Definitions:

Termination

∃ terminating computation

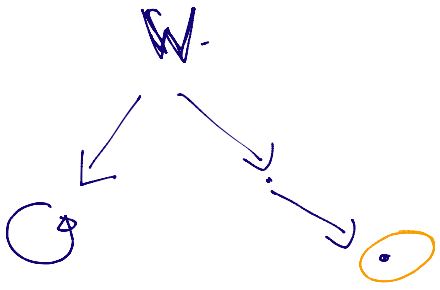
∀ computations terminate

(important in the presence of non-determinism)

(A, →)

An ARS is terminating ("if all computations terminate")

if there is no infinite sequence $a_0 \to a_1 \to \ldots$



w has a n.f.

Q2

$ab \to ba$

$ba \to ab$

$aaa \to$

$bbb \to$

Remark:

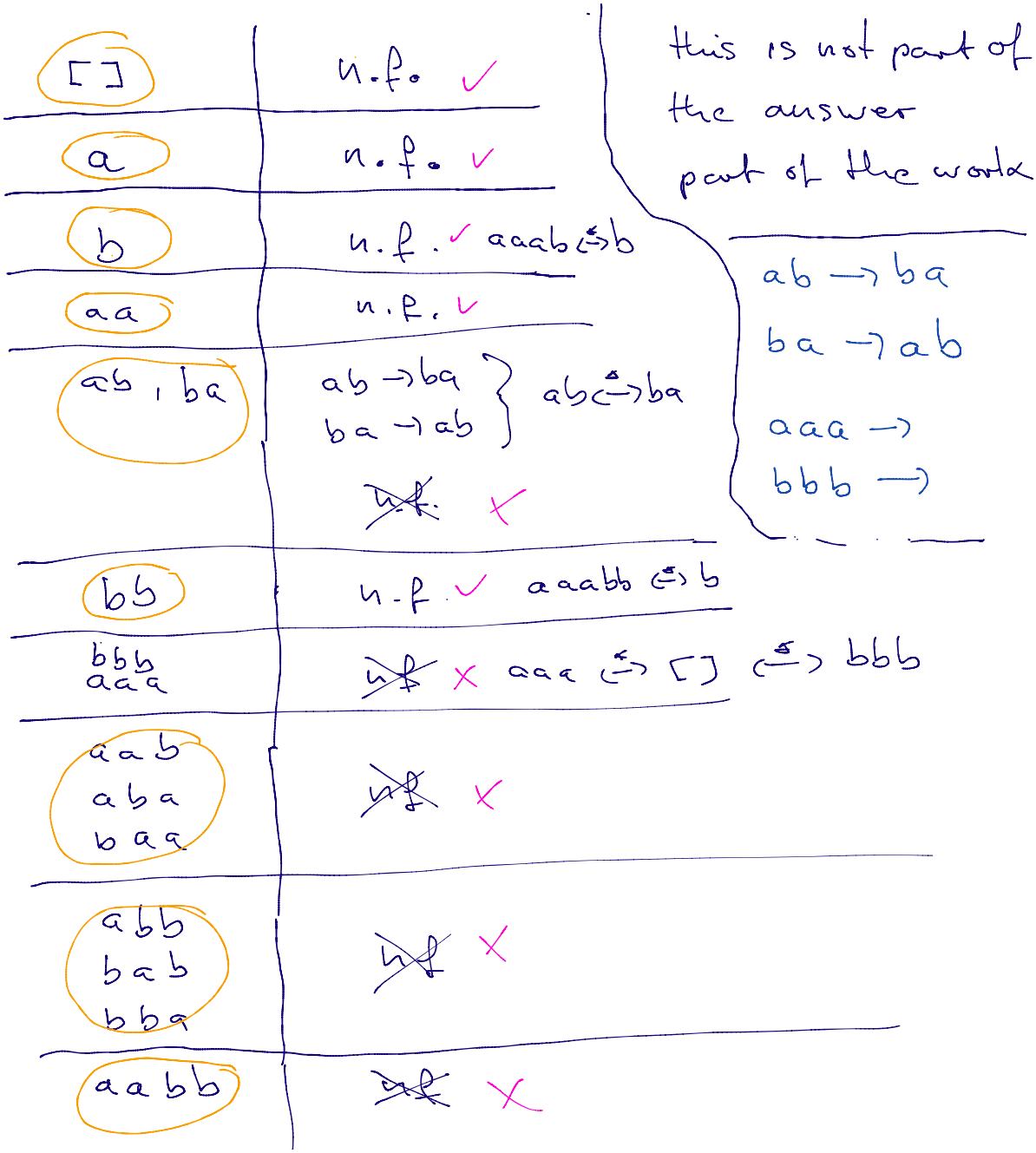
string words lists	bags
ababa	$3a+2b$
↓	=
aabba	$3a+2b$
↓	=
aabab	$3a+2b$
↓	=
aaabb	$3a+2b$
↓	↓
bb	$2b$

$ab \to ba$

$ba \to ab$

Remark: "gather data"

We know that every word w is in one, and only one, equ. class



every word with 5 letters has at least 3 a's or 3 b's, hence it is equivalent to a shorter word

"we collected enough data"

"the table is completed: every eq. class in the table"

Q 2.1

We can eliminate "aaa", "bbb".

