Design of an Underactuated Self Balancing Robot using Linear Quadratic Regulator and Integral Sliding Mode Controller

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Abstract: **- This paper describes the design procedure of an underactuated self balancing robot using LQR and ISMC. The two wheeled self balancing robot works on the principle of inverted pendulum concept so it is otherwise referred to as two wheeled inverted pendulum mobile robot. The two WMR is widely used in many applications such as a personal transport system (Segway), robotic wheelchair, baggage transportation and navigation etc.LQR and ISMC are introduced into the system in order to achieve the set point control task. That is the 2 WMR should reach the desired set point and then stops while keeping the balance. Both the controller will track the system but the performance of the controller slows some slight differences. By using the MATLAB simulation, two methods are compared and discussed. Also the load transportation task is also assigned to this 2 WMR and controlled using LQR controller.**

Keywords--Wheeled mobile robot (WMR), Integral sliding mode controller (ISMC), Linear quadratic regulator (LQR), Linear quadratic tracking (LQT).

I. INTRODUCTION

THE two wheeled self balancing mobile robot is a highly unstable system and proper control is needed in order to maintain in its equilibrium position. Due to this the self balancing robot is considered as one of the challenging topics in the control system [1]-[10].Inverted pendulum problem is currently being used as the scale for testing the control algorithms. The two WMR consists of two parallel wheels and an inverted pendulum, which is unstable. It has two degrees of freedom so it controls the linear motion of robot in the X-axis and the rotational motion of the pendulum in the X-Y plane. The control of this two WMR is done using LQR and ISMC. This paper focused on the motion control of the wheels of a 2WMR to stabilize the pendulum in its unstable equilibrium position. This 2 WMR is underactuated hence by using lesser number of actuator the system ought to be controlled properly.

In robotics the function of actuator is to induce motion or to stop it. In this system the DC brushless motor will do all those activities. The function of motor is to produce the control signal needed to the system. So it will generate torque according to the controller output. The controller will decide the amount of torque to the system. This torque will act on both the wheels and to the pendulum also.

The 2 WMR has numerous advantages. It has only two wheels hence there will be two points touching the ground. So a 2 WMR requires less space. Tilt angle is the angle made by the inverted pendulum with the vertical axis. This self balancing robot requires continuous correction of its tilt angle. So it exhibits improved dynamic behavior and mobility. Due to this feature it can very easily turn sharp corners and can make U-turns also. These types of robots are maneuverable since it reduces the turn radius to zero. So it can rotate in a position, to change its direction of motion and can travel through tight spaces which the three or four wheeled robots cannot.

Integral sliding-mode controller (ISMC) was implemented on a 2 wheeled self balancing mobile robot which has both matched and unmatched uncertainties [1]. The system is underactuated so using the minimum number of actuators all the states are properly controlled. Underactuated systems have many advantages. It reduces cost, weight and energy consumption. ISMC is better for the control of an underactuated system since it provide an extra degree of freedom in control. A human transporter model is developed and controlled using PID and LQR controller [2]. Then compared the performance of these controllers with the help of simulation results. The results depicts that LQR shows better

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performance than a PID controller. The prototype of a revolutionary Two-wheeled vehicle with 3 DOF is developed which is capable of making stationary U-turns [3].

Each of the wheels is coupled to a dc motor. The control input to the system is nothing but the torque generated by the motor. The paper mainly focusing on the velocity tracking of a self balancing robot using a sliding mode controller (SMC) [4]. One of the challenging problem in this work is to completely nullify the model uncertainties that may affect the system. In order to compensate these uncertainties 2 SMC's are used which will consider both parametric uncertainty and external disturbances. A decoupled control algorithm was implemented on a unicycle robot, which is controlled by two different controllers.[5].One for the control of pitch axis and other for the control of roll axis. Fuzzy sliding mode controller is used for the control of roll axis and LQR for pitch axis. To reduce the effect of chattering signum function is used.

