Carrier Tracking Loop; Loop Filter

GPS Signals And Receiver Technology MM12

Darius Plaušinaitis
dpl@gps.aau.dk

Today's Subjects

- Demodulation of the GPS signal
- Tracking loop introduction
- Carrier tracking
  - Phase Lock Loop (PLL)
  - Frequency Lock Loop (FLL)
- Loop filters

Demodulation

Or how to turn the radio waves back into the data message that we are interested in

Demodulation

- **The transmitted signal from satellite k is:**

$$
\begin{aligned}
S^k(t) &= \sqrt{2P_c}\,(C^k(t)D^k(t))\cos(2\pi f_{L1}t) && \text{L1 C/A signal}\\
&+\sqrt{2P_{PL1}}\,(P^k(t)D^k(t))\sin(2\pi f_{L1}t) && \text{L1 P(Y) signal}\\
&+\sqrt{2P_{PL2}}\,(P^k(t)D^k(t))\cos(2\pi f_{L2}t) && \text{L2 P(Y) signal}
\end{aligned}
$$

D – data
C – C/A code
P – P(Y) code

Demodulation

DANISH GPS CENTER

- **The transmitted signal from satellite k is:**

$$S^k(t) = \sqrt{2P_c}\,(C^k(t)D^k(t))\cos(2\pi f_{L1}t) + \sqrt{2P_{PL1}}\,(P^k(t)D^k(t))\sin(2\pi f_{L1}t)$$
$$+ \sqrt{2P_{PL2}}\,(P^k(t)D^k(t))\cos(2\pi f_{L2}t)$$

- **After an RF front-end (L1 only):**

$$S^k(t) = \sqrt{2P_c}\,(C^k(t)D^k(t))\cos(\omega_{IF}t) + \sqrt{2P_{PL1}}\,(P^k(t)D^k(t))\sin(\omega_{IF}t)$$

- **The signal from one satellite after the ADC (narrow filters and low sampling frequency):**

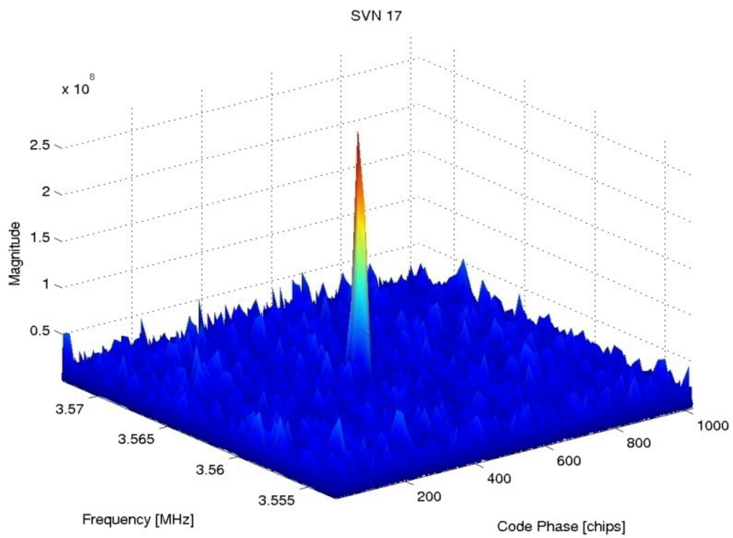
$$S^k(n) = C^k(n)D^k(n)\cos(\omega_{if}n) + e(n)$$

Demodulation

- **The signal from two satellites after the ADC:**

$$S(n) = C^1(n)D^1(n)\cos(\omega_{if1}n) + C^2(n)D^2(n)\cos(\omega_{if2}n) + e(n)$$

- **The code phase and the intermediate frequency of the carrier wave should be known parameters to demodulate the navigation data from e.g. satellite 1.**



Demodulation

- **Convert the signal down to baseband:**

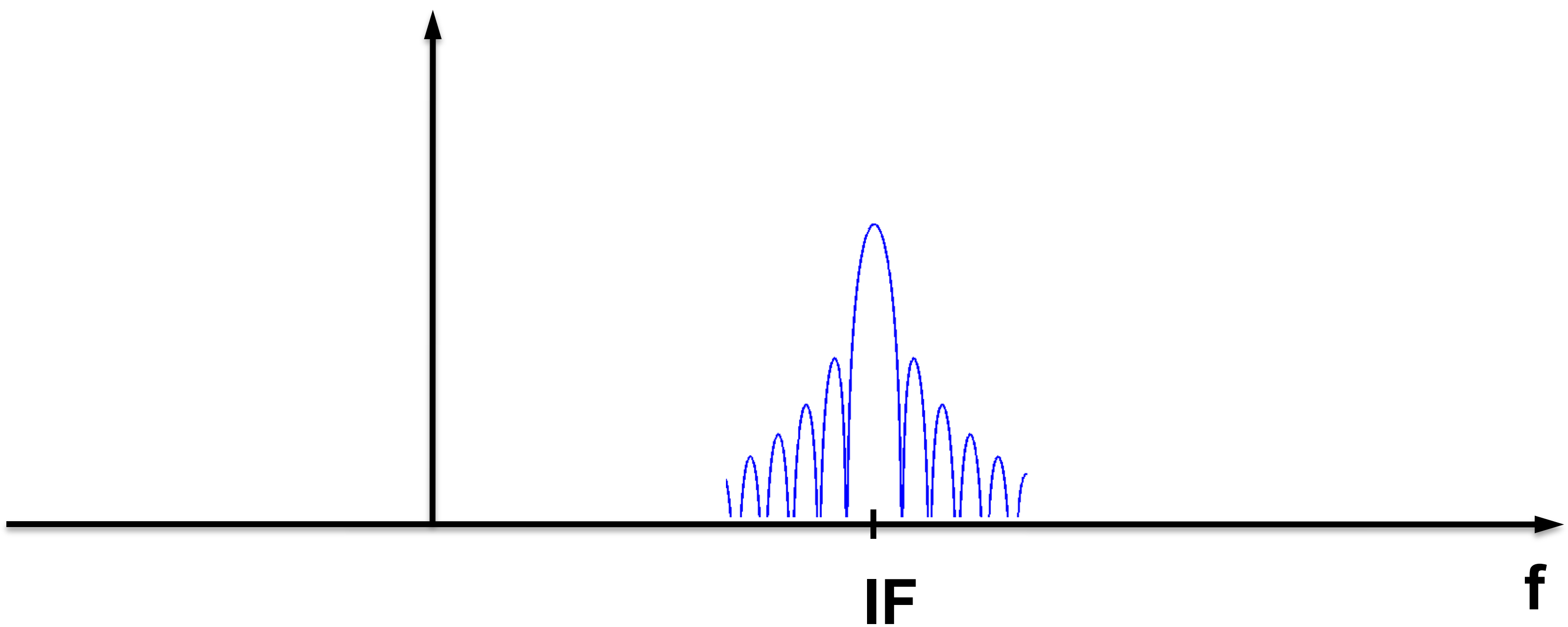
$$S(n)\cos(\omega_{if1}n) = C^1(n)D^1(n)\cos(\omega_{if1}n)\cos(\omega_{if1}n) + C^2(n)D^2(n)\cos(\omega_{if2}n)\cos(\omega_{if1}n) + e(n)$$

cos(a)*cos(b) = ½cos(a+b) + ½cos(a-b)

$$S(n)\cos(\omega_{if1}n) = \boxed{\tfrac{1}{2}C^1(n)D^1(n)} + \tfrac{1}{2}C^1(n)D^1(n)\cos(2\omega_{if}) + C^2(n)D^2(n)\cos(\omega_{if2}n)\cos(\omega_{if1}n) + e(n)$$

$$S(n)\cos(\omega_{if1}n) = \tfrac{1}{2}C^1(n)D^1(n) + \tfrac{1}{2}C^1(n)D^1(n)\cos(2\omega_{if}) + \tfrac{1}{2}C^2(n)D^2(n)\cos(\omega_{if2} - \omega_{if1}n) + \tfrac{1}{2}C^2(n)D^2(n)\cos(\omega_{if2} + \omega_{if1}n) + e(n)$$