$w \xrightarrow{\delta} []$ if, and only if, $\#a \bmod 3 = 0$

#a in w is a multiple of 3

#b ——— " ———

Q 2.2

From the table we guess that it is 9 equ. classes.

From Q 2.1 we guess the invariant involves $\#a \bmod 3$ and $\#b \bmod 3$

We guess that

$$(\#a \bmod 3, \#b \bmod 3)$$

is an invariant.

Verification:

	before	after
$ab \to ba$	$(1,1)$	$(1,1)$
$ba \to ab$	$(1,1)$	$(1,1)$
$bbaaa \to bb$	$(0,\overset{2}{0})$	$(0,\overset{2}{0})$

abbb → a | (0,0) | (0,0)

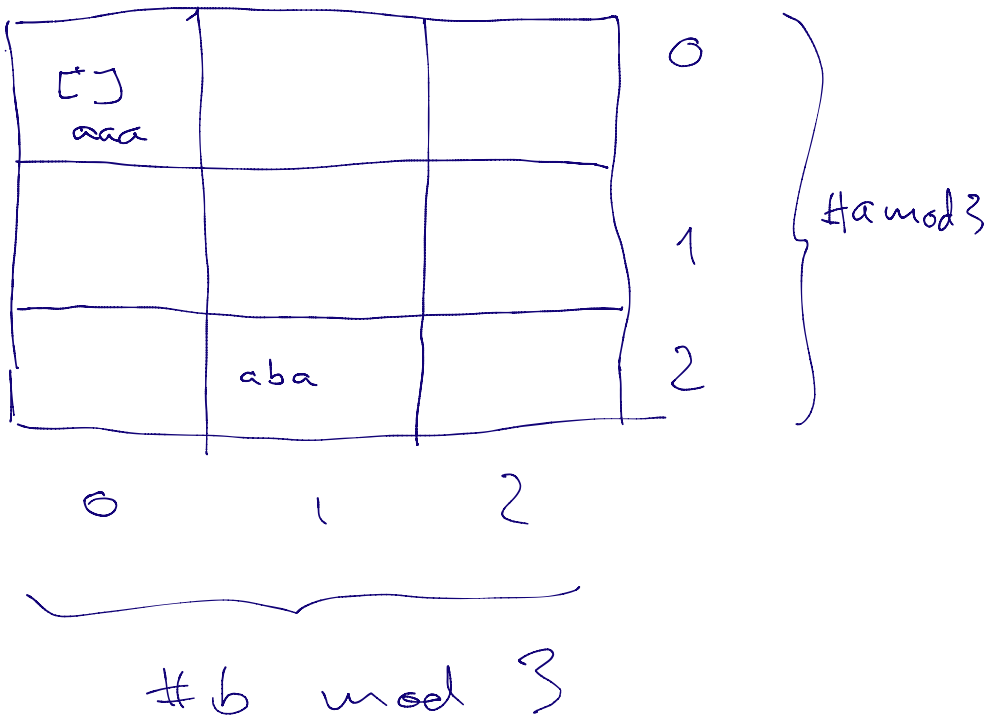
#b mod 3

3 mod 3 = 0

Summary: We showed that

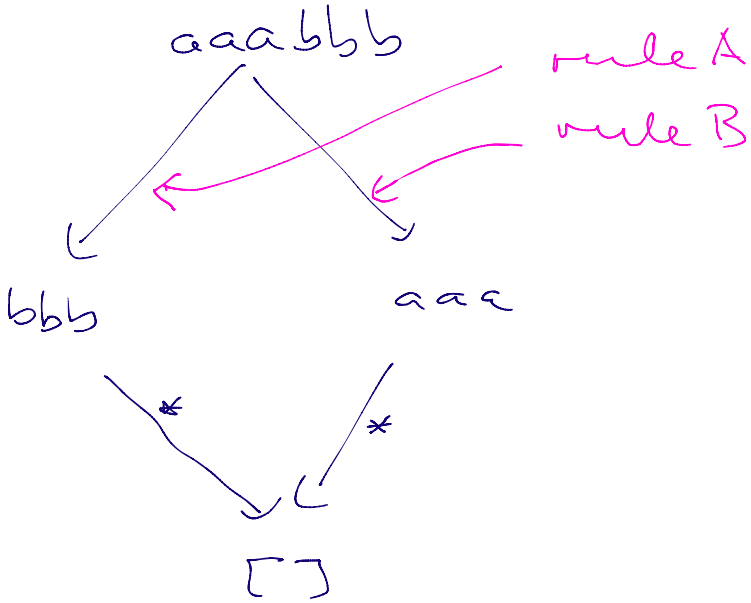
(#a mod 3, #b mod 3)

