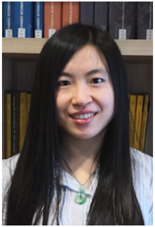
Fast Beam Alignment via Pure Exploration in Multi-Armed Bandits

Yi Wei Zixin Zhong **Vincent Y. F. Tan**



National University of Singapore

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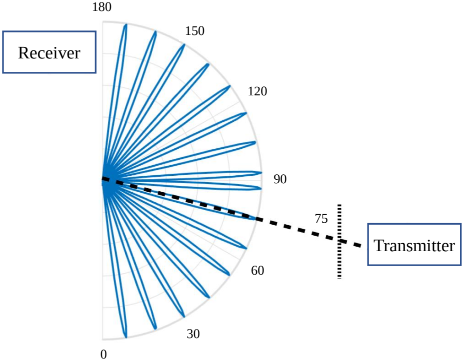
The Beam Alignment Problem



- mmWave Communications
- Beams at Tx and Rx are narrow directional

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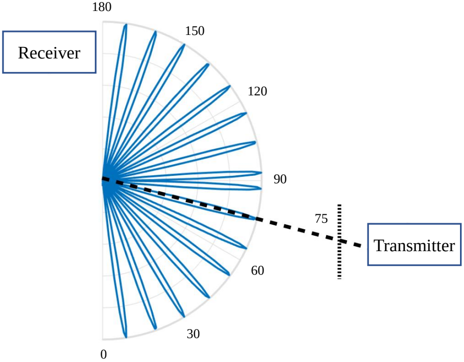

- mmWave Communications
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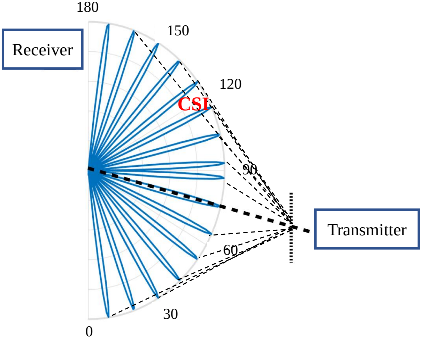
Beam alignment (BA) is to ensure the transmitter and receiver beams are accurately aligned to establish a reliable communication link.

Some Fundamental Challenges in Beam Alignment



- To find optimal Rx/Tx pair, CSI corresponding to each pair is measured
- Frequency of each measurement is high
- High beam alignment latency

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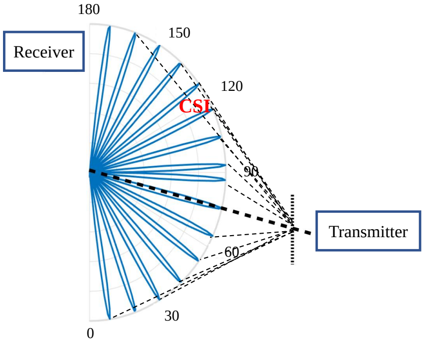

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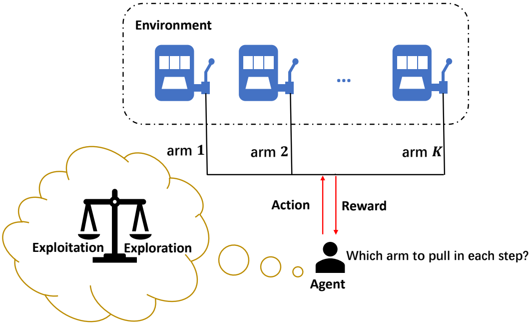
Beam alignment latency will increase with the number of antennas at the receivers and transmitters.

