Chapter 1:

Exercise 1.1

1.

(i) $f(3 + h) = 4 - 3(3 + h) = 4 - 9 - 3h = -5 - 3h$

and $f(3) = 4 - 3 \times 3 = 4 - 9 = -5$

$\therefore \dfrac{(3+h) - f(3)}{h} = \dfrac{-5 - 3h + 5}{h} = -5$

(ii) $\dfrac{f(x) - f(1)}{x - 1} = \dfrac{\frac{x+3}{x+1} - 2}{x - 1} = \dfrac{x + 3 - 2x - 2}{x^2 - 1} = \dfrac{-x + 1}{x^2 - 1} = \dfrac{-(x - 1)}{(x - 1)(x + 1)} = \dfrac{-1}{x + 1}$

2.

(i) For domain, $x^2 - 9 = 0 \Rightarrow x = \pm 3$

f(x) is not exist for $x = \pm 3$, hence domain is set of all real number except 3 and − 3

So, domain is $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

(ii) For domain, $x^2 + x - 6 = 0$

$x^2 + 3x - 2x - 6 = 0$

$x(x + 3) - 2(x + 3) = 0$

$(x - 2)(x + 3) = 0$

$\therefore \quad x = 2, -3$

f(x) is not exist for $x = 2$ and $x = -3$.

Hence domain is set of all real number except 2 and −3.

So domain $(-\infty, -3) \cup (-3, 2) \cup (2, \infty)$

(iii) Here, f(t) is exist for all value of t. Thus, domain is $(-\infty, \infty)$

(iv) For domain $3 - t \geq 0$ and $2 + t \geq 0$

i.e. $t - 3 \leq 0$ and $t \geq -2$

i.e. $t \leq 3$ and $t \geq -2$

$\therefore$ domain is $-2 \leq t \leq 3$

(v) For domain, $2 - \sqrt{P} \geq 0$

$\Rightarrow 0 \leq P \leq 4$

(vi) Domain is set of all real number except 0.

i.e. $(-\infty, 0) \cup (0, \infty)$

3.

(i) For domain, $4 - x^2 \geq 0$

$x^2 - 4 \leq 0$

$(x - 2)(x + 2) \leq 0$

$\therefore$ Domain is $-2 \leq x \leq 2$.

For Range, $h(x) = \sqrt{4 - x^2}$

$y = \sqrt{4 - x^2}$

A complete solution of Mathematics-I (CSIT)

$y^2 = 4 - x^2$

$x^2 = 4 - y^2$

$x = \pm\sqrt{4 - y^2}$

Here, $4 - y^2 \geq 0$

i.e. $y^2 - 4 \leq 0$

i.e. $(y - 2)(y - 2) \leq 0$

$\therefore \quad -2 \leq y \leq 2$

But by the question is always positive.

Hence, range is $0 \leq y \leq 2$.

(ii) For domain, $x - 5 \geq 0$

i.e. $x \geq 5$

For range, $f(x) = \sqrt{x - 5}$

$y = \sqrt{x - 5}$

$y^2 = x - 5$

$x = y^2 + 5$

Here y is set of all real numbers. But by the question y is positive. Hence range is $0 \leq y < 0$.

(iii) Given, $g(x) = \dfrac{2x + 3}{x - 3}$

For domain, $x - 3 = 0$

i.e. $x = 3$

$\therefore$ Domain n set of all real numbers except 3.

i.e. $(-\infty, 3) \cup (3, \infty)$

For range, $y = \dfrac{2x + 1}{x - 3}$

$xy - 3y = 2x + 1$

$x(y - 2) = 1 + 3y$

$x = \dfrac{1 + 3y}{y - 2}$

$\therefore$ Value of y is set of all real number except 2.

Here, range is $(-\infty, 2) \cup (2, \infty)$.

4.

(i) $y = x + 2$ is graph of function because, for any vertical line $x = 1$, $y = 1 + 2 = 3$. Vertical line meet on only one point.

(ii) $x = y^2$

Let $x = 4$, then $y = \pm 2$. Here vertical line $x = 4$ meet the curve at two points $y = \pm 2$. So $x = y^2$ is not function.

(iii) $y = x^2$. Let $x = 1$ then $y = 1$. Here vertical line $x = 1$ meet the curve at only one point. So it is function.

(iv) $y = -\sqrt{x+2}$. Let x = 2, then y = –2. Here the vertical line x = 2 meet the curve at only one point y = –2. So it is function.

(v) $x^2 + y = 5$, let x = 1, then y = 4. Here the vertical line x = 1 meet the curve at only one point. y = 4 so it is function.

(vi) $x = y^2 - 2$. Let x = 2 then y = ± 2. Here the vertical line x = 2 meet the curve at two points y = ± 2. So it is not function.

5.

(i) $f(-x) = \frac{x^2}{x^4+1} = f(x)$

∴ f(x) is even function.

(ii) $g(-x) = -x\,|x| = -g(x)$

∴ f(x) is odd function.

(iii) $h(-x) = |-x^3 + x^5| = -(-1 + x^3 - x^5)$

∴ h(x) is neither even nor odd function.

(iv) $f(-x) = 2|x| + 1 = f(x)$

∴ f(x) is even function.

(v) $g(-x) = 3 = g(x)$

∴ f(x) is even function.

7.

Perimeter = $2(l + w)$

$20 = 2(l + w)$

∴ $l + w = 10$ .... (1)

Since $A = l\,w$

$A = l\,(10 - l)$

$A = 10\,l - l^2$

8.

