CS 40 Homework 3

Viraj Zaveri

Tuesday, February 14th

1 Inclusion

a. G: A
H: A, C, and G
I: A, B, C, D, and G

b. G: A
H: A, C, F, and G
I: A, C, E, F, G, and H

2 Counting

$(25)^3 * 2^{2^{25^2}}$

Given $P^3 \times \wp(\wp(P \times P))$, we can work on each side of the final Cartesian Product by starting with P^3. We're given that the cardinality of P, $|P|$, is 25.

The cardinality of the Cartesian Product of two sets is equivalent to the product of the cardinality of the individual sets. Thus, P^3 is $(25)^3$.

The $\wp$(N) will yield $2^{|N|}$ elements; thus, $\wp$(P $\times$ P) results in 2^{25^2} elements.

The final Cartesian Product allows us to simply multiply the cardinalities of the left and right sets, which therefore yields $(25)^3 * 2^{2^{25^2}}$.

3 Powersets

a. Goal: Prove $\forall$A, B : $\wp(A) \cap \wp(B) = \wp(A \cap B)$

Method: Direct Proof

1. By definition, for some arbitrary sets S and X, if $X \subseteq S \leftrightarrow X \in \wp(S)$

2. For all $X \in \wp(A \cap B)$, then $X \subseteq (A \cap B)$

3. If $X \subseteq (A \cap B)$, then $X \subseteq A$ and $X \subseteq B$

4. Since $X \subseteq A$, X is therefore part of $\wp(A)$. The same case applies for set B.

5. Therefore, $X \in \wp(A)$ and $X \in \wp(B)$

6. Hence, $X \in \wp(A) \cap \wp(B)$

7. Using our initial definition, X must be a subset of $\wp(A \cap B)$

Conclusion: Thus, $\wp(A) \cap \wp(B) = \wp(A \cap B)$

b. Suppose we have sets $A = \{a, b\}$ and $B = \{x, y\}$
$\wp(A) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \}$
$\wp(B) = \{\emptyset, \{x\}, \{y\}, \{x, y\}\}$
$\wp(A) \cup \wp(B) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{x\}, \{y\}, \{x, y\}\}$
$A \cup B = \{a, b, x, y\}$
$\wp(A \cup B) = \{\emptyset, \{a\}, \{b\}, \{x\}, \{y\}, \{a, b\}, \{a, x\}, \{a, y\}, \{b, x\}, \{b, y\}, \{x, y\},$
$\{a, b, x\}, \{a, b, y\}, \{a, x, y\}, \{b, x, y\}, \{a, b, x, y\}\}$

It is clear from the power sets above that the two aren't equal. Specifically, we can see that the cardinality of $\wp(A \cup B)$ is significantly greater. The two sets aren't equal; however, $\wp(A) \cup \wp(B)$ is a proper subset of $\wp(A \cup B)$.

4 Tuples

a. Person and their corresponding displacement, in feet, relative to where they woke up that day, rounded to the nearest integer.

b. City and its corresponding **average** flux of residents within the last year.

c. A person and someone they know.

d. How long a person has lived in that corresponding city for, rounded to the nearest integer.

e. Two cities and the number of people who moved from the first to the second city.

f. City and its corresponding **average** flux of residents within the last year and its elevation relative to sea level in feet.

5 Relations

a. $\wedge$: $\{(T, T)\}$
$\vee$: $\{(T, T), (T, F), (F, T)\}$
$\oplus$: $\{(T, F), (F, T)\}$
$\rightarrow$: $\{(T, T), (F, T), (F, F)\}$
$\leftrightarrow$: $\{(T, T), (F, F)\}$

b. $\wedge$: Symmetric, Transitive, and Antisymmetric
$\vee$: Symmetric
$\oplus$: Irreflexive, Symmetric
$\rightarrow$: Reflexive, Antisymmetric, and Transitive
$\leftrightarrow$: Reflexive, Symmetric, Antisymmetric, and Transitive

6 Equivalence

By definition, a relation is an equivalence relation if it satisfies the properties of being reflexive, symmetric, and transitive. Suppose we have two real numbers, a, b $\in \mathbb{N}$ such that a $\approx$ b. If two numbers are equal, they are also approximately equal. a $\approx$ a and b $\approx$ b; hence the reflexive property is satisfied. Since this is true, we can also say that b $\approx$ a. Therefore, the symmetric property for this relation holds. However, the transitive property doesn't hold because we ultimately compare numbers of significantly different magnitudes. For example, if we consider the gravitational constant g $\approx$ 10 and 10 $\approx$ 11, we imply that g $\approx$ 11. Furthermore, if 11 $\approx$ 12, then g $\approx$ 12. There is no quantitative definition of what it means to be approximately equal, so this chain of comparing numbers is indefinite. Thus, the transitive property doesn't apply and the relation can't be equivalent.

7 Cycles

Goal: Prove that if $\succ$ is a strict partial order, it cannot contain a cycle.

Method: Proof by contradiction

1. Assume for the sake of contradiction that $\succ$ has a cycle.
2. Although the original cycle is longer, we can repeatedly apply the transitive property to reduce the size of the cycle. Therefore, by the definition of a cycle, WLOG $\exists a, b, c(a \succ b \succ c \succ a)$
3. Since $\succ$ is a strict partial order, it satisfies the properties of being asymmetric and transitive.
4. Since a $\succ$ b and b $\succ$ c, then a $\succ$ c through transitivity.

5. It is clear from the end of the cycle that c $\succ$ a, so we have the conclusions that a $\succ$ c and c $\succ$ a.
6. This means that nodes a and c are not asymmetric since there exists a bidirectional relation between the two: a is related to c and c is related to a.
7. This contradicts our premise that $\succ$ is a strict partial order since $\succ$ is transitive but not asymmetric.
8. Therefore our initial assumption of $\succ$ having a cycle is false.

Conclusion: Therefore, by proof by contradiction, if $\succ$ is a strict partial order, it cannot contain a cycle.