The Sender-Excited Secret Key Agreement Model: Capacity and Error Exponents

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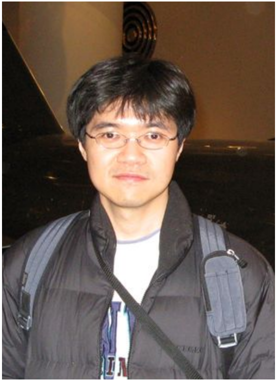
Allerton (Sep 2011)

Joint work with



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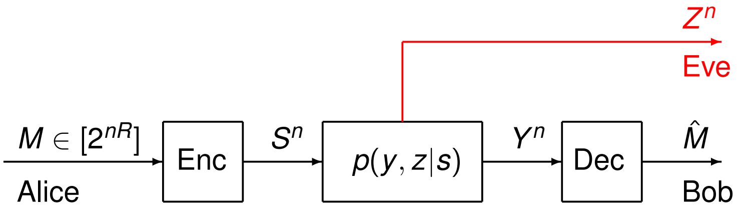
  - Channels with action-dependent states [Weissman]
- Main contributions:
  - Secret key capacity
  - Inner bound for rate-reliability-secrecy-exponent region

Wiretap Channel [Wyner, Csiszár and Körner]



- Want to transmit message reliably to Bob but keep Eve ignorant
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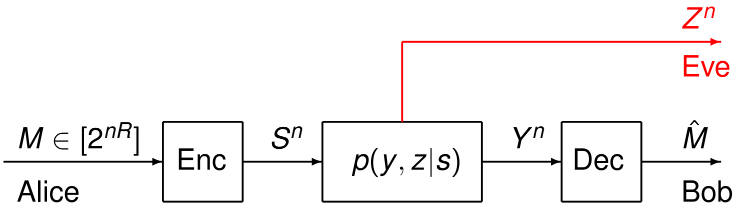

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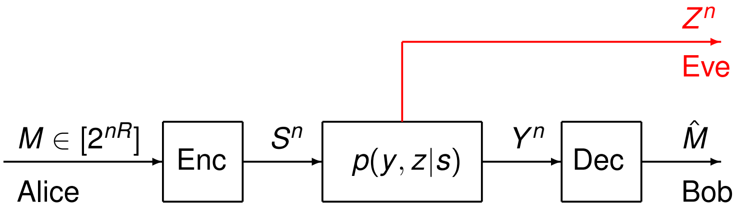

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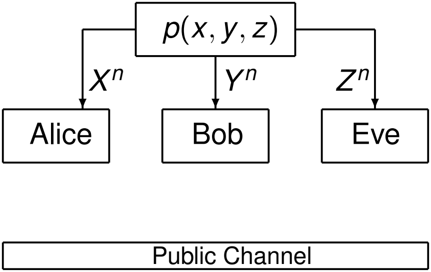
- Channel-type model

Secret Key Generation [Maurer, Ahlswede & Csiszár]



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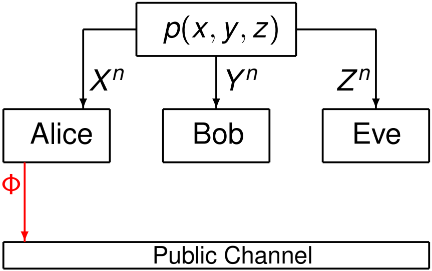

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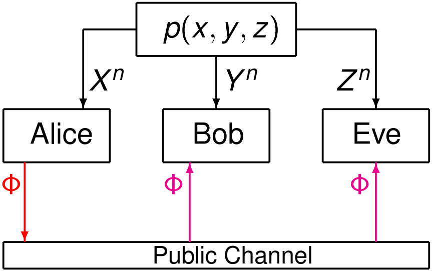

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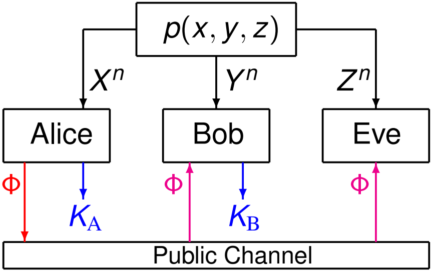

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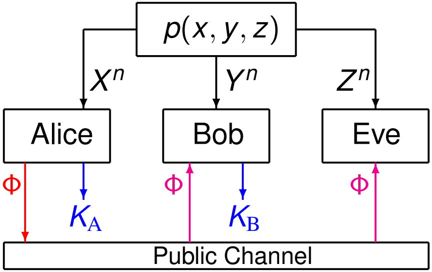

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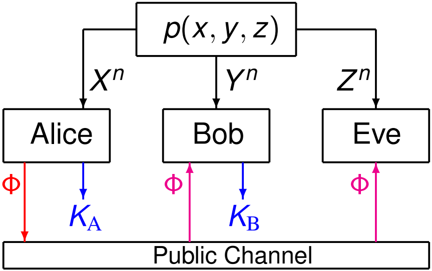

