Convolutional Neural Networks

Bùi Tiến Lên

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Contents

Notation

symbol	meaning
$a, b, c, N \ldots$	scalar number
$\boldsymbol{w}, \boldsymbol{v}, \boldsymbol{x}, \boldsymbol{y} \ldots$	column vector
$\boldsymbol{X}, \boldsymbol{Y} \ldots$	matrix
$\mathbb{R}$	set of real numbers
$\mathbb{Z}$	set of integer numbers
$\mathbb{N}$	set of natural numbers
$\mathbb{R}^D$	set of vectors
$\mathcal{X}, \mathcal{Y}, \ldots$	set
$\mathcal{A}$	algorithm

operator	meaning
$\boldsymbol{w}^\mathsf{T}$	transpose
$\boldsymbol{X}\boldsymbol{Y}$	matrix multiplication
$\boldsymbol{X}^{-1}$	inverse
$\boldsymbol{X} \odot \boldsymbol{Y}$	an element-wise matrix-vector multiplication

Architecture

Convolutional neural network

Convolutional neural networks (CNNs) use three basic ideas:

- *local receptive fields*
- *shared weights*
- *pooling*

What are learnt in CNNs?

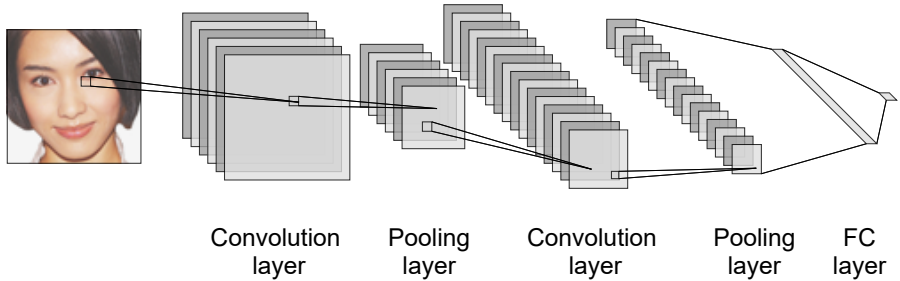
CNNs has two interesting properties

- The patterns they learn are translation invariant
- They can learn spatial hierarchies of patterns

A basic architecture of CNN

- A common pattern

$$\text{Convolution} + \text{ReLU} \rightarrow \text{Pool} \rightarrow \text{Fully connected} \rightarrow \text{Softmax loss}$$



Convolutional layer

- Single Input, Single Output
- Multiple Input, Multiple Output
- Tensor Operation

Convolutional layer

The fundamental difference between a densely connected layer and a convolution layer is this:

- Dense layers learn **global patterns** in their input feature space
- Convolutional layers learn **local patterns** in their input feature space

Convolutional types

There are three common kinds of convolution

- 1D convolution
- **2D convolution**
- 3D convolution

Image processing

In image processing, an input image is convolved with a kernel to create an output image

- **Input image** (*input feature map*) *In*
- **Output image** (*output feature map*) *Out*
- **Filter** (*kernel*) requires three parameters
  - The *spatial extent* (kernel size) F
  - The *stride* S (distance between two consecutive positions of the kernel)
  - The amount of *zero padding* P (number of zeros concatenated at the beginning and at the end of an axis)

Convolution vs. Cross-correlation

- Convolution operation (used in digital processing)

$$Out(x,y) = (K * In)(x,y) = \sum_m \sum_n In(x-m, y-n)K(m,n) \quad (1)$$

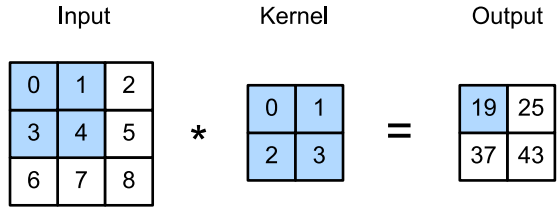
- Cross-correlation operation (used in CNN)

$$Out(x,y) = (K * In)(x,y) = \sum_m \sum_n In(x+m, y+n)K(m,n) \quad (2)$$

Convolution vs. Cross-correlation (cont.)

The following figure illustrates 2D cross-correlation operation.

- Input size is 3×3
- Output size is 2×2
- Kernel has $F = 2, S = 1, P = 0$

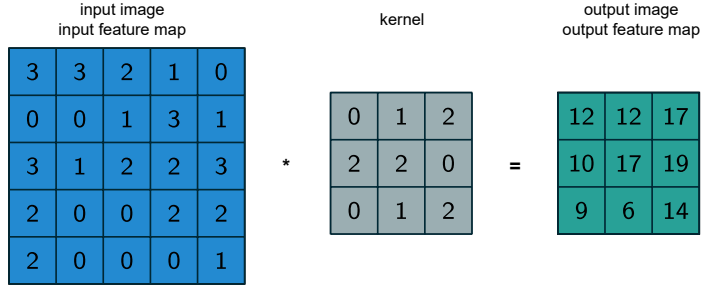


- The shaded portions are the first output element as well as the input and kernel tensor elements used for the output computation

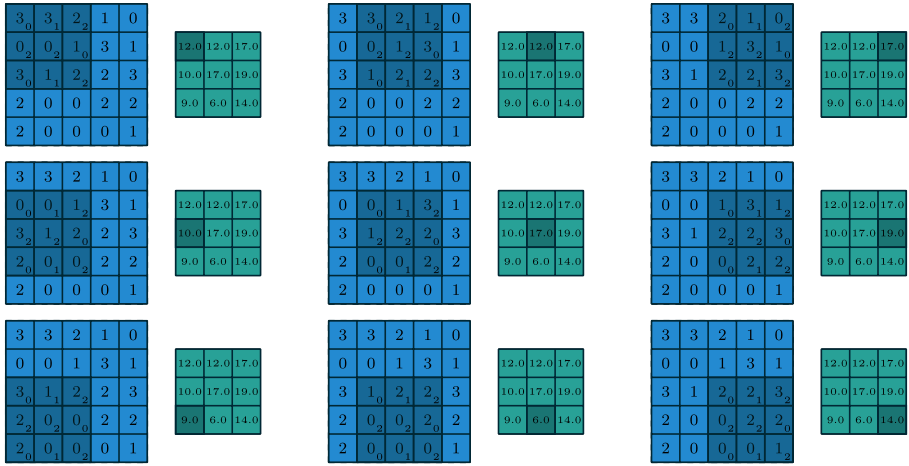
$$0 \times 0 + 1 \times 1 + 3 \times 2 + 4 \times 3 = 19$$

