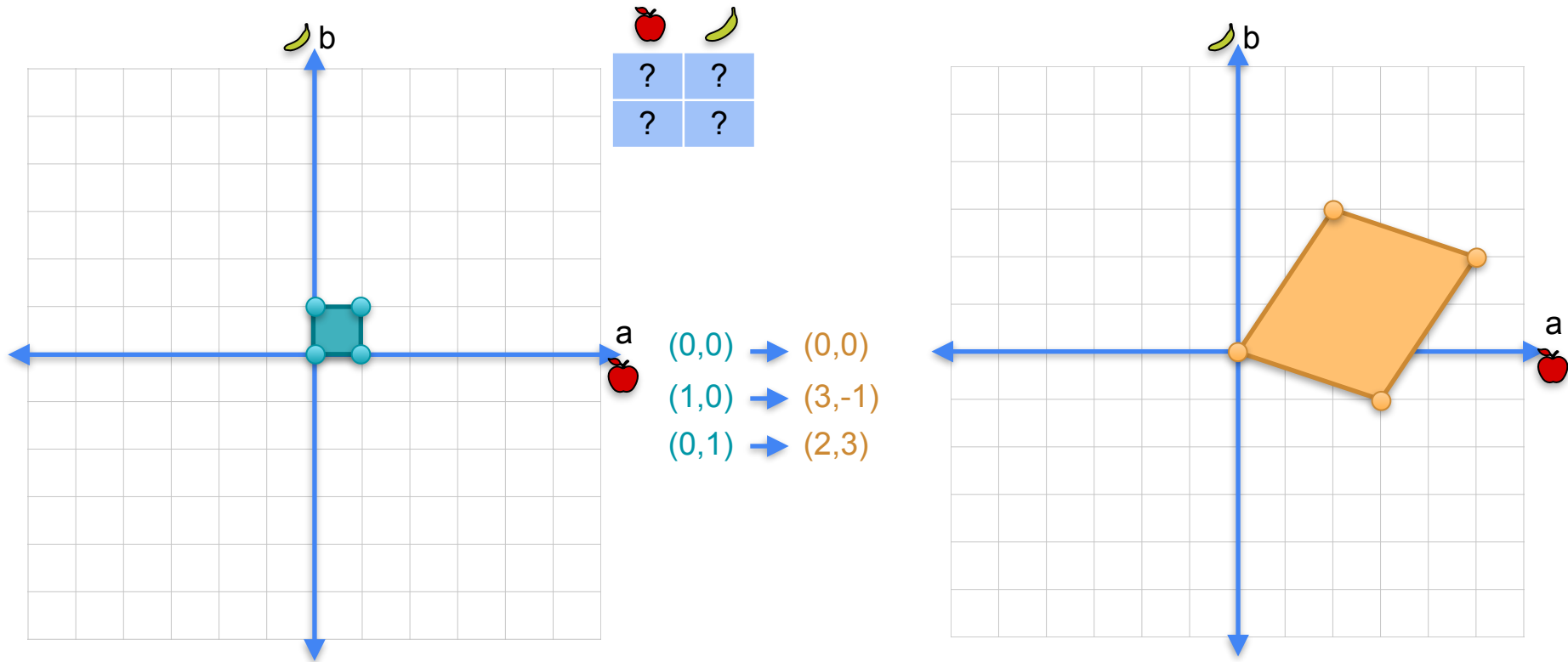




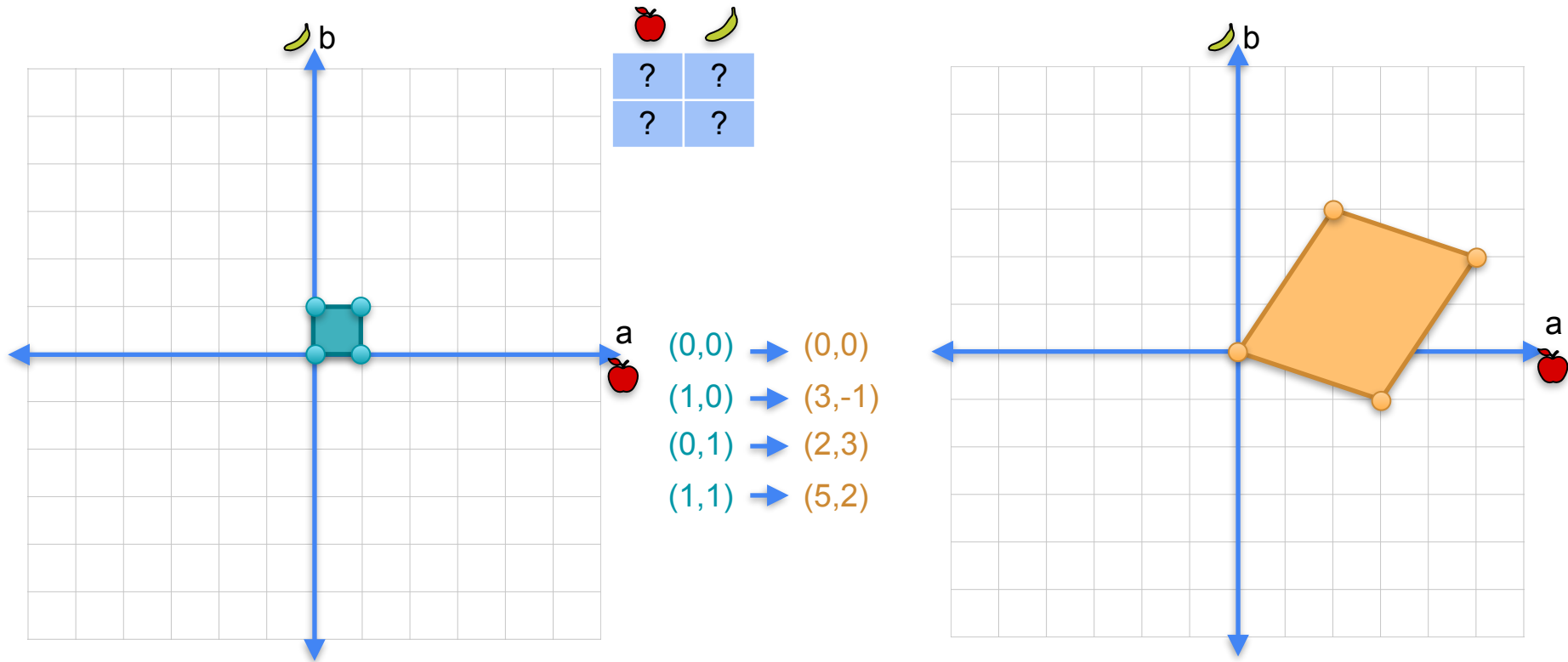
# Linear transformations as matrices



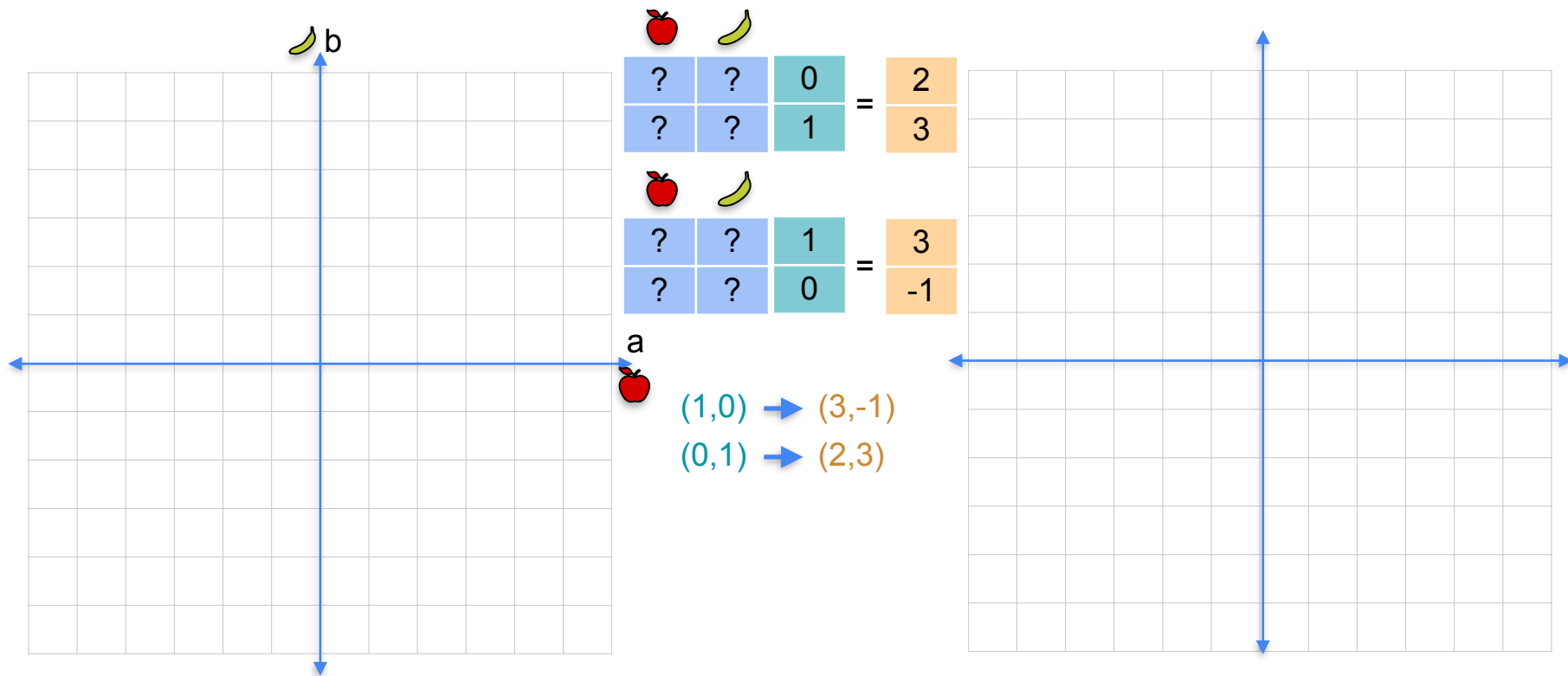
# Linear transformations as matrices



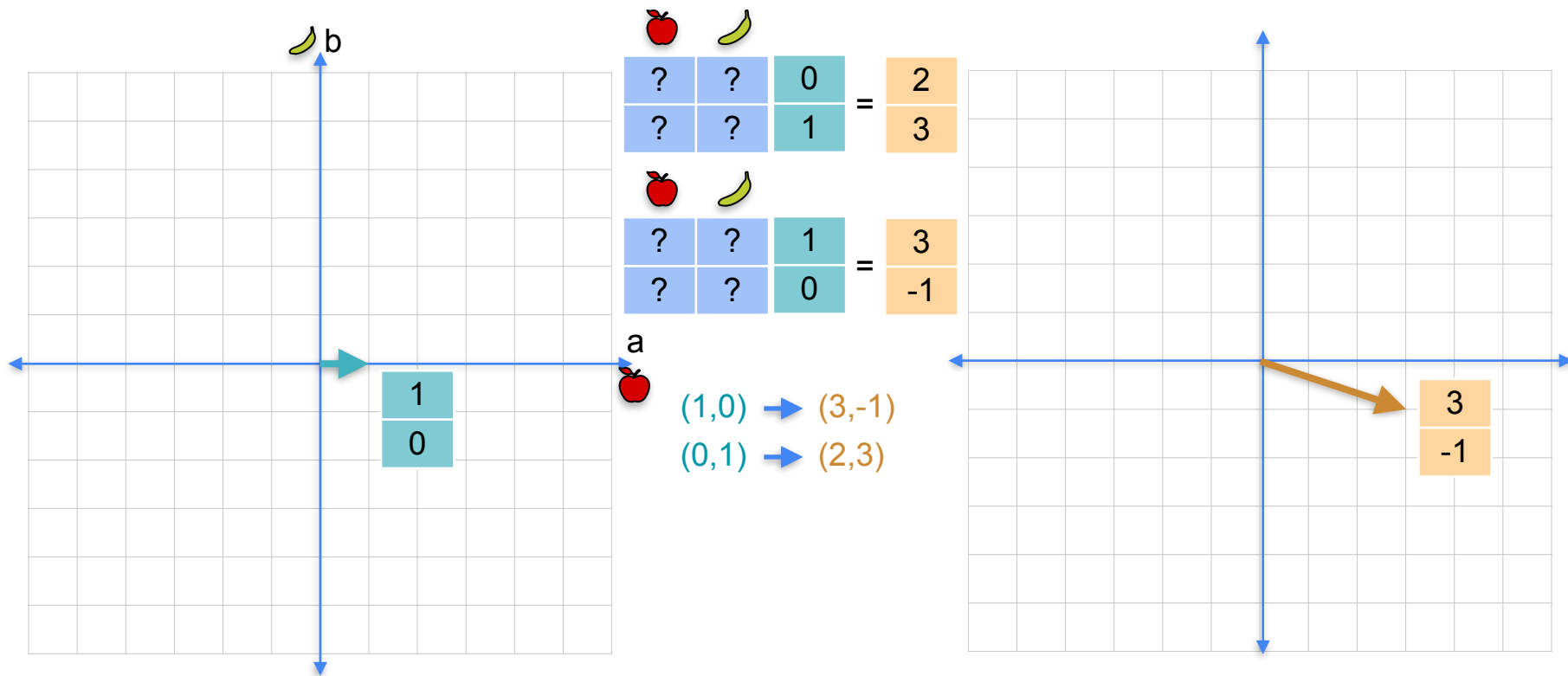
# Linear transformations as matrices



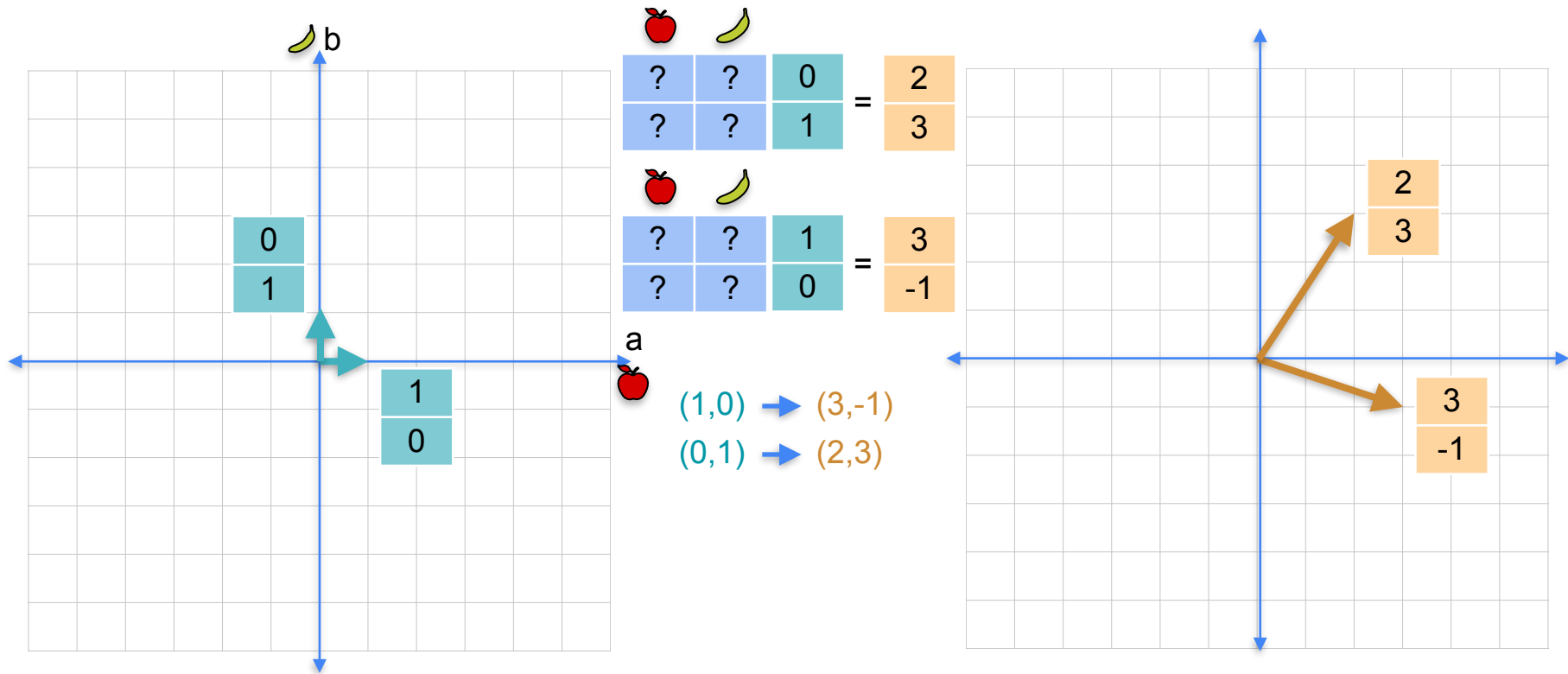
# Linear transformations as matrices



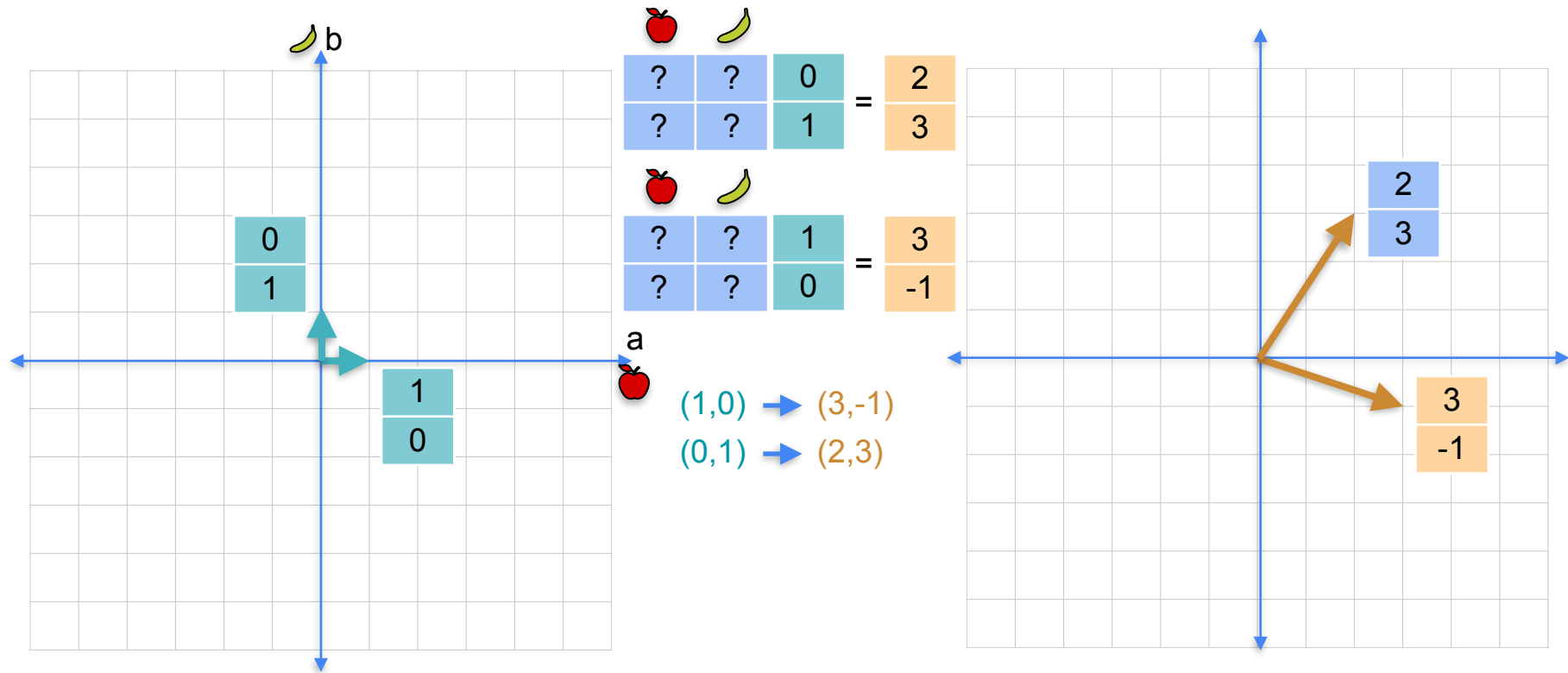
# Linear transformations as matrices



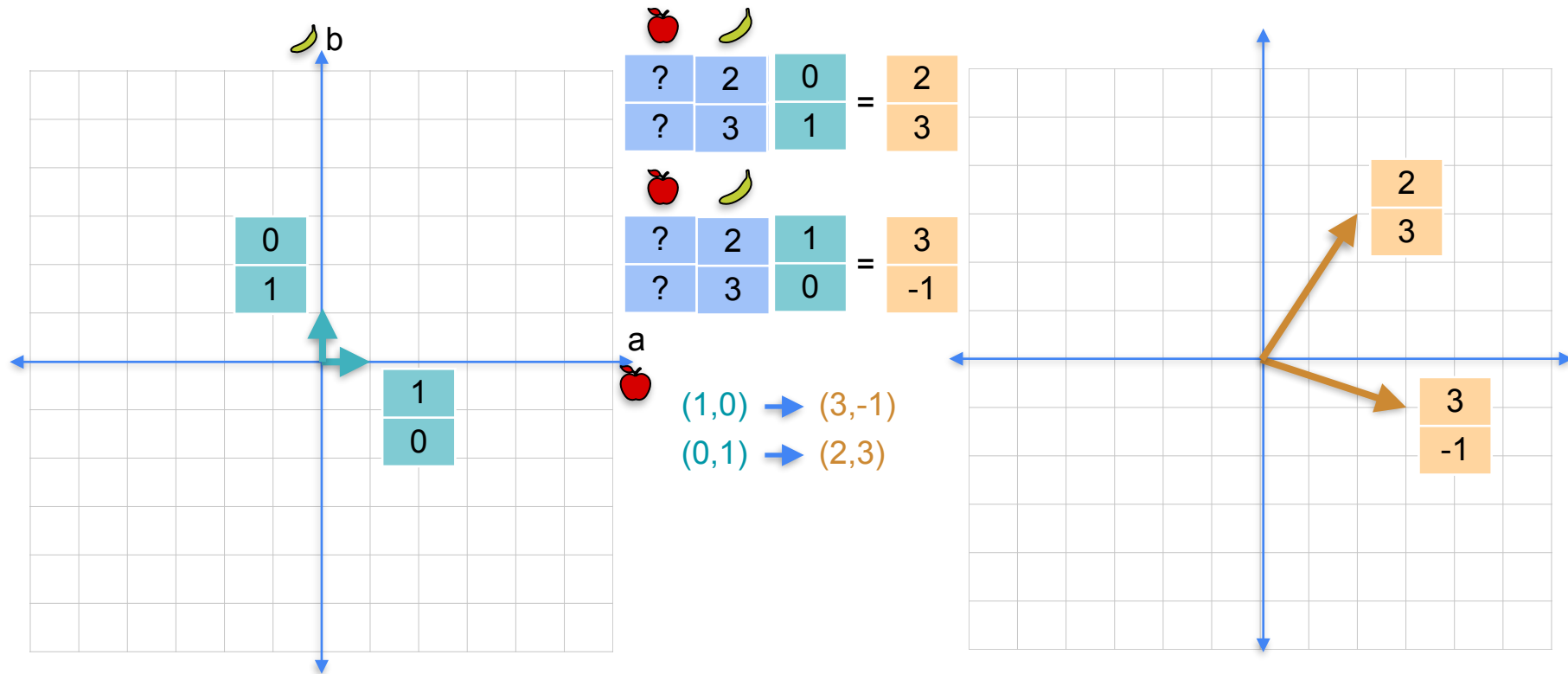
# Linear transformations as matrices



# Linear transformations as matrices

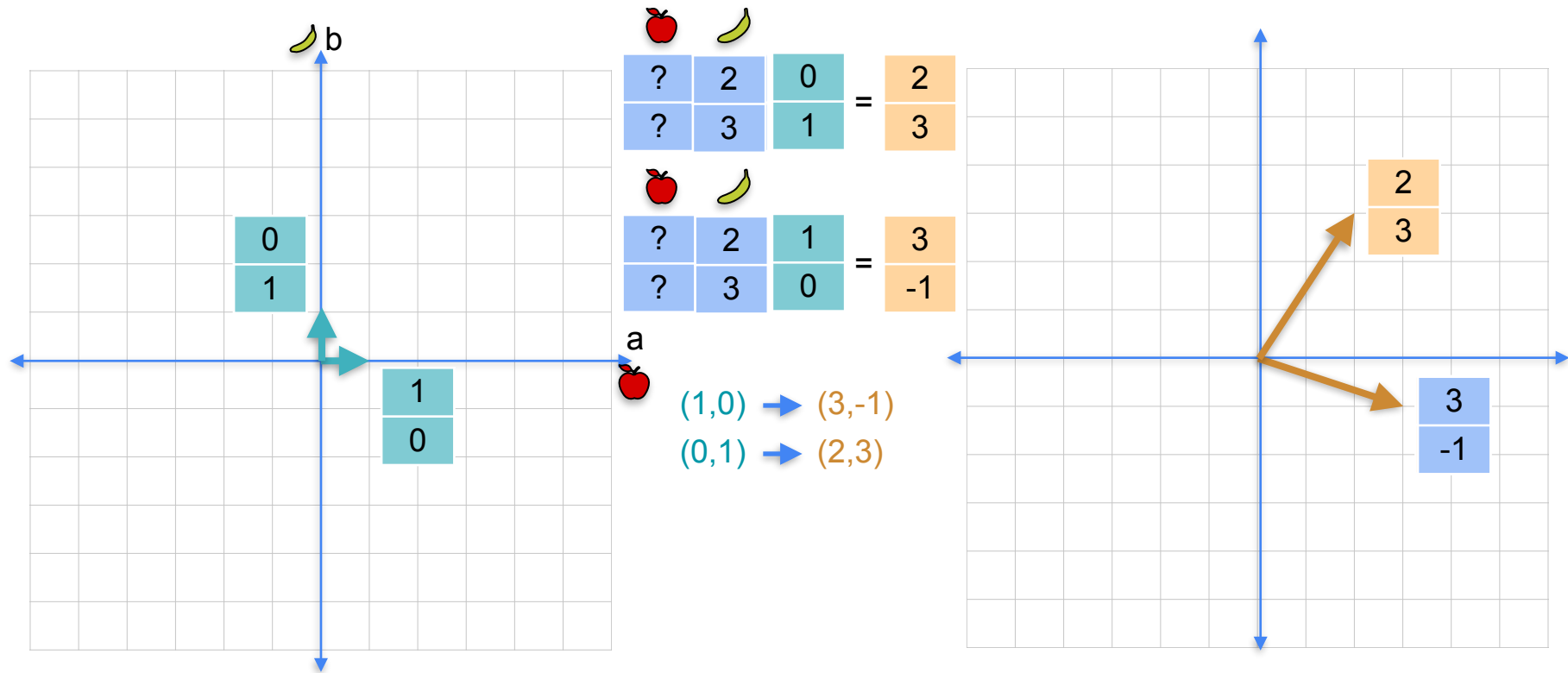


# Linear transformations as matrices

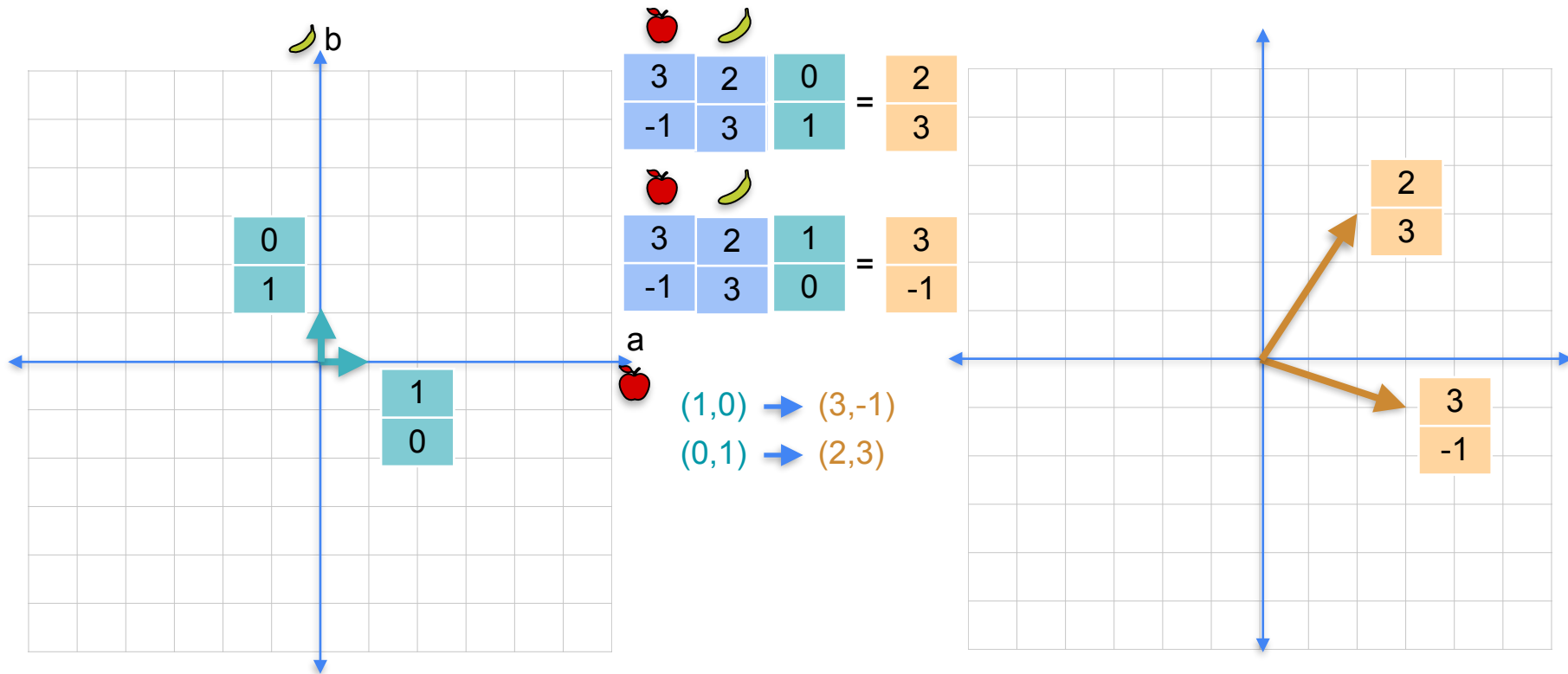




# Linear transformations as matrices



# Linear transformations as matrices





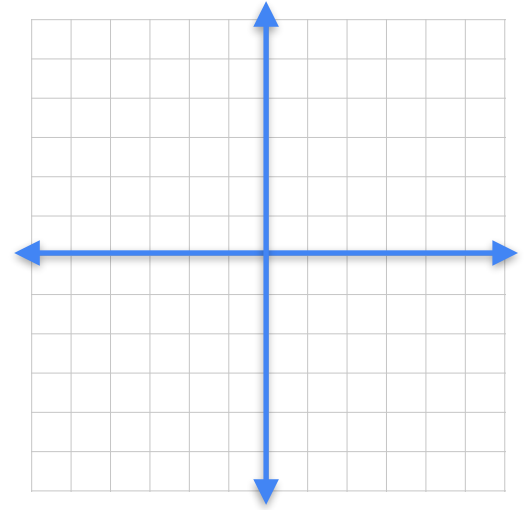
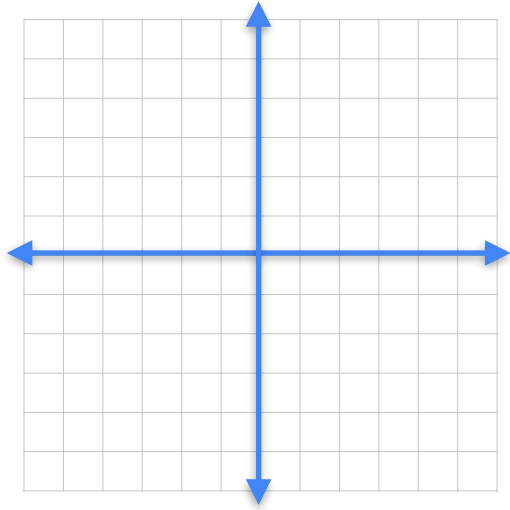