Demodulation Visualized



Demodulation

- **Code wipe off:**

$$S(n)\cos(\omega_{if1}n)C^1(n) = \tfrac{1}{2}D^1(n) + \tfrac{1}{2}D^1(n)\cos(2\omega_{if}) + e(n)$$

- **After low-pass filtering (integration):**

$$S(n)\cos(\omega_{if1}n)C^1(n) = I = \tfrac{1}{2}D^1(n) + e(n)$$

- **Received signal amplitude vs. tracking errors**

$$I_i = \frac{\sin(\pi\Delta f_i T)}{(\pi\Delta f_i T)}\sqrt{2\frac{S}{N_0}}R(\tau_i)D_i\cos(\Delta\phi_i) + e_i$$

- **Conclusion: perfectly aligned code and carrier replicas are required to do demodulation. These replicas can be tracked using two tracking loops.**

Tracking Loop

A way to generate exact copy of the received signal

What does tracking do and why?

- The main goal is to receive the GNSS signal ”as clear and loud” as possible – the local carrier and spreading code must be well aligned with the ones in the signal
- GNSS adds one more requirement: to track signal arrival (time) as precise as possible
- Advanced receivers can detect multipath to some extent
- Additional task can be signal quality monitoring

The Tracking Loop Idea

DANISH GPS CENTER

- Generate a local signal
- Correlate it with the received signal
- Measure (time/code-phase, frequency, phase) error between the local and the received signals
- Steer local signal generators to minimize the error
- Pass demodulated data bit value stream to the data processing task
- Repeat procedure

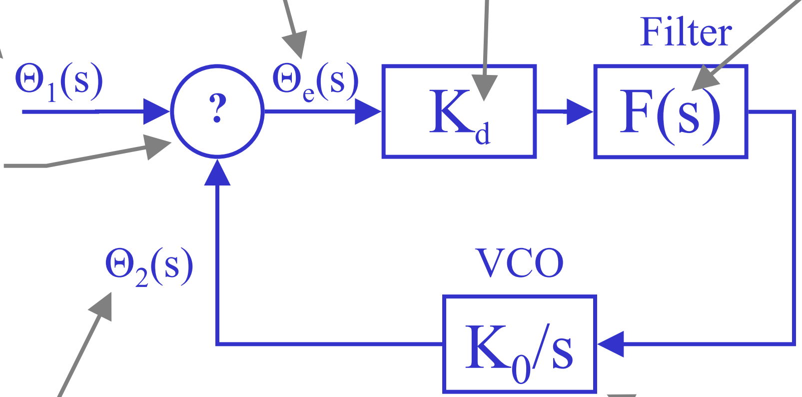
Main Parts Of A Tracking Loop

The error measurement

Scaling of error – used to control sensitivity to the tracking error

Tracked signal

Error measurement unit (called error detector or discriminator)



Filter has two main tasks:

- To filter noise in the error measurements (due to noise in the tracked signal)
- To shape the tracking loop's response to the tracking error

Generated signal

Signal generator generates replica of the tracked signal

Types Of Tracking Loops

- There are 3 main types of the tracking loops depending on the tracked property of the tracked signal:
  - Phase lock loop (PLL)
  - Frequency lock loop (FLL)
  - Delay lock loop (DLL)
- There are few error detectors for each type of the tracking loop with different properties
- Variations of filter parameters will shape the filter response and amount of noise filtering

Plan For The Tracking Topic

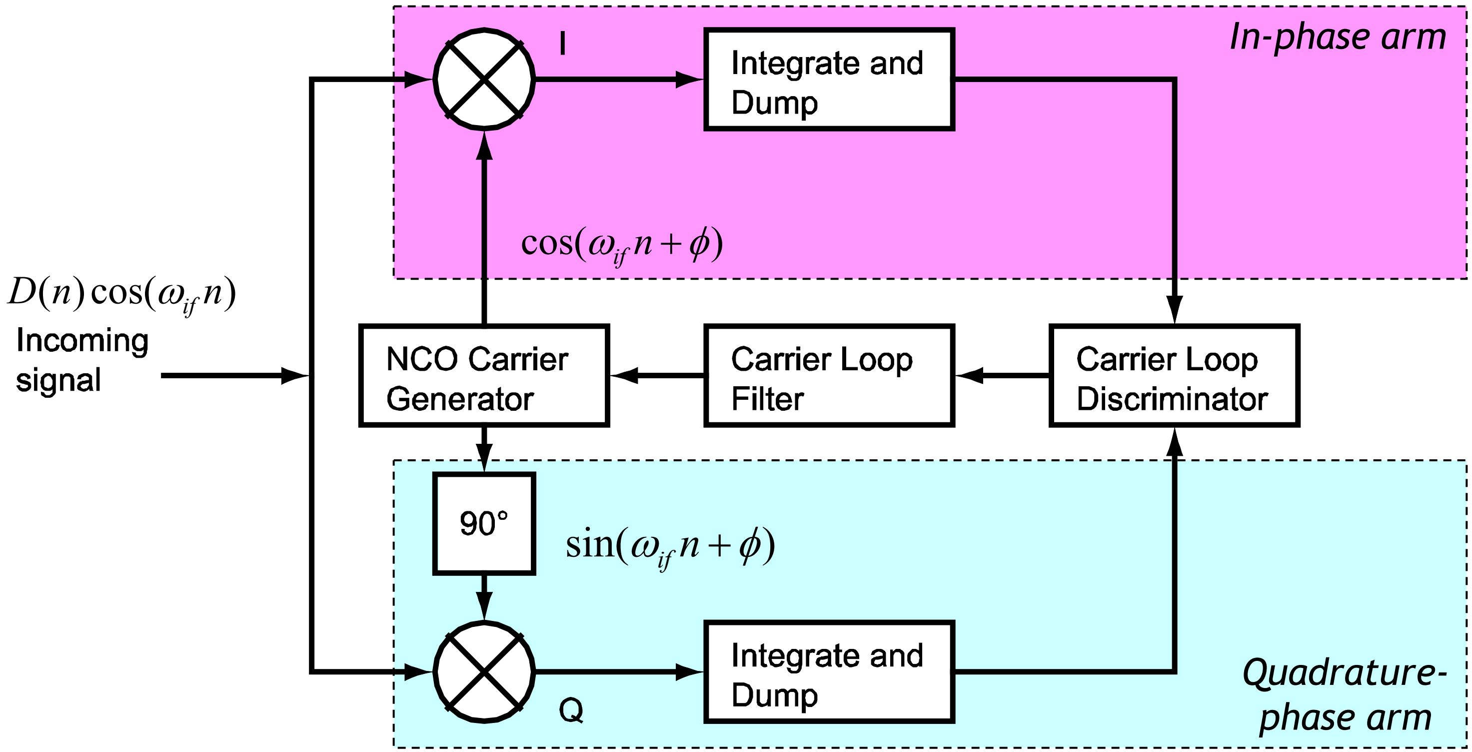
- PLL (FLL) and DLL are explained in sections on carrier and code tracking
- Each section will also cover a set of error detectors applicable in a given tracking loop
- Tracking loop filter is explained in a separate section as the same theory is used for all types of tracking loops