Example 1

- Input image size is 5×5
- Output image size is 3×3
- Kernel has $F = 3, S = 1, P = 0$

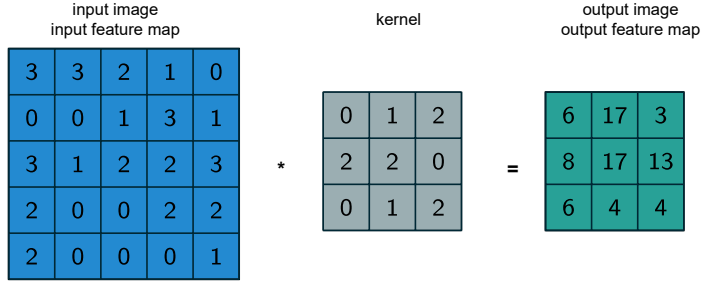


Example 1 (cont.)

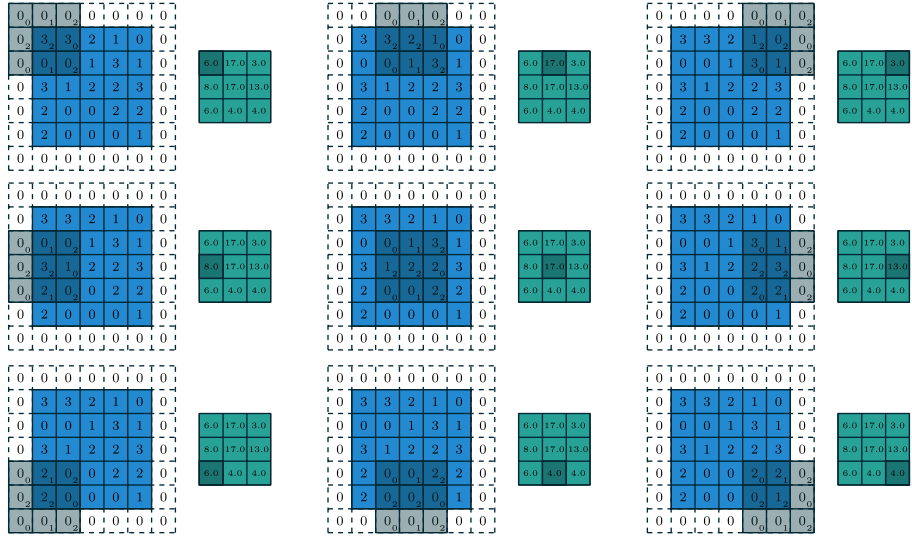


Example 2

- Input image size is 5×5
- Output image size is 3×3
- Kernel has $F = 3, S = 2, P = 1$

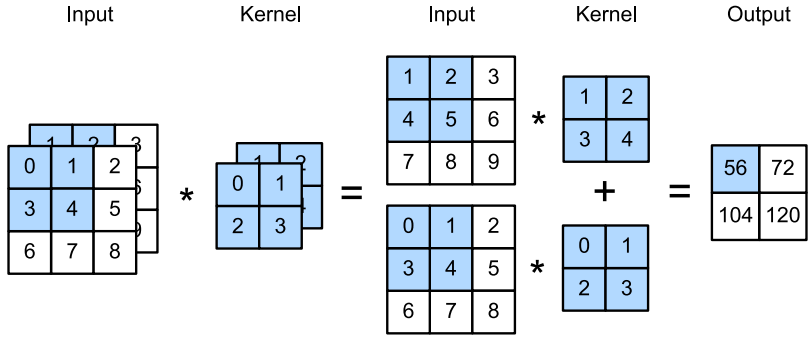


Example 2 (cont.)



Multiple Input Channels

- Convolution computation with 2 input channels.

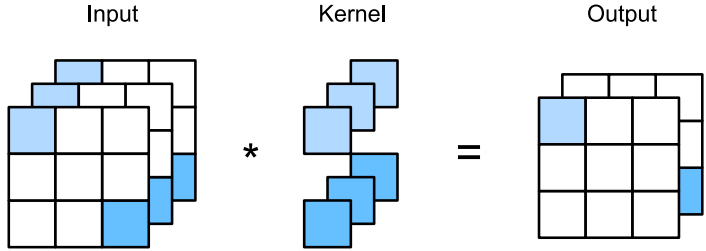


- The shaded portions are the first output element as well as the input and kernel tensor elements used for the output computation

$$(1 \times 1 + 2 \times 2 + 4 \times 3 + 5 \times 4) + (0 \times 0 + 1 \times 1 + 3 \times 2 + 4 \times 3) = 56$$

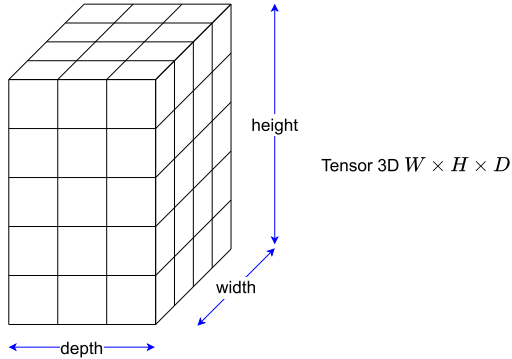
Multiple Input, Output Channels

- Convolution computation for 3 input channels and 2 output channels.



