

$$1) a) \sigma(a) = \frac{1}{1 + e^{-a}}$$

$$\sigma(-a) = 1 - \sigma(a)$$

$$\Leftrightarrow \frac{1}{1 + e^a} = 1 - \frac{1}{1 + e^{-a}} \quad | \cdot 1 + e^a$$

$$\Leftrightarrow 1 = 1 + e^a - \frac{1 + e^a}{1 + e^{-a}}$$

$$\Leftrightarrow 0 = e^a - \frac{1 + e^a}{1 + e^{-a}} \quad | \cdot 1 + e^{-a}$$

$$\Leftrightarrow 0 = e^a + e^{a-a} - 1 + e^a$$

$$b) \frac{d}{da} \sigma(a) = (1 - \sigma(a)) \sigma(a)$$

Kettenregel:  $(U \circ V)'(x_0) = U'(V(x_0)) \cdot V'(x_0)$

$$u(x) = \frac{x}{1+x} \quad v(a) = e^a$$

↓ Quotientenregel

$$u'(x) = \frac{1}{x+1} - \frac{x}{(x+1)^2} \quad v'(a) = e^a$$

$$\frac{d}{da} \cdot \frac{1}{1 + e^{-a}} = \frac{d}{da} \cdot \frac{e^a}{1 + e^a}$$

$$= \frac{e^a}{1 + e^a} \cdot \frac{1}{e^{a+1}}$$

$$= \frac{1}{1 + e^{-a}} \cdot \left( \frac{e^a + 1}{e^{a+1}} - \frac{e^a}{e^{a+1}} \right)$$

$$= \sigma(a) \cdot (1 - \sigma(a))$$

$$c) \sigma^{-1}(b) = \log \frac{b}{1-b} \quad (\Rightarrow) \log \frac{\sigma(a)}{1-\sigma(a)}$$

$$\log \frac{\frac{1}{1+e^{-a}}}{1 - \frac{1}{1+e^{-a}}} = \log \frac{1}{1+e^{-a}-1}$$

$$= \log \frac{1}{e^{-a}} = \log \frac{e^a}{1} = 1$$

2)