Given A = 16

∴ $lb = 16$

∴ $b = \frac{16}{l}$

Also, $P = 2b + 2l$

$= 2 \times \frac{16}{l} + 2l$

$P = \frac{32}{l} + 2l$

9.

Volume = a × a × h

$= a^2 h$; where a is length of base and h is height

∴ $2 = a^2h$

i.e. $a^2h = 2$

$h = \frac{2}{a^2}$

Surface area, A = area of base + Area of 4 sides

$= a^2 + 4ah$

$= a^2 + 4a \times \frac{2}{a^2}$

$A = a^2 + \frac{8}{a}$

10. Let x = Width of rectangle window

$\frac{x}{2}$ = Radius of circle

l = Length of rectangle

Area of window, A = Area of rectangle + Area of semicircle

$= lx + \frac{\pi}{2}\left(\frac{x}{2}\right)^2$

$= lx + \frac{\pi x^2}{8}$

The perimeter of window = Perimeter of semi-circle + Perime

$30 = \frac{2\pi}{2}\frac{x}{2} + x + 2l$

$30 = \frac{\pi x}{2} + x + 2l$

∴ $l = 15 - \frac{\pi x}{4} - \frac{x}{2}$

Thus (1) becomes,

$A = \left(15 - \frac{\pi x}{4} - \frac{x}{2}\right)x + \frac{\pi x^2}{8} = 15x - \frac{x^2(\pi + 4)}{8}$

11. Here height is x, cutout length 2x and cutout width 2x.

∴ Volume of box, V = (Area of base) × height

$= (20 - 2x)(12 - 2x) \times x$

$= 4x^3 - 64x^2 + 240x$

12. Here $f(x) = \begin{cases} 15(40 - x) & \text{for } 0 \le x < 40 \\ 0 & \text{for } 40 \le x \le 65 \\ 15(x - 65) & \text{for } x > 65 \end{cases}$

13. $E = \begin{cases} 10 + 0.06x & \text{for } 0 \le x \le 1200 \\ 82 + 0.07(x - 1200) & \text{for } x > 1200 \end{cases}$

Exercise 1.2

Equation of linear function is $f(x) = mx + b$ .... (1)

Given, $f(x) = 1$

So, $f(x) = 2m + b$

$\Rightarrow 1 = 2m + b$

$\therefore\ b = 1 - 2m$

Hence, (1) is $f(x) = mx + (1 - 2m)$ be the family of linear function.

Given $T = 0.02\,t + 8.50$

(a) Slope is $m = 0.02$

T-intercept is, $T = 8.50$ (put $t = 0$)

$m = 0.2$ means rate of change of temperature is 0.02 (i.e. when 1 year increase temp is increased by 0.02°C)

$T = 8.50$ means, in 1900, temperature of is 8.50°C

(b) In 2100 means, $t = 200$ year

So, $T = 0.02 \times 200 + 8.50$

$= 12.5°$

$\therefore$ In 2100, temperature of earth is 12.5°C

Let $y = mx + b$ (A) be linear cost function.

Where y is cost in \$ to produce x number chairs in one day.

Given, $y = 2200$ for $x = 100$

So, $2200 = m \cdot 100 + b$

i.e. $100\,m + b = 2200$ .... (1)

Also, given $y = 4800$ for $x = 300$

$\therefore\ 300\,m + b = 4800$ .... (2)

Solving (1) and (2)

$100\,m + b = 2200$

$\underline{-300\,m \pm b = -4800}$

$-200\,m = -2600$

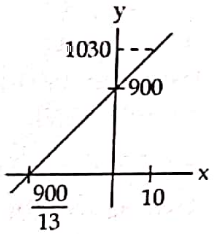
$m = 13$

using on (1), we get

$100 \times 13 + b = 2200$

$b = 2200 - 1300$

$b = 900$



(A) is $y = 13x + 900$

Slope $m = 13$, it means rate of change of cost of with respect to number of chair is 13 i.e. to produce a more chair \$ 13 additional money is required.

y - intercept is, $y = 900$. It means fixed cost (there is no production of chair) is 900 \$.

4.

(a) Let linear model is $T = Nm + b$ .... (A)

Given that $N = 113$ when $T = 70°$ C and

$N = 173$ when $T = 80°$

Using it on (A) and solving

$113\,m + b = 70$ ....(1)

$\underline{-173\,m \pm b = -80}$ ....(2)

$-60\,m = -10$

$m = \frac{1}{6}$

Using (i) we get,

$113 \times \frac{1}{6} + b = 70$

$b = 70 - \frac{113}{6} = \frac{420 - 113}{6} = \frac{303}{6} = \frac{101}{2}$

$\therefore$ (A) is $T = \frac{1}{6}N + \frac{101}{2}$ is required model.

(b) Slope $m = \frac{1}{6}$. Rate of change of temperature with respect chirps is 1 more chirps per minutes rise the temperature $\frac{1}{6}$ by $\frac{1}{6}$°F.

(c) Find T when $N = 150$

Since, $T = \frac{1}{6} \times 150 + \frac{101}{2}$

$T = \frac{151}{2}$°C

$T = 75.5°$

5.

Let $P = mt + b$ .... (A)

where P is the price and t be time (Here $t = 0$, first of January)

Given that $m = 2$ cent/month

Thus, $P = 2t + b$ .... (1)

Also given, By Nov. fist $p = 1.56$ \$ i.e. $p = 156$ cent

i.e. when $t = 10$, $p = 156$

$\therefore$ Using (1) we get,

$156 = 2 \times 10 + b$

$\therefore$ $b = 136$

Hence (A) is

$P = 2t + 136$ is required linear function.

