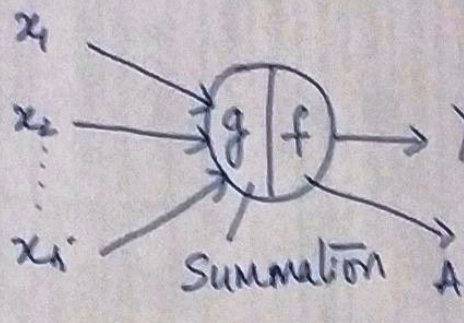
McCulloch Pitts Neuron / Thresholding Logic

* Muculloch & pitts proposed a highly simplified Computational Model of neuron (1943).



g aggregates the input.

$$g(x) \Rightarrow \sum_{x=i}^{n} x_i \quad \text{—— (1)}$$

$$\Rightarrow x_1 + x_2 + \cdots - x_n \quad \text{— (2)}$$

function f takes decision based on aggregation (g).

function f is threshold function in this case.

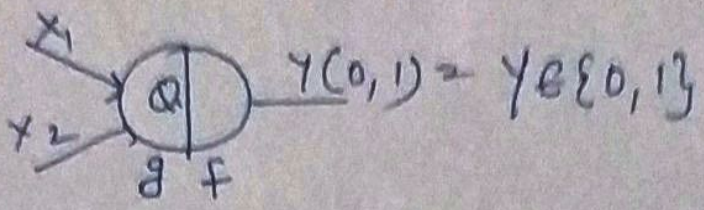
$$y = 1 \quad \text{if } g(x) \geq \theta \quad \text{— (3)}$$

$$y = 0 \quad \text{if } g(x) < \theta \quad \text{— (4)}$$

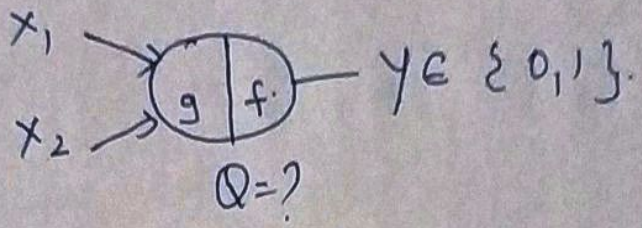
θ is called the thresholding parameter.

(1) (1)

Let us complement some Boolean function Using this MuCulloch Pitts (MP) neuron



Implement AND Gate



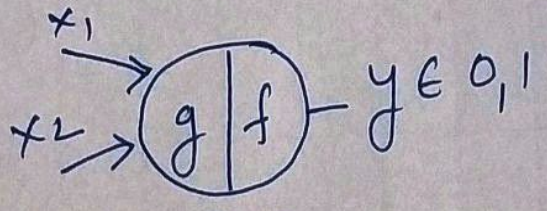
Truth Table for AND.

x_1	x_2	Y	$g = x_1 + x_2$
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	2

Data set

Target O/p.

When both input are 1 we get o/p = 1 so the Q = 2.

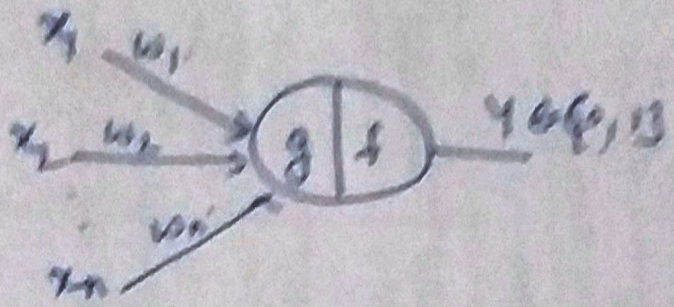


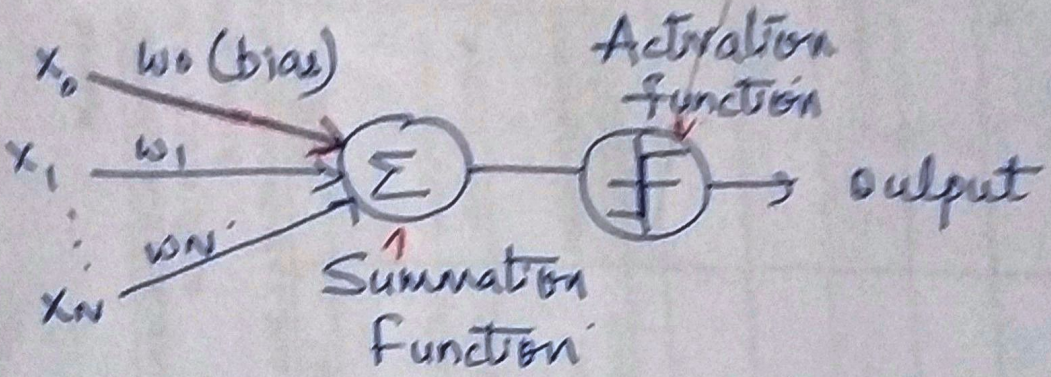
Q=2

Single MuCulloch neuron can be used to represent boolean function which are linearly separable. (2)

Perceptron

Frank Rosenblatt, an American psychologist proposed the classical perceptron model (1958).





Perceptron has 4 important Components

1. Inputs - - - - -> * Numerical data Points, called input are fed into the neuron.
2. Weight & Bias
3. Summation Function
4. Activation Function

* The weight of each input data indicates how important that input is predicting the outcomes.

⇒ The bias parameter allow you to adjust the activation function curve in such a way that a precise output is achieved

⇒ Summation function ⇒ The product of the respective input & weight is taken Adding all these products gives us the weighted sum.

$$\text{Weighted sum} = \sum_{i=1}^{n} x_i w_i + \text{bias} \quad \text{— (1)}$$

(expand) — eq (1)

$$= x_1 w_1 + x_2 w_2 + \cdots x_n w_n + \text{bias}$$

where,

⇒ Activation function :→ To map the weight sum to the output.

for ex

$$\sum_{i=1}^{n} x_i w_1 + \text{bias} \geq 0 \quad y = 1$$

$$\sum_{i=1}^{n} x_i w_1 + \text{bias} < 0 \quad y = 0$$

P-2

Example

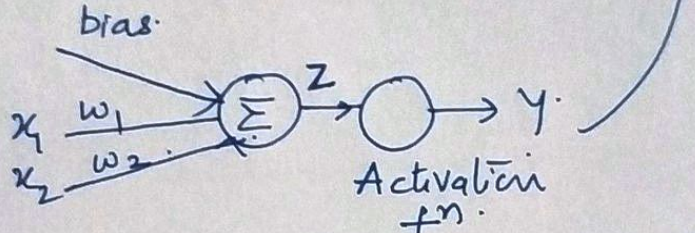
* Let us see how does a perceptron model reconstruct AND operation

x_1	x_2	O/P Label
0	0	0
0	1	0
1	0	0
1	1	1

Activation f^n.

$$f(z) = \begin{cases} 1 & z \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

We Want.



To

$z = w_1x_1 + w_2x_2 + b = \text{Label}$ (we want) — (1)

$w_1 \cdot 0 + w_2 \cdot 0 + b = 0$ — (2)

$w_1 \cdot 0 + w_2 \cdot 1 + b = b$ — (3)

$w_1 \cdot 1 + w_2 \cdot 0 + b = 0$ — (4)

$w_1 \cdot 1 + w_2 \cdot 1 + b = 1$ — (5)

Can we suitably adjust weight & bias parameters so that all eq^n (2 to 5) are satisfied.

Yes choose $w_1 = 0.3$

$w_2 = 0.3$ & $b = 0$.

(P-3)