II. SYSTEM MODELLING

Fig.1 indicates the model of an underactuated self balancing robot. The control input is represented as u which is nothing but the torque acting on the wheels with clockwise rotation taken as positive. The motor has the connection to the wheels and to the pendulum also. And hence a reaction torque$-u$ will act on the pendulum.x,$\dot{x}$, $\ddot{x}$are the displacement,velocity and acceleration of the wheel. Likewiseθ ,$\dot{\theta}$, $\ddot{\theta}$ represents the angular displacement, velocity and acceleration of pendulum. r is the radius of wheel and l is the distance from center of wheel to the center of the gravity of pendulum. Moment of inertia of wheel is denoted as I_w and moment of inertia of pendulum is denoted as I_p . $t_f$is the joint friction and f is the ground friction.

By using the Lagrange equation the dynamic equation of wheel is derived as

$$a\ddot{x} + b\ddot{\theta} - m_p l \sin\theta\, \dot{\theta}^2 = \frac{1}{r}(u + t_f - rf) \qquad (1)$$

Where

$$a = m_w + m_p + \left(\frac{I_w}{r^2}\right),\ b = m_p l \cos\theta,\ c = I_P + m_p l^2$$

Dynamic equation for pendulum is obtained as

$$b\ddot{x} + c\ddot{\theta} - m_p \lg \sin\theta = -u - t_f \qquad (2)$$

The objective of this work is the set point control of 2 WMR. So the robot should reach the desired destination and then stops. When it reaches the destination it will be in its equilibrium point ($\theta = 0$). Taking the state variables as $X = [x, \dot{x}, \theta, \dot{\theta}]^T$ and the reference signal is defined as r =$[x_r, v_r, \theta_r, 0]^T$.

The error state is obtained as e = $[e_1, e_2, e_3, e_4]^T$

$$= [x_1 - x_r, x_2 - v_r, x_3 - \theta_r, x_4]^T$$

The value of error should be zero in order to obtain desired response. From (1) and (2), the error dynamic equations can be derived as

$$\dot{e} = A(e) + B(e)[u + u_m(e,t)] + u_u(e,t) \qquad (3)$$

Where

$$A(e) = [e_2 a_1(e) e_4 a_2(e)]^T$$
$$B(e) = [0\ b_1(e)\ 0\ b_2(e)]^T$$
$$u_m = t_f$$

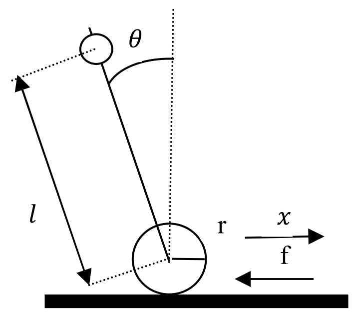

Fig.1. Model of the self balancing robot

$$u_u = [0\ \ u_{u1}(e,t)\ 0\ \ u_{u2}(e,t)]$$
$$a_1(e) = \frac{m_p l}{ac - b^2}[ce_4^2 \sin(e_3) - bg\sin(e_3)]$$
$$a_2(e) = \frac{m_p l}{ac - b^2}[-be_4^2 \sin(e_3) + ag\sin(e_3))]$$
$$b_1 = \frac{1}{r}\left[\frac{c}{ac - b^2} + \frac{b}{ac - b^2}\right]$$
$$b_2 = \frac{1}{r}\left[\frac{-b}{ac - b^2} + \frac{-a}{ac - b^2}\right]$$
$$u_{u1} = \frac{-c}{ac - b^2}f$$
$$u_{u2} = \frac{b}{ac - b^2}$$

III. CONTROLLER DESIGN

A. Linear Quadratic Regulator (LQR)

The linear-quadratic regulator technique is widely used in control system since it incorporates all the state variables to calculate the control signal value. This feature helps the system to hold their position and tilting angle at desired values.

