Modelo de carro y péndulo invertido

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FDS Control Systems

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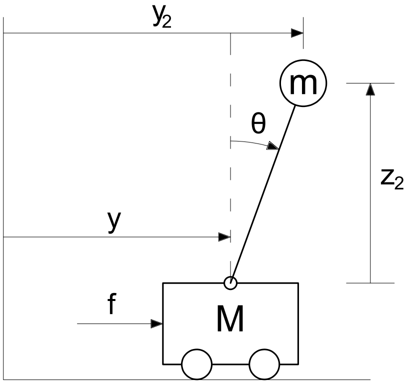
Derivación del modelo



$$T_1 = \frac{1}{2} M \dot{y}^2 \qquad (1)$$

$$T_2 = \frac{1}{2} m(\dot{y}_2^2 + \dot{z}_2^2) \qquad (2)$$

$$y_2 = y + l \sin\theta \qquad \dot{y}_2 = \dot{y} + l\dot{\theta}\cos\theta$$
$$z_2 = l\cos\theta \qquad \dot{z}_2 = -l\dot{\theta}\sin\theta$$

Recuerda: $\sin^2\theta + \cos^2\theta = 1$

$$T = T_1 + T_2 = \frac{1}{2} M\dot{y}^2 + \frac{1}{2} m[(\dot{y} + l\dot{\theta}\cos\theta)^2 + l^2\dot{\theta}^2\sin^2\theta] \qquad (3)$$

Derivación del modelo

$$T = T_1 + T_2 = \frac{1}{2}M\dot{y}^2 + \frac{1}{2}m[\dot{y}^2 + 2\dot{y}\dot{\theta}l\cos\theta + l^2\dot{\theta}^2] \tag{4}$$

$$V = mgz_2 = mgl\cos\theta \tag{5}$$

$$\boxed{L = T - V = \frac{1}{2}(M+m)\dot{y}^2 + m\dot{y}\dot{\theta}l\cos\theta + \frac{1}{2}ml^2\dot{\theta}^2 - mgl\cos\theta} \tag{6}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}}\right) - \frac{\partial L}{\partial y} = f \qquad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0 \tag{7}$$

Derivación del modelo

$$\boxed{L = T - V = \frac{1}{2}(M+m)\dot{y}^2 + m\dot{y}\dot{\theta} l \cos\theta + \frac{1}{2} m l^2 \dot{\theta}^2 - mgl\cos\theta}$$

$$\frac{\partial L}{\partial \dot{y}} = (M+m)\dot{y} + \dot{\theta} m l \cos\theta$$

$$\frac{\partial L}{\partial y} = 0$$

$$\frac{\partial L}{\partial \dot{\theta}} = ml\cos\theta\dot{y} + ml^2\dot{\theta}$$

$$\frac{\partial L}{\partial \theta} = mgl\sin\theta - ml\dot{y}\dot{\theta}\sin\theta$$

Derivación del modelo

$$(M+m)\ddot{y} + \ddot{\theta} ml \cos\theta - ml\dot{\theta}^2 \sin\theta = f \tag{8}$$

$$ml\cos\theta\ddot{y} + ml^2\ddot{\theta} - mgl\sin\theta = 0 \tag{9}$$

for small enough values of θ

$$\sin\theta \approx \theta \qquad \cos\theta \approx 1 \tag{10}$$

the system simplifies to

$$(M+m)\ddot{y} + ml\ddot{\theta} = f \tag{11}$$

$$m\ddot{y} + ml\ddot{\theta} - mg\theta = 0 \tag{12}$$

Representación en espacio de estados

$$x = \begin{bmatrix} y \\ \theta \\ \dot{y} \\ \dot{\theta} \end{bmatrix} \tag{13}$$

$$\frac{dy}{dt} = \dot{y} \qquad \frac{d\theta}{dt} = \dot{\theta} \tag{14}$$

$$\frac{d}{dt}(\dot{y}) = \ddot{y} = \frac{f}{M} - \frac{mg}{M}\theta \tag{15}$$

$$\frac{d}{dt}(\dot{\theta}) = \ddot{\theta} = -\frac{f}{Ml} + \frac{M+m}{Ml}\theta \tag{16}$$

$$\dot{x} = Ax + Bu \tag{17}$$

Representación en espacio de estados

$$\dot{x} = Ax + Bu$$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -mg/M & 0 & 0 \\ 0 & (M+m)g/Ml & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ 1/M \\ -1/Ml \end{bmatrix} \tag{18}$$

$$u = f = \text{fuerza externa} \tag{19}$$