Multi-Armed Bandit



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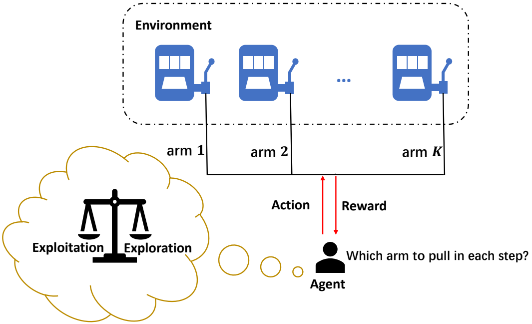

Pure Exploration: Identify the best arm (arm with largest mean) as quickly as possible.

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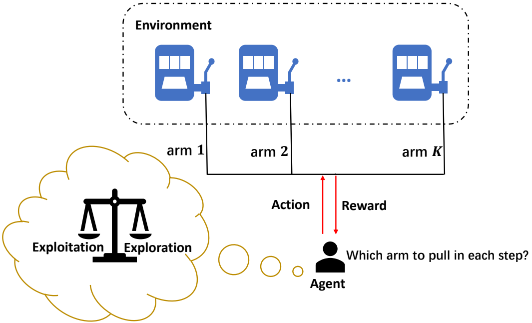

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Main Contribution: Formulate and solve the beam alignment problem using ideas in pure exploration in the fixed-confidence setting.

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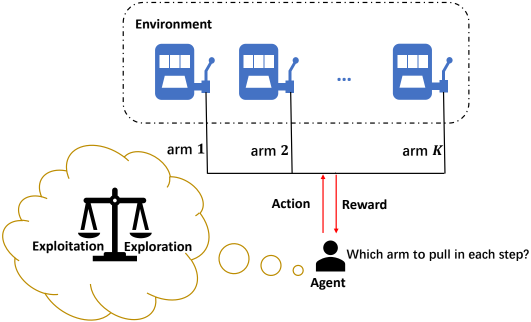

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Exploit structure in the beam alignment problem.

System Model

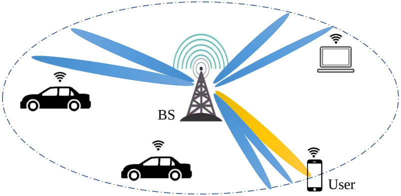

Figure: A mmWave massive MISO system system.

- Massive mmWave MISO system: A base station (BS) equipped with N transmit antennas serves a single-antenna user

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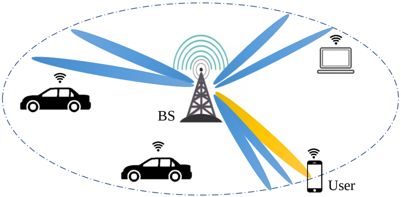


Figure: A mmWave massive MISO system system.

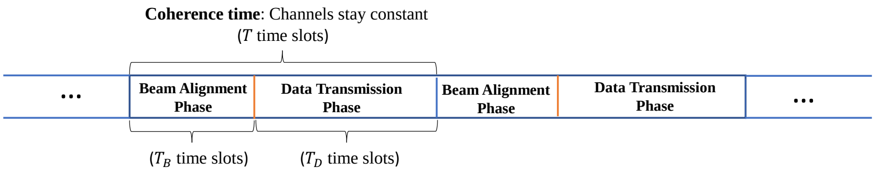
- Massive mmWave MISO system: A base station (BS) equipped with N transmit antennas serves a single-antenna user
- Adopt the widely-used Saleh–Valenzuela channel model

$$\mathbf{h} = \beta^{(1)}\mathbf{a}\left(\theta^{(1)}\right) + \sum_{l=2}^{L} \beta^{(l)}\mathbf{a}\left(\theta^{(l)}\right)$$

Amplitude

1 line-of-sight (LoS) path $\geq$ L − 1 non-LoS (NLoS) paths

Transmission Scheme



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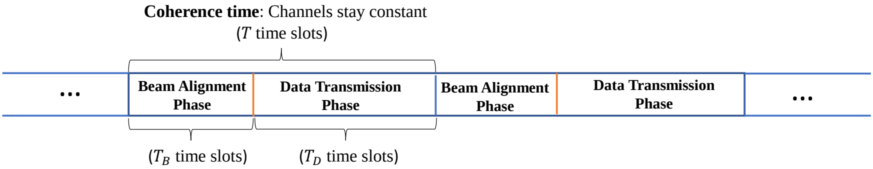