The Carrier Tracking Loop

The Phase Locked Loop (PLL)

Carrier Tracking Loop

- **The goal of the Carrier Tracking Loop is to produce a perfectly aligned carrier replica. The most common way is to use a PLL:**



Carrier Tracking Loop

- **The demodulation in the In-phase (I) branch:**

$$D(n)\cos(\omega_{if}n)\cos(\omega_{if}n+\phi)=\tfrac{1}{2}D(n)\cos(\phi)+\tfrac{1}{2}D(n)\cos(2\omega_{if}n+\phi)$$

- **The demodulation in the Quadrature-phase (Q) branch:**

$$D(n)\cos(\omega_{if}n)\sin(\omega_{if}n+\phi)=\tfrac{1}{2}D(n)\sin(\phi)+\tfrac{1}{2}D(n)\sin(2\omega_{if}n+\phi)$$

- **The I signal:**

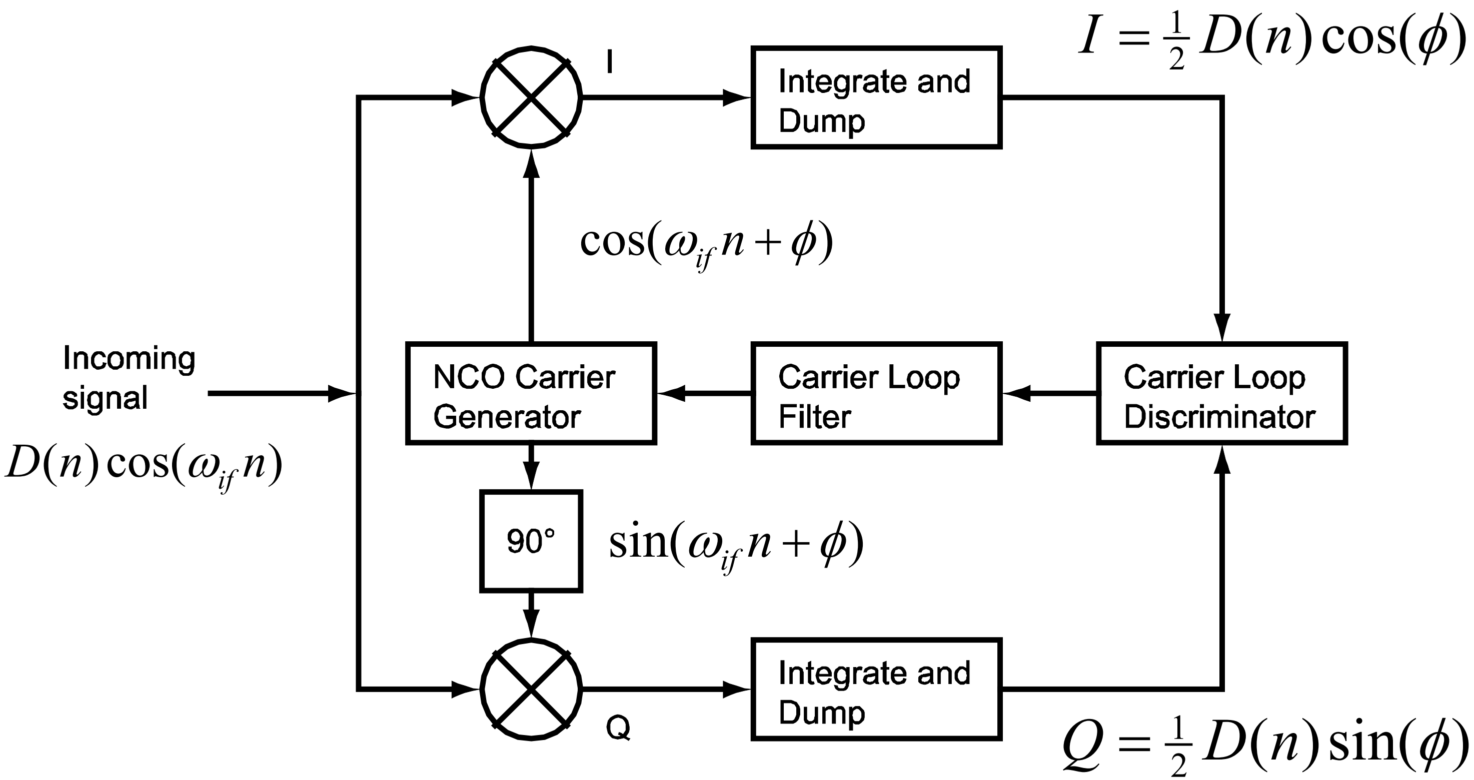
$$I=\tfrac{1}{2}D(n)\cos(\phi)$$

- **The Q signal:**

$$Q=\tfrac{1}{2}D(n)\sin(\phi)$$

Carrier Tracking Loop

- **Costas loop:**



Carrier Tracking Loop

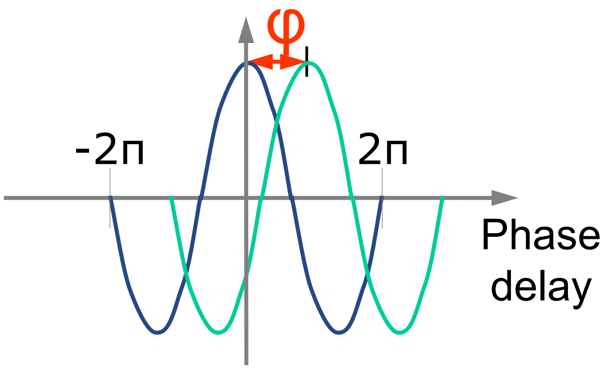
DANISH GPS CENTER

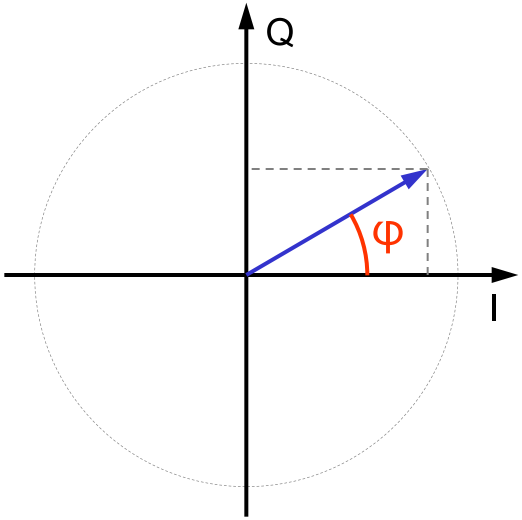
- **To find the phase error:**

$$\frac{Q}{I} = \frac{\frac{1}{2}D(n)\cos(\phi)}{\frac{1}{2}D(n)\sin(\phi)} = \tan(\phi)$$

