1.

- a) M -> P, N -> MQR, O -> S all violate BCNF because LHS are not superkeys
- b) BCNF decomposition is applied as the following:
 - a. Our first violating FD is M -> P, so we'll use it to split R into R1 = MP and R2 = LMNOQRS
 - b. FDs that still apply to each relation: R1 {M -> P}, R2 {O -> S, N -> MQR, L -> NO}
 - c. R1 is clearly in BCNF, and R2 clearly not
 - d. Decomposing R2 using O -> S, we get R2-1 = OS and R2-2 = LMNOQR
 - e. FDs that still apply to each relation: R2-1 {O -> S} R2-2 {N -> MQR, L -> NO}
 - f. R2-1 is clearly in BCNF, R2-2 still isn't because N is not a superkey
 - g. We decompose again, R2-2 into R2-2-1 with N->MQR and R2-2-2 with L->NO
 - h. Now both are in BCNF
 - i. Final Relations:
 - i. R1 = MP | FD: M -> P
 - ii. R2-1 = OS | FD: O -> S
 - iii. R2-2-1 = NMQR | FD: N -> MQR
 - iv. R2-2-2 = LNO | FD: L -> NO
- c) Yes, even though we know that BCNF does not guarantee FD preservation, in this specific case it does because none of the FDs have more than 1 item on the left side, meaning that we did not split the LHS of any FDs.
- d) Chase test

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	L	М	N	0	Р	Q	R	S
MP	1	М	2	3	Р	4	5	6
OS	7	8	9	0	10	11	12	S
MNQR	13	М	N	14	15	Q	R	16
LNO	L	17	N	0	18	19	20	21

Applying M -> P

	L	М	N	0	Р	Q	R	S
MP	1	М	2	3	Р	4	5	6
OS	7	8	9	0	10	11	12	S
MNQR	13	М	N	14	Р	Q	R	16
LNO	L	17	N	0	18	19	20	21

Applying O -> S

	L	М	N	0	Р	Q	R	S
MP	1	М	2	3	Р	4	5	6
OS	7	8	9	0	10	11	12	S
MNQR	13	М	N	14	Р	Q	R	16
LNO	L	17	N	0	18	19	20	S

Applying N -> MQR

	L	М	N	0	Р	Q	R	S
MP	1	М	2	3	Р	4	5	6
OS	7	8	9	0	10	11	12	S
MNQR	13	М	N	14	Р	Q	R	16
LNO	L	М	N	0	18	Q	R	S

Applying M -> P again

	L	М	N	0	P	Q	R	S
MP	1	М	2	3	Р	4	5	6
OS	7	8	9	0	10	11	12	S
MNQR	13	М	N	14	Р	Q	R	16
LNO	L	<mark>M</mark>	N	<mark>O</mark>	<mark>P</mark>	Q	R	S

We got {I, m, n, o, p, q, r, s}, thus it must be lossless

2. A)

a) Firstly, we note that B+ = BCDAEF and D+ = DABCEF, so B and D is powerful, and we can keep this in mind to quickly reduce LHS.

b) Splitting FDs:

ACD -> E

B -> C

B -> D

BE -> A

BE -> C

BE -> F

D -> A

D -/ A

D -> B

E -> A

E -> C

c) using what we learned in a), we apply the following:

ACD -> E because of D+

B -> C

B -> D

BE -> A because of B+

BE > C because of B+

BE -> F because of B+

D -> A

D -> B

E -> A

E -> C

All the other ones only have 1 element on the LHS, so they can't be reduced d) now we remove redundant FDs,

('it' refers to the current FD being examined and previously removed FDs)

D -> E cannot be removed because without it D+ does not include E

B -> C can be removed because without it B+ still includes C

B -> D cannot be removed because without it FDs B+ does not contain D

B -> A can be removed because without it B+ still includes A

B -> F cannot be removed because without it B+ does not include F

D -> A can be removed because without it D+ still includes A

D -> B cannot be removed because without it D+ does not include B

E -> A cannot be removed because without it E+ does not include A

E -> C cannot be removed because without it E+ does not include C

e) the following FDs remain:

$$B \rightarrow DF$$
, $D \rightarrow BE$, $E \rightarrow AC$

2 B)

Since G and H does not show up at all in the FDs, they have to be a part of any key.

From the FDs in the minimal basis, ACF is only on the right side, so they won't be in the key Nothing appears on the left side only.

BDE are the only attributes that appear on both sides, so we will need to compute closure with GH attached.

BGH+ = ABCDEFGH

DGH+ = ABCDEFGH

EGH+ = ACEGH

Thus EGH is not a key, BGH and DGH are the only possible keys.

2 C)

We first take the minimum basis FDs from 2A) and turn into relations.

We know that none of these are super keys through 2B), so we'll add a super key relation R4 = BGH

We'll add all applicable FDs to each relation

 $R1 = BDF \mid FD = \{B->DF, D->B\}$

 $R2 = BDE \mid FD = \{B->D, D->BE\}$

 $R3 = ACE \mid FD = \{E->AC\}$

 $R4 = BGH \mid No FDs.$

2 D) Yes it does allow redundancy, specifically since D and B determines each other, the fact that B determines D in both R1 and R2 (vice versa for D -> B) shows that something is redundant somewhere.