Pandemics under lockdown policies*

Chen Wei, 120090218

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Abstract

In this article, we build up an SIR model with lockdown policy and explore how the tightness of the lockdown policies can affect the trajectories of the susceptibles, infectives, and removed, the duration of the pandemic, the total number of infections at the end of the pandemics, the largest number of infections at the same time, and the effect of the lockdown policy on the economy. We also consider different types of thresholds in lockdown policies and compare their efficiency under different lockdown tightness. We analyze the sensitivity of the model we build and try to fit this model into the data in the coronavirus outbreak in Shanghai on March 2022.

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Contents

1 Introduction and Modeling **1**

2 Lockdown policy I: threshold in increment **3**

2.1 Trajectories in different lockdown tightness 4

2.2 The duration of the pandemic 5

2.3 Total number of infections 6

2.4 Largest number of infectives 7

2.5 Duration of lockdown and its impact on the economy 9

3 Lockdown policy II: threshold in infections **10**

4 Sensitivity analysis **12**

5 Fitting into datas **13**

6 Conclusions **17**

7 Postscript: the end of the lockdown policies **19**

1 Introduction and Modeling

By December 2022, it has been almost three years since the coronavirus outbreak. During this time, governments have dealt with this infectious disease in various ways, including quarantine and vaccination. Among these methods, one of the most influential and controversial ways is to lock down the district where the pandemic breaks out, which gives rise to discussion throughout the country. As a result, we are interested in the efficiency of the lockdown policy and its effect on the economy. In this article, we quantify the efficiency of lockdown policies as the difference between the duration of the pandemics when there are no/some lockdown policies. Also, we are concerned with other quantities, such as the total number of infections, which will help us understand the effect of the policy on reducing the impact of the sequela on people, and the largest number of infectives during the outbreak of the disease, which would help us understand its effect on reducing the strain of the medical system.

We apply the SIR model introduced in lectures with slight modifications to solve these problems: in the modified model, we consider the proportion of each component S, I, R instead of the actual number and allow the transmission rate β to be dependent on time t to indicate the effect of the lockdown policy. To achieve this, consider the original SIR model:

$$\frac{dS(t)}{dt} = -\beta S(t)I(t)$$
$$\frac{dI(t)}{dt} = \beta S(t)I(t) - \gamma I(t)$$
$$\frac{dR(t)}{dt} = \gamma I(t)$$

By substituting $S' = S/N$, $I' = I/N$, $R' = R/N$, $\beta' = \beta/N$ and replace the constant β by a function $\beta' = \beta'(t)$ of time t, we obtain the system of differential equations for this problem. The transmission rate β' now no longer depends on the size of the population. We assume that it only depends on the frequency of interactions between people and the property of the disease:

$$\begin{aligned} \frac{dS'(t)}{dt} &= -\beta'(t)S(t)I(t) \\ \frac{dI(t)}{dt} &= \beta'(t)S(t)I(t) - \gamma I(t) \\ \frac{dR(t)}{dt} &= \gamma I(t) \end{aligned} \tag{1}$$

For simplicity, the parameter $\beta'(t)$ can only take two values: one is the transmission rate without any lockdown, denoted as β_0, and the other is the transmission rate under lockdown, denoted as β_1. By our assumption, these values only depend on the frequency of interactions between people and the property of the disease, and it should be proportional to the frequency of interactions between people. As a result, we assume that $\beta_1 = c\beta_0$, and the parameter c represents the tightness of the lockdown policy. For ease of notation, we will use $\beta(t), S, I, R$ to describe the corresponding parameter or values $\beta'(t), S', I', R'$ throughout the rest of this article.

A lot of work has been done by researchers to estimate the set of parameters (β_0, γ), including [1] [2] [3]. As for the parameter β_1, for the efficiency of the lockdown so that it can ultimately eliminate the epidemic, there is a restriction that $\frac{\beta_1}{\gamma} \leq R_0$, where R_0 is the basic reproductive ratio for the disease.

