# Turing Machine

$Q, q_0, \Sigma, \Gamma, B, \delta, F$

$B$ → Blank Symbol

$\Gamma$ → Tape symbol

$\delta : Q \times \Gamma \rightarrow Q \times \Gamma \times L | R | S$

$L$ → left, $R$ → Right, $S$ → stationary

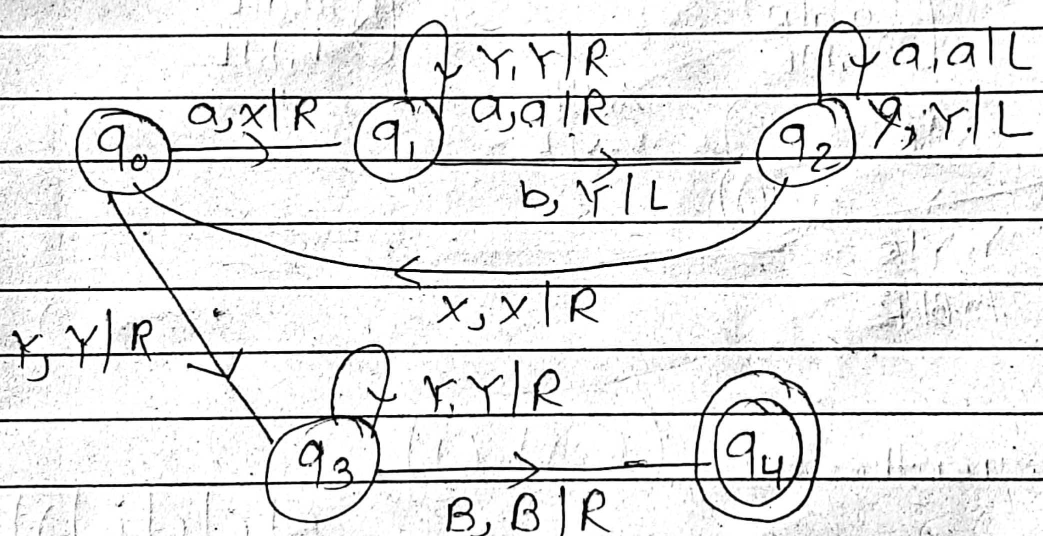
**Example:**

(*) $L = \{a^n b^n \mid n \geq 1\}$

$L = \{ab, aabb, aaabbb, \ldots\}$

aaaabbbb :

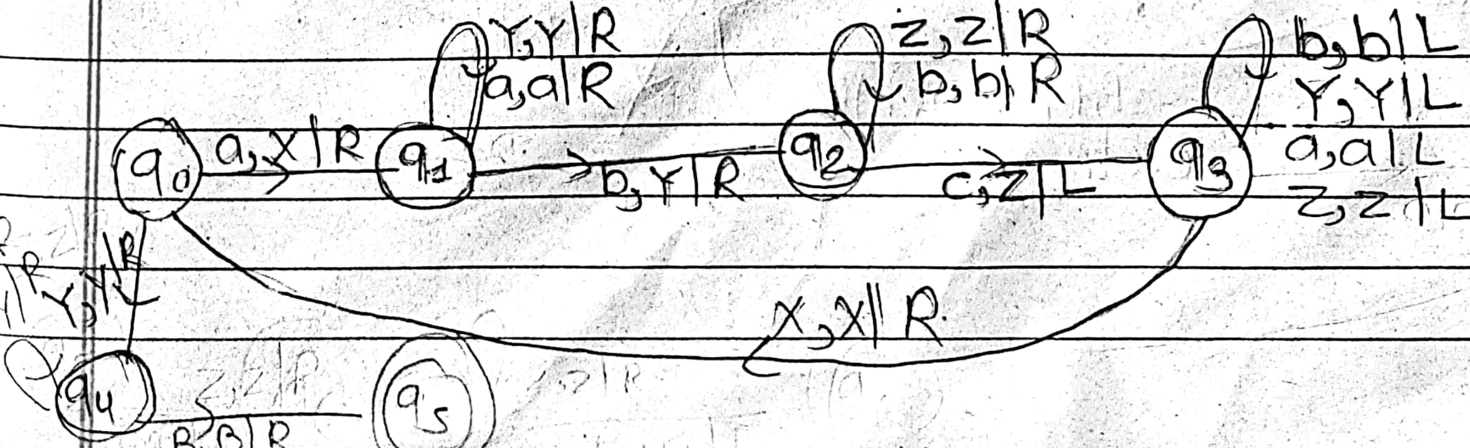
| | X | X | X | X | Y | Y | Y | Y | |
|---|---|---|---|---|---|---|---|---|---|
| B | a | a | a | a | b | b | b | b | B |




