Stabilization Control of Double Inverted Pendulum System

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Abstract

The work deal with the stabilization control of double inverted Pendulum system .Inverted Pendulum system is a complicated, nonlinear, unstable system of high order. Fuzzy control research for stabilization a double inverted pendulum at an upright position successfully is proposed based on weight variable fuzzy inputs. The weight variable fuzzy inputs is gained by combining the fuzzy control theory with the optimal control theory. The fuzzy control rules of a double inverted pendulum are given. In order to consider cart, the lower pendulum, the upper pendulum error information, three different fuzzy controller were designed in this paper. Simulation results show that the controller, which the upper pendulum is considered as main control variable, has higher accuracy and quicker convergence speed and higher precision, and simulation result is promising. The control result can be expanded the control of multilevel inverted Pendulum, and have a guiding meaning in the control of other unstable system.

1. Introduction

The double inverted pendulum[1,2,3] is a multivariate nonlinear fast reaction and unstable system. To stabilize a double inverted pendulum is not only a challenging problem but also a useful way to show the power of the control method. As a typical unstable nonlinear system, inverted pendulum system is often used as a benchmark for verifying the performance and effectiveness of a new control method because of the simplicities of the structure. Since the system has strong nonlinearity and inherent instability, the variable structure control system[4] had to linear the mathematical model of the object near upright position of the pendulum.

Recently, a lot of research on control of the inverted pendulum system by using fuzzy control systems [5-13] containing fuzzy inference have been done. It has been known as a more difficult problem to design a fuzzy controller for double inverted pendulum by utilizing fuzzy control theory.

In this paper, three different fuzzy controller for the stabilization control of inverted pendulum systems is presented based on weight variable fuzzy inputs. Weight variable is given by optimal feedback matrix and the two new fuzzy input variables x1 and x2 are constructed by the weight variable. A fuzzy controller is designed by using x1 and x2. A new fuzzy controller with 2 input items and 1 output item is applied for stabilizing a double inverted pendulum. In order to consider cart, the lower pendulum, the upper pendulum error information, three different fuzzy controller were designed in this paper. The proposed fuzzy controller has a simple and intuitively understandable structure.

2. System model

The inverted pendulum system defined here is shown in Figure 1, which is formed from a cart, two pendulums and a rail for defining the position of the cart. The pendulum is hinged in the center of the top surface of the cart and can rotate around the pivot in the same vertical plane with the rail. The cart can move right or left on the rail freely. It is given that no friction exists in the system between the cart and the rail or between the cart and the pendulum. The dynamic equation of the inverted pendulum system can be expressed as[14].

Consider the system of Figure 1, in which a horizontally translating inverted pendulum of two links l_1, l_2 moves under the action of a single control input, Generalized coordinates $x_1(t)$, $\theta_1(t), \theta_2(t)$ are attached to the system, and coordinates $\theta_1(t)$, $\theta_2(t)$ measured positive clockwise from a local vertical. The cart is assumed to have mass M (kg), with link li of mass mi (kg) and length li (m); i=1; 2;the unit of time is the second(s). Acceleration due to gravity, which acts downwards in the plane, is written as g with units m/s^2.

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System motion is assumed to be damped by viscous friction throughout.

The state vector x(t) comprises the three generalized coordinates and their time derivatives

$$x(t)=\begin{bmatrix}x_1(t) & \theta_1(t) & \theta_2(t) & \dot{x}_1(t) & \dot{\theta}_1(t) & \dot{\theta}_2(t)\end{bmatrix}^T.$$

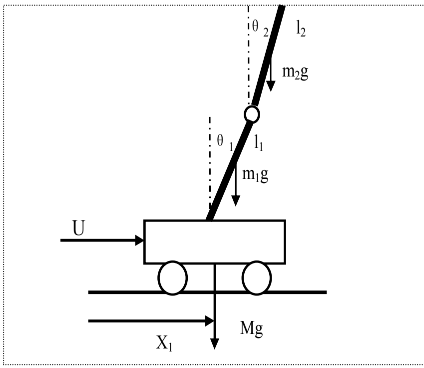

Figure 1. Schematic diagram of double inverted pendulum system.

Thus, a mathematical model of the double-inverted pendulum has been derived. The symbols are defined and the parameters are given above.