is an invariant



The invariant guarantees that all 9 classes are different.

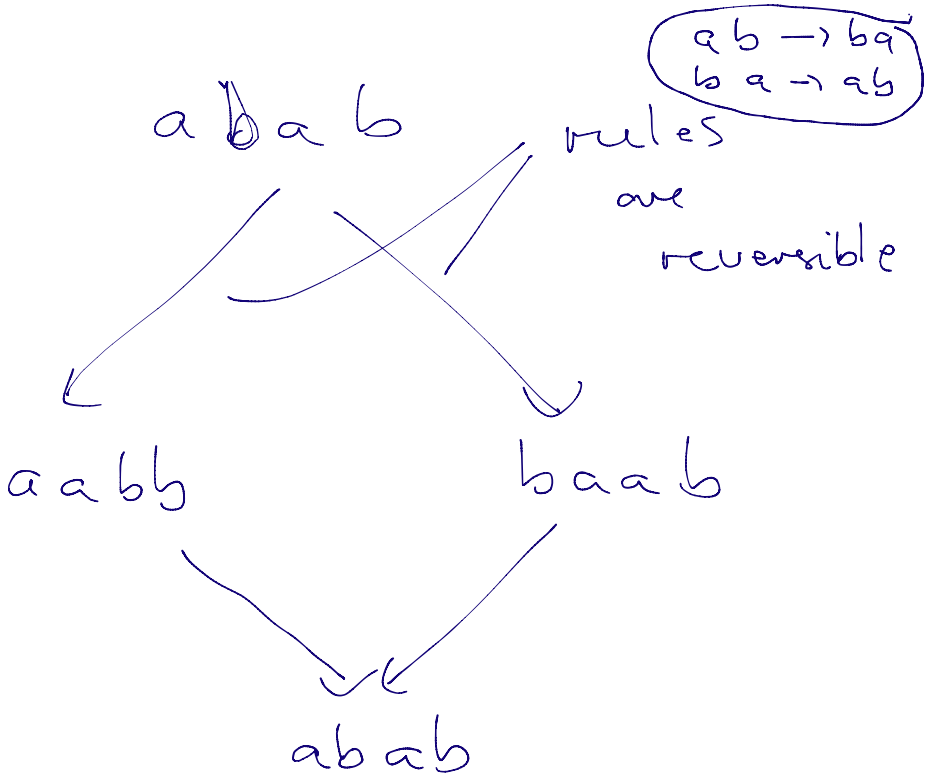
Q 2.3 confluence



the rules do not overlap

are independent

are orthogonal



Q2.4 see table

Q2.5 ~~drop~~ keep ba → ab

drop ab → ba

ba → ab

aaa →

bbb →

Q2.6 now all equal have normal forms

Q2.7 $2\,1 > 1\,2$
$b\,a \rightarrow a\,b$

$1\,1\,1 > 0$
$a\,a\,a \rightarrow$

$2\,2\,2 > 0$
$b\,b\,b \rightarrow$

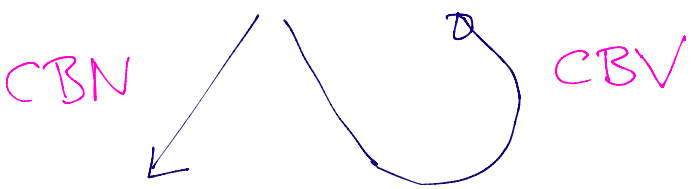
Q3

$$(\lambda a.(\lambda b.(\lambda c\,((a\ b)(\lambda x.(x\ y\ c))))))$$

$$Exp ::= (\lambda x.Exp) \mid (Exp\ Exp)$$

Q4

$$(\lambda x.\lambda y.x)\ (\lambda x.x)\ ((\lambda x.xx)(\lambda x.xx))$$



$$(\lambda y.(\lambda x.x))\ ((\lambda x.xx)(\lambda x.xx))$$

$$\downarrow$$

$\lambda x.x$ normal function