By what we have mentioned in lectures, the epidemic would ultimately disappear with the limits

$$S_\infty = \lim_{t\to\infty} S(t),\ I_\infty = \lim_{t\to\infty} I(t) = 0,\ R_\infty = \lim_{t\to\infty} R(t)$$

The quantities we are interested in include the time t_d that the epidemic disappears, defined as $t_d = \inf(t > 0 : I(t) < \epsilon)$, the total number of infections R_∞, the largest number of infectives $I_{max} = \sup(I(t) : t \in \mathbb{R}^+)$, and the duration of lockdown t_l. The threshold for the epidemic to disappear ϵ we choose here is 10^{-5}, as the population of Shenzhen is 1.2×10^7, and we allow approximately 100 infections remained when declaring that the pandemic disappears.

2 Lockdown policy I: threshold in increment

In this and the following section, we will choose appropriate parameters β_0, β_1, γ, and a lockdown policy and apply numerical methods to solve the ODE system and compare the quantities that we are interested in with the case that there is no lockdown policy.

The parameters β_0 and γ have been estimated by many researchers mentioned in the introduction, and here we use the result of Ianni and Rossi in [1], which was obtained from the data in Germany in early 2020. Their result claimed that $1/\beta_0 = 2.2$ days, $1/\gamma = 15$ days, and the basic reproductive ratio $R_0 = \frac{\beta}{\gamma} = 6.6$. The initial number of infectives $I(0)$ we choose here is the same as the threshold for eliminating the pandemic: 10^{-5}.

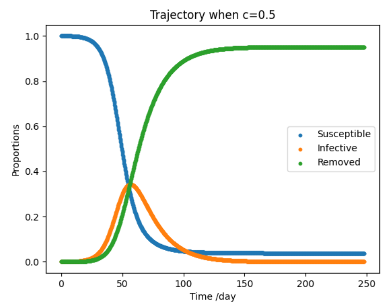
The tightness of the lockdown policy is determined by the parameter c, which determines the transmission rate β_1 after lockdown. We are interested in exploring the relationship between the tightness of lockdown c and the quantities we are interested in, which are mentioned in the introduction.

The lockdown policy we choose here is that the government set up a threshold for the number of increased infections. After the increased infectives achieve this threshold, the district would be put in lockdown until this wave of epidemic is eliminated. With reference to the data in [4] from the outbreak of the pandemic in Shanghai on March 2022, the threshold we choose here is 0.0003 for the increase of $I(t)$ in a day as this is the proportion of increment in the number of infectives when Shanghai implement the lockdown policy.

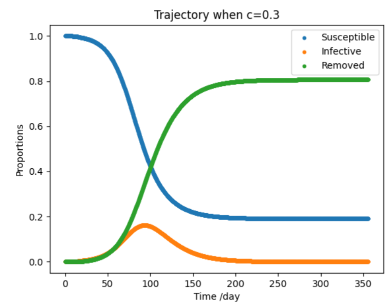
After choosing the parameters, to apply Euler's method to solve this ODE system, we need to do time discretization: we only detect the function values S, I, R and detect whether to implement lockdown at certain times in a day. We assume that the change of S, I, R is linear with the increase rate given by the ODE system between two successive detects. After making these assumptions, we can solve the ODE system numerically and compare the quantities we are concerned with if there is no lockdown policy.

2.1 Trajectories in different lockdown tightness

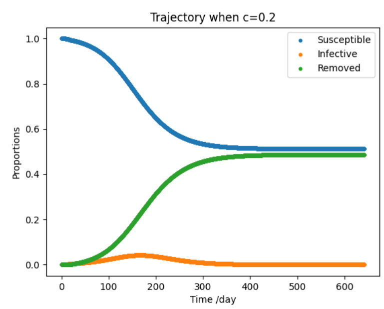
By running the code in the attachment, we can obtain the solution trajectory of the ODE system with varying values of parameters $c \in \{0.5, 0.3, 0.2, 0.1\}$ as follows:



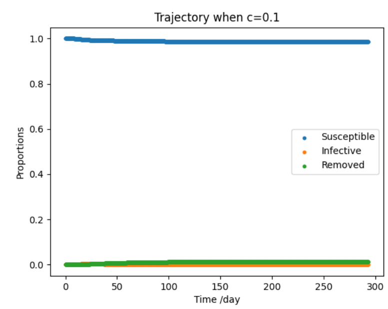
(a) c=0.5



(b) c=0.3



(c) c=0.2



(d) c=0.1

Figure 1: Trajectory of the ODE system with different c

It can be observed clearly from the trajectory that a large proportion of people are infected when $c = 0.5$ or $c = 0.3$. Approximately half of the people are infected when $c = 0.2$, and only a tiny amount of people are infected if $c = 0.1$. Also, from the trajectory, we can see that the peak of the number of infections will appear earlier with a loose lockdown policy. When there is no lockdown, which corresponds to the case that $c = 1$, almost all people are infected. The peak of the number of infectives appears approximately 40 days after the pandemic breaks out, as illustrated below:

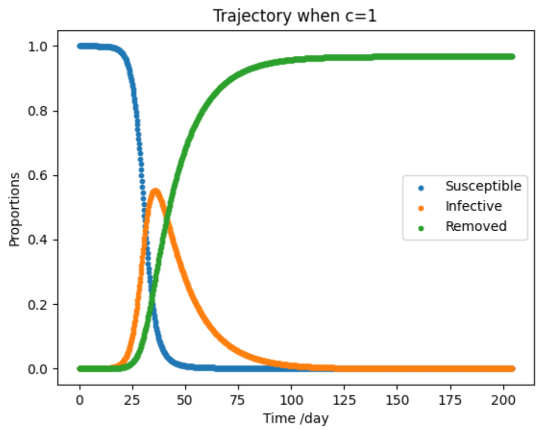


Figure 2: Trajectory of the ODE system when $c = 1$

2.2 The duration of the pandemic

As the parameter c varies, the change in the duration of the pandemic is illustrated as follows:

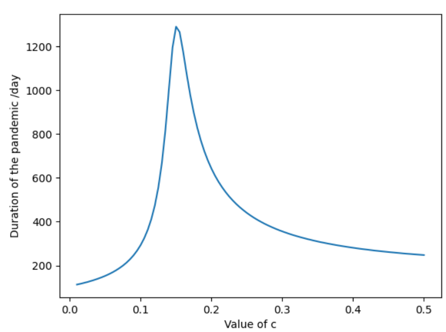


Figure 3: The duration of the pandemic with different c

Tightness c	0.1	0.15	0.2	0.25	0.3	0.35	0.4
Duration (/days)	293.4	1290.2	642	440	355.8	309.6	280.1

Table 1: Duration of pandemic when c changes

We can observe that the peak of the duration of the pandemic appears when c is approximately 0.15, which lies in the middle of the test interval $(0, 0.5]$ for c. The reason is that when c is large and near 0.5, which corresponds to the case that the lockdown policy is loose, the number of infectives grows fast, and then these infectives would also recover relatively quickly. When c is small and near 0, which corresponds to the case that the lockdown policy is tight, the number of infections is small, and they would recover in a short time. For the above reasons, the duration of the pandemic is long only when the tightness of the lockdown policy is medium. In this case, the number of infections grows and decreases slowly, resulting in a long-duration pandemic. The numerical algorithm shows that the peak of the duration is 1290 days with $c = 0.15$.

2.3 Total number of infections

As the parameter c varies, the change in the number of total infections is illustrated below:

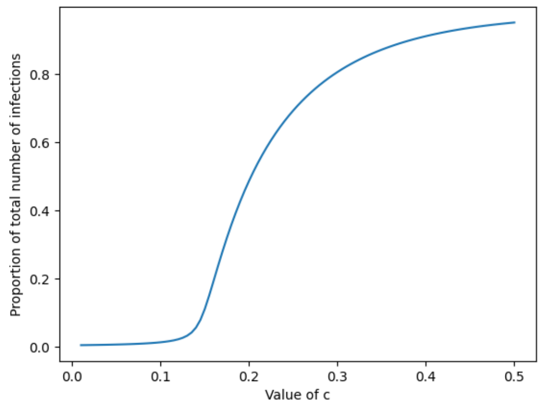


Figure 4: Proportions of total infections with different c

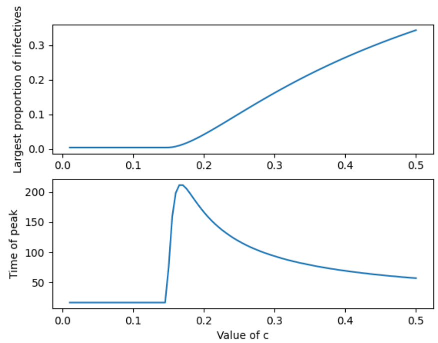
Tightness c	0.1	0.15	0.2	0.25	0.3	0.35	0.4
Total infections	1.37%	11.17%	48.74%	69.4%	80.67%	87.23%	91.2%