DeepLearning.AI

# Vectors and Linear Transformations

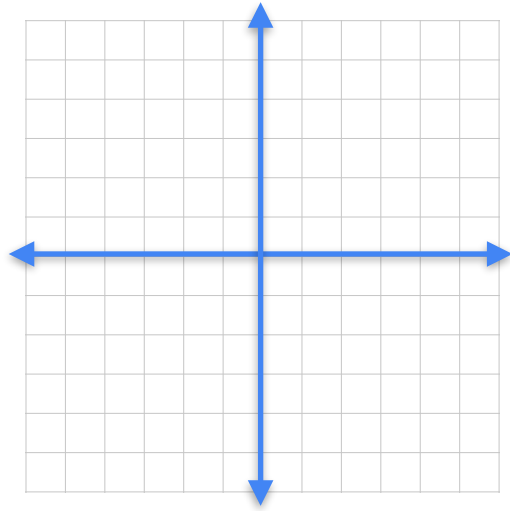
---

## **Matrix multiplication**

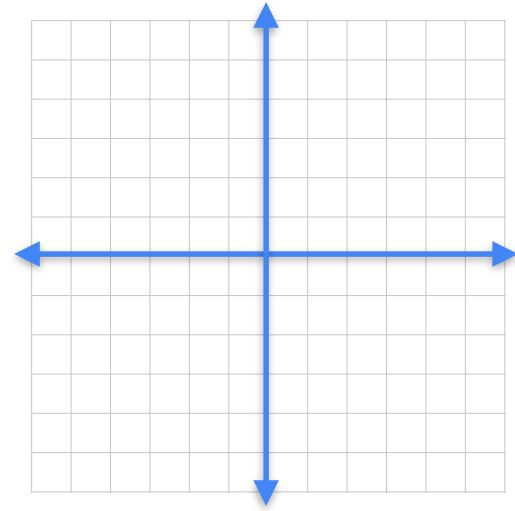
# Combining linear transformations



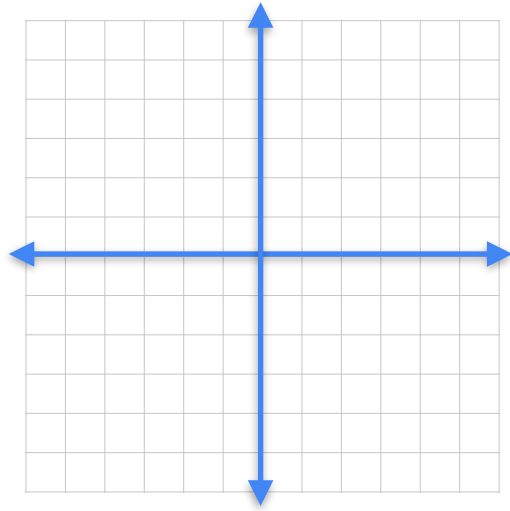
# Combining linear transformations



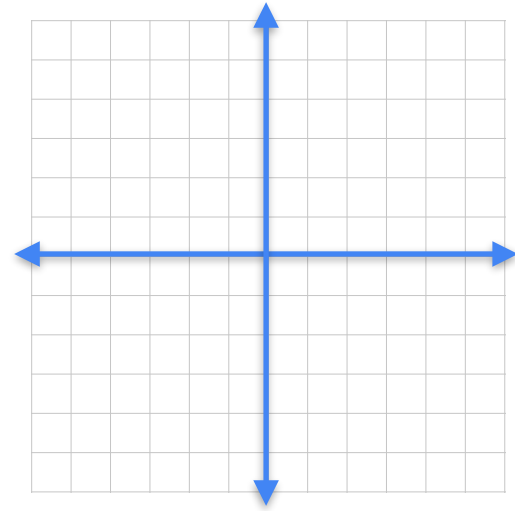
|   |   |
|---|---|
| 3 | 1 |
| 1 | 2 |



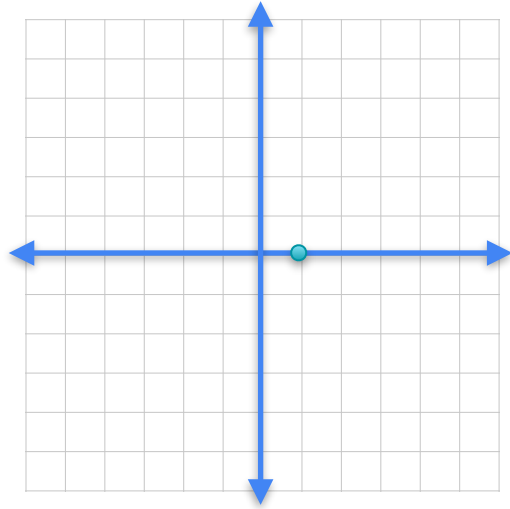
# Combining linear transformations



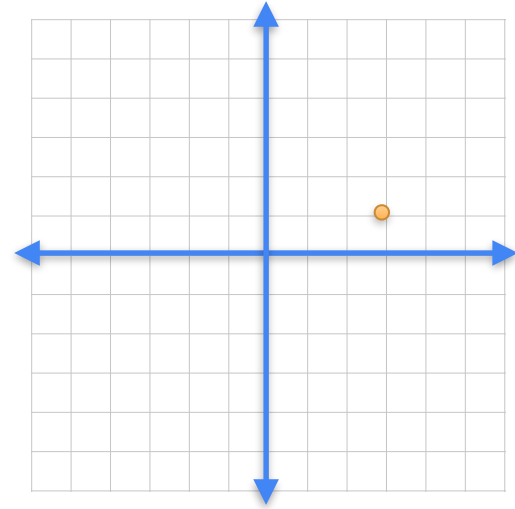
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$



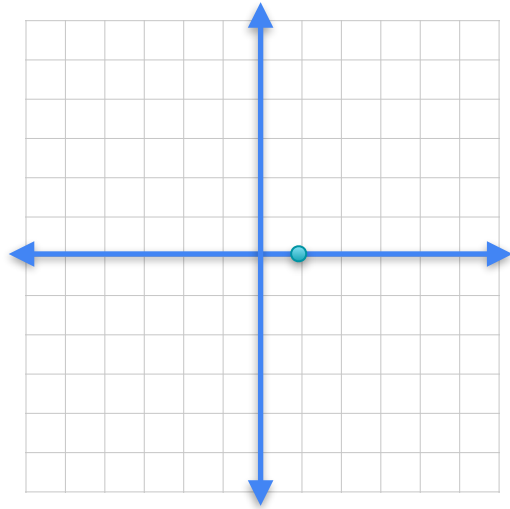
# Combining linear transformations



$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

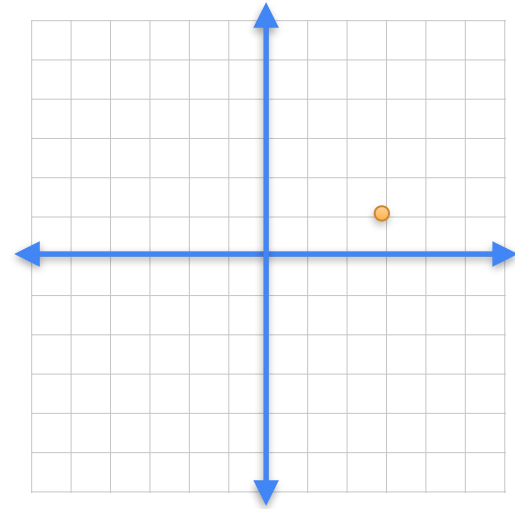


# Combining linear transformations



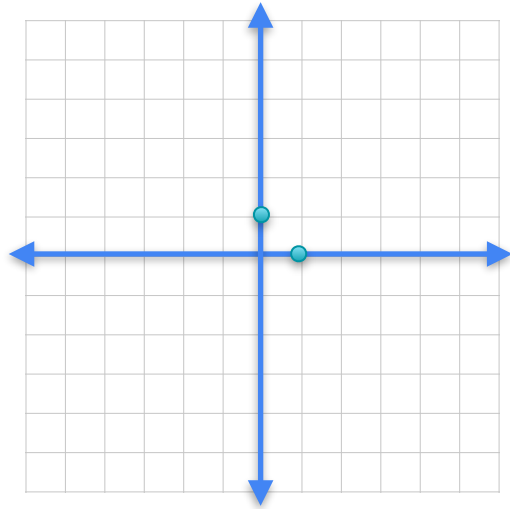
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$