The dynamic model can be linearized in the form

$$\dot{X} = AX + BU \qquad (4)$$

Where X represents the states of the system and U the control input. A and B are 2 constant matrices which is obtained from the system dynamics. To linearize the given model some assumptions are taken such as $\sin e_3 \approx e_3, e_4^2 \approx 0, \cos e_3 \approx 1$

$$\dot{X} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & a_{23} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & a_{43} & 0 \end{bmatrix} \begin{bmatrix} x \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ b_1 \\ 0 \\ b_2 \end{bmatrix} u \qquad (5)$$

Where

$$a_{23} = \frac{-m_p lbg}{ac - b^2}$$

$$a_{43} = \frac{m_p lag}{ac - b^2}$$

By substituting the values the state space model is obtained as

$$\dot{X} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -2.5083 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 47.418 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 5.2216 \\ 0 \\ -38.605 \end{bmatrix} u \tag{6}$$

So the matrices A and B be

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -2.5083 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 47.418 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 5.2216 \\ 0 \\ -38.605 \end{bmatrix}$$

LQR offers an optimal control over the system's input by taking the states of the system and the control input into consideration. And it provides a feedback gain that will reduces the integral quadratic cost function.

$$J = \frac{1}{2}\int_0^\infty e'(t)Qe(t) + U'(t)RU(T)dt \tag{7}$$

Control law for LQR is obtained as

$$u = -R^{-1}BPX \tag{8}$$

With controller feedback gain as

$$K = R^{-1}BP \tag{9}$$

The value of K determines the amount of control fed back into the system. The Q and R matrix should be positive definite matrices. The size of Q matrix depends on the size of the system's state matrix and R matrix depends on the number of control input to the system. Q matrix assumes the form of

$$Q = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & d \end{bmatrix}$$

Where a, b, c, d are the weightages for the respective states$x, \dot{x}, \theta, \dot{\theta}$. Weighting matrix R is a scalar value as there is only one control input to the system. The value of Q matrix is adjusted according to the required response. P is the solution of the Riccati equation.

$$AP + PA + Q - PBR^{-1}B^T P = 0 \tag{10}$$

Choosing Q as

$$Q = \begin{bmatrix} 50 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0 & 0 & 500 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

R = 1 and the initial states of the 2 wheeled mobile robot are chosen as x= $[0; 0; 0.1; 0]$. Using MATLAB, algebraic riccati equation is solved and the control gain K is obtained as K= [−7.0711, −9.6708, −27.0228,−2.8418].

The simulation results using an LQR controller are shown in Fig.3 and Fig.4. The 2WMR reaches the desired set point smoothly with a small overshoot, while balancing the entire system.

B. Integral Sliding Mode Controller (ISMC)

Integral sliding mode controller is a robust controller which is an advanced technique for the control of nonlinear systems. Since it is a robust controller it will adjust the parameters if it deviates from its desired value. Reachability condition is eliminated by using an Integral sliding mode controller. By choosing a suitable sliding surface the unmatched uncertainty can be eliminated. While designing a system the effect of friction should be considered to obtain desired response.. In this system ground friction as well as joint frictions is considered. Joint friction is one of the matched uncertainty acting on a system and ground friction is an unmatched uncertainty. An integral sliding surface is proposed to handle both matched and unmatched uncertainties.

$$\sigma(e,t) = Se(t) - Se(t_0) - \int_{t0}^{t} [S\,A(e) + sB(e)k(e,t)]d\tau = 0 \tag{11}$$

Where k(e, t) is the ISMC gain, **s** is the projection vector and it is a 1 × 4 matrix. And by ISMC the control law that should be provided to the system is derived as

$$u(t) = k(e,t) - \rho(e,t)\mathrm{sgn}(sg\sigma) \tag{12}$$

where the switching gain function is

$$\rho = \rho_m + \rho_u + \rho_0 \tag{13}$$

Upper bound of matched uncertainty is represented as ρ_{m}. The upper bound of $\{sg\}^{-1}su_u$is denoted asρ_u. ρ_0 is a constant.Differentiating the sliding surface with respect to time *t* the equation becomes,

$$\dot{\sigma}(t) = s\dot{e}(t) - sA(e) - sB(e)k(e) \tag{14}$$

Substituting the value for $\dot{e}$, the equation modified into

$$\dot{\sigma} = sg(u_m + \frac{su}{sg} + u - k) \tag{15}$$

Then choose a quadratic function

$$v = {\sigma^2}/{2} \tag{16}$$

Differentiating this equation with respect to time

$$\dot{v} = \sigma\dot{\sigma} \tag{17}$$

Substitute for $\dot{\sigma}$ then the equation becomes

$$\dot{v} = \sigma sg(u_m + \frac{su_u}{sg} + u - k) \tag{18}$$

Finally substitute the control law into this

$$\begin{aligned} \dot{v} &= \sigma sg\left[u_m + \frac{su_u}{sg} - \rho sgn(sg\sigma)\right] \\ &\le |\sigma sg|\left(|u_m| + \left|\frac{su_u}{sg}\right| - \rho\right) \\ &\le -\rho_0|\sigma sg| < 0 \end{aligned} \tag{19}$$

