1) Weight (kg) of sample of 35 workers in a company are found as follows -

56	78	65	49	63	58	70	61	53	69	57	69	90	78	64
71	65	49	56	59	50	57	62	70	68	54	49	87	68	71
55	78	80	73	85										

a) Is mean weight of workers 64 at 1% level of significance?
b) Find 99% confidence limit of mean weight of workers in company.

<u>Working Steps:</u>

1) Calculate the sample mean ($\bar{x}$):

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

2) Calculate the sample standard deviation (S):

$$S = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$$

3) Calculate the standard error of the mean $S.E.(\bar{x})$:

$$S.E.(\bar{x}) = \frac{S}{\sqrt{n}}$$

4) Calculate Z:

$$Z = \frac{\bar{x} - \mu_0}{S.E.(\bar{x})}$$

where, μ_0 is the hypothesized pop^n mean.

5) Calculate p value:

$$p = Prob(Z > |Z_{calc}|)$$

6) Compare p value with given level of significance,
If $p \leq \alpha$; Reject H_0.
If $p > \alpha$; Accept H_0.

7) For Confidence limit (C.I.):

$$C.I. = \bar{x} \pm S.E.(\bar{x}) \times Z_\alpha$$

$$= \bar{x} \pm \frac{S}{\sqrt{n}} \times Z_\alpha$$

1

(a)

Ans Problem to test :

$H_0 : \mu = 64$

$H_1 : \mu \neq 64$

Output :

One Sample t-test

data : weight

t = 0.71176 , d.f. = 34 , p-value = 0.4815

alternative hypothesis : true mean is not equal to 64

99 percent confidence interval :

60.19525 70.49047

sample estimates:

mean of x = 65.34286

From above result,

Decision:

Since, $p = 0.4815 > \alpha = 0.01$,

Accept H_0 at $\alpha = 0.01$.

Conclusion:

Mean weight of workers is 64 kg at 1% level of significance.

(b)

Ans From above result,

99% Confidence Limit for mean weight of workers is (60.19525, 70.49047).

12/16

2) Marks secured in statistics by a sample of 33 and 36 students of section A and section B are found as follows

Section A

37	29	50	58	24	41	25	49	56	49	17	56	48
35	29	43	28	7	21	33	40	38	50	17	32	50
18	44	49	47	38	51	51						

Section B

38	57	37	43	55	53	48	40	50	34	24	15	46
58	55	49	53	58	50	37	44	52	40	56	51	15
10	37	51	38	42	57	13	7	32	42			

i) Is there any significant difference in mean marks in statistics of statistic section A and section B at 1% level of significance?

ii) Find 99% confidence limit for difference of mean marks.

Working Steps:

1) Calculate sample means and standard deviations:

- For Section A,

  Sample mean: $\bar{x}_A = \frac{\Sigma x_i}{n_A}$

  Standard deviation: $S_A = \sqrt{\frac{\Sigma (x_i - \bar{x}_A)^2}{n_A - 1}}$

- For Section B,

  Sample mean: $\bar{x}_B = \frac{\Sigma x_i}{n_B}$

  Standard deviation: $S_B = \sqrt{\frac{\Sigma (x_i - \bar{x}_B)^2}{n_B - 1}}$

2) Calculate Z:

$$Z = \frac{\bar{x}_A - \bar{x}_B}{\sqrt{\frac{S_A^2}{n_A} + \frac{S_B^2}{n_B}}}$$

3) Calculate P value:

$$p = \text{Prob}(Z > |Z_{cal}|)$$

4) Compare p-value with α; $p \leq \alpha \Rightarrow$ Reject H_0.
$p > \alpha \Rightarrow$ Accept H_0.

5) Calculate $S.E.(\bar{x})$:

$$S.E.(\bar{x}) = \sqrt{\frac{S_A^2}{n_A} + \frac{S_B^2}{n_B}}$$

6) Calculate C.I:

$$C.I. = (\bar{x}_A - \bar{x}_B) \pm Z_\alpha \cdot S.E(\bar{x})$$

(2)

(a)

Ans Problem to test:

$H_0 : \mu_A = \mu_B$

$H_1 : \mu_A \neq \mu_B$

Output:

Two Sample t-test

data : section A and section B

$t = -0.9329$, d.f. $= 67$, p-value $= 0.3542$

H_1: true difference in means is not equal to 0.

99% Confidence Interval:

-12.001101 5.753626

sample estimates:

mean of x = 38.18182

mean of y = 41.30556

From above result,

$p = 0.3542$

$\alpha = 0.01$

Decision:

Since, $p > \alpha$; Accept H_0 at $\alpha = 0.01$.

Conclusion:

There is no significant difference in mean marks in statistics of section A and section B at 1% level of significance.

(b)

Ans From above result,

Confidence Limit is -12.001101 to 5.753626.

12/16

3) Following information represents result of a sample of 32 students of BSc. CSIT II Semester.

P	P	P	F	P	F	P	P	F	F	P	P	P	P	F	P
P	P	F	P	P	P	F	P	P	F	P	P	P	F	P	P

i) Is pass percentage of BSc. CSIT II Semester 80% ?
Use 5% level of significance.

ii) On basis of sample pass percentage what is sample size required to study result of BSc. CSIT II Semester students at 95% confidence limit with 5% margin of error.

Working Steps:

1) Calculate the sample pass percentage (p):

$$p = \frac{\text{no. of students passed}}{\text{Total sample size}}$$

2) Calculate the standard error $S.E(p)$:

$$S.E.(p) = \sqrt{\frac{pq}{n}}$$

where, $q = 1-p$

3) Calculate Z :

$$Z = \frac{p-P}{S.E.(p)}$$

4) Calculate p-value :

$$p = \text{Prob}\,(Z > |Z_{calc}|)$$

5) Compare p-value with given level of significance.

If $p \leq \alpha$, Reject H_0.

If $p > \alpha$, Accept H_0.

6) Calculate sample size (n) by using :

$$n = \frac{Z_\alpha^2 \times pq}{e^2}$$

where, $q = 1-p$

e = margin of error

(i)

Ans Problem to test:

$H_0: P = 0.8$

$H_1: P \neq 0.8$

Output:

1- sample proportion test

data: 23 out of 32

null probability = 0.8

X-squared = 0.86133, d.f. = 1, p-value = 0.3534

alternative hypothesis: true p is not equal to 0.8.

95 percent confidence interval:

0.5302034 0.8560142

sample estimates:

p = 0.71875

sample size = 310.6294

From above result:

$p = 0.3534$

$\alpha = 0.05$

Decision:

Since $p > \alpha$, accept H_0 at $\alpha = 0.05$.

