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Fast Acquisition of GPS Signal using FFT Decomposition

Shaik Fayaz Ahamed^{a*}, G Sasibhushana Rao^b, L Ganesh^c

^{a*}*Department of Electronics and Communications Engineering, V R Siddhartha Engineering College, Vijayawada, India.*

^b*Department of Electronics and Communications Engineering, Andhra University, Visakhapatnam, India.*

^c*Department of Electronics and Communications Engineering ANITS, Visakhapatnam, India.*

Abstract

Acquisition in Global Positioning System (GPS) is primary and important step to measure the code phase of Pseudo Random Noise (PRN) code and Doppler shift in carrier frequency of received GPS signal. The performance of a GPS receiver system depends on the fast and accurate measure of the code phase and Doppler shift in the received signal. A new GPS signal acquisition method based on decomposition of FFT is proposed to improve the acquisition performance. Conventional GPS receivers use Fast Fourier Transform (FFT) to carry out correlation in the acquisition process. However, this paper proposes a new method that reduces the complexity of FFT computation. The proposed algorithm is implemented, validated and compared the performance with conventional serial search and radix2 FFT search algorithms using Intermediate Frequency (IF) GPS signal. Acquisition algorithms are implemented on 1.25MHz signal data of one millisecond period, sampled at 5MHz. For the sampled data, the proposed algorithm uses 5000 samples instead of number of samples raised to nearest power of 2 i.e. 8192 samples as in the case of radix2 FFT search method to compute FFT. Performance of the algorithms are compared based on computational time and correlation peak magnitude.

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1. Introduction

Global Positioning System (GPS) is a satellite based navigational system¹. In GPS, the transmitter uses the Direct Sequence Code Division Multiple Access (DS-CDMA) technique. It utilizes a Pseudorandom Noise (PRN) sequence of 1.023MHz to expand 50Hz navigational data signal over a wider spectrum. This signal is broadcasted

* Corresponding author. Tel.: +919866462333;
E-mail address: skfayaz01@gmail.com

over a carrier frequency of 1575.42MHz using Binary Phase Shift Keying (BPSK) modulation. The Coarse/Acquisition (C/A) code (PRN) is 1023 chips with 1ms period and used by different satellites are quasi-orthogonal².

A GPS receiver receives RF signal through antenna and the signal is pre processed in the RF frontend block to proper amplitude and given to Down-converter. The carrier frequency of the GPS signal is converted to a desired output intermediate Frequency (IF) using a Local Oscillator (LO). An Analog to Digital Converter (ADC) is used to digitize the IF signal. After the signal is digitized, software is used to process. Acquisition is the first signal processing operation performed on IF GPS signal. Acquisition process identifies the satellites that are visible to the receiver and provides the measurement of Doppler shift in carrier frequency and delay in the C/A code of the incoming GPS signal. Doppler shift in carrier frequency of GPS signal is due to the relative velocity of satellite with receiver and C/A code delay is due to the transit time of satellite signal from GPS satellite to the receiver. These parameters are very important to synchronise the locally generated signal with received signal and extract the navigational data. After identifying the available satellites and acquiring the parameters, parallel channels are used to track the each satellite. In each channel, tracking loops are used and extract the navigational data. In tracking loops C/A code and carrier are removed by refining the code phase and Doppler frequency. So receiver performance depends on the accuracy of acquisition process.

Acquisition is carried by synchronizing the locally generated C/A code and carrier with the received signals³. The acquisition is usually performed on a block of data received from the satellite signal. In general the block size of data is period of C/A code i.e. one millisecond. The size of the data block used for acquisition depends on the Carrier to Noise Ratio (CNR) of the received signal. If the CNR is low, then the receiver must process the signal for more than a single C/A code period.

There are three basic signal acquisition techniques in GPS systems. They are the serial search method, Circular correlation and Delay-Multiply technique. In serial search method, searches sequentially at all possible combinations of code delay and Doppler shift one by one⁴. For this, it generates a Doppler compensated replica code shifted by a possible code delay, multiplies it with the IF signal and the result is integrated. This process continues until the result exceeds the threshold correlation peak magnitude. However, this sequential searching method is a time consuming process. In Circular correlation method, FFT is used to perform integration for all code delays at each Doppler shift in a single step⁴. Thus it requires less number of computations compared to conventional method. In delay-multiply technique the acquisition processes is carried by eliminating the carrier signal information from the input received signal by multiplying the complex received signal with the complex conjugate of a delayed version of itself⁴. It uses the locally generated C/A code to find the code delay in the carrier removed signal. And then FFT is used to find the Doppler shift in the received signals frequency.