The control is obtained when substituting$\dot{\sigma} = 0$,

$$u_{eq}(t) = k - u_m - \frac{su_u}{sg} \tag{20}$$

Substitute this in error dynamics equation,

$$\dot{e_d}(t) = A(e_d) + B(e_d)k(e_d) + \delta \tag{21}$$

δ is the remaining unmatched uncertainty and is given by

$$\delta = [0\ \delta_1\ 0\ \ \delta_2]^T = \left(I - \frac{gs}{sg}\right) u_u = \frac{b_2u_{u1} - b_1u_{u2}}{s_2b_1 + s_4b_2} [0\ s_4 0\ s_2]^T$$

To minimize the unmatched uncertainty s_2 and s_4 should be chosen properly. If $s_2 = 0$ and $s_4 \neq 0$ then the uncertainty exist only in the wheels. Or if choosing $s_2 \neq 0$ and $s_4 = 0$ the uncertainty exist only in the pendulum.Compared to the wheel pendulum is more sensitive to uncertainties so it is recommended that s should be$s_2 = 0$ and $s_4 \neq 0$.

IV. EFFECT OF ADDING LOAD TO A 2 WMR

To make this 2 WMR to carry the load, the dynamic equations will change completely. The dynamic equations of the 2 WMR changes when load or equipment on it changes. This is due to the change in center of gravity when adding load on it. So that to contend with the changing center of gravity the dynamic equations are needed. The angular displacement θ is replaced by (θ + β).β represents the changing COG.

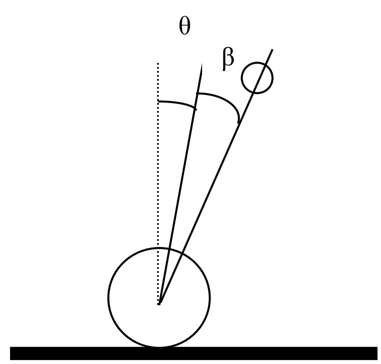

Fig.2. Effect of adding load

The dynamic equations for wheel and pendulum are obtained by taking the lagrangian equation with respect to the β and θ_w. The wheel angle is represented by θ_w.Thus the dynamic quations of a 2 wheeled self balancing robot is obtained as

$$(-m_b rl\sin\theta)\,\dot{\theta}^2 + (l^2m_b + m_b rl\cos\theta + I_w)\ddot{\theta} + (r^2(m_w + m_b) + m_b lr\cos\theta + I_b)\ddot{\varphi} + \mu_g\dot{\varphi} - m_b gl\sin\varphi = 0 \tag{22}$$

$$(l^2m_b + I_b + I_m\eta^2)\ddot{\theta} + (m_b rl\cos\theta - I_m\eta^2)\ddot{\varphi} + \mu_s\dot{\theta} - \mu_s\dot{\varphi} - m_b gl\sin\theta = -u \tag{23}$$

When these equations are linearized in the form

$$\dot{X} = AX + BU \tag{24}$$

Where $X = \begin{bmatrix}\theta \\ \dot{\theta} \\ \dot{\varphi}\end{bmatrix}$

And the constant matrixes A and B will be

$$A = \frac{1}{\Delta}\begin{bmatrix} 0 & \Delta & 0 \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33}\end{bmatrix}, \quad B = \begin{bmatrix}0 \\ Q_2 \\ Q_3\end{bmatrix}$$