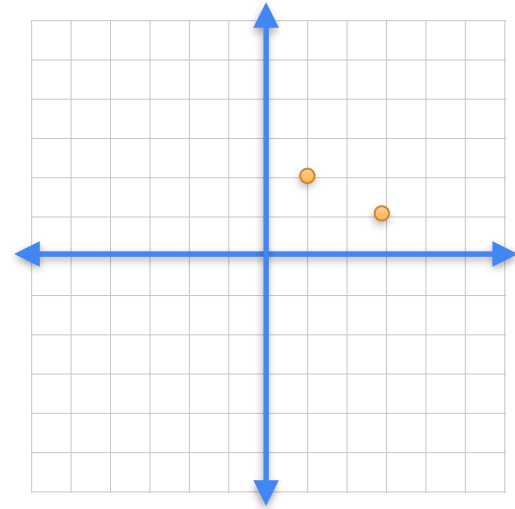


# Combining linear transformations

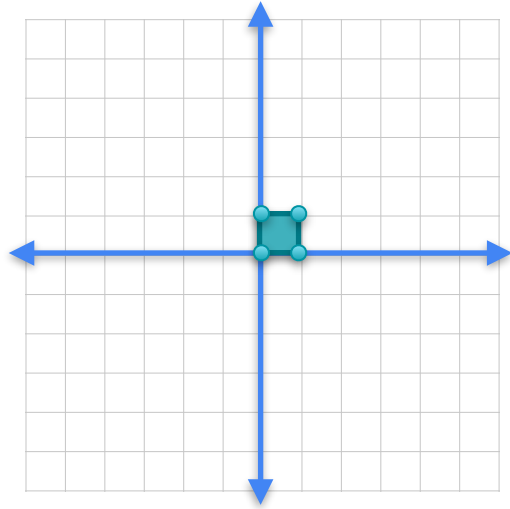


$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

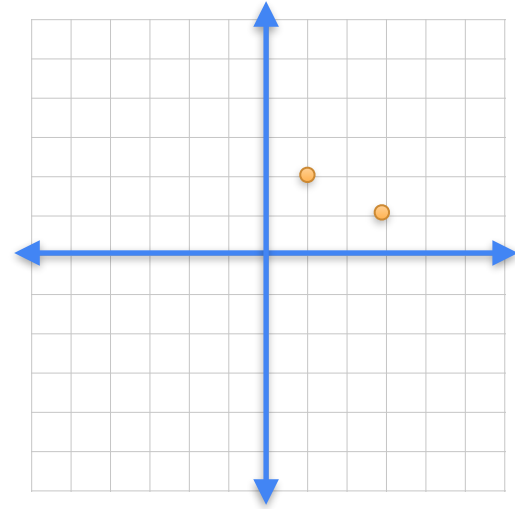


# Combining linear transformations

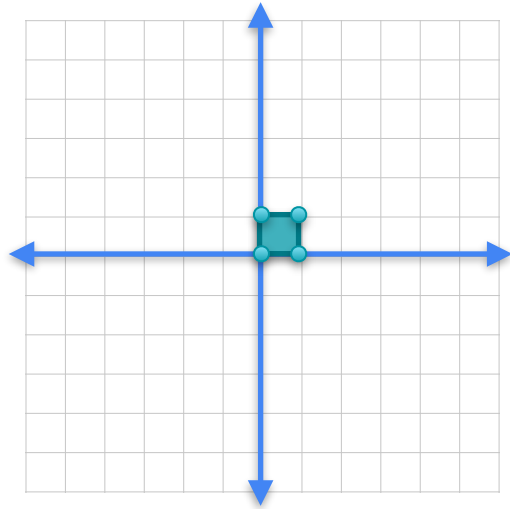


$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

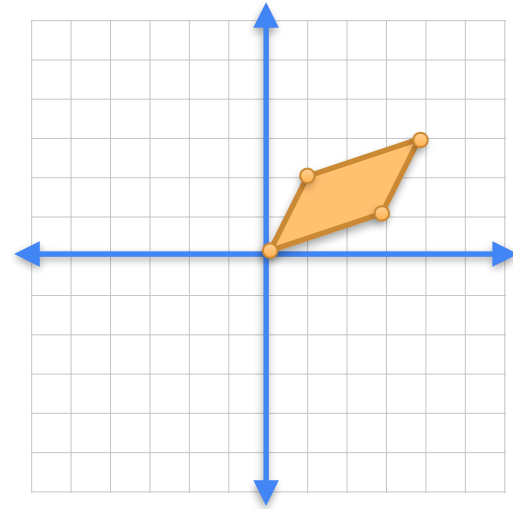


# Combining linear transformations

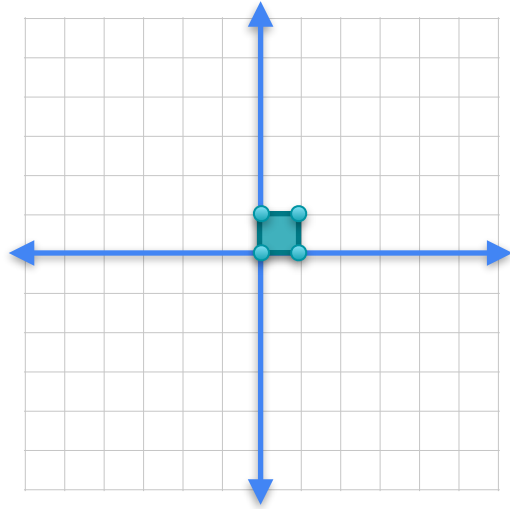


$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

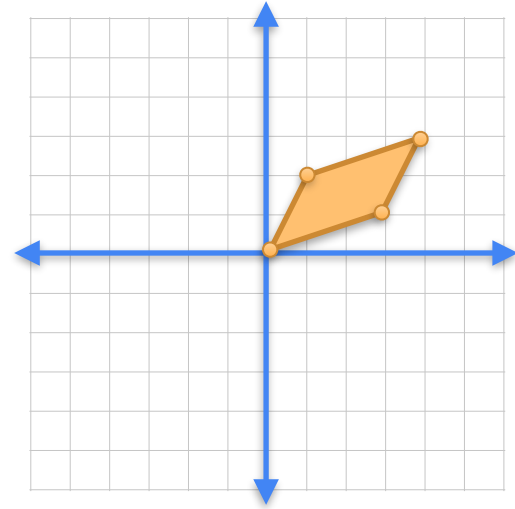


# Combining linear transformations

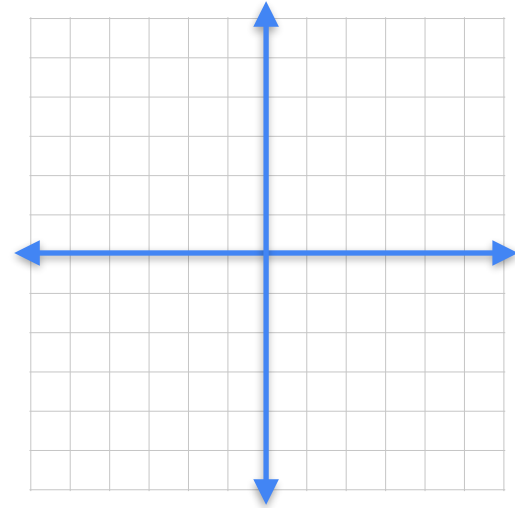
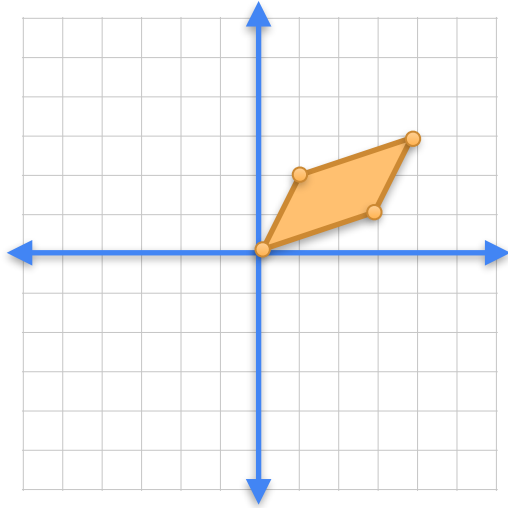


$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

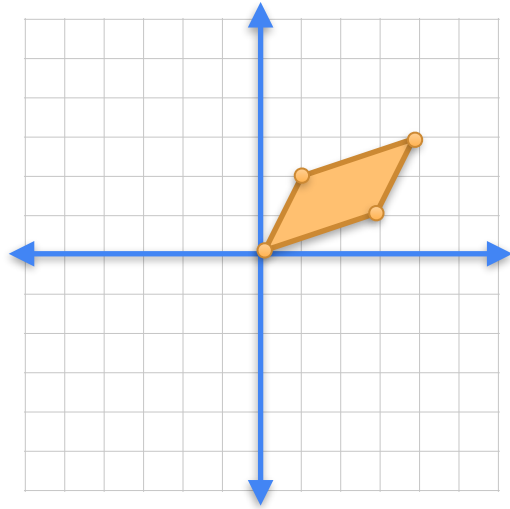
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$



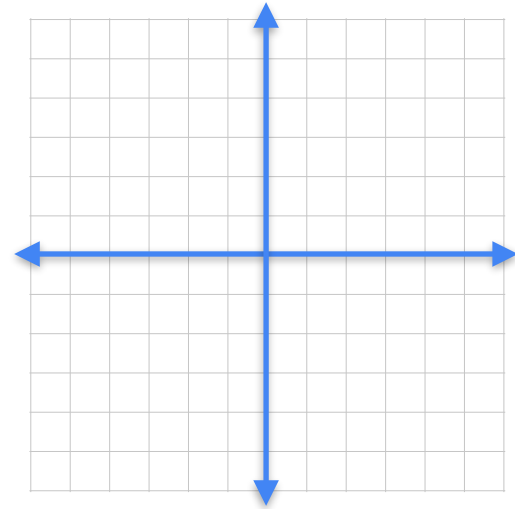
# Combining linear transformations



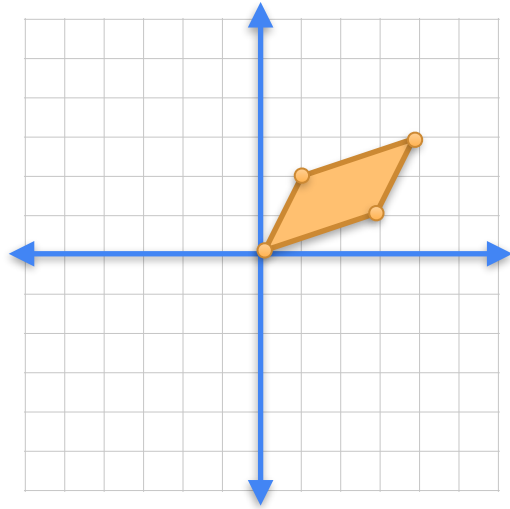
# Combining linear transformations



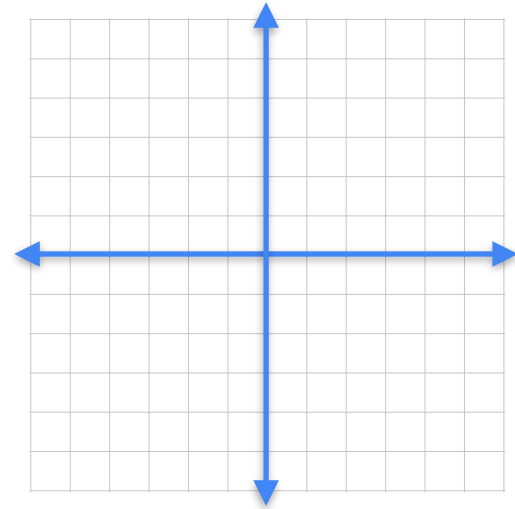
|   |    |
|---|----|
| 2 | -1 |
| 0 | 2  |



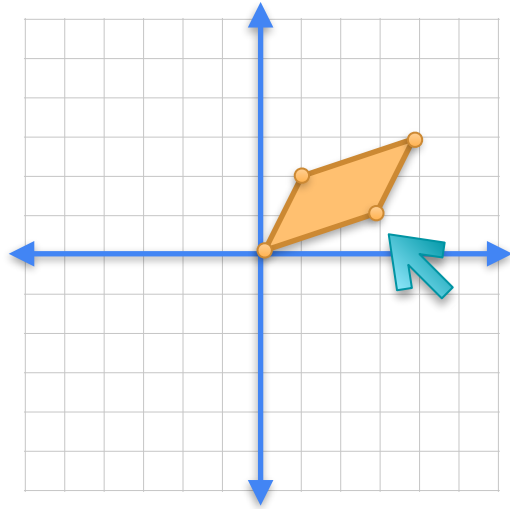
# Combining linear transformations



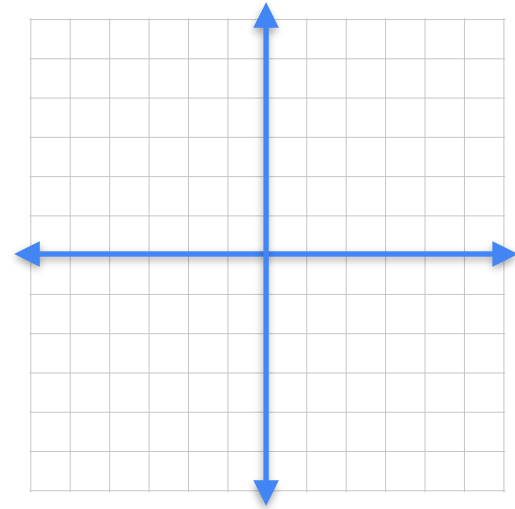
$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$