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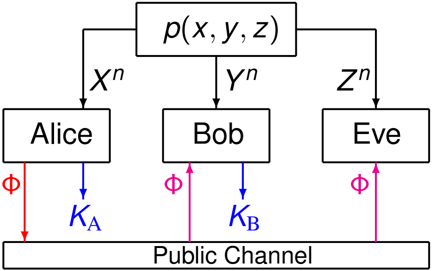

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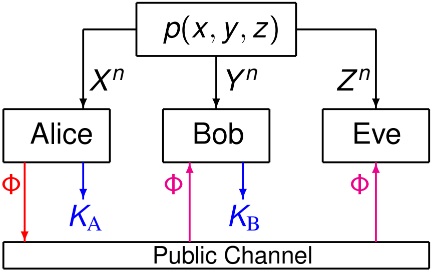

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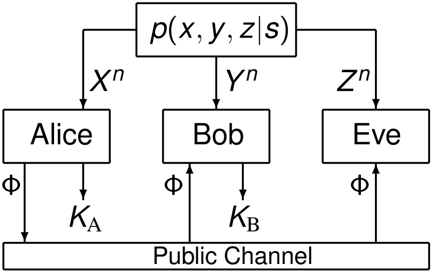

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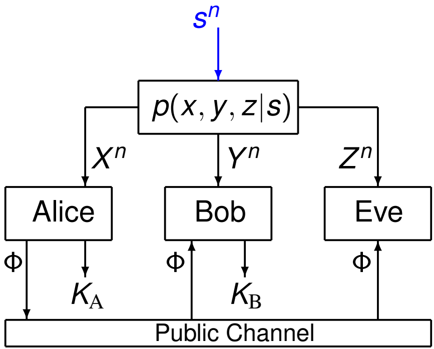

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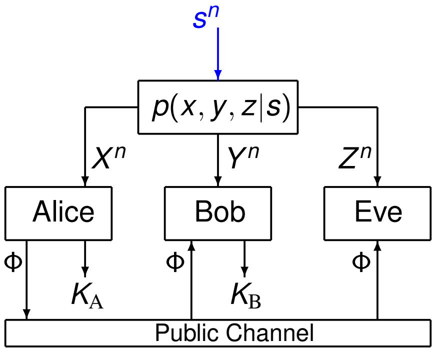

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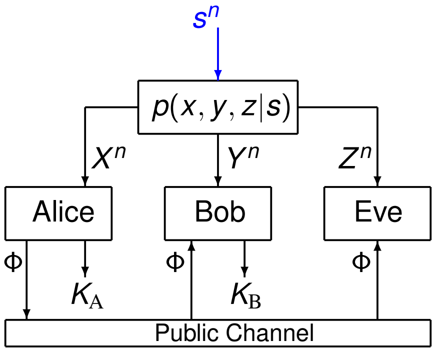

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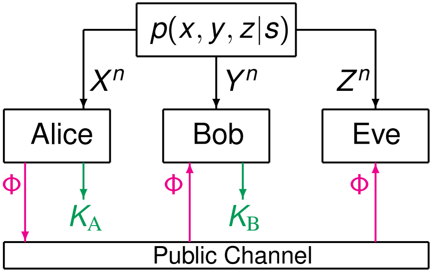
Our model: Sender-Excited Secret Key Agreement



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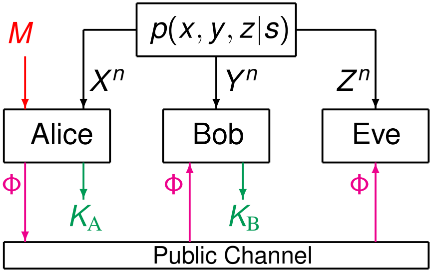

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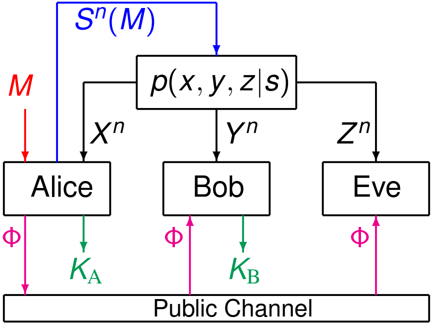

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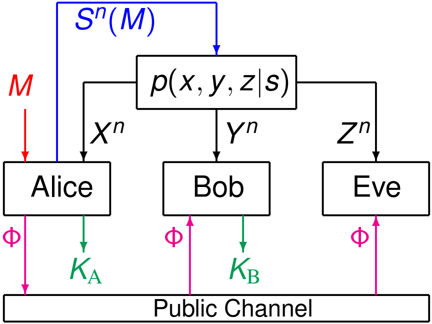