Conclusion:

Pass percentage of B Sc. CSIT II Semester is 80% at 5% level of significance.

(ii)

Ans From the above result:

Sample Size = 310.6294 ≈ 311

at 95% conf. level with 5% margin of error.

12/16

4) Following information represent result of a sample of 32 BIT I student and 36 B.Sc. CSIT I Semester Students.

BIT I ~~Stud~~ Semester

P	F	P	P	P	P	F	F	F	F	P	P	P	P	P	P
P	P	F	F	P	P	F	P	P	F	F	P	P	F	P	P

B.Sc. CSIT I Semester

P	P	F	P	P	P	P	F	P	P	P	F	P	F	P	P
P	P	F	P	P	P	F	F	P	P	P	P	F	P	P	P
F	P	P	P												

Is there any significant difference in pass percentage of BIT I Semester and B.Sc. CSIT I Semester? Use 5% LOS.

Working Steps:

1) Calculate sample proportion of pass for each group:

$$p = \frac{\text{no. of pass}}{\text{Total Sample Size}}$$

2) Calculate overall proportion of pass across both group.

$$P = \frac{\text{Total no. of passed students}}{\text{Total no. of students}}$$

3) Calculate S.E. of diff^r bet^n prop^n:

$$S.E. = \sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

where, $Q = 1 - P$
n_1 = total students in BIT I Semester
n_2 = total students in B.Sc. CSIT I Semester

4) Calculate Z:

$$Z = \frac{p_1 - p_2}{S.E.}$$

5) Calculate p-value:

$$p = \text{Prob}(Z > |Z_{calc}|)$$

6) Compare P-value with α,
If $p \le \alpha$, Reject H_0.
If $p > \alpha$, Accept H_0.

4

Solution:

Problem to test:

$H_0: p_1 = p_2$

$H_1: p_1 \neq p_2$

Output:

2- sample test for equality of propⁿ

BIT result

F	P
11	21

BSc. CSIT result

F	P
9	27

data: c(21, 27) out of c(32, 36)

X-squared = 0.3367, d.f. = 1, p-value = 0.5167

alternative hypothesis: two sided

95 % confidence interval:

-0.34026 0.152762

sample estimates:

prop 1	prop 2
0.6525	0.75000

From the above result;

Decision:

Since, $p = 0.5617 > \alpha = 0.05$,

Accept H_0 at $\alpha = 0.05$.

Conclusion:

Here is no significant difference in pass percentage of BIT I Semester and BSc. CSIT I Semester.

12/1/6

5) Marks secured by a sample of 22 students in final examination of Statistics I are found as 43, 52, 34, 56, 28, 12, 46, 38, 10, 51, 49, 38, 46, 24, 38, 44, 38, 46, 49, 27, 35, 41.

i) Is average marks in statistics I 30 at 5% level of significance using parametric test.

ii) Obtain 95% confidence limit for average marks of statistics I for all students appeared in examination.

iii) On basis of sample standard deviation obtained from marks of students in Statistics I, What is sample size required for the study of marks distribution of students at 5% level of significant with 10% margin of error.

Working Steps:

1) Calculate sample mean ($\bar{x}$):

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

2) Calculate sample standard deviation (S):

$$S = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$$

3) Calculate S.E. ($\bar{x}$):

$$S.E.(\bar{x}) = \frac{S}{\sqrt{n}}$$

4) Calculate Z:

$$Z = \frac{\bar{x} - \mu_0}{S.E.(\bar{x})}$$

where, μ_0 is hypothesized pop^n mean.

5) Calculate p-value:

$$p = Prob(Z > |Z_{calc}|)$$

6) Compare p-value with α.

If $p \leq \alpha$; Reject H_0

If $p > \alpha$, Accept H_0.

7) For Confidence Interval (C.I.):

$$CI = \bar{x} \pm S.E.(\bar{x}) \times Z_\alpha$$

8) Calculate sample size (n):

$$n = \frac{Z_\alpha^2 \times S^2}{e^2}$$

where,

e = margin of error.

⑤

(i)

Ans Problem to test:

$H_0: \mu = 30$

$H_1: \mu \neq 30$

Output:

One Sample t-test

data : marks

t = 3.2129, d.f. = 21, p-value = 0.004177

alternative hypothesis: true mean is not equal to 30.

95 % Confidence interval:

32.93404 43.70232

sample estimates:

mean of x = 38.31818

sd = 12.14353

sample size = 56650.28

From above result;

Decision:

Since, $p = 0.004177 < \alpha = 0.05$,

Reject H_0 at $\alpha = 0.05$

Conclusion:

Average marks in statistics I is not 30 at 5% level of significance.

(ii)

Ans From above result;

95% Confidence Interval for average marks of students in Statistics I is (32.93404, 43.70232).

(iii)

Ans From above result,

Sample size of students at 5% level of significant with 10% margin of error on basis of sample standard deviation i.e. sd = 12.14353 obtained from given marks is -

$$n = 56650.28 \approx 56650$$

12/1/6

6) Following are marks secured by 14 students of section A and 15 students of section B of DWIT in final examination of Digital Logic are found as -

Section A	34	48	21	52	31	43	29	37	24	52	49	34	40	48	
Section B	11	53	27	38	47	50	26	38	44	33	27	33	41	40	28

i) Is mean marks of section A and section B identical at 1% level of significance?

ii) Obtain 99% confidence limit for difference of mean.

Working Steps :

1) Calculate the mean $(\bar{x})$:

$$\bar{x}_A = \frac{\Sigma x_A}{n_A} \quad \text{and} \quad \bar{x}_B = \frac{\Sigma x_B}{n_B}$$

2) Calculate sample standard deviation :

$$S_A = \sqrt{\frac{\sum_{i=1}^{n_A}(x_{Ai} - \bar{x}_A)^2}{n_A - 1}} \quad \text{and} \quad S_B = \sqrt{\frac{\sum_{i=1}^{n_B}(x_{Bi} - \bar{x}_B)^2}{n_B - 1}}$$

3) Compute the test statistic :

$$t = \frac{\bar{x}_A - \bar{x}_B}{\sqrt{\frac{S_A^2}{n_A} + \frac{S_B^2}{n_B}}}$$

4) Calculate p-value :

$$p = Prob(t > |t_{calc}|)$$

5) Compare p-value with α,

If $p \le \alpha$, Reject H_0.

If $p > \alpha$, Accept H_0.

