Tribhuvan University

Institute of Science and Technology

2069

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Bachelor Level/First Year/ Second Semester/ Science

Computer Science and Information Technology

(MTH.155 – Linear Algebra)

Full Marks: 80

Pass Marks: 32

Time: 3hours

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all questions:

<u>Group A</u> **(10 x 2 = 20)**

1. What do you mean by linearly independent set and linearly dependent set of vectors?

2. Verify that $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ is an eigen vector of $\begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix}$.

3. What do you mean by consistent equations? Give suitable examples.

4. What do you mean by change of basis in R^n?

5. Find the dimension of the vector spanned by (1, 1, 0) and (0, 1, 0).

6. When is a linear transformation invertible.

7. Find the rank of AB where

   $A = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$ $and\ B = [1 \quad 4 \quad 5]$.

8. Is $\lambda_1 = -2$ an Eigen value of $\begin{pmatrix} 7 & 3 \\ 3 & -1 \end{pmatrix}$?

9. Define Kernel and image of linear transformation.

10. What is meant by Discrete dynamical system? Give suitable example.

Group B (5 x 4 = 20)

11. Let $T: R^3 \rightarrow R^3$ be the linear transformation defined by T(x, y, z) = (x, y, x-2y). Find a basis and dimension of (a) Ker T (b) Im T.

12. Show that the following vectors are linearly independent: (1, 1, 2), (3, 1, 2), (0, 1, 4).

13. Find the matrix representation of linear transformation $T: R^2 \longrightarrow R^2$ defined by T(x, y) = (x + 2y) relative to the standard basis.

14. Is the set of vectors {91, 0, 1), (0, 1, 0), (-1, 0, 1)} orthogonal? Obtain the corresponding orthonomal set in R^3.

15. Let the four vertices O(0, 0), A(1, 0), B(0, 1) and C(1, 1) f a unit square be represented by a 2 x 4 matrix : $\begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$. Investigate and interpret geometrically the effect of pre-multiplication of this matric by the 2 x 2 matrix: $\begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$.

OR

State and prove orthogonality property for any two non-zero vectors in $\mathbf{R}^n$.

Group C (5 x 8 = 40)

16. Find a matrix A whose inverse is

$$A^{-1} = \begin{bmatrix} 1 & -3 & 2 \\ -3 & 3 & -1 \\ 2 & -1 & 0 \end{bmatrix}$$

17. Test the consistency and solve

$$x + y + z = 4$$
$$x + 2y + 2z = 2$$
$$2x + 2y + z = 5$$

OR

Verify Cayley Hamilton theorem for matrix $A = \begin{bmatrix} 6 & 2 & -1 \\ -6 & -1 & 2 \\ 7 & 2 & -2 \end{bmatrix}$.

18. The set of matrices of the form

$$\begin{bmatrix} 0 & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & a_{32} & a_{33} \end{bmatrix}$$

is a subspace of the vector 3 x 3 matrices. Verify it.

19. Let V and W be vector spaces over a field F of real numbers. Let dim V =n and dim W =m. Let {e_1, e_2, …….., e_n} be a basis of V and {f_1, f_2, ………., f_m} be a basis of W. Then, prove that each linear transformation T:V→W can be represented by an $m \times n$ matrix A with elements from F such that

Y = AX

Where $X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ $and\ Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix}$

Are column matrices of coordinates of $v \in V$ relative to its basis and coordinates of $w \in W$ relative to its basis respectively.

OR

Compute the multiplication partitioned matrices for

$$A = \begin{bmatrix} 2 & -3 & 1 & 0 & -4 \\ 1 & 5 & -2 & 3 & -1 \\ 0 & 4 & -2 & 7 & -1 \end{bmatrix} \ and\ B = \begin{bmatrix} 6 & 4 \\ -2 & 1 \\ -3 & 7 \\ -1 & 3 \\ 5 & 2 \end{bmatrix}$$

20. Find the equation $y = a_0 + a_1 x$ for the least squares line that best fits the data points (2, 1), (5, 2), (7, 3), (8, 3).