(*) $L = \{a^n b^n c^n \mid n \geq 1\}$

aaabbbccc

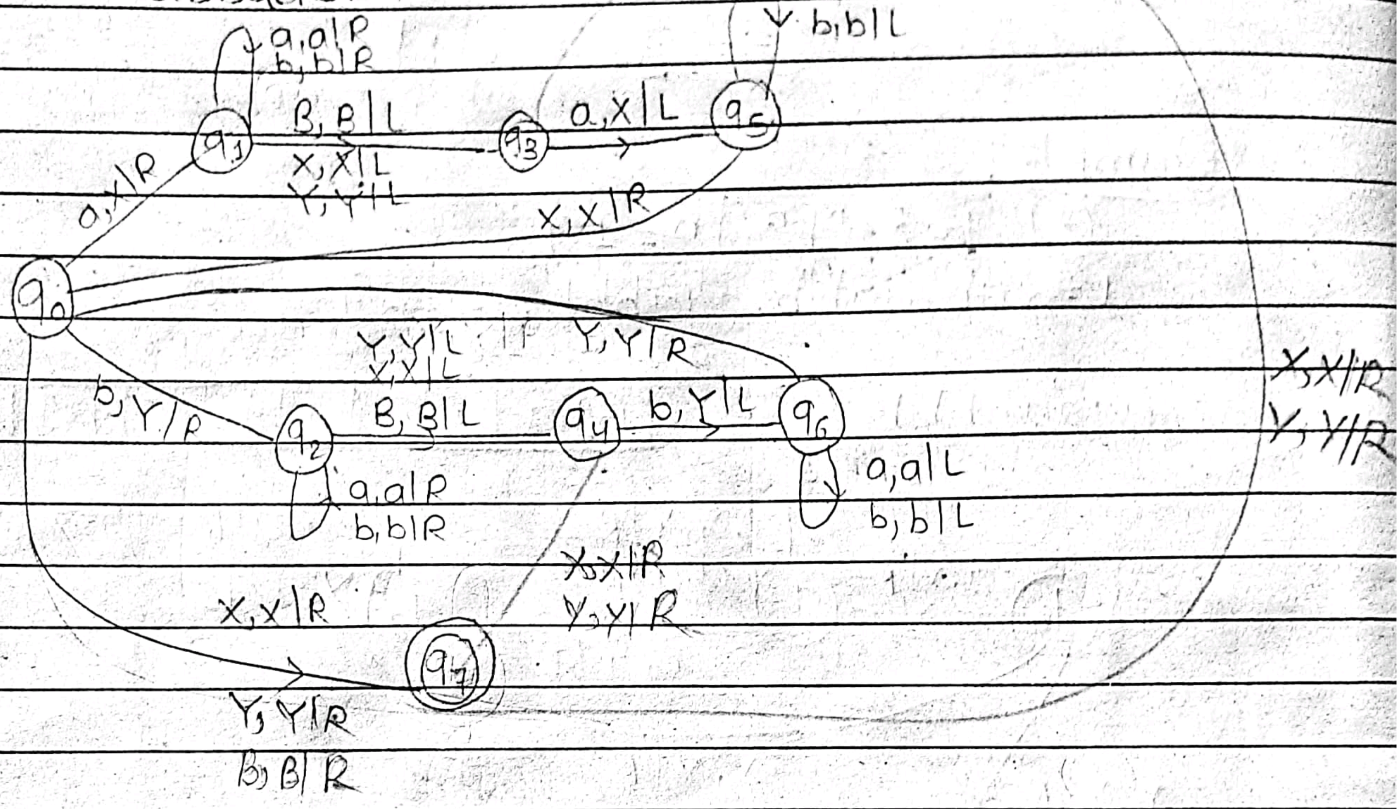
| B | $\cancel{a}$ | $\cancel{a}$ | $\cancel{a}$ | b | b | b | c | c | c | B |
|---|---|---|---|---|---|---|---|---|---|---|