This paper presents the serial search method and the circular correlation method for the GPS signal acquisition process. In order to compute the FFT operation in Circular correlation method this paper proposes a new method based decomposition of FFT. Acquisition is also performed using radix2 FFT to compute the FFT in circular correlation. Performance of the proposed method is compared with the radix2 FFT and serial search methods.

2. Signal acquisition

The received GPS signal after digitization of IF signal can be represented as

$$r[n] = \sqrt{A} d[n] c[n - \tau] \cos[2\pi(f_{IF} + f_d)n T_s - \phi] + N \quad (1)$$

Where, A is the carrier power; d[n] is the navigational data, c[n] is the C/A code, f_{IF} , f_d , denote the Intermediate Frequency (IF) and Doppler shift (Hz) respectively, $T_s = 1/F_s$ stands for the sampling period (seconds), F_s is the sampling frequency (Hz), ϕ is the initial carrier phase, τ is the initial code delay (samples), and N is the Additive White Gaussian Noise.

As a first step, acquisition process removes the carrier from the signal by mixing it with carrier replica. The nominal value of the signal carrier frequency f_{IF} is the IF frequency of the GPS signal. The Doppler shift f_d , is defined by satellite movement. Then the Doppler shift is in the range ± 6 KHz for a static or low dynamics user and in the range ± 10 KHz for a high dynamic user⁵. This limits the search range of the Doppler frequency of the IF signal. After the carrier frequency is removed, the signal is correlated with the replica of satellite C/A code. The initial delay τ in C/A code is unknown. So the correlation process is carried with all possible shifts in replica C/A

code. Therefore, the acquisition process is conducted in two-dimensional space: code delay and Doppler shift to the IF frequency and it is as shown in Fig. 1

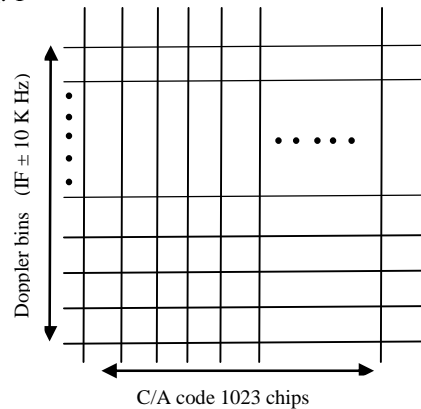


Fig. 1 Signal acquisition search process

3. Serial Search method

Serial search method is a conventional algorithm for signal acquisition in CDMA receiver system⁶. Fig.2 depicts the operation of Serial search method. It involves multiplication of IF signal with the locally generated golden code sequence and carrier signal. Initially, the IF GPS signal is multiplied with C/A code. As second step, the output of first step is multiplied with locally generated carrier signal which results two components. First component i.e. in-phase signal (I) is generated by multiplying signal with the carrier, and the second component i.e. quadrature signal (Q) is generated by multiplying the signal with a 90° phase-shifted version of carrier, which gives the navigational data from GPS signal. The 'I' and 'Q' branch signals are integrated over 1millisecond, squared and summed⁷. An accumulated output is compared with the threshold, if it is exceeded, then it is considered as the acquisition process is completed. Otherwise, the C/A code replica is shifted by one chip and repeat the process until the whole code domain search is finished. If the acquisition is not completed then the same process is repeated with change in the frequency in steps of 500Hz.

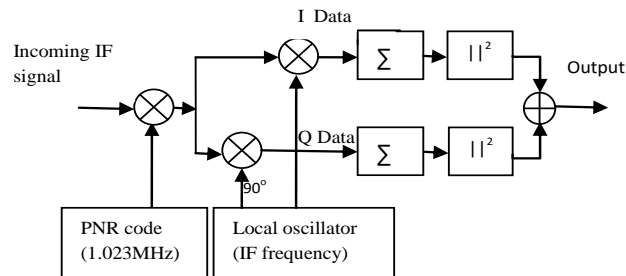


Fig. 2 Block diagram of Serial search acquisition

4. FFT search algorithm

Acquisition is a process of searching sequentially through two dimensions combining frequency and code phase. It is an exhaustive process to search the correct Doppler and code shifts. If one of the parameter could be eliminated from the search procedure, it will speed up the process significantly⁸. Circular correlation search scheme based on FFT and Inverted FFT (IFFT) are being used traditionally for acquisition. FFT search algorithm converts the GPS signal from time domain into frequency domain and thus it eliminates one parameter. Fig. 3 illustrates the block diagram of FFT search algorithm.