$$\phi = \tan^{-1}\frac{\frac{1}{2}D(n)\sin(\phi)}{\frac{1}{2}D(n)\cos(\phi)}$$

$$\phi = \tan^{-1}\frac{Q}{I}$$

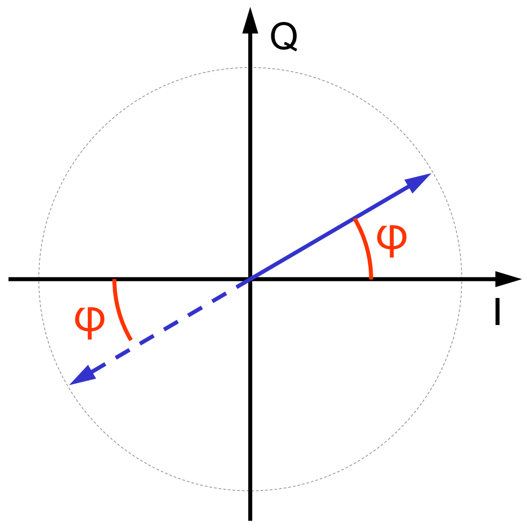


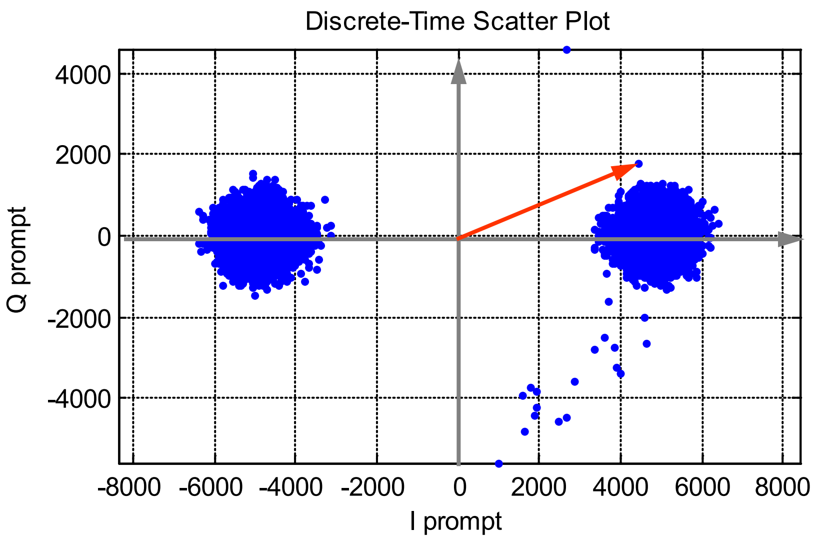


Carrier Tracking Loop

- **The Costas loop is independent on the phase shifts caused by the data bits**

- **Phasor diagram:** **An output example:**





Carrier Tracking Loop

- **Different kinds of phase lock loop discriminators:**
  - **Arctan**

$$D = \tan^{-1}\frac{Q}{I}$$

    - **Much time consuming (not a big problem today)**
    - **The output is the real phase error**

