Syllabus

Full Marks: 60 + 20 + 20
Pass Marks: 24 + 8 + 8
Credit Hrs: 3

Course No: CSC317
Nature of the Course: Theory + Lab

Course Description: The syllabus consists of introduction to system modeling and simulation of different types of systems. It includes the modeling of systems, its validation, verification and analysis of simulation output. It comprises the concept of queuing theory, random number generation as well as study of some simulation languages.

Course Objective: To make students understand the concept of simulation and modeling of real time systems.

Course Details

Unit 1: Introduction to Simulation **(6 Hours)**

System and System Environment, Components of System, Discrete and Continuous System, System Simulation, Model of a System, Types of Model, Use of Differential and Partial differential equations in Modeling, Advantages, Disadvantages and Limitations of Simulation, Application Areas, Phases in Simulation Study

Unit 2: Simulation of Continuous and Discrete System **(7 Hours)**

Continuous System Models, Analog Computer, Analog Methods, Hybrid Simulation, Digital-Analog Simulators, Feedback Systems Discrete Event Simulation, Representation of time, Simulation Clock and Time Management, Models of Arrival Processes - Poisson Processes, Non-stationary Poisson Processes, Batch Arrivals; Gathering statistics, Probability and Monte Carlo Simulation

Unit 3: Queuing System **(6 Hours)**

Characteristics and Structure of Basic Queuing System, Models of Queuing System, Queuing notation, Single server and Multiple server Queueing Systems, Measurement of Queueing System Performance, Elementary idea about networks of Queuing with particular emphasis to computer system, Applications of queuing system

Unit 4: Markov Chains **(2 Hours)**

Features, Process Examples, Applications

Unit 5: Random Numbers **(7 Hours)**

Random Numbers and its properties, Pseudo Random Numbers, Methods of generation of Random Number, Tests for Randomness - Uniformity and independence, Random Variate Generation

Unit 6: Verification and Validation **(4 Hours)**

Design of Simulation Models, Verification of Simulation Models, Calibration and Validation of the models, Three-Step Approach for Validation of Simulation Models, Accreditation of Models

Unit 7: Analysis of Simulation Output **(4 Hours)**

Confidence Intervals and Hypothesis Testing, Estimation Methods, Simulation run statistics, Replication of runs, Elimination of initial bias

Unit 8: Simulation of Computer Systems **(9 Hours)**

Simulation Tools, Simulation Languages: GPSS, Case Studies of different types of Simulation Models and Construction of sample mathematical models

Laboratory Work:

Practical should include the simulation of some real time systems (continuous and discrete event systems), Queuing Systems, Random Number generations as well as study of Simulation Tools and Language

Text Book:

1. Jerry Banks, John S. Carson, Barry L. Nelson, David M. Nicole, "Discrete Event system simulation", 5th Edition, Pearson Education

Reference Books:

1. Geoffrey Gordon: System Simulation
2. Law, "Simulation Modeling and Analysis", 5th Edition, McGraw-Hill

Table

1 Chapter INTRO

2 Chapter SIMUL

SYSTEM

A system is defined as an aggregation or assemblage of objects joined in some regular interaction or interdependence toward the accomplishment of some purpose.

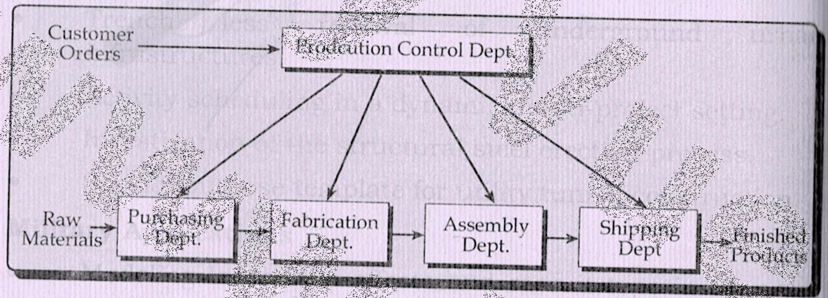

Fig 1.1: Production system

Example: Production System

In the above system there are certain distinct objects, each of which possesses properties of interest. There are also certain interactions occurring in the system that cause changes in the system.

Example: An aircraft under autopilot control

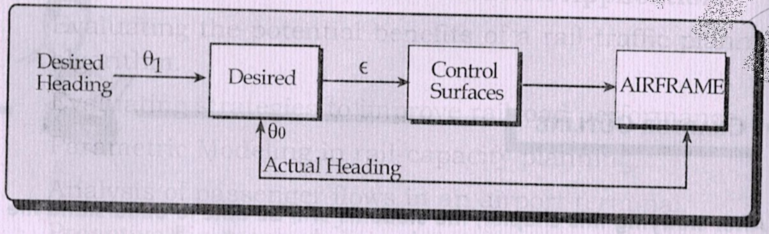

Fig 1.2: An Aircraft under autopilot control

COMPONENTS OF A SYSTEM

Entity

An entity is an object of interest in a system. For example in the above example in the factory system, departments, orders, parts and products are the entities.

Attribute

An attribute denotes the property of an entity.

Ex: Quantities for each order, type of part, or number of machines in a Department are attributes of factory system.

Activity

Any process causing changes in a system is called as an activity.

Ex: Manufacturing process of the department.

State of the System

The state of a system is defined as the collection of variables necessary to describe a system at any time, relative to the objective of study. In other words, state of the system mean a description of all the entities, attributes and activities as they exist at one point in time.

Event

An event is define as an instantaneous occurrence that may change the state of the system.

System Environment

The external components which interact with the system and produce necessary changes are said to constitute the system environment.

In modeling systems, it is necessary to decide on the boundary between the system and its environment. This decision may depend on the purpose of the study.

Ex: In a factory system, the factors controlling arrival of orders may be considered to be outside the factory but yet a part of the system environment. When, we consider the demand and supply of goods, there is certainly a relationship between the factory output and arrival of orders. This relationship is considered as an activity of the system.