- Beam alignment phase: Fast beam alignment algorithm searches the optimal beam from a given codebook

$$\mathcal{C} \triangleq \{\mathbf{f}_k = \mathbf{a}(-1 + 2k/K) \mid k = 0, 1, \ldots, K - 1\}$$

where the array response vector:

$$\mathbf{a}(x) = \frac{1}{\sqrt{N}} \left[1, e^{j\frac{2\pi}{\lambda}dx}, e^{j\frac{2\pi}{\lambda}2dx}, \cdots, e^{j\frac{2\pi}{\lambda}(N-1)dx}\right]^{\mathrm{H}} \in \mathbb{C}^{N\times 1}$$

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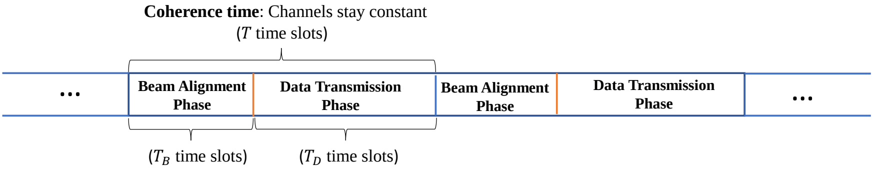

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- Data transmission phase: BS transmits effective data using the selected $\mathbf{f}^*$. Received signal at the user in time slot t is

$$y_t = \sqrt{p}\, \mathbf{h}^{\mathrm{H}}\mathbf{f}^* s_t + n_t,$$

Beam Alignment Phase

- **System Throughput Performance:** effective achievable rate

$$R_{\text{eff}} \triangleq \left(1 - \frac{T_{\mathrm{B}}}{T_{\mathrm{D}}}\right) \log\left(1 + \frac{p|\mathbf{h}^{\mathrm{H}}\mathbf{f}^{*}|^2}{\sigma^2}\right)$$

Time to search for optimal beam T_{B} to be minimized for high R_{eff}

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- **Measurement:** The received signal power

$$R(\mathbf{f}_k) = |\sqrt{p}\mathbf{h}^{\mathrm{H}}\mathbf{f}_k + n|^2 = p|\mathbf{h}^{\mathrm{H}}\mathbf{f}_k|^2 + 2\sqrt{p}\Re(\mathbf{h}^{\mathrm{H}}\mathbf{f}_k n^*) + |n|^2$$

Heteroscedastic Gaussian Variable $\mathcal{N}(p|\mathbf{h}^{\mathrm{H}}\mathbf{f}_k|^2, 2p|\mathbf{h}^{\mathrm{H}}\mathbf{f}_k|^2\sigma^2)$

Gamma Variable $\Gamma(1, 1/\sigma^2)$

Approximate (Because: noise power $\ll$ transmit power)

$$r_k = p|\mathbf{h}^{\mathrm{H}}\mathbf{f}_k|^2 + 2\sqrt{p}\Re(\mathbf{h}^{\mathrm{H}}\mathbf{f}_k n^*)$$

Relation to MABs and Properties

Beam Alignment	→	Pure Exploration in MAB
Find the optimal beam as soon as possible	→	Find the optimal beam as soon as possible
beams	→	base arms
received signal power	→	rewards

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Property: Let $\boldsymbol{\mu} = (\mu_1, \cdots, \mu_K)$, and let $\mu_{(1)} \geq \mu_{(2)} \geq \mu_{(3)} \geq \ldots \geq \mu_{(K)}$ be the sorted sequence of means.