Table 2: Total proportions of infections when c changes

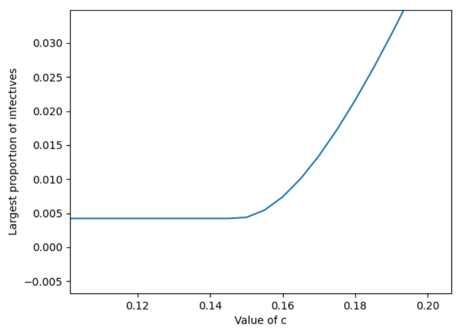
A natural observation is that the total number of infections decreases as the lockdown policy becomes tight, which coincides with our intuition that the lockdown policy can prevent the spread of diseases. Also, we notice that the number of infectives changes most rapidly when $c = 0.16$, which means that near this value, the number of total infectives would be very sensitive to the tightness of the lockdown policy, which indicates that a slight relaxation in executing the policy near this value will result in a large increase of the total number of infections.

2.4 Largest number of infectives

The largest number of infections here is an important indicator of the pressure this epidemic brings to the medical system. The change in the peak of the number of infectives is illustrated here:



(a) Proportion of largest number of infectives and the time when peak appears



(b) The rapid increase when c is near 0.15

Figure 5: The largest number of infectives and the time it appears

Tightness c	0.1	0.15	0.2	0.25	0.3	0.35	0.4
Largest infections	0.42%	0.44%	4.21%	10.23%	16.21%	21.62%	26.4%

Table 3: Largest proportions of infections when c changes

The observation that the largest proportion of infectives increases as c increases also corresponds to our intuition, and there is a rapid increase in the largest proportion of infectives near $c = 0.15$, which corresponds to our observation that the duration also changes rapidly when c is near 0.15. The reason for their correspondence is that a longer duration of the pandemic would provide more time for the infectives to increase; hence, a rapid change in the duration would result in a rapid change of the largest proportion of the infectives.

2.5 Duration of lockdown and its impact on the economy

The impact of the lockdown on the economy is not easy to quantify. Here we assume that the implementation of the lockdown policy does not affect the relative frequency of the economic activities in all human interactions, and the amount of economic loss is proportional to the economic activities eliminated. Then the economic loss caused by the lockdown can be quantified as $M(1-c)t$, where M is the constant that relates the eliminated economic activities to the actual economic loss, and t is the duration of the lockdown. The numerical results of t and $(1-c)t$ is illustrated here:

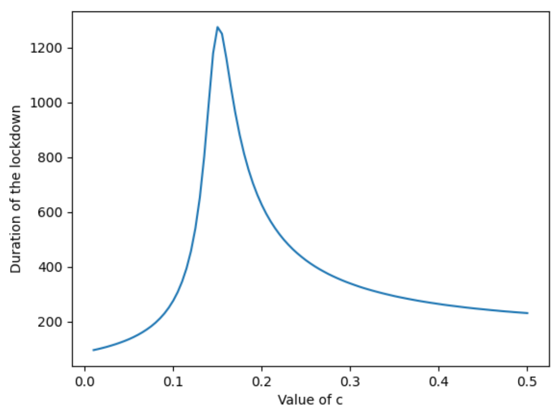


Figure 6: Duration of lockdown with different c

Tightness c	0.1	0.15	0.2	0.25	0.3	0.35	0.4
Lockdown time (/days)	277.2	1274	625.8	423.8	339.6	293.4	264.8