3D tensor

- Convolutions operate over 3D tensors, called *feature maps*, with two spatial axes (*width* W and *height* H) as well as a *depth* axis D (also called the *channels* axis)



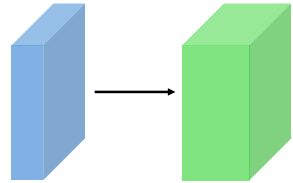
Convolution operator

- Accepts a volume of size $W_{in} \times H_{in} \times D_{in}$ and a group of filters with four hyperparameters:
  - Number of filters K
  - Their spatial extent F
  - The stride S
  - The amount of zero padding P

Convolution operator (cont.)

- Produces a volume of size $W_{out} \times H_{out} \times D_{out}$ where:

$$
\begin{aligned}
W_{out} &= \frac{W_{in} - F + 2P}{S} + 1 \\
H_{out} &= \frac{H_{in} - F + 2P}{S} + 1 \\
D_{out} &= K
\end{aligned}
\tag{3}
$$



Convolution operator (cont.)

In summary,

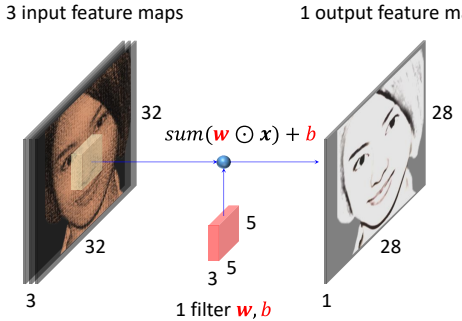
- With parameter sharing, it introduces $F \times F \times D_{in}$ weights per filter, for a total of $F \times F \times D_{in} \times K$ **weights** and K **biases**.
- The filters depth is always equal to the input volume depth.
- The output volume depth is equal to the number of filters
- In the output volume, the d-th depth slice (of size $W_{out} \times H_{out}$ is the result of performing a valid convolution of the d-th filter over the input volume with a stride of S, and then offset by d-th bias.
- Normally, in the same conv layer, all filters have the same dimensions so that special optimized routines can be invoked.

Explanation

- We can imagine a filter as a neuron.
- In a convolution layer we have filters.
  - Each filter is a weight matrix which is convoluted over the input volume.
  - Each time the filter is applied it outputs a single value.
  - The result of convolving the filter over all the input volume (*input feature maps*) is another volume called **activation map** (*output feature map*).
  - A filter's depth is always equal to the input volume depth.
- Instead of having only one filter, we have a stack of filter which produce a stack of activation maps (one for each filter)

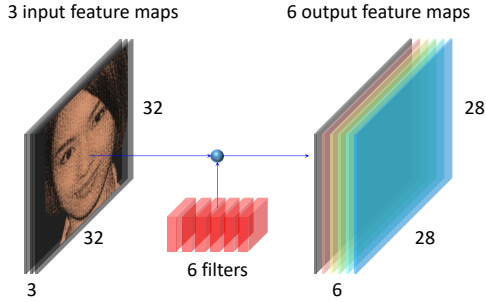
Explanation (cont.)

- The input volume $32 \times 32 \times 3$ is convolved with the filter $5 \times 5 \times 3$ ($S = 1$ and $P = 0$) and the result is the activation map $28 \times 28 \times 1$



Explanation (cont.)

- The input volume $32 \times 32 \times 3$ is convolved with the stack of 6 filter $5 \times 5 \times 3$ and the result is the stack of 6 activation map $28 \times 28 \times 1$ ($28 \times 28 \times 6$)



Pooling layer

Pooling layer

- The reason to use pooling layer (downsampling) is to reduce the number of feature-map coefficients to process
- Pooling layers also provide invariance to small translations of the input
- The most common kind of pooling is *max pooling*
- Another kind of pooling is average pooling; however, it's more informative to look at the *maximal presence* of different features than at their *average presence*

Pooling operator

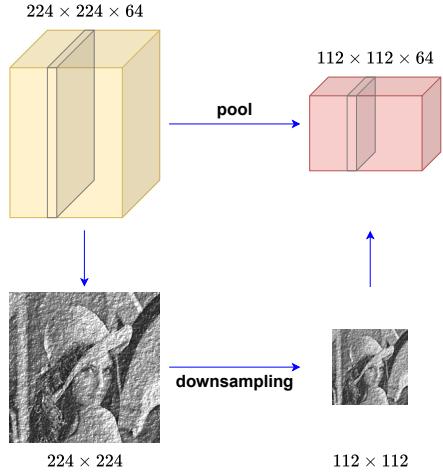
- Accepts a volume of size $W_{in} \times H_{in} \times D_{in}$ and one filter with two hyperparameters:
  - Their spatial extent F
  - The stride S
- Produces a volume of size $W_{out} \times H_{out} \times D_{out}$ where:

$$\begin{aligned} W_{out} &= \frac{W_{in} - F}{S} + 1 \\ H_{out} &= \frac{H_{in} - F}{S} + 1 \\ D_{out} &= D_{in} \end{aligned} \tag{4}$$

- The most common form is the filter of size 2×2 applied with a stride of 2.

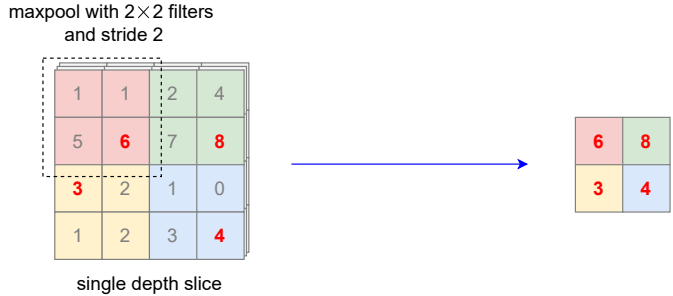
Example

- The input volume of size $[224 \times 224 \times 64]$ is pooled with filter size 2, stride 2 into output volume of size $[112 \times 112 \times 64]$



Example (cont.)