Endogenous System

The term endogenous is used to describe activities and events

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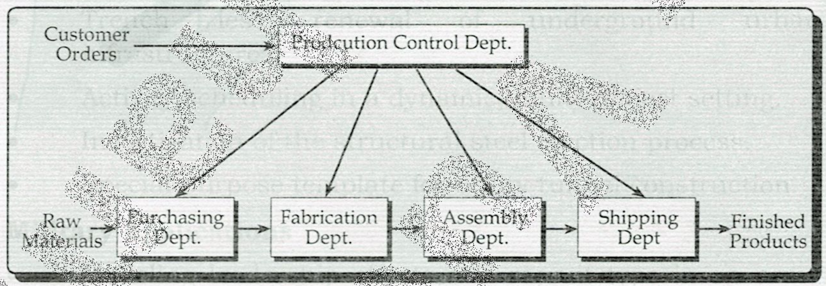

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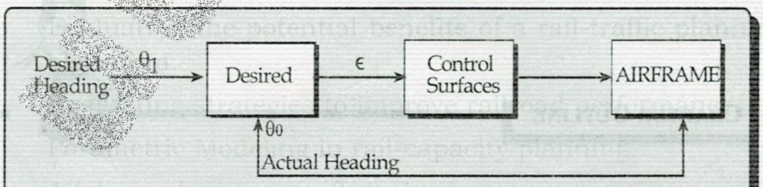

Fig 1.2: An Aircraft under autopilot control

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Endogenous System

The term endogenous is used to describe activities and events occurring within a system. Ex: Drawing cash in a bank.

Exogenous System

The term exogenous is used to describe activities and events in the environment that affect the system. Ex: Arrival of customers.

Closed System

A system for which there is no exogenous activity and event is said to be a closed. Ex: Water in an insulated flask.

Open system

A system for which there is exogenous activity and events sad to be an open. Ex: Bank system.

DISCRETE AND CONTINUOUS SYSTEMS

Continuous Systems

Systems in which the changes are predominantly smooth are called continuous system. Ex: Head of a water behind a dam.

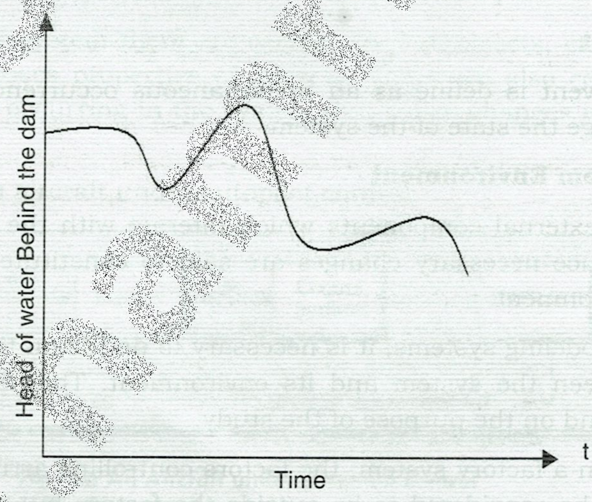

Figure 1.3: Continuous system state variable

DISCRETE SYSTEMS

Systems in which the changes are predominantly discontinuous are called discrete systems. Ex: Bank – the number of customer changes only when a customer arrives or when the service provided a customer is completed.

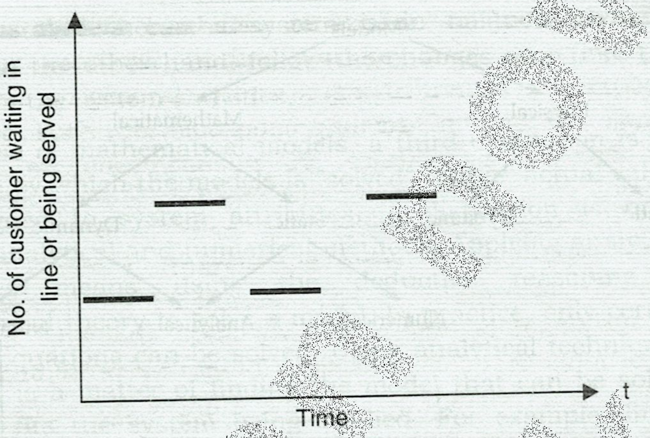

Figure 1.4: Discrete-system state variable

MODEL OF A SYSTEM

A model is defined as a representation of a system for the purpose of studying the system. It is necessary to consider only those aspects of the system that affect the problem under investigation. These aspects are represented in a model, and by definition it is a simplification of the system.

TYPES OF MODELS

The various types of models are

- Mathematical or Physical Model
- Static Model
- Dynamic Model
- Deterministic Model
- Stochastic Model
- Discrete Model
- Continuous Model

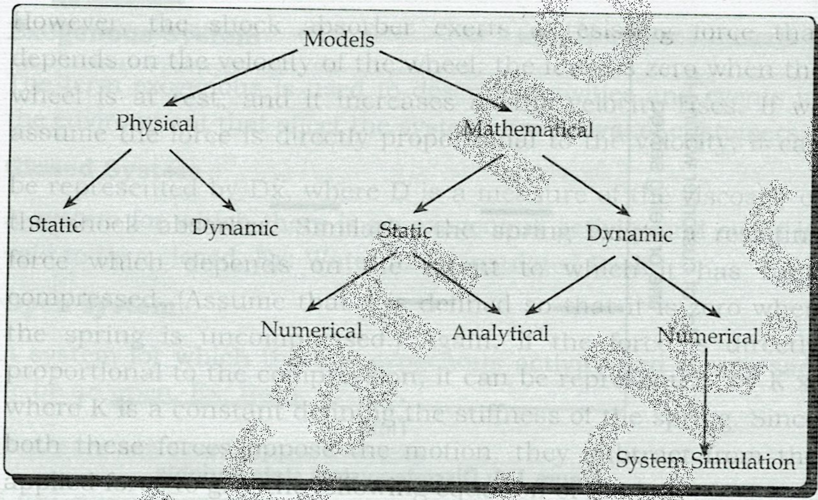

Fig 1.5: Types of Model

Models used in system studies have been classified in many ways. The classification that will be used her are illustrated in fig 1.5. Models will be separated into physical models or mathematical models.

Physical models are based on some analogy between such system as mechanical and electrical, or electrical and hydraulic. In a physical model of a system, the system attributes are represented by such measurements as a voltage or the position of a shaft. The system activities are reflected in the physical laws that drive the model. For example, the rate at which the shaft of a direct current motor turns depends upon the voltage applied to the motor. If the applied voltage is used to represent the velocity of a vehicle, then the number of revolutions of the shaft is a measure of the distance the vehicle has traveled; the higher the voltage, or velocity, the greater is the buildup of revolutions, or distance covered, in a given time.

Mathematical models, of course, use symbolic notation and mathematical equations to represent a system. The stem attributes are represented by variables, and the activities are represented by mathematical functions that interrelate the variables.