6) Calculate Confidence Limit :

$$CI = \bar{x}_A - \bar{x}_B \pm S.E(\bar{x}_A - \bar{x}_B) \times t_{\frac{\alpha}{2}}$$

⑥

i)

Ans Problem to test:

$H_0 : \mu_A = \mu_B$

$H_1 : \mu_A \neq \mu_B$

Output:

Two sided t-test

data : sectionAmarks and sectionBmarks

$t = 1.1608$, $d.f. = 26.497$, p-value = 0.2561

H_1 : true difference in means, is not equal to 0

99% Confidence Interval :

-6.92487 16.886732

Sample estimates:

mean of x and y

38.71429 33.7333

From above result,

Decision:

Since, $p = 0.2561 > \alpha = 0.01$

Accept H_0 at $\alpha = 0.01$

Conclusion:

Mean marks of section A and section B is identical.

ii)

Ans From above result,

Confidence Limit for difference of means is (-6.92, 16.88)

12/16

7) Marks secured by a sample of 15 students of a college in first test and second test of Statistics II are found as -

Student	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Test I	12	7	15	11	17	19	5	13	17	6	9	18	14	10	8
Test II	14	5	17	13	12	18	9	10	18	12	3	14	16	16	8

Is there improvement in marks in test II as comparared to test I? Use parametric test at 1% level of significance.

Working Steps:

1) Calculate the differences (d):

$$d = \text{test II score} - \text{test I score}$$

2) Compute the mean difference $(\bar{d})$:

$$\bar{d} = \frac{\Sigma d}{n}$$

3) Compute the sum of square of difference.

4) Calculate the SD of the difference:

$$\text{Sample SD} = \sqrt{\frac{1}{n-1}\left(\Sigma d^2 - n\bar{d}^2\right)}$$

5) Compute the Standard error

$$SE = \frac{S}{\sqrt{n}}$$

6) Calculate test statistic:

$$t = \frac{\bar{d}}{SE}$$

7) Calculate p-value:

$$p = \text{Prob}(t > |t_{calc}|)$$

8) Compare p-value with α,

If $p \leq \alpha$, Reject H_0.

If $p > \alpha$, Accept H_0.

⑦

Problem to test:

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 < \mu_2$

Output:

Paired t- test

data: mark_test_I and mark_test_II
t = - 0.27678, d.f = 14 , p-value = 0.786
H_1: true mean difference is not equal to 60
99% Confidence Interval:
-3.134734 2.601400
Sample estimate:
mean difference
-0.2666667

From above result

$p = 0.786 > \alpha = 0.01$

Decision:

Since $p > \alpha$, Accept H_0 at $\alpha = 0.01$.

Conclusion:

There is no improvement in marks in test II as compared to marks in test I.

12/1/6

8) On tossing a coin 30 times outcomes of head and tail are found as :-

Head, Tail, Head, Head, Tail, Head, Head, Tail, Tail, Head, Tail, Head, Head, Tail, Tail, Head, Head, Head, Tail, Head, Tail, Head, Head, Tail, Tail, Had, Tail, Tail, Tail, Head.

i) Are outcomes in random order?

ii) Is coin unbiased?

Using 1% level of significance.

Working Steps :

* For Qno. i)

1) Calculate the no. of head and tail.

2) Calculate no. of run.

3) Calculate population mean (μ_1) using

$$\mu_r = \frac{2n_1n_2}{n_1+n_2} + 1$$

where,

n_1 = no. of head.

n_2 = no. of tail.

4) Calculate pop^n SD (σ_r) :

$$\sigma_r = \sqrt{\frac{2n_1n_2}{(n_1+n_2)^2} \times \frac{(2n_1n_2 - n_1 - n_2)}{(n_1+n_2-1)}}$$

5) Calculate Z :

$$Z = \frac{|r - \mu_r|}{\sigma_r} - 0.5$$

6) Calculate p-value :

$$p = Prob(Z > |Z_{calc}|)$$

7) Compare p-value with α.

If $p \leq \alpha$, Reject H_0.

If $p > \alpha$, Accept H_0.

* For Qno. ii)

1) Count no. of head and tail and denoted by some name.

   Let n_1 = no. of head.
   n_2 = no. of tail.

2) Calculate x_0 :

   $x_0 = \min[x_1, n_2]$

3) Calculate pop^n mean (μ):

   $\mu = np$

4) Calculate p-value

   $$p = Prob(x \leq x_0) = \sum_{x=0}^{x_0} C(n,x)\, p^x q^{n-x}$$

5) Calculate pop^n SD (σ):

   $\sigma = npq$

6) Calculate Z:

   $$Z = \frac{x_0 - \mu}{\sigma}$$

7) Calculate p-value

   $p = Prob(Z > |Z_{calc}|)$

8) Compare p-value with α

   If $p \leq \alpha$, Reject H_0.
   If $p > \alpha$, Accept H_0.

⑧

i) Problem to test:

H_0: Samples are random.
H_1: Samples are not random.

Run Test

	Outcome
Test value (median)	1.00
Cases < Test value	14
cases > Test value	16
Total cases	30
No. of ~~cases~~ runs	19
Z	0.96
Asymp. sig (2-tailed)	0.338

From above result,
$p = 0.338 > \alpha = 0.01$
Accept H_0.

Conclusion:
Samples are random.

ii) Binomial test

		Category	N	Observed prop.	Test Prop.	p-value
outcome	Group1	Head	16	0.53	0.50	0.856
	Group2	Tail	14	0.47		
	Total		30	1.00		

Problem to test:
$H_0: P = 0.5$
$H_1: P \neq 0.5$

Decision:
Since $p = 0.856 > \alpha = 0.01$, Accept H_0 at $\alpha = 0.01$.

Conclusion:
Coin is unbiased.

12/1/16

9) Marks secured by a sample of 32 students in Final examination of Statistics I are found as 43, ~~4~~52, 34, 56, 28, 12, 46, 38, 10, 51, 49, 38, 46, 24, 36, 44, 38, 46, 49, 27, 35, 41, 11, 23, 35, 42, 52, 49, 20, 35, 43, 37.

i) Are samples selected in random order?
ii) Are marks uniformly distributed? Use Kolmogorov Smirnov.
iii) Are marks uniformly distributed? Use Chi square test.
Using 5% level of significance.

Working Steps:

For i)

1) Calculate median from given data then assign a symbol say A if observation > Md
say B if observation < Md
omit if observation = Md

2) Calculate the number of run.

3) Calculate no. of A and B and denoted by n_1 and n_2.