# Combining linear transformations

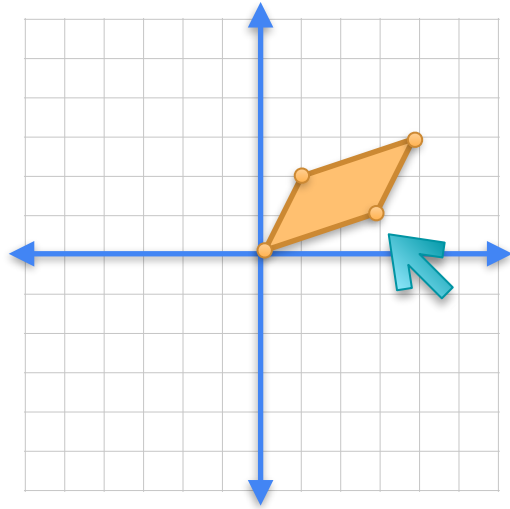


$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

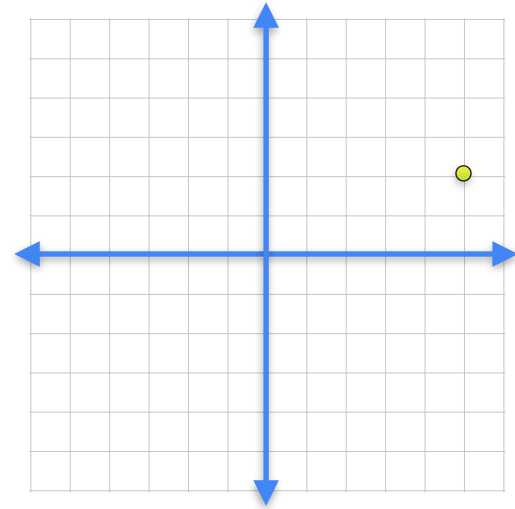




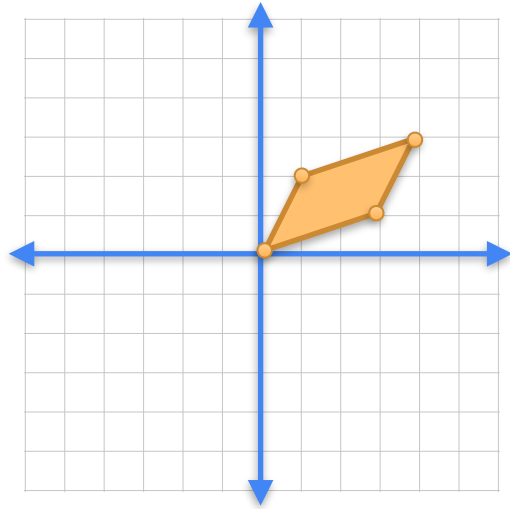
# Combining linear transformations



$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

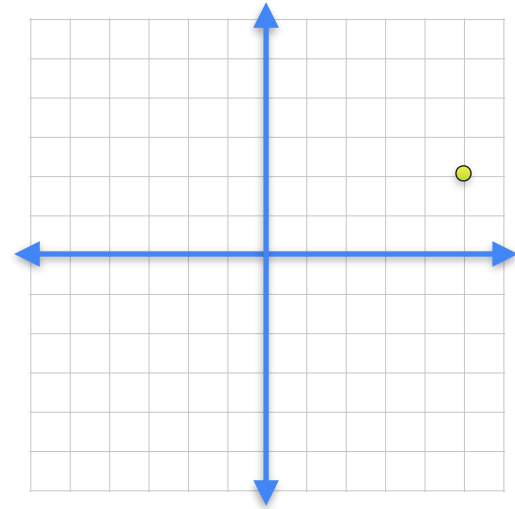


# Combining linear transformations

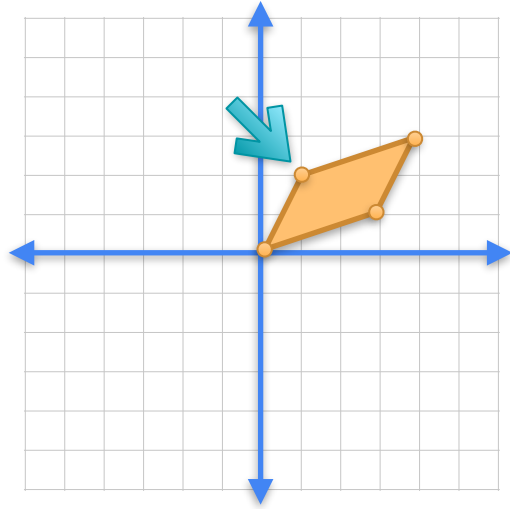


$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

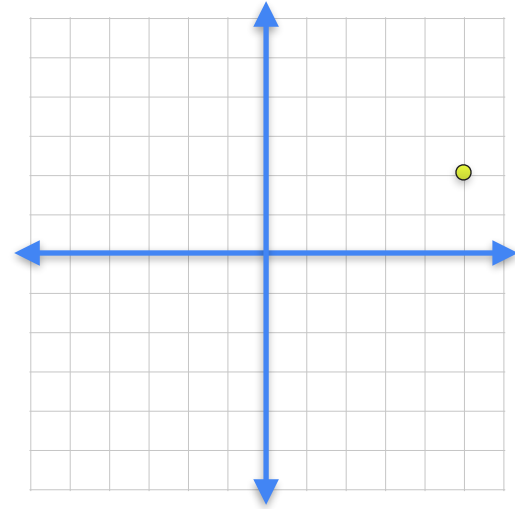


# Combining linear transformations

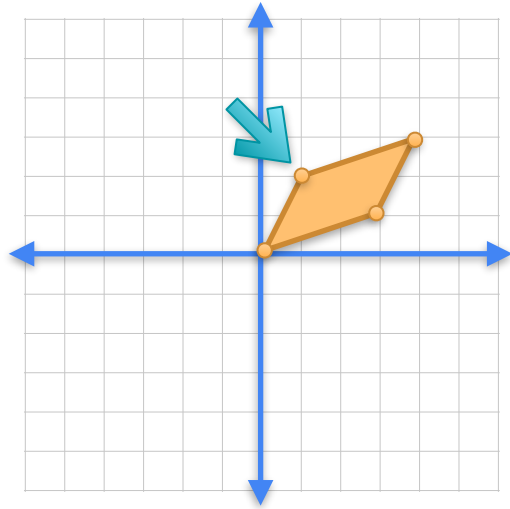


$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

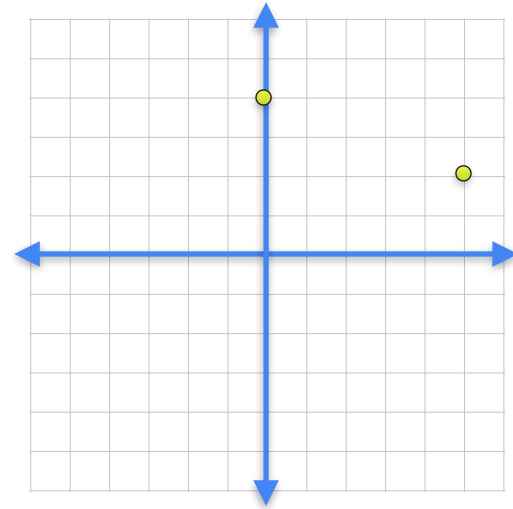


# Combining linear transformations

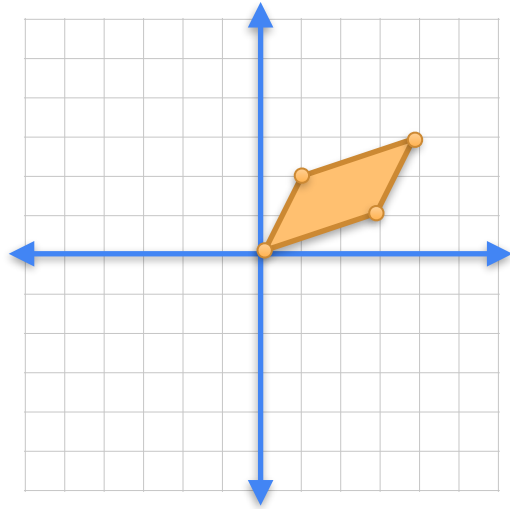


$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

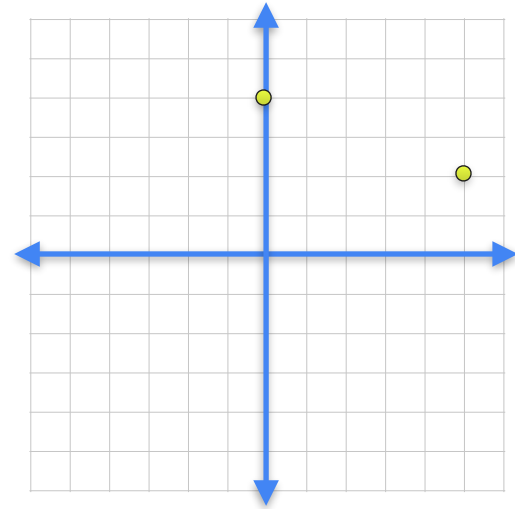


# Combining linear transformations

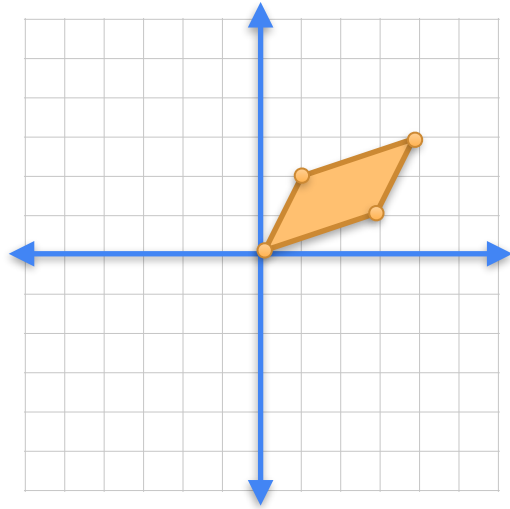


$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

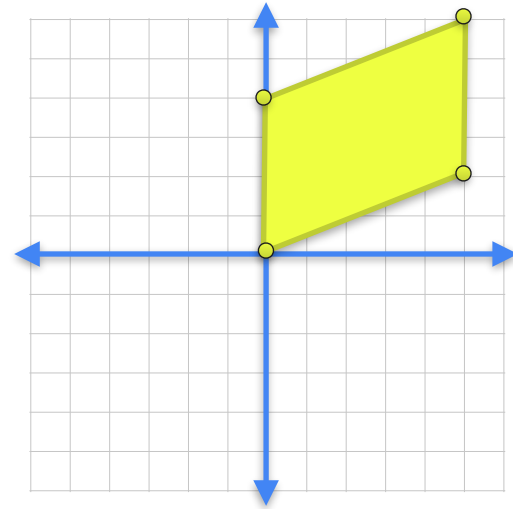


# Combining linear transformations

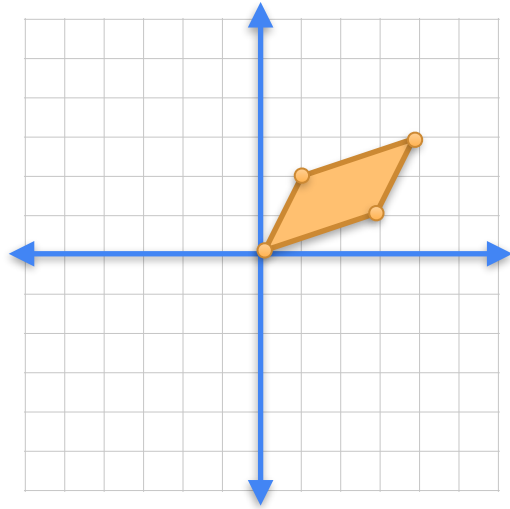


$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

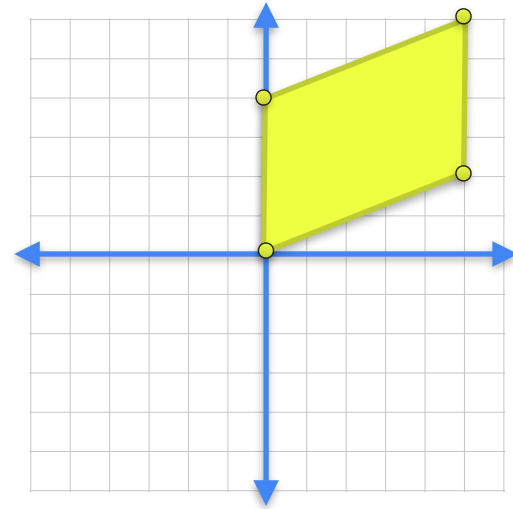


# Combining linear transformations

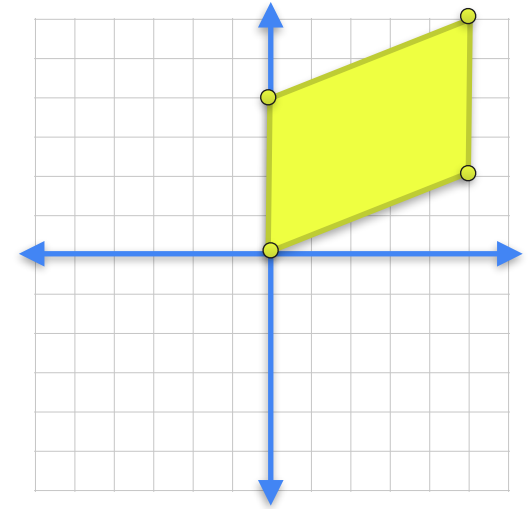
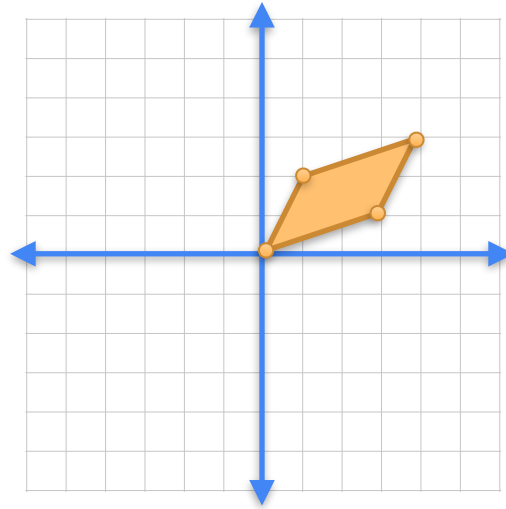
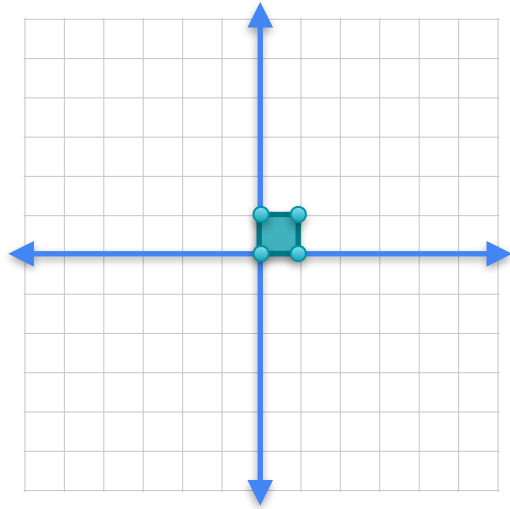