A second distinction will be between static models and dynamic models. Static models can only show the values that system attributes take when the system is in balance. Dynamic models, on the other hand, follow the changes over time that result from the system activities.

In the case of mathematical models, a third distinction is the technique by which the models is "solved" that is, actual values are assigned to system attributes. A distinction is made between analytical and numerical methods. Applying analytical techniques means using the deductive reasoning of mathematical theory to solve a model. In practice, only certain forms of equations can be solved. Using analytical techniques, therefore, is a matter of finding the model that can be solved and best fits the system being studied. For example, linear differential equations can be solved. Knowing this, an engineer who restricts the description of a system to that form will derive a model that can be solved analytically.

Numerical methods involve applying computational procedures to solve equations. To be strictly accurate, any assignment of numerical values that us mathematical tables involves numerical methods, since tables are derived numerically. The distinction being drawn here is that analytical methods produce solutions in tractable form, meaning a form where values can be assigned from available tables. Making use of an analytical solution m, in fact require a considerable amount of computation. For example, the solution may be derived in the form of a complicated integral which then needs to be expanded as a power series for evaluation. However, mathematical theory for making such expansions exists, and, in principle, any degree of accuracy in the solution is obtainable if sufficient effort is expended.

PHYSICAL MODEL

Static Physical Model

The best known examples of physical models are scale models. In shipbuilding, making a scale model provides a simple way of determining the exact measurements of the plates covering the hull, rather than having to produce drawings of complicated, three-dimensional shapes. Scientists have used models in which spheres represent atoms, and rods or specially shaped sheets of metal connect the spheres to represent atomic bonds. A model of this nature played an important role in the deciphering of the DNA molecule, work that was the subject of a Nobel Prize award. These models are static physical models. They are sometimes said to be iconic models, a term meaning "look-alike" .

Scale models are also used in wind tunnels and water tanks in the course of designing aircraft and ships. Although air is blown over the model, or the model is pulled through the water, these are static physical models because the measurements that are taken represent attributes of the system being studied under one set of equilibrium conditions. In this case, the measurements do not translate directly into system attribute values. Well known laws of similitude are used to convert measurements on the scale model to the values that would occur in the real system.

Sometimes, a static physical model is used as a means of solving equations with particular boundary conditions. There are many examples in the field of mathematical physics where the same equations apply to different physical phenomena. For example, the flow of heat and the distribution of electric charge through space can be related by common equations. In general, these equations can only be solved for simple-shaped bodies. In practice, solutions are needed for specific, complicated shapes. The distribution of heat in a body can be predicted by enclosing a space that has the same shape as the body, and measuring the charge in the space when the surface of the space has been electrified in a manner that reflects the way heat will be injected into the body.

Dynamic Physical Model

Dynamic physical models rely upon an analogy between the system being studied and some other system of a different nature, the analogy usually depending upon an underlying similarity in the forces governing the behavior of the systems. To illustrate this type of physical model, consider the two systems shown in Fig. 1.6.

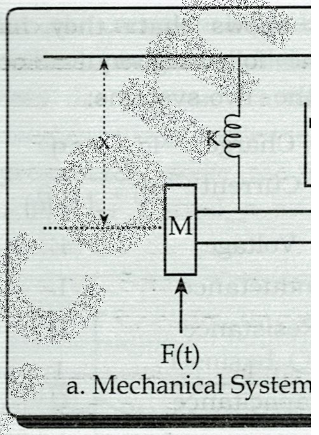

Fig: 1.6: Analogy betwe

Figure 1-6(a) represents a
force F(t) varying with
proportional to its exter
absorber that exerts a d
velocity of the mass. The s
the suspension of an auto
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$$M\ddot{x} + D\dot{x} + Kx = KF(t)$$

where X is the distan

M is the mass.

K is the stiffne

D is the damp

Figure 1.6(b) represents
inductance L, a resistance
in series with a voltage sou
the function E(t). If q is the
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following differential equat

$$L\ddot{q} + R\dot{q} + \frac{q}{C} = \frac{E(t)}{C}$$

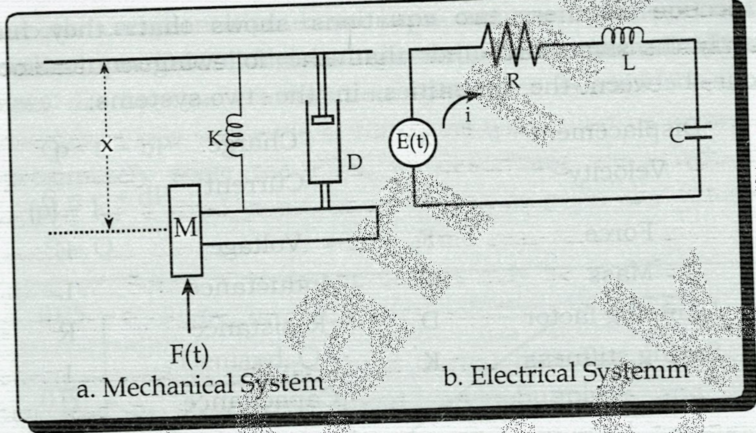

Fig: 1.6: Analogy between mechanical and electrical systems

Figure 1-6(a) represents a mass that is subject to an applied force F(t) varying with time, a spring whose force is proportional to its extension or contraction, and a shock absorber that exerts a damping force proportional to the velocity of the mass. The system might, for example, represent the suspension of an automobile wheel when the automobile body is assumed to be immobile in a vertical direction. It can be shown that the motion of the system is described by the following differential equation:

$$M\ddot{x} + D\dot{x} + Kx = KF(t)$$

where X is the distance moved,

M is the mass..,

K is the stiffness of the spring,

D is the damping factor of the shock absorber.

Figure 1.6(b) represents an electrical circuit with an inductance L, a resistance R, and a capacitance C, connected in series with a voltage source that varies in time according to the function E(t). If q is the charge on the capacitance, it can be shown that the behavior of the circuit is governed by the following differential equation:

$$L\ddot{q} + R\dot{q} + \frac{q}{C} = \frac{E(t)}{C}$$

Inspection of these two equations shows that they have exactly the same form and that the following equivalences occur between the quantities in the two systems:

Displacement	x	Charge	q
Velocity	$\dot{x}$	Current	$I = (\dot{q})$
Force	F	Voltage	E
Mass	M	Inductance	L
Damping factor	D	Resistance	R
Spring stiffness	K	$\frac{1}{\text{Capacitance}}$	$\frac{1}{C}$

MATHEMATICAL MODEL

Static Mathematical Model

A static model gives the relationships between the system attributes when the system is in equilibrium. If the point of equilibrium is changed by altering any o the attribute values, the model enables the new values for all the attributes to be derived but does not show the way in which they changed to their new values.