At beginning of the year means first of Jan, i.e. $t = 0$. So $P = 136$.

So at beginning of the year price of a bottle of soda is 136 cent i.e. 1.36 \$.

6.

(a) Let linear relationship is c = md + b .... (A)

Where C is monthly cost in \$ and d is mile drive.

By the question, when c = 380 \$, d = 480 ml

and c = 460 \$, d = 800 ml

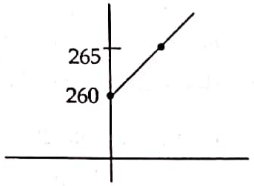
Thus, (A) becomes $480\,d + b = 380$ .... (1)

$800\,d + b = 460$ ....(2)

Solving (1) and (2)

$320\,d = 80$

$\therefore \quad d = \frac{1}{4} = 0.25$



Substituting value of d = 0.25 in (1) we get

$480 \times 0.25 + b = 380$

$b = 380 - 120$

$b = 260$

$\therefore$ (A) is $\overline{C} = 0.25\,d + 260$ is required model.

(b) C = ? When d = 1500 ml

Since, $C = 0.25\,d + 260$

$\therefore \quad C = 0.25 \times 1500 + 260$

$C = \$\,536$

(c) Slope m = 0.25

m = 0.25 mean, when a more mile car drive paid 0.25 \$.

(d) C-intercept = 260. It means fixed cost of driving per month is 260\$.

7.

Let linear function, F = mc + b .... (A)

Where F be temperature in °F and C be the temperature in °C.

Given, C = 0 then F = 32 and C = 100 then F = 212

So, from (A) using 1st condition

$32 = m\,0 + b$

i.e. $b = 32$

and using 2nd condition

$212 = m \times 100 + 32$

$100m = 180 \qquad m = \frac{9}{5}$

$\therefore$ (A) is $F = \frac{9}{5}C + 32$

(a) When C = 15°C find F = ?

So, using model,

$F = \frac{9}{5} \times 15 + 32$

$F = 59°C$

(b) When F = 68°F find C = ?

So, using model,

$68 = \frac{9}{5}C + 32$

$\frac{9}{5}C = 36$

$C = 20°$

(c) When F = C,

So, using model

$F = \frac{9}{5}F + 32$

$-\frac{4}{5}F = 32$

$F = -40$

$\therefore \quad F = C = -40$

8.

Let functional relation between N and x is

a. $N = mx + b$

Given that N = 97, when x = 100 and

N = 110 when x = 500

So, using it on (A) we get,

$100\,m + b = 97$

$500\,m + b = 110$

Solving equation (1) and (2)

$-400\,m = -13$

$m = \frac{13}{400}$

From equation (1)

$100 \times \frac{13}{400} + b = 97$

$\frac{13}{4} + b = 97$

$b = 97 - \frac{13}{4}$

$b = \frac{375}{4}$

$\therefore$ (A) is $N = \frac{13}{400}x + \frac{375}{4}$

(b) Find N when x = 300.

Since $N = \frac{13}{400} \times 300 + \frac{375}{4} = \frac{39}{4} + \frac{375}{4} = \frac{414}{4} \approx 104.$

9. Because of constant rate 50 is linear model and is $y = mt + b$ ....(A), where y is score and t is year from.

a. In 1995, given that $t = 0$, $y = 545$ and $t = 5$ (in 2000) $y = 545$

using this data, we get

$b = 575$ (from 1st condition)

and $545 = m \times 5 + 575$

$\therefore\ m = -6$

$\therefore$ (A) is $y = -6t + 575$

b. In 2005, i.e. $t = 10$

$y = -6t + 575 = -6 \times 10 + 575 = -60 + 575 = 515$

c. Find $t = ?$ when $y = 527$

Since, $y = -6t + 575$

$527 = -6t + 575$

or $6t = 575 - 527$

or $t = 8$

$\therefore$ In 1995 + 8 = 2003, the score is 527.

Exercise 1.3

1.

(i) $(f + g)\,x = f(x) + g(x) = x^3 + 5x^2 - 1$

$(f - g)\,x = f(x) - g(x) = x^3 - x^2 + 1$

$(fg)(x) = f(x)\,g(x) = (x^3 + 2x^2)(3x^2 - 1) = 3x^5 + 6x^4 - x^3 - 2x^2$

$\frac{f}{g}(x) = \frac{f(x)}{g(x)} = \frac{x^3 + 2x^2}{3x^2 - 1}$

The domain of f(x) is $A = (-\infty, \infty)$ and domain of g(x) is $B = (-\infty, \infty)$.

$A \cap B = (-\infty, \infty)$.

The domain for f + g, f − g, fg is $A \cap B = (-\infty, \infty)$.

The domain for f/g is $(-\infty, \infty) - \left\{\pm\frac{1}{\sqrt{3}}\right\}$.

