$$((A\ B)\ C)\ D$$

$$(\text{add}\ e1)\ e2$$

$$\underbrace{((\lambda x.\lambda y.x+y)\ 2)}_{(\lambda y.2+y)}\ 3$$

$$(\lambda f.\lambda x.f\ f\ x)\ \lambda x.\lambda y.x\,y$$

$$(\lambda f.(\lambda x.(f\ f)\ x))\ (\lambda x\ (\lambda y.x\,y))$$

$$(\lambda f.(\lambda x.f\ (f\ x)))\ (\lambda x.x)$$

- $\lambda x.(\lambda y.\ A\ B)$
- $\lambda x.(\lambda y.\ A)\ B$

two different parse trees

$aaa \to$
$aa \to b$
$ab \to$
$bb \to a$

INVARIANT:
$(\#a + 2 \cdot \#b) \bmod 3$

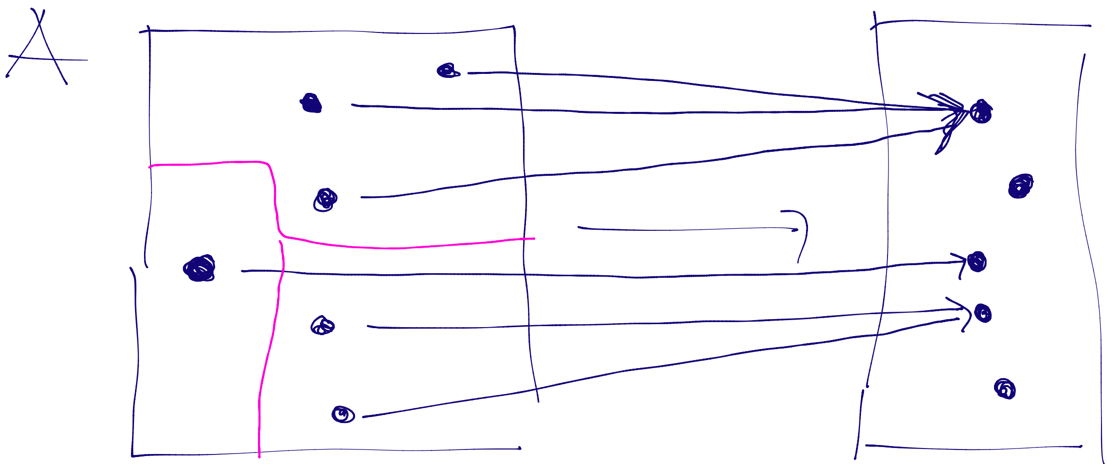
1) $\{w \mid w \overset{*}{\longleftrightarrow} a\}$ = "the set of all words such that $(\#a + 2 \cdot \#b) \bmod 3 = 1$"

2) $\{w \mid w \overset{*}{\longleftrightarrow} []\} = \ldots$

3) $\{w \mid w \overset{*}{\longleftrightarrow} b\} = \ldots$

Discrete Math : Relations

$f : \underset{\text{domain}}{A} \longrightarrow \underset{\text{co-domain}}{B}$



the equivalence class of $a \in A$

is $\{a' \mid f(a) = f(a')\}$

every function partitions its domain into equ. classes

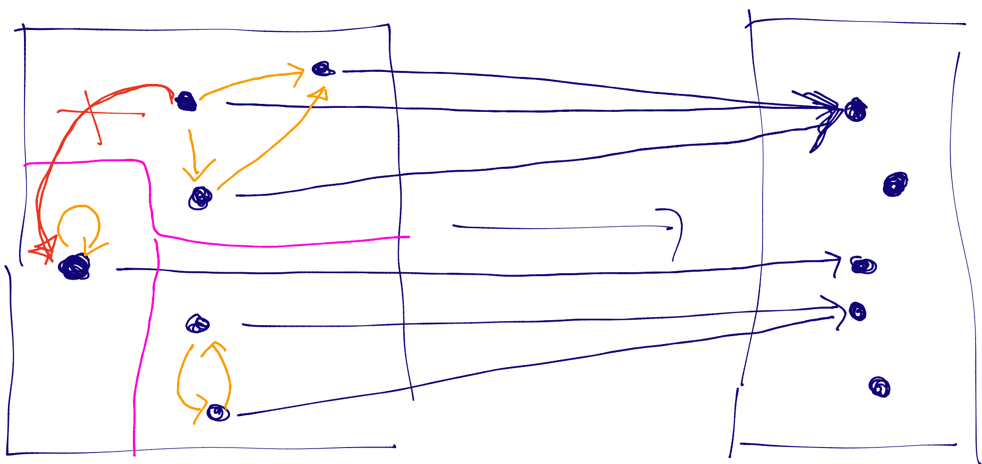
f could be an invariant

$(A, \to)$

f invariant

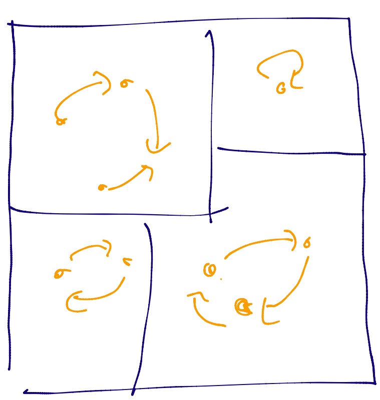
$\to$ $\boxed{\text{does not connect}}$ two different equ classes

