CS 40 Homework 2

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1 Decode

a. Lisa likes nothing.

b. All meats are edible things, and all edible things may be meats.

c. Lisa likes everything that are in both poisons and yogurts.

d. There are non-edible yogurts.

e. Lisa likes edible things that are not meats or poisons.

f. Meats and yogurts share no items in common.

2 Encode

The following are sets.

- L = Living.
- F = Flying.
- C = Crawling.
- A = Airplanes.
- B = Bees.
- D = Dirigibles.

a. $L \cap F \neq \emptyset$

b. $F \setminus L \neq \emptyset$

c. $C \subseteq L$

d. $A \subseteq (F \setminus L)$

e. $B = F \cap C \cap L$

f. $D \cap (C \cup L) = \emptyset$

3 Proofreading

a. (1) is invalid. The conclusion assumes that the inverse of premise (1) is true, which isn't a guarantee. The inverse would translate to **if it doesn't have four legs, then it's not a mammal**. However, we can't prove this in any manner; therefore, this argument is not sound.

b. The argument doesn't provide any information about Bob playing tennis on Sundays. Given that it's a sunny weekend where Bob is playing tennis, and we know that this is true on Saturdays, nothing stops the day from being a Sunday. Bob's tendency to play tennis on Sunday is unknown; thus we can't explicitly conclude what day of the weekend it is.

c. The omission of the universal quantifier makes this an unsound argument. Using a value of 13 is just one specific case of the argument, but there exist other cases which the argument must also include.

4 Addition

Assume for the sake of contradiction that x $\in \mathbb{Q}^+$, y $\in (\mathbb{R} \setminus \mathbb{Q})^+$, and z $\in \mathbb{Z}^+$

z = x+y

$\frac{a}{b}$ - $\frac{c}{d} = \frac{ad}{b}$ - $\frac{bc}{d} = \frac{ad-bc}{bd}$

Therefore, by the definition of a rational number, x can be expressed as $\frac{c}{d}$, where c $\in \mathbb{Z}$ and d $\in \mathbb{Z}$. z, an integer, can also be expressed as $\frac{a}{b}$, where a $\in \mathbb{Z}$ and b $\in \mathbb{Z}$.

The difference of z and x, respectively, results in an expression of y of $\frac{ad-bc}{bd}$. It is known that *a*, *b*, *c*, and *d* are all integers; therefore, *ad*-*bc* as well as *bd* are both integers. This implies that the difference is a positive rational number.

However, this contradicts the requirement that y is a positive *irrational* number.

This means our initial assumption of x being rational is false. Therefore, by proof of contradiction, the sum of two positive irrational numbers is an integer.

5 Division

Assume for the sake of contradiction that x $\in \mathbb{Q}$ and y $\in \mathbb{R} \setminus \mathbb{Q}$ and that $\frac{x}{y}$ is a rational number; thus, $\frac{rational}{irrational} = rational$.

Rearranging the terms, it is clear that $\frac{rational}{rational} = irrational$. Since x is rational, it can be expressed as $\frac{a}{b}$ where a $\in \mathbb{Z}$ and b $\in \mathbb{Z}$ where b $\neq 0$. We can define another rational number z as $\frac{c}{d}$ where c $\in \mathbb{Z}$ and d $\in \mathbb{Z}$. Dividing the two yields $\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{ad}{bc}$. It is known that a, b, c, and d are all integers, so ad and bc are integers too. Thus, $\frac{ad}{bc}$ is rational, so a rational divided by a rational is rational.

However, this contradicts the statement above that $\frac{rational}{irrational} = rational$.

This means our initial assumption of $\frac{rational}{irrational} = rational$ is false. Therefore, by proof of contradiction, a rational number divided by an irrational number is irrational.

6 Multiplication

Given S is a set of integers, with a size of at least 3, there exists a non empty subset of S, S', such that the product of all elements in the subset is a positive integer. We can conduct a proof by cases with 3 cases.

Case (i): Assuming S contains all positive integers, we can take a subset of S of any size. The product of all positive integers is a positive integer itself and S' is non-empty, so the conclusion holds.

Case (ii): Assuming S contains all negative numbers, we can take a subset of S such that its cardinality is even. The product of an even number of negative integers is a positive integer and S' is non-empty, so the proof holds.

Case (iii): Assuming S contains a mix of positive and negative integers, we can take a subset of S such that there are an even number of negative integers. From Case (ii), we know that this product will be positive. Multiplying that result with more positive integers yields another positive integer and S' is non-empty, so the conclusion holds.

Case (iv): Assuming S contains the element 0 and has a cardinality of at least 3, we can take a subset of S according to the previous cases such that S' doesn't include 0 and S' is non-empty.

Therefore, by proof by cases, there exists a set S', a subset of S, such that the product of its elements is a positive integer and S' is non-empty.