Where,

$$P_{21} = m_b gl(d - b)$$
$$P_{22} = -\mu_g b$$
$$P_{23} = -(\mu_g d + \mu_s b)$$
$$P_{31} = m_b gl(a - c)$$
$$P_{32} = -\mu_s a$$
$$P_{33} = -(\mu_g c + \mu_s a)$$
$$a = l^2m_b + m_b rl + I_b$$
$$b = r^2(m_b + m_w) + m_b rl + I_w$$
$$c = l^2m_b + I_b + I_m\eta^2$$
$$d = m_b rl - I_m\eta^2$$
$$\Delta = ad - bc$$

V. SIMULATION AND RESULTS

Controlled the two wheeled mobile robot using LQR and ISMC using the MATLAB software. All the parameters in dynamic equation is listed in the table 1. First of all the 2WMR is controlled using LQR. The feedback gain of LQR is obtained as K = [-7.0711,-9.6,-27.02,-2.8].

TABLE.1. PARAMETERS USED IN 2 WMR

Parameters	Value
Without load	
Mass of wheel (m_w)	1.551kg
Mass of pendulum (m_p)	1.6kg
Length of pendulum (l)	0.13m
Radius of wheel (r)	0.08m
Moment of inertia of wheel (I_w)	0.005kgm^2
Moment of inertia of pendulum (I_p)	0.027kgm^2
With load	
Mass of wheel (m_w)	0.33kg
Mass of pendulum (m_b)	8.91kg
Radius of wheel (r)	0.062m
Length of pendulum (l)	0.1844m
Moment of inertia of wheel (I_w)	0.392×10^{-3}kgm^2
Moment of inertia of pendulum (I_p)	8.736×10^{-3}kgm^2

Moment of inertia of motor axis (I_p)	$10.3 \times 10^{-4}\text{kg}m^2$
Reduction ratio of gears (η)	39.5
Torque constant of motor (t_0)	0.0234Nm/a
Friction between wheel and ground (μ_g)	0.00425
Friction between motor and gear (μ_g)	0.00576

A. *LQR simulation results (Without load)*

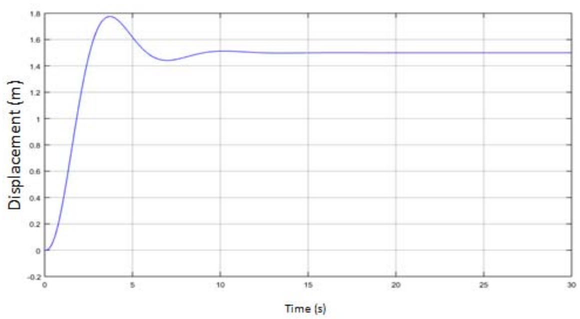

Fig.3. wheel displacement of 2WMR without carrying load

Since it is a tracking problem the 2 WMR is supposed to reach a desired positionx_d, and stop there. Here the value of x_d is provided as 1.5m. So the WMR starts from an initial condition and stops after travelling 1.5 m. The response will shows some overshoot as shown above.

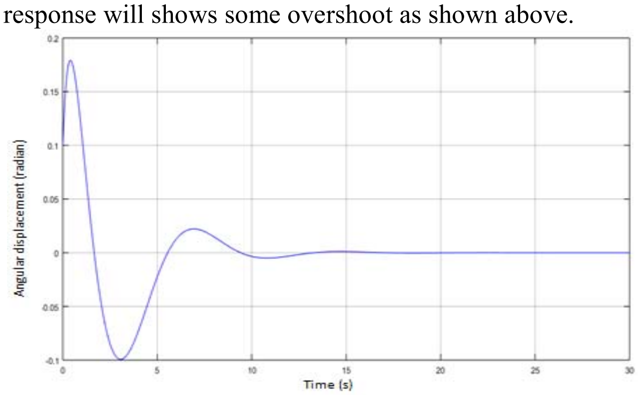

Fig.4. Angular displacement of 2WMR without carrying load

The objective of this work is to make angular displacement of the pendulum to zero. The Initial condition of angular displacement is provided as 0.1 radian. So proper control action is provided in order to make it zero radian.