Table 4: Lockdown time when c changes

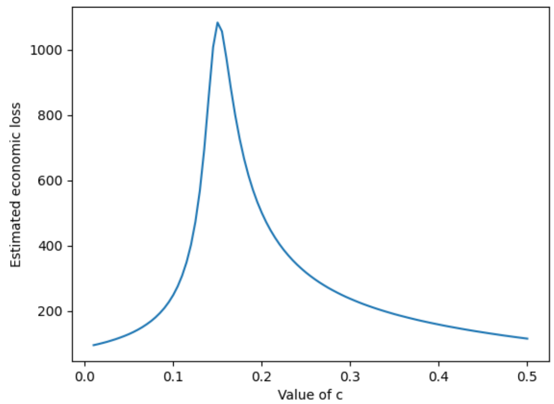


Figure 7: Relative economic loss with different c

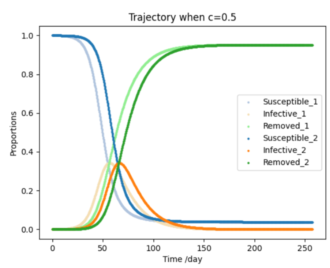
Tightness c	0.1	0.15	0.2	0.25	0.3	0.35	0.4
Economic loss	249.5	1082.9	500.64	317.84	237.72	190.71	158.88

Table 5: Relative economic loss when c changes

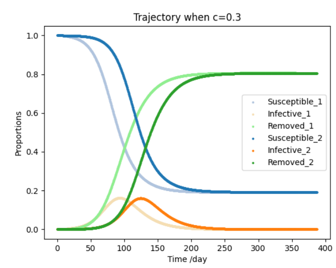
The result that the duration of lockdown and the economic loss are maximized when c is near 0.15 is not surprising, as when c takes these values, the pandemic is also long. Also, we can see that there is no way to balance the economic loss and the tightness of the lockdown: to achieve minimal financial loss, the policy has either to be tight enough to eliminate the pandemic in a short time so that it does not affect the economics seriously, or to be loose enough so that the economic activities are not affected by the lockdown policy to a significant extent. This observation may be the most important indication of this simple model to the real world.

3 Lockdown policy II: threshold in infections

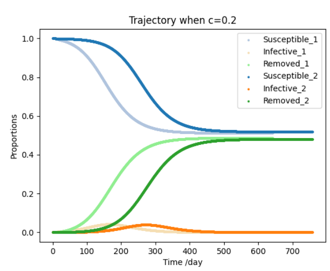
Another lockdown policy that is frequently applied is to set up a threshold for the number of total infections. When the number of infections achieves this threshold, a lockdown is implemented. As for our model, the only difference between these two policies is the time to implement the lockdown. We can understand this difference by plotting the trajectory of the ODE system with different lockdown policies and the same lockdown tightness c. The threshold we choose here refers again to the data in [4] from the breakout of the pandemics in Shanghai on March 2022: the total proportion of infectives when the lockdown policy is implemented is approximately 0.002. The comparison when $c \in \{0.5, 0.3, 0.2, 0.1\}$ is illustrated as follows, where policy one uses the increment as the threshold and policy 2 uses the total infectives as the threshold:



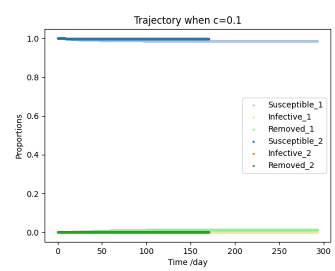
(a) c=0.5



(b) c=0.3



(c) c=0.2



(d) c=0.1

Figure 8: Trajectory of the ODE system with different c

The quantities that we are concerned with are listed as follows. In each entry, the value on the left is the result of lockdown policy one, and the right is the result of lockdown policy two:

Tightness	Uninfected	Pandemic time	Lockdown time	Economic loss
c=0.5	0.038/0.038	248/257	231/248	115.8/124.1
c=0.3	0.191/0.191	356/386	340/378	237.7/264.9
c=0.2	0.512/0.519	642/758	626/749	500/600
c=0.1	0.987/0.999	293/171	277/161	249.5/145

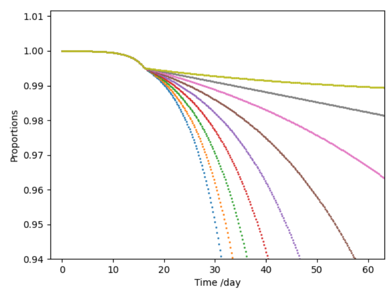
Table 6: Value of quantites when policy and c changes

We can see that only when the lockdown is tight, the second policy will behave better than the first one. It is because when c is small, only a tiny proportion of people would be infected throughout the pandemic, and S is always near 1. Then the increase of infectives $\frac{dI}{dt} = I(\beta S - \gamma)$ is mainly determined by the number of infectives, which means that controlling the number of infectives would be efficient in preventing the disease from spreading. This phenomenon indicates that only when the lockdown policy is tight, we should use the total number of infectives as the threshold; otherwise, using the increase in the infectives as the threshold would be a better choice.

