Researching Of Two-Wheeled Self-Balancing Robot Base On LQR Combined With PID

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***Abstract*—Two-wheeled self-balancing robot is a non-stable, non-linear, strong coupling system.On the basis of building up the system structure model,kinetic equation is built up by using the Lagrange' s method,then obtaining the linearizing model in the vicinity of the balance.The control method of combining LQR and PID can effectively overcome the impact of the constraints on the system in the linearization process,and whit the controller core of DSP TMS320LF2812,the two-wheeled self-balancing robot can achieve dynamic balance whitin a larger angle range. The physical experiments results demonstrate that it can control the two-wheeled robot system in a very short period of time to get a stable dynamic equilibrium,validity and rationality of the designed controller are verified through the performance experiments of the prototype.**

Keywords-Two-wheeled self-balancing robot;Lagrange method;LQR;PID;DSP2812

I. INTRODUCTION

The two-wheeled self-balancing robot has become a hot subject in verifying various control theories since it occurred, which mainly due to its unstable dynamic performance and strong nonlinearity.In addition it has some other advantages for autonomous mobile, small size,simple structure and flexibility of action,so it can suitable for some jobs in narrow space or dangerous work.Overall,two-wheeled self-balancing robot is an ideal platform to verify various control algorithms and a tool in the civilian and military[1].

Researchers at home and abroad have done a great deal of research on two-wheeled self-balancing robot,and proposed a number of methods to linearize the nonlinear system. And many of them have used the approximate linearization method to linearize the nonlinear model of the robot,then, analysised the linearization method by means of the modern control theory such as pole placement or LQR.The simulation result can get very good effects.However,this linear method assumes that the inclination of robot keep within the scope of a small angle,but in practice,the scope of the controllable angle is much larger.The linearization method can obtain very good result in the vicinity of the balance point,but can not keep balance when far from the balance point.

This paper introduces the structural design of the two-wheeled self-balancing robot,builds up kinetic equation using the Lagrange method and proposes a control method combining of LQR and PID to control the robot inclination.The physical experimental demonstrate that this method not only have a good effect in the vicinity of the balance point,but also can keep the robot balance in a larger angle range.

II. DYNAMICS MODEL OF THE TWO SELF-BALANCE ROBOT

A. System Structure

Figure 1 shows the structure of the robot.The mechanical frame of system is layered structure.In bottom layer there are the same type and coaxial DC motor,posture detection sensors,servo drives and power source.In the middle layer there are power monitoring and switching module,controller module.The robot head is on the top,which can place the vision sensors and other extension components.

The control core of the two-wheeled self-balancing robot is DSP TMS320F2812.The ADC module sample and convert the signal of posture detection sensors for determining the robot pose.And the QEP module collect motor encoder signals to determine the robot velocity and displacement,then send motor voltage calculating by some sort of control slgorithm to servo drives.

Fig.1 Structure of Two-wheel Robot

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B. Dynamics Model

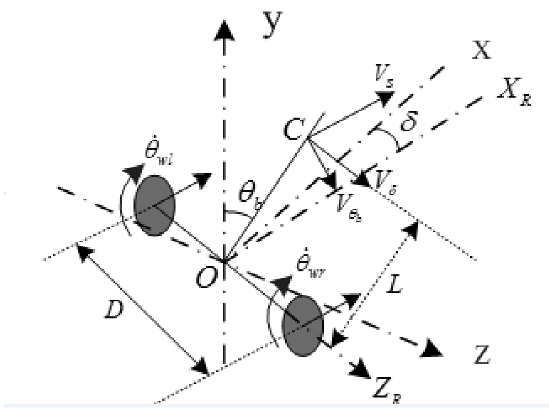

Fig.2 Structure figure of Robot

Building the system dynamics by lagrange equation[2], the mathematical expression is as follows:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right)-\frac{\partial T}{\partial q_i}+\frac{\partial V}{\partial q_i}+\frac{\partial D}{\partial \dot{q}_i}=F_{q_i}$$

Where q_i is generalized coordinates, F_{q_i} is external force, T is kinetic energy, V is potential energy, D is dissipated energy.

And q_i choose ϕ, ψ, θ_b, F_{q_i} is about motor output torque M_l, M_r.