As shown in Fig.2, the input GPS IF signal is expressed by Eq. 1 and the locally generated carrier signal expressed by Eq. 2 are used for acquisition process using FFT.

$$y_c[n] = c[n - \tau] \exp[j2\pi(f_{IF} + f_d)nT_s] \quad (2)$$

Where, $y_c[n]$ is the locally generated carrier signal at an instant ' n '; $c[n]$ represents the C/A code of one GPS satellite, τ is the delay. f_d is the Doppler shift in carrier frequency, f_{IF} is the incoming digitized GPS signal IF frequency; $T_s = 1/F_s$ stands for the sampling period (seconds), F_s is the sampling frequency (Hz).

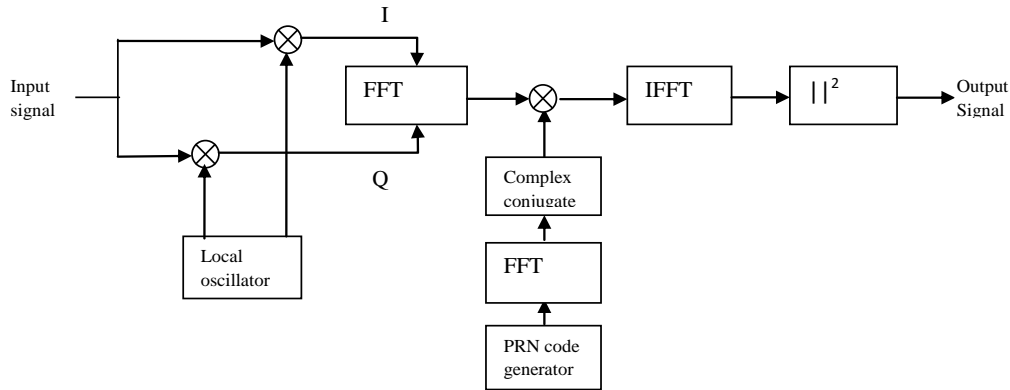


Fig. 3 Block diagram of FFT based acquisition

The correlation operation between digitised carrier signal replica and the digitised input GPS IF signal is defined as the following equation.

$$z(n) = \sum_{m=0}^{N-1} y_c(m) y_{IF}(m+n) \quad (4)$$

Where, m represents the index of the sampling time sequence and N represents the number of samples. The circular correlation is defined in FFT as

$$\begin{aligned} \text{FFT}[z(n)] &= \text{FFT}\left[\sum_{m=0}^{N-1} y_c(m) y_{IF}(m+n)\right] \\ &= \text{FFT}[y_c(n)] \text{FFT}^*[y_{IF}(n)] \end{aligned} \quad (5)$$

Where, FFT is Fast Fourier Transform and FFT^* is complex conjugate of FFT. Equation 5 can be written in frequency domain as

$$Z(K) = Y_c(K) Y_{IF}^*(K) \quad (6)$$

Correlation function in time domain can be written as

$$z(n) = \text{IFFT}(Z(K)) = \text{IFFT}(Y_c(K) Y_{IF}^*(K)) = \text{IFFT}(\text{FFT}[y_c(n)] \text{FFT}^*[y_{IF}(n)]) \quad (7)$$

The highest absolute value of correlation power obtained from Eq. 7 gives the required Doppler shift and code phase of GPS signal.

4.1. Radix2 FFT algorithm

The N-point DFT is defined as⁹

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{nk} \quad \text{Where, } k \in [0, N-1] \text{ and } W_N = e^{\frac{-j2\pi}{N}}$$

Suppose that $N = N_1 \times N_2$, the index of DFT is expressed as shown below

$$n = N_2 n_1 + n_2 \quad (8)$$

$$k = N_1 k_1 + k_2 \quad (9)$$

$$X(k_1 + N_1 k_2) = \sum_{n_2=0}^{N_2-1} \sum_{n_1=0}^{N_1-1} x(n_1 N_2 + n_2) W^{(k_1 + N_1 k_2)(n_1 N_2 + n_2)} \quad (10)$$

Equation 9 is also represented in two dimensional as

$$X(k_1, k_2) = \sum_{n_2=0}^{N_2-1} W^{n_2 k_2} \left[W^{n_2 k_1} \sum_{n_1=0}^{N_1-1} W^{n_1 k_1} x(n_1, n_2) \right] \quad (11)$$