4) Calculate pop^n mean :

$$\mu = \frac{2n_1n_2}{n_1+n_2} + 1$$

5) Calculate pop^n SD (σ) :

$$\sigma = \sqrt{\frac{2n_1n_2\,(2n_1n_2 - n_1 - n_2)}{(n_1+n_2)^2\,(n_1+n_2-1)}}$$

6) Calculate Z:

$$Z = \frac{r-\mu}{\sigma}$$

7) Calculate p-value :

$$p = Prob(Z > |Z_{calc}|)$$

8) Compare p-value with α.

If $p \le \alpha$, Reject Ho.

If $p > \alpha$, Accept Ho.

For ii)

1) Calculate Cf_o and relative Cf_o (Cf_o/n) for given freq.
where, f_o = given frequency

2) Calculate probability according to pop^n distribution, then obtain frequency (f_e) then Cf_e and and relative Cf_e (Cf_e/n)

3) Calculate absolute value of difference of Cf_o/n and Cf_e/n.
It is test statistic
$$D = \max \left| \frac{Cf_o}{n} - \frac{Cf_e}{n} \right|$$

4) Calculate p-value :
$$p = P(D_o > |D_{calc}|)$$

5) Compare p-value with α, then.
If $p \leq \alpha$, Reject H_o.
If $p > \alpha$, Accept H_o.

For iii)

1) Calculate exp. frequency (E_i) :
$$E_i = Np_i$$
where, P_i = Prob. of happening.
N = No. of data given.

2) Calculate test statistic
$$\chi^2 = \sum_i \frac{(O_i - E_i)^2}{E_i}$$
where, O_i = observed $freq^n$, which is given.

3) Calculate p-value
$$p = Prob(\chi^2 > |\chi^2_{calc}|)$$

4) Compare p with α.
$p \leq \alpha$, Reject H_o.
$p > \alpha$, Accept H_o.

9

i)

⇒ Problem to test:

H_0: Samples are random.

H_1: Samples are not random.

Run Test:

	Marks
Test value (median)	38.00
Cases < Test value	14
cases ≥ Test value	18
Total cases	32
No. of Runs	16
Z	−0.09
Asym-sig (2 tailed)	0.927

From above result

Decision:

Since $p = 0.927 > \alpha = 0.05$

Accept H_0 at $\alpha = 5\%$.

Conclusion:

Sample are random.

ii)

⇒ Problem to test:

H_0: Marks are uniformly distributed.

H_1: Marks are not " ".

One Sample Kolmogorov-Smirnove Test

		Marks
N		32
Uniform parameters	Min.	10.00
	Max.	56.00
Most Extreme Differences	Absolute	0.27
	Positive	0.06
	Negative	−2.7
Kolmogorov-Smirnov Z		1.57
Asymp. sig (2 tailed		0.012

From result

$p = 0.012 < \alpha = 0.05$

Reject H_0 at $\alpha = 5\%$

Conclusion

Marks are not uniformly distributed.

iii)

⇒ Problem to test:

H_0 : Marks are uniformly distributed.

H_1 : Marks are not " ".

Test Statistic:

	Chi-square	d.f.	Asymp sig.
Marks	9.25	21	0.987

From above result

$p = 0.987 > \alpha = 0.05$

Accept H_0 at $\alpha = 5\%$.

Conclusion:

Marks are uniformly distributed.

12/1/16

10) Following are marks secured by 14 students of section A and 15 students of section B of DWIT in final examination of Digital Logic are found as:-

Section A	34	48	21	52	31	43	29	37	24	52	49	34	40	48	
Section B	11	53	27	38	47	50	26	38	44	33	27	33	41	10	28

Is median marks of section A and section B identical at 5% level of significance using?

i) Median test
ii) Mann Whitney U test
iii) Kolmogorov Sinirnov test

Working Steps:

For i) & ii)

1) Calculate difference d of related samples.
2) Rank d irrespective of sign omit if d = 0. If two or more d are same then assign average rank.
3) Calculate rank of d according to sign to get S(+) and S(-).
4) Calculate test statistic:
   $T = \min\{S(+) \text{ and } S(-)\}$
5) Calculate p using
   $p = Prob(T > |T_{calc}|)$
6) Compare p-value and α,
   If $p \le \alpha$, reject H_0.
   If $p > \alpha$, acept H_0.

~~7) Compare~~

For iii)

1) Calculate Cf_A and relative Cf_A (Cf_A/n_A) for given freq^n.
2) Calculate propability according to pop^n distribution, then obtain frequency (Cfe) then Cfe and relative Cfe (Cfe/n)
3) Calculate test statistic
   $$D = \max \left| \frac{Cf_A}{n_A} - \frac{Cf_B}{n_B} \right|$$
4) Calculate p-value:
   $p = Prob(D > |D_{calc}|)$
5) Compare p-value with α.
   If $p \le \alpha$, reject H_0.
   If $p > \alpha$, accept H_0.

10)

For qn. i) and ii)

Problem to test:

$H_0: Md_A = Md_B$

$H_1: Md_A \neq Md_B$

Wilxoxon rank sum test
data: Section_A_marks and Section_B_marks
W = 128 , p = 0.3258
H_1 : true location shift is not equal to 0.

From above result,

$p = 0.325 > \alpha = 0.05$

Accept H_0 at $\alpha = 0.05$.

Conclusion:

Yes, the median of section A and section B are identical.

For qn. iii)

Problem to test:

$H_0: Md_A = Md_B$

$H_1: Md_A \neq Md_B$

Exact two-sample Kolmogorov-Smirnov Test
data: Section A marks and Section B marks
D = 0.25714 , p-value = 0.5416
H_1 : two-sided

From above result,

$p = 0.5416 > \alpha = 0.05$

Accept H_0 at $\alpha = 5\%$.

Conclusion:

The median of section A and section B are identical.

12/1/6

11) Following information are obtaining from locality related to gender and eye color.

Person	Gender	Eye Color
A	Male	Black
B	Female	Black
C	Male	Brown
D	Male	Black
E	Female	Blue
F	Male	Brown
G	Female	Black
H	Male	Black
I	Female	Black
J	Female	Brown
K	FeMale	Black
L	~~Fe~~male	Black
M	Female	Blue
N	FeMale	Brown
O	Male	Black
P	Female	Black
Q	Male	Brown
R	Female	Black
S	Female	Black
T	Male	Brown

Is there any associated between gender and eye color? Use 5% level of significance.

11)

Working Steps:

1) Calculate Expected frequency (E_i):

$$E_i = NP_i$$

where, P_i = Prob. of happening.
N = No. of data given

2) Calculate test Statistic

$$\chi = \sum_i \frac{(O_i - E_i)^2}{E_i}$$

Where O_i = observed frequency which will be given.

