Homework 3 writeup

Starting with the general expression for a line in Cartesian coordinates:

$$ax + by = c$$

I need to derive the equation below from the equation above:

$$x\cos\theta + y\sin\theta = \rho$$

Starting with Pythagorean theorem $a^2 + b^2 = c^2 \rightarrow \sqrt{(\,a^2 + b^2\,)} = hypothenuse$ the trigonometric identities are defined as:

$$\cos\theta = \frac{adjacent}{hypothenuse} \qquad \sin\theta = \frac{opposite}{hypothenuse}$$

We rearrange the original equation to better form the derivation first we divide both sides by the hypothenuse $\sqrt{(\,a^2 + b^2\,)}$

$$\frac{a}{\sqrt{a^2+b^2}}x + \frac{b}{\sqrt{a^2+b^2}}y = \frac{c}{\sqrt{a^2+b^2}}$$

Now $\frac{a}{\sqrt{a^2+b^2}}$ can be replaced with $\cos\theta = \frac{adjacent}{hypothenus}$ and $\frac{b}{\sqrt{a^2+b^2}}$ can be replaced with $\sin\theta = \frac{opposite}{hypothenus}$ this leaves the form:

$$x\cos\theta + y\sin\theta = \frac{c}{\sqrt{a^2+b^2}}$$

From this we define ρ as $= \frac{c}{\sqrt{a^2+b^2}}$ which results in the desired equation $x\cos\theta + y\sin\theta = \rho$.

Questions:

What is the geometric significance of terms a,b? (Hint: think intercepts)

When I set y to zero, I get the equation ax = c and when I set x to zero, I get by = c in either case a or b scales c through division resulting in a x or y intercept at some constant. The significance of the terms a and b is that they determine where the x and y intercept of a line is located at.

How is ρ related to a, b, c?

ρ is radius it forms the direct relationship of equaling $\frac{c}{\sqrt{a^2+b^2}}$

How is θ related to a, b, c?

θ is angle this variable ensures that the x and y components in the cartesian coordinates follows the same direction in the polar coordinates it forms a direct relation relationship with a and b by

using $\cos\theta$ and $\sin\theta = \frac{a}{\sqrt{a^2+b^2}}$ and $\frac{b}{\sqrt{a^2+b^2}}$ respectively. When these two components are summed together the they form another relationship directly to equaling $\frac{c}{\sqrt{a^2+b^2}}$.

Questions:

Explain why the expressions for a, b have different sign for $\rho(\cos\theta)$ and $\rho(\sin\theta)$?

The reason these two expressions have different signs is due to how the coordinate system is defined with the x axis moving from left to right of positive values while the y is defined as increasing from moving top to bottom of positive values. This is different from the standard cartesian coordinates we are use to seeing.

Were you able to detect the 2 circles in the image **FaceCloseUp.jpg**? What is your estimate of the radius of the eye, and how did you identify this value?

Yes, My estimate for the radius of the eye was 35 pixels, I estimated this value by inspecting the image with a pointer on either side of the eye to determine the diameter and then divided by 2.

Were you able to detect the 4 circles in the image **PokerChips.png**? What is your estimate of the radius of the eye, and how did you identify this value?

Yes, My estimate for the radius of the Poker chips was 180 pixels, I did the same procedure as with the picture above except I only took the dimensions of the first chip and assumed the rest of the chips had the same dimensions.

What happens to the number of detected circles when you pick a large value for the threshold in `houghpeaks`?

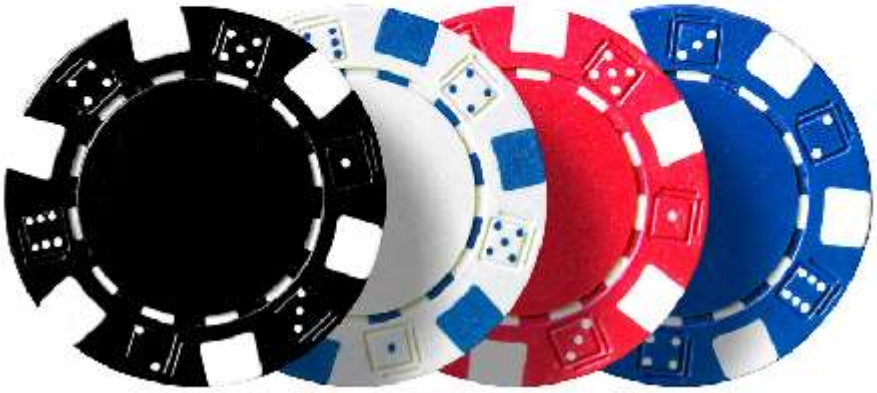
The number of detected circles drop as the threshold removes certain centers as possibilities. Due to too high of a threshold.

