Tutorial Fiskom 1

Diferensial Numerik

Soal

1. Tentukan turunan dari

$$f(x) = \frac{x^2 - 4x \, sin(x^2)}{8xe^x - 5ln(x)}$$

   Di x = 1 dan h = 0.01

2. Buat program 5-titik simetris sebagai tambahan dari program 3-titik simetris!

3. Hitung turunan dari

$$y = sin(x)$$

   Di x = 1 dengan kedua metode (5 titik dan 3 titik simetris) diatas! Bandingkan!

4. Buat program untuk menghitung turunan ke 2 dengan metode di atas!

Penyelesaian

1. Metode 2 titik

   - maju

   Persamaan umum

$$f'(x) = \frac{f(x_{i+1}) - f(x_i)}{h}$$

   Dengan $x_{i+1} = x_i + h$

$$f'(1) = \frac{f(1 + 0.01) - f(1)}{0.01}$$

$$= \frac{f(1.01) - f(1)}{0.01}$$

   - $$f(1.01) = \frac{1.01^2 - 4(1.01) \, sin(1.01^2)}{8(1.01)e^{1.01} - 5ln(1.01)}$$

$$= -0.10898843606322459$$

   - $$f(1) = \frac{1^2 - 4(1) \, sin(1^2)}{8(1)e^1 - 5ln(1)}$$

$$= -0.10879500768012582$$

$$f'(1) = \frac{-0.10898843606322459 + 0.10879500768012582}{0.01}$$

$$= -0.019342838309877042$$

- Mundur

  Persamaan umum

  $$f'(x) = \frac{f(x_i) - f(x_{i-1})}{h}$$

  Dengan $x_{i-1} = x_i - h$

  $$f'(1) = \frac{f(1) - f(1 - 0.01)}{0.01}$$

  $$= \frac{f(1) - f(0.99)}{0.01}$$

  - $$f(0.99) = \frac{0.99^2 - 4(0.99)\,sin(0.99^2)}{8(0.99)e^{0.99} - 5ln(0.99)}$$

    $$= -0.10852842230384027$$

  - $$f(1) = \frac{1^2 - 4(1)\,sin(1^2)}{8(1)e^1 - 5ln(1)}$$

    $$= -0.10879500768012582$$

  $$f'(1) = \frac{-0.10898843606322459 + 0.10852842230384027}{0.01}$$

  $$= -0.026658537628554413$$

Metode 3 titik

- Tengah

  Persamaan umum

  $$f'(x) = \frac{f(x_{i+1}) - f(x_{i-1})}{2h}$$

  Dengan $x_{i+1} = x_i + h$ dan $x_{i-1} = x_i - h$

  $$f'(1) = \frac{f(1 + 0.01) - f(1 - 0.01)}{2(0.01)}$$

$$= \frac{f(1.01) - f(0.99)}{2(0.01)}$$

- $$f(1.01) = \frac{1.01^2 - 4(1.01)\,sin(1.01^2)}{8(1.01)e^{1.01} - 5ln(1.01)}$$

$$= -0.10898843606322459$$

- $$f(0.99) = \frac{0.99^2 - 4(0.99)\,sin(0.99^2)}{8(0.99)e^{0.99} - 5ln(0.99)}$$

$$= -0.10852842230384027$$

$$f'(1) = \frac{-0.10898843606322459 \; + \; 0.10852842230384027}{2(0.01)}$$

$$= -0.023000687969215727$$

2. Metode 5 titik simetris

$$f'(x) \; = \; \frac{1}{12h}(f(x_{i-2}) - 8f(x_{i-1}) + 8f(x_{i+1}) - f(x_{i+2}))$$

"the program is in another application"

3. Penyelesaian secara analitik

$$y' \; = \; cos(1)$$
$$= 0.5403023058681398$$

Penyelesaian secara numerik

- 3 titik simetris

$$y'(1) \; = \; \frac{y(1 + 0.01) - y(1 - 0.01)}{2(0.01)}$$

$$= \frac{y(1.01) - y(0.99)}{2(0.01)}$$

- $$y(1.01) = \; sin(1.01)$$

$$= 0.8468318446180152$$

- $y(0.99) = \sin(0.99)$

$$= 0.8360259786005205$$

$$y'(1) = \frac{0.8468318446180152 - 0.10852842230384027}{2(0.01)}$$

$$= 0.5402933008747335$$

- 5 titik simetis

$$y'(1) = \frac{1}{12(0.01)}(y(1 - 2(0.01)) - 8y(1 - 0.01) + 8y(1 + 0.01) - y(1 + 2(0.01)))$$

$$= \frac{1}{12(0.01)}(y(0.98) - 8y(0.99) + 8y(1.01) - y(1.02))$$

- $y(0.98) = \sin(0.98)$

$$= 0.8304973704919705$$

- $y(0.99) = \sin(0.99)$

$$= 0.8360259786005205$$

- $y(1.01) = \sin(1.01)$

$$= 0.8468318446180152$$

- $y(1.02) = \sin(1.02)$

$$= 0.852108021949363$$

$$y'(1) = \frac{1}{12(0.01)}(0.8304973704919705 - 8(0.8360259786005205) + 8(0.8468318446180152) - 0.852108021949363)$$

$$y'(1) = \frac{1}{0.12}(0.8304973704919705 - 6.688207828804164 + 6.774654756944122 - 0.852108021949363)$$

$$y'(1) = 0.5403023056880424$$

Perbandingan dengan error relatif

$$error = \left| \frac{nilai\ eksak - nilai\ pendekatan}{nilai\ eksak} \right| \times 100\%$$

Perbandingan untuk metode 3 titik

$$error = \left| \frac{0.5403023058681398 - 0.5402933008747335}{0.5403023058681398} \right| \times 100\%$$

$$= 0.0016666583333957608\ \%$$

Perbandingan untuk metode 5 titik

$$error = \left| \frac{0.5403023058681398 - 0.5403023056880424}{0.5403023058681398} \right| \times 100\%$$

$$= 3.3332706430551304 \times 10^{-08}\ \%$$

4. Turunan kedua

$$y''(x) = \frac{y(x_{i-1}) - 2y(x_i) + y(x_{i-1})}{h^2}$$

"the program is in another application"

Penyelesain secara analitik

$$y''(1) = \frac{y(1 + 0.01) - 2y(1) + y(1 - 0.01)}{h^2}$$

$$= \frac{y(1.01) - 2y(1) + y(0.99)}{h^2}$$

- $y(1.01) = 0.8468318446180152$

- $y(1) = 0.8414709848078965$

- $y(0.99) = 0.8360259786005205$

$y''(1)$

$$= \frac{0.8468318446180152 - 2(0.8414709848078965) + 0.8360259786005205}{0.01^2}$$

$$y''(1) = -0.8414639725728978$$