3) Calculate p-value:

$$p = Prob(\chi^2 > |\chi^2_{calc}|)$$

4) Compare p-value with α;
If $p \leq \alpha$, reject H_0.
If $p > \alpha$, accept H_0.

Problem to test:
H_0: No association bet^n gender and eye color.
H_1: Association bet^n " " " ".

Eye color			
Gender	Black	Blue	Brown
F	7	2	2
M	5	0	4

Pearson's Chi-Squared Test

data : table (Gender, Eyecolor)
χ squared = 2.8283, d.f. = 2, p-value = 0.2431

From above result
$p = 0.24 > \alpha = 0.05$
Accept H_0 at $\alpha = 0.05$.

Conclusion :

There is no any association between gender and eye color.

12/1/6

12) Marks secured by a sample of 15 students of a college in test I and test II of Stat II are -

Student	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Test I	12	7	15	11	17	19	5	13	17	6	9	18	14	10	8
Test II	14	5	17	13	12	18	9	10	18	12	3	14	16	16	8

Is there improvement in marks in test II as compared to test I? Use non-parametric test at 5% level of significance.

Working Steps:

1) Calculate difference d of related samples.

2) Rank d irrespective of sign omit if $d=0$. If two or more d are same then assign average rank.

3) Calculate rank of d according to sign to get -

$S(+)$ and $S(-)$

4) Calculate test statistic :

$$T = \min\{S(+) \text{ and } S(-)\}$$

5) Calculate p-value.

$$p = Prob(T > |T_{calc}|)$$

6) Compare p-value with α.

If $p \le \alpha$, reject H_0.

If $p > \alpha$, accept H_0.

Problem to test

$H_0: Md_I = Md_{II}$

$H_1: Md_I < Md_{II}$

Wilxcon Signal rank test
data : Test I marks and Test II marks
$V = 48$, p-value = 0.4002
H_1: true location shift is less than 0.

From above table,

$p = 0.4002 > \alpha = 0.05$

Accept H_0 at $\alpha = 5\%$.

Conclusion

There is no significant improvement in marks in test II as compared to test I.

12/1/16

⑬ Four diets are fed to 9 cows, each diet for a month and the result of increase (I) and decrease (D) of milk given by differente cows are found as follows: -

Cow Diet	I	II	III	IV	V	VI	VII	VIII	IX
D1	I	I	D	I	D	I	I	D	I
D2	D	D	I	D	I	D	D	I	I
D3	I	D	I	D	D	I	I	D	D
D4	I	I	I	D	D	I	I	D	I

Test whether diets are equally effective or not at 1% level of significance.

Working Step:

1) Calculate ΣR_i and ΣR_i^2 where,
R_i = sum of presence according to treatment.

2) Calculate ΣC_j and ΣC_j^2,
where,
C_j = sum of presence acc. to block.

3) Calculate test Statistic:

$$Q = \frac{(K-1)(K \Sigma R_i^2 - (\Sigma R_i)^2)}{K \Sigma C_j - \Sigma C_j^2}$$

where, k = no. of samples in which test will be done.
Here, $n = 4$ acc. to this given question.

4) Calculate p-value:
$p = Prob(Q > |Q_{calc}|)$

5) Compare p-value with α,
If $p \le \alpha$, Reject H_0.
If $p > \alpha$, accept H_0.

Problem to test:

$H_0: D_1 = D_2 = D_3 = D_4$

H_1: At least one D_i is different, $i = 1, 2, 3, 4$

NPAR Test

/ COCHRAN = D1 D2 D3 D4

Frequencies

	Value	
	Success (1)	Failure (0)
D_1	6	3
D_2	4	5
D_3	4	5
D_4	6	3

Test Statistic

	Value
N	9
Cochran's Q	1.714
d.f.	3
Asym sig	0.634

From above result,

$p = 0.634 > \alpha = 0.01$

Accept H_0 at $\alpha = 1\%$

Conclusion:

Diets are equally effective.

12/1/C

(14) Following data represent the operating times in hours for four types of laptop before a charge is required.

Dell	5.3	4.8	6.1	3.5			
Acer	5.2	5.8	3.9	4.6	4.9	5.1	5.6
HP	4.5	5.2	3.8	4.8	5.3		
Lenovo	4.7	6.2	5.7	5.5	3.9	4.8	

Are operating time for all laptops equal at 5% level of significance. Use non-parametric test?

Working Steps:

1) Rank all the given data, assigning smallest value rank 1, the second smallest rank 2 and so on. If there are ties, then assign the average rank to each tied observation.

2) Calculate R_i^2.

3) Calculate test Statistics

$$H = \frac{\frac{12}{n(n+1)} \sum \frac{R_i^2}{n_i} - 3(n+1)}{1 - \frac{\sum t_i^3 - t_i}{n^3 - n}}$$

where, n = no. of observations
t_i = no. of times i^{th} rank is repeated

4) Calculate p-value :
$p = \text{Prob}(H > |H_{calc}|)$

5) Compare p with α,
If $p \leq \alpha$, reject H_0
If $p > \alpha$, accept H_0

Problem to test:

$H_0: Md_{Dell} = Md_{Acer} = Md_{HP} = Md_{Lenovo}$

H_1: At least one Md_i is different

i = Dell, Acer, HP, Lenovo

NPAR Test

(Kruskal - WALLIS = Operating Time By Laptop (1, 4)

Ranks

		N	Mean Rank
Operating Time	Acer	7	12
	Dell	4	11.63
	HP	5	9.00
	Lenovo	6	12.92
	Total	22	

Test Statistic

	Operating Time
Chi-square	1.07
d.f.	3
Asymp sig.	0.783

From above result,

$p = 0.783 > \alpha = 0.05$

Accept H_0 at $\alpha = 5\%$.

Conclusion:

Operating time for all laptops are equal.

12-11-6

15) The scores of 7 students in Statistics II in three test are found as-

Test \ Student	A	B	C	D	E	F	G
I	15	13	8	12	9	16	13
II	14	16	12	10	14	11	6
III	10	12	5	16	8	14	16

i) Is there any significant difference in marks in three test?

ii) Is there any significant difference in marks of seven students?

Use non-parametric test at 1% level of significance?

Working Steps

1) Rank the data within each group (test) separately. Assign ranks to each observations within each group.

2) Calculate R_i^2 which the square of rank of each observation.