(*) {L is a palindrome over {a,b}}

L = {aa, bb, aba, bab, bbb, abba, . . . }

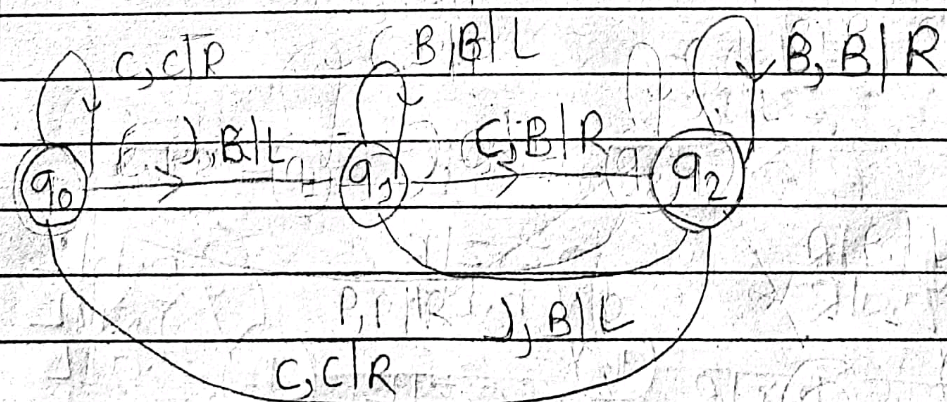
abbaabba




(*) Balance parenthesis (( ( ) ) ( ))

B | ( | ( | ( | ) | ) | ) | ( | ) | )