- Max pooling with filter size 2, stride 2



Fully connected layer

Fully connected layer

- Fully connected layers (FC layers), also called *affine layers*, produce the *high-level reasoning* in the DNN.
- Neurons in a fully connected layer have full connections to all activations in the previous layer, as in regular Neural Networks.
- Their activations can hence be computed with a matrix multiplication followed by a bias offset.

VGGNet

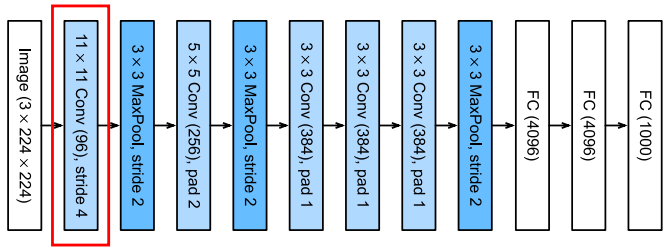
Layer name	Variables		Parameters	
	Layout	Size	Weights	Biases
INPUT	[224x224x3]	224*224*3=150K		
CONV3-64	[224x224x64]	224*224*64=3.2M	(3*3*3)*64=1,728	64
CONV3-64	[224x224x64]	224*224*64=3.2M	(3*3*64)*64=36,864	64
POOL2	[112x112x64]	112*112*64=800K		
CONV3-128	[112x112x128]	112*112*128=1.6M	(3*3*64)*128=73,728	128
CONV3-128	[112x112x128]	112*112*128=1.6M	(3*3*128)*128=147,456	128
POOL2	[56x56x128]	56*56*128=400K		
CONV3-256	[56x56x256]	56*56*256=800K	(3*3*128)*256=294,912	256
CONV3-256	[56x56x256]	56*56*256=800K	(3*3*256)*256=589,824	256
CONV3-256	[56x56x256]	56*56*256=800K	(3*3*256)*256=589,824	256
POOL2	[28x28x256]	28*28*256=200K		
CONV3-512	[28x28x512]	28*28*512=400K	(3*3*256)*512=1,179,648	512
CONV3-512	[28x28x512]	28*28*512=400K	(3*3*512)*512=2,359,296	512
CONV3-512	[28x28x512]	28*28*512=400K	(3*3*512)*512=2,359,296	512
POOL2	[14x14x512]	14*14*512=100K		
CONV3-512	[14x14x512]	14*14*512=100K	(3*3*512)*512=2,359,296	512
CONV3-512	[14x14x512]	14*14*512=100K	(3*3*512)*512=2,359,296	512
CONV3-512	[14x14x512]	14*14*512=100K	(3*3*512)*512=2,359,296	512
POOL2	[7x7x512]	7*7*512=25K		
FC	[1x1x4096]	4096	7*7*512*4096=102,760,448	4096
FC	[1x1x4096]	4096	4096*4096=16,777,216	4096
FC	[1x1x1000]	1000	4096*1000=4,096,000	1000

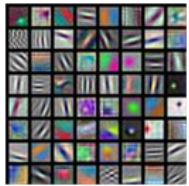
- Total variables (memory): 24M $\times$ 4 bytes $\simeq$ 93MB/image (only forward! $\sim \times 2$ for backward)
- Total parameters: 138M parameters
- The most of the network parameters are in the FC layer and most of the variables (memory) required by the network is used in the first 2 Conv Layers.

Visualizing and Understanding

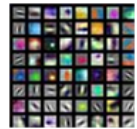
First Layer

- Visualize first-layer filters/kernels (raw weights) directly

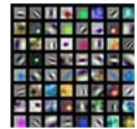




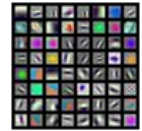
AlexNet:
64 x 3 x 11 x 11



ResNet-18:
64 x 3 x 7 x 7



ResNet-101:
64 x 3 x 7 x 7



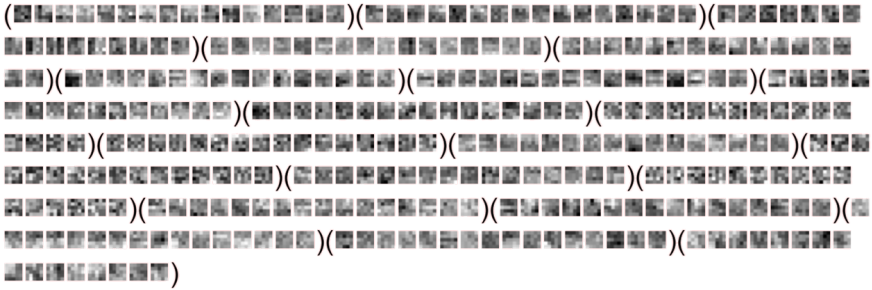
DenseNet-121:
64 x 3 x 7 x 7

First Layer (cont.)

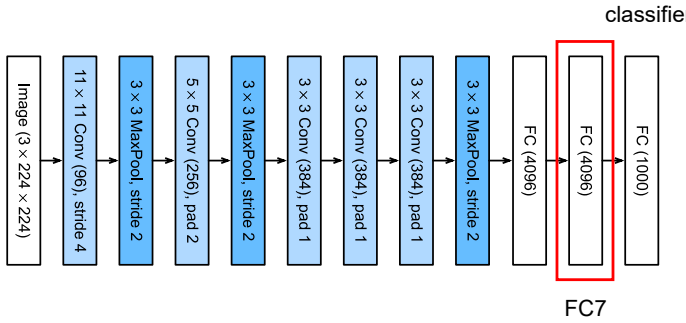
- We can visualize filters at higher layers, but not that interesting (these pictures are taken from ConvNetJS CIFAR-10 demo)