$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

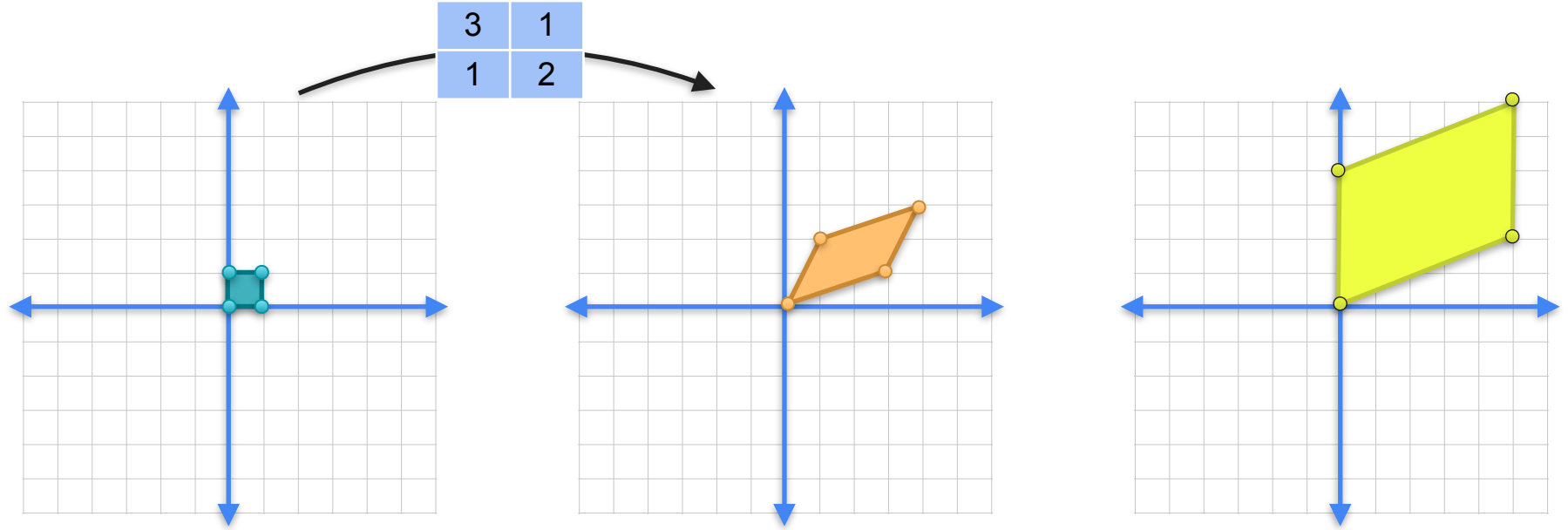


# Combining linear transformations

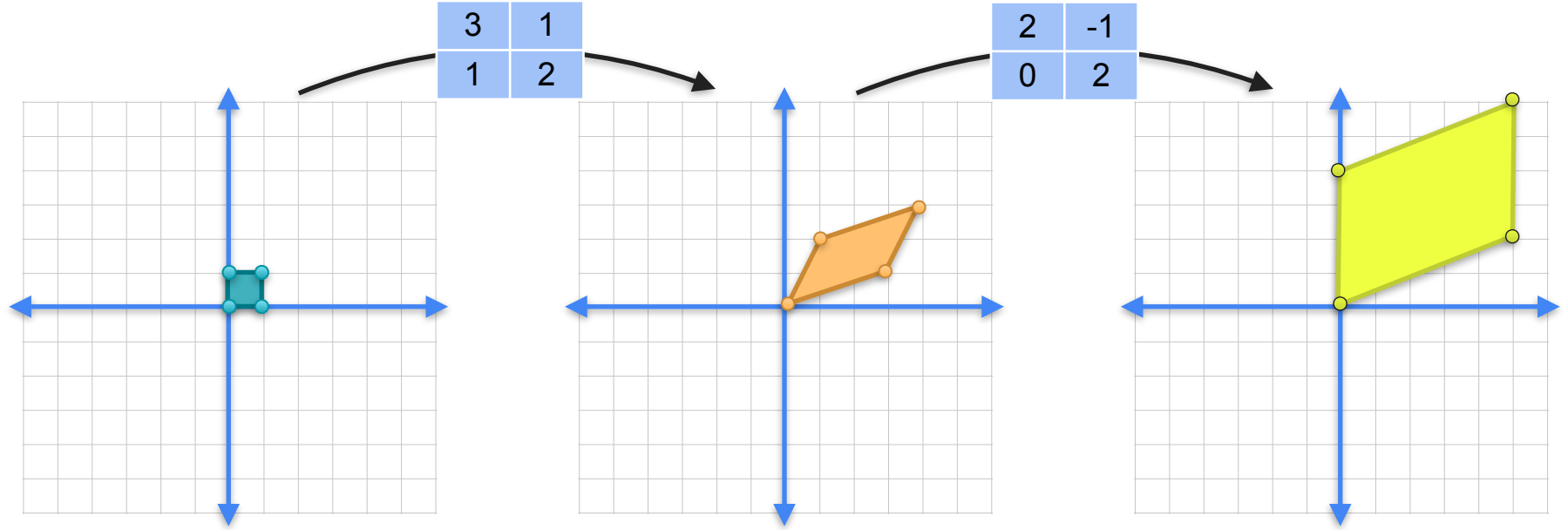




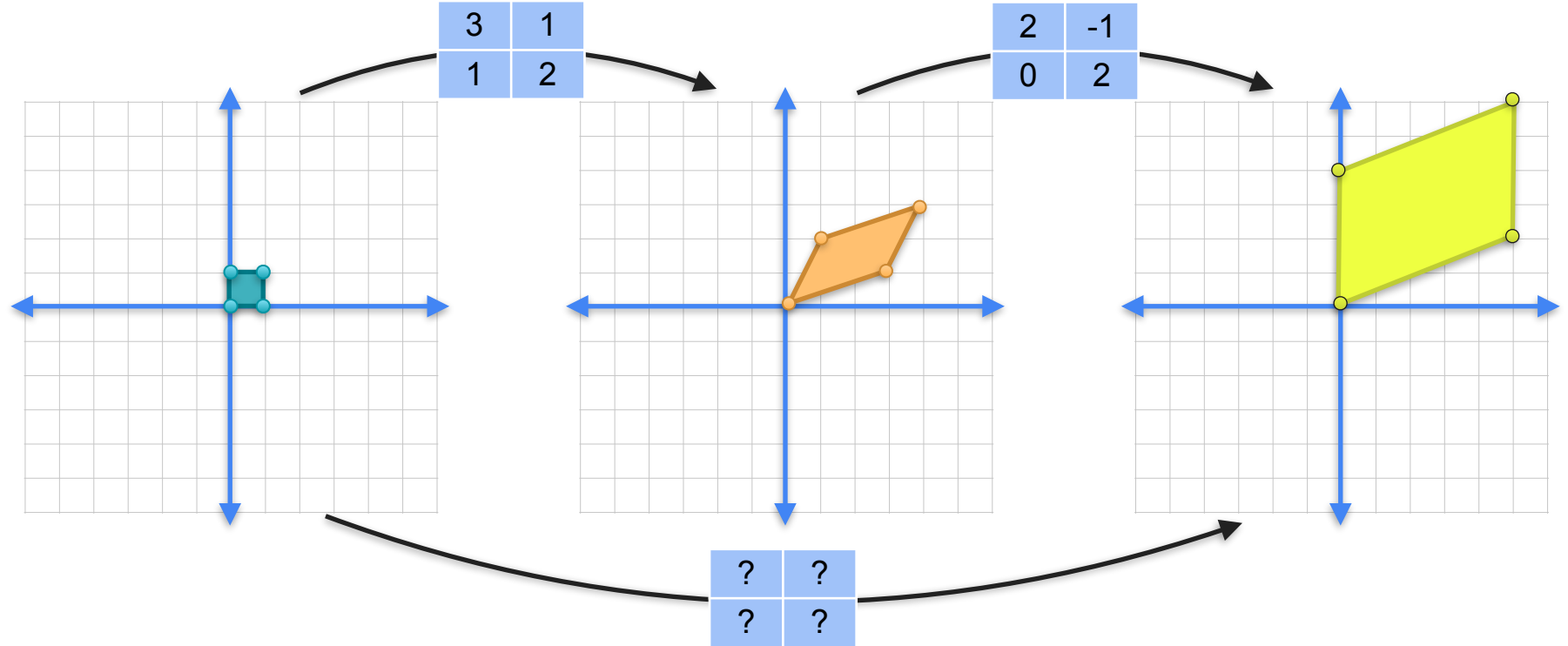
# Combining linear transformations



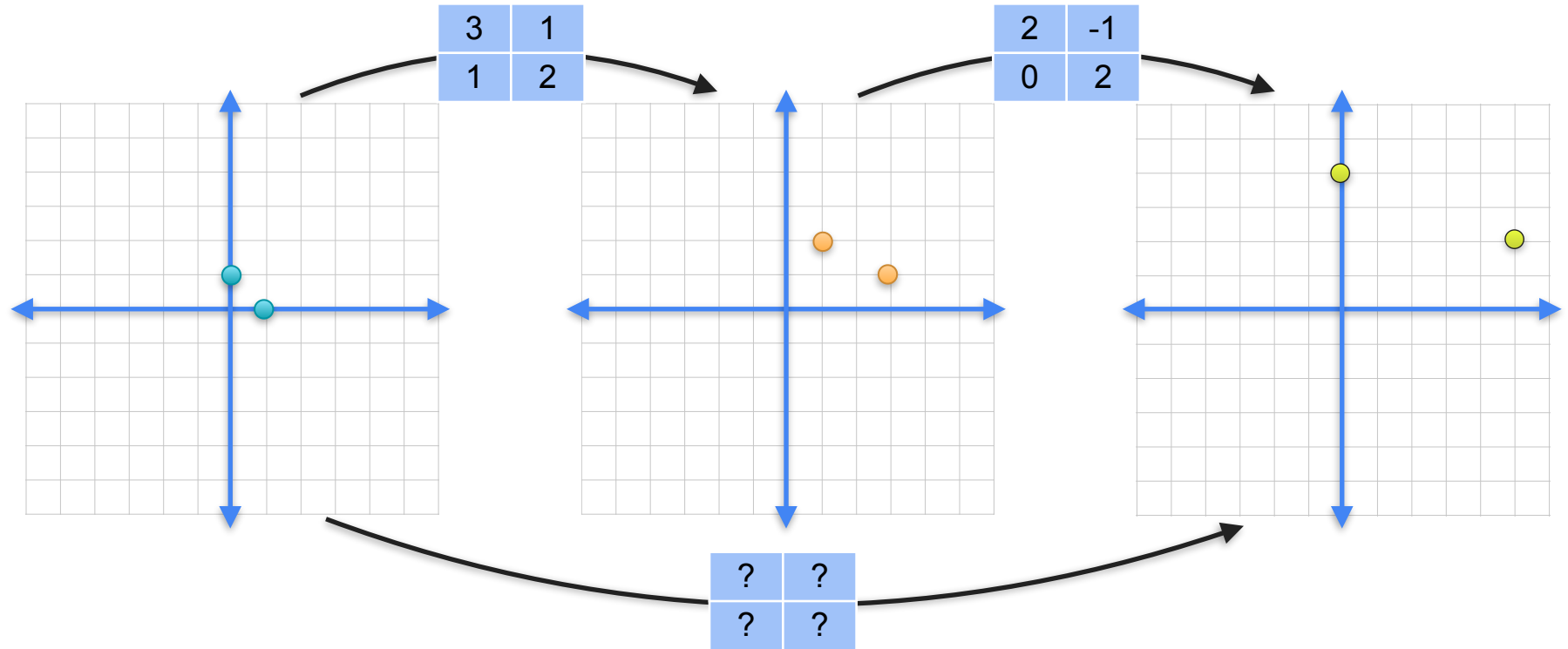
# Combining linear transformations



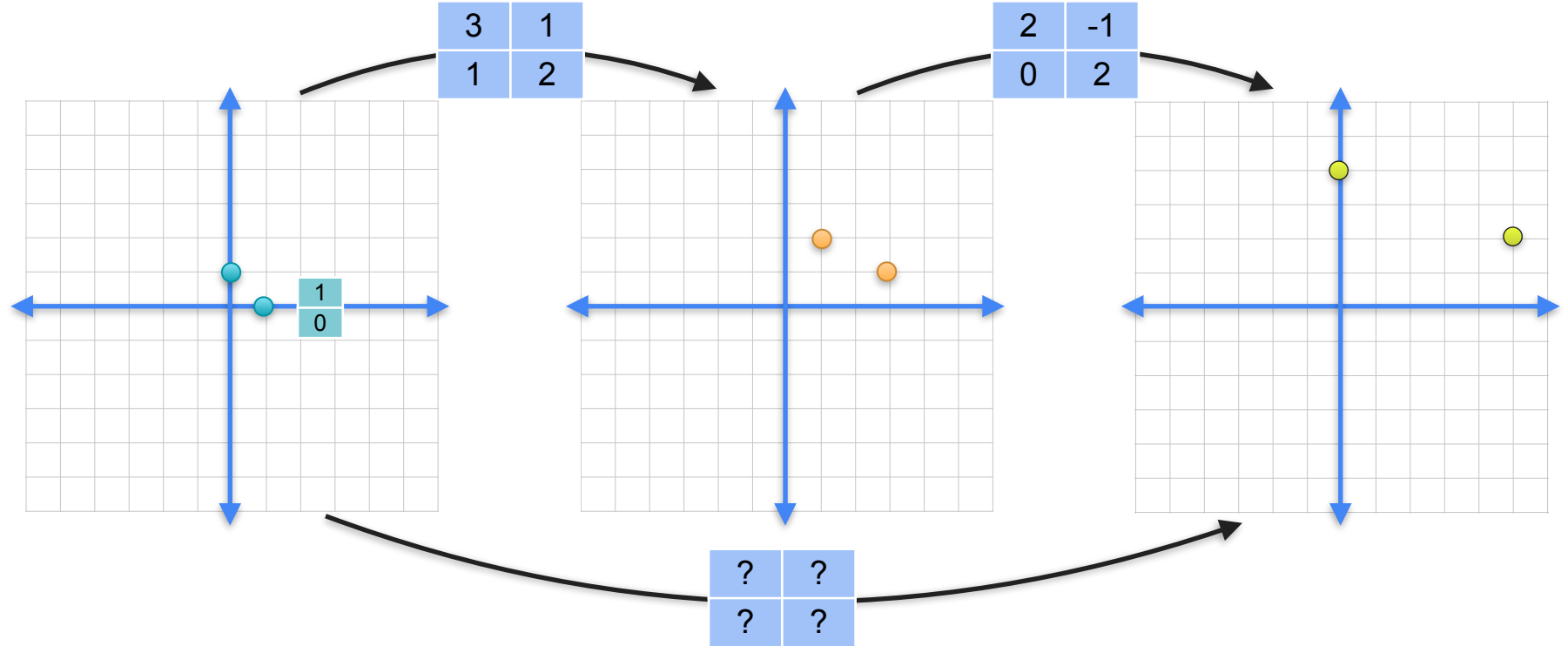
# Combining linear transformations



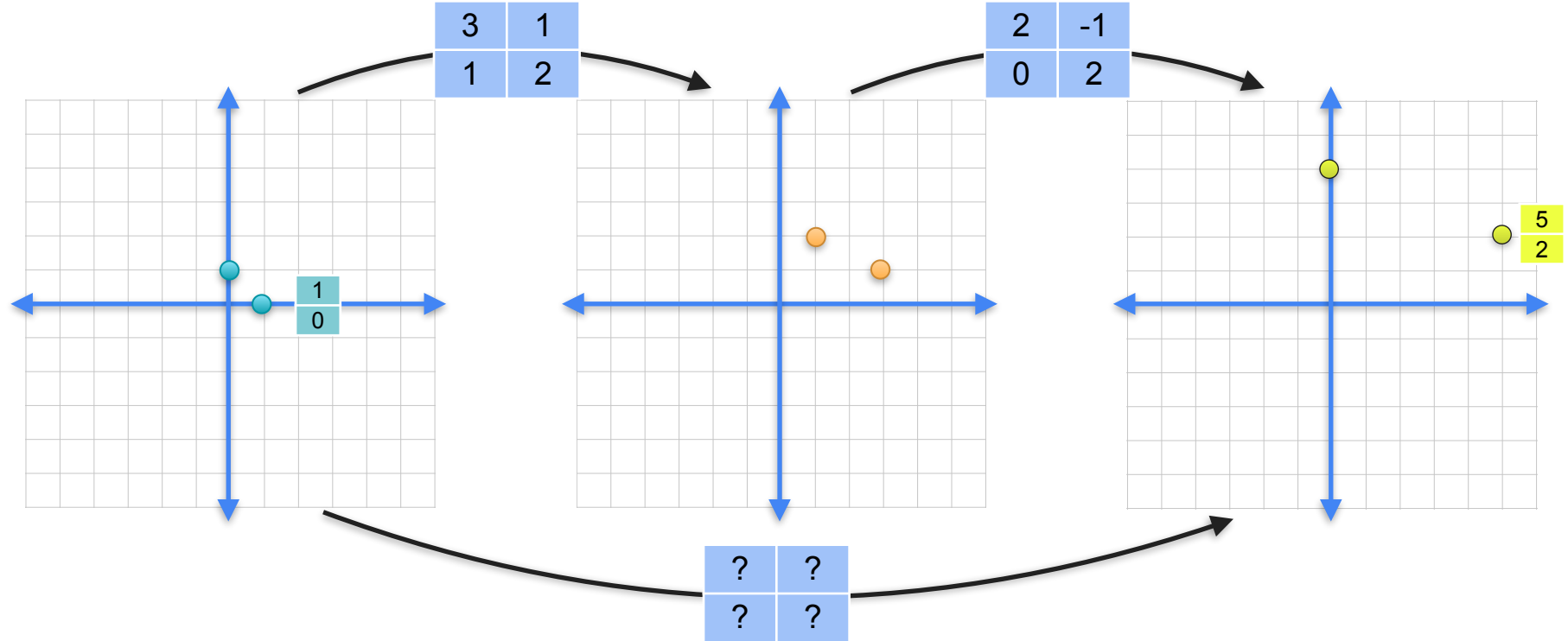
# Combining linear transformations



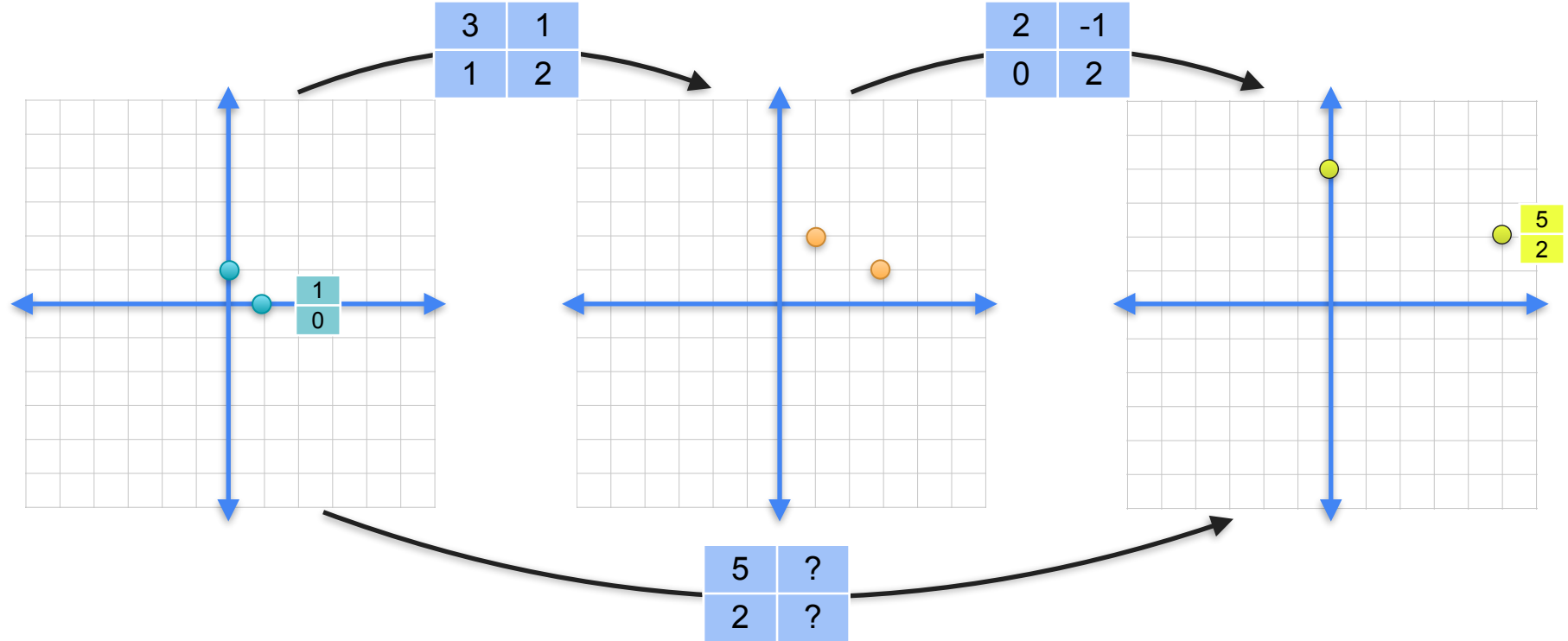
# Combining linear transformations



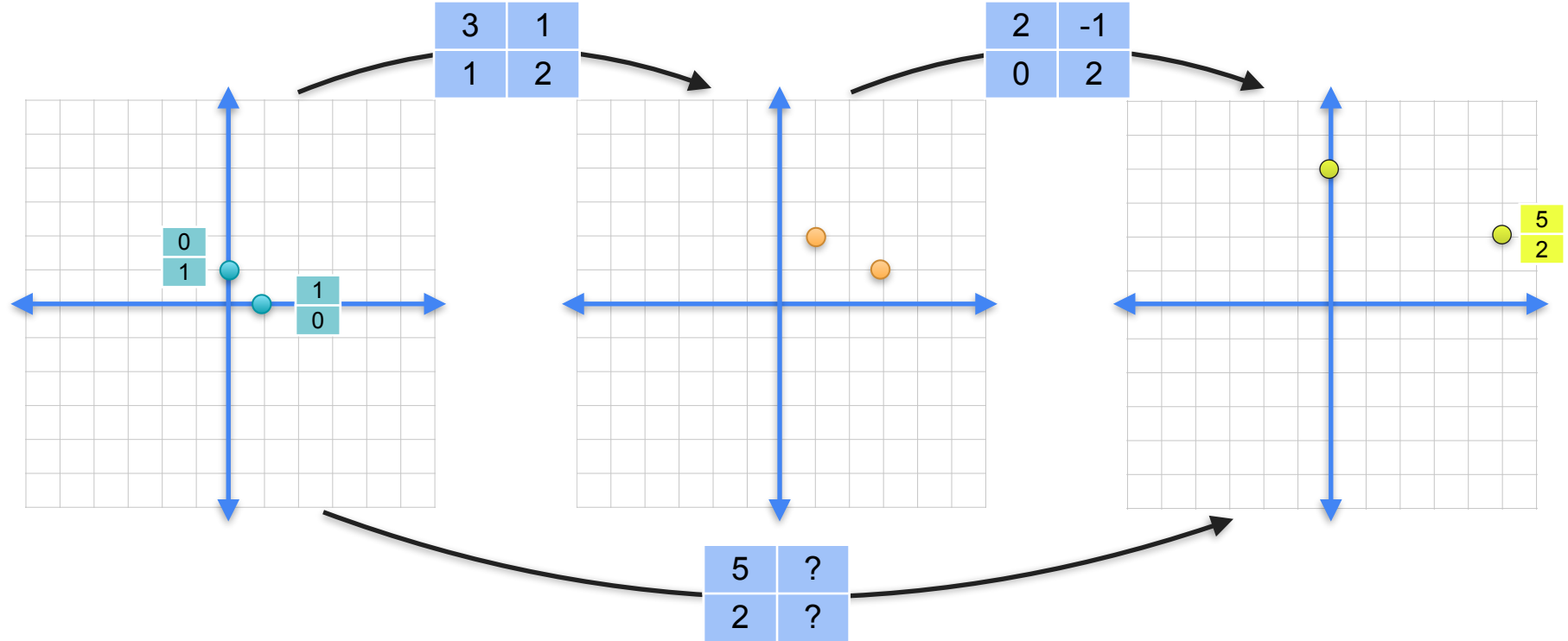
# Combining linear transformations



# Combining linear transformations

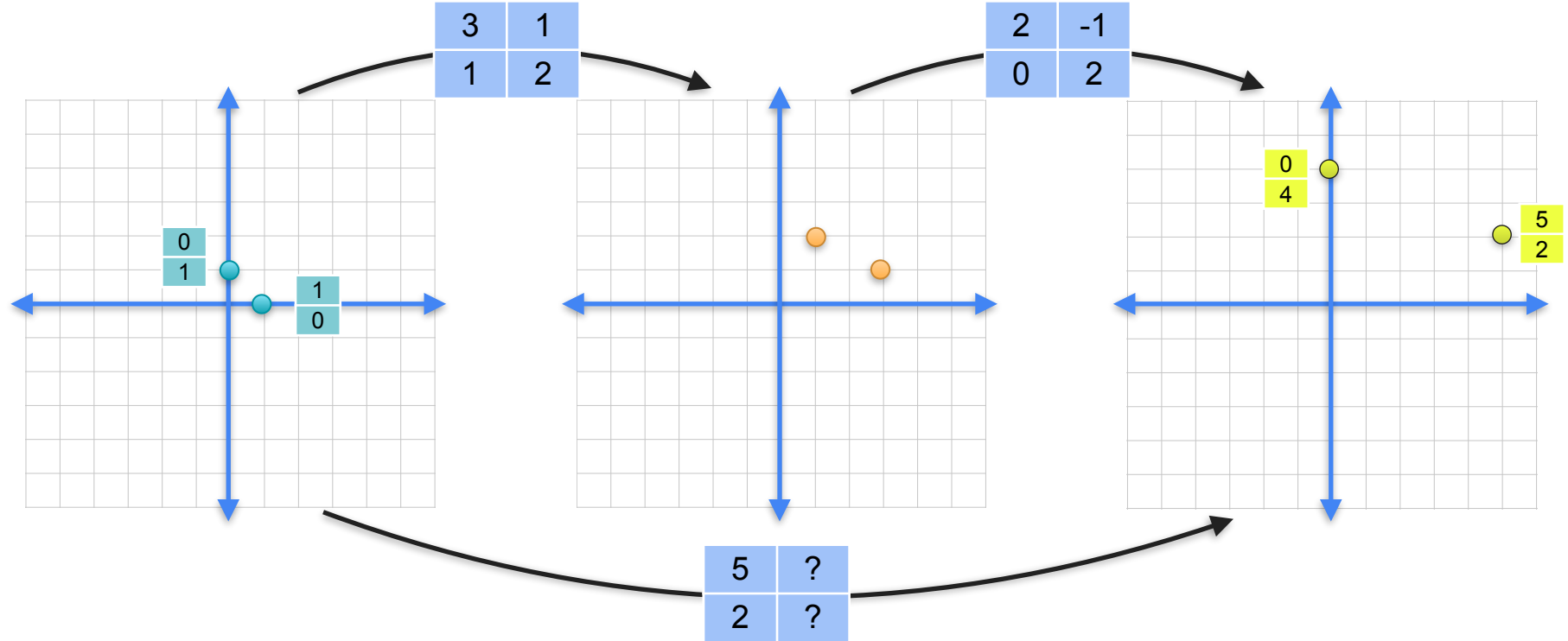


# Combining linear transformations

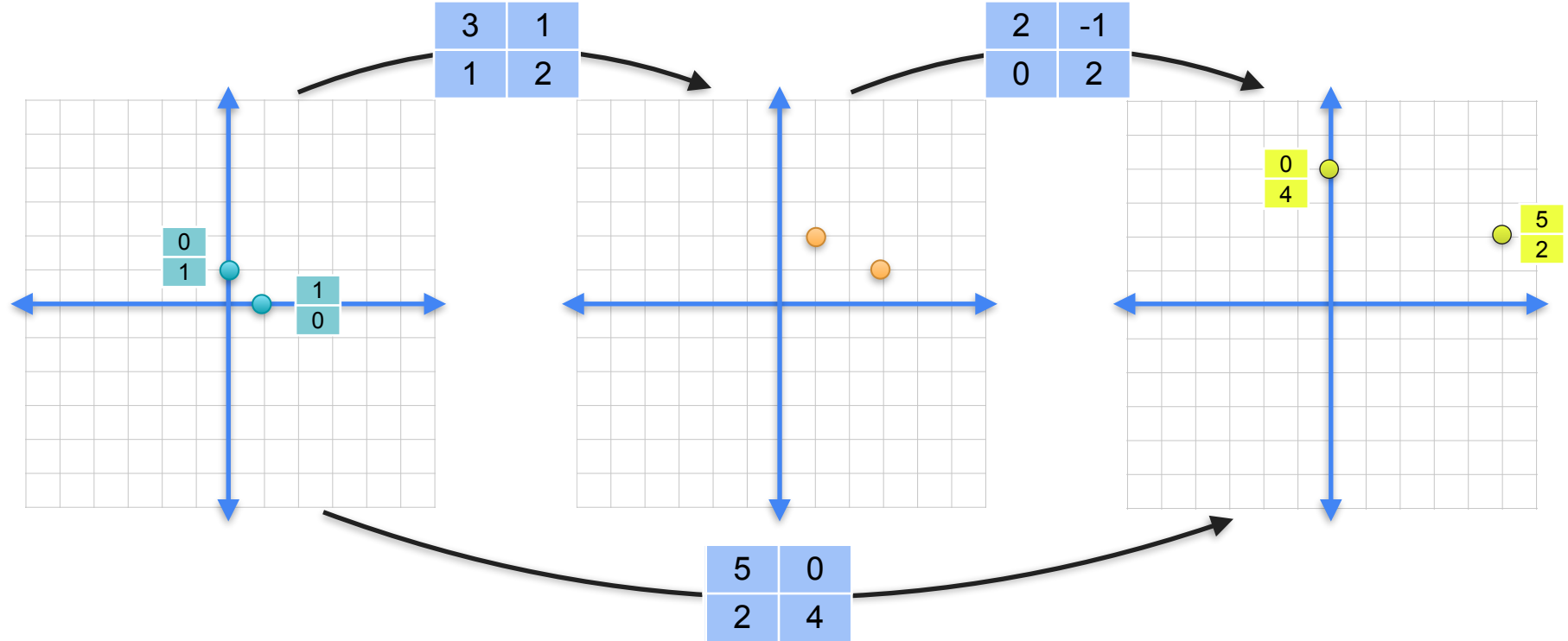




# Combining linear transformations



# Combining linear transformations



# Combining linear transformations

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 2 & 4 \end{bmatrix}$$

# Combining linear transformations

First

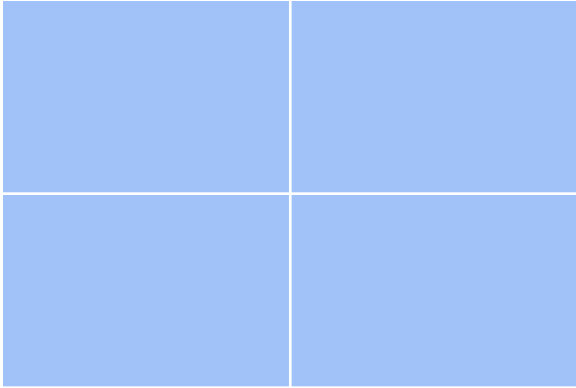
↓

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 2 & 4 \end{bmatrix}$$

# Combining linear transformations

$$\begin{array}{c} \text{Second} \\ \downarrow \\ \begin{array}{|c|c|} \hline 2 & -1 \\ \hline 0 & 2 \\ \hline \end{array} \end{array} \cdot \begin{array}{c} \text{First} \\ \downarrow \\ \begin{array}{|c|c|} \hline 3 & 1 \\ \hline 1 & 2 \\ \hline \end{array} \end{array} = \begin{array}{|c|c|} \hline 5 & 0 \\ \hline 2 & 4 \\ \hline \end{array}$$

# Multiplying matrices

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 5 & 0 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} & \\ & \end{bmatrix}$$


# Multiplying matrices

The diagram illustrates the process of multiplying two 2x2 matrices. The first matrix (left) has rows  $[2, -1]$  and  $[0, 2]$ . The second matrix (middle) has columns  $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$  and  $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ . The result is a 2x2 matrix (right) where each element is the dot product of a row from the first matrix and a column from the second matrix.

Matrix 1 (Teal):

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix}$$

Matrix 2 (Orange):

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$$

Result Matrix (Teal):

$$\begin{bmatrix} 2 \cdot 3 + (-1) \cdot 1 & 2 \cdot 1 + (-1) \cdot 2 \\ 0 \cdot 3 + 2 \cdot 1 & 0 \cdot 1 + 2 \cdot 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 2 & 4 \end{bmatrix}$$

# Multiplying matrices

Diagram illustrating the multiplication of two 2x2 matrices:

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & \begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} \\ \begin{bmatrix} 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} & \begin{bmatrix} 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} \end{bmatrix}$$



# Multiplying matrices

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ \begin{bmatrix} 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} & \begin{bmatrix} 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} \end{bmatrix}$$

# Multiplying matrices

The diagram illustrates the multiplication of two 2x2 matrices. The first matrix (teal) contains the values 2, -1, 0, and 2. The second matrix (orange) contains the values 3, 1, 1, and 2. These are multiplied to produce a 2x2 result matrix (blue) with values 5, 0, 2, and a highlighted 2x2 sub-matrix (teal and orange) containing the values 0, 2, 1, and 2.