What happens to the number of detected circles when you pick a small value for the threshold in `houghpeaks`?

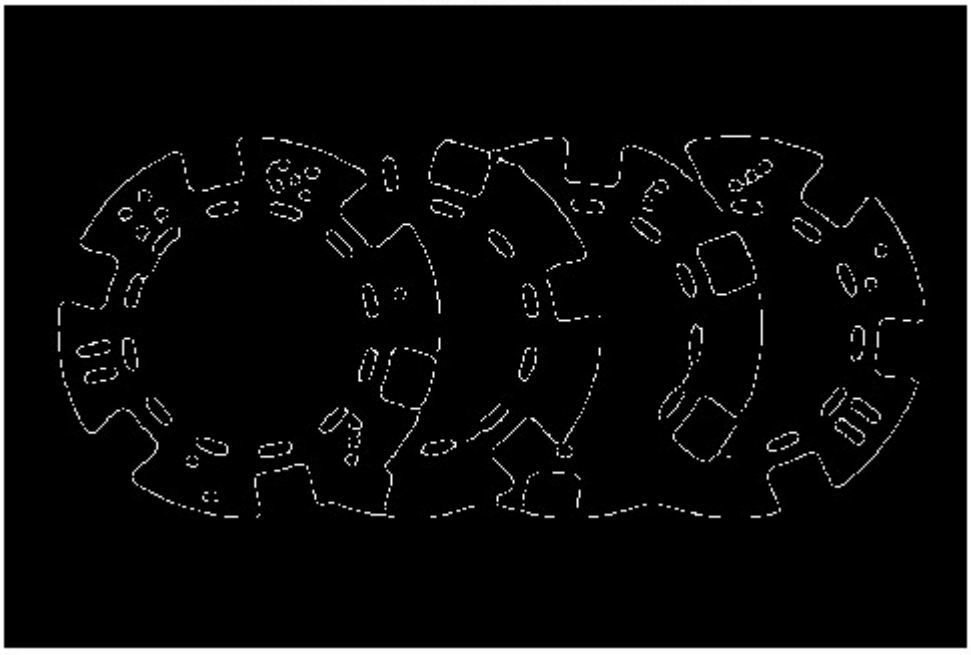
The number of detected circles remains the same since the houghpeaks command lets the user define how many peaks to define if it can find them.

Poker Chip Results:

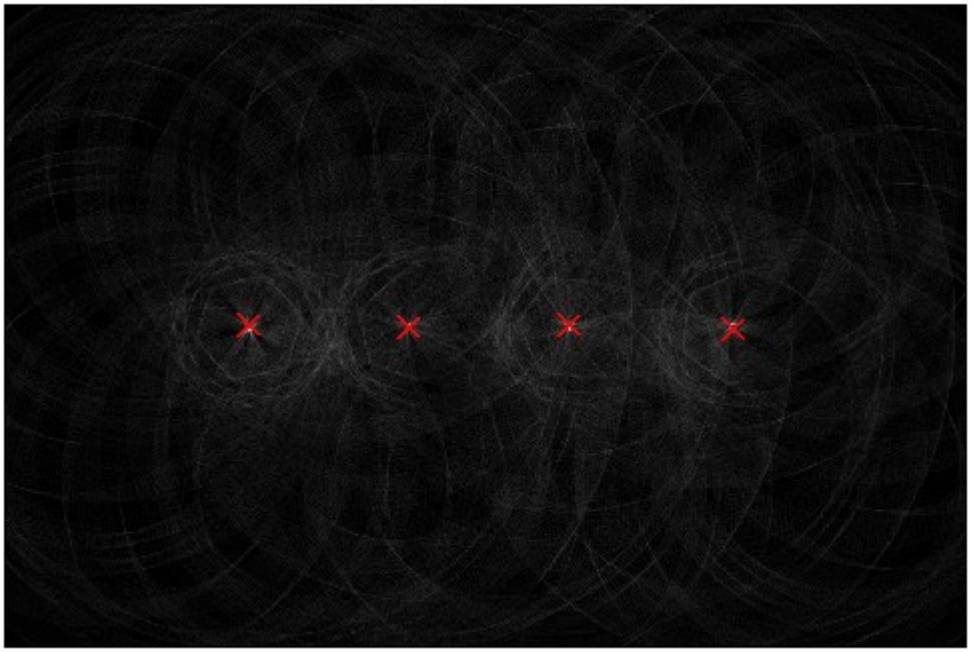
Original



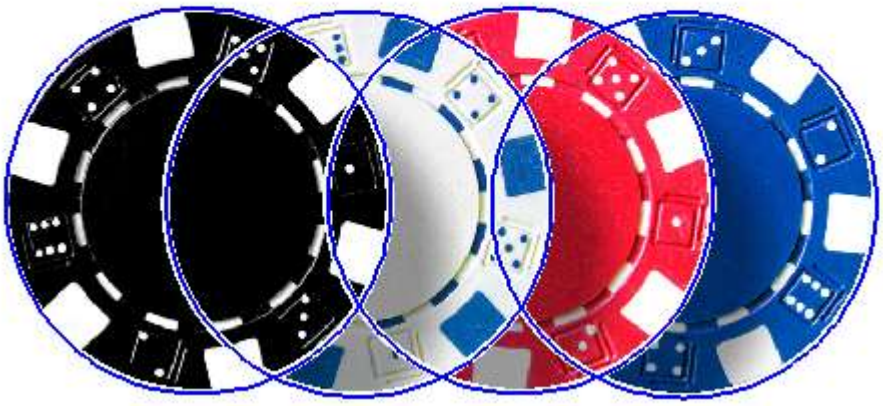
Output of Canny edge Detector



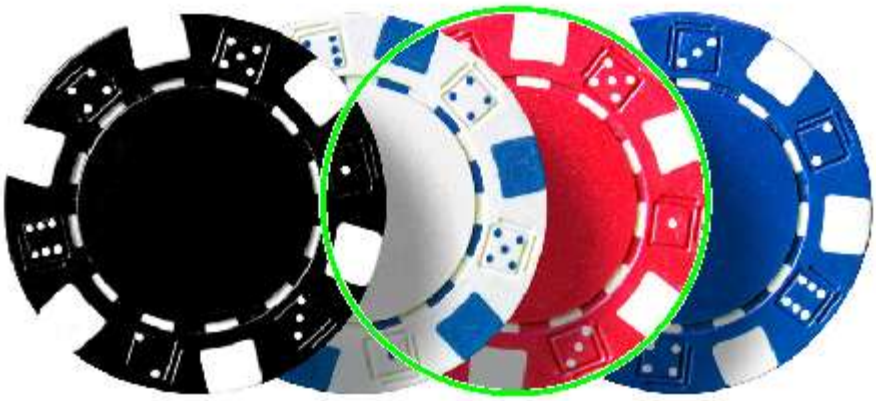
HoughSpace With Peaks



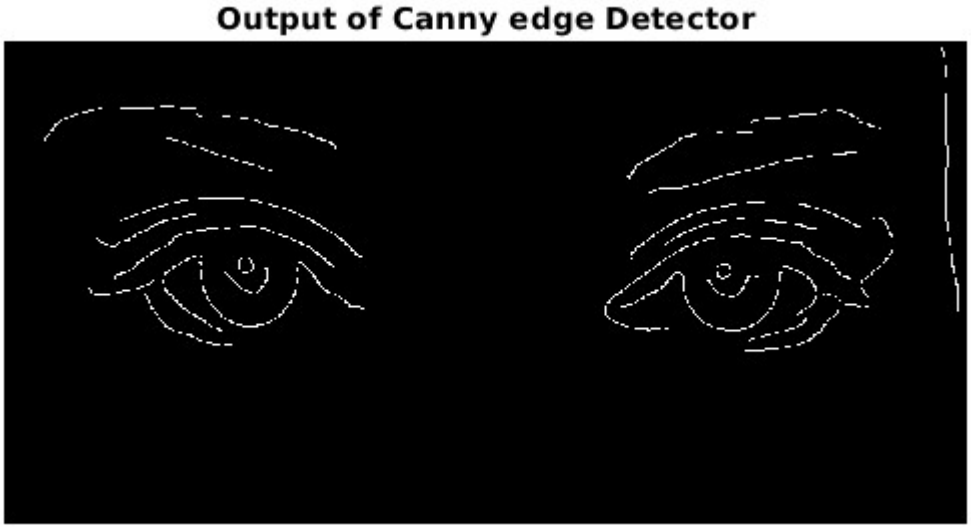
Original With Overlays



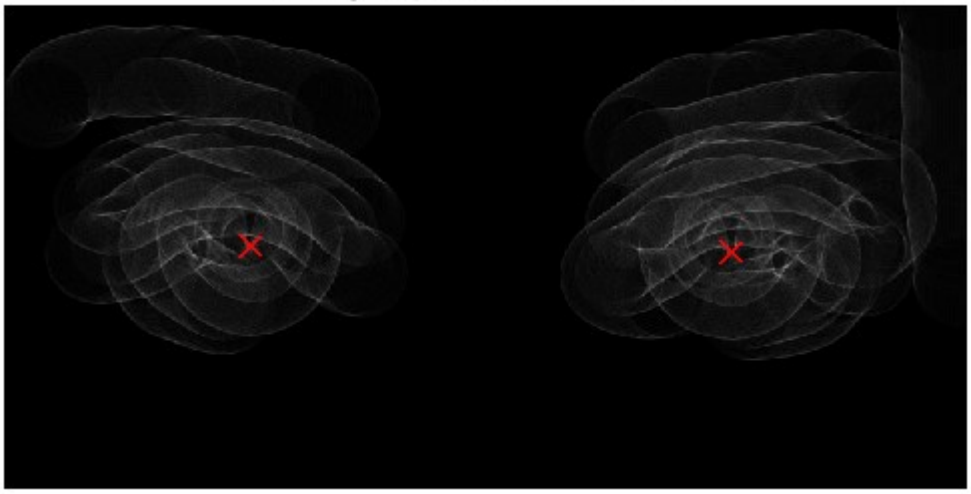
Red Only



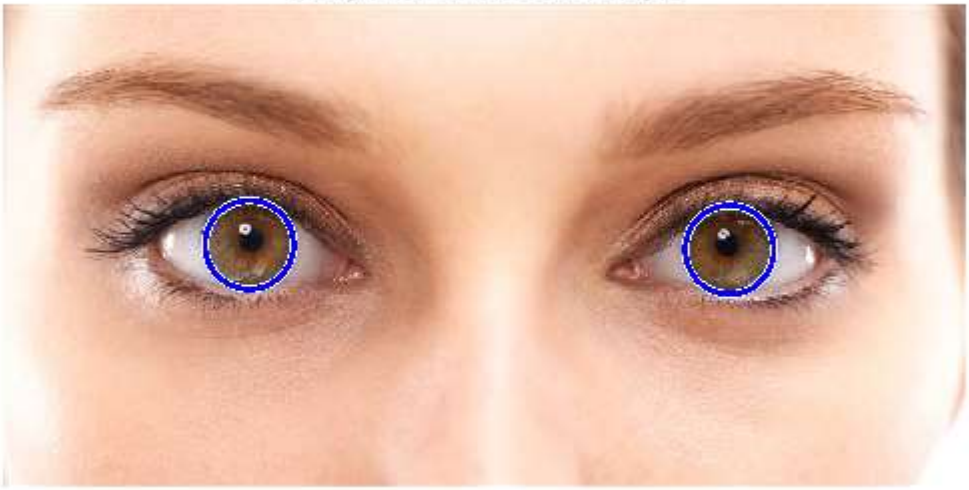
Face Close Up Results:



HoughSpace With Peaks



Original With Overlays



Computationally Efficient Ellipse Fitting using HT Questions:

Why is the Hough space for ellipse fitting 5 dimensional?

A ellipse needs 5 parameters to define it those being two parameters for the center point one parameter for the orientation of the ellipse and two more parameters for the major and minor axes of the ellipse.

In your own words, describe the central premise of the paper. Refrain from copying text in the paper.

The central premise of the paper is given three points all five parameters of the ellipse can be determined. We can determine 4 of the 5 parameters straight away using the first two points. The 5^{th} parameter can be determined through voting and using a 3^{rd} point which can be narrowed down by comparing the length it forms to the midpoint of first two points and the length of the assumed major axis. If we follow their algorithm, then we can skip calculating all possible ellipses from each point and only calculate and cast votes on points which the length from the mid point to the 3^{rd} point is shorter then the major axis.

The authors make specific assumptions that permit reduction of the dimensionality of the Hough space. Enumerate these assumptions?

They first assume each pair of pixels are vertices on the major axis of a ellipse and they ignore the assumption that they could be vertices on the minor axis to save computation cost. Using those two points the center two parameters can be calculated the orientation parameter can be calculated and the major axis half length can be calculated which eliminates 4 of the 5 parameters leaving the 5^{th} parameter being length of the minor axis. The length of the minor axis can be calculated given a 3^{rd} point.