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1. The means of the rewards associated with close-by arms are close.
2. The variance of the reward of an arm is linearly related to its mean

$$\sigma_k^2 = 2\mu_k\sigma^2.$$

Beam Codebooks Possess the Group Property

$\frac{1}{J}$-lower resolution beam codebook

- Constructed by grouping the nearby beams in the codebook $\mathcal{C}$

$$\mathcal{C}_{(J)} \triangleq \left\{ \mathbf{b}_g = \sum_{k=J(g-1)+1}^{Jg} \mathbf{f}_k \ : \ g = 0, 1, \ldots, G-1 \right\}$$

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$$R_g = p|\mathbf{h}^{\mathrm{H}}\mathbf{b}_g|^2 + 2\sqrt{p}\Re(\mathbf{h}^{\mathrm{H}}\mathbf{b}_g n^*),$$

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- Information of a **set** of beams can be obtained at each time step

Problem Setup for Bandit Beam Alignment

Bandit BA Problem

- K base arms $[K] \triangleq \{1, \cdots, K\}$: associated with the beam $\mathbf{f}_k$
- $\{[K], J\}$: set of all non-empty consecutive tuples of length $\leq J$

  $\{[6], 2\} = \{\{1\}, \{1, 2\}, \{2\}, \{2, 3\}, \{3\}, \{3, 4\}, \{4\}, \{4, 5\}, \{5\}, \{5, 6\}, \{6\}\}$
- (K, J)-super arm: each tuple in $\{[K], J\}$, associated with a "grouped beam" $\mathbf{b}_g \in \mathcal{C}_{(J)}$.

Bandit BA Problem Setup

In time step t

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where $F(\mathbf{f}, p, \mathbf{h}, n) = p|\mathbf{h}^{\mathrm{H}}\mathbf{f}|^2 + 2\sqrt{p}\Re(\mathbf{h}^{\mathrm{H}}\mathbf{f}n^*)$

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- Each observed reward $R(t)$ also follows a heteroscedastic Gaussian distribution.

Bandit BA Problem Setup

Algorithm: $\pi := \{(\pi_t)_t, \tau^\pi, \psi^\pi, J\}$

- Sampling rule π_t: Determines the $([K], J)$-super arm $A(t)$ to pull at time step t based on the observation history and the arm history $\{A(1), R(1), A(2), R(2), \cdots, A(t-1), R(t-1)\}$
- Stopping rule: Leads to a stopping time τ^π satisfying $\mathbb{P}(\tau^\pi < \infty) = 1$
- Recommendation rule ψ^π: Outputs a base arm $k^\pi \in [K]$.

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Aim: Use as few samples as possible to output a guess of the optimal arm with probability at least $1 - \delta$.

Some Notations for the General Lower Bound

- Heteroscedastic Gaussian bandit instance:

$$\nu = \{\mathcal{N}(\mu_1^\nu, 2\mu_1^\nu\sigma^2), \ldots, \mathcal{N}(\mu_K^\nu, 2\mu_K^\nu\sigma^2)\}$$

- Optimal arm $A^*(\nu)$:

$$\mu_{A^*(\nu)}^\nu = \operatorname*{argmax}_{k\in[K]} \mu_k^\nu$$

- Set of probability distributions

$$\mathcal{W}_K := \left\{ \mathbf{w} \in \mathbb{R}_+^K : \sum_{k=1}^K w_k = 1 \right\}$$

- Alternative Set

$$\mathrm{Alt}(\nu) := \{\mathrm{u} \in \mathcal{V} : A^*(\mathrm{u}) \neq A^*(\nu)\}$$

