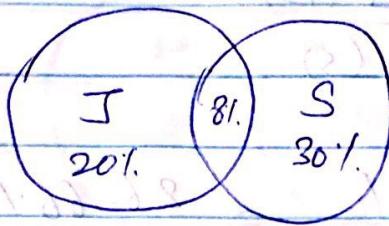


CS 513

VYOM A-SHAH
10446209

HW 01.

1.1



$$(S \cap J) = (S \cup J) - (S \cup J)^c$$

| | Susan at Bank (S) | Susan not in Bank (S') |
|------------------------|-------------------|------------------------|
| Jerry at Bank (J) | 8% | 12% |
| Jerry not at Bank (J') | 22% | 58% |

$$(S \cap J) = 8\%$$

$$P(J \cap S) = 8\%$$

$$P(J - S) = 20 - 8 = 12\%$$

$$P(S - J) = 30 - 8 = 22\%$$

$$P(J' \cup S') = 100\% - (J \cap S)$$

$$= 100\% - \{ P(S) + P(J) - P(J \cap S) \}$$

$$= 100\% - \{ 20 + 30 - 8 \}$$

$$= 100 - 42$$

$$= 58\%$$

$$\textcircled{a} \quad P(I|S) = \frac{P(I \cap S)}{P(S)}$$

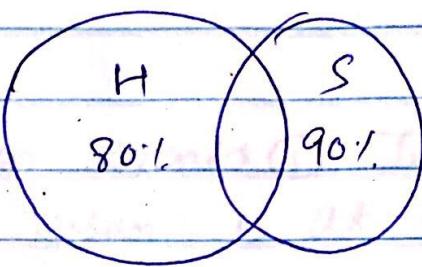
$$= \frac{8\%}{30\%} = \underline{26.66\%}$$

$$\textcircled{b} \quad P(I|S') = \frac{P(I \cap S')}{P(S')} = \frac{12\%}{70\%} = \underline{17.14\%}$$

$$\textcircled{c} \quad P(I \cap S | I \cup S) = \frac{P(I \cap S) \cap (I \cup S)}{P(I \cup S)}$$

$$= \frac{P(I \cap S)}{P(I \cup S)} = \frac{8\%}{42\%} = \underline{19.04\%}$$

1. 2



H - Harold
S - Sharon

Given - $P(H \cup S) = 91\%$

$$P(H \cup S) = P(H) + P(S) - P(H \cap S)$$

$$\begin{aligned}P(H \cap S) &= P(H) + P(S) - P(H \cup S) \\&= 80\% + 90\% - 91\% \\&= 79\%\end{aligned}$$

(a) $P(H) = P(H) - P(H \cap S)$
 $= 80\% - 79\% = 1\%$

(b) $P(S) = P(S) - P(H \cap S)$
 $= 90\% - 79\% = 11\%$

(c) $P(H' \cup S') = 100\% - P(H \cup S)$
 $= 100\% - 91\% = 9\%$

1. 3

The given events ① Jerry is at the bank and ② Susan is at the bank are not independent.

Because the probability of occurrence of the events is given 8%.

Also,

According to independent probability,

$$\begin{aligned} P(J \cap S) &= P(J) * P(S) \\ &= 20\% * 30\%, \\ &= \underline{\underline{6\%}}. \end{aligned}$$

The above equation is not equal to the given 8%. Hence, the events are not independent.

1.4

@ $P(\text{Sum is } 6) = \frac{5}{36}$

$P(\text{Second die is } 5) = \frac{1}{6}$

\leftarrow ~~out of 36 outcomes~~ $\frac{1}{6}$ outcome

Joint prob states that -

$P(\text{Sum is } 6 \cap \text{Second die shows } 5) = \frac{1}{36}$

$(2, 2) + (3, 1) + (4, 0) + (5, 1) + (6, 0)$

According to

Independent Probability \rightarrow

$P(\text{Sum is } 6 \cap \text{Second die shows } 5)$

$= P(\text{Sum is } 6) * P(\text{Second die shows } 5)$

$= \frac{1}{6} * \frac{1}{6}$

The above equations are not equal, therefore, these two events are not independent.

$P(A \cap B) = 1/36$

So, $P(A \cap B) = P(A) * P(B)$

\Rightarrow Independent

$$\textcircled{b} \quad P(\text{Sum is } 7) = \frac{6}{36} = \frac{1}{6} \quad [A.1]$$

$$P(\text{Sum is } 7) = \frac{6}{36} = \frac{1}{6}$$

$$P(\text{first die shows } 5) = \frac{6}{36} = \frac{1}{6}$$

$$P(\text{Sum is } 7 \cap \text{first die shows } 5) = ?$$

According to joint probability \Rightarrow

$$P(\text{Sum is } 7 \cap \text{first die shows } 5) = \frac{1}{36}$$

- both events are independent

Acc. to Independent Probability \Rightarrow

$$P(\text{Sum is } 7 \cap \text{first die shows } 5) = P(\text{Sum is } 7) + P(\text{first die shows } 5)$$

$$= \frac{1}{6} * \frac{1}{6}$$

- independent events are multiplied at probability

$$P(\text{Sum is } 7) = \frac{1}{6} \quad P(\text{first die shows } 5) = \frac{1}{6}$$

$$(\frac{1}{6}) * (\frac{1}{6}) = \frac{1}{36}$$

$$= \frac{1}{36}$$

The equations are equal, implies,

these events are independent.