B. *ISMC Simulation result (without load)*

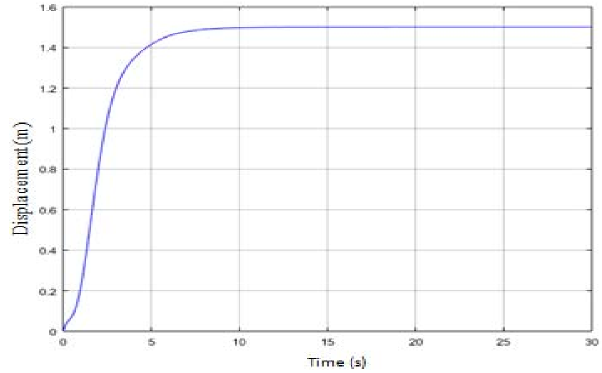

Fig.5. Wheel displacement of 2WMR without carrying load

The 2 WMR will start moving at time t = 0 sec and settled at the desired set point 1.5m at time t = 10 sec. There will not be any overshoot in the response.

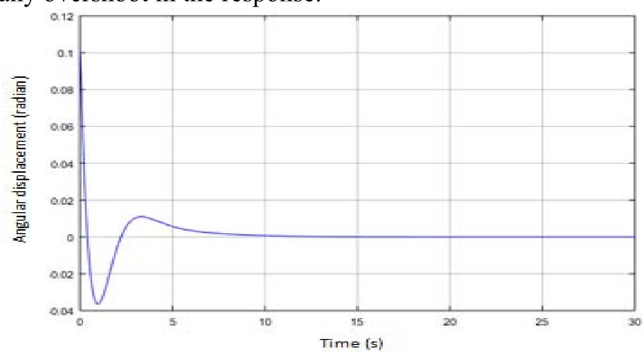

Fig.6. Angular displacement of 2WMR without carrying load

The initial condition for angular displacement is provided as 0.1 radian. Hence the angular displacement will starts from 0.1 and will finally converge to zero as the system gets stabilized. And it will settled at time t = 10 sec.

C. *LQR simulation (with load)*

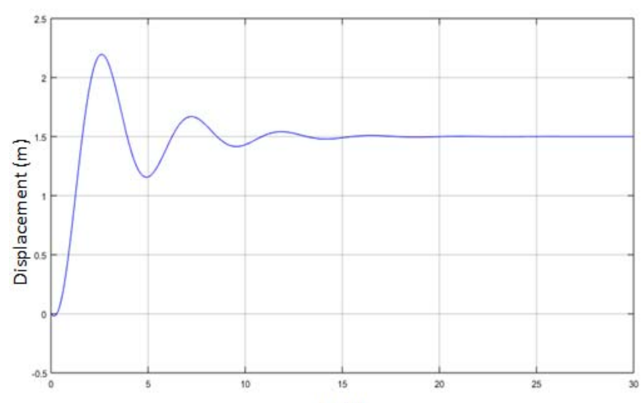

Fig.7. Wheel displacement of 2WMR with load

The 2 WMR starts at time t = 0 and finally it reaches the desired set point 1.5m at time t =17s. The results show some overshoot.

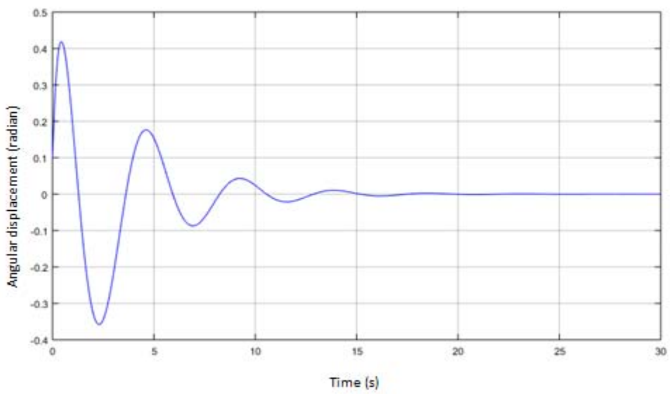

Fig.8. Angular displacement of 2WMR with load

The initial condition for the angular displacement is provided as 0.1 radian. Hence it will starts from 0.1 and finally converges to zero. The result will show some overshoot.