Layer 1 filters

Layer 2 filters

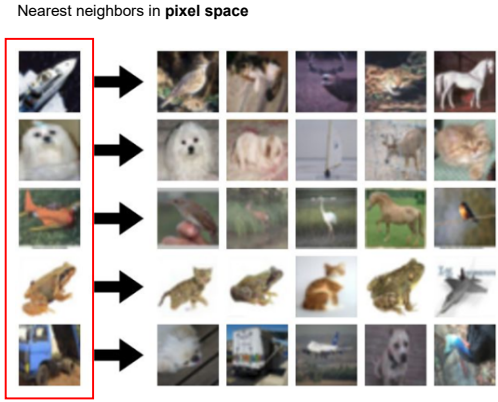


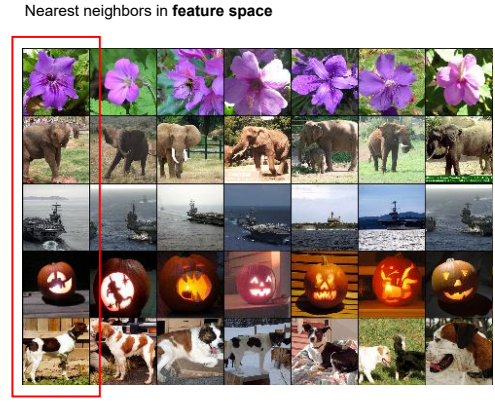
Last Layer



- FC7: 4096-dimensional feature vector for an image (layer immediately before the classifier)
- Run the network on many images, collect the feature vectors

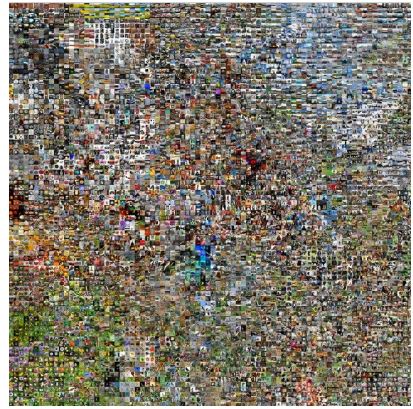
Last Layer: Nearest Neighbors





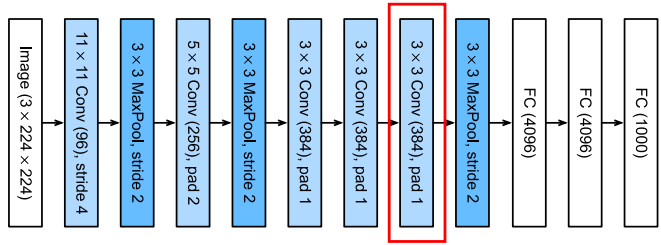
Last Layer: Dimensionality Reduction

- Visualize the "space" of FC7 feature vectors by reducing dimensionality of vectors from 4096 to 2 dimensions
  - Simple algorithm: Principal Component Analysis (PCA)
  - More complex: t-SNE



Visualizing Activations

- A view of the 13×13 activation of the 151^{st} channel on the conv5 layer



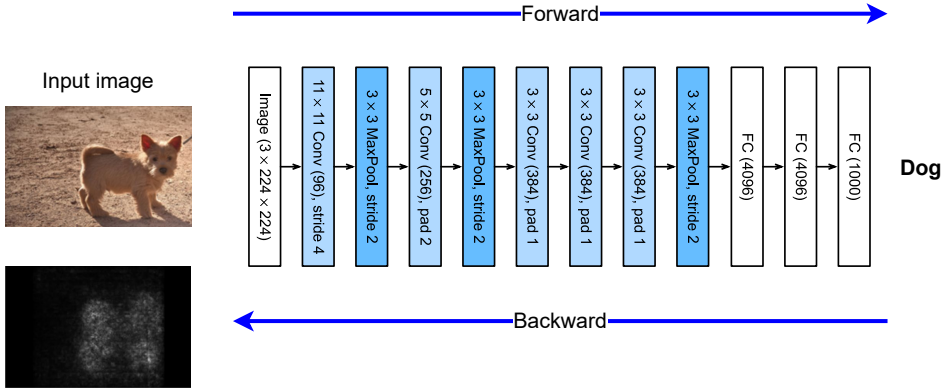
Mapping activations back to pixels

(Guided) backpropagation method

- Forward an image through the network
- Choose a feature map and an activation
- Zero out all values except for the one of interest
- Propagate that value back to the image

Which pixels matter: Saliency

- Compute gradient of (unnormalized) class score with respect to image pixels,
- Take absolute value and max over RGB channels



Which pixels matter: Saliency (cont.)

- Superimposing the class activation heatmap on the original picture

Generate Images: Gradient Ascent

(Guided) backpropagation
Find the part of an image that a neuron responds to

Gradient ascent
Generate a synthetic image that maximally activates a neuron

$$I_{syn} = \arg\max_I S_c(I) - \lambda \|I\|_2^2 \tag{5}$$

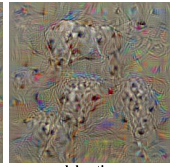
where S_c score for class c (before Softmax)

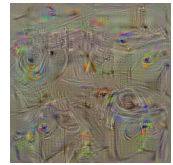
1. Initialize image I to zeros
2. Repeat:
   - 2.1 Forward image to compute current scores
   - 2.2 Backprop to get gradient of neuron value with respect to image pixels
   - 2.3 Make a small update to the image

