Task 2

given:

$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m}\left[-y^i \log(h_\theta(x^i)) - (1-y^i)\log(1-h_\theta(x^i))\right]$$

$$h_\theta(x) = g(\theta^T x)$$

$$g(z) = \frac{1}{1+e^{-z}}$$

to show:

$$\frac{\partial}{\partial\theta_j} J(\theta) = \frac{1}{m}\sum_{i=1}^{m}\left[(h_\theta(x^i) - y^i)\, x_j^i\right]$$

supportiv definitions:

$$\theta^T x = \theta_0 x_0 + \theta_1 x_1 + \dots + \theta_n x_n$$

$$h_\theta(x) = g(\theta^T x) = g(z) = \frac{1}{1+e^{-z}} = g \mid z = \theta_0 x_0 + \theta_1 x_1 + \dots + \theta_n x_n$$

$$\cancel{L(x,y,\theta) = -y \log(g(z))}$$

$$L(x,y,\theta) = -y\log(g) - (1-y)\log(1-g)$$

$$\Rightarrow J(\theta) = \frac{1}{m}\sum_{i=1}^{m}\left[-y^i \log(h_\theta(x^i)) - (1-y^i)\log(1-h_\theta(x^i))\right]$$

$$= \frac{1}{m}\sum_{i=1}^{m}\left[-y^i \log(g) - (1-y^i)\log(1-g)\right] \mid x = x^i$$

$$= \frac{1}{m}\sum_{i=1}^{m} L(x^i, y^i, \theta)$$

derivate:

$$\frac{\partial}{\partial \theta_j} J(\theta) = \frac{\partial}{\partial \theta_j}\left(\frac{1}{m}\sum_{i=1}^{m} L(x^i, y^i, \theta)\right)$$

$$= \frac{1}{m}\sum_{i=1}^{m}\left[\frac{\partial}{\partial \theta_j}\left(L(x^i, y^i, \theta)\right)\right] \quad \text{sum rule}$$

$$\frac{\partial}{\partial \theta_j} L(x, y, \theta) = \frac{\partial L}{\partial \theta_j}$$

chain rule: $\frac{d}{dx}\left[f(g(x))\right] = f'(g(x))\, g'(x)$

$$\Rightarrow \quad \frac{\partial L}{\partial \theta_j} = \frac{\partial L}{\partial g} \frac{\partial g}{\partial z} \frac{\partial z}{\partial \theta_j}$$

$\frac{\partial L}{\partial g}$:

$$\frac{\partial L}{\partial g} = \frac{\partial}{\partial g}\left(-y \log(g) - (1-y)\log(1-g)\right)$$

$$= \frac{\partial}{\partial g}\left(-y \log(g)\right) - \frac{\partial}{\partial g}\left((1-y)\log(1-g)\right)$$

$$\frac{\partial}{\partial g}\left(\log(1-g)\right) = -\frac{1}{1-g}$$

$$= -\frac{y}{g} + \frac{1-y}{1-g}$$

$\frac{\partial g}{\partial z}$:

$$\frac{\partial g}{\partial z} = \frac{\partial}{\partial z}\left(\frac{1}{1+e^{-z}}\right)$$

$$= \frac{-(-e^{-z})}{(1+e^{-z})^2}$$

$$= \frac{e^{-z}}{(1+e^{-z})^2}$$

$$= \frac{1+e^{-z}-1}{(1+e^{-z})^2} \qquad | +1 -1$$

$$= \frac{1+e^{-z}}{(1+e^{-z})^2} - \frac{1}{(1+e^{-z})^2}$$

$$= \frac{1}{1+e^{-z}} - \frac{1}{(1+e^{-z})^2}$$

$$= \frac{1}{1+e^{-z}}\left(1 - \frac{1}{1+e^{-z}}\right)$$

$$= \underline{g(1-g)} \qquad | \; g = \frac{1}{1+e^{-z}}$$

$\frac{\partial z}{\partial \theta_j}$

$$\frac{\partial z}{\partial \theta_j} = \frac{\partial}{\partial \theta_j}\left(\theta_0 x_0 + \theta_1 x_1 + \dots + \theta_n x_n\right)$$

$$= \underline{x_j}$$ all others are ~~constants~~ constants

$\frac{\partial L}{\partial \theta_j}$:

$$\frac{\partial L}{\partial \theta_j} = \frac{\partial L}{\partial g} \frac{\partial g}{\partial z} \frac{\partial z}{\partial \theta_j}$$

$$= \left(-\frac{y}{g} + \frac{1-y}{1-g}\right) g(1-g) \; x_j$$

$$= \left(-\frac{y g(1-g)}{g} + \frac{(1-y) g (1-g)}{1-g}\right) x_j$$

$$= (-y(1-g) + (1-y)g) \; x_j$$

$$= (-y + yg + g - yg) \; x_j$$

$$\underline{= (g-y) \; x_j}$$

$\frac{\partial J}{\partial \theta_j}$:

$$\frac{\partial J}{\partial \theta_j} = \frac{1}{m} \sum_{i=1}^{m} \left[\frac{\partial}{\partial \theta_j} \left(L(x^i, y^i, \theta)\right)\right]$$

$$= \frac{1}{m} \sum_{i=1}^{m} \left[(g - y^i) \; x_j^i \right]$$

$$\underline{\underline{= \frac{1}{m} \sum_{i=1}^{m} \left[(h_\theta(x^i) - y^i) \; x_j^i \right]}}$$

$[\; g = h_\theta(x)$
see supportiv definitions