3) Calculate test statistic

$$Fr = \frac{\frac{12}{nK(K+1)} \sum R_i^2 - 3n(K+1)}{1 - \frac{\sum t_i^3 - t_i}{n(K^3 - K)}}$$

Where K = no. of group (test)
n = no. of obs.

4) Calculate p-value
$p = Prob(Fr > |F_{calc}|)$

5) Compare p with α.
If $p \le \alpha$, reject H_0.
If $p > \alpha$, accept H_0.

(15)

i)

Problem to test:

$H_0: Md_{I} = Md_{II} = Md_{III}$

$H_1:$ At least one Md_{i} is different.

$i = I, II, III$

NPAR TEST

(FRIEDMAN = Test I Test II Test III)

Ranks

	Mean Rank
Test I	2.29
Test II	2.00
Test III	1.71

Test Statistic

N	7
Chi-square	1.14
df	2
Asymp sig.	0.565

From above result

$p = 0.565 > \alpha = 0.01$

Accept H_0 at $\alpha = 0.01$.

Conclusion:

There is no significant difference in three test.

15

ii)

Problem to test:

$H_0: Md_1 = Md_2 = Md_3 = Md_4 = Md_5 = Md_6 = Md_7$

H_1: At least one Md_i is diff^r. $i = 1,2,3,4,5,6,7$.

Ranks

	Mean Ranks
A	4.83
B	5.17
C	2.00
D	3.83
E	3.17
F	5.00
G	4.00

Test Statistic

N	3
Chi-square	5.03
d.f.	6
Asymp sig.	0.532

From above result,

$p = 0.532 > \alpha = 0.01$

Accept H_0 at 1 % α.

Conclusion:

There is no any significant diff^r in marks of seven students

12-11

16) The following information has been gathered from a random sample of apartment renters in a city. We have information of rent in (000 Rs per month) based on the size of apartment, (number of rooms) and the distance from downtown (in KM).

Rent (000 Rs)	16	20	25	22	20	25
No. of rooms	4	6	3	4	5	3
Distance from downtown	8	10	4	6	2	1

a) Obtain the multiple regression models that best relate these variables.

b) Interpret the obtained regression coefficients.

c) If someone is looking for two bed apartment 8 km from downtown, what rent should he expect to pay?

d) Obtain residuals.

e) Calculate standard error of estimate.

f) Test the significance of regression coefficient at 5% level of significance.

g) Test overall significance of regression equation at 5% level of significance.

Ans **Working Steps :-**

1) Find out dependent and independent variable.
Here, Rent is dependent (say y), No. of rooms and Distance from downtown (say x_1 and x_2) respectively independent.

2) Regression Equation of y on x_1 and x_2.
To fit, $y = b_0 + b_1x_1 + b_2x_2$
where, b_0, b_1, b_2 are parameters of 3 variables.

$\sum y = nb_0 + b_1\sum x_1 + b_2 \sum x_2$

$\sum yx_1 = b_0 \sum x_1 + b_1 \sum x_1^2 + b_2 \sum x_1 x_2$

$\sum y x_2 = b_0 \sum x_2 + b_1 \sum x_1 x_2 + b_2 \sum x_2^2$

3) Use Cramer's Rule for calculating the value of parameters a_0, a_1, a_2.

4) Residual Calculation

R_i = Observed rent − Predicted rent

5) Standard Error Calculation

$$S_e = \sqrt{\frac{SSE}{n-k-1}}$$ where, $SSE = \Sigma(x_1 - \bar{x}_1)^2$
$= \Sigma x_1^2 - a\Sigma x_1 - b_2 \Sigma x_1 x_2 - b_3 \Sigma x_1 x_3$

6) Test Statistic Calculation,

$$t = \frac{b_i}{S_{b_i}}$$ where, b_i = Sample regression coeff.
S_{b_i} = Standard error of regression coefficient

7) Test of significance for Regression Coefficients.

- Check the t-statistics and p-value for each coefficient
- if p value > α level of significance then the coefficient is significant.

8) Calculate, test Statistics

$$F = \frac{MSR}{MSE}$$ where, MSR = Mean Sum of Square due to regression.
MSE = Mean Sum of Square due to error.

9) Test of overall significance of the regression Coefficients.

- Check f-statistic and p-value if pvalue > α then regression model is significant.

Output

Residuals:

1	2	3	4	5	6
-4.3168	2.3942	2.0852	0.8549	-1.8603	0.8428

Coefficients:

	Estimate	std. Error	t-value	Pr(>|t|)
Intercepts	27.3954	5.6128	4.881	0.0164
no. of rooms	-0.9414	1.5725	-0.599	0.5916
distance	-0.4141	0.5270	-0.786	0.4893

Residual Standard Error : 3.348 on 3 d.f.

Multiple R-squared : 0.4334

Adjusted R-squared : 0.05569

F-Statistic : 1.147 on 2 and 3 d.f.

p-value : 0.4265

Predicted rent
22.1998

Conclusion

a) The multiple regression model obtained from data is:-

Rent = 27.3954 - 0.9414 No. of rooms - ~~No of rooms~~ 0.4141 Distance

$$y = 27.3954 - 0.9414x_1 - 0.4141x_2$$

b) Interpretation of Coefficients:

- The intercept of 27.3954 represents the expected rent when the number of rooms and distance from downtown are both zero.

- For each additional rooms, the rent decreases by approximately 0.9414 Rs with constant distance from downtown.
- For each additional kilogmeter away from downtown, the rent decreases by approximately 0.4141 Rs with constant no. of rooms.

c) If someone is looking for a two-bedroom apartment 8 km from downtown, the predicted rent would be approximately 22.1998 Rs.

d) Residuals

1	2	3	4	5	6
-4.3168	2.3942	2.0852	0.8249	-1.8603	0.8428

e) The standard error of estimate is approximately 3.348 Rs.

f) Testing the significance of regression coefficients:

- The coefficients b_1 and b_2 are insignificant at 5% level, as indicated by their p-values being greater than 0.05. And Intercept (b_0) is significant.

g) Testing the overall significance of regression equation:-

- The p-value for the F-statistic is 0.4265, indicating that the regression equation as a whole is statistically insignificant at the 5% level. (p-value = 0.4265 > α = 0.05)

17) A developer of food for pig would like to determine, what relationship exists among the age of a pig, when it starts receiving a newly developed food supplement, the initial weight of the pig and the amount of weight it gains in a week period with the food supplement. The following information is the result of study of eight piglets.