General Lower Bound

Theorem

For any $(\delta, J)-$*PAC algorithm where* $\delta \in (0,1)$, *it holds that*

$$\mathbb{E}_\pi[\tau] \geq c^*(\nu) \ln\left(\frac{1}{4\delta}\right),$$

where

$$c^*(\nu)^{-1} = \sup_{\mathbf{w}\in\mathcal{W}_K} \inf_{\mathrm{u}\in\mathrm{Alt}(\nu)} \sum_{k=1}^{K} w_k D_{\mathrm{HG}}(\mu_k^\nu, \mu_k^{\mathrm{u}})$$

and the heteroscedastic relative entropy is

$$D_{\mathrm{HG}}(\mu_i, \mu_j) = \frac{1}{2}\ln\left(\frac{\mu_j}{\mu_i}\right) + \frac{\mu_i}{2\mu_j} + \frac{(\mu_j - \mu_i)^2}{4\mu_j\sigma^2} - \frac{1}{2}.$$

Two-Phase Heteroscedastic Track & Stop (2PHT&S)

Main Idea

to exploit the **prior knowledge** which have not been considered by existing bandit-based beam alignment algorithms:

- Close-by correlation
- Heteroscedasticity
- Group property

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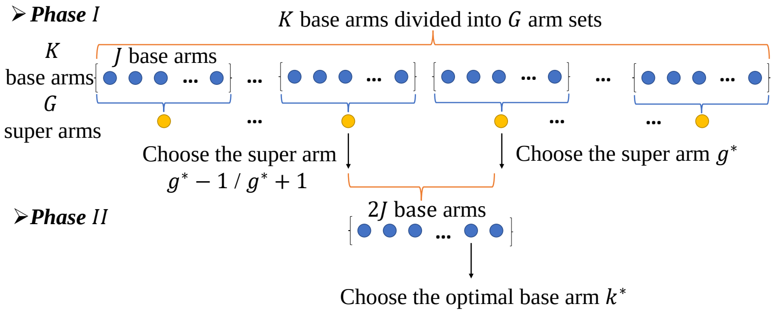
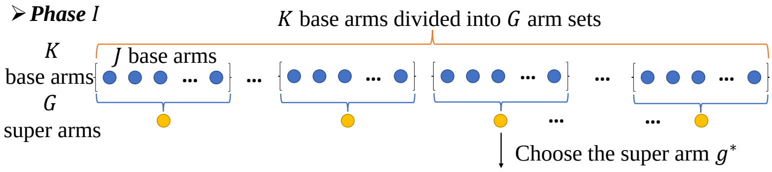


2PHT&S Algorithm



Phase I: Search for the optimal super arm w.p. $\geq 1 - \delta_1$

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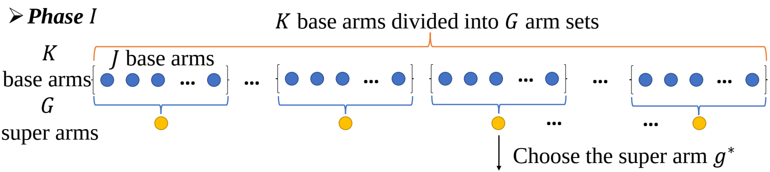

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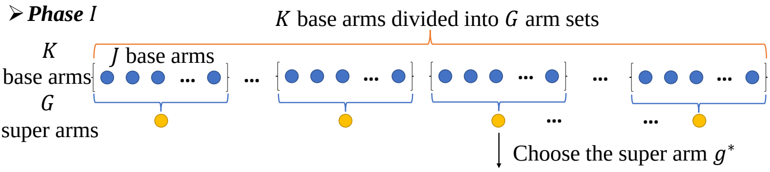

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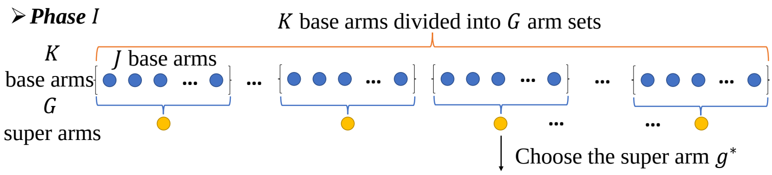