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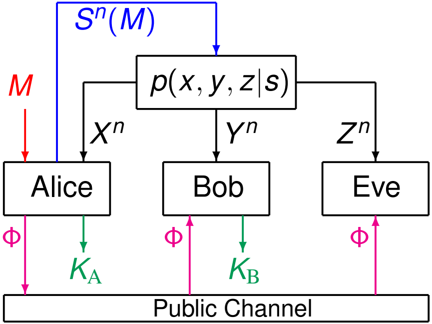

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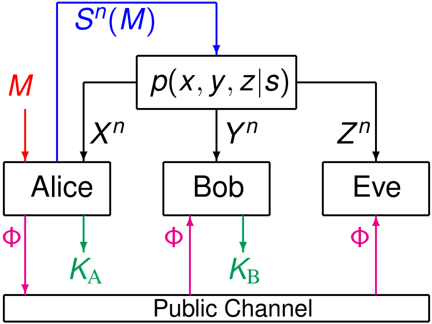

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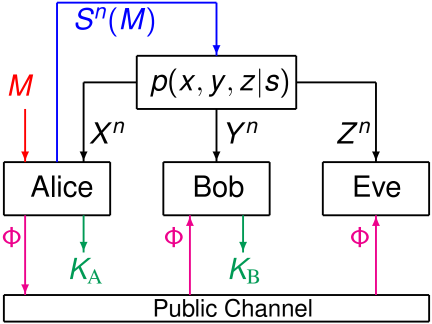

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- Key Generation: $k_{\mathrm{A}} = k_{\mathrm{A}}(m, x^n) \in \mathbb{N}$ and $k_{\mathrm{B}} = k_{\mathrm{B}}(\phi, y^n) \in \mathbb{N}$

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  - Key generation when encoder and decoder have state information [Khisti, Diggavi, Wornell 2011]

Weak Achievability

The rate R_{SK} is weakly-achievable if there exists a sequence of $(2^{nR_M}, 2^{nR_\Phi}, n, \Gamma)$ codes such that

$$\lim_{n\to\infty} \mathbb{P}(K_\mathrm{A} \neq K_\mathrm{B}) = 0$$

$$\lim_{n\to\infty} \frac{1}{n} I(K_\mathrm{A}; Z^n, \Phi) = 0$$

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But weak secrecy $\frac{1}{n} I(K_{\mathrm{A}}; Z^n, \Phi) \to 0$ is usually not good enough

[Maurer & Wolf 2000], [Watanabe et al. 2009], [Bloch & Barros 2011], [Bloch & Laneman 2011],

Strong Achievability

The rate-exponent triple (R_{SK}, E, F) is achievable if there exists a sequence of $(2^{nR_M}, 2^{nR_\Phi}, n, \Gamma)$ codes such that

$$\liminf_{n\to\infty} -\frac{1}{n}\log \mathbb{P}(K_{\mathrm{A}} \neq K_{\mathrm{B}}) \geq E, \quad \Leftrightarrow \quad \mathbb{P}(K_{\mathrm{A}} \neq K_{\mathrm{B}}) \stackrel{.}{\leq} 2^{-nE}$$

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Definition (Capacity-reliability-secrecy region)

$$\mathcal{R}^*(p(x,y,z|s)) := \overline{\left\{(R_{\mathrm{SK}}, E, F) \in \mathbb{R}_+^3 \;:\; (R_{\mathrm{SK}}, E, F) \text{ achievable}\right\}}$$

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Definition (Strong-achievability)

R_{SK} is strongly-achievable if (R_{SK}, E, F) is achievable for some $E, F > 0$

Capacity Result

Theorem (Secret Key Capacity for Sender-Excited Model)

The secret key capacity is

$$C_{\mathrm{SK}}(\Gamma) = \max \left\{ I(U, V; Y|W) - I(U, V; Z|W) \right\}$$

where the max is over all joints

$$p(w, u, v, s, x, y, z) = p(w, u)p(s|u)p(v|w, u, x)p(x, y, z|s)$$

such that $\mathbb{E}\Lambda(S) \leq \Gamma$.