For example, in marketing a commodity there is a balance between the supply and demand for the commodity. Both factors depend upon price: a simple market mode! will show what is the price at which the balance occurs.

Demand for the commodity will be low when the price is high, and it will increase as the price drops. The relationship between demand, denoted by Q, and price, denoted by P, might be represented by the straight line marked "Demand" in Fig. 1.7 On the other hand, the supply can be expected to increase as the price increases, because the suppliers see an opportunity for more revenue. Suppose supply, denoted by S, is plotted against price, and the relationship is the straight line marked "Supply" in Fig. 1.7. If conditions remain stable, the price will settle to the point at which the two lines cross, because that is where the supply equals the demand.

Since the relationships
complete market mod
follows:

$Q = a - bP$

$S = c + dP$

$S = Q$

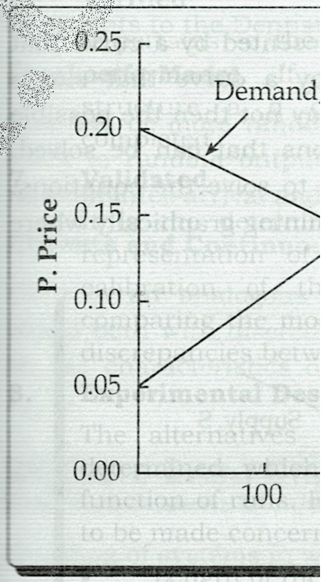

Figure 1

The last equation states
cleared; it says supply e
price to which the marke
For the model to corre
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For realistic, positive r
be positive. Figure 1.7 ha
of the coefficients:

$a = 60$

$b = 3,000$

$c = -100$

$d = 2,000$

Since the relationships have been assumed linear, the complete market model can be written mathematically as follows:

$$Q = a - bP$$

$$S = c + dP$$

$$S = Q$$

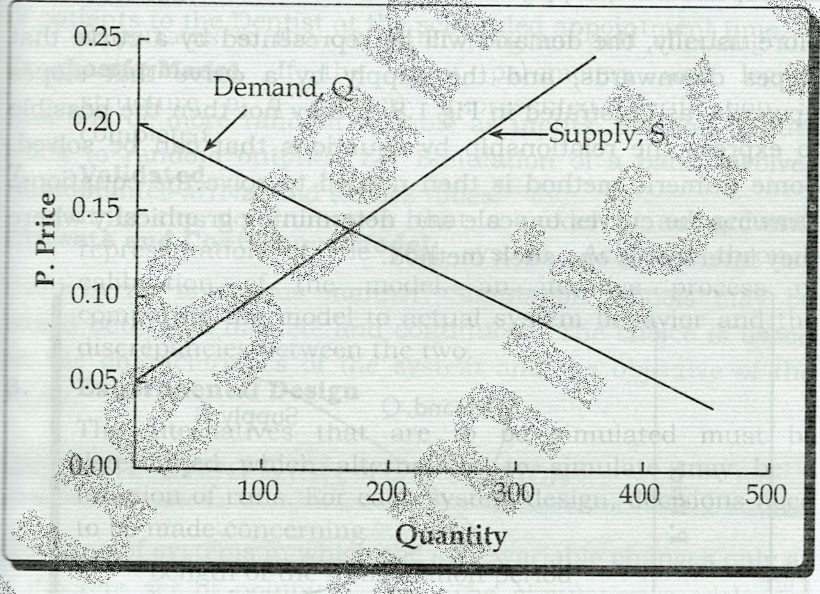

Figure 1.7: Linear Market model

The last equation states the condition for the market to be cleared; it says supply equals demand and, so, determines the price to which the market will settle.

For the model to correspond to normal market conditions in which demand goes down and supply increases as price goes up the coefficients b and d need to be positive numbers. For realistic, positive results, the coefficient a must also be positive. Figure 1.7 has been plotted for the following values of the coefficients:

$a = 60$

$b = 3{,}000$

$c = -100$

$d = 2{,}000$

The fact that linear relationships have been assumed allows the model to be solved analytically. The equilibrium market price, in fact, is given by the following expression:

$$P = \frac{a - c}{b + d}$$

With the chosen values, the equilibrium price is 0.14, which corresponds to a supply of 180.

More usually, the demand will be represented by a curve that slopes downwards, and the supply by a curve that slopes upwards, as illustrated in Fig 1.8. It may not then the possible to express the relationship by equations that can be solved. Some numeric method is then needed to solve the equations. Drawing the curves to scale and determining graphically where they intersect is one such method.

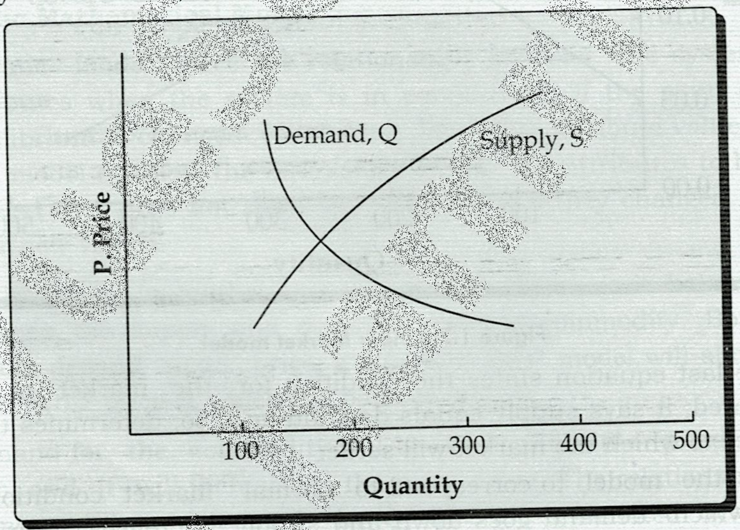

Figure 1.8: Non-linear Market model

Mathematical Model

Uses symbolic notation and the mathematical equations to represent a system.

Static Model

Represents a system at a particular point of time and also known as Monte-Carlo simulation.