B B B B B B B B

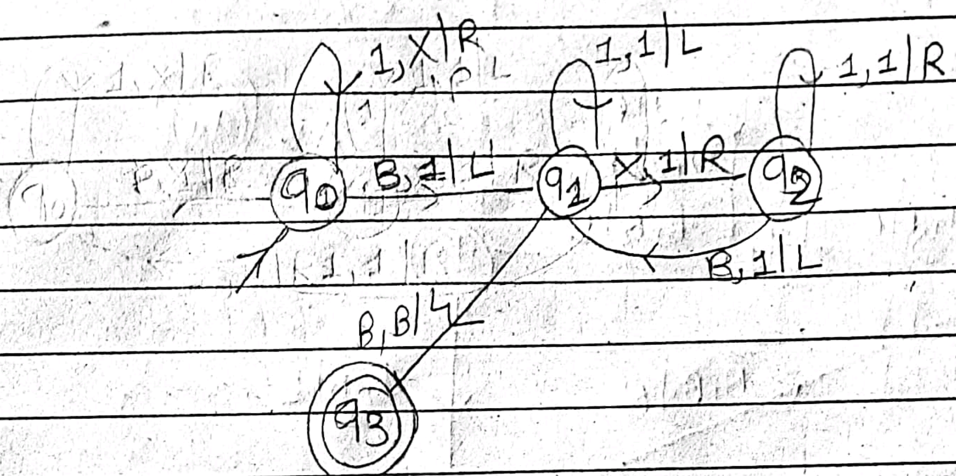


$f(x) = 2x$

Eg:- B | 1 | 1 | 1 | B | B | B | B

X X X

1 1 1 1 1 1



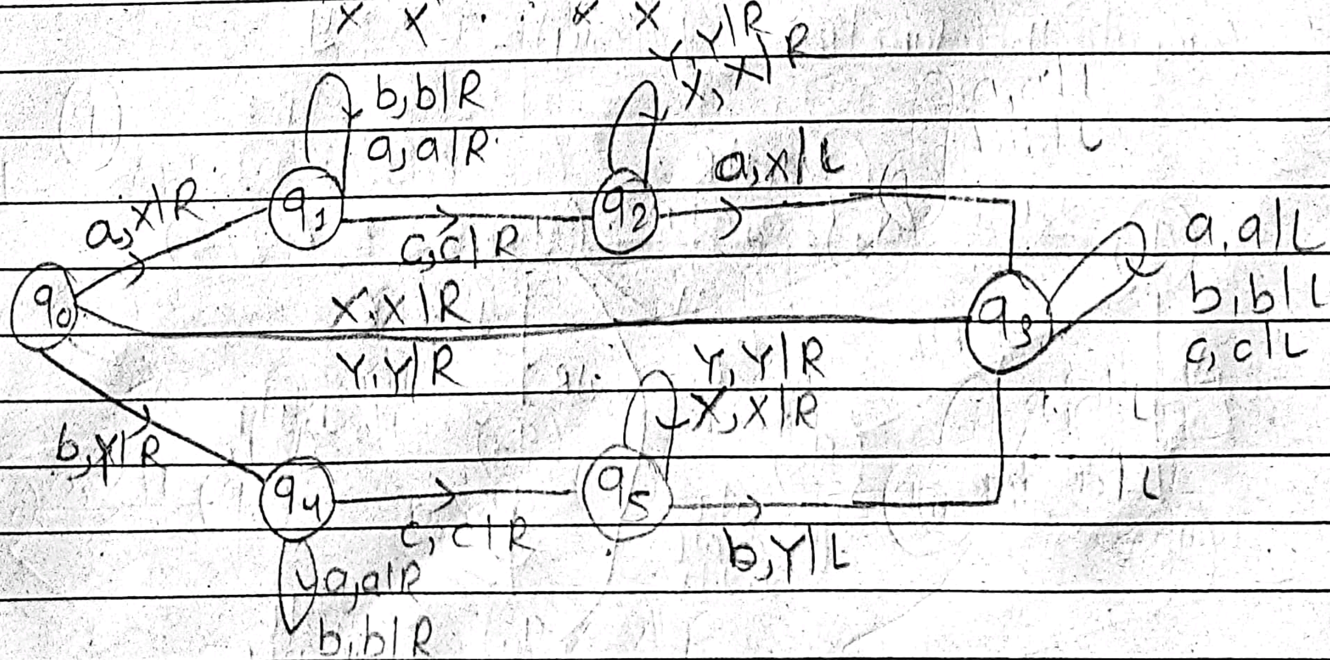

$L = \{wcw \mid w \in (a,b)^*\}$

$L = \{aca, bcb, aacaa, abcab, abacaba \ldots\}$

B | b | a | b | c | b | a | b | B

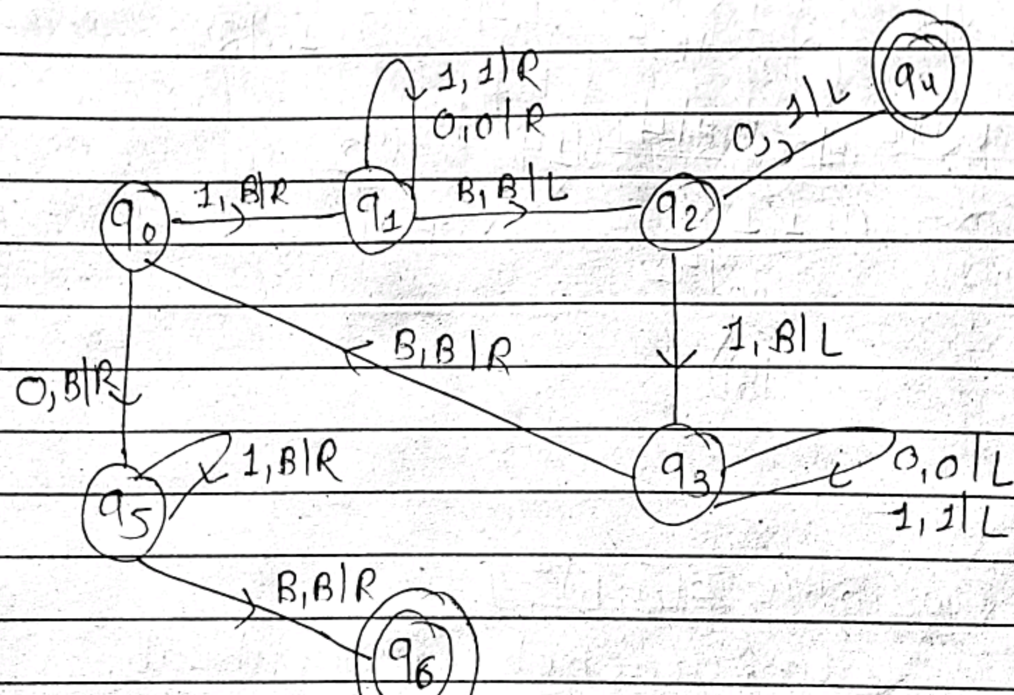
B | a | b | a | c | a | b | a | B

X X X X

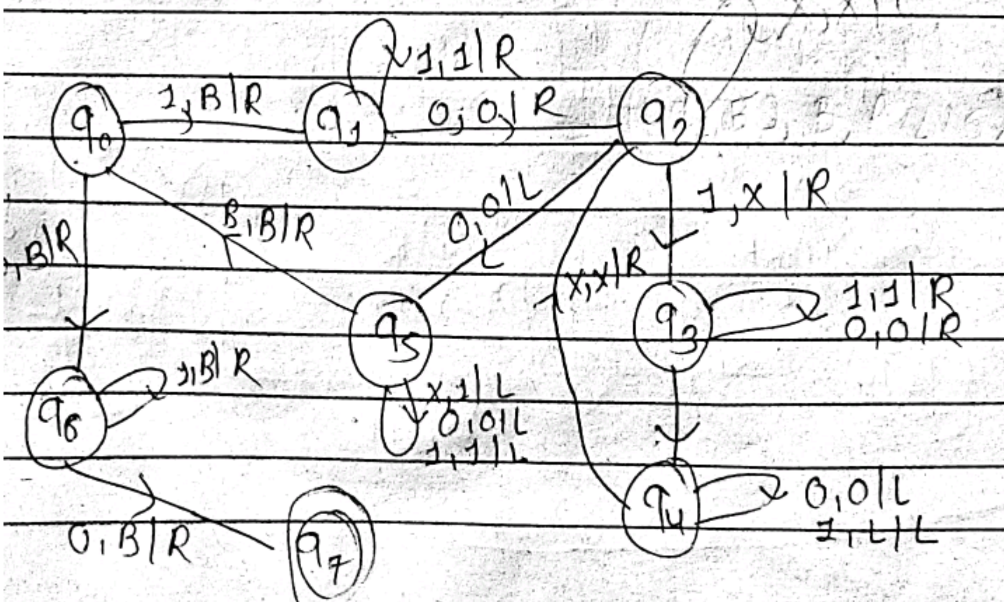


Compute the fx

$$f(x,y) = \begin{cases} x-y & \text{if } x>y \\ 0 & \text{if } x \le y \end{cases}$$