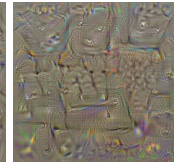
Generate Images: Gradient Ascent

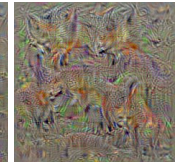

dumbbell


cup

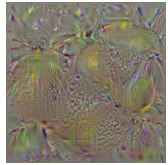

dalmatian

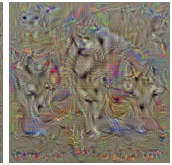

washing machine


computer keyboard


kit fox

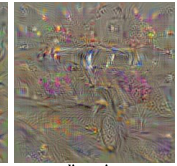

bell pepper


lemon


husky


goose


ostrich


limousine

Network Design

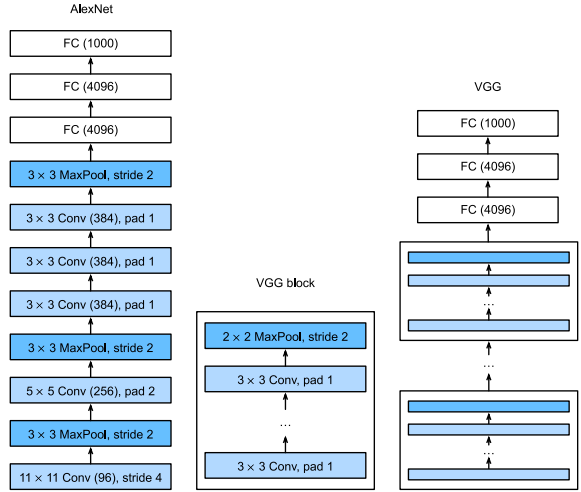
- Networks Using Blocks
- Networks with Parallel Concatenations
- Residual Networks

Blocks

- The design of neural network architectures had grown progressively more abstract, moving from thinking in terms of individual neurons to whole layers, and now to blocks, repeating patterns of layers.
- The basic building block of classic CNNs is a sequence of the following:
  - a convolutional layer with padding to maintain the resolution
  - a nonlinearity such as a ReLU
  - a pooling layer such as a max pooling layer

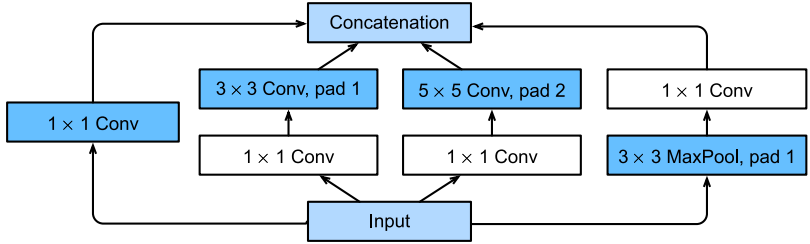
VGG Network

- From AlexNet to VGG that is designed from building blocks.



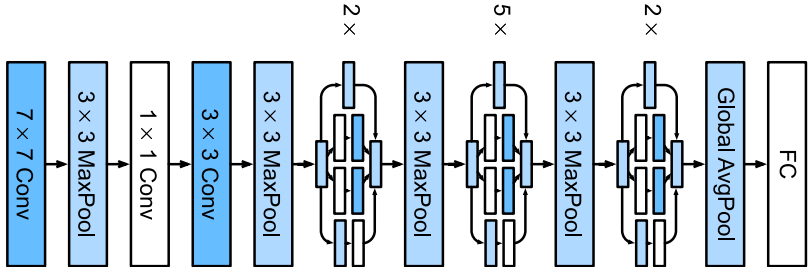
Inception Blocks

- Inception block employs a combination of variously-sized kernels



GoogleNet

- GoogLeNet uses a stack of a total of 9 inception blocks and global average pooling to generate its estimates.



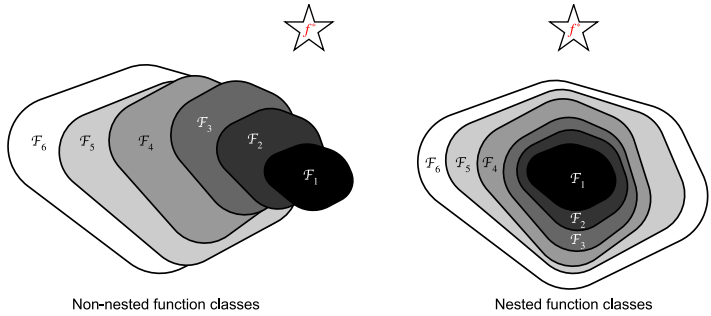
Learning Model

- Consider $\mathcal{F}$, **the class of functions** (hypothesis set) that a specific network architecture can reach.
- Given a data set $\mathcal{D}$, algorithm A finds the best estimated $\hat{f}$ of the "truth" function f^*

$$\hat{f} = \arg\min_f L(f \mid \mathcal{D}) \text{ subject to } f \in \mathcal{F}. \tag{6}$$

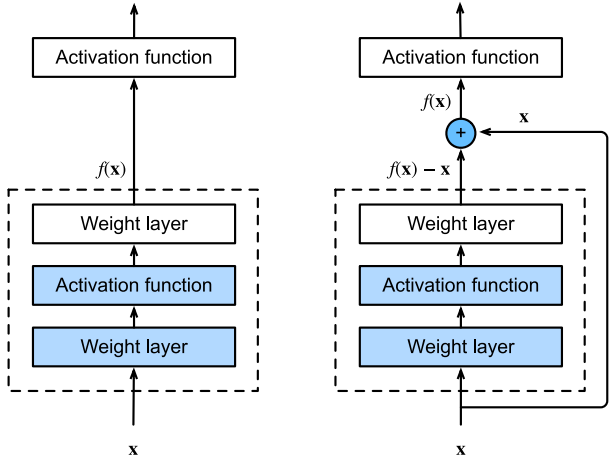
Learning Model (cont.)

- For non-nested function classes, a larger (indicated by area) function class does not guarantee to get closer to the "truth" function f^*. This does not happen in nested function classes.



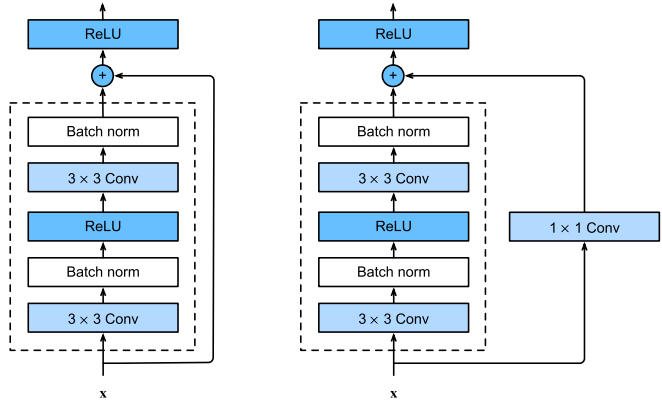
Residual Blocks

- A regular block (left) and a residual block (right).