Dynamic Model

Represents systems
of a bank

Deterministic Mod

Contains no rando
inputs which will re
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Stochastic Model

Has one or more ra
leads to random ou
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Discrete and Conti

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DISCRETE-EVENT

Modeling of systems
a discrete set of po
analyzed by numeric

Analytical methods
mathematics to solv
be used to determ
inventory models.

Numerical methods
'runs', which is gene
observations are col
true system perform

Real-world simulati
with the help of con
simulation manually

Dynamic Model

Represents systems as they change over time. Ex: Simulation of a bank

Deterministic Model

Contains no random variables. They have a known set of inputs which will result in a unique set of outputs. Ex: Arrival of patients to the Dentist at the scheduled appointment time.

Stochastic Model

Has one or more random variable as inputs. Random inputs leads to random outputs. Ex: Simulation of a bank involves random inter-arrival and service times.

Discrete and Continuous Model

Used in an analogous manner. Simulation models may be mixed both with discrete and continuous. The choice is based on the characteristics of the system and the objective of the study.

DISCRETE-EVENT SYSTEM SIMULATION

Modeling of systems in which the state variable changes only at a discrete set of points in time. The simulation models are analyzed by numerical rather than by analytical methods.

Analytical methods employ the deductive reasoning of mathematics to solve the model. Eg: Differential calculus can be used to determine the minimum cost policy for some inventory models.

Numerical methods use computational procedures and are 'runs', which is generated based on the model assumptions and observations are collected to be analyzed and to estimate the true system performance measures.

Real-world simulation is so vast, whose runs are conducted with the help of computer. Much insight can be obtained by simulation manually which is applicable for small systems.

PHYSICAL MODEL

Simulation

Simulation is the imitation of the operation of a real-world process or system over time. Simulation involves the generation of an artificial history of the system and the observation of that artificial history to draw inferences concerning the operating characteristics of the real system that is represented. Simulation is an indispensable problem solving methodology for the solution of many real-world problems. Simulation is used to describe and analyze the behavior of a system, ask what-if questions about the real system, and aid in the design of real systems. Both existing and conceptual systems can be modeled with simulation.

The assumptions are expressed in

- Mathematical relationships
- Logical relationships
- Symbolic relationships between the entities of the system.

The model solved by mathematical methods such as differential calculus, probability theory, algebraic methods has the solution usually consists of one or more numerical parameters which are called measures of performance.

When Simulation is the Appropriate Tool

- Simulation enables the study of and experimentation with the internal interactions of a complex system, or of a subsystem within a complex system.
- Informational, organizational and environmental changes can be simulated and the effect of those alternations on the model's behavior can be observer.
- The knowledge gained in designing a simulation model can be of great value toward suggesting improvement in the system under investigation.

- By changing simulation inputs and observing the resulting outputs, valuable insight may be obtained into which variables are most important and how variables interact.
- Simulation can be used as a pedagogical device to reinforce analytic solution methodologies.
- Simulation can be used to experiment with new designs or policies prior to implementation, so as to prepare for what may happen.
- Simulation can be used to verify analytic solutions.
- By simulating different capabilities for a machine, requirements can be determined.
- Simulation models designed for training, allow learning without the cost and disruption of on-the-job learning.
- Animation shows a system in simulated operation so that the plan can be visualized.
- The modern system (factory, water fabrication plant, service organization, etc.) is so complex that the interactions can be treated only through simulation.

When Simulation is Not Appropriate

- Simulation should be used when the problem cannot be solved using common sense.
- Simulation should not be used if the problem can be solved analytically.
- Simulation should not be used, if it is easier to perform direct experiments.
- Simulation should not be used, if the costs exceeds savings.
- Simulation should not be performed, if the resources or time are not available.
- If no data is available, not even estimate simulation is not advised.

- If there is not enough time or the person are not available, simulation is not appropriate.
- If managers have unreasonable expectation say, too much soon – or the power of simulation is over estimated, simulation may not be appropriate.
- If system behavior is too complex or cannot be defined, simulation is not appropriate.

Advantages of Simulation

- Simulation can also be used to study systems in the design stage.
- Simulation models are run rather than solver.
- New policies, operating procedures, decision rules, information flow, etc can be explored without disrupting the ongoing operations of the real system.
- New hardware designs, physical layouts, transportation systems can be tested without committing resources for their acquisition.
- Hypotheses about how or why certain phenomena occur can be tested for feasibility.
- Time can be compressed or expanded allowing for a speedup or slowdown of the phenomena under investigation.
- Insight can be obtained about the interaction of variables.
- Insight can be obtained about the importance of variables to the performance of the system.
- Bottleneck analysis can be performed indication where work-in-process, information materials and so on are being excessively delayed.
- A simulation study can help in understanding how the system operates rather than how individuals think the system operates.
- "what-if" questions can be answered. Useful in the d sign of new systems.

Disadvantages of simulation

- Model building requires special training.
- Simulation results may be difficult to interpret.
- Simulation modeling and analysis can be time consuming and expensive.
- Simulation is used in some cases when an analytical solution is possible or even preferable.

APPLICATIONS OF SIMULATION

Manufacturing Applications

- Analysis of electronics assembly operations
- Design and evaluation of a selective assembly station for high-precision scroll compressor shells.
- Comparison of dispatching rules for semiconductor manufacturing using large facility models.
- Evaluation of cluster tool throughput for thin-film head production.
- Determining optimal lot size for a semiconductor backend factory.
- Optimization of cycle time and utilization in semiconductor test manufacturing.
- Analysis of storage and retrieval strategies in a warehouse.
- Investigation of dynamics in a service oriented supply chain.
- Model for an Army chemical munitions disposal facility.

Semiconductor Manufacturing

- Comparison of dispatching rules using large-facility models.
- The corrupting influence of variability.
- A new lot-release rule for wafer fabrications.
- Assessment of potential gains in productivity due to proactive retied management.
- Comparison of a 200 mm and 300 mm X-ray lithography cell.

- Capacity planning with time constraints between operations.
- 300 mm logistic system risk reduction.

Construction Engineering

- Construction of a dam embankment.
- Trench less renewal of underground urban infrastructures.
- Activity scheduling in a dynamic, multi-project setting.
- Investigation of the structural steel erection process.
- Special purpose template for utility tunnel construction.

Military Applications

- Modeling leadership effects and recruit type in an army recruiting station.
- Design and test of an intelligent controller for autonomous underwater vehicles.
- Modeling military requirements for non-war fighting operations.
- Multi trajectory performance for varying scenario sizes.
- Using adaptive agents in U.S. Air Force retention.