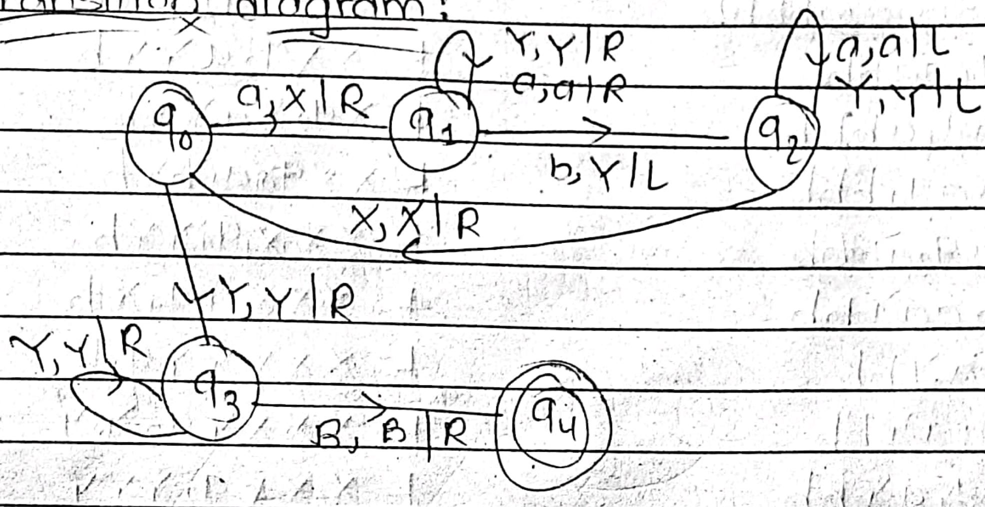



Compute the function: $f(x,y) = x * y$



$L = \{a^n b^n \mid n \geq 1\}$

Transition diagram:




Transition table:

| States | a | b | X | Y | B |
|---|---|---|---|---|---|
| $q_0$ | $(q_1, X, R)$ | | | $(q_3, Y, R)$ | |
| $q_1$ | $(q_1, a, R)$ | $(q_2, Y, L)$ | | $(q_1, Y, R)$ | |
| $q_2$ | $(q_2, a, L)$ | | $(q_0, X, R)$ | $(q_2, Y, L)$ | |
| $q_3$ | | | | $(q_3, Y, R)$ | $(q_4, B, R)$ |
| $q_4$ | | | | | |