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 2 & \begin{bmatrix} 0 & 2 \\ 1 & 2 \end{bmatrix} \end{bmatrix}$$

# Multiplying matrices

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 2 & 4 \end{bmatrix}$$



DeepLearning.AI

# Vectors and Linear Transformations

---

## **The identity matrix**

# The identity matrix

|   |   |   |   |   |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 0 | 1 |

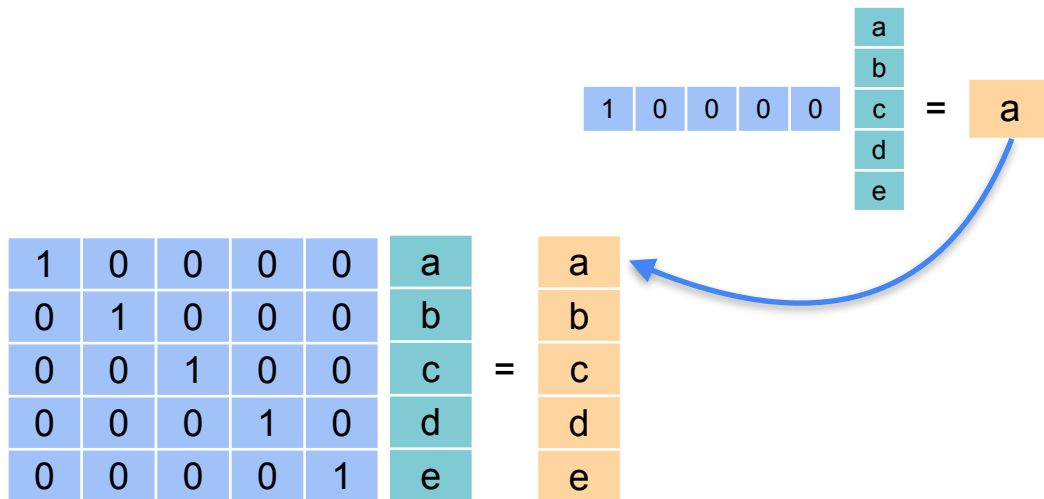
# The identity matrix

|   |   |   |   |   |   |
|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | a |
| 0 | 1 | 0 | 0 | 0 | b |
| 0 | 0 | 1 | 0 | 0 | c |
| 0 | 0 | 0 | 1 | 0 | d |
| 0 | 0 | 0 | 0 | 1 | e |

# The identity matrix

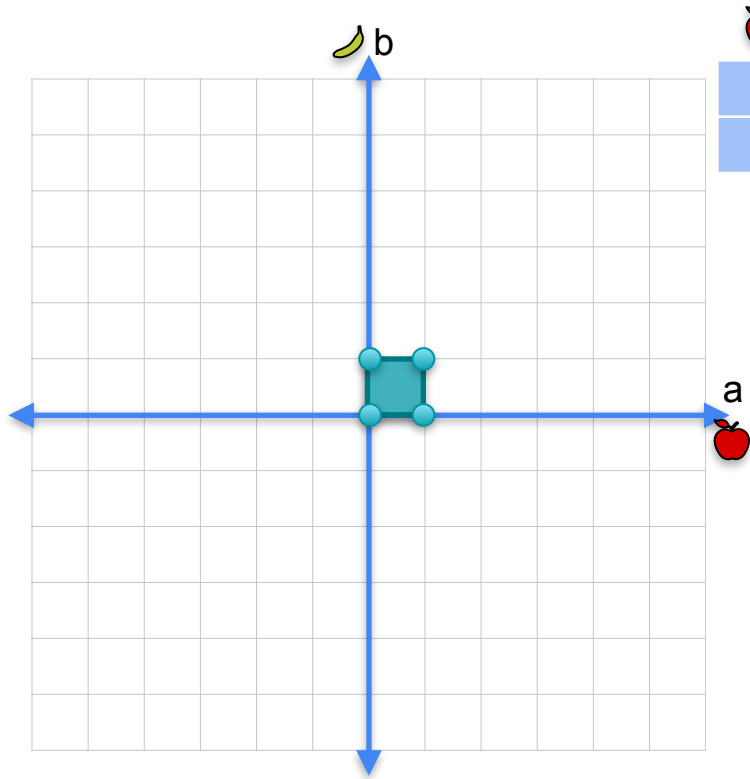
|   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | a | a |
| 0 | 1 | 0 | 0 | 0 | b | b |
| 0 | 0 | 1 | 0 | 0 | c | c |
| 0 | 0 | 0 | 1 | 0 | d | d |
| 0 | 0 | 0 | 0 | 1 | e | e |

# The identity matrix

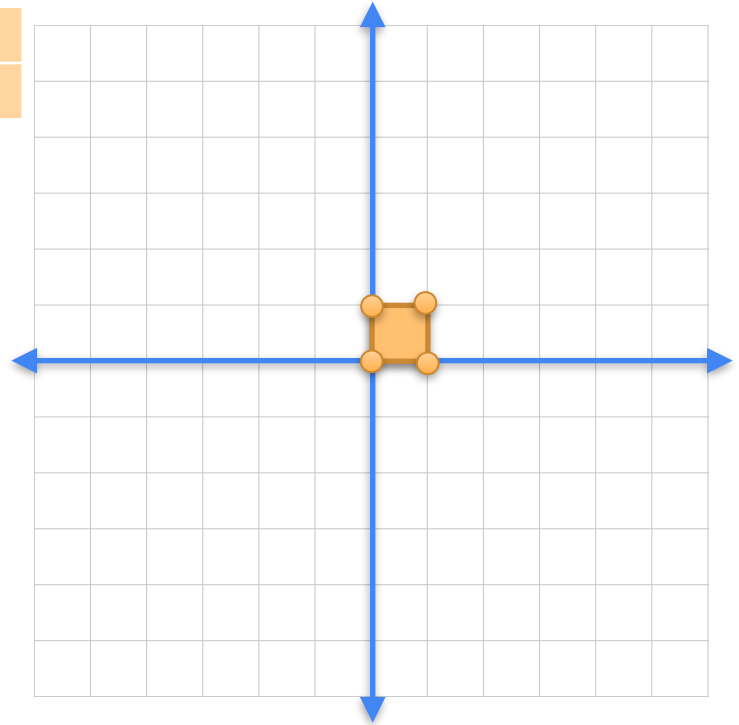




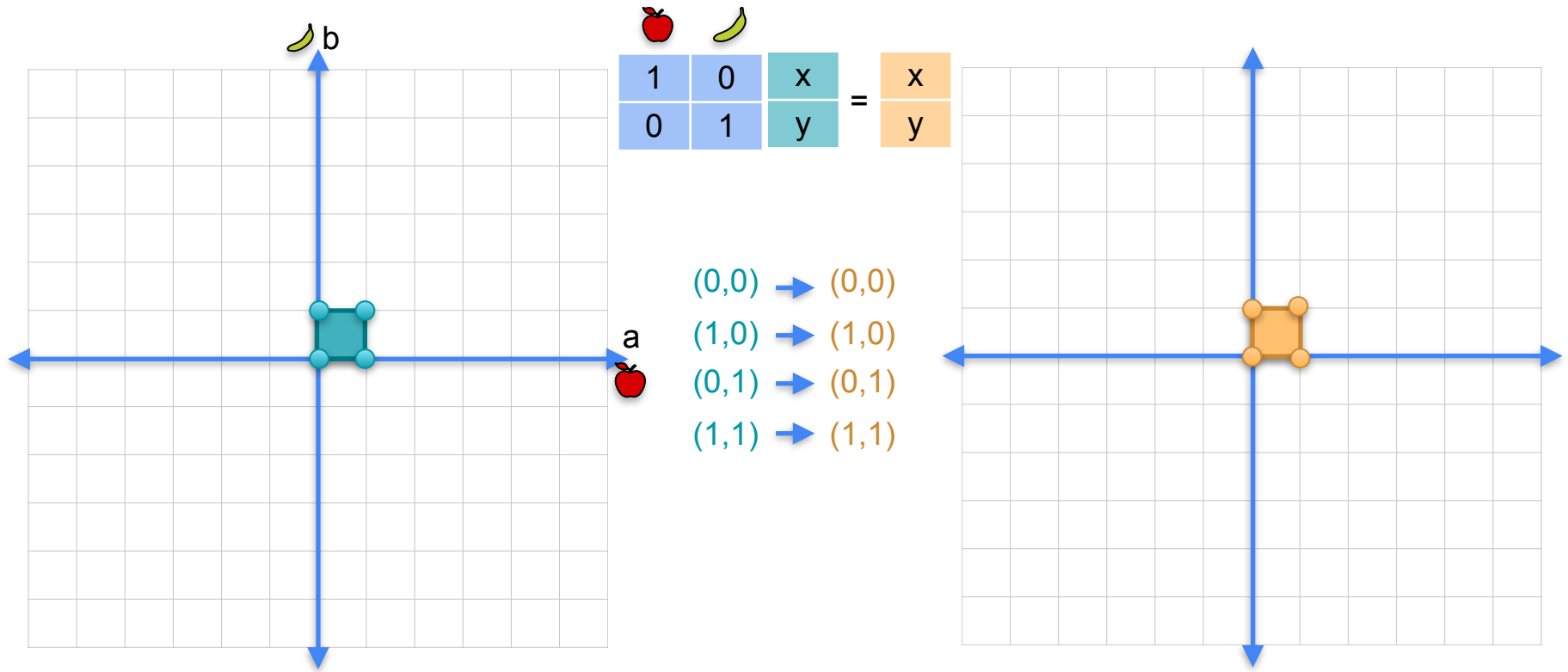
# The identity matrix



$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{matrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$



# The identity matrix





DeepLearning.AI

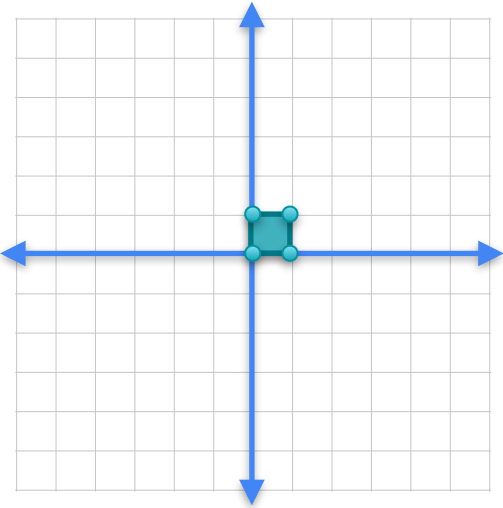
# Vectors and Linear Transformations

---

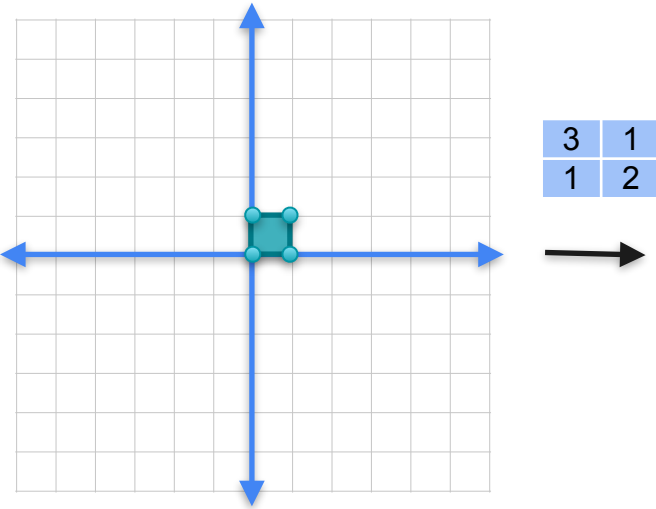
## **Matrix inverse**

# Matrix inverses

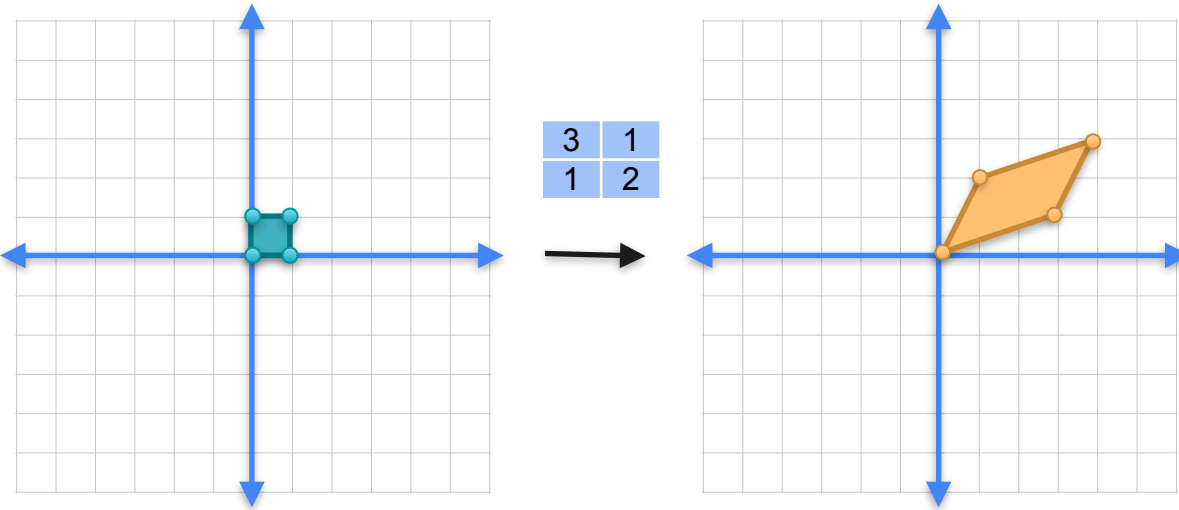
# Matrix inverses



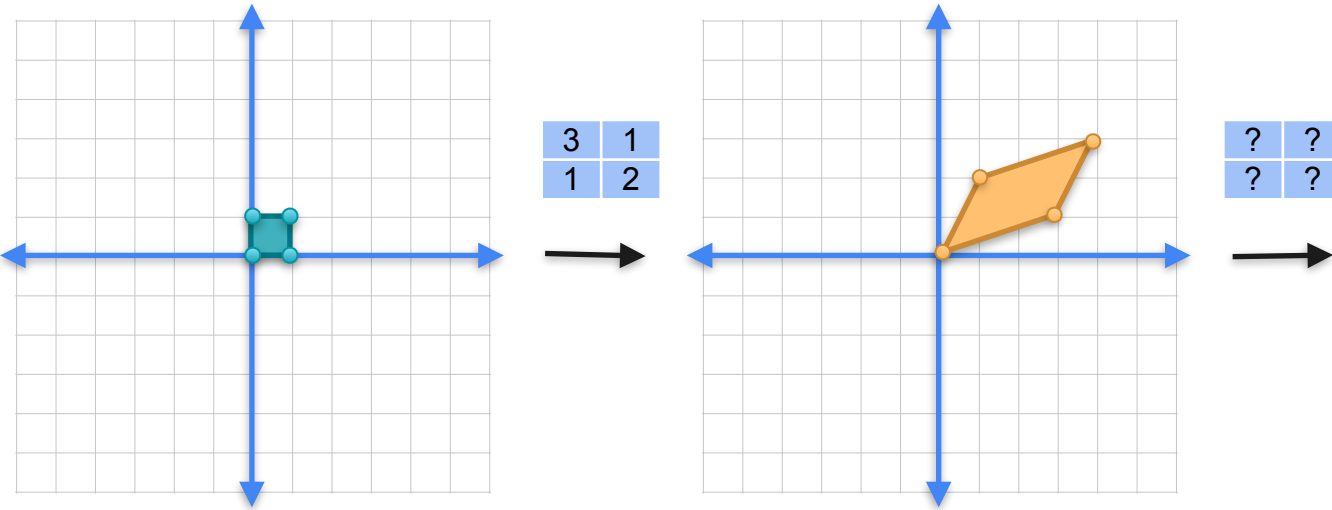
# Matrix inverses



# Matrix inverses

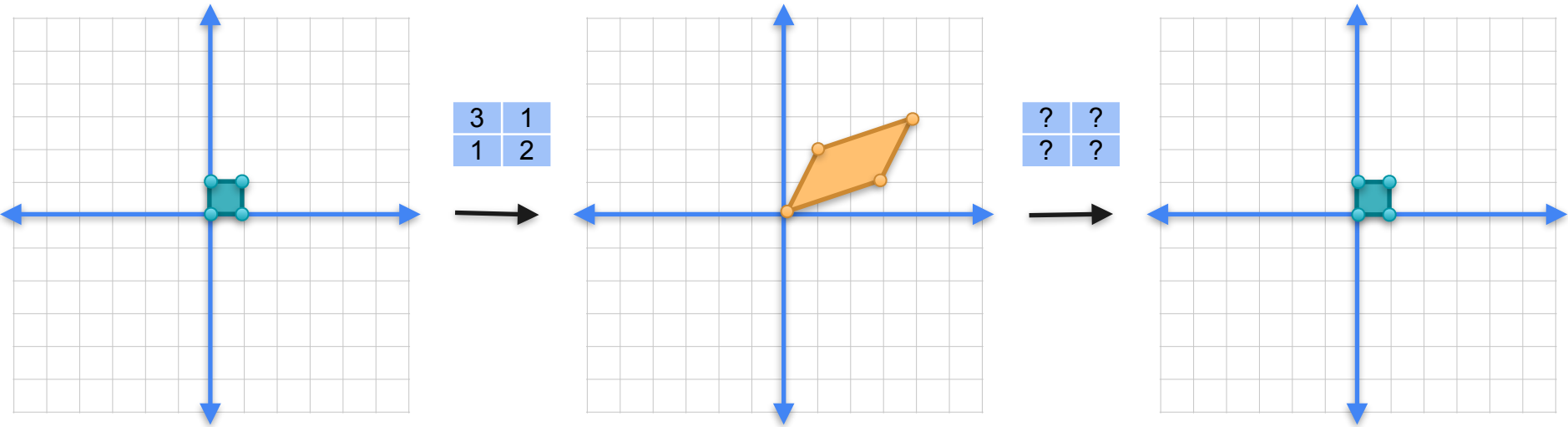


# Matrix inverses

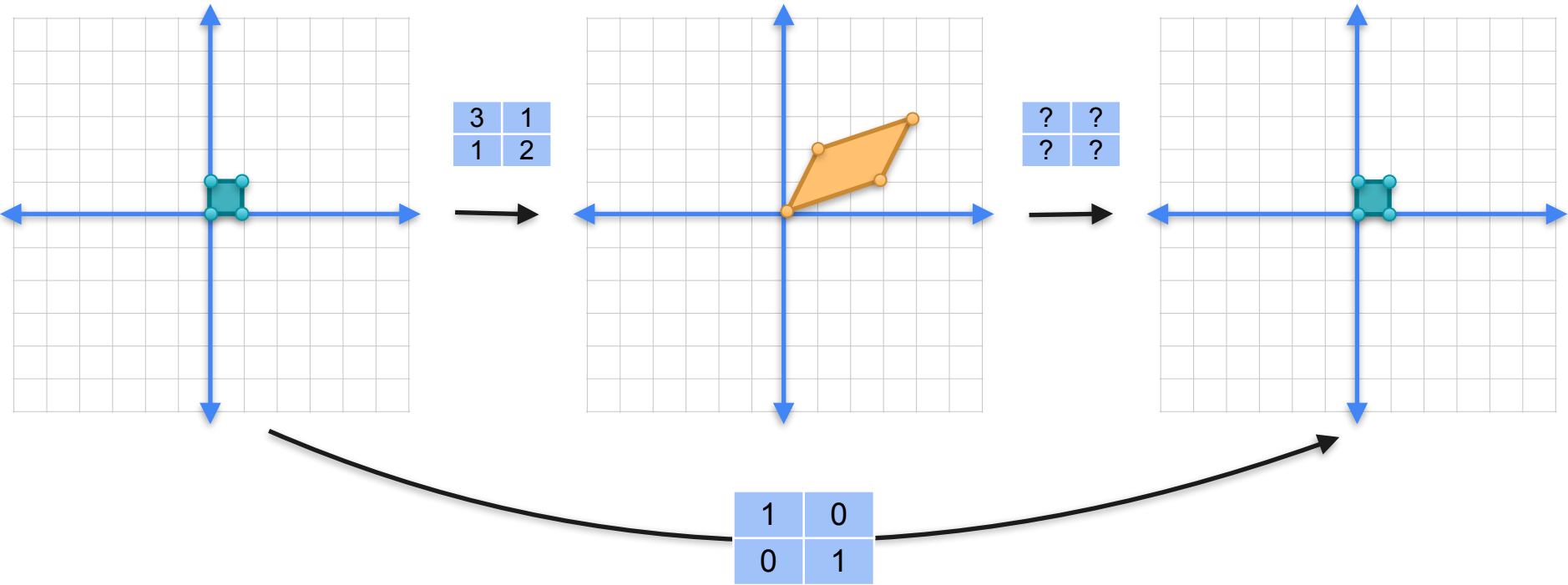




# Matrix inverses



# Matrix inverses



# Multiplying matrices

# Multiplying matrices

|   |   |
|---|---|
| a | b |
| c | d |

# Multiplying matrices


$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$$

# Multiplying matrices

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$


# Multiplying matrices

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1}$$

# Multiplying matrices

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
  
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 2/5 & -1/5 \\ -1/5 & 3/5 \end{bmatrix}$$




# How to find an inverse?

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

# How to find an inverse?

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{array}{l} \begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = 1 \\ \begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 0 \\ \begin{bmatrix} c & d \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = 0 \\ \begin{bmatrix} c & d \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 1 \end{array}$$

# How to find an inverse?

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = 1$$

$$3a + 1b = 1$$

$$\begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 0$$

$$1a + 2b = 0$$

$$\begin{bmatrix} c & d \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = 0$$

$$3c + 1d = 0$$

$$\begin{bmatrix} c & d \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 1$$

$$1c + 2d = 1$$

# How to find an inverse?

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = 1$$

$$3a + 1b = 1$$

$$a = \frac{2}{5}$$

$$\begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 0$$

$$1a + 2b = 0$$

$$b = -\frac{1}{5}$$

$$\begin{bmatrix} c & d \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = 0$$

$$3c + 1d = 0$$

$$c = -\frac{1}{5}$$

$$\begin{bmatrix} c & d \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 1$$

$$1c + 2d = 1$$

$$d = \frac{3}{5}$$

# Quiz

- Find the inverse of the following matrix. If you find that the task is impossible, feel free to click on “I couldn’t find it”

|   |   |
|---|---|
| 5 | 2 |
| 1 | 2 |

# Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 1$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 0$$

# Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 1$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 1$$

# Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 1$$

$$\bullet 5a + 2c = 1$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 0$$

$$\bullet 5b + 2d = 0$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 0$$

$$\bullet a + 2c = 0$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 1$$

$$\bullet b + 2d = 1$$



# Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 1$$

$$\bullet 5a + 2c = 1$$

$$\bullet a = 1/4$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 0$$

$$\bullet 5b + 2d = 0$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 0$$

$$\bullet a + 2c = 0$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 1$$

$$\bullet b + 2d = 1$$

# Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 1$$

$$\bullet 5a + 2c = 1$$

$$\bullet a = 1/4$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 0$$

$$\bullet 5b + 2d = 0$$

$$\bullet b = -1/4$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 0$$

$$\bullet a + 2c = 0$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 1$$

$$\bullet b + 2d = 1$$

# Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 1$$

$$\bullet 5a + 2c = 1$$

$$\bullet a = 1/4$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 0$$

$$\bullet 5b + 2d = 0$$

$$\bullet b = -1/4$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 0$$

$$\bullet a + 2c = 0$$

$$\bullet c = -1/8$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 1$$

$$\bullet b + 2d = 1$$

# Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 1$$

$$\bullet 5a + 2c = 1$$

$$\bullet a = 1/4$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 0$$

$$\bullet 5b + 2d = 0$$

$$\bullet b = -1/4$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 0$$

$$\bullet a + 2c = 0$$

$$\bullet c = -1/8$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 1$$

$$\bullet b + 2d = 1$$

$$\bullet d = 5/8$$

# Quiz

- Find the inverse of the following matrix. If you find that the task is impossible, feel free to click on “I’m reaching a dead end”

|   |   |
|---|---|
| 1 | 1 |
| 2 | 2 |

# Solutions

- The inverse doesn't exist!