Logistics, Transportation and Distribution Applications

- Evaluating the potential benefits of a rail-traffic planning algorithm.
- Evaluating strategies to improve railroad performance.
- Parametric Modeling in rail-capacity planning.
- Analysis of passenger flows in an airport terminal.
- Proactive flight-schedule evaluation.
- Logistic issues in autonomous food production systems for extended duration space exploration.
- Sizing industrial rail-car fleets.
- Production distribution in newspaper industry.
- Design of a toll plaza
- Choosing between rental-car locations.
- Quick response replenishment.

Business Process Simulation

- Impact of connection bank redesign on airport gate assignment.
- Product development program planning.
- Reconciliation of business and system modeling.
- Personal forecasting and strategic workforce planning.

Human Systems

- Modeling human performance in complex systems.
- Studying the human element in our traffic control.

DIFFERENTIAL EQUATIONS

Modeling of a system whether system is continuous or discrete heavily used the concepts of Ordinary Differential Equation (ODE), Partial Differential Equation (PDE) and probability and statistics. Even we can say that without these two branch of mathematics modeling is just impossible.

Continuous processes occur everywhere as we will learn in unit V. Here, we are interested in cases with discrete variables, some examples of continuous process are

- An object falling to the ground
- The motion of the planets orbiting the sun
- The current and voltage in an electrical circuit
- The level of alcohol in my blood on January 1st, 2005
- The populations of a predator and its prey

In almost all above cases, the relationships between the variables and its rate of change i.e. its derivative are defined by an Ordinary Differential Equations (ODEs) are very important in all branches of Science and Engineering. ODEs form the basis for the simulation of almost all continuous phenomena. Understanding ODEs is essential for understanding natural and technical processes.

An example of a linear differential equation with consta coefficients to describe the wheel suspension system of a automobile. The equation derived was

$$M\ddot{x} + D\dot{x} + Kx = KF(t)$$

Note that the dependent variable x appears together with its first and second derivatives $\ddot{x}$ and $\dot{x}$, and that the term involving these quantities are multiplied by constan coefficients and added. The quantity F(t) is an input t the system , depending upon the independent variable t. A linear differential equation with constant coefficients is always of this form, although derivatives of any order may enter the equation. If the dependent variable or any of its derivatives appear in any other form, such as being raised t a power, or are combined in any other way- for example, by being multiplied together, the differential equation is said to be nonlinear.

When more than one independent variable occurs in a differential equation, the equation is said to be a partial differential equation. It can involve the derivatives of the same dependent variable with respect to each of the independent variables. An example is an equation describing the flow of heat in a three dimensional body. There are four independent variables, representing the three dimensions and time, and one dependent variable, representing temperature. The general method of solving such equations numerically is to use finite differences to convert the equations into a set of ordinary (that is, non-partial) differential equations, which can be solved by the methods that are about to be described. There are programming languages specially designed for this type of equation which will perform the function of constructing the finite differences.

Differential equations, both linear and nonlinear, occur repeatedly in scientific and engineering studies. The reason for this prominence is that most physical and chemical processes involve rates of change, which require differential equations for their mathematical description. Since a differential coefficient can also represent a growth rate,

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would be as follows:

$$M\ddot{x} = KF(t)$$

continuous models can also be applied to problems of a social or economic nature where there is a need to understand the general effects of growth trends. We shall be discussing such socio-economic systems in the next chapter. For now, we concentrate on the solution of scientific and engineering types of problems which generally require a higher degree of accuracy than just establishing trends. This, however, is only a separation of types of application which have different emphases. The continuous system simulation methods that will be described in this chapter can be used for either type of application.

To illustrate how differential equations can represent engineering problems we will show how the equation describing the automobile wheel suspension system is derived from mechanical principles. If we pick a point of the wheel as a reference point from which to measure the vertical displacement of the wheel, the variable x can represent the displacement of the point, taking x to be positive for an upward movement. (See Fig. 1.6.) The velocity of the wheel, in the vertical direction, is the rate of change of position, which is the first differential, $\dot{x}$. The acceleration of the wheel, in the vertical direction, is the rate of change of the velocity, which is the second differential, $\ddot{x}$.

The mechanical law that determines the relationship between applied force and movement of a body states that the acceleration of the body is proportional to the force. In particular, if there is no force there is no acceleration. The body then remains stationary, if no force has been acting upon it, or it continues moving at a constant velocity-a fact that, at one time, would have seemed to be merely an abstraction but is familiar in this age of space travel, since the ideal condition of no force acting on a body is obtainable in space.

The coefficient of proportionality between the force and acceleration is the mass of the body ; so, in the case of the automobile wheel, where the mass is M and the applied force is KF(t), the equation of motion in the absence of other forces would be as follows:

$$M\ddot{x} = KF(t)$$

However, the shock absorber exerts a resisting force that depends on the velocity of the wheel: the force is zero when the wheel is at rest, and it increases as the velocity rises. If we assume the force is directly proportional to the velocity, it can be represented by $D\dot{x}$, where D is a measure of the viscosity of the shock absorber. Similarly, the spring exerts a resisting force which depends on the extent to which it has been compressed. (Assume that x is defined so that it is zero when the spring is uncompressed.) Again, if the force is directly proportional to the compression, it can be represented by K x, where K is a constant defining the stiffness of the spring. Since both these forces oppose the motion, they subtract from the applied force to give the following equation of motion:

$$M\ddot{x} = KF(t) - D\dot{x} - Kx$$

This is a linear differential equation, with constant coefficients, and, as mentioned before, it can be solved analytically. A more accurate model, however, would not assume the restraining forces are linear functions of the motion variables, and so the coefficients would not be linear. Also, there are physical limits to the amount of movement that is possible, which places another nonlinearity on the equation. The more realistic model is unlikely to be soluble by analytic methods, so it would become the basis of a continuous system simulation study.

In Sec. 1-8 the equation that has just been derived was also used to describe an electrical system. We will not derive the equation for that interpretation, but the same general principles apply. The quantity being measured in that case is the electrical charge of the condenser. The physical laws of electricity relating current and voltage to charge introduce the first two derivatives of the charge, and so lead to a linear differential equation of the same form as that for the automobile wheel, but with different interpretations of the constant coefficients. Similar examples can be taken from many other fields of engineering.