While comparing the 2 controllers it is found that the LQR shows some overshoot that can be nullified by using the ISMC. Also the settling time needed for the ISMC is less than that of LQR.

TABLE.2. COMPARISON OF THE PERFORMANCE OF LQR AND ISMC

CONTROLLER	OVERSHOOT	SETTLING TIME	RISE TIME
LQR without load	1.79	15	3.6
ISMC without load	0	10	4.6
LQR with load	2.2	17	3.5

VI. CONCLUSION

One of the challenging tasks concerning two wheeled inverted pendulum (TWIP) mobile robot is balancing its tilt to upright position, this is due to its highly unstable nature. In this work regulation and set point control of an underactuated 2 WMR without carrying load are done using Linear quadratic regulator and Integral sliding mode controller. Hence the 2 WMR travels 1.5m and then stops at the same time its tilting angle converges to zero. Also the control of an underactuated 2 WMR carrying a load is done using LQR.

LQR is an optimal controller and has been widely used in many applications. When tracking is needed LQT is used so that it will finally converge to the set point. But the response will show some overshoot.

ISMC is an advanced technique for the control of nonlinear system. The ISMC is constructed by a nominal part and a switching term. For this nominal part a linear controller is used which will stabilize the sliding surface. There will not be any overshoot in the response and the settling time is less compared to that of an LQR response.

REFERENCES

[1] Jian-XinXu, Zhao-Qin Guo, and Tong Heng Lee," Design and Implementation of Integral Sliding-Mode Control on an Underactuated Two-Wheeled Mobile Robot" IEEE Trans. on industrial electronics, vol. 61, no. 7, July 2014.

[2] Wei An and Yangmin Li, "Simulation and Control of a Two Wheeled Self Balancing Robot" IEEE Int. conference on Robotics and Biomimetics (ROBIO), Shenzhen, China, December 2013.

[3] Felix Grasser, Aldo D'Arrigo, Silvio Colombi, and Alfred C. Rufer, "JOE: A Mobile, Inverted Pendulum" IEEE Trans. on industrial electronics, vol. 49, no. 1, February 2002.

[4] Jian Huang, Zhi-Hong Guan,Takayuki Matsuno,Toshio Fukuda and KosukeSekiyama, " Sliding-Mode Velocity Control of Mobile-Wheeled Inverted-Pendulum Systems" IEEE transactions on robotics, vol. 26, no. 4, August 2010.

[5] J.Lee, S. Han, and J. Lee, "Decoupled Dynamic Control for Pitch and Roll Axes of the Unicycle Robot" IEEE Trans. Ind. Electron., vol. 60, no. 9,pp. 3814–3822, Sep. 2013.

[6] Nandagopal J L and Letha Kumari R, "Optimal Control of Spacecraft Docking System using Integral LQR controller", International Conference on Technological Advancements in Power and Energy (TAP Energy), 2015.

[7] Cheng-Hao Huang, Wen-June Wang and Chih-Hui Chiu, "Design and Implementation of Fuzzy Control on a Two-Wheel Inverted Pendulum" IEEE Trans. on industrial electronics, vol. 58, no. 7, July 2011.

[8] K. Pathak, J. Franch, and S. K. Agrawal, "Velocity and Position Control of a Wheel Inverted Pendulum by Partial Feedback Linearization" IEEE Trans. Robot., vol. 21, no. 3, pp. 505–513, June 2005.

[9] L. Ouyang, B. Du, A. Peng, and Y Ou, "Design of a Wheeled Inverted Pendulum as a Platform for Learning based Control" Proc. of the IEEE Int. Conf. on Information and Automation, Shenyang, China, June 2012.

[10] Mun-Soo Park and Dongkyoung Chwa, "Swing-Up and Stabilization Control of Inverted Pendulum Systems Via Coupled Sliding Mode Control Method", IEEE Trans.on industrial electronics, vol. 56, NO. 9, Sept. 2009.

[11] P R Jayasree, Jithin Sathyan and MK Sreenivasan, "Sliding Mode Control of Coupled Inductor Buck Converter", International Conference on Circuit, Power and Computing Technologies [ICCPCT], 2016.