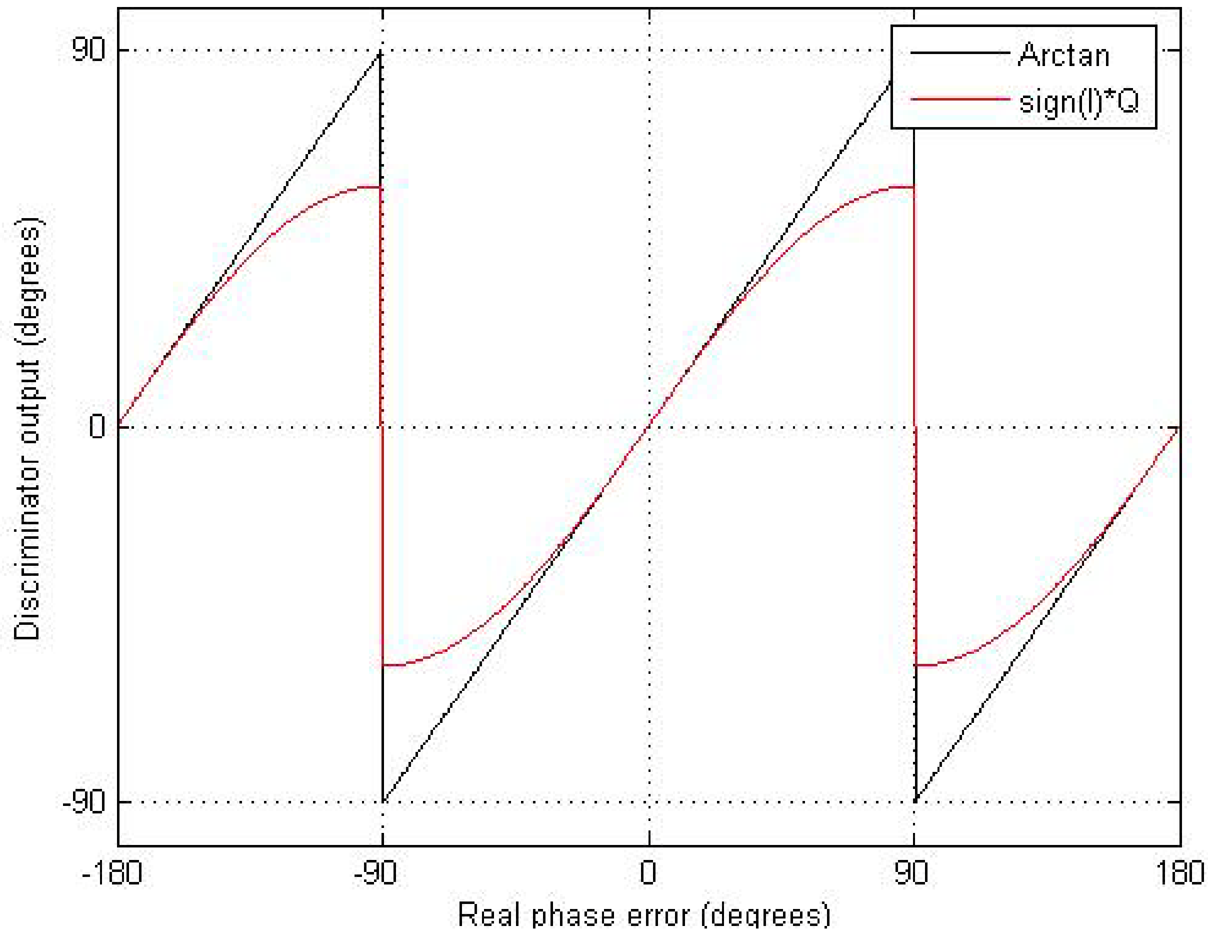
  - **Sign product**

$$D = Q \bullet sign(I)$$

    - **Fast method**
    - **The discriminator output is proportional to *sin(φ)***

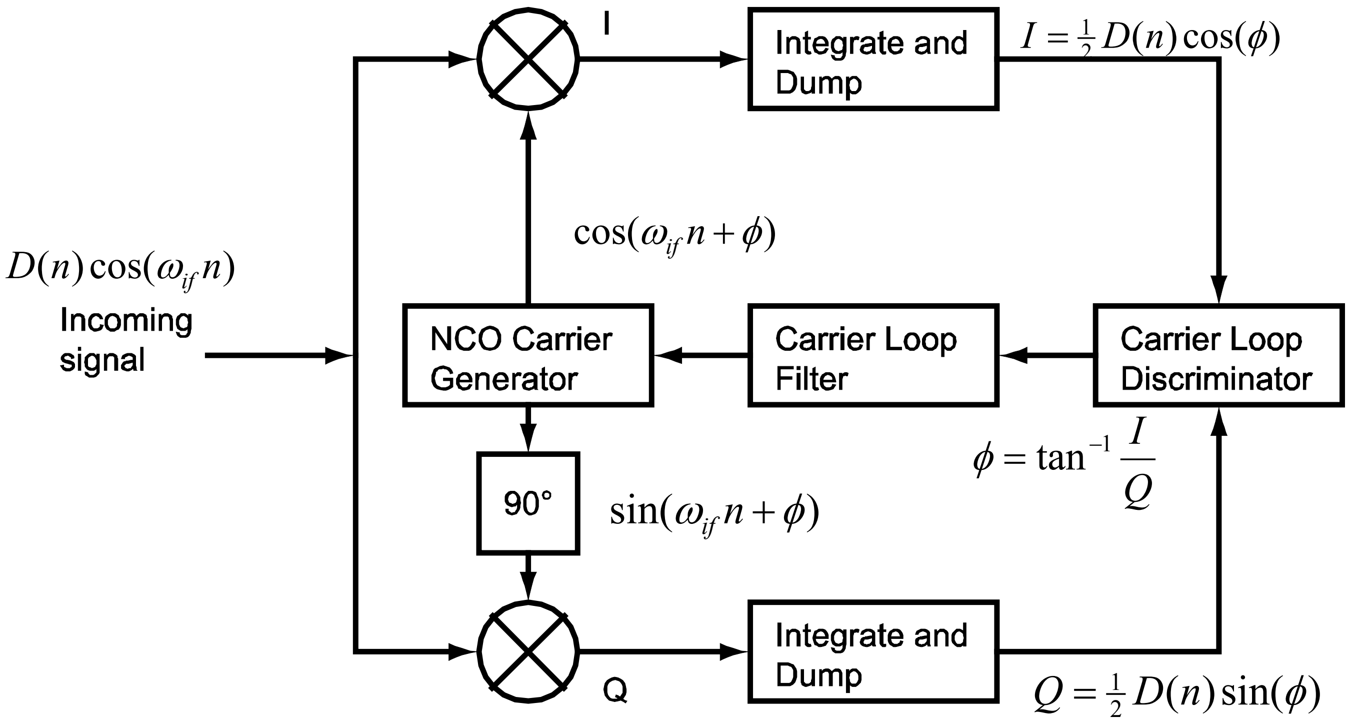
Carrier Tracking Loop

- **Different kinds of phase lock loop discriminators:**



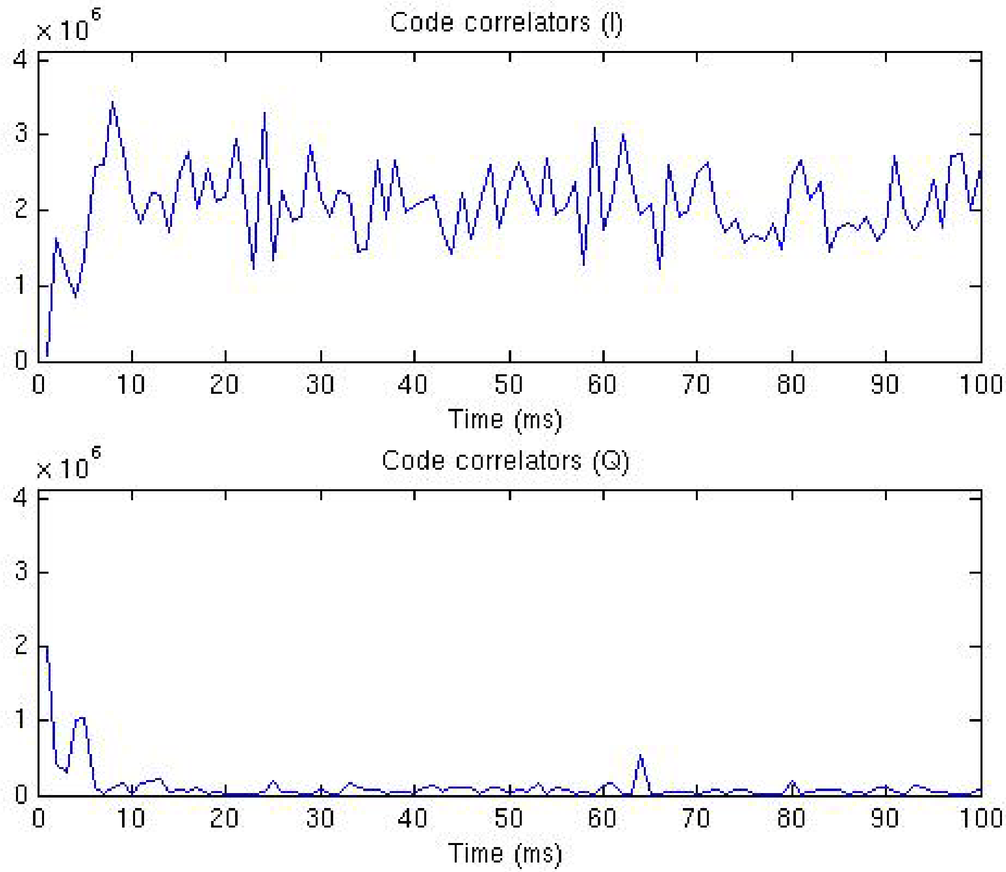
Carrier Tracking Loop

- **Costas loop:**



Carrier Tracking Loop

- **The signal energy in I and Q when PLL has locked on the signal:**



The Carrier Tracking Loop

The Frequency Locked Loop (FLL)

Frequency Lock Loop

- The discriminators are a bit different than in PLL. They measure change in carrier phase over an interval of time.
- Less noise sensitive than PLL – it can track at lower SNR
- The tracking loop has more noise than PLL
- Can be used for the re-acquisition or pull-in states due to bigger frequency lock range

Loop Filters

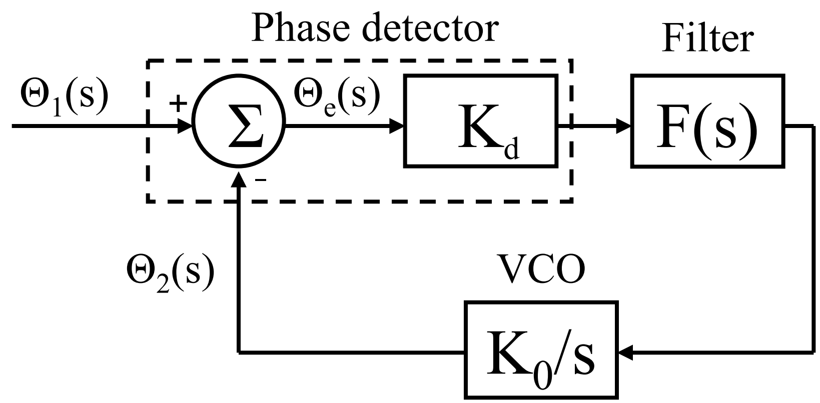
Why The Filter Is Needed Anyway?

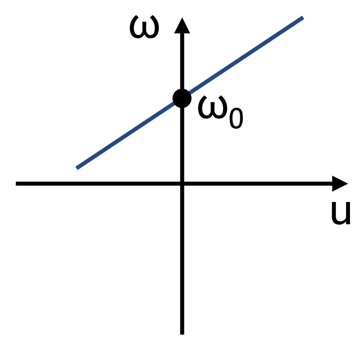
DANISH GPS CENTER

- The error measurement is there, so just correct the generator frequency and job is done, right?
- The answer is NO:
  - There is an error measurement noise (even at good SNR)
  - There is a stady state error caused by Doppler

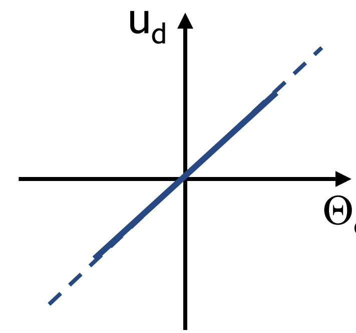
The Typical Tracking Loop

- Phase error detector has gain K_d. The transfer function is showed in figure a)
- The VCO has a center frequency ω_0 and gain K_0. The transfer function is showed in figure b)
- The filter coefficients depend on K_d and K_0