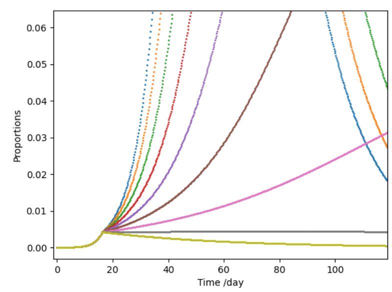
4 Sensitivity analysis

The numerical stability of the SIR model, which means that little perturbations in its parameters would not result in a significant change in its trajectories, has been explored by many researchers, including [5] [6]. The model presented here is just a slight modification of the original SIR model, which gives rise to a jump discontinuity of the parameter $\beta(t)$ as a result of the implementation of the lockdown policy. However, as this parameter only appears in the derivatives of an ODE system, this discontinuity in the parameter does not violate the continuity of the trajectory, as shown in section 2. As a result, we may claim that this model is numerically stable in the parameters (β_0, γ).

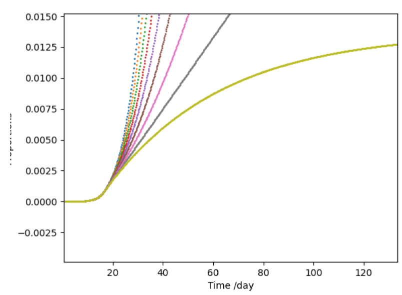
Now we would like to investigate the sensitivity of the parameter c: the trajectories are continuous with respect to parameter c in the sense that slight changes in the parameter c would not result in great changes in the trajectory, as illustrated below:



(a) Change in susceptibles when c changes



(b) Changes in infectives when c changes



(c) Changes in removed when c changes

Figure 9: Change in the trajectories

We can observe that all solution trajectories change continuously when c changes, which indicates that this model is numerically stable concerning the parameter c. As a result, this model is numerically stable in all its parameters.

5 Fitting into datas

In this section, we will fit the data in [4] from the outbreak of the pandemic in Shanghai on March 2022 into our model. We want to determine the parameters β_0, β_1, and γ in the model so that the mean square error between estimated value of the model and the data is minimized.

We refer to the data and policy in [4]: there was no lockdown between March 1^{st} to March 31^{st} and there was lockdown between April 1^{st} to June 30^{st}. The initial data we choose here is the data on March 2^{nd} with 10 infections ad we set $t = 0$ on March 2^{nd}. We start with estimating the parameters separately in the first 29 days and last 60 days to give an initial approximation for the parameters. To estimate β_0 and γ, we run the code to find the optimal β_0, γ such that the mean square error $\frac{1}{29}\sum_{i=1}^{29}(Y(i) - Y\hat{(i)})^2$ is minimized, where $Y(i)$ is the number of infectives and removed obtained from the data, and $\hat{Y}(i)$ is the value estimated by the model. The results are $\beta_0 = 1.48$ and $\gamma = 1.255$, and the comparison between the number of infections and removed predicted by the model and the number of infections in the data is illustrated below:

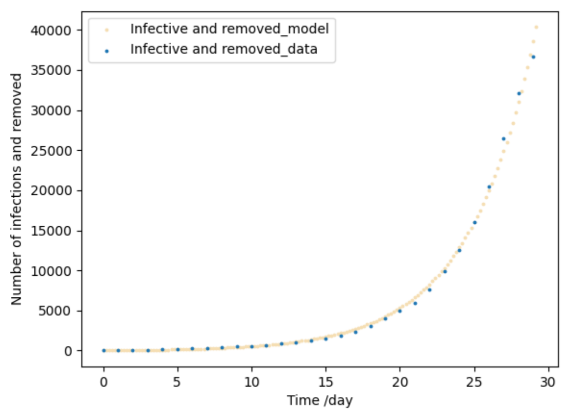


Figure 10: Fitted value compared to data in the first 29 days

Time (day)	5	10	15	20	25	29
Actual No.	176	544	1472	4990	16027	36641
No. in model	142	538	1729	5307	16036	40352