(ii) $(f + g)(x) = f(x) + g(x) = \sqrt{3 - x} + \sqrt{x^2 - 1}$

$(f - g)(x) = f(x) - g(x) = \sqrt{3 - x} - \sqrt{x^2 - 1}$

$(f.g)(x) = f(x)\,g(x) = \sqrt{(3 - x)(x^2 - 1)}$

$\frac{f}{g}(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{3 - x}}{\sqrt{x^2 - 1}}$

The domain of f(x) is $3 - x \geq 0$

i.e. $x - 3 \leq 0$

i.e. $x \leq 3$

$\therefore$ $(-\infty, 3] = A$

The domain for $g(x) = \sqrt{x^2 - 1}$

$x^2 - 1 \geq 1$

$(x - 1)(x + 1) \geq 0$

$\therefore$ Domain is $(-\infty, -1] \cup [1, \infty) = B$

$\therefore$ Domain for f + g, f − g, f.g is $A \cap B = (-\infty, -1] \cup [1, 3]$

Domain f/g is $(-\infty, -1) \cup (1, 3]$

$(f + g)(x) = \sqrt{3 - x} + \sqrt{x^2 - 1}$

$(f - g)(x) = \sqrt{3 - x} - \sqrt{x^2 - 1}$

$(f.g)(x) = \sqrt{(3 - x)(x^2 - 1)}$

$(f/g)(x) = \sqrt{\frac{3 - x}{x^2 - 1}},\quad x \neq \pm 1$

For Domain of $f(x) = \sqrt{3 - x}$

$3 - x \geq 0$

$x - 3 \leq 0$

$x \leq 3$ i.e. $(-\infty, 3]$

For domain of $g(x) = \sqrt{x^2 - 1}$

$x_2 - 1 \geq 0$

$(x - 1)(x + 1) \geq 0$

$\therefore$ $B = (-\infty, -1]\ \theta\ [1, \infty)$

Thus Domain of (f + g), (f − g), and (f/g) is $B = (-\infty, -1]\ \theta\ [1, 3]$

The domain of f/g is $(-\infty, -1)\ \theta\ (1, 3]$

(iii) $(f + g)(x) = \sqrt{x} + \sqrt{1 - x}$

$(f - g)(x) = \sqrt{x} - \sqrt{1 - x}$

$(f \,.\, g)(x) = f(x)\,.\,g(x) = \sqrt{x\,(1 - x)^2}$

$\frac{f}{g}(x) = \frac{f(x)}{g(x)} = \sqrt{\frac{x}{1 - x}};\quad x \neq 1$

Domain for $f(x) = \sqrt{x}$ is $x \geq 0$

i.e. $[0, \infty) = A$

Domain for $g(x) = \sqrt{1 - x}$ is $1 - x \geq 0$

i.e. $x = 1 \leq 0$

i.e. $x \leq 1$

i.e. $(-\infty, 1] = B$

$\therefore$ Domain for f + g, f − g, f.g is $A \cap B = [0, 1]$.

Domain $f_o \frac{f}{g}$ is [0, 1)

(iv) $(f + g)(x) = f(x) + g(x) = x + \sqrt{x-1}$

$(f - g)(x) = f(x) - g(x) = x - \sqrt{x-1}$

$(f g)(x) = f(x)\, g(x) = x\sqrt{x-1}$

$\frac{f}{g}(x) = \frac{f(x)}{g(x)} = \frac{x}{\sqrt{x-1}};\quad x \neq 1$

Domain for $f(x) = x$ is $(-\infty, \infty) = A$

Domain for $g(x) = \sqrt{x-1}$ is $x - 1 \geq 0$

i.e. $x \geq 1$

$[1, \infty) = B$

$\therefore$ Domain for f + g, f − g, $f_o g$ is $A \cap B = [1, \infty)$.

Domain for $\frac{f}{g}(x)$ is $(1, \infty)$

(v) $(f + g)(x) = f(x) + g(x) = \sqrt{x+1} + \sqrt{x-1}$

$(f - g)(x) = f(x) - g(x) = \sqrt{x+1} - \sqrt{x-1}$

$(fg)(x) = f(x) \cdot g(x) = \sqrt{x^2-1}$

$\frac{f}{g}(x) = \frac{f(x)}{g(x)} = \sqrt{\frac{x+1}{x-1}},\quad x \neq 1$

Domain of f(x) is $x + 1 \geq 0$

i.e. $x \geq -1$

i.e. $[-1, \infty) = A$

Domain for g(x) is $x - 1 \geq 0$

i.e. $x \geq 1$

i.e. $[1, \infty) = B$

Domain for f + g, f − g and f . g is $A \cap B = [1, \infty)$.

Domain for f/g (x) is $(1, \infty)$.

2.

(i) $f(x) = \sqrt{x}$, $g(x) = x + 1$

$f_o g(x) = f(g(x)) = f(x+1) = \sqrt{x+1}$

Domain is $x + 1 \geq 0$, i.e. $x \geq 0$

So domain is $[-1, \infty)$.

$g_o f(x) = g(\sqrt{x}) = \sqrt{x} + 1$

Domain is $x \geq 0$ i.e. $[0, \infty)$

$f_o f(x) = f(f(x)) = f(\sqrt{x}) = \sqrt{\sqrt{x}} = (x)^{\frac{1}{4}}$

Domain is $x \geq 0$ i.e. $[0, \infty)$

$g_o g(x) = g(g(x)) = g(x + 1) = x + 1 + 1 = x + 2$

Domain is $(-\infty, \infty)$

(ii) $f(x) = x^2 - 1$, $g(x) = 2x + 1$

$f_o g(x) = f(2x + 1) = (2x + 1)^2 - 1 = 4x^2 + 4x$

Domain is $(-\infty, \infty)$

$g_o f(x) = g(f(x)) = g(x^2 - 1) = 2(x^2 - 1) + 1 = 2x^2 - 1$

Domain is $(-\infty, \infty)$

$g_o g(x) = g((g(x)) = g(2x + 1) = 2(2x + 1) + 1 = 4x + 3$

Domain is $(-\infty, \infty)$

(iii) $f(x) = \sqrt{x}$; $g(x) = \sqrt[3]{1-x}$

$f_o g(x) = f(g(x)) = f(\sqrt[3]{1-x}) = \left[(1-x)^{\frac{1}{3}}\right]^{\frac{1}{2}} = (1-x)^{\frac{1}{6}}$