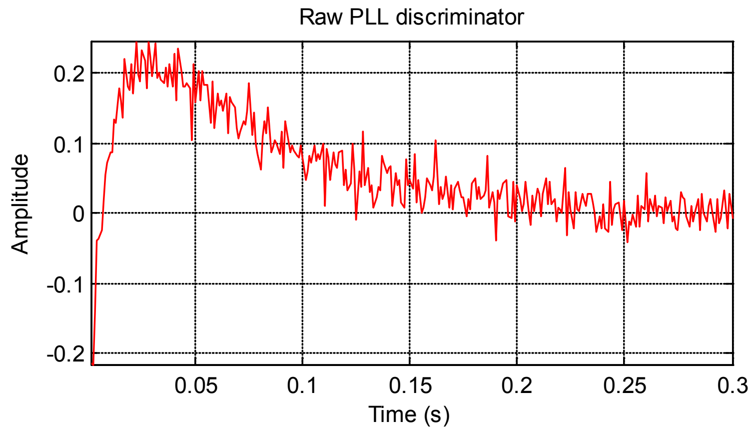


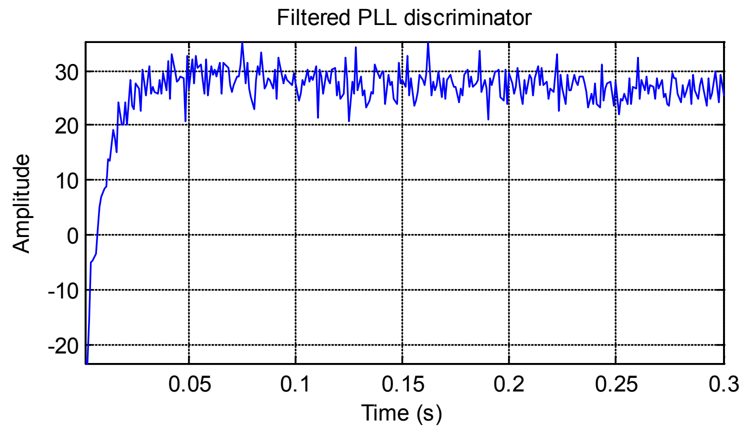
a)



b)

An Example Of Filter Output vs. Input

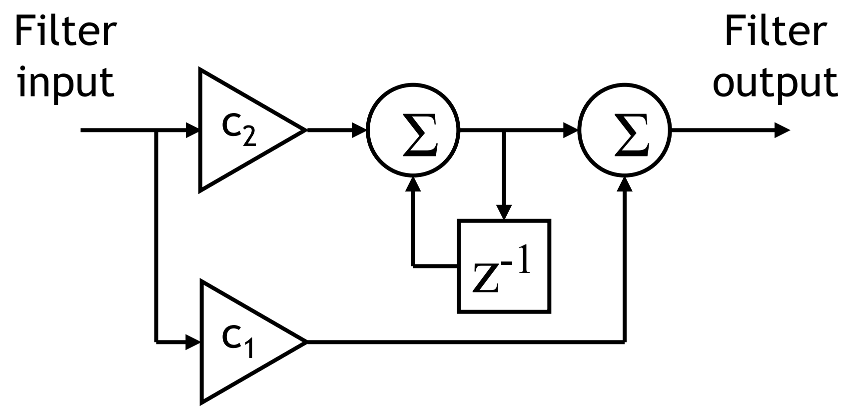




- There is an initial frequency between tracked and generated signals (27Hz here)
- Figures show:
  - The filter “accumulates” offset over time and keeps it
  - The result of damping – different convergence times

A Simple Digital Loop Filter

- The C_1 and C_2 depend on loop noise bandwidth B_L, VCO and PD gains and loop damping factor ζ.
- Damping factor controls how fast the filter reaches its settle point
- Noise bandwidth controls the amount of allowed noise in the filter



$$C_1 = \frac{1}{K_0 K_D} \frac{8\zeta\omega_n T}{4 + 4\zeta\omega_n T + (\omega_n T)^2}$$

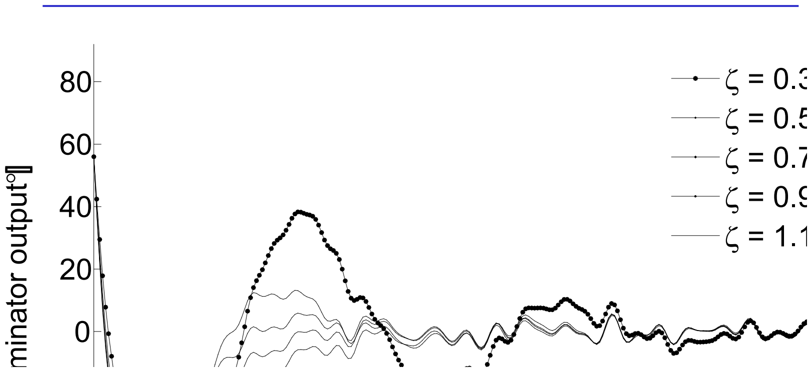
$$C_2 = \frac{1}{K_0 K_D} \frac{4(\zeta\omega_n T)^2}{4 + 4\zeta\omega_n T + (\omega_n T)^2}$$

$$\omega_n = \frac{8\zeta\, B_L}{4\zeta^2 + 1}$$

Damping Factor

Different loop responses depending on the damping factor

(first 20ms are due to loop filter initialization)

