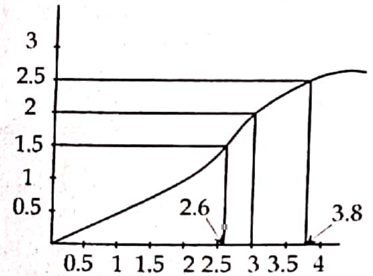
Chapter 2:

Exercise 2.1

1. Here $|f(x) - 2| < 0.5$
$-0.5 < f(x) - 2 < 0.5$
$1.5 < f(x) < 2.5$
$2.6 < x < 3.8;$
Since in the graph we see that in order to have $1.5 < f|x| < 2.5$.
Now,
$2.6 - 3 < x . 3 < 3.8 - 3$; we know that $-0.4 < x . 3 < 0.8$ because $|x - 3| < \delta$
So, first we try to show this expression by adding -3 in each side.
$-0.4 < -\delta$ and $\delta \le 0.8$
$\delta \le 0.4$ and $\delta \le 0.8$
In order for x to verify $-0.4 < x - 3 < 0.8$, δ must verify $-0.4 \le -\delta$ and $\delta < 0.4$.
So, we can choose $\delta = 0.4$ in order to be sure that the 2 conditions due satisfied. Of course we can choose $\delta = 0.3$ or any value that fulfill $\delta = 0.3$ or any value that fulfill $\delta \le 0.4$.
Alternative: Another quick graphical method: you can find the answer using the graph of you follow these steps.
Find the intervals where f(x) and x must be: in our case for f(x) [1.5, 2.5]; for x[2.6, 3.8].
Find the middle of the first interval: in our case it is 2. $\frac{1.5 + 2.5}{2} = 2$.
Find x_0 where $\lim_{x \to x_0} f(x)$ = middle: 2, in our case $x_0 = 3$.
Calculate the distance between x_0 and the end points of the second interval, in our case $|x_0 - 2.6| = |3 - 2.6| = 0.4$ and $|x_0 - 3.8| = |3 - 3.8| = 0.8$.
δ-the minimum of these two values in our case it's 0.4



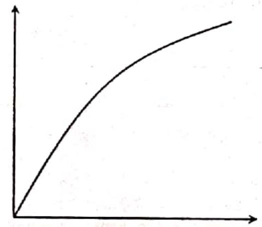
Here, $y = f(x) = x^2$ and $x^2 = \frac{1}{2} \Rightarrow x = \frac{1}{\sqrt{2}}$ and $x^2 = \frac{3}{2} \Rightarrow x = \frac{\sqrt{3}}{\sqrt{2}}$

$|x - 1| < \left|\frac{1}{\sqrt{2}} - 1\right| \approx 0.292$

$|x - 1| < \left|\frac{\sqrt{3}}{2} - 1\right| \approx 0.224$

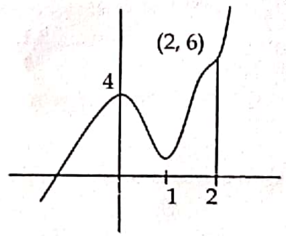
$\delta = 0.224$

3. Simply use the graph of $f(x) = \frac{2x}{x^2 + 4}$ to find the x values. Given, a = 1, L = 0.4 and epsilon 0.1 .

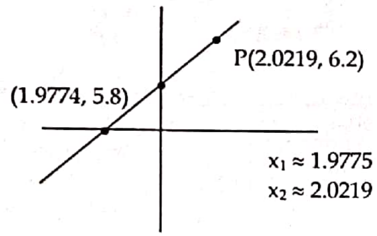


$x - 1 < \delta \Rightarrow 0.1 < \frac{2x}{x^2 + 1} - 0.4 < 0.1$
$\Rightarrow 0.3 < \frac{2x}{x^2 + 1} < 0.5$
$\Rightarrow 3 < \frac{20x}{x^2 + 1} < 0.5$
$\Rightarrow 3x^2 + 3 < 20x < 5x^2 + 5$
Middle: $\frac{0.3 + 0.5}{2} = 0.4$
Here, $\lim_{x \to x_0} f(x) = 0.4$ i.e. $x_0 = 1$ in our case.
Now, $\delta = \min\left\{|1 - 2|, \left|1 - \frac{2}{3}\right|\right\} = \frac{1}{3}$