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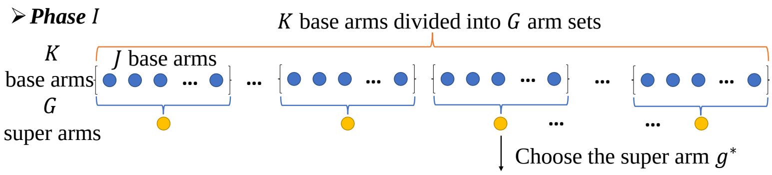

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- Select the optimal super arm

$$g^* = \operatorname*{argmax}_{g \in [G]} \mathbb{E}[R_g(t)].$$

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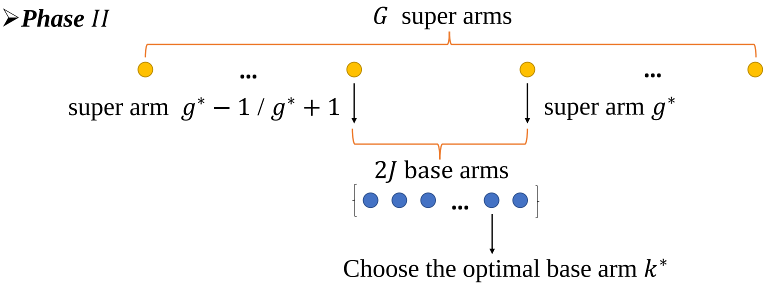

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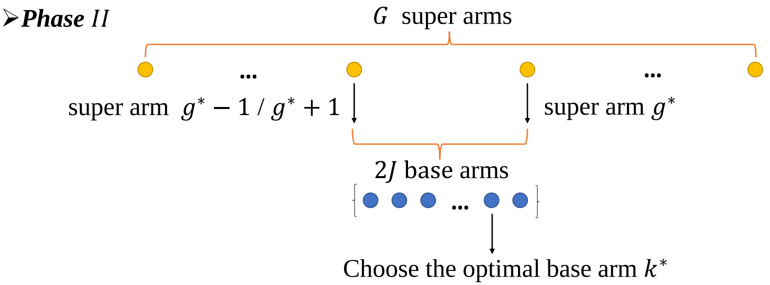

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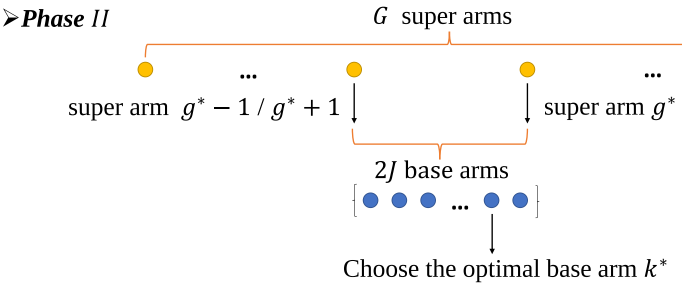

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- Construct a **base arm set** (including the optimal super arm and its neighboring super arm)
- Search for the optimal base arm in the **base arm set** using the HT&S algorithm

HT&S: An Improved Track & Stop Algorithm

- **Sampling Rule:** Estimate the number of times each arm should be sampled

$$Q(t) = \begin{cases} \mathrm{argmin}_i \, T_i(t-1), & \text{if } \min_i T_i(t-1) \le \sqrt{t}, \\ \mathrm{argmax}_i \, t\hat{w}_i^*(t-1) - T_i(t-1), & \text{otherwise.} \end{cases}$$

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- **Stopping Rule:** Stop when the number of pulls of all arms meet a certain requirement

  Threshold to be $\beta(t,\delta,\alpha) = \ln(\alpha t/\delta)$, and the stopping rule is

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- **Heteroscedasticity:** Considered explicitly in the calculations of estimated reward $\hat{w}_i^*(t-1)$ and statistic $Z(t)$