- independent more no. of events exist

L.5

(making jointed set)

| Joint road | TX | AK | NJ | |
|------------|-----|-----|-----|------|
| Oil | 18% | 6% | 1% | 25% |
| No oil | 42% | 24% | 9% | 75% |
| | 60% | 30% | 10% | 100% |

The probability of finding oil in TX,

$$P(\text{Oil}/\text{TX}) = 30\% \quad \textcircled{2}$$

The probability of choosing TX,

$$P(\text{TX}) = 60\%$$

$$(100\%) \cdot 9 = (10\%) \cdot 9 \quad \textcircled{3}$$

$$P(\text{Oil}/\text{TX}) = \frac{P(\text{Oil} \cap \text{TX})}{P(\text{TX})}$$

$$\begin{aligned} P(\text{Oil} \cap \text{TX}) &= P(\text{Oil}/\text{TX}) * P(\text{TX}) \\ &= 30\% * 60\% \\ &= 18\% \end{aligned}$$

For finding $P(\text{Oil}/\text{NJ})$ we need,

$$P(\text{Oil}/\text{NJ}) = 10\%$$

$$P(\text{NJ}) = 10\%$$

$$\begin{aligned} \text{So, } P(\text{Oil}/\text{NJ}) &= P(\text{Oil}/\text{NJ}) * P(\text{NJ}) \\ &= 10\% * 10\% \\ &= 1\% \end{aligned}$$

Ques

for finding $P(\text{oil} \cap \text{AC})$,

$$P(\text{oil} \cap \text{AC}) = 100\% - (60\% + 10\%)$$

$$= 100\% - 70\% = 30\%$$

$$P(\text{oil} \cap \text{AC}) = 30\% \times 20\%$$

$$= 6\%$$

(a) $P(\text{oil}) = 100\% - 30\% = 70\%$

$P(\text{AC}) = 25\% \text{ and } P(\text{oil} \cap \text{AC}) = 6\%$

(b) $P(\text{TX} | \text{oil}) = P(\text{TX} \cap \text{oil})$

$$\frac{P(\text{oil} \cap \text{TX})}{P(\text{oil})} = 70\%$$

$$70\% = \frac{10\%}{P(\text{oil})} = 70\%$$

$$70\% \times 100\% = 70\%$$

$$100\% =$$

$P(\text{TX} | \text{AC}) = 25\% \text{ and } P(\text{AC}) = 6\%$

$$25\% = P(\text{TX} \cap \text{AC})$$

$$25\% = P(\text{TX})$$

$$P(\text{TX}) \times P(\text{TX} \cap \text{AC}) = P(\text{TX} \cap \text{AC}) = 0.02$$

$$0.02 \times 0.06 =$$

$$0.012 =$$

11.6

1. $P(\text{Passenger did not survive}) =$

No. of not survived

Number of not survived = $(x) 9 + (x) 9 = (\text{Total}) 18$

$$= \frac{1490}{2201}$$

$$= 0.676$$

2. $P(\text{Passenger staying in first class}) = \frac{325}{2201}$

$$= \frac{1058}{2201}$$

$$= 0.477$$

$$= \frac{147}{2201}$$

3. A: Passenger Survived

B: Passenger (was) staying in first class.

$$P(B|A) = P(B \cap A)$$

$$P(A) = \frac{1058}{2201} \approx 0.477$$

$$= \frac{203}{2201} \approx 0.285$$

$$= \frac{711}{2201} \approx 0.329$$

$$(0.477)(0.285) = 0.136$$

$$= 0.136$$

$$= \frac{1058}{2201}$$

$$= 0.481$$

4.

Let $X = \text{Survival}$

$\therefore Y = \text{Staying in First Class}$

If $P(X \cap Y) = P(X) \cdot P(Y)$ then X & Y are independent

$$P(X \cap Y) = \frac{203}{2201} = 0.092$$

$$P(X) = \frac{711}{2201} = 0.323 \quad P(Y) = \frac{325}{2201} = 0.147$$

$$P(X) \cdot P(Y) = 0.323 \times 0.147 \\ = 0.047$$

Since $P(X \cap Y) \neq P(X) \cdot P(Y)$
 $\Rightarrow X$ and Y are not independent

5. Let, $A = \text{Staying in first class}$

$B = \text{Passenger was a child}$

$C = \text{Passenger Survived.}$

$$P(A \cap B | C) = \frac{P(A \cap B \cap C)}{P(C)} \\ = \frac{6}{2201} = 0.008$$

- 6 Let $X = \text{Passenger Survived}$ (a) 9, (b) 9
 $Y = \text{Passenger was an adult}$

$$P(Y|X) = \frac{P(Y \cap X)}{P(X)} = \frac{654/2201}{71/2201} = 0.919$$

(a) 9, (b) 9 + (c) 9 = ?

7. Let $A = \text{Passenger Survived}$, $B = \text{Adult}$
 $C = \text{Child}$

$D = \text{First class}$

$$P(B \cap D) = \frac{197}{711} = 0.277 \text{ and } 0.277$$

(Survived and First class)

$$P(C \cap D) = \frac{6}{711} = 0.009$$

$$P(C) = \frac{57}{711} = 0.680$$

$$P(D) = \frac{203}{711} = 0.285$$

$$P(B) = \frac{654}{711} = 0.919$$

$$P(C) \cdot P(D) = \frac{57 \times 203}{711 \times 711} = 0.022$$

$$P(B) \cdot P(D) = \frac{654 \times 203}{711 \times 711} = 0.262$$

Since $P(B \cap D) \neq P(B) \cdot P(D)$

Adult and first class are not independent
(Given that passenger survived)

Since $P(C \cap D) \neq P(C) \cdot P(D)$

Child and first class are not independent
(Given that passenger survived)