ϕ :corner of right wheel relative to the robot body (rad)

ψ :corner of left wheel relative to the robot body (rad)

θ_b :the angle between the robot body and the vertical axis(rad)

We obtain the system kinetic energy,potential energy and dissipated by analysising the kinematics of the robot,model the robot by using lagrange equation,then substitute actual arguments,linearize the system in the vicinity of equilibrium,and get eventually:

$$\begin{bmatrix}\dot{\theta}_b\\ \dot{\phi}\\ \dot{\psi}\\ \ddot{\theta}_b\\ \ddot{\phi}\\ \ddot{\psi}\end{bmatrix}=\begin{bmatrix}0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\\ 18.8936 & 0 & 0 & 0.2895 & 0.1532 & 0.1532\\ -33.0814 & 0 & 0 & -1.5746 & -1.3039 & -0.3269\\ -33.0814 & 0 & 0 & -1.5746 & -0.3269 & -1.3039\end{bmatrix}\begin{bmatrix}\theta_b\\ \phi\\ \psi\\ \dot{\theta}_b\\ \dot{\phi}\\ \dot{\psi}\end{bmatrix}+\begin{bmatrix}0 & 0\\ 0 & 0\\ 0 & 0\\ -2.6518 & -2.6518\\ 0.9488 & 0.7390\\ 0.7390 & 0.9488\end{bmatrix}\begin{bmatrix}M_l\\ M_r\end{bmatrix} \quad (1)$$

In order to expression the relationship between the robot control system more intuitive,doing the following variable substitution:

$$x=\frac{x_{wl}+x_{wr}}{2}=\frac{R\theta_{wl}+R\theta_{wr}}{2}=\frac{R}{2}(\phi+\psi)+R\theta_b$$
$$\delta=\frac{x_{wl}-x_{wr}}{D}=\frac{R\theta_{wl}-R\theta_{wr}}{D}=\frac{R}{D}(\phi-\psi) \quad (2)$$

where, x :the displacement of robot axle center(m)

x_{wl} :the displacement of robot left wheel (m)

x_{wr} :the displacement of robot right wheel (m)

So we get:
$$\begin{cases}\phi=\dfrac{1}{R}x-\dfrac{1}{2R}\theta_b+\dfrac{D}{2R}\delta\\ \psi=\dfrac{1}{R}x-\dfrac{1}{2R}\theta_b-\dfrac{D}{2R}\delta\end{cases} \quad (3)$$

We can find the relationship between motor armature voltage and output torque from motor reducer,as follow:

$$M_l=-\frac{30\cdot n^2\cdot K_m}{\pi\cdot R_m\cdot K_m}\dot{\phi}+\frac{n\cdot K_m}{R_m}u_{al}$$
$$M_r=-\frac{30\cdot n^2\cdot K_m}{\pi\cdot R_m\cdot K_m}\dot{\psi}+\frac{n\cdot K_m}{R_m}u_{ar} \quad (4)$$

Where, u_{al} :armature voltage of left motor(v)

u_{ar} :armature voltage of right motor(v)

Then:

$$\ddot{\theta}_b=18.8936\theta_b-10.3111\dot{\theta}_b+70.6712\dot{x}-6.5684u_{a1}$$
$$\ddot{x}=-2.1282\theta_b+2.7197\dot{\theta}_b-19.4162\dot{x}+1.7316u_{a1}$$
$$\ddot{\delta}=-10.2274\dot{\delta}+3.5414u_{a2}$$

where:
$$u_{a1}=u_{al}+u_{ar}$$
$$u_{a2}=u_{al}-u_{ar} \quad (5)$$

Therefore,we convert original multi-input system to two single-input subsystems, u_{a1} control the displacement and tilt angle of robot, u_{a2} control yaw angle of robot.For controling robot balance,we only need to consider the tilt angle and displacement,so we select state variable:angle,angular velocity,displacement and speed,then write state equation as follow:

$$\begin{bmatrix}\dot{\theta}_b\\ \ddot{\theta}_b\\ \dot{x}\\ \ddot{x}\end{bmatrix}=\begin{bmatrix}0 & 1 & 0 & 0\\ 18.8936 & -10.3111 & 0 & 70.6712\\ 0 & 0 & 0 & 1\\ -2.1282 & 2.7197 & 0 & -19.4162\end{bmatrix}\cdot\begin{bmatrix}\theta_b\\ \dot{\theta}_b\\ x\\ \dot{x}\end{bmatrix}+\begin{bmatrix}0\\ -6.5684\\ 0\\ 1.7316\end{bmatrix}\cdot u_{a1} \quad (6)$$

$$y=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\theta_b\\ \dot{\theta}_b\\ x\\ \dot{x}\end{bmatrix} \quad (7)$$

According to controllabe and observabe theory,we obtain result that the system is controllability and observability by calculating in matlab.