Time Complexity Analysis of 2PHT&S

Theorem

Let the means of the super and base arms be respectively

$$\mathrm{s} := \{\mathcal{N}(\mu_1^{\mathrm{s}}, 2\mu_1^{\mathrm{s}}\sigma^2), \ldots, \mathcal{N}(\mu_G^{\mathrm{s}}, 2\mu_G^{\mathrm{s}}\sigma^2)\} \qquad \textit{and}$$

$$\mathrm{b} := \{\mathcal{N}(\mu_{\mathcal{S}_f(1)}^{\mathrm{b}}, 2\mu_{\mathcal{S}_f(1)}^{\mathrm{b}}\sigma^2), \ldots, \mathcal{N}(\mu_{\mathcal{S}_f(2J)}^{\mathrm{b}}, 2\mu_{\mathcal{S}_f(2J)}^{\mathrm{b}}\sigma^2)\}$$

where

$$\mu_g^{\mathrm{s}} = p\left|\mathbf{h}^{\mathrm{H}}\Big(\sum_{k\in\mathcal{S}_g}\mathbf{f}_k\Big)\right|^2.$$

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where

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Under 2PHT&S, we have

$$\limsup_{\delta\to 0} \frac{\mathbb{E}[\tau^{\mathit{2PHT\&S}}]}{\ln(1/\delta)} \le C_{\mathrm{s}} + C_{\mathrm{b}},$$

where C_{s} and C_{b} represent time complexities of Phases I and II resp.

Simulation Results

Experiment Setup

- Massive mmWave MISO system
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Baseline Algorithms

- Original Track-and-Stop (T&S) algorithm (Garivier et al. 2016)
- Two-Phase Track-and-Stop (2PT&S) algorithm
- Heteroscedastic Track-and-Stop (HT&S) algorithm

A. Garivier and E. Kaufmann, “Optimal best arm identification with fixed confidence,” in PMLR, 2016, pp. 998-1027.

Numerical Simulations on Synthetic Data

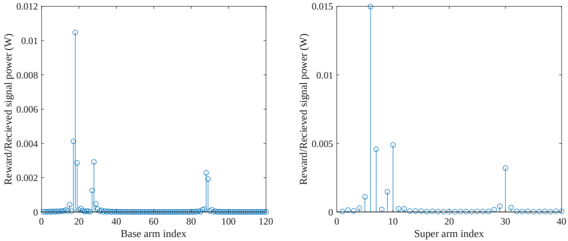

Figure: Mean of the reward of each base arm and super arm (p = 10 dBm).

Table: Average time complexities over 100 experiments when $\delta = 0.1$

Power	4	6	8	10	12
T&S	1154.3 $\pm$338.7	654.6 $\pm$212.1	382.5 $\pm$129.6	209.4$\pm$68.6	133.7 $\pm$8.9
HT&S	473.2 $\pm$275.5	271.4 $\pm$143.4	175.6 $\pm$69.2	133.2 $\pm$24.1	123.9 $\pm$6.5
2PT&S	206.2 $\pm$60.4	120.2 $\pm$35.0	68.4 $\pm$19.4	49.1 $\pm$4.6	45.2 $\pm$1.1
2HPT&S	84.3 $\pm$41.5	58.0 $\pm$19.6	48.4 $\pm$6.3	45.5 $\pm$1.6	45 $\pm$0

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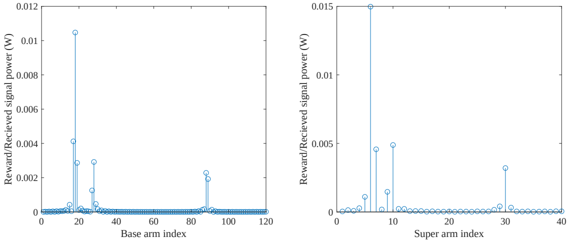

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Experiments on real data for a practical scenario in a city available in the longer version of our paper.

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