Partial Differential Equations

Like Ordinary Differential Equations, Partial Differential Equations are equations to be solved in which the unknown element is a function, but in PDEs the function is one of several variables, and so of course the known information relates the function and its partial derivatives with respect to the several variables. Again, one generally looks for qualitative statements about the solution. For example, in many cases, solutions exist only if some of the parameters lie in a specific set (say, the set of integers). Various broad families of PDE's admit general statements about the behaviour of their solutions. This area has a long-standing close relationship with the physical sciences, especially physics, thermodynamics, and quantum mechanics, for many of the topics in the field, the origins of the problem and the qualitative nature of the solutions are best understood by describing the corresponding result in physics.

Roughly corresponding to the initial values in an ODE problem, PDEs are usually solved in the presence of *boundary conditions*. For example, the Dirichlet problem (actually introduced by Riemann) asks for the solution of the Laplace condition on an open subset D of the plane, with the added condition that the value of u on the boundary of D was to be some prescribed function f. (Physically this corresponds to asking, for example, for the steady-state distribution of electrical charge within D when prescribed voltages are applied around the boundary.) It is a nontrivial task to determine how much boundary information is appropriate for a given PDE.

Linear differential equations occur perhaps most frequently in applications (in settings in which a superposition principle is appropriate.) When these differential equations are first-order, they share many features with ordinary differential equations. (More precisely, they correspond to *families* of ODEs, in which considerable attention must be focused on the dependence of the solutions on the parameters.)

Historically, three equations were of fundamental interest and exhibit distinctive behaviour. These led to the clarification of three types of second-order linear differential equations of great interest. The Laplace equation

$$\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} = 0$$

applies to potential energy functions u=u(x,y) for a conservative force field in the plane. PDEs of this type are called ***elliptic***. The Heat Equation

$$\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} = \frac{du}{dt}$$

applies to the temperature distribution u(x,y) in the plane when heat is allowed to flow from warm areas to cool ones. PDEs of this type are ***parabolic***. The Wave Equation

$$\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} = \frac{d^2u}{dt^2}$$

applies to the heights u(x,y) of vibrating membranes and other wave functions. PDEs of this type are called ***hyperbolic***. The analyses of these three types of equations are quite distinct in character. Allowing non-constant coefficients, we see that the solution of a general second-order linear PDE may change character from point to point. These behaviors generalize to nonlinear PDEs as well.

A general linear PDE may be viewed as seeking the kernel of a linear map defined between appropriate function spaces. (Determining which function space is best suited to the problem is itself a nontrivial problem and requires careful functional analysis as well as a consideration of the origin of the equation. Indeed, it is the analysis of PDEs which tends to drive the development of classes of topological vector spaces.) The perspective of differential operators allows the use of general tools from linear algebra, including eigen space decomposition (spectral theory) and index theory.

Modern approaches seek methods applicable to non-linear PDEs as well as linear ones. In this context existence and uniqueness results, and theorems concerning the regularity of solutions, are more difficult. Since it is unlikely that explicit solutions can be obtained for any but the most special of problems, methods of "solving" the PDEs involve analysis within the appropriate function space: for example, seeking convergence of a sequence of functions which can be shown to approximately solve the PDE, or describing the sought for function as a fixed point under a self-map on the function space, or as the point at which some real-valued function is minimized. Some of these approaches may be modified to give algorithms for estimating numerical solutions to a PDE.

Generalizations of results about partial differential equations often lead to statements about function spaces and other topological vector spaces. For example, integral techniques (solving a differential equation by computing a convolution, say) lead to integral operators (transforms on functions spaces); these and differential operators lead in turn to general pseudo differential operators on function spaces.

Applications and Related Fields of PDE

1. Differential geometry
2. Global analysis, analysis on manifolds
3. Probability theory and stochastic processes
4. Numerical analysis
5. Mechanics of solids
6. Fluid mechanics
7. Optics, electromagnetic theory
8. Classical thermodynamics, heat transfer
9. Quantum Theory
10. Statistical mechanics, structure of matter
11. Relativity and gravitational theory
12. Geophysics
13. Biology and other natural sciences
14. Systems theory; control

STEPS IN A SIMULATION STUDY

1. **Problem formulation**

   Every study begins with a statement of the problem, provided by policy makers. Analyst ensures its clearly understood. If it is developed by analyst policy makers should understand and agree with it.

2. **Setting of objectives and overall project plan**

   The objectives indicate the questions to be answered by simulation. At this point a determination should be made concerning whether simulation is the appropriate methodology. Assuming s appropriate, the overall project plan should include

   - A statement of the alternative systems
   - A method for evaluating the effectiveness of these alternatives
   - Plans for the study in terms of the number of people involved
   - Cost of the study
   - The number of days required to accomplish each phase of the work with the anticipated results.

3. **Model conceptualization**

   The construction of a model of a system is probably as much art as science. The art of modeling is enhanced by an ability

   - To abstract the essential features of a problem
   - To select and modify basic assumptions that characterize the system
   - To enrich and elaborate the model until a useful approximation results

   Thus, it is best to start with a simple model and build toward greater complexity. Model conceptualization enhance the quality of the resulting model and increase the confidence of the model user in the application of the model.

4. **Data collection**

   There is a constant interplay between the construction of model and the collection of needed input data. Done in the early stages. Objective kind of data are to be collected.

5. **Model translation**

Real-world systems result in models that require a great deal of information storage and computation. It can be programmed by using simulation languages or special purpose simulation software. Simulation languages are powerful and flexible. Simulation software models development time can be reduced.

6. **Verified**

It pertains to the computer program and checking the performance. If the input parameters and logical structure a d correctly represented, verification is completed.

7. **Validated**

It is the determination that a model is an accurate representation of the real system. Achieved through calibration of the model, an iterative process of comparing the model to actual system behavior and the discrepancies between the two.

8. **Experimental Design**

The alternatives that are to be simulated must be determined which alternatives to simulate may be a function of runs. For each system design, decisions need to be made concerning

- Length of the initialization period
- Length of simulation runs
- Number of replication to be made of each run

9. **Production runs and analysis**

They are used to estimate measures of performance for the system designs that are being simulated.

10. **More runs**

Based on the analysis of runs that have been completed. The analyst determines if additional runs are needed and what design those additional experiments should follow.

11. **Documentation and reporting**

Two types of documentation.

- Program documentation
- Process documentation

Program documentation

Can be used again by the same or different analysts to understand how the program operates. Further modification will be easier. Model users can change the input parameters for better performance.