Domain is $1 - x \geq 0$ i.e. $x - 1 \leq 0$ i.e. $x \leq 1$. So domain $(-\infty, 1]$

$g_o f(x) = g(f(x)) = g(\sqrt{x}) = \sqrt[3]{1-x} = (1 - \sqrt{x})^{\frac{1}{3}}$

Domain is $x \geq 0$ i.e. $[0, \infty)$

$f_o f(x) = f(f(x)) = f(\sqrt{x}) = \sqrt{\sqrt{x}} = (x)^{\frac{1}{4}}$

Domain is $x \geq 0$ i.e. $[0, \infty)$

$g_o g(x) = g(g(x)) = g((1-x)^{\frac{1}{3}}) = \sqrt[3]{1 - \sqrt[3]{1-x}}$

Domain is $(-\infty, \infty)$

(iv) $f(x) = x + \frac{1}{x}$, $g(x) = \frac{x+1}{x+2}$

$f_o g(x) = f\left(\frac{x+1}{x+2}\right) = \frac{x+1}{x+2} + \frac{1}{\frac{x+1}{x+2}} = \frac{x+1}{x+2} + \frac{x+2}{x+1} = \frac{2x^2+6x+5}{(x+2)(x+1)}$

Domain $g(x) = \frac{x+1}{x+2}$ is $R - \{-2\} = A$

Domain of $f(g(x) = \frac{2x^2+6x+5}{(x+2)(x+1)}$ is $R - \{-1, -2\} = B$

$\therefore$ Domain of $f_o g$ is $A \cap B = R - \{-1, -2\}$

$g_o f(x) = g(f(x)) = g\left(x + \frac{1}{x}\right) = \frac{x + \frac{1}{x} + 1}{x + \frac{1}{x} + 2} = \frac{x^2 + 1 + x}{x^2 + 1 + 2x} = \frac{x^2 + x + 1}{(x+1)^2}$

Domain of $f(x) = x + \frac{1}{x}$ is $R - \{0\} = A$

Domain of $f(g(x)) = \frac{x^2 + x + 1}{(x+1)^2}$ is $R - \{0, -1\}$.

$$f_o f(x) = f(f(x)) = f\left(x+\frac{1}{x}\right) = x+\frac{1}{x}+\frac{1}{x+\frac{1}{x}} = \frac{\left(x+\frac{1}{x}\right)^2+1}{\left(x+\frac{1}{x}\right)} = \frac{x^4+3x^2+1}{x(x^2+1)}$$

Domain of f(x) is R – {0} = A
Domain of f(f(x)) is R – {0} = B
Domain of $f_o f$ is $A \cap B$ = R – {0}

$$g_o g(x) = g(g(x)) = g\left(\frac{x+1}{x+2}\right) = \frac{\frac{x+1}{x+2}+1}{\frac{x+1}{x+2}+2} = \frac{2x+3}{3x+5}$$

Here domain of g(x) is R – {–2} = A

domain of g(g(x)) is R – $\left\{-\frac{5}{3}\right\}$ = B

Domain of $g_o g$ is $A \cap B = R - \left\{-2, -\frac{5}{3}\right\}$

$f(x) = \sqrt{x+1}$; $g(x) = \frac{1}{x}$

Here $f_o g(x) = f(g(x)) = f\left(\frac{1}{x}\right) = \sqrt{\frac{1}{x}+1}$

Domain of g(x) is R – {0} = A
Domain of f(g(x)) is R – (–1, 0] = B
Domain of $f_o g$ is $A \cap B$ = R – (–1, 0]
Now,

$$g_o f(x) = g(f(x)) = g(\sqrt{x+1}) = \frac{1}{\sqrt{x+1}}$$

Domain for $f(x) = \sqrt{x+1}$ is $x + 1 \geq 0$ i.e. $x \geq -1$, i.e. [–1, ∞) = A

Domain for $g(f(x)) = \frac{1}{\sqrt{x+1}}$ is $x + 1 > 0$ i.e. $x > -1$, i.e. (–1, ∞) = B

Thus, domain for $g_o f$ is $A \cap B$ = (–1, ∞).

$$f_o f(x) = f(f(x)) = f(\sqrt{x+1}) = (x+1)^{\frac{1}{4}}$$

Domain of $f(x) = \sqrt{x+1}$ is [–1, ∞) = A and

Domain of $f(f(x)) = (x+1)^{\frac{1}{4}}$ (–1, ∞) = B
Thus,
Domain of $f_o f$ is $A \cap B$ = [–1, ∞).

$$g_o g(x) = g(g(x)) = g\left(\frac{1}{x}\right) = \frac{1}{\frac{1}{x}} = x$$

Domain of $g(x) = \frac{1}{x}$ is R – {0} = A

Domain for g(g(x)) = x is (– ∞ ∞) = B

∴ Domain for $g_o g$ is $A \cap B$ = R – {0}

Domain of $(g.f)(x) = \frac{1}{\sqrt{x+1}}$

Here $g.f(x) = g(f(x)) = g(\sqrt{x+1}) = \frac{1}{\sqrt{x+1}}$

Domain of $f(x) = \sqrt{x+1}$ is $x + 1 \geq 0$ i.e. $x \geq -1$ so, A = [-1, ∞)