In the neighborhood of unstable balance point ,

$$x(t)=\begin{bmatrix}x_1(t) & \theta_1(t) & \theta_2(t) & \dot{x}_1(t) & \dot{\theta}_1(t) & \dot{\theta}_2(t)\end{bmatrix}^T=\begin{bmatrix}0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}^T$$

, By substituting the parameters, the following linear model is gained:

$$\begin{cases}\dot{x}=Ax+Bu\\ Y=Cx+Du\end{cases}\qquad(1)$$

where

$$A=\begin{bmatrix}0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 86.6907 & -21.6172 & 0 & 0 & 0\\ 0 & -40.31121 & 39.45 & 0 & 0 & 0\end{bmatrix}\qquad B=\begin{bmatrix}0\\ 0\\ 0\\ 1\\ 6.64015\\ 0.087668\end{bmatrix}$$

$$C=\begin{bmatrix}1 & 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\end{bmatrix}\qquad D=\begin{bmatrix}0\\ 0\\ 0\\ 0\\ 0\\ 0\end{bmatrix}$$

3. Fuzzy control of double inverted pendulum system

3.1. fuzzy control method

In order to resolve the question, six input items is weighted as two input items and one out items is concluded by fuzzy inference.

The weight variable is derived by using the state-space theory.

The optimal control variable is

$$U=-KX\qquad(2)$$

The quadratic criterion function is given by

$$J=\frac{1}{2}\int_0^{\infty}\left[X^TQX+U^TRU\right]dt\qquad(3)$$

where Q--- state variable weight matrix

R--- input variable weight matrix

According to the quadratic criterion function the positive-definite symmetric matrix P can be solved by the Riccati equation of linear optimal control system

$$PA+A^TP-PBR^{-1}B^TP+Q=0\qquad(4)$$

The composition coefficient K can be derived by

$$K^T=R^{-1}B^TP=[k1\ k2\ k3\ k4\ k5\ k6]\qquad(5)$$

If the positive-definite matrix

$$Q=\begin{bmatrix}100 & 0 & 0 & 0 & 0 & 0\\ 0 & 100 & 0 & 0 & 0 & 0\\ 0 & 0 & 200 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix},$$

$R=0.01$; the composition coefficient K is

K^T =[100 242.397 -637.7204 94.6427 -4.2122 -104.5292] by means of MATLAB lqr command.

Considering the error information from cart, the lower pendulum and the upper pendulum, we choose different variable as main control variable, and so in this paper three new fuzzy controller is designed.

We define variable distance of cart, velocity cart , the lower pendulum angle, the lower pendulum angle velocity , the upper pendulum angle, the upper pendulum angle velocity as

$$x(t)=\begin{bmatrix}x_1(t) & \theta_1(t) & \theta_2(t) & \dot{x}_1(t) & \dot{\theta}_1(t) & \dot{\theta}_2(t)\end{bmatrix}^T.$$

We assume that two new state variables, the composition error X1 and the rate of error X2 are gained with the weight variable K. Let

$$x1=[x_1\ \ \theta_1\ \ \theta_2]*\begin{bmatrix}100\\ 242.397\\ -637.7204\end{bmatrix}\qquad(6)$$

In order to consider the cart as the main control variable, we transform (6) into (7)

$$x1=[x_1\ \ \theta_1\ \ \theta_2]*\begin{bmatrix}1\\ \dfrac{242.397}{100}\\ \dfrac{-637.7204}{100}\end{bmatrix}\qquad(7)$$

Definition x2 as

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$$x2=[\dot{x}_1\ \ \dot{\theta}_1\ \ \dot{\theta}_2]*\begin{bmatrix}96.6427\\-4.2122\\-104.5292\end{bmatrix} \tag{8}$$

In order to consider the cart as the main control variable, we transform (8) into (9)

$$x2=[\dot{x}_1\ \ \dot{\theta}_1\ \ \dot{\theta}_2]*\begin{bmatrix}1\\ \dfrac{-4.2122}{96.6427}\\ \dfrac{-104.5292}{96.6427}\end{bmatrix} \tag{9}$$

The input items of fuzzy controller was reduced to two, so reduced the design difficulty of fuzzy controller and retained all system state variable information at the same time. Considering cart variable as main control variable, weight value of cart is 1.

First we construct seven membership functions for x1,x2,u on its universe, that is, for the values positive big(PB), positive medium (PM), positive small(PS), zero (ZO), negative small(NS), negative medium (NM),negative big (NB),

After the stage of constructing membership functions, we construct a fuzzy control rule Table for the rule base.The rule base is formed by forty nine rules to control the inverted pendulum system (see Table 1).

Table 1fuzzy control system rule base for cart

X2 X1	NB	NM	NS	ZO	PS	PM	PB
NB	NB	NB	NB	NB	NM	NS	ZO
NM	NB	NB	NM	NM	NS	ZO	ZO
NS	NB	NM	NM	NS	ZO	ZO	PS
ZO	NM	NS	NS	ZO	PS	PS	PM
PS	NS	ZO	ZO	PS	PM	PM	PB
PM	ZO	ZO	PS	PM	PM	PB	PB
PB	ZO	PS	PM	PB	PB	PB	PB

3.2. Simulation

The initial conditions are determined as $x=[0\ 0\ 0\ 0\ 0\ 0]^T$. The desired conditions are determined as $x=[0.01\ 0\ 0\ 0\ 0\ 0]^T$.