A



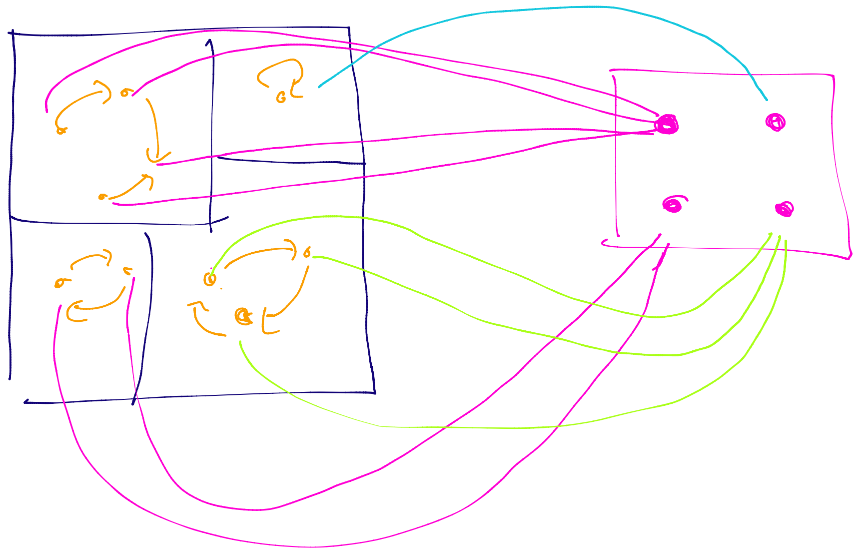
Given $(A, \to)$

the equ. classes are already determined by $\to$



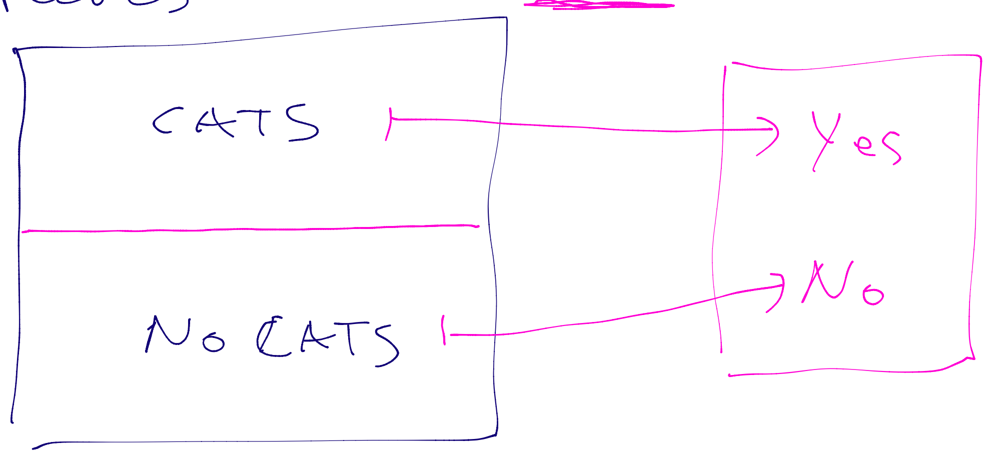
FIND INVARIANT MEANS TO FIND f

such that:



pictures

classifier



- idea
- articulate a guess
- prove that it's an invariant

idea 1

aaa → mod 3

ab →

aa → b

bb → a

mod 3

three times table

3	6	9	12	15	...

sequence of threes

ab → ba	ba → ab

order doesnt matter

aabacb → 4a+2b

abaaba

aaaabb   4a+2b

concatenation   plus

idea 2

ab →

1+2   o

articulate guess

$$(\#a + 2 \cdot \#b) \bmod 3$$

prove

	$(\#a + 2\#b) \bmod 3$ before	after
$aaa \to$	0	0
$ab \to$	0	0
$aa \to b$	2	2
$bb \to a$	1	1

it is invariant because the "before" and "after" columns agree

(SR2) $ab \to ba$ and $ba \to ab$

* $aa \to$
* $ba \to bbaa$

equ. class of [ ] ? $\boxed{\#a \bmod 2 = 0 \text{ and no } b\text{'s}}$

equ. class of a ? $\boxed{\#a \bmod 2 = 1 \text{ and no } b\text{'s}}$

can ignore $ba \to bbaa$ ... why?

what happens if we have b's?

$$b = baa = bbaaa = bba$$
$$= bbbaa = bbb = bbbbb = b^7$$

notation
$$b = b^3 = b^7 = b^9 = \dots$$
$$b = 3b = 7b = 9b = \dots$$

- idea — we are still here
- guess ...
- proof ...