Q1. Is the following function a proper distance function? Why? Explain your answer.

$$d(\mathbf{x},\mathbf{y}) = (\sum_i | x_i - y_i |)^3$$

Hint: Measure the distance between (0,0), (0,1) and (1,1)

Answer-

Let uss assume that X(0,0), Y(0,1) & Z(1,1)

For any distance function to work, the following conditions must be satisfied-

1. d(x,y)>=0 Non-negativity

2. d(x,y)=0⇔ x=y identity of indiscernibles

3. d(x,y)=d(y,x) symmetry

4. d(x,z)<=d(x,y)+d(y,z) triangle inequality

The distance x(0,0) and y(0,1) =>d(x,y)

D=$((0-0)+(0-1))^3$ = 1

Which is equal to d(y,x)

The distance y(0,1) and z(1,1) =>d(y,z)

D=$((0-1)+(1-1))^3$ = 1

Which is equal to d(z,y)

The distance z(1,1) and x(0,0) =>d(z,x)

D=$((1-0)+(1-0))^3$ = 8

Which is equal to d(x,z)

We will check the validity of distance function-

1.) d(x,y)>=0, d(y,x)>=0, d(y,z)>=0,

d(z,y)>=0, d(z,x)>=0, d(x,z)>=0

Clearly d(x,y)>=0 and d(x,y)=0 ⇔x=y are satisfied

2.)

$d(x,y)=d(y,x)$

$d(y,z)=d(z,y)$

$d(z,x)=d(x,z)$

clearly $d(x,y)=d(y,x)$ is satisfied

3.)

$d(x,z)=8$, $d(x,y)=1$, $d(y,z)=1$

$d(x,z)<=d(x,y)+d(y,z)$

$8<=1+1$

$8<=2$ **Which is False**

Condition 4 fails

$d(z,x)<=d(z,y)+d(y,x)$

$8<=1+1$

$8<=2$ **Which is false**

Condition 4 fails here as well.

Hence, the given function is not a proper distance function.