In the Eq. 10, inner summation gives N_1 point DFT and the outer summation represents N_2 point DFT. This is known as Cooley-Tukey algorithm⁹. From the Eq. 10, if N is in the form of power of 2 then N_1 is $N/2$ and N_2 is 2. N point DFT is represented as matrix given below

$$\begin{bmatrix} X(k_1, 0) \\ X(k_1, 1) \end{bmatrix} = \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} W^0 X(k_1) \\ W^{k_1} X(k_1) \end{bmatrix} \quad (12)$$

Where, k_1 ranges from 0 to $N/2$

Radix2 FFT algorithm is derived by dividing the N-point sequence (assume $N = 2^l$, l is integer) into two $N/2$ point sequences as even number samples and odd number samples. Then N-point DFT is computed by computing DFTs of these two divided sequences using Eq.11. By doing this it results in some reduction of arithmetic operations of N-point FFT⁸. Further reduction in number of arithmetic operation can be achieved by dividing each of the two $N/2$ -point sequences into two $N/4$ -point sequences and calculating the $N/2$ -point DFTs in terms of the corresponding two $N/4$ -point DFTs using Eq.11. Like this divide the sequence in each stage till the two-point sequences are obtained. The algorithm using this process used to compute DFT is known as radix2 FFT. This process reduces the number of multiplications and additions required to perform DFT. But the number of samples (N) must be the power of 2. To implement this algorithm for acquisition process the number of samples are make equal to nearest power of 2. For example GPS signal is sampled at 5MHz then 5000 samples exist in 1ms. To implement radix2 FFT, 5000 samples are converted to 8192 samples by appending zeros.

4.2. Proposed algorithm

Let the IF signal consists N number of samples in one C/A code period. Usually the number of samples N is not a power of 2 and to compute DFT using radix2 FFT, it needs appending of zeros to make number of samples to nearest power of 2. Thus acquisition process becomes complex and time consuming. To avoid this, an algorithm is proposed to compute N point DFT whose length is not a power of 2. In this Cooley-Tukey and Winograd Fourier Transform (WFTA) algorithms are used. In this algorithm N is represented as the product of any two factors N_1 and N_2 . Now use Cooley-Tukey algorithm to compute N point DFT using N_1 point FFT and N_2 point FFT. Further N_1 and N_2 are decomposed as factors and use Cooley-Tukey algorithm to compute FFT. This process continues till the factors are prime numbers (p). Now use WFTA to compute DFT of prime number p - point FFT. Cooley-Tukey algorithm is used to compute the DFT of longer length sequence. In order to perform correlation in the acquisition, WFTA and Cooley-Tukey FFT are used. For example, if sampling frequency (f_s) is 5MHz, then there are 5000 samples in 1ms, C/A code period. The 5000-point DFT can be computed using proposed algorithm as shown in

Fig.4.

Proposed algorithm is summarized as follows

- Reshape the 5000 samples data into 125×40 data matrix using Eq. 8
- Compute the 125 point DFT by writing 125 as 5×25 and using WFTA, for each column of matrix.
- Multiply the result with rotating factor (W_{5000}^{ij}) as per Cooley-Tukey algorithm
- Compute the 40 point DFT by writing 40 as 5×8 and using WFTA, for each row.
- Perform index mapping of output sequence as per Eq. 9

Winograd Fourier Transform is used to compute 5 point FFT, 8 point FFT and Cooley-Tukey FFT is applied to compute 40 point and 125 point FFT.

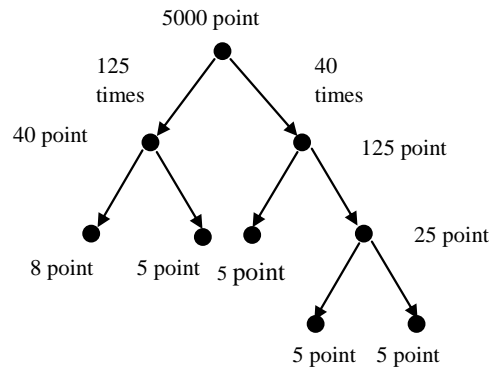


Fig. 4 Structure of a 5000 point FFT

5. Results and Discussions

The IF signal used for the implementation of the proposed algorithm consists of 1ms duration, carrier of 1.25MHz and sampled at 5MHz.