Domain of $g(f(x)) = \frac{1}{\sqrt{x+1}}$ is $x + 1 > 0$ i.e. $x > -1$ so B = (-1, ∞)

Hence domain of g.f(x) is $A \cap B$ = (-1, ∞)

Domain of $f.f(x) = (x+1)^{1/4}$

Domain of $f(x) = \sqrt{x+1}$ is $x + 1 \geq 0$ i.e. $x \geq -1$ so B = [-1, ∞)

Thus domain of f.f(x) is $A \cap B$ = [- 1, ∞)

(vi) $f(x) = x^2$, $g(x) = 1 - \sqrt{x}$

$f_o g(x) = f(g(x)) = f(1-\sqrt{x}) = (1-\sqrt{x})^2 = 1 - 2\sqrt{x} + x = 1 + x - 2\sqrt{x}$

Domain is $x \geq 0$ i.e. [0, ∞)

$g_o f(x) = g(f(x)) = g(x^2) = 1 - \sqrt{x^2} = 1 - x$

Domain of $g_o f$ is (– ∞, ∞)

$f_o f(x) = f(f(x)) = f(x^2) = x^4$

Domain is (– ∞, ∞)

$g_o g(x) = g(g(x)) = g(1-\sqrt{x}) = 1 - \sqrt{1-\sqrt{x}}$

Domain for $g(x) = 1 - \sqrt{x}$ is $x \geq 0$ i.e. [0, ∞) = A

Domain for $g(g(x)) = 1 - \sqrt{1-\sqrt{x}}$ is [0, 1] = B

Thus, domain of $g_o g$ is $A \cap B$ = [0, 1]

3.

i. $f(x) = 3x - 2$, $g(x) = \sin x$, $h(x) = x^2$

$f_o g_o h(x) = f_o g(h(x)) = f_o g(x^2) = f(g(x^2)) = f(\sin x^2) = 3 \sin x^2 - 2$

ii. $f(x) = |x - 4|$, $g(x) = 2^x$, $h(x) = \sqrt{x}$

$f_o g_o h(x) = f_o g(h(x)) = f_o g(\sqrt{x}) = f(g(\sqrt{x})) = f(2^{\sqrt{x}}) = |2^{\sqrt{x}} - 4|$

4.

i. $F(x) = (2x + x^2)^4$

Let $g(x) = 2x + x^2$ and $f(x) = x^4$

so, $f_o g(x) = f(g(x)) = f(2x + x^2) = (2x + x^2)^4 = F(x)$

$\therefore$ $F = f_o g$ where $f(x) = x^4$ and $g(x) = 2x + x^2$

ii. $F(x) = \cos^2 x = (\cos x)^2$

Let $f(x) = x^2$

$g(x) = \cos x$

$\therefore$ $f_o g(x) = f(g(x)) = f(\cos x) = \cos^2 x = F(x)$

$\therefore$ $f_o g = F$ where $f(x) = x^2$ and $g(x) = \cos x$

iii. $v|t| = \sec(t^2)\tan(t^2)$

Let $g|t| = \sec t \tan t$

$f_o g(t) = f(g(t)) = f(t^2) = \sec t^2 . \tan t^2 = v|t|$

$\therefore$ $f_o g = v$

So $f = \sec t \tan t$ and $g(t) = t^2$

5.

Express the function in form $f_o g_o h$ if

i. $R(x) = \sqrt{\sqrt{x} - 1}$

$f(x) = x^{1/2}$

$g(x) = x - 1$

$h(x) = \sqrt{x}$. So that

$f_o g_o h(x) = f_o g(h(x)) = f_o g(\sqrt{x}) = f_o(g(\sqrt{x})) = f(\sqrt{x} - 1) = \sqrt{\sqrt{x} - 1} = R(x)$

$\therefore$ $f_o g_o h(x) = R(x)$

Where, $f(x) = x^{1/2}$, $g(x) = x - 1$ and $h(x) = \sqrt{x}$

ii. $H(x) = \sqrt[8]{2 + |x|}$

Here, $f(x) = \sqrt[8]{x}$

$g(x) = 2 + x$

$h(x) = |x|$

So, that

$\therefore$ $f_o g_o h(x) = f_o g(h(x)) = f_o g(|x|) = f(g(|x|)) = f(2 + |x|) = \sqrt[8]{2 + |x|} = H(x)$

$\therefore$ $f_o g(x) = H(x)$

Where $f(x) = \sqrt[8]{x}$, $g(x) = 2 + x$ and $h(x) = |x|$

iii. $H(x) = \sec^4(\sqrt{x})$

Let $f(x) = x^4$, $g(x) = \sec x$, $h(x) = \sqrt{x}$

So that,

$f_o g_o h(x) = f_o g(h(x)) = f_o g(\sqrt{x}) = f(\sec\sqrt{x}) = \sec^4(\sqrt{x}) = H(x)$

$\therefore$ $f_o g_o h(x) = H(x)$, where $f(x) = x^4$, $g(x) = \sec x$ and $h(x) = \sqrt{x}$.

6.

(i) Given, $y = f(x)$

For shift 3 unit upward, $y = f(x) + 3$

(ii) Given, $y = f(x)$, for shift 2 unit to the right, $y = f(x - 2)$

(iii) Given, $y = f(x)$, for reflect about y-axis, $y = f(-x)$

(iv) Given, $y = f(x)$, for stretch vertically by a factor of 3, $y = 3 f(x)$

(v) Given, $y = f(x)$, for compressed horizontally by a factor 1, $y = f(1x) = f(x)$

7.