III. Design Controller based on LQR and PID

A. Analysis the controllable angle of the robot

The model analysis shows that the system is controllable.While the model is linearized in the equilibrium point and the angle is $\pm 5^{\circ}$,but the actual robot tilt angle is much greater than this value.And the linear detection range of the inclinometer is $\pm 20^{\circ}$ .The physical experiment shows that when the tilt angle of robot is $\pm 20^{\circ}$ and the motor provides the greatest torque,the robot can back to the equilibrium point.Therefore,the range of controllable angle is set to $\pm 20^{\circ}$ which is much larger than the constraint in linearization.So,the hybrid controller researched in this paper has a very important practical significance[3].

B. Controller Design

1) LQR Controller[4]

For the robot system described by $X = AX + BU, Y = CX$,with a quadratic cost function defined as $J=\frac{1}{2}\int_0^{\infty}[e^T(t)Qe(t)+u^T(t)Ru(t)]dt$,where the weight matrices Q and R are symmetric positive-definite matrices.The control method is to find the best vector control of matrix K,then the feedback control law that minimizes the value of the cost is $U = -K \cdot X$,where Q=[100 0 0 0;0 100 0 0;0 0 100 0;0 0 0 100],R=1.So we can get the system feedback gain $K = [-40.9839 \quad -13.8993 \quad -10 \quad 24.6190]$ by using of matlab commands.

For the selected state variables,the θ_b is defined as 3° or 20° ,and the other variables are defined as zero.The zero input response are followed:

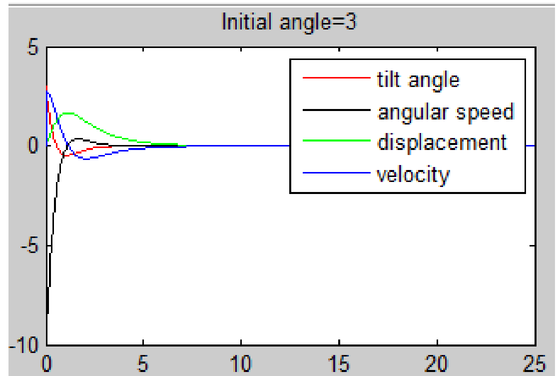

Fig.3 LQR control when initial angle is 3

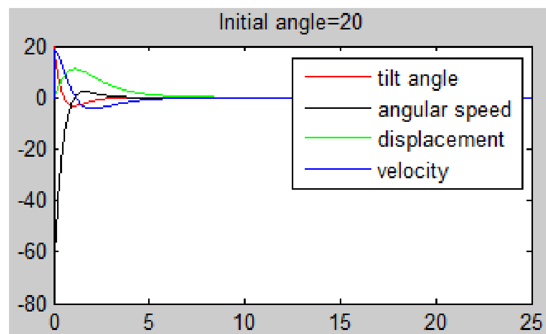

Fig.4 LQR control when initial angle is 20

The compare of different initial conditions shows that there is not great change in the situation of large tilt angle change in theoretical simulation . However in the physical system,the situation is quite different.Along with the tile angle increace,the approximate linearization error is also increased, and it is impossible to achieve the dynamic balance of robots under the control of LQR. In addition a superior LQR controller must have a accurate system modeling and strict physical system parameters.

2) PID Controller

The classic PID controller is the main technology in industrial control.In the application of Self-Balancing Two-Wheeled Robot,the main advantage is that the controller structure is simple ,robustness, easy to implement with a very good control effect.And it do not rely on its linearization model.However,the introduction of integral action into classic PID which achieves the non-poor regulation results inevitably overshoot in the adjustment process.Moderate overshoot is beneficial to improve response rapidity of the system.But in the process of physical control,the overshoot makes robot oscillation and the static performance is poor.Figure 5 shows the simulation curve of step response.

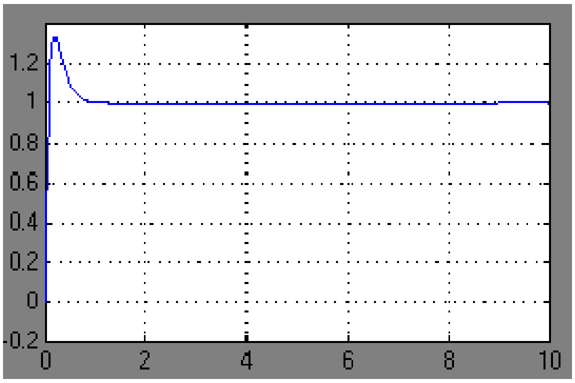

Fig.5 PID Control

3) Hybrid Controller of LQR and PID

The pluses and minuses of LQR and PID controller using in robot control was stated clearly in the above analysis.For the robot,because of its non-linear and uncertainties,it can not achieve the desired result by using any one of the controllers.In order to get faster control speed and better effect of dynamic balance,the paper design a composite controller according to the characteristics of the two controllers:each of which have a controlling factor to limit the output of two controllers,and the control variable is the sum of two output.