The acquisition of GPS signal is implemented using serial search method, radix2 FFT based search and decomposed FFT based search algorithms. In all the acquisition algorithms, the Doppler frequency search range and code phase observed are $\pm 10\text{KHz}$ in steps of 500Hz and 1 to 5000 samples respectively. The search index for Doppler frequency is from 1 to 41 (1.24MHz – 1.26MHz). It is found that a maximum correlation peak is detected at 222 C/A code phase sample and 26th frequency bin (i.e. 1.252MHz), which specify that starting of C/A code is located at this sample and received signal carrier frequency is 1.252M Hz. For all the algorithms the acquired Doppler shift and code phase are observed to be the same. However, the magnitude of correlation peak obtained is different.

Figure 5 and 6 shows the correlation in 2D plot as a function of code phase, frequency and correlation peak magnitude for serial search, radix2 FFT based search and decomposed FFT based search algorithms. From these figures it is found that the acquired correlation peak magnitude obtained using the proposed algorithm and conventional serial search algorithm is same (2571) and its value is greater than the value obtained using radix2 FFT search method (2491). Hence the proposed and serial search methods (Fig. 6 and 5(a)) give more accurate peak than the one obtained using radix2 FFT search algorithm (Fig. 5(b)).

The computational time of different acquisition algorithms are also compared. Table 1 shows the performance of the acquisition algorithms in respect of their computational time. Hundred monte-carlos simulations are carried to compute the computational time, and then average is taken. For serial search method the computational time obtained is 49.83s and this value is far greater than the computational time obtained using radix2 FFT based search (5.44s) and the proposed method (0.71s).

Table 1 Performance comparison of various algorithms

| Acquisition algorithm | Autocorrelation peak magnitude | Computational time for computing 1ms data in second | | |
|-----------------------------|--------------------------------|---|-----------------|------------------------------|
| | | Single frequency point | 41 Doppler bins | 100 Monte- Corlon iterations |
| Serial search algorithm | 2571 | 1.201 | 49.8323 | 4983.23 |
| Radix2 FFT algorithm | 2571 | 0.132 | 5.4445 | 544.45 |
| FFT decomposition algorithm | 2491 | 0.016 | 0.7108 | 71.08 |

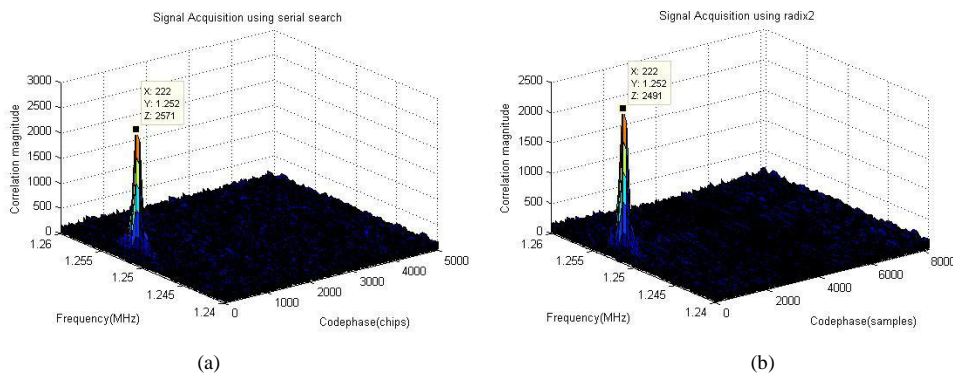


Fig. 5 Signal acquisition using (a) serial search algorithm (b) radix2 FFT search algorithm

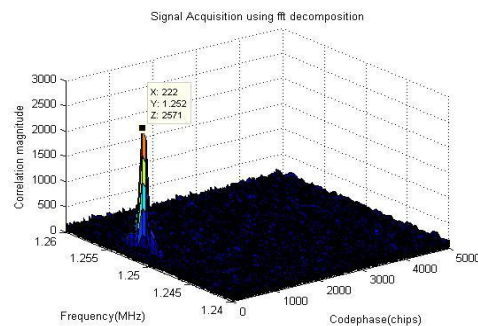


Fig. 6 Signal acquisition using proposed search algorithm

4. Conclusions

The proposed decomposed FFT based acquisition algorithm is validated using one millisecond duration IF signal data. The data corresponding to 1.25 MHz sampled at 5MHz. The performance of the proposed algorithm is also compared with the two other algorithms (serial search, and radix2 FFT search). It is found that the code offset and Doppler shift acquired from GPS signal are same in all the methods. However, the correlation peak magnitude obtained using the proposed algorithm is high (2571). Also it is also found that the computational time for the proposed method is (0.71 s) less than the radix2 FFT search (5.44 s) and serial search methods (49.88 s). Hence, the proposed method is very useful to implement fast and accurate acquisition in real time GPS receivers.

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