(i) $y = f(x) + 8$

Graph is obtained by shifting the graph $y = f(x)$ distance 8 unit upward.

(ii) $y = f(x + 8)$

Graph is obtained by shifting the graph $y = f(x)$ a distance 8 unit to the left.

(iii) $y = f(8x)$

Graph is obtained by compressing the graph $y = f(x)$ horizontally by a factor of 8.

(iv) Graph is obtained by stretching horizontally by a factor of 8 followed by stretching vertically by a factor of 8.

8.

i. $f(x) = -\sqrt{x}$ shifted right by 3

Since, $y = f(x - c)$ is graph shifted by c unit to right of graph $y = f(x)$

So, required function if $f(x) = f(x - 3) = -\sqrt{x - 3}$

ii. $y = 2x - 7$ shifted up by 7.

Since, $y = f(x) + c$ is graph shifted by c unit up of graph $y = f(x)$.

So, required function is $y = 2x - 7 + 7$

i.e. $y = 2x$

iii. $y = x^2 - 1$ stretched vertically by a factor of 3.

Since, $y = c\, f(x)$ is graph stretched vertically of graph of $y = f(x)$.

So, required function is $y = 3(x^2 - 1)$ i.e. $y = 3x^2 - 3$.

iv. $y = \sqrt{x + 1}$ compressed horizontally by factor 4.

Since, $y = f(x)$ is graph compressed horizontally by factor c.

So, required function is $y = f(4x)$ i.e. $y = \sqrt{4x + 1}$

v. $y = \frac{1}{2}(x + 1) + 3$

Since, $y = f(x) - C$ is graph shifted by c unit down, so required equation is

$y = \frac{1}{2}(x + 1) + 3 + 5$

$y = \frac{1}{2}x + \frac{1}{2} + 8$

$y = \frac{1}{2}x + \frac{17}{2}$

Since, $y = f(x - c)$ is graph shifted by c down, so required equation is

$y = \frac{1}{2}(x - 1) + \frac{17}{2}$

$y = \frac{1}{2}x - \frac{1}{2} + \frac{17}{2}$

$y = \frac{1}{2}x + 8$

(vi) $f(x) = \frac{1}{x^2}$

For shifted left to c is $f(x) = f(x + c)$. So,

Required function is $f(x) = \frac{1}{(x+2)^2}$

For shifted down to c is $y = f(x) - c$. Thus,

Required function is $f(x) = \frac{1}{(x+2)^2} - 1$.

(vii) $f(x) = x^3 - 4x^2 - 10$ compress vertically by 2 followed by reflection about x-axis.

Compress vertically factor 2. So, required equation is

$f(x) = \frac{1}{2}f(x)$

$y = \frac{1}{2}(x^3 - 4x^2 - 10)$

$y = \frac{1}{2}x^3 - 2x^2 - 5$

and followed by reflection about x-axis, is

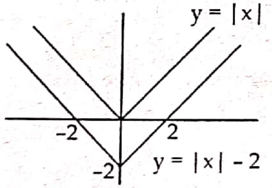
$y = -f(x)$ Or, $y = -\frac{1}{2}x^3 + 2x^2 + 5$

Required equation is $y = -\frac{1}{2}x^3 + 2x^2 + 5$

(i) $y = |x| - 2$

Original function is $y = |x|$

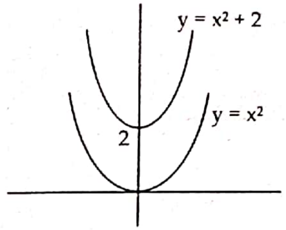
New function $y = |x| - 2$ is obtained by shifting the graph of $y = |x|$ a distance 2 unit down ward.



(ii) $y = x^2 + 2$

Original function is $y = x^2$.

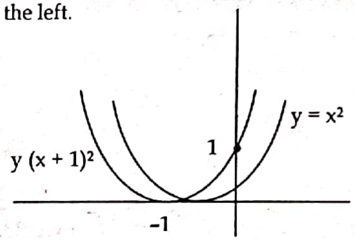
New function $y = x^2 + 2$ is obtained by shifting the graph of $y = x^2$ at distance 2 unit upward.



(iii) $y = (x + 1)^2$

Original function is $y = x^2$

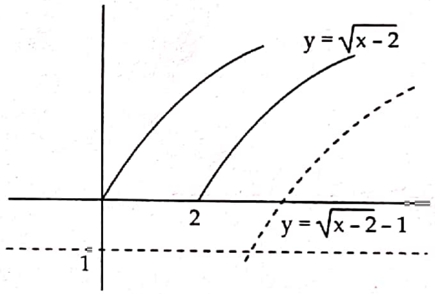
New function $y = (x + 1)^2$ is obtained by shifting the graph of $y = x^2$ at distance 1 unit to the left.



(iv) $y = \sqrt{x - 2} - 1$

The original function is $y = \sqrt{x}$.

The new function $y = \sqrt{x-2} - 1$ is obtained by shifting the graph $y = f(x)$ a distance 2 unit to the right followed by shifting a distance 1 unit down ward.



