# Exam

The exam lasts 1 hour.

- I only accept hand written solutions.
- Take pictures when you are finished and send them to me via email (use Chapman email on both ends).
- **No communication:** You have to work alone.
- You can use everything in the github repo of this course as well as your own notes.

There is a total of 10 marks.

## Question 1 (1 point)

For an ARS (A,->), define the notions of

– confluence

– terminating

## Question 2 (7 points)

Consider the ARS (A,->) defined by the schemas of rules

```
ba -> ab
ab -> ba

aaa ->
bbb ->
```

- Which words are in the equivalence class of the empty word? (1 point)

- How many equivalences classes are there? Find invariants describing the equivalence classes. (2 points)

- Is the ARS confluent? Justify your answer in one sentence. (1 point)

- Which equivalence classes have normal forms and which do not? (1 point)

- Modify the definition of the relation -> so that the ARS becomes terminating without changing the equivalence relation. (0.5 point)

- Revisit the question about normal forms again. (0.5 point)

- Find a measure function that proves termination of your modified ARS. Briefly justify why it is a measure function. (1 point)

## Question 3 (1 point)

Put all parentheses `(` and `)` into the following lambda expression to indicate the correct parsing.

$$\lambda a . \lambda b . \lambda c . a\ b\ \lambda x . x\ y\ c$$

## Question 4 (1 point)

Does the following lambda expression have a normal form?

$$(\lambda x . \lambda y . x)\ (\lambda x . x)\ ((\lambda x . x\ x)(\lambda x . x\ x))$$

If yes, give a reduction to normal form.