We need to solve the following system of linear equations:

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$a + c = 1$$

$$2b + 2d = 1$$

$$2a + 2c = 0$$

$$b + d = 0$$

This is clearly a contradiction, since equation 1 says  $a+c=1$ , and equation 3 says  $2a+2c=0$ .



DeepLearning.AI

# Vectors and Linear Transformations

---

**Which matrices have an  
inverse?**

# Which matrices have inverses?



# Which matrices have inverses?

$$5^{-1} = 0.2$$

# Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

# Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

# Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

# Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

# Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

# Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Non-singular matrix

# Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Non-singular matrix

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Non-singular matrix

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$



# Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Non-singular matrix

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Non-singular matrix

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Singular matrix

# Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Non-singular matrix  
Invertible

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Non-singular matrix

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Singular matrix

# Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Non-singular matrix  
Invertible

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Non-singular matrix  
Invertible

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Singular matrix

# Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Non-singular matrix  
Invertible

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Non-singular matrix  
Invertible

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Singular matrix  
Non-invertible

# Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

|   |   |
|---|---|
| 3 | 1 |
| 1 | 2 |

<sup>-1</sup> = 

|      |      |
|------|------|
| 0.4  | -0.2 |
| -0.2 | 0.6  |

Non-singular matrix  
Invertible

$$\text{Det} = 5$$

|   |   |
|---|---|
| 5 | 2 |
| 1 | 2 |

<sup>-1</sup> = 

|        |       |
|--------|-------|
| 0.25   | -0.25 |
| -0.125 | 0.625 |

Non-singular matrix  
Invertible

|   |   |
|---|---|
| 1 | 1 |
| 2 | 2 |

 = 

|   |   |
|---|---|
| ? | ? |
| ? | ? |

Singular matrix  
Non-invertible

# Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Non-singular matrix  
Invertible

$$\text{Det} = 5$$

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Non-singular matrix  
Invertible

$$\text{Det} = 8$$

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Singular matrix  
Non-invertible

# Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Non-singular matrix  
Invertible

$$\text{Det} = 5$$

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Non-singular matrix  
Invertible

$$\text{Det} = 8$$

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Singular matrix  
Non-invertible

$$\text{Det} = 0$$

# Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Non-singular matrix  
Invertible

$$\text{Det} = 5$$

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Non-singular matrix  
Invertible

$$\text{Det} = 8$$

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Singular matrix  
Non-invertible

$$\text{Det} = 0$$

Non-zero determinants



# Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

|   |   |
|---|---|
| 3 | 1 |
| 1 | 2 |

<sup>-1</sup> = 

|      |      |
|------|------|
| 0.4  | -0.2 |
| -0.2 | 0.6  |

Non-singular matrix  
Invertible

$$\text{Det} = 5$$

|   |   |
|---|---|
| 5 | 2 |
| 1 | 2 |

<sup>-1</sup> = 

|        |       |
|--------|-------|
| 0.25   | -0.25 |
| -0.125 | 0.625 |

Non-singular matrix  
Invertible

$$\text{Det} = 8$$

Non-zero determinants

|   |   |
|---|---|
| 1 | 1 |
| 2 | 2 |

 = 

|   |   |
|---|---|
| ? | ? |
| ? | ? |

Singular matrix  
Non-invertible

$$\text{Det} = 0$$

Zero determinant



DeepLearning.AI

# Vectors and Linear Transformations

---

**Neural networks and  
matrices**



# Quiz: Natural language processing

| Spam | Lottery | Win |
|------|---------|-----|
| Yes  | 1       | 1   |
| Yes  | 2       | 1   |
| No   | 0       | 0   |
| Yes  | 0       | 2   |
| No   | 0       | 1   |
| No   | 1       | 0   |
| Yes  | 2       | 2   |
| Yes  | 2       | 0   |
| Yes  | 1       | 2   |

# Quiz: Natural language processing

| Spam | Lottery | Win |
|------|---------|-----|
| Yes  | 1       | 1   |
| Yes  | 2       | 1   |
| No   | 0       | 0   |
| Yes  | 0       | 2   |
| No   | 0       | 1   |
| No   | 1       | 0   |
| Yes  | 2       | 2   |
| Yes  | 2       | 0   |
| Yes  | 1       | 2   |

**Scores:**

Lottery: \_\_\_\_ points

Win: \_\_\_\_ points

# Quiz: Natural language processing

| Spam | Lottery | Win |
|------|---------|-----|
| Yes  | 1       | 1   |
| Yes  | 2       | 1   |
| No   | 0       | 0   |
| Yes  | 0       | 2   |
| No   | 0       | 1   |
| No   | 1       | 0   |
| Yes  | 2       | 2   |
| Yes  | 2       | 0   |
| Yes  | 1       | 2   |

## Scores:

Lottery: \_\_\_\_ points

Win: \_\_\_\_ points

## Examples

Lottery: 3 point

Win: 2 points

“Win, win the lottery!” : 7points

# Quiz: Natural language processing

| Spam | Lottery | Win |
|------|---------|-----|
| Yes  | 1       | 1   |
| Yes  | 2       | 1   |
| No   | 0       | 0   |
| Yes  | 0       | 2   |
| No   | 0       | 1   |
| No   | 1       | 0   |
| Yes  | 2       | 2   |
| Yes  | 2       | 0   |
| Yes  | 1       | 2   |

## Scores:

Lottery: \_\_\_\_ points

Win: \_\_\_\_ points

## Examples

Lottery: 3 point

Win: 2 points

“Win, win the lottery!” : 7points

## Rule:

If the number of points of the sentence is bigger than \_\_\_\_,  
then the email is spam.

# Quiz: Natural language processing

| Spam | Lottery | Win |
|------|---------|-----|
| Yes  | 1       | 1   |
| Yes  | 2       | 1   |
| No   | 0       | 0   |
| Yes  | 0       | 2   |
| No   | 0       | 1   |
| No   | 1       | 0   |
| Yes  | 2       | 2   |
| Yes  | 2       | 0   |
| Yes  | 1       | 2   |

## Scores:

Lottery: \_\_\_\_ points

Win: \_\_\_\_ points

## Examples

Lottery: 3 point

Win: 2 points

“Win, win the lottery!” : 7points

## Rule:

If the number of points of the sentence is bigger than \_\_\_\_, then the email is spam.

## Goal: Find the best points and threshold

Lottery: \_\_\_\_ point

Win: \_\_\_\_ point

Threshold: \_\_\_\_ points



# Quiz: Natural language processing

| Spam | Lottery | Win |
|------|---------|-----|
| Yes  | 1       | 1   |
| Yes  | 2       | 1   |
| No   | 0       | 0   |
| Yes  | 0       | 2   |
| No   | 0       | 1   |
| No   | 1       | 0   |
| Yes  | 2       | 2   |
| Yes  | 2       | 0   |
| Yes  | 1       | 2   |

| Score | > 1.5? |
|-------|--------|
| 2     | Yes    |
| 3     | Yes    |
| 0     | No     |
| 2     | Yes    |
| 1     | No     |
| 1     | No     |
| 4     | Yes    |
| 2     | Yes    |
| 3     | Yes    |

## Solution:

Lottery: 1 point

Win: 1 point

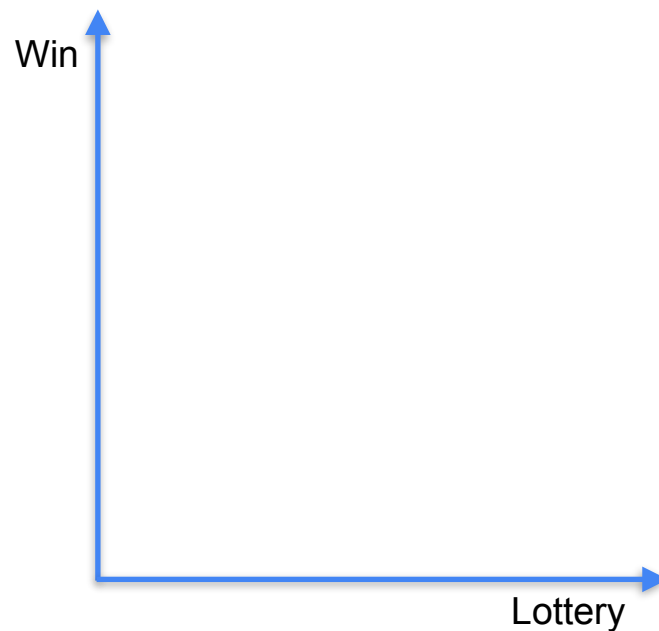
Threshold: 1.5 points

# Graphical natural language processing

| Spam | Lottery | Win |
|------|---------|-----|
| Yes  | 1       | 1   |
| Yes  | 2       | 1   |
| No   | 0       | 0   |
| Yes  | 0       | 2   |
| No   | 0       | 1   |
| No   | 1       | 0   |
| Yes  | 2       | 2   |
| Yes  | 2       | 0   |
| Yes  | 1       | 2   |

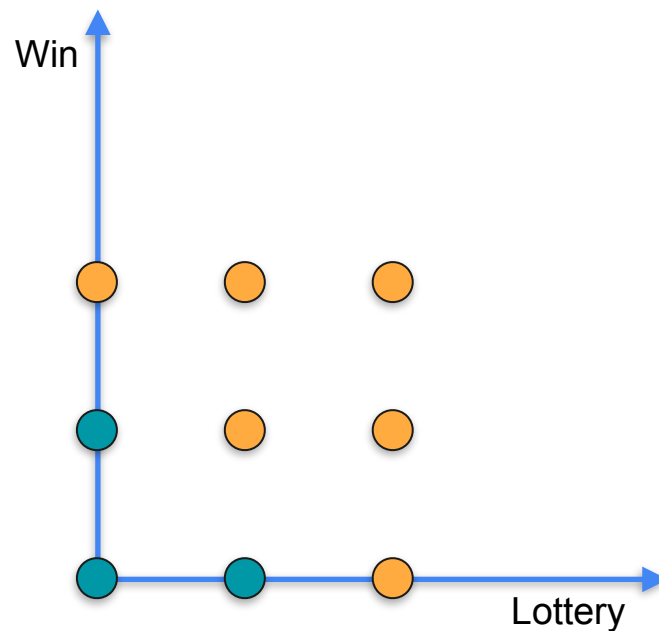
# Graphical natural language processing

| Spam | Lottery | Win |
|------|---------|-----|
| Yes  | 1       | 1   |
| Yes  | 2       | 1   |
| No   | 0       | 0   |
| Yes  | 0       | 2   |
| No   | 0       | 1   |
| No   | 1       | 0   |
| Yes  | 2       | 2   |
| Yes  | 2       | 0   |
| Yes  | 1       | 2   |



# Graphical natural language processing

| Spam | Lottery | Win |
|------|---------|-----|
| Yes  | 1       | 1   |
| Yes  | 2       | 1   |
| No   | 0       | 0   |
| Yes  | 0       | 2   |
| No   | 0       | 1   |
| No   | 1       | 0   |
| Yes  | 2       | 2   |
| Yes  | 2       | 0   |
| Yes  | 1       | 2   |

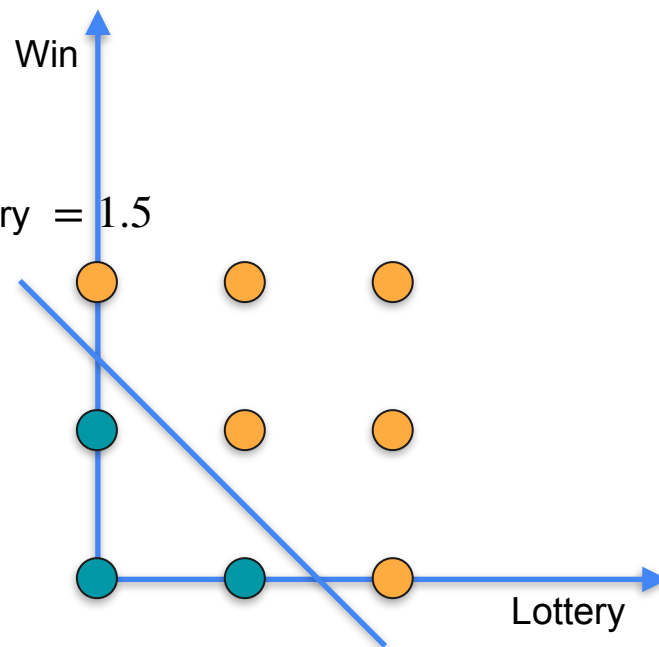


# Graphical natural language processing

| Spam | Lottery | Win |
|------|---------|-----|
| Yes  | 1       | 1   |
| Yes  | 2       | 1   |
| No   | 0       | 0   |
| Yes  | 0       | 2   |
| No   | 0       | 1   |
| No   | 1       | 0   |
| Yes  | 2       | 2   |
| Yes  | 2       | 0   |
| Yes  | 1       | 2   |

Line:

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} = 1.5$$

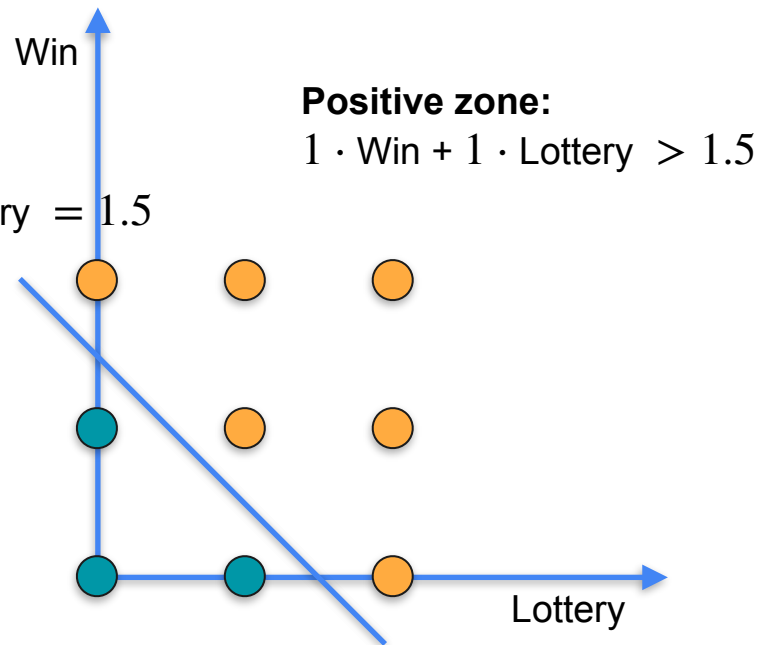


# Graphical natural language processing

| Spam | Lottery | Win |
|------|---------|-----|
| Yes  | 1       | 1   |
| Yes  | 2       | 1   |
| No   | 0       | 0   |
| Yes  | 0       | 2   |
| No   | 0       | 1   |
| No   | 1       | 0   |
| Yes  | 2       | 2   |
| Yes  | 2       | 0   |
| Yes  | 1       | 2   |

**Line:**

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} = 1.5$$



# Graphical natural language processing

| Spam | Lottery | Win |
|------|---------|-----|
| Yes  | 1       | 1   |
| Yes  | 2       | 1   |
| No   | 0       | 0   |
| Yes  | 0       | 2   |
| No   | 0       | 1   |
| No   | 1       | 0   |
| Yes  | 2       | 2   |
| Yes  | 2       | 0   |
| Yes  | 1       | 2   |

**Line:**

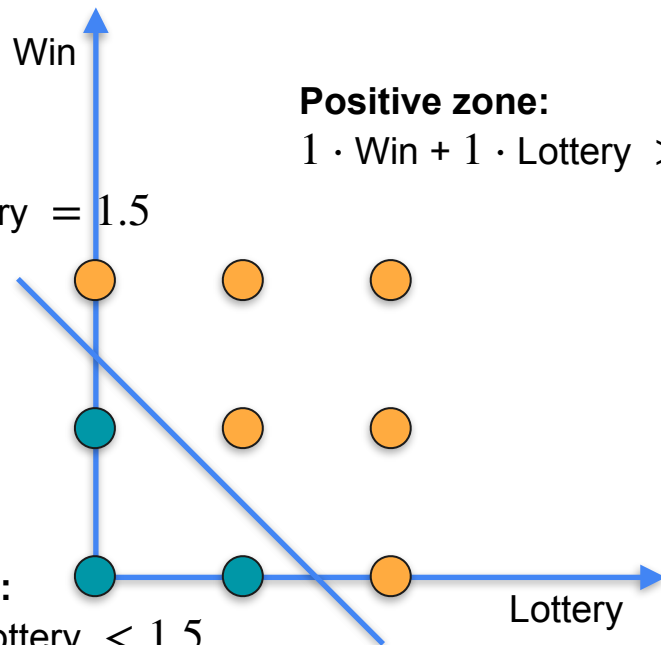
$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} = 1.5$$

**Negative zone:**

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} < 1.5$$

**Positive zone:**

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} > 1.5$$



# Graphical natural language processing

| Spam | Lottery | Win |
|------|---------|-----|
| Yes  | 1       | 1   |
| Yes  | 2       | 1   |
| No   | 0       | 0   |
| Yes  | 0       | 2   |
| No   | 0       | 1   |
| No   | 1       | 0   |
| Yes  | 2       | 2   |
| Yes  | 2       | 0   |
| Yes  | 1       | 2   |

| Model |
|-------|
| 1     |
| 1     |

Check: > 1.5?



# Graphical natural language processing

| Spam | Lottery | Win |
|------|---------|-----|
| Yes  | 1       | 1   |
| Yes  | 2       | 1   |
| No   | 0       | 0   |
| Yes  | 0       | 2   |
| No   | 0       | 1   |
| No   | 1       | 0   |
| Yes  | 2       | 2   |
| Yes  | 2       | 0   |
| Yes  | 1       | 2   |

|   |   |
|---|---|
| 2 | 1 |
|---|---|

| Model |
|-------|
| 1     |
| 1     |

Check: > 1.5?

# Graphical natural language processing

| Spam | Lottery | Win |
|------|---------|-----|
| Yes  | 1       | 1   |
| Yes  | 2       | 1   |
| No   | 0       | 0   |
| Yes  | 0       | 2   |
| No   | 0       | 1   |
| No   | 1       | 0   |
| Yes  | 2       | 2   |
| Yes  | 2       | 0   |
| Yes  | 1       | 2   |

2

1

| Model |
|-------|
| 1     |
| 1     |

= 3

Check: > 1.5?

# Graphical natural language processing

| Spam | Lottery | Win |
|------|---------|-----|
| Yes  | 1       | 1   |
| Yes  | 2       | 1   |
| No   | 0       | 0   |
| Yes  | 0       | 2   |
| No   | 0       | 1   |
| No   | 1       | 0   |
| Yes  | 2       | 2   |
| Yes  | 2       | 0   |
| Yes  | 1       | 2   |

2

1

| Model |
|-------|
| 1     |
| 1     |

= 3

Check: > 1.5?



Spam

# Dot product between vectors

| Spam | Lottery | Win |
|------|---------|-----|
| Yes  | 1       | 1   |
| Yes  | 2       | 1   |
| No   | 0       | 0   |
| Yes  | 0       | 2   |
| No   | 0       | 1   |
| No   | 1       | 0   |
| Yes  | 2       | 2   |
| Yes  | 2       | 0   |
| Yes  | 1       | 2   |

| Model |
|-------|
| 1     |
| 1     |

Check:  $> 1.5$ ?

# Dot product between vectors

| Spam | Lottery | Win |
|------|---------|-----|
| Yes  | 1       | 1   |
| Yes  | 2       | 1   |
| No   | 0       | 0   |
| Yes  | 0       | 2   |
| No   | 0       | 1   |
| No   | 1       | 0   |
| Yes  | 2       | 2   |
| Yes  | 2       | 0   |
| Yes  | 1       | 2   |

|   |   |
|---|---|
| 0 | 1 |
|---|---|

| Model |
|-------|
| 1     |
| 1     |

Check:  $> 1.5$ ?

# Dot product between vectors

| Spam | Lottery | Win |
|------|---------|-----|
| Yes  | 1       | 1   |
| Yes  | 2       | 1   |
| No   | 0       | 0   |
| Yes  | 0       | 2   |
| No   | 0       | 1   |
| No   | 1       | 0   |
| Yes  | 2       | 2   |
| Yes  | 2       | 0   |
| Yes  | 1       | 2   |

0

1

| Model |
|-------|
| 1     |
| 1     |

= 1

Check: > 1.5?

# Dot product between vectors

| Spam | Lottery | Win |
|------|---------|-----|
| Yes  | 1       | 1   |
| Yes  | 2       | 1   |
| No   | 0       | 0   |
| Yes  | 0       | 2   |
| No   | 0       | 1   |
| No   | 1       | 0   |
| Yes  | 2       | 2   |
| Yes  | 2       | 0   |
| Yes  | 1       | 2   |

0

1

| Model |
|-------|
| 1     |
| 1     |

= 1

Check: > 1.5?