Define the outputs of the controllers U1 and U2 respectively and controlling factor for K1 and K2.Consider the following:when the tilt angle is $|\theta_b| \le 5^\circ$,the robot close to equilibrium point,linearization error of the system state equation is very small,so the effect of LQR controller is very well.When the tilt angle is $5^\circ \le |\theta_b| \le 10^\circ$,the robot begin to deviate from the vicinity of equilibrium point,the linearization error enlarge,then the control parameters are in error,so the LQR controller alone can not fully satisfy the requirements of balance control,at this point the introduction of PID controller can play their respective advantages.When the tilt angle is $10^\circ \le |\theta_b| \le 20^\circ$,the robot has deviated far from the equilibrium point ,the linearization error isgreat,the LQR control parameters no longer meet the requirements of actual control,at the moment,PID control algorithm is called only in order to make the angle of robot rapid recovery to range of 10° [5].

As a result,define K1,K2 as follow:

$$K1 = \begin{cases} 1, \left(|\theta_b| \le 5^\circ\right) \\ \dfrac{10 - |\theta_b|}{15}, \left(5^\circ \le |\theta_b| \le 10^\circ\right), U1 = U1 \times K1; \\ 0, \left(10^\circ \le |\theta_b|\right) \end{cases}$$

$$K2 = \begin{cases} 0, \left(|\theta_b| \le 5^\circ\right) \\ \dfrac{|\theta_b| - 5}{5}, \left(5^\circ \le |\theta_b| \le 10^\circ\right), U2 = U2 \times K2; \\ 1, \left(10^\circ \le |\theta_b|\right) \end{cases}$$

IV. PHYSICAL CONTROL OF TWO ROBOTIC

The control core of two robotic used in experiments is TI DSP2812.The software development environment is CCS2.0.First,initialize the various units of the DSP , configurethe appropriate port and open break,wait for interrupt.The sensor signal is sampled in interrupt program and obtain the posture and position by calculating the sampling data.Then calling control algorithm controls the robot,and sending sensor data to the host computer.Through physical experiments to obtain data and draw curves as follow.

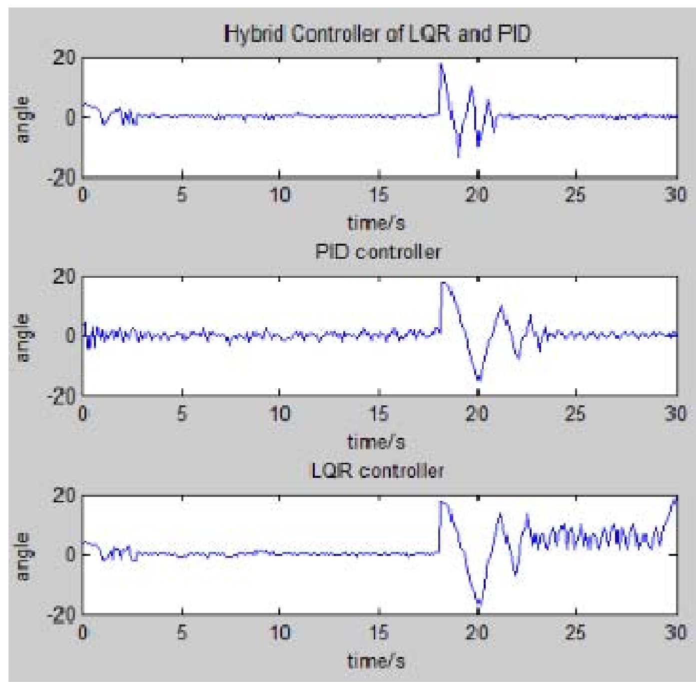

Fig.6 Hybrid Controller

Experiment summary:the control algorithm combined by LQR and PID can obtain a good balance effect and has a good anti-disturbance effects,can restore dynamic fast.For the PID controller,it can control the robot balance but the robot vibrate larger in the vicinity of balance point,the static performance is poor.As for the linear controller LQR,it has a good control effect in small-angle scope,but for larger disturbance,when the angle beyond the linearization constraint conditions,the controller do not have good control effect,even can not keep robot balance.

CONCLUSION

In this paper,we analyzed the important problem of balance control in two-wheeled self-balancing robot control.By analyzing the control method of approximate linearize the nonliear system and the advantages and disadvantages of the classic PID,we designed composite controller based on LQR and PID,then verified the effectiveness of this control method by physical experiments,and achieved robot dynamic equilibrium within the scope of a larger inclination angle.

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