Intial weight (Pounds)	39	52	49	46	61	36	28	57
Initial age (weeks)	8	7	6	11	8	7	9	5
Weight gain	8	7	6	9	10	6	4	5

a) Determine multiple correlation coefficient and partial correlation coefficients of dependent variable with independent variables.

b) Determine multiple coefficient of determination and interpret.

c) Determine adjusted multiple coefficient of determination.

Working Steps

1) Multiple Correlation Coefficient

- It is lies between 0 and 1.
  i.e. $0 < R_{1.23} \leq 1$, $0 \leq R_{2.13} \leq 1$, $0 \leq R_{3.12} \leq 1$
- It is not less than simple correlation
  $R_{1.23} \geq r_{12}, r_{13}, r_{23}$
- If $R_{1.23} = 0$, then $r_{12} = 0$, $r_{13} = 0$
  $R_{1.23} = R_{1.32}$

2) Partial Correlation Coefficients.

Let, three variables X_1, X_2 & X_3 then, partial correlation Coefficients betn X_1 and X_2 keeping X_3 constant is denoted by $r_{12.3}$.

$$r_{12.3} = \frac{r_{12} - r_{13} r_{23}}{\sqrt{1 - r_{13}^2} \sqrt{1 - r_{23}^2}}$$

same way,

$$r_{13.2} = \frac{r_{13} - r_{12} r_{32}}{\sqrt{(1 - r_{13}^2)(1 - r_{32}^2)}}, \quad r_{23.1} = \frac{r_{23} - r_{21} r_{31}}{\sqrt{1 - r_{23}^2} \sqrt{1 - r_{31}^2}}$$

Calculate,

3) Multiple Coefficient of determination

$$R_{1.23}^2 = \left(\sqrt{\frac{r_{12}^2 + r_{13}^2 - 2 r_{12} r_{13} r_{23}}{1 - r_{23}^2}} \right)^2$$

$$= \frac{r_{12}^2 + r_{13}^2 - 2 r_{12} r_{13} r_{23}}{1 - r_{23}^2}$$

4) Calculate adjusted multiple coefficient of determination

$$R^2_{adjusted} (\bar{R}_2) = 1 - \frac{n-1}{(n-k-1)} [1 - R^2]$$

where, n = no. of pair of observation
k = no. of independent variables

Output:

Coefficients:

	Estimate std.	Error	t-value	Pr(>|t|)
(Intercept)	−5.50561	3.77392	−1.459	0.2044
Initial weight	0.13775	0.04926	2.797	0.0381
Initial age	0.79269	0.29630	2.675	0.0441

Multiple R-squared : 0.6865
Adjusted R-squared : 0.561
sqrt (0.6865)
0.828553

$ estimate

	Weight gain	Initial weight	Initial age
Weight gain	1.000000	0.7810314	0.7672784
Initial weight	0.7810314	1.00000	−0.7470585
Intial age	0.7672784	−0.7470585	1.000000

a) Multiple Correlation Coefficient and partial correlation coefficients:-

⊙ Multiple Correlation Coefficient

- The multiple correlation coefficient R is 0.828553 indicating a strong positive linear relationship between the dependent variable (weight gain) and the independent variables (initial weight and initial age).

⊙ Partial Correlation Coefficient

- The partial correlation coefficients between weight gain and initial weight is 0.7810314 and between weight gain and initial age is 0.7672784. These values indicate strength and direction of the relationships between weight gain & each independent variable, controlled for the effect of the other independent variable.

b) Multiple Coefficient of Determination (R^2) & interpretation:

⊙ Multiple Coefficient of determination (R^2): 0.6865, which means that approximately 68.65 % of the variance in weight gain can be explained by the linear relationship with initial weight and initial age.

c) Adjusted Multiple Coefficient of determination

⊙ The adjusted $R^2_{adj.} = 0.561$

This indicates that approximately 56.1 % of the variance in weight gain can be explained by the linear relationship with initial weight and initial age, adjusting for the number of predictors and sample size.

18) Let A, H, D and L represents Acer, HP, Dell, and Lenovo laptop and following information represents their operating time in hours before charge is required.

A	H	D	H	D	L
5.2	3.8	4.6	5.2	3.6	4.4
L	A	H	L	L	A
5.6	3.9	4.6	6.2	4.8	3.5
H	D	L	D	A	D
4.4	3.6	5.2	4.8	4.2	5.4
A	L	A	H	D	H
6.1	4.7	3.2	5.3	4.8	3.9

Carryout analysis of the design at 1% α.

⇒

1) Find out which design of experiement and calculate.

- It is CRD design of experiment.

So, calculate treatment $(t) = 4$ (A, H, D, L)

Replication $(r) = 6$

2) Mathematical Model

$y_{ij} = \mu + T_i + e_{ij}$

where, $i = 1, 2, \ldots, t$; $j = 1, 2, \ldots, r$

y_{ij} = Obs. or yield due to ith treatment & jth replication

μ = General mean

T_i = Effect due to i^{th} treatment

e_{ij} = Error due to i^{th} treatment & j^{th} replication.

3) Calculate

$TSS = SST + SSE$ where, $TSS = \sum_i \sum_j (y_{ij} - \bar{y}_{..})^2$

$SST = \sum_i \sum_j (\bar{y}_{i.} - \bar{y}_{..})^2$

$SSE = \sum_i \sum_j (y_{ij} - \bar{y}_{i.})^2$

4) ANOVA Table

SV	d.f	SS	MS	Fcal.	Ftab.
Treatment	$t-1$	SST	$MST = \frac{SST}{t-1}$	$F_T = \frac{MST}{MSE}$	$F_{\alpha}(t-1, t(r-1))$
Error	$t(r-1)$	SSE	$MSE = \frac{SSE}{t(r-1)}$		
Total	$rt-1$	TSS			

5) Check F-value with a corresponding p-value, if p-value > α then accept Ho. Else Reject.

Output

	Df	Sum Sq	Mean Sq	Fvalue	Pr(>F)
Laptop	3	2·308	0·7694	1·197	0·336
Residuals	20	12·857	0·6428		

Here, from the above result,

- The design used in the analysis is a completely randomized Design (CRD), as laptops of different brands were randomly assigned to the experimental unit without any specific blocking or grouping criteria.
- The value of F is 1·197 with a corresponding p-value is 0·336. Since, the p-value > α = 0·01. So, accept Ho at α = 0·01.

Problem to test

H_0 : ~~Design~~ Treat is insignificant.

H_1 : ~~Design~~ " is significant.

Conclusion: ~~Laptop~~

The design is insignificant.

19) Let A, H, D and L represents Acer, HP, Dell & Lenovo laptop and following information represents their operating time in hours before charge is required.