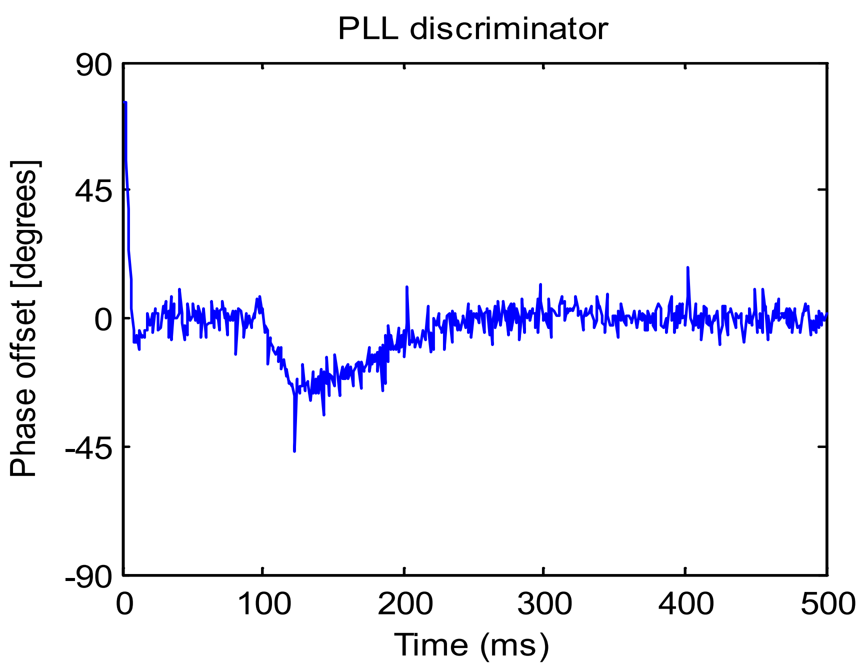


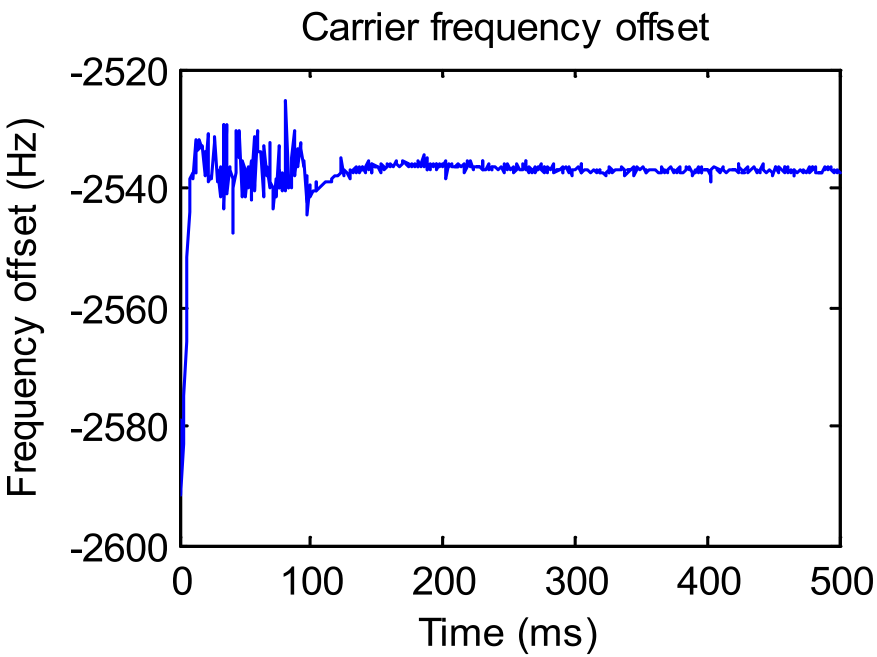
- **Determines how much the loop filter ”resists” to the control signal:**
  - **On one hand – how fast the loop will ”fix” the tracking error**
  - **On other hand – how much the loop will overshoot 0 error point**
- **A compromise value is used or few values are used for different receiver modes**

Noise Bandwidth

DANISH GPS CENTER

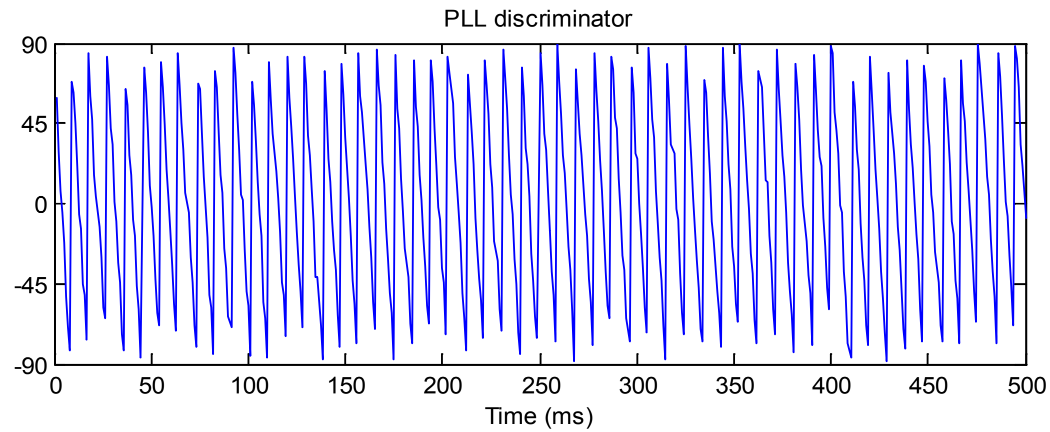
- Narrow noise bandwidth decreases noise in the tracking loop, AND – response speed
- At 100ms the loop noise bandwidth is switched from about 100Hz to 15Hz

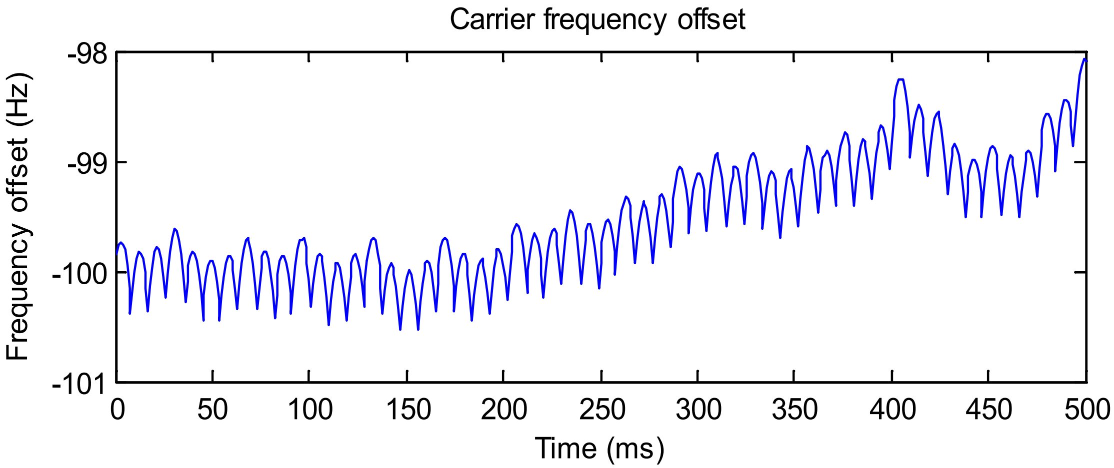




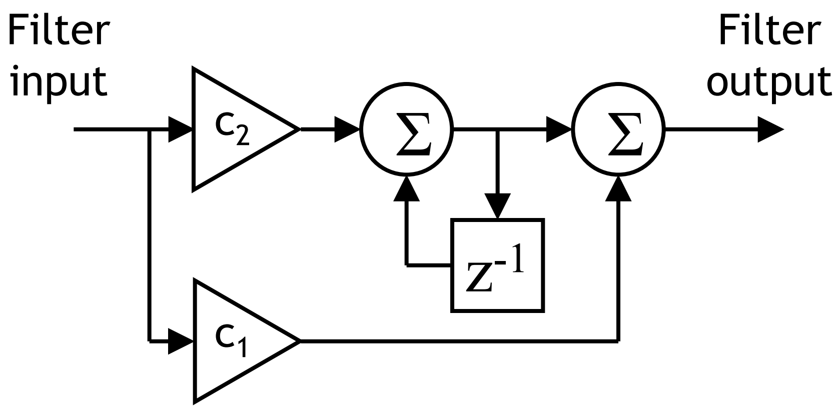
Noise Bandwidth

- **Loop noise bandwidth also determines maximum Doppler offset and rates tolerated by the loop**
- **Figures show a case of too big initial frequency error in acquisition**





Loop Order

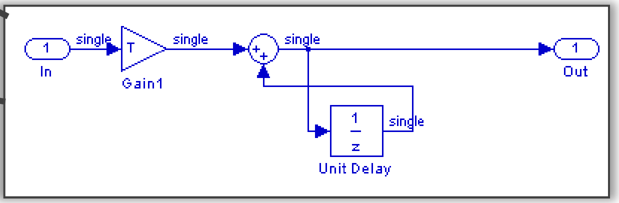


- The filter is a first order filter
- The tracking loop (excluding filter) is a first order system, therefore the tracking loop is second order
- Higher order filters approximate the error dynamics better (e.g. to be used for ships etc.)