(v) $y = 1 - 2\sqrt{x + 3}$

The original function is $y = \sqrt{x}$

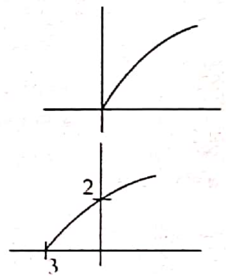
The new function $y = 1 - 2\sqrt{x + 3}$

New function is $y = \sqrt{x}$

Shift the graph $y = \sqrt{x}$ to left by 3 unit

$y = \sqrt{x + 3}$

Stretch the graph vertically by 2



$y = 2\sqrt{x+3}$

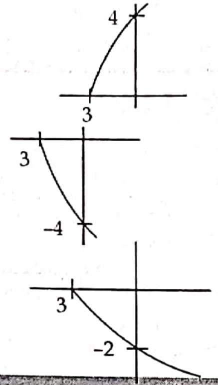
Reflect the graph about x-axis

$y = -2\sqrt{x+3}$

Shift the graph 1 unit upward.

$y = -2\sqrt{x+3} + 1$

i.e. $y = 1 - 2\sqrt{x+3}$

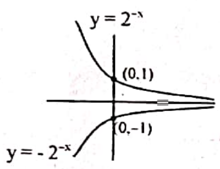


Exercise 1.4

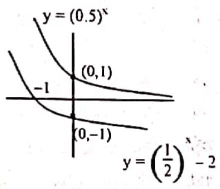
1. (i) $y = e^x - 2$ (ii) $y = e^{x-2}$ (iii) $y = -e^x$ (iv) $y = e^{-x}$ (v) $y = -e^{-x}$

2.

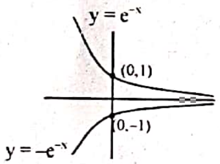
(i)

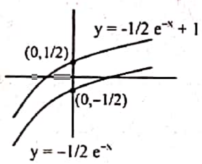


(ii)



(iii)





3.

(i) Since $f(x) = ca^x$ is passing through (1, 6) so, $6 = ca$ .... (1)

Also it is passing through (3, 24) so, $24 = ca^2$ .... (2)

Solving (1) and (2)

$a = 4$

use on (i), we get $c = \frac{3}{2}$

Hence, required exponential function's, $f(x) = \frac{3}{2}(4)^x$

(ii) Since, $f(x) = ca^x$ is passing through (1, 4/3) so, $\frac{4}{3} = ca$

i.e. $4 = 3ca$ .... (1)

Also it is passing through (–1, 4) so, $4 = \frac{c}{a}$

i.e. $c = 4a$ .... (2)

Solving (1) and (2),

$4 = 3 \times 4a \times a$

i.e. $a = \sqrt{3}$

Hence, $c = 4\sqrt{3}$

Thus, exponential function is $f(x) = 4\sqrt{3}\,(\sqrt{3})^x$

$f(x) = 4(\sqrt{3})^{x+1}$

$f(x) = 4(3)^{\frac{x+1}{2}}$

4.

(i) $f(x) = x^2 - 2x$, this is one to one function, because

$f(x_1) = f(x_2)$

$\Rightarrow x_1^2 - 2x_1 = x_2^2 - 2x_2$

$\Rightarrow x_1^2 - x_2^2 = 2(x_1 - x_2) = 0$

$\Rightarrow (x_1 - x_2)(x_1 + x_2 - 2) = 0$

$\Rightarrow (x_1 - x_2) = 0$

$\Rightarrow x_1 = x_2$

$\therefore$ f is one to one.

(ii) $f(x) = 10 - 3x$ is one to one, because

Let $f(x_1) = f(x_2)$

$\Rightarrow 10 - 3x_1 = 10 - 3x_2$

$\Rightarrow -3x_1 = -3x_2$

$\Rightarrow x_1 = x_2$

(iii) $g(x) = \frac{1}{x}$ is one to one, because

$g(x_1) = g(x_2)$

$\Rightarrow \frac{1}{x_1} = \frac{1}{x_2}$

$\Rightarrow x_1 = x_2$

(iv) $h(x) = 2 + |x|$ is not one to one because

for $1 \neq -1$

But $h(1) = h(-1) = 3$

$f(x) = 1 + \sqrt{2 + 3x}$

$y = 1 + \sqrt{2 + 3x}$

$\sqrt{2 + 3x} = y - 1$

$2 + 3x = (y - 1)^2$

$x = \dfrac{(y - 1)^2 - 2}{3}$

$f^{-1}(y) = \dfrac{y^2 - 2y - 1}{3}$

$f^{-1}(x) = \dfrac{x^2 - 2x - 1}{3}$

(ii) $f(x) = \dfrac{4x - 1}{2x} + 3$

$y = \dfrac{4x - 1}{2x} + 3$

$y - 3 = \dfrac{4x - 1}{2x}$

$2xy - 6x = 4x - 1$

$10x - 2xy = 1$

$x(10 - 2y) = 1$

$x = \dfrac{1}{2(5 - y)}$

$f^{-1}(y) = \dfrac{1}{2(5 - y)}$

$\therefore \quad f^{-1}(x) = \dfrac{1}{2(5 - y)}$

$f(x) = e^{2x-1}$

$y = e^{2x-1}$

$2x - y = \ln y$

$x = \dfrac{1 + \ln y}{2}$

$f^{-1}(y) = \dfrac{1 + \ln y}{2}$

$f^{-1}(x) = \dfrac{1}{2}(1 + \ln x)$

(iv) $y = \dfrac{e^x}{1 + 2e^x}$

$y + 2ye^x = e^x$

$e^x(1 - 2y) = y$

$e^x = \dfrac{y}{1 - 2y}$

$x = \ln\left(\dfrac{y}{1 - 2y}\right)$

$f^{-1}(y) = \ln\left(\dfrac{y}{1 - 2y}\right)$

$\therefore \quad f^{-1}(x) = \ln\left(\dfrac{x}{1 - 2x}\right)$

•••