A	H	D	A	D	L
5.0	3.6	4.8	4.2	3.8	4.6
L	A	H	L	L	A
5.4	4.9	4.3	5.2	5.8	5.5
H	D	L	D	A	D
4.8	4.6	5.5	4.6	5.2	5.0
D	L	A	H	H	H
6.0	4.5	3.9	5.1	4.9	4.9

Carryout analysis of the design at 1% α.

⇒ <u>Working Steps:</u>

1) Find out that which design of experiment is it.
   - If design is RBD then, calculate treatment $(t) = 4$ and Block $(b) = 6$.

2) Mathematically Model

$$y_{ij} = \mu + T_i + B_j + e_{ij}$$

where, $i = 1, 2, 3, \ldots, t$
$j = 1, 2, 3, \ldots, r$

y_{ij} = Yield or obserbation due to i^{th} treatment and j^{th} block.
μ = General Mean
T_i = Effect due to i^{th} treatment
B_j = Effect due to j^{th} block
e_{ij} = Error due to i^{th} treatment and j^{th} block.

3) Calculate,

$$TSS = SST + SSB + SSE$$

where,

$$TSS = \sum_i \sum_j y_{ij}^2 - \frac{G^2}{N}$$

$$SST = \sum_i \sum_j (\bar{y}_{i.} - \bar{y}_{..})^2 = \sum_i \frac{T_{i.}^2}{r} - \frac{G^2}{N}$$

$$SSB = \sum_j \frac{T_{.j}^2}{t} - \frac{G^2}{N}$$

$$SSE = TSS - SST - SSB$$

4) ANOVA Table

SV	df	SS	MS	Fcal	Ftab
Treatment	$t-1$	SST	$MST = \frac{SST}{t-1}$	$F_T = \frac{MST}{MSE}$	$F_\alpha(t-1,(t-1)(r-1))$
Block	$r-1$	SSB	$MSB = \frac{SSB}{r-1}$	$F_B = \frac{MSB}{MSE}$	
Error	$(r-1)(t-1)$	SSE	$MSE = \frac{SSE}{(t-1)(r-1)}$		
Total	$rt-1$	TSS			

5) Check F_T value and F_B value with a corresponding p-value if p-value > α then accept H_0T and H_0B. Otherwise Reject.

Output

	Df.	Sum sq	Mean sq	F val.	Pr(>F)
Laptop	3	1.015	0.3382	0.945	0.44
Block	5	1.954	0.3908	1.092	0.405
Residuals	15	5.368	0.3579		

Problem to test

H_0T: Treatment of design is insignificant
H_1T: Treatment of design is significant
H_0B: Block of design is insignificant
H_1B: Block of design is significant

- F-val = 0.945 (Treatment)
  P-val = 0.44
  Since P-val > α, Accept H_0T at $\alpha = 0.01$.

- F-val = 1.092 (Block)
  P-val = 0.405
  Since, P-val > α, Accept H_0B at $\alpha = 0.01$.

Hence,
Treatment of design is insignificant.
Block of design is insignificant.

20) Let A, H, D, L represents Acer, HP, Dell and Lenovo Laptop and following information represents their operating time in hours before charge is required.

A 4.2	H 4.8	D 4.2	L 6.2
L 4.6	A 5.9	H 4.8	D 5.2
H 5.4	D 5.6	L 5.6	A 4.8
D 4.1	L 5.7	A 4.2	H 4.3

Carryout analysis of the design at 5% α.

Working Steps:

1) Find out which design of experiment is it.
   - If design is LSD then calculate, treatment $(t) = 4$, row $= 4$, column $= 4$

2) Mathematical Model

$$y_{ijk} = \mu + r_i + c_j + T_k + e_{ijk}$$

where, $i = 1, 2, 3, \ldots, m$
$j = 1, 2, 3, \ldots, m$
$k = 1, 2, 3, \ldots, m$

y_{ijk} = Observation due to i^{th} row, j^{th} column and k^{th} treatment

μ = General mean

r_i = Effect due to i^{th} row

c_j = Effect due to j^{th} column.

T_k = Effect due to k^{th} treatment

e_{ijk} = Error due to i^{th} row, j^{th} column, & k^{th} treatment.

3) Calculate

$$TSS = SSR + SSC + SST + SSE$$

where,

$$TSS = \sum_{i,j,k} y_{ijk}^2 - \frac{G^2}{N}$$

$$SSR = \sum_i \frac{T_{i..}^2}{m} - \frac{G^2}{N}$$

$$SSC = \sum_j \frac{T_{.j.}^2}{m} - \frac{G^2}{N}$$

$$SST = \sum_k \frac{T_{..k}^2}{m} - \frac{G^2}{N}$$

$$SSE = TSS - SSR - SSC - SST$$

4) ANOVA Table

SV	d.f.	SS	MS	Fcal
Row	$m-1$	SSR	$MSR = SSR/m-1$	$F_R = MSR/MSE$
Column	$m-1$	SSC	$MSC = SSC/m-1$	$F_C = MSC/MSE$
Treatment	$m-1$	SST	$MST = SST/m-1$	$F_T = MST/MSE$
Error	$(m-1)(m-2)$	SSE	$MSE = SSE/(m-1)(m-2)$	
Total	m^2-1	TSS		

5) Check F_T value, F_r value, F_c value with a corresponding p-value, if p-value $> \alpha$ then accept H_0T, H_0r & H_0c, Reject otherwise.

Output

	Df	Sum sq	mean sq	F-value	Pr(>F)
Laptop	3	1.620	0.5400	1.588	0.288
row	3	1.355	0.4517	1.328	0.350
column	3	2.135	0.7117	2.093	0.203
Residuals	6	2.040	0.3400		

Problem to test:

H_0T : Treatment of design is insignificant
H_1T : Treatment of design is significant
H_0R : Row of design is insignificant
H_1R : Row of design is significant
H_0C : Column of design is insignificant
H_1C : Column of design is significant

- The f-value of treatment is 1.588 with corresponding p-value is 0.288. Since, p-value $= 0.288 > \alpha = 0.05$. Accept H_0T at $\alpha = 0.05$, level of significance.
- The F-value of row is 1.328 with a corresponding p-value is 0.350. Since, p-value $> \alpha = 0.05$. Accept H_0R at $\alpha = 0.05$,
- The F-value of column is 2.093 with a corresponding p-value is 0.203. Since, p-value $> \alpha = 0.05$. Accept H_0C at $\alpha = 0.05$

Hence,

Treatment of design is insignificant.
Row of design is insignificant.
Column of design is insignificant.

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