Process documentation

Gives the history of a simulation project. The result of all analysis should be reported clearly and concisely in a final report. This enable to review the final formulation a d alternatives, results of the experiments and the recommended solution to the problem. The final report provides a vehicle of certification.

12. Implementation

Success depends on the previous steps. If the model user has been thoroughly involved and understands the nature of the model and its outputs, likelihood of a vigorous implementation is enhanced.

The simulation model building can be broken into 4 phases.

I Phase

- Consists of steps 1 and 2
- It is period of discovery/orientation
- The analyst may have to restart the process if it is not fine-tuned
- Recalibrations and clarifications may occur in this phase or another phase.

II Phase

- Consists of steps 3,4,5,6 and 7
- A continuing interplay is required among the steps
- Exclusion of model user results in implications during implementation

III Phase

- Consists of steps 8,9 and 10
- Conceives a thorough plan for experimenting
- Discrete-event stochastic is a statistical experiment
- The output variables are estimates that contain random error and therefore proper statistical analysis is required.

IV Phase

- Consists of steps 11 and 12
- Successful implementation depends on the involvement of user and every steps successful completion.

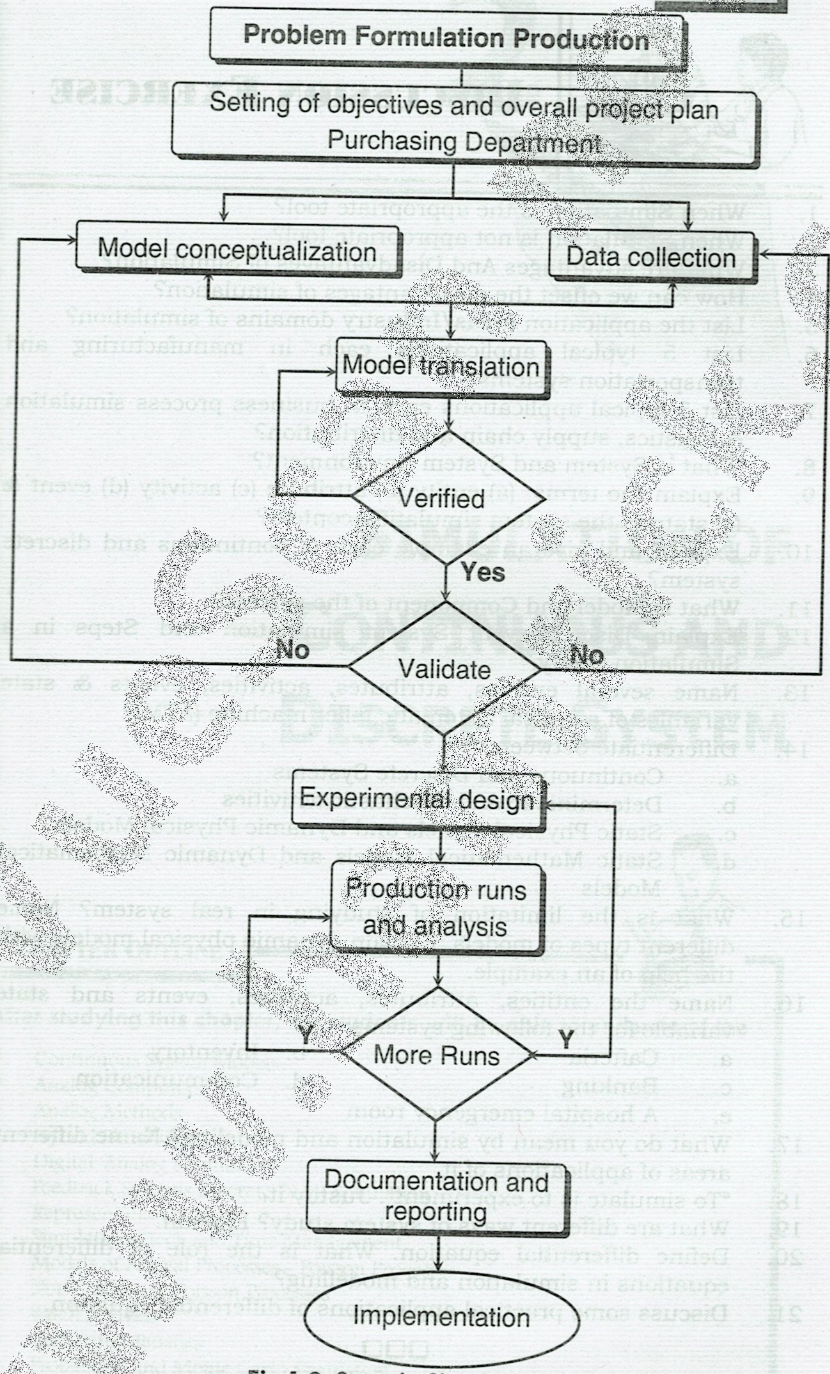

Fig 1.9: Steps in Simulation

DISCUSSION EXERCISE

1. When Simulation is the appropriate tool?
2. When Simulation is not appropriate tool?
3. What are advantages And Disadvantages of Simulation?
4. How can we offset the disadvantages of simulation?
5. List the application areas/Industry domains of simulation?
6. List 5 typical applications each in manufacturing and transportation systems?
7. List 5 typical applications each in business process simulation & logistics, supply chain and distribution?
8. What is System and System Environment?
9. Explain the terms: (a) entity (b) attribute (c) activity (d) event & (e) state in the system simulation context?
10. Explain and give an example each of continuous and discrete system?
11. What is Model and Component of the system?
12. Explain Discrete-event System simulation and Steps in a Simulation Study.
13. Name several entities, attributes, activities, events & state variables of a typical automatic teller machine (ATM)?
14. Differentiate between:
    a. Continuous and Discrete Systems
    b. Deterministic and Stochastic activities
    c. Static Physical Models and Dynamic Physical Models
    d. Static Mathematical Models and Dynamic Mathematical Models
15. What is the limitation of studying in real system? Name different types of models. Explain dynamic physical models with the help of an example.
16. Name the entities, attributes, activities, events and state variable for the following systems:
    a. Cafteria
    b. Inventory
    c. Banking
    d. Communication
    e. A hospital emergency room
17. What do you mean by simulation and modeling? Name different areas of applications of it.
18. "To simulate is to experiment". Justify it.
19. What are different ways of system study? Explain.
20. Define differential equation. What is the role of differential equations in simulation and modelling?
21. Discuss some practical applications of differential equation.

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