



ASSIGNMENT OF BACHELOR'S THESIS

Title: Timing Attack on the RSA Cipher
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Instructions

Review known timing side channel attacks on RSA decryption and signing operations. Create a demonstration application that will perform timing attack on RSA in order to determine the private key. The application will be used in courses on cryptology and computer security as a part of laboratory exercises. Consider an attack on a local computer or over the network and evaluate its time complexity.

References

Will be provided by the supervisor.

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Bachelor's thesis

Timing Attack on the RSA Cipher

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12th May 2017

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Declaration

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In Prague on 12th May 2017

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Abstrakt

Tato práce se zabývá útokem na šifru RSA časovým postranním kanálem. Pomocí měření času podepisování předgenerovaných zpráv, je útočník schopen postupně uhádnout každý bit soukromého klíče. Výsledkem práce je demonstrační aplikace, která bude použita ve výuce předmětu, zabývajícím se počítačovou bezpečností.

Klíčová slova Replace with comma-separated list of keywords in Czech.

Abstract

This thesis is focused on replication of timing attack on RSA cipher, which is done by measuring time of square and multiply algorithm. Implementation should be used for education purposes, mainly in security courses.

Keywords RSA, cryptanalysis, timing attack, side channel, square and multiply

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Introduction

State-of-the-art

RSA

RSA is public-key cryptosystem which was invented by Ron Rivest, Adi Shamir and Leonard Adleman. The cryptosystem was published in the 1977.

2.1 Principle

The cipher is based on modular exponentiation. The whole process of crypting message is divided to four steps

2.1.1 Key generation

- Generate p and q , which have to be distinct prime numbers.
- Compute n , where $n = p * q$
- Compute Euler's totient function $\Phi(n)$. Because we know p and q it is simple to compute it.

$$\Phi(n) = (p - 1) * (q - 1)$$

- Generate e such as $\gcd(e, \Phi(n)) = 1$
- Compute $d = e^{-1} \bmod \Phi(n)$
- The pair (e, n) is released as public key
- The pair (d, n) is secret private key

2.1.2 Key distribution

Alice would like to send Bob secret message. Bob generates public key (e, n) and his private key (d, n) . Bob sends Alice public key using reliable route (it has not to be secret route), Alice uses it to encrypt her message and sends it to Bob. Bob decrypts her message using his private key.

2. RSA

2.1.3 Encryption

Encryption is done by using public keypair (e, n) :

$$c = |m^e|_n$$

where m is plaintext message and c is encrypted message which will be sent to receiver.

2.1.4 Decryption

Decryption is done similar thanks to relation $e * d \equiv 1 \pmod{\Phi(n)}$. We can simply power ciphertext to our private exponent d to obtain original message.

$$|c^d|_n = |(m^e)^d|_n = |m^{e*d}|_n = |m^1|_n = m$$

2.2 Optimization

Because we generally use high value of modulus n . The exponentiation of such high numbers is very time consuming so there are some algorithms to increase speed of computation

2.2.1 Square and Multiply

This optimization uses bitwise representation of the exponent we use. Cycling through all bits from MSB (most significant bit) we determine which operation will be performed for each bit. For bits equal to 1 we perform squaring preset value then we multiply it with the base of exponent. For bits equal to 0 we just perform squaring part.

Algorithm 1 Square & Multiply algorithm

```
1: function SQUARE_AND_MULTIPLY( $m, e, n$ )
2:    $c \leftarrow 1$ 
3:    $k \leftarrow \text{BitLen}(e)$ 
4:   for  $i \leftarrow k - 1, 0$  do
5:      $c \leftarrow c^2$ 
6:     if  $e[i] == 1$  then                                     ▷  $i$ th bit of exponent  $e$ 
7:        $c \leftarrow c * m$ 
8:     end if
9:   end for
10:  return  $c$ 
11: end function
```

2.2.2 Chinese remainder theorem

Attacks

3.1 *Attack on multiply*

3.2 *Attack on square*

Realisation

Conclusion

Bibliography

Acronyms

MSB Most significant bit

LSB Least significant bit

Contents of enclosed CD

	readme.txt	the file with CD contents description
	exe	the directory with executables
	src	the directory of source codes
	wbdcm	implementation sources
	thesis	the directory of \LaTeX source codes of the thesis
	text	the thesis text directory
	thesis.pdf	the thesis text in PDF format
	thesis.ps	the thesis text in PS format