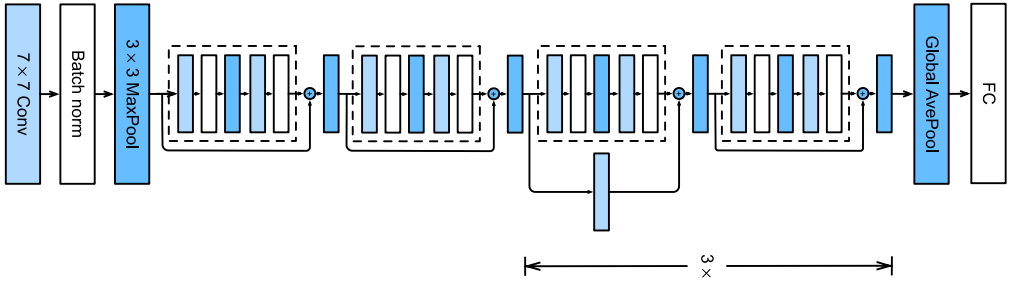
Residual Blocks (cont.)

- ResNet block with and without 1×1 convolution.



ResNet Model

- The ResNet-18 architecture



Miscellaneous convolutions

- Transposed convolution
- Dilated convolutions

Transposed convolution

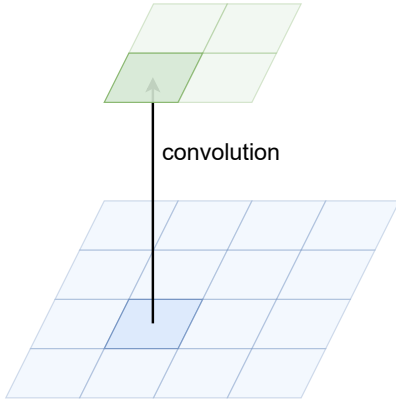
- Convolution can be interpreted as a matrix multiplication

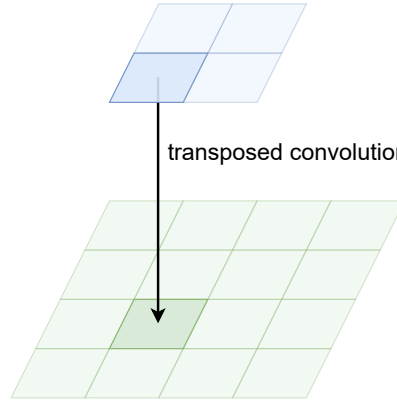
$$\text{kernel } w \rightarrow \text{matrix } C$$

- The need for **transposed convolutions** (often called **deconvolution**) generally arises from the desire to use a transformation going in the opposite direction of a normal convolution

$$\text{convolution } C \longleftrightarrow \text{transposed convolution } C^{\mathsf{T}}$$

Transposed convolution (cont.)

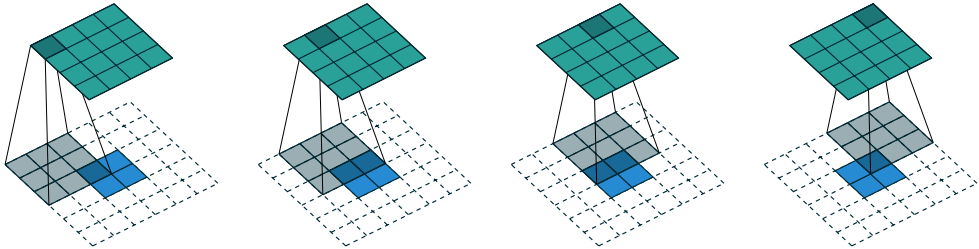




Transposed Convolution (cont.)

- No zero padding, unit strides, transposed

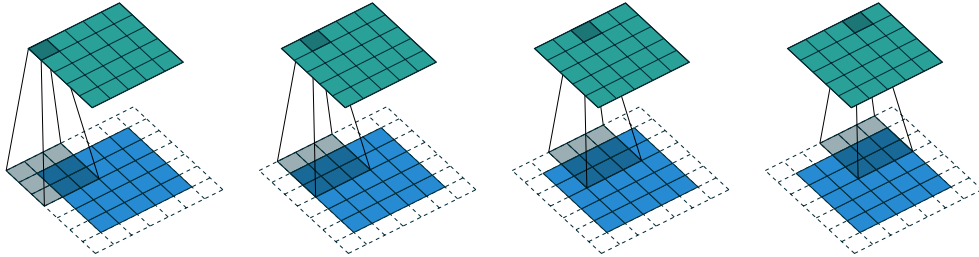
	convolution	transposed convolution
kernel size	k	$k' = k$
stride	$s = 1$	$s' = 1$
padding	$p = 0$	$p' = k - 1$
output size		$o' = i' + (k - 1)$



Transposed Convolution (cont.)

- Zero padding, unit strides, transposed

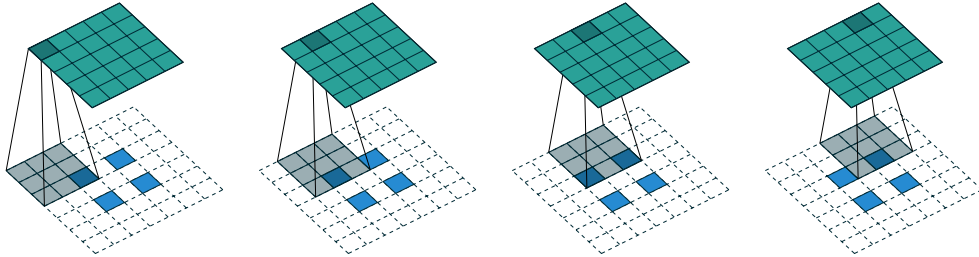
	convolution	transposed convolution
kernel size	k	$k' = k$
stride	$s = 1$	$s' = 1$
padding	p	$p' = k - p - 1$
output size		$o' = i' + (k - 1) - 2p$



Transposed Convolution (cont.)