The simulation result for cart main control is shown as Figure 2.

According to the results, originally the amplitude of all state variable fluctuated intensively, and overshoot become bigger. The time that system research the equilibrium state is about six seconds.

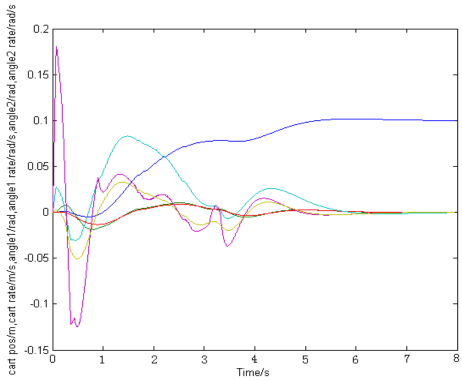

Figure 2 The simulation result for cart main control variable

In order to consider the lower pendulum as the main control variable, Let

$$x1=[x_1\ \ \theta_1\ \ \theta_2]*\begin{bmatrix}\dfrac{100}{242.397}\\ 1\\ \dfrac{-637.7204}{242.397}\end{bmatrix} \tag{10}$$

$$x2=[\dot{x}_1\ \ \dot{\theta}_1\ \ \dot{\theta}_2]*\begin{bmatrix}\dfrac{96.6427}{-4.2122}\\ 1\\ \dfrac{-104.5292}{-4.2122}\end{bmatrix} \tag{11}$$

The simulation result is shown as Figure 3.

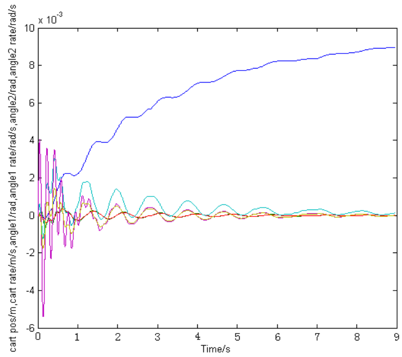

Figure 3 The simulation result for the lower pendulum main control variable

According to the results, originally the fluctuation of all state variable changed intensively, and overshoot become small. The time that system research the equilibrium state is about nine seconds.

In order to consider the upper pendulum as the main control variable, Let

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$$x1=[x_1\ \ \theta_1\ \ \theta_2]*\begin{bmatrix}100\\ -637.7204\\ 242.379\\ -637.7204\\ 1\end{bmatrix} \tag{12}$$

$$x2=[\dot{x}_1\ \ \dot{\theta}_1\ \ \dot{\theta}_2]*\begin{bmatrix}96.6427\\ -104.5292\\ -4.2122\\ -104.5292\\ 1\end{bmatrix} \tag{13}$$

The simulation result is shown as Figure 4.

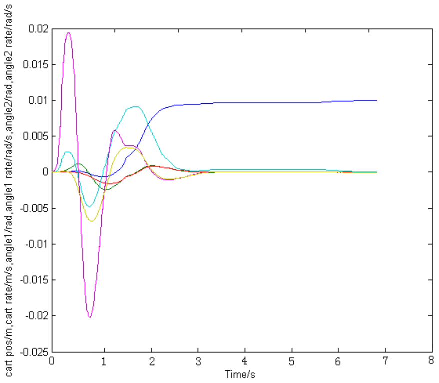

Figure 4. The simulation result for the upper pendulum main control variable

After these experimental results, we decided to compare the results of three fuzzy controller. This fuzzy controller of the upper pendulum main control variable have best result. It takes about 3s to stabilize the pendulum system completely. The fluctuation of all state variable changed , and overshoot become smallest. The time that system research the equilibrium state is the shortest.

4. Conclusions

Three different fuzzy controller based on the weight variable is compared for the stabilization control of the inverted pendulum systems. The input items of fuzzy controller was reduced to two, so reduced the design difficulty of fuzzy controller and retained all system state variable information at the same time. Considering the error information from cart, the lower pendulum and the upper pendulum, we choose different variable as main control variable, and so in this paper three new fuzzy controller is designed.

Comparing the simulation results of three fuzzy controller, this fuzzy controller of the upper pendulum main control variable have best result. The proposed fuzzy controller has a high generalization ability to stabilize completely a wide range of the inverted pendulum systems .

Moreover, the proposed fuzzy controller has a simple and intuitively understandable structure. The fluctuation of all state variable and overshoot become smallest. The time that system reach the equilibrium state is the shortest. It has higher accuracy and quicker convergence speed and higher precision The control result can be expanded the control of multilevel inverted Pendulum, and have a guiding meaning in the control of other unstable system.

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