Remarks on Capacity Result

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$$R_{\mathrm{ch}} = I(U;Y|W) - I(U;Z|W),$$
$$R_{\mathrm{src}} = I(V;Y|W,U) - I(V;Y|W,U)$$

- $R_{\mathrm{ch}} =$ Confidential message rate of wiretap channel $p(y,z|s)$
- $R_{\mathrm{src}} =$ Secret key rate of excited source $p(x,y,z|s)$ [Chou et al.]
    - Sounding signal s^n deterministic
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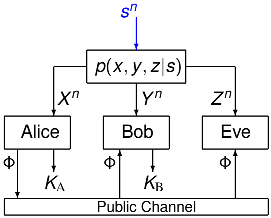
$$I(V;Y|W,S) - I(V;Z|W,S)$$



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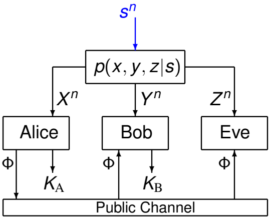

- Capacity: Find optimal sum rate $R_{\mathrm{ch}} + R_{\mathrm{src}}$

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Theorem (Secret Key Capacity for Degraded Sender-Excited Model)

If the DM-BC $p(x, y, z|s)$ is degraded the secret key capacity is

$$C_{\mathrm{SK}}(\Gamma) = C_{\mathrm{SK}}^{(\mathrm{Weak})}(\Gamma) = \max_{p(s):\mathbb{E}\Lambda(S)\le\Gamma} \{I(X, S; Y) - I(X, S; Z)\}$$

Also, $C_{\mathrm{SK}}^{(\mathrm{Weak})}(\Gamma) = C_{\mathrm{SK}}^{(\mathrm{Strong})}(\Gamma)$

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$$R_{\mathrm{ch}} = I(S; Y) - I(S; Z), \qquad R_{\mathrm{src}} = I(X; Y|S) - I(X; Z|S)$$

Binary Example

Consider the case where $S, X, Y, Z \in \mathbb{F}_2$:

$$X = (H \cdot S) \oplus N_1, \qquad Y = (H \cdot S) \oplus N_2, \qquad Z = (\tilde{H} \cdot H \cdot S) \oplus N_3$$

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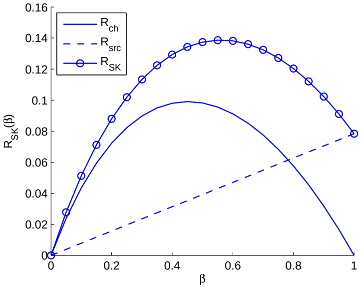

- Noises N_i indep
- H and $\tilde{H}$ indep
- $S \sim \mathrm{Bern}(\beta)$
- $R_{\mathrm{ch}} = I(S; Y) - I(S; Z)$
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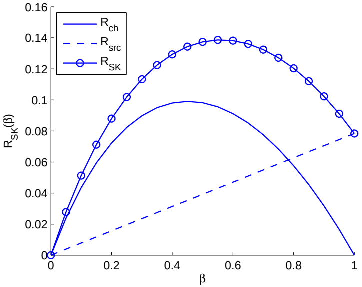

- Noises N_i indep
- H and $\tilde{H}$ indep
- $S \sim \mathrm{Bern}(\beta)$
- $R_{\mathrm{ch}} = I(S; Y) - I(S; Z)$
- $R_{\mathrm{src}} = I(X; Y|S) - I(X; Z|S)$

- Interplay between common randomness and wiretap rate

Error Exponents: Setup

- Recall that (R_{SK}, E, F) is achievable if

$$H(K_{\mathrm{A}}) \geq n(R_{\mathrm{SK}} - \epsilon), \quad \mathbb{P}(K_{\mathrm{A}} \neq K_{\mathrm{B}}) \stackrel{.}{\leq} 2^{-nE}, \quad I(K_{\mathrm{A}}; Z^n, \Phi) \stackrel{.}{\leq} 2^{-nF}$$

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Let $\mathcal{R}(p(s), R_\Phi, R_M) := \big\{(R_{\mathrm{SK}}, E, F) \in \mathbb{R}_+^3 :$

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- Reliability exponent E:
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- Secrecy exponent F:
  - Hayashi's wiretap channel exponents [2006, 2011]
  - Chou's key agreement model with external excitation [In Press]
- Strongly-achievable rates for degraded case [Preprint]

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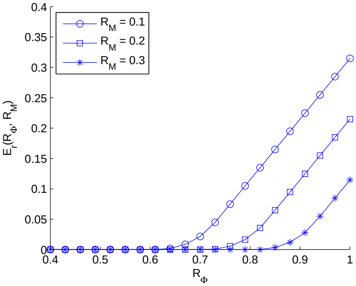

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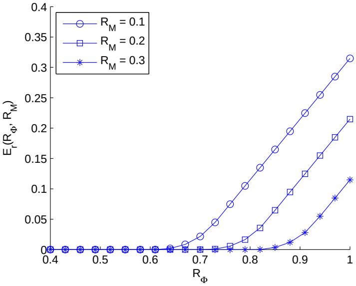

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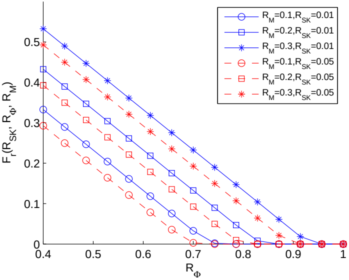

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