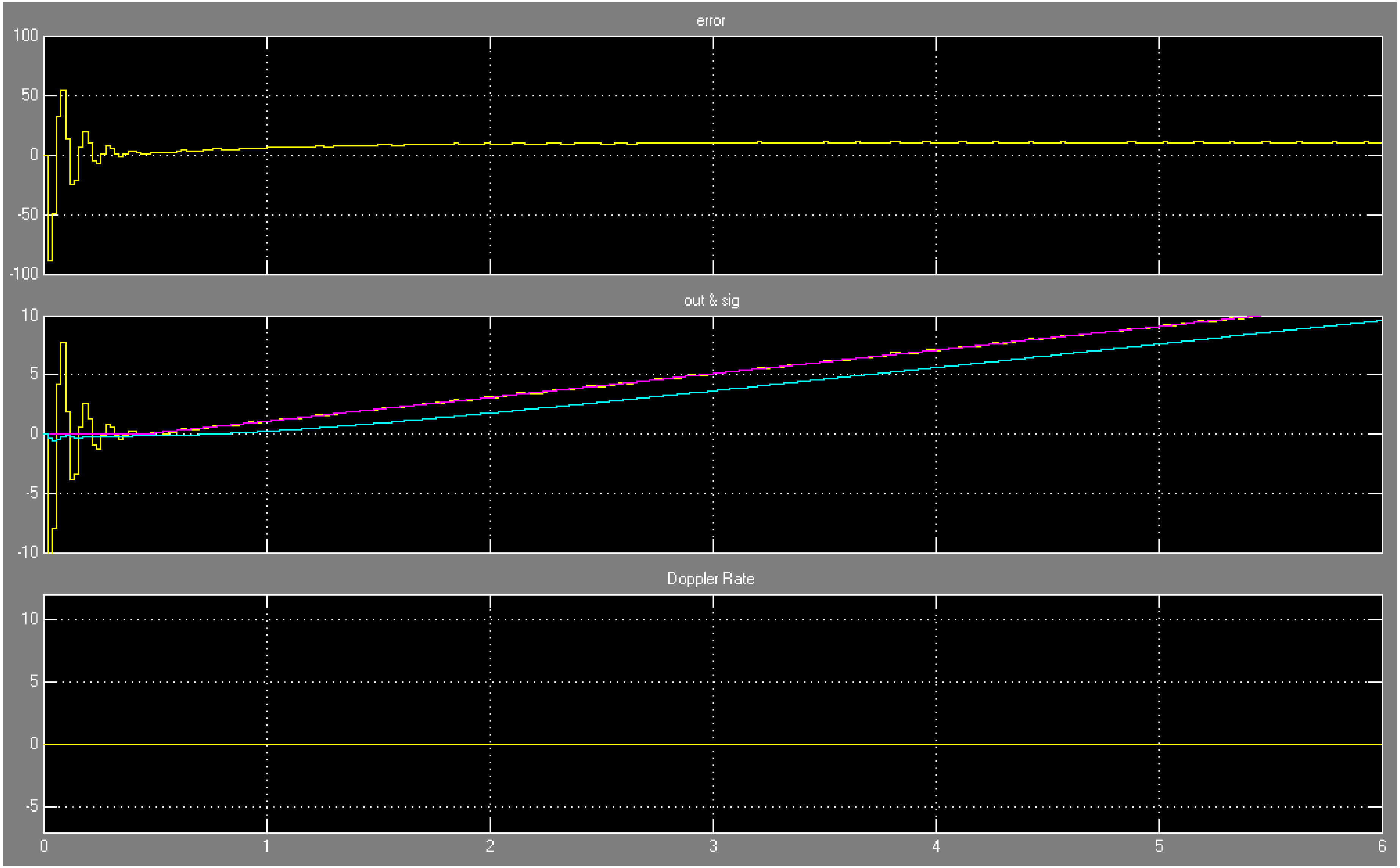
An Example Of A PLL

- ω=20/0.7845; T=0.02;
  - k1= 2.4 * ω
  - k2= (1.1 * ω ^2)/20
  - k3= ω ^3/20
- ω =20/0.53; T=0.02;
  - k1= 1.414 * ω
  - k2= ω ^2/20
  - k3= 0

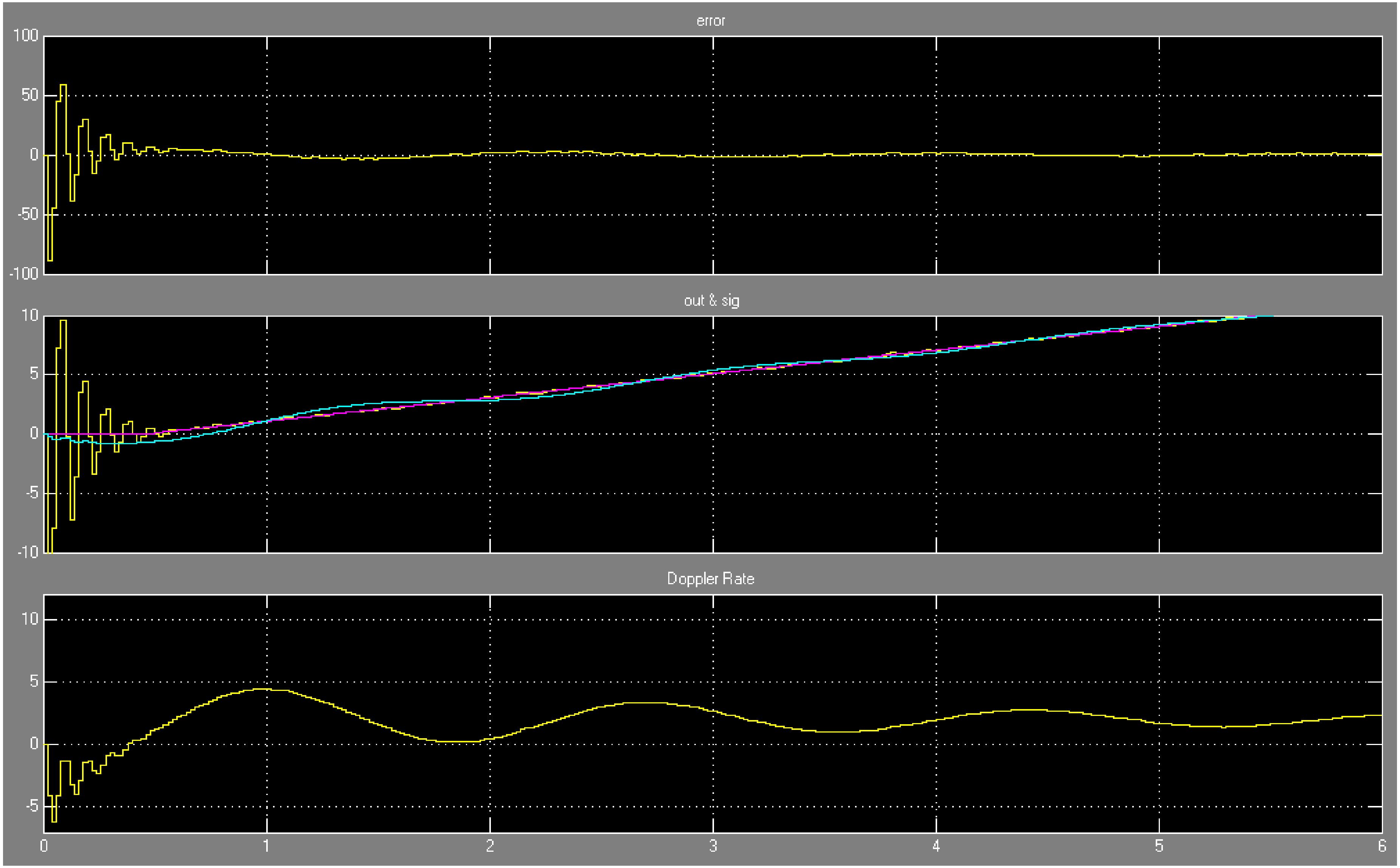
This slide contents is only available to the listeners of our courses



2-nd Order Loop Responce



3-rd Order Loop Responce



Questions and Exercises