DSLs in finance, an overview

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Outline

Introduction

Contract languages

Certified symbolic management cite:Bahr:2015hm

- Contracts are expressed in relative times
- Can describe stipulation between multiple parties (& operator)
- Can describe *observable external decisions*:

$$\text{if } \underbrace{\text{Obs(X exercises option}, 0)}_{\text{eval'd at the beginning of each day}} \text{ in } \overbrace{90}^{\text{days' range}} \text{ then } 100 * \underbrace{\text{DKK}(Y \to X)}_{\text{atomic exchange contract}} \text{ else } 0$$

Findel cite:Biryukov:2017ip

- Absolute times;
- Syntax similar to others but no intuitive reference to parties involved as contracts change over time:

$$\underbrace{Give(10 * One(USD))}_{\text{Bob pays 10 now}} \wedge \overbrace{At(now + 1\ years, 11 * One(USD))}^{\text{Bob expects to receive 11 in a year}}$$

Semantics

Certified symbolic management cite:Bahr:2015hm

Subdivides semantics into contract and expression (denotational) semantics.

Contract semantics maps expressions into a cash-flow trace:

$$C : [\![\Gamma]\!] \to \overbrace{\mathbb{N} \to \underbrace{Party \times Party \times Asset \to \mathbb{R}}_{\text{transaction}}}^{\text{trace}}$$

for example (note the delay $\uparrow$ and *unit transfer* $\to$ operators)[1]:

$$\begin{aligned}
\mathcal{C}[\![0]\!] &= \lambda n.\lambda t.0 \\
\mathcal{C}[\![c1\&c2]\!] &= \mathcal{C}[\![c1]\!] + \mathcal{C}[\![c2]\!] \\
\mathcal{C}[\![d \uparrow c]\!] &= \lambda n.\mathcal{C}[\![c]\!](n-d) \\
\mathcal{C}[\![a(p_1 \to p_2)]\!] &= \lambda n.\lambda t.\delta_{0,(p_1,p_2,a)}(n,t) - \delta_{0,(p_2,p_1,a)}(n,t)
\end{aligned} \tag{1}$$

[1] δ = Kronecker's delta

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Contract transforms consist in specialisation and advancement, i.e., instantiation of a contract to a concrete starting time or simplification. Consider the following contract:

$$\mathrm{DKK}(Y \to Z)\ \&\ \text{if Obs(X defaults}, 0)\ \text{in } 30\ \text{then DKK}(Z \to Y)\ \text{else } 0$$

and assume that

$$\mathit{default}(X, i) = \top\ \mathit{if}\ i = 15,\ \perp\ \mathit{otherwise}$$

Then, at time $i = 16$, the contract can be transformed into:

$$\mathrm{DKK}(Y \to Z)\ \&\ \mathrm{DKK}(Z \to Y) \sim 0$$

Software verification and certified software

Type systems

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- Problem: Simple expressions could involve non-causality, e.g.:

$$\underbrace{\mathbf{obs}(FX(USD, DKK), 1)}_{\text{tomorrow's observation}} \times \underbrace{DKK(X \to Y)}_{\text{pay today}}$$

- Solution: time-indexed types;

Examples of typing rules using time-indexed types:

- an observation at time t is available at all times t' after t:

$$\frac{t \leq t'}{\Gamma \vdash \mathbf{Obs}(l, t) : \tau^{t'}}$$

- an expression e can only meaningfully scale a contract c if e is available at some time t' and c makes no stipulations strictly before t':

$$\frac{\Gamma \vdash e : \mathit{Real}^{t'} \qquad \Gamma \vdash c : \mathit{Contr}^{t'} \qquad t \leq t'}{\Gamma \vdash e \times c : \mathit{Contr}^{t}}$$

Temporal logic cite:Davies:2017gk

- Temporal logic is an extension of logic to include proofs that formulas are valid at particular times.
- Inference rules are time-indexed and there is a new operator $\bigcirc A$ which stands for the value of A at the "next" (discrete) time:

$$\frac{\Gamma \vdash^{(n)} A \to B \qquad \Gamma \vdash^{(n)} A}{\Gamma \vdash^{(n)} B}$$

or,

$$\frac{\Gamma \vdash^{(n)} \bigcirc A}{\Gamma \vdash^{(n+1)} A}$$

References

Papers

bibliographystyle:unsrt bibliography:biblio.bib