CS 40 Homework 6

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Wednesday, March 15th

1 Dice I

a. 0

b. $\frac{1}{12}$

c. $\frac{3(3!)+3(\frac{3!}{2!})}{216} = \frac{27}{216} = \frac{1}{8}$

2 Dice II

a.
- Die X: $\frac{2}{6}(1) + \frac{1}{6}(3) + \frac{2}{6}(5) + \frac{1}{6}(6) = 3.5$
- Die Y: $\frac{1}{6}(2) + \frac{2}{6}(3) + \frac{2}{6}(4) + \frac{1}{6}(5) = 3.5$
- Die Z: $\frac{1}{6}(1) + \frac{2}{6}(2) + \frac{1}{6}(4) + \frac{2}{6}(6) = 3.5$

b.
- X and Y:
  - Win: $\frac{2}{6}(0) + \frac{1}{6}(\frac{1}{6}) + \frac{2}{6}(\frac{5}{6}) + \frac{1}{6}(\frac{6}{6}) = \frac{17}{36}$
  - Tie: $\frac{2}{6}(0) + \frac{1}{6}(\frac{2}{6}) + \frac{2}{6}(\frac{1}{6}) + \frac{1}{6}(0) = \frac{4}{36}$
  - Lose: $\frac{15}{36}$
- X and Z:
  - Win: $\frac{2}{6}(0) + \frac{1}{6}(\frac{3}{6}) + \frac{2}{6}(\frac{4}{6}) + \frac{1}{6}(\frac{4}{6}) = \frac{15}{36}$
  - Tie: $\frac{2}{6}(\frac{1}{6}) + \frac{1}{6}(0) + \frac{2}{6}(0) + \frac{1}{6}(\frac{2}{6}) = \frac{4}{36}$
  - Lose: $\frac{17}{36}$
- Y and Z:
  - Win: $\frac{1}{6}(\frac{1}{6}) + \frac{2}{6}(\frac{3}{6}) + \frac{2}{6}(\frac{3}{6}) + \frac{1}{6}(\frac{4}{5}) = \frac{17}{36}$
  - Tie: $\frac{1}{6}(\frac{2}{6}) + \frac{2}{6}(\frac{0}{6}) + \frac{2}{6}(\frac{1}{6}) + \frac{1}{6}(\frac{0}{6}) = \frac{4}{36}$
  - Lose: $\frac{15}{36}$

c. Alice can give herself an advantage by letting Bob choose his die first then accordingly choosing the die with the highest winning percentage against Bob's.

3 Coins I

a. $(0.55)^7 = 0.015$

b. $1 - (0.45)^7 = 0.996$

c. $(0.55^3)(0.45^4)(\frac{7!}{3!*4!}) = 0.239$

d. $(0.55^4)(0.45^3)\binom{7}{3} + (0.55^5)(0.45^2)\binom{7}{2} + (0.55^6)(0.45)\binom{7}{6} + 0.55^7 = 0.608$

4 Coins II

We can analyze the winnings for each game by cases:

T $\rightarrow$ 0
HT $\rightarrow$ 1
HHT $\rightarrow$ 2
HHHT $\rightarrow$ 3

The sequence will continue infinitely, but we can then take the weighted sum of each by multiplying the probability of that sequence with the winnings to find the expected value. Therefore we can define a random variable X to be the amount of points won for each game. $E(X)$, the expected value of X, then becomes

$$X = \frac{1}{2}(0) + \frac{1}{4}(1) + \frac{1}{8}(2) + \frac{1}{16}(3) + \frac{1}{32}(4) + ... + \frac{1}{2^n}(n-1)$$

$$\frac{X}{2} = \frac{0}{4} + \frac{1}{8} + \frac{2}{16} + \frac{3}{32} + ...$$

$$X - \frac{X}{2} = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + ...$$

$$\frac{X}{2} = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + ...$$

$$X = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + ...$$

$$X = \frac{1}{2}\sum_{n=0}^{\infty} \frac{1}{2^n} = 1$$

Thus, the fee that would set the expected value of this game to be zero is the value of the infinite sum above, which is $1.

5 Doors

The calculation of the expected value is incorrect. I can never double the amount of money and win half the money at the same time: I can either keep the current amount of money or lose half of it, or keep the current amount of money or double it. The expected value is the sum of the product of probabilities and their values; however, winning $\frac{n}{2}$ dollars isn't an option. There needs to be some define value for n. If n is the amount of money at the door with the lower value, then the expected value is

$$\frac{1}{2}(n) + \frac{1}{2}(2n) = \frac{3n}{2}$$

6 Werewolf

a. $\frac{(0.02)(0.0001)}{(0.02)(0.0001)+(0.97)(0.9999)} = 2.06(10^{-6})$

b. $\frac{(0.98)(0.0001)}{(0.98)(0.0001)+(0.03)(0.9999)} = 0.003$

c. $\frac{(0.98)^3(0.0001)}{(0.98)^3(0.0001)+(0.03)^3(0.9999)} = 0.777$