(*) Instantaneous description of TM

$w = aabb$

$\delta(q_0, aabb)$

$\vdash X q_1 abb$

$\vdash X a q_1 bb$

$\vdash X q_2 a Y b$

$\vdash q_2 X a Y b$

$\vdash X q_0 a Y b$

$\vdash X X q_1 Y b$

$\vdash X X Y q_1 b$

$\vdash X X q_2 Y Y$

$\vdash X q_2 X Y Y$

$\vdash X X q_0 Y Y$

$\vdash X X Y q_3 Y$

$\vdash X X Y Y q_3 B$

$\vdash X X Y Y B q_4$

#

$w = aaabbb$

$\delta(q_0, aaabbb)$

$\vdash X q_1 aabbb$

$\vdash X a q_1 abbb$

$\vdash X a a q_1 bbb$

$\vdash X a q_2 a Y bb$

$\vdash X q_2 a a Y bb$

$\vdash q_2 X a a Y bb$

$\vdash X q_0 a a Y bb$

$\vdash X X q_1 a Y bb$

$\vdash X X a q_1 Y bb$

$\vdash X X a Y q_1 bb$

$\vdash X X a q_1 Y Y b$

$\vdash X X q_2 a Y Y b$

$\vdash X q_2 X a Y Y b$

$\vdash X X q_0 a Y Y b$

$\vdash X X X q_1 Y Y b$

$\vdash X X X Y q_1 Y b$

$\vdash X X X Y Y q_1 b$

$\vdash X X X Y q_2 Y Y$

$\vdash X X X q_2 Y Y Y$

$\vdash X X q_2 X Y Y Y$

$\vdash X X X q_0 Y Y Y$

$\vdash X X X Y q_3 Y Y$

$\vdash X X X Y Y q_3 Y$

$\vdash X X X Y Y Y q_3 B$

$\vdash X X X Y Y Y B q_4$ #

## Turing machine having storage in the state

$L = 01^* + 10^*$

$\delta([q_0, B], 0) \Rightarrow ([q_1, 0], 0, R)$

$\delta([q_1, 0], 1) \Rightarrow ([q_1, 0], 1, R)$

$\delta([q_1, 0], B) \Rightarrow ([q_2, B], B, R/L/S)$

## Universal TM $\exists$ (Tu)

1) Tu accepts the string $e(T)\, e(\omega)$ if and only if T accepts $\omega$.
2) If T accepts $\omega$ and produce output y then Tu produce output $e(y)$.

# Encoding transition of TM

$$\delta(q, a) = (P, b, D)$$

$$e(m) = S(q)\, 1\, S(a)\, 1\, S(P)\, 1\, S(b)\, 1\, S(D)\, 1$$

# Encoding the TM

$$e(T) = \underbrace{S(q)}_{\text{initial state}}\, 1\, e(m_1)\, 1\, e(m_2)\, 1 --- 1\, e(m_K)\, 1$$

# $e(\omega) = 1\, S(\omega_1)\, 1\, S(\omega_2)\, 1 --- 1\, S(\omega_K)\, 1$

$$\boxed{\begin{aligned} &S(B) = 0 \\ &S(a_i) = 0^{i+1} \ \forall\, a_i \in \Sigma \\ &S(q_i) = 0^{i+2} \ \forall\, q_i \in Q \\ &S(S) = 0 \\ &S(L) = 00 \\ &S(R) = 000 \end{aligned}}$$

Question:

$\delta(q_1, b) \exists (q_3, a, R)$

$\delta(q_3, a) \exists (q_1, b, R)$

$\delta(q_3, b) \exists (q_2, a, R)$

$\delta(q_3, B) \exists (q_3, b, L)$

Solution:- Step 1: encoding transition of TM

$\delta(q_1, b) \exists (q_3, a, R)$

move 1 : $m_1$ = 0001 0001 00000 1001 0001

move 2 : $m_2$ = 00000100 1 0001 0001 0001

move 3 : $m_3$ = 000001 000 1 0000 100 1 0001

$m_4$ = 000 001 01 0000010001 001

Step 2 : Encoding TM

e(T) = 000 1 0001 0001 00000 1001 0001 1

000001001000100010001 1 000001000

000010010001 1 000001 01 0000010001

0011