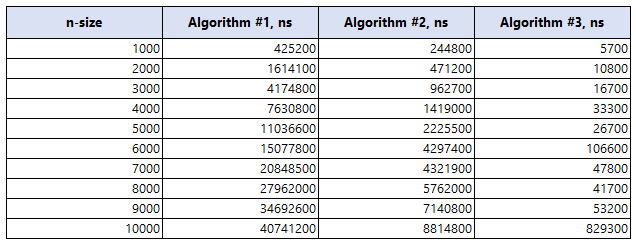
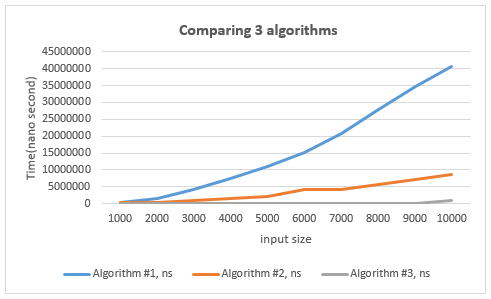
Q1

1. **The result of comparing 3 algorithms**



1. **Display result by graph**

****

**Conclusion**

The graphs indicate that as n increases, the runtime of Algorithm #1 grows linearly with O(n), while Algorithm #2 and Algorithm #3 exhibit quadratic O(n^2) and cubic O(n^3) growth rates, respectively.

Q2

**Step 1.** Prove the base cases n = 5, n = 6

F(5) > (4/3)^5, LHS = 5, RHS = ~ 4.21, It is true

F(6) > (4/3)^6, LHS = 8, RHS = ~ 5.62, It is true

**Step 2.**

Assume it is true for n = k and n = k - 1, when k > 4, then it is also true for n = k + 1

**Step 3.**

F(k + 1) > (4/3)^(k+1)

Using F(k+1) = F(k) + F(k-1)

F(k) = (4/3)^k

F(k-1) = (4/3)^(k-1)

F(k + 1) > (4/3)^k + (4/3)^(k-1)

= (4/3)^k \* 1 + (4/3)^k \* (4/3)^-1

= (4/3)^k \* (1 + (4/3)^-1))

= (4/3)^k \* (1 + ¾)

= (4/3)^k \* 7/4

= (4/3)^k \* 1.75

Since (4/3)^(k+1) = (4/3)^k \* 4/3 ~ (4/3)^k \* 1.33, so 1.75 > 1.33, so:

F(k + 1) > (4/3)^k \* 1.75 > (4/3)^(k+1)

F(k) > (4/3)^k is true for all k > 4