Not spam

# Matrix multiplication

| Spam | Lottery | Win |
|------|---------|-----|
| Yes  | 1       | 1   |
| Yes  | 2       | 1   |
| No   | 0       | 0   |
| Yes  | 0       | 2   |
| No   | 0       | 1   |
| No   | 1       | 0   |
| Yes  | 2       | 2   |
| Yes  | 2       | 0   |
| Yes  | 1       | 2   |

| Model |
|-------|
| 1     |
| 1     |



# Matrix multiplication

| Spam | Lottery | Win | Model | = | Prod |
|------|---------|-----|-------|---|------|
| Yes  | 1       | 1   |       |   | 2    |
| Yes  | 2       | 1   | 1     |   | 3    |
| No   | 0       | 0   | 1     |   | 0    |
| Yes  | 0       | 2   |       |   | 2    |
| No   | 0       | 1   |       |   | 1    |
| No   | 1       | 0   |       |   | 1    |
| Yes  | 2       | 2   |       |   | 4    |
| Yes  | 2       | 0   |       |   | 2    |
| Yes  | 1       | 2   |       |   | 3    |

# Matrix multiplication

| Spam | Lottery | Win |
|------|---------|-----|
| Yes  | 1       | 1   |
| Yes  | 2       | 1   |
| No   | 0       | 0   |
| Yes  | 0       | 2   |
| No   | 0       | 1   |
| No   | 1       | 0   |
| Yes  | 2       | 2   |
| Yes  | 2       | 0   |
| Yes  | 1       | 2   |

| Model |
|-------|
| 1     |
| 1     |

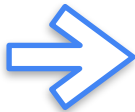
=

| Prod |
|------|
| 2    |
| 3    |
| 0    |
| 2    |
| 1    |
| 1    |
| 4    |
| 2    |
| 3    |

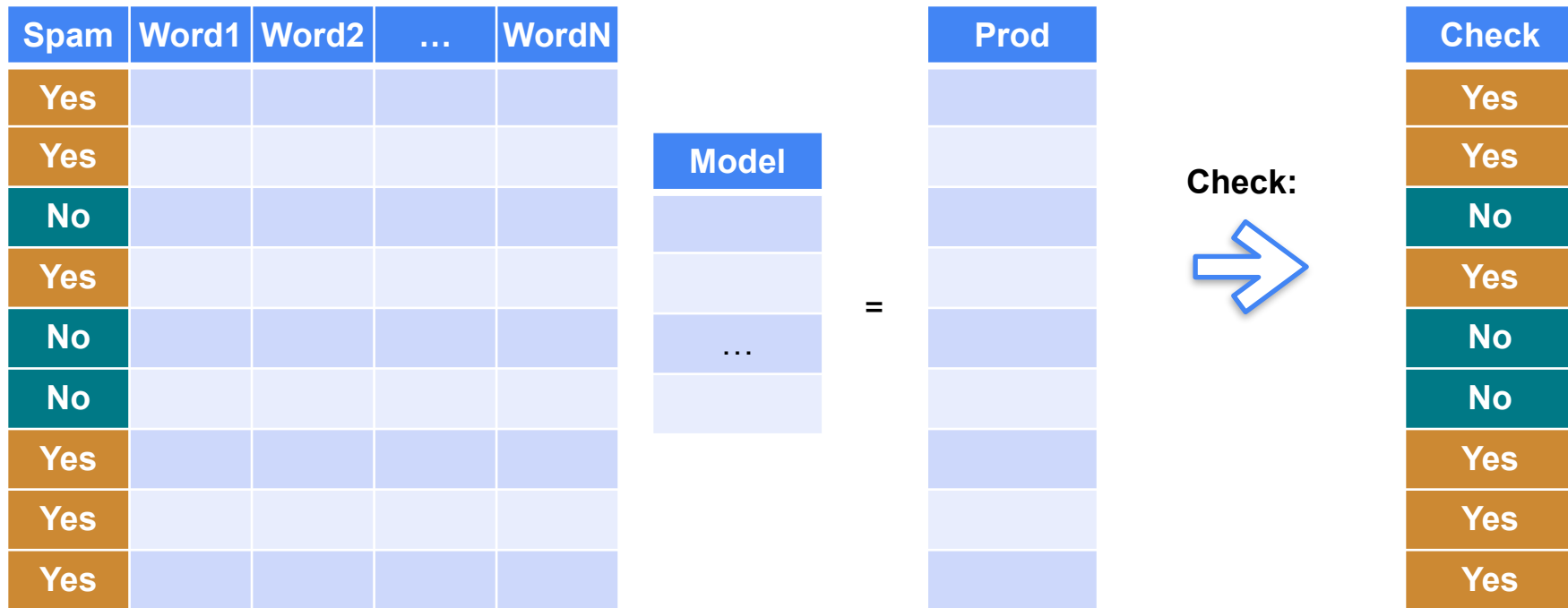
Check: >1.5?



# Matrix multiplication

| Spam | Lottery | Win |       |   | Prod |              |   | Check |
|------|---------|-----|-------|---|------|--------------|---|-------|
| Yes  | 1       | 1   | Model | = | 2    | Check: >1.5? |  | Yes   |
| Yes  | 2       | 1   |       |   | 3    |              |   | Yes   |
| No   | 0       | 0   | 1     |   | 0    |              |   | No    |
| Yes  | 0       | 2   |       |   | 2    |              |   | Yes   |
| No   | 0       | 1   | 1     |   | 1    |              |   | No    |
| No   | 1       | 0   |       |   | 1    |              |   | No    |
| Yes  | 2       | 2   |       |   | 4    |              |   | Yes   |
| Yes  | 2       | 0   |       |   | 2    |              |   | Yes   |
| Yes  | 1       | 2   |       |   | 3    |              |   | Yes   |

# Perceptrons



# Threshold and bias

| Spam | Lottery | Win |
|------|---------|-----|
| Yes  | 1       | 1   |
| Yes  | 2       | 1   |
| No   | 0       | 0   |
| Yes  | 0       | 2   |
| No   | 0       | 1   |
| No   | 1       | 0   |
| Yes  | 2       | 2   |
| Yes  | 2       | 0   |
| Yes  | 1       | 2   |

| Model |
|-------|
| 1     |
| 1     |

Check:  $> 1.5$ ?

# Threshold and bias

| Spam | Lottery | Win |
|------|---------|-----|
| Yes  | 1       | 1   |
| Yes  | 2       | 1   |
| No   | 0       | 0   |
| Yes  | 0       | 2   |
| No   | 0       | 1   |
| No   | 1       | 0   |
| Yes  | 2       | 2   |
| Yes  | 2       | 0   |
| Yes  | 1       | 2   |

**Check**

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

 Threshold

| Model |
|-------|
| 1     |
| 1     |

**Check: > 1.5?**

# Threshold and bias

| Spam | Lottery | Win |
|------|---------|-----|
| Yes  | 1       | 1   |
| Yes  | 2       | 1   |
| No   | 0       | 0   |
| Yes  | 0       | 2   |
| No   | 0       | 1   |
| No   | 1       | 0   |
| Yes  | 2       | 2   |
| Yes  | 2       | 0   |
| Yes  | 1       | 2   |

**Check**

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

Threshold

Bias

| Model |
|-------|
| 1     |
| 1     |

**Check: > 1.5?**

# Threshold and bias

| Spam | Lottery | Win | Bias |
|------|---------|-----|------|
| Yes  | 1       | 1   | 1    |
| Yes  | 2       | 1   | 1    |
| No   | 0       | 0   | 1    |
| Yes  | 0       | 2   | 1    |
| No   | 0       | 1   | 1    |
| No   | 1       | 0   | 1    |
| Yes  | 2       | 2   | 1    |
| Yes  | 2       | 0   | 1    |
| Yes  | 1       | 2   | 1    |

**Check**

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

Threshold

Bias

| Model |
|-------|
| 1     |
| 1     |

**Check: > 1.5?**



# Threshold and bias

| Spam | Lottery | Win | Bias |
|------|---------|-----|------|
| Yes  | 1       | 1   | 1    |
| Yes  | 2       | 1   | 1    |
| No   | 0       | 0   | 1    |
| Yes  | 0       | 2   | 1    |
| No   | 0       | 1   | 1    |
| No   | 1       | 0   | 1    |
| Yes  | 2       | 2   | 1    |
| Yes  | 2       | 0   | 1    |
| Yes  | 1       | 2   | 1    |

**Check**

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

**Threshold**

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

**Bias**

| Model |
|-------|
| 1     |
| 1     |
| -1.5  |

**Check: > 1.5?**

**Bias**

# Threshold and bias

| Spam | Lottery | Win | Bias |
|------|---------|-----|------|
| Yes  | 1       | 1   | 1    |
| Yes  | 2       | 1   | 1    |
| No   | 0       | 0   | 1    |
| Yes  | 0       | 2   | 1    |
| No   | 0       | 1   | 1    |
| No   | 1       | 0   | 1    |
| Yes  | 2       | 2   | 1    |
| Yes  | 2       | 0   | 1    |
| Yes  | 1       | 2   | 1    |

**Check**

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

Threshold

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

Bias

| Model |
|-------|
| 1     |
| 1     |
| -1.5  |

**Check: > 0?**

Bias

# The AND operator

| AND | x | y |
|-----|---|---|
| No  | 0 | 0 |
| No  | 1 | 0 |
| No  | 0 | 1 |
| Yes | 1 | 1 |

# The AND operator

| AND | x | y |
|-----|---|---|
| No  | 0 | 0 |
| No  | 1 | 0 |
| No  | 0 | 1 |
| Yes | 1 | 1 |

| Model |
|-------|
| 1     |
| 1     |

# The AND operator

| AND | x | y |  | Dot prod |
|-----|---|---|--|----------|
| No  | 0 | 0 |  | 0        |
| No  | 1 | 0 |  | 1        |
| No  | 0 | 1 |  | 1        |
| Yes | 1 | 1 |  | 2        |

| Model |
|-------|
| 1     |
| 1     |

=

| Dot prod |
|----------|
| 0        |
| 1        |
| 1        |
| 2        |

# The AND operator

| AND | x | y |
|-----|---|---|
| No  | 0 | 0 |
| No  | 1 | 0 |
| No  | 0 | 1 |
| Yes | 1 | 1 |

| Model |
|-------|
| 1     |
| 1     |

=

| Dot prod |
|----------|
| 0        |
| 1        |
| 1        |
| 2        |

Check: >1.5?



# The AND operator

| AND | x | y |
|-----|---|---|
| No  | 0 | 0 |
| No  | 1 | 0 |
| No  | 0 | 1 |
| Yes | 1 | 1 |

| Model |
|-------|
| 1     |
| 1     |

=

| Dot prod |
|----------|
| 0        |
| 1        |
| 1        |
| 2        |

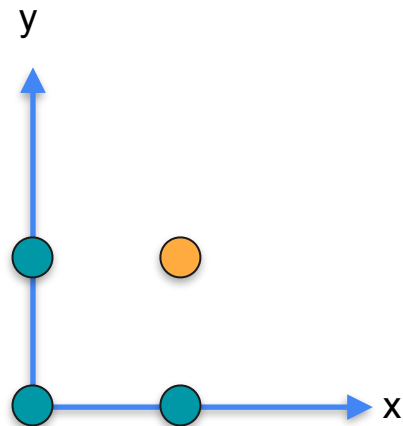
Check: >1.5?



| Check |
|-------|
| No    |
| No    |
| No    |
| Yes   |

# The AND operator

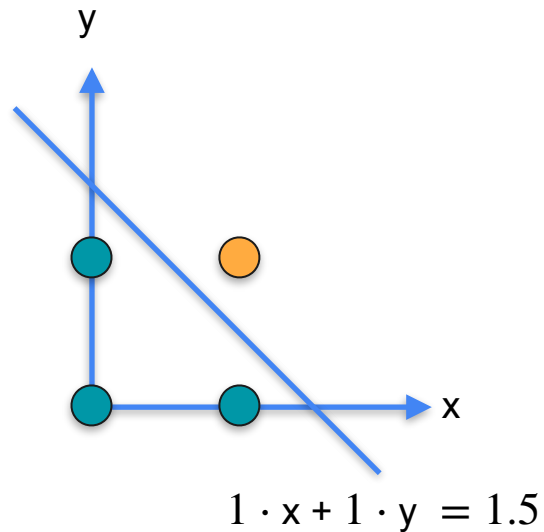
| AND | x | y |
|-----|---|---|
| No  | 0 | 0 |
| No  | 1 | 0 |
| No  | 0 | 1 |
| Yes | 1 | 1 |



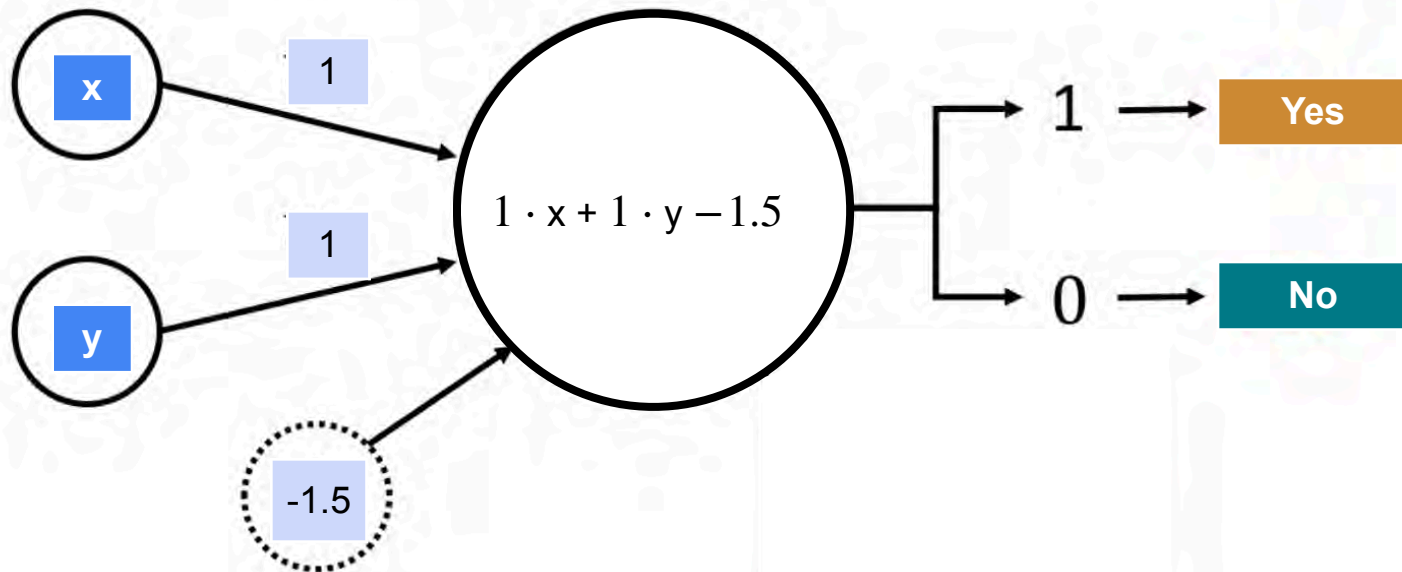


# The AND operator

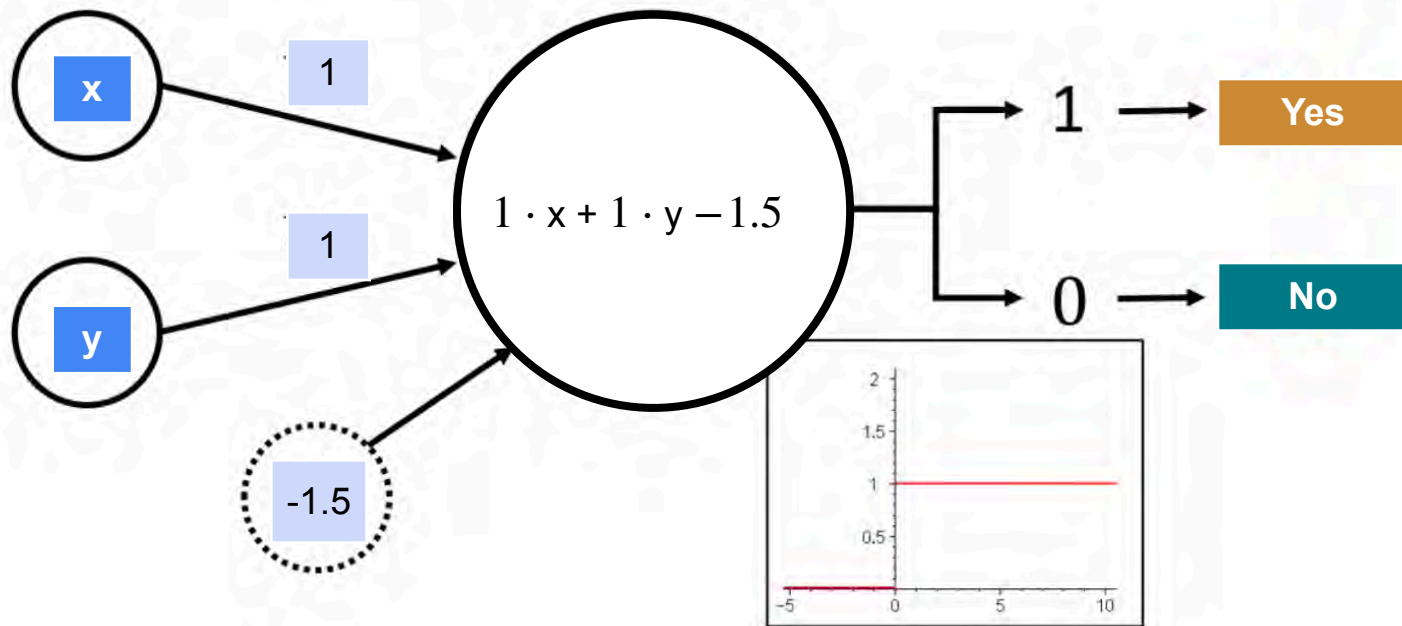
| AND | x | y |
|-----|---|---|
| No  | 0 | 0 |
| No  | 1 | 0 |
| No  | 0 | 1 |
| Yes | 1 | 1 |

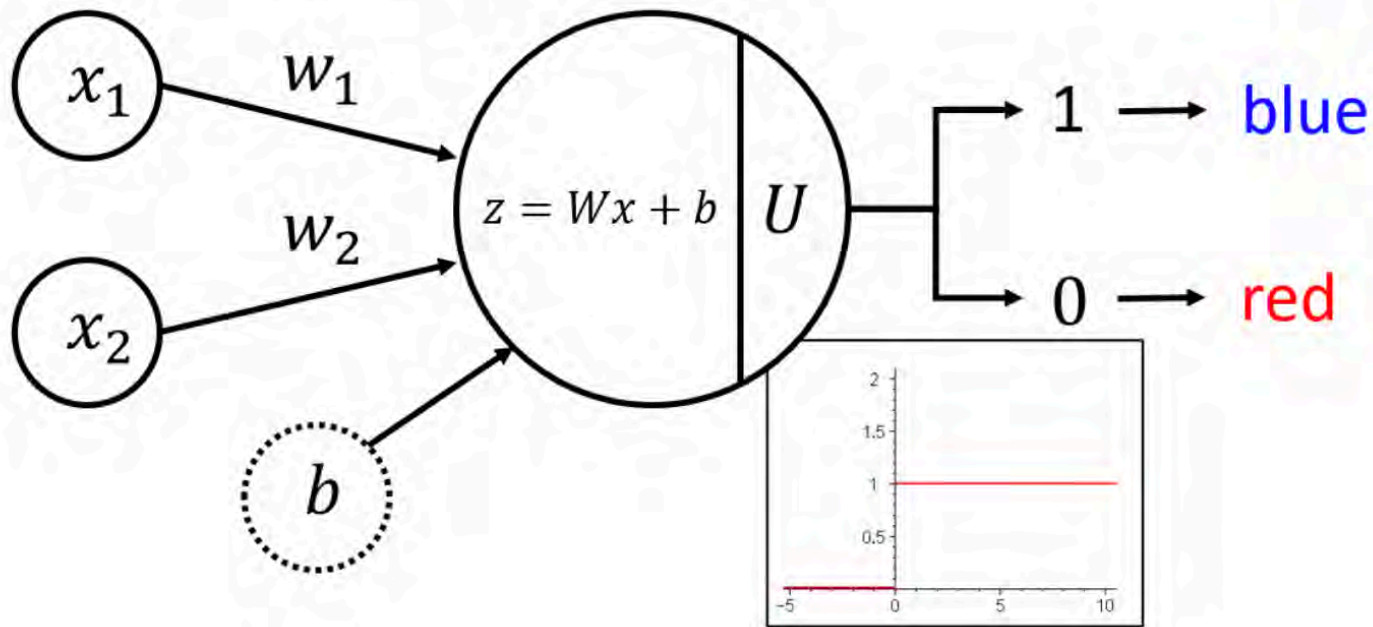


# The perceptron



# The perceptron







DeepLearning.AI

# Vectors and Linear Transformations

---

## Conclusion