CS 40 Homework 1

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1 Encode

a. P = Amy is a banker, $\neg P$

b. P = Bob speaks French, Q = Bob speaks Spanish, $\neg(P \vee Q)$

c. P = Carrie likes Derek, Q = Derek likes Carrie, P $\wedge$ Q

d. P = Erika will buy a boat, Q = Erika will buy a plane, P $\oplus$ Q

e. P = George will come to dinner, Q = Fred will come to dinner, P $\rightarrow$ Q

f. P = Hailey will go to Macau, Q = Isabel will go to Macau, P $\leftrightarrow$ Q

2 Decode

a. A particle was not detected

b. Either the cat is dead or the poison was not released

c. If the particle was detected, then the poison was released

d. If the cat is not dead, then the poison was not released

e. The poison was not released if and only if a particle was not detected

f. A particle was detected and a particle was not detected (contradiction)

3 Truth Tables

a.

p	q	$p \rightarrow q$	$\neg q$	$\neg q \wedge p$	$(p \rightarrow q) \leftrightarrow (\neg q \wedge p)$
T	T	T	F	F	F
T	F	F	T	T	F
F	T	T	F	F	F
F	F	T	T	F	F

Contradiction, each entry of the expression column is False

b.

p	q	$p \to q$	$\neg q$	$\neg p$	$\neg q \to \neg p$	$(p \to q) \leftrightarrow (\neg q \to \neg p)$
T	T	T	F	F	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

Tautology, each entry of the expression column is True

c.

p	q	$p \to q$	$q \to p$	$(p \to q) \to (q \to p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

Contingency, mix of True and False values

d.

P	Q	$\neg Q$	$P \vee Q$
T	T	F	T
T	F	T	T
F	T	F	T
F	F	T	F

Valid deduction, there exists only one row where ¬Q, P ∨ Q, and P are true

e.

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

Valid deduction, there exists only one row where P, Q, and P → Q are true

f.

P	Q	$P \to Q$	$\neg P$	$\neg Q$
T	T	T	F	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

Valid deduction, there exists only one row where P → Q, ¬P, and ¬Q are true

4 Operators

a. $\neg R = \neg(R \vee R) = R \downarrow R$

b. $L \wedge R = \neg(\neg L \vee \neg R) = (L \downarrow L) \downarrow (R \downarrow R)$

c. $L \vee R = \neg(\neg L \wedge \neg R) = \neg(\neg(L \vee R)) = \neg\ (L \downarrow R) = (L \downarrow R) \downarrow (L \downarrow R)$

d. $L \oplus R = \neg(L \leftrightarrow R) = \neg((L \wedge R) \vee (L \downarrow R)) = \neg((L \downarrow L) \downarrow (R \downarrow R) \vee (L \downarrow R)) = (L \downarrow L) \downarrow (R \downarrow R) \downarrow (L \downarrow R)$

e. $L \rightarrow R = R \vee \neg\ L = (L \downarrow L) \vee R = ((L \downarrow L) \downarrow R) \downarrow ((L \downarrow L) \downarrow R)$

f. $L \leftrightarrow R = (L \rightarrow R) \wedge (R \rightarrow L) = (((L \downarrow L) \downarrow R) \downarrow ((L \downarrow L) \downarrow R) \downarrow ((L \downarrow L) \downarrow R) \downarrow ((L \downarrow L) \downarrow R)) \downarrow (((R \downarrow R) \downarrow L) \downarrow ((R \downarrow R) \downarrow L) \downarrow ((R \downarrow R) \downarrow L) \downarrow ((R \downarrow R) \downarrow L))$

5 Limitations

P = I have at least five dollars
Q = I can buy cake
R = I can buy beer
S = I have exactly five dollars

$P \rightarrow Q$
$P \rightarrow R$
$S \rightarrow P$
S

Since S is True, through deduction, P must be True to satisfy the premise that $S \rightarrow P$ is True. This implies that both atomic propositions Q and R are True too.

Thus, we can construct the argument that $\mathbf{(S \rightarrow P) \rightarrow ((P \rightarrow Q) \wedge (P \rightarrow R))}$

Translating this back into English, we get that **If I have exactly five dollars, then I have at least five dollars, then I can buy cake and I can buy beer.**

However, this is a contradiction since I can only buy either cake or beer, but not both since I only have five dollars. Therefore, this is an unrealistic conclusion.