- No zero padding, non-unit strides, transposed:
  - adding $s - 1$ zeros between each input unit

	convolution	transposed convolution
kernel size	k	$k' = k$
stride	s	$s' = 1$
padding	$p = 0$	$p' = k - 1$
output size		$o' = s(i' - 1) + k$



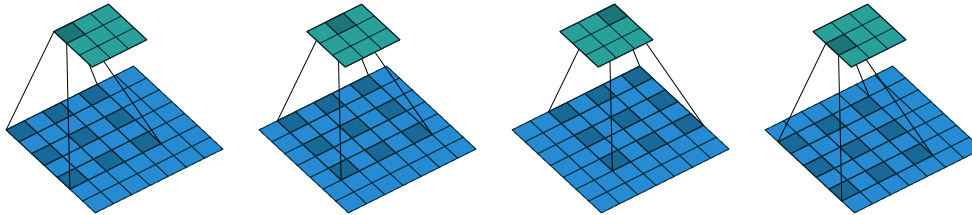
Dilated convolutions

Concept 1

Dilated convolutions "inflate" the kernel by inserting spaces between the kernel elements. The dilation "rate" is controlled by an additional hyperparameter d.

- For any i, k, s and p, and for a dilation rate d,

$$o = \left\lfloor \frac{i + 2p - k - (k-1)(d-1)}{s} \right\rfloor + 1. \tag{7}$$

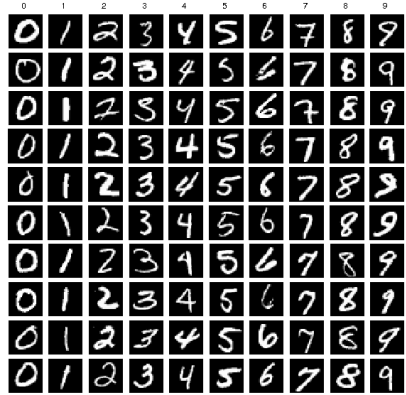


Applications

- Datasets
- Applications

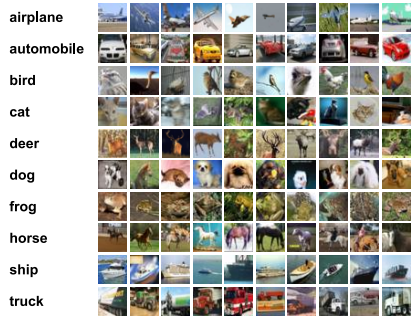
MNIST

- The MNIST database is a large database of handwritten digits that is commonly used for training various image processing systems
- The MNIST database contains 60,000 training images and 10,000 testing images



CIFAR

- The CIFAR-10 and CIFAR-100 are labeled subsets of the 80 million tiny images dataset. They were collected by Alex Krizhevsky, Vinod Nair, and Geoffrey Hinton.
- The CIFAR-10 dataset consists of 60000 32x32 colour images in 10 classes, with 6000 images per class. There are 50000 training images and 10000 test images.



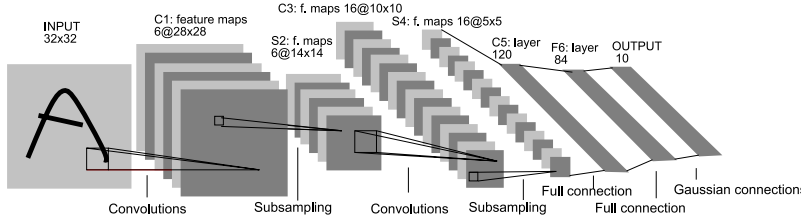
ImageNet

- The ImageNet project is a large visual database designed for use in visual object recognition software research

Image classifcation

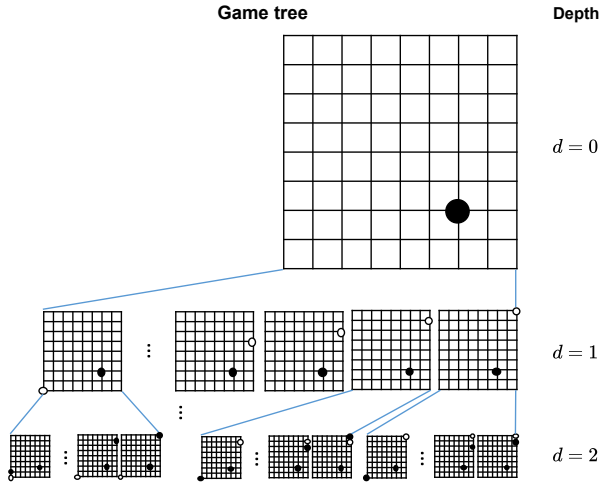
- Architecture of LeNet-5 a Convolutional Neural Network, here for digit recognition, but is extended to character recognition.

:

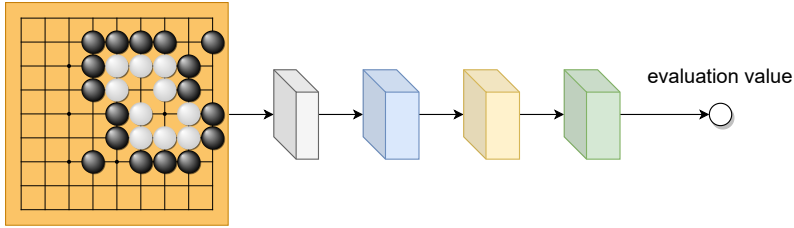


Games

- Go game



Games (cont.)



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