Table 7: Number of infectives and removed predicted by the model

Similarly, starting from the data we observed on day 30 in the previous approximation, we can also find the optimal parameters β_1 and γ such that the mean square error $\frac{1}{60}\sum_{i=1}^{60}(Y(i)-\hat{Y(i)})^2$ is minimized, where $Y(i)$ is the total number of infectives and removed obtained from the data, and $\hat{Y(i)}$ is the value estimated by the model. The results are $\beta_1 = 1.015$ and $\gamma = 1.04$, and the comparison between the number of infections and removed predicted by the model and the number of infections in the data is illustrated below:

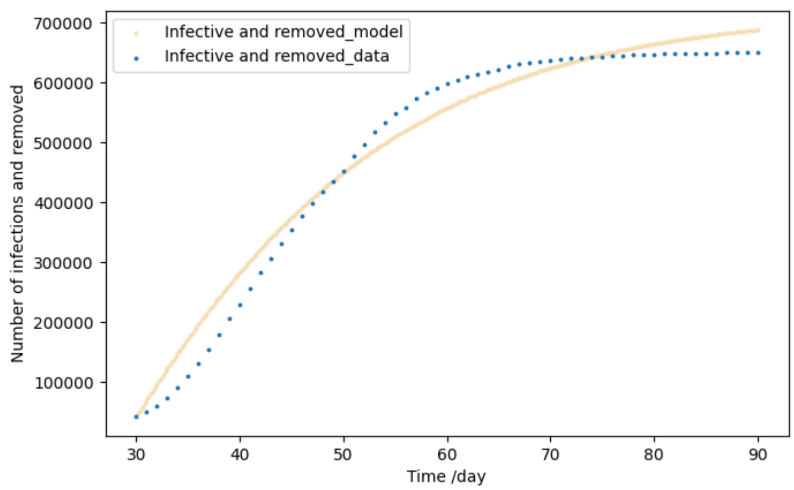


Figure 11: Fitted value compared to data in the last 60 days

Time (day)	35	40	45	50	55	60
Actual No.	110597	229815	355269	452958	547383	598423
No. in model	172685	283182	374689	449583	509574	557179
Time (day)	65	70	75	80	85	90
Actual No.	622208	636080	642987	646909	648936	649341
No. in model	594602	623805	646463	663964	677437	687780

Table 8: Number of infectives and removed predicted by the model

The parameter γ we obtain here has a slightly different meaning from the original model. In the coronavirus outbreak in Shanghai on March 2022, most of the infectives were sent to hospitals and separated from the susceptibles once they were detected. As a result, the parameter γ here indicates the average time it takes for an individual from being infected to develop symptoms of the pandemic and be sent to the hospital (in short, the incubation period of the virus) instead of the recovery rate in the original model.

Starting from these initial approximations, we can obtain the optimal paramters for the model: $(\beta_0, \beta_1, \gamma) = (1.374, 1.101, 1.11)$. The comparison between the number of infections and removed predicted by the model and the number of infections in the data is illustrated below:

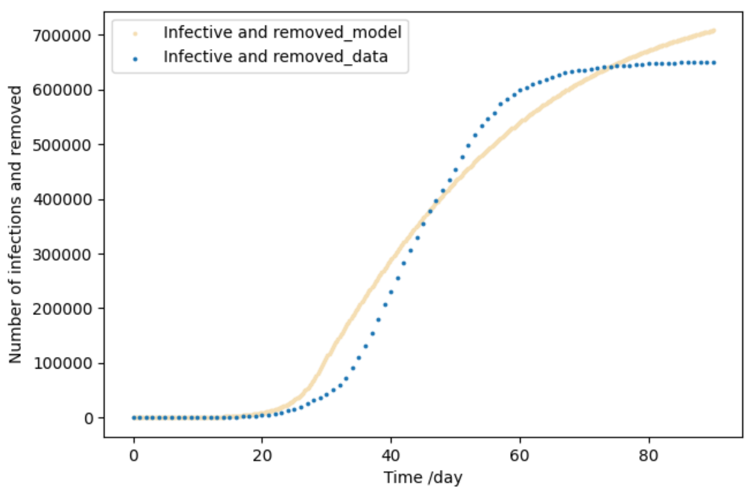


Figure 12: Fitted value compared to data

Time (day)	5	10	15	20	25	30
Actual No.	176	544	1472	4990	16027	42952
No. in model	146	640	2425	8881	32163	109802
Time (day)	35	40	45	50	55	60
Actual No.	110597	229815	355269	452958	547383	598423
No. in model	203425	288712	364981	432068	490231	540026
Time (day)	65	70	75	80	85	90
Actual No.	622208	636080	642987	646909	648936	649341
No. in model	582202	617602	647088	671493	691587	708060

Table 9: Number of infectives and removed predicted by the model

The estimation is poor when the size of infected population is small and is relatively accurate when the size of infected population is large because we use the mean square error criterion here and lay more emphasis on the case when the number of infections is large.

The tightness of lockdown is $c = \frac{\beta_1}{\beta_0} = 0.8$, which is not so high as there was regular prevention and control of the disease throughout early 2022.

6 Conclusions

In this article, we build up an SIR model with lockdown policy and apply it to explore how the tightness of the lockdown policies can affect several quantities we are concerned with, including the trajectories of the susceptibles, infectives, and removed, the duration of the pandemic, the total number of infections at the end of the pandemics, the largest number of infections at the same time, and the effect of the lockdown policy on the economy. We see that the duration of the pandemic grows when the lockdown tightness c is less than 0.15 and decreases when c is larger than 0.15. The largest number of infectives grows as c increases and grows most rapidly when c is around 0.16. The effect of the lockdown policy on the economy is most significant when $c = 0.15$.

We also consider different types of thresholds in lockdown policies and compare which is more efficient under different lockdown tightness. The result is that only when the lockdown policy is tight, a threshold in the total number of infectives would be more efficient than the threshold in the increment of the infectives.

Then we analyze the sensitivity of the model we built and find that this model is numerically stable. We also try to fit the model to data obtained in the real world: with our estimated parameters, this model provides a relatively good approximation for the data in Shanghai.

However, this model is relatively simple, and some of our assumption in the article is sketchy. In the model, we assume that the transmission rate $\beta(t)$ is a step function with only one discontinuity, which is not always the case in reality: when the pandemic breaks out, people would automatically avoid interactions with other people, so the transmission rate should decrease as time goes on even if there is no lockdown, and in [1] $\beta(t)$ is modeled as decreasing exponentially. We also assume that the recovery rate γ is constant throughout the pandemic, which results in an error when predicting the dynamics of the number of infectives, as shown in section 6 (there, the recovery rate is smaller in the last 60 days). The reason is that in a relatively short period, the recovery rate should decrease as the number of infectives decreases, fewer medical resources are available. In a relatively long period, the recovery rate should increase as time goes on as new medicines and treatments are provided for the patients, and they would recover in a shorter time. Another simple and naive assumption we made is that the effect of the lockdown on the economy is directly proportional to its effect on the interactions between people, which overestimates the effect of the lockdown as we can proceed with many economic activities online nowadays. Such assumptions make our model simple, but they also give rise to errors when predicting the dynamics of the pandemic.

7 Postscript: the end of the lockdown policies

Surprisingly, this article's accomplishment has witnessed the end of lockdown policies. On Dec 7^{th} 2022, *the ten new regulations on prevention and control of the coronavirus* was announced by the State Council, which marked the end of the three-year-long regular control and prevention of the pandemic (i.e., dynamic zero-COVID policy). After that, most cities canceled the constraints on the nucleic acid test result when entering public places. In the meantime, the pandemic has broken out in many cities throughout the country, including Beijing, Baoding, and Guangzhou, and also spread to our campus. It is hard to say whether the lockdown policy affects the economy's growth significantly, but it does prevent most of the population from being infected by the disease. As predicted at the beginning of this article, most people would get infected without any lockdown policies. Would this be the case in 2023 in our country? Would the economy resume back to pre-pandemic? Only time will tell.

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