4. $\lim_{x \to 2} x^3 - 3x + 4 = 6$, $a_1 = 0.2$ and $a_2 = 0.1$



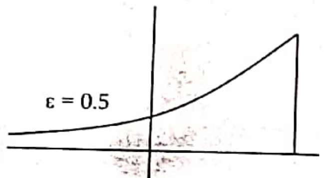
Now, $|x - 2| < \delta_1 \Rightarrow |x^3 - 3x + 4| - 6| < \varepsilon_1$
We have to find suel δ_1.
$|x^3 - 3x + 4 - 6| < 0.2$
$6 - 0.2 < x^3 - 3x + 4 < 0.2 + 6$
$5.8 < x^3 - 3x + 4 < 6.2$
We are interested in the region near the point (2, 6) so we hare to determine the values of x for which the curve $y = x^3 - 3x + 4$ lies between the lines y = 5.8 and y = 6.2. Therefore, we graph the curves $y = x^3 - 3x + 4$, y = 5.8 and y = 6.2 near the point (2, 6).

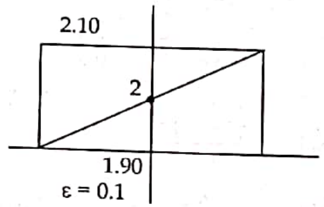


Use the cursor to estimate the x-coordinate of the intersection of the line y = 5.8 with the curve $y = x^2 - 3x + 4$ and similarly the x-coordinate of the intersection of the line y = 6.2 when the curve $y = x^2 - 3x + 4$.
The interval (1.9775, 2.0219) is not symmetric about x = 2. The distance from x = 1.9775 to the left end point is 2 - 1.9775 = 0.0225 and the distance from the right end point is 2.0219 - 2 = 0.0219. We can chose s_1 to be smallest of each numbers. When $\varepsilon_2 = 0.01$, we have
$5.9 < x^3 - 3x + 4 < 6.1$
So we need to determine the values of x for which the curve $y = x^3 - 3x + 4$ lies between the lines y = 5.9 and y = 6.1. Therefore, we graph the curves.
$y = x^3 - 3x + 4$, y = 5.9 and y = 6.1 near the point (2, 6).

$x_1 \approx 1.9889$, $x_2 \approx 2.0110$. Since the interval (1.9889 , 2.0110) is not symmetrical about x = 2. The distance from x = 1.9889 to the left end point is 2 – 1.9889 = 0.0111 and the distance from the right end point is 2.0110 – 2 = 0.0110. We choose $S_2 = 0.01$.

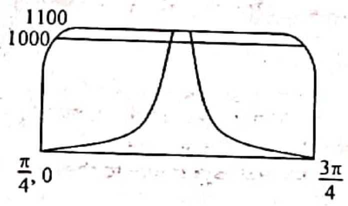
5.





when ε = 0.5, the values of x range from – 0.3 and 0.2 thus we choose the smallest δ = 0.2.
Likewise, when ε = 0.1, – 0.05 < x < 0.05, therefore δ = 0.05.

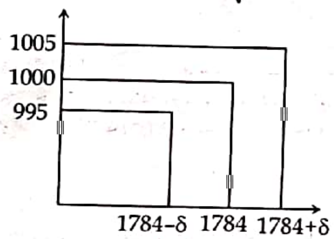
6.



From the graph we can see that $y = \tan^2 x = 10,000$. When $x \approx 1.561$ and $x \approx 1.581$ for x near $\frac{\pi}{2}$, Thus we have gave $\delta \approx 1.581 - \frac{\pi}{2} \approx 0.010$ for M = 10,000.

7. (a) $A = \pi r^2 = 1000\ cm^2 = 1\ r = \sqrt{\frac{1000}{\pi}}$ $\therefore r \approx 17.8412$ cm

(b)



$995 = n\ [1784 - \delta]^2$ and $1005 = 1\ (17.84 + \delta)^2$

$\sqrt{\frac{995}{11}} = 1784 - \delta$ $\delta = 0.0458$

$\therefore$ $\delta = 0.04341 = 0.043$

8. For ε = 0.1

a. we have, $|4x - 8| = 4\ |x - 2| < 0.1 \Rightarrow |x\ .2\ | < \frac{0.1}{4}$ so $\delta = \frac{0.1}{4} = 0.025$

b. For ε = 0.01 then δ = 0.0025

Since $|4x - 8| = 4\ |x\ .\ 2| < 0.01 \Rightarrow |x - 2| < \frac{0.01}{4} = 0.0025$

Hence, δ = 0.0025.

9. For ε = 0.1, if $0 < |x - 2| < \delta$. The $\left|\frac{5x - 7}{-3}\right| < 0.1$

We want to find a numbers for a = 2, c = 3, ε = 0.1

$|5x - 7 - 3| < 0.1$

$|5x - 10| < 0.1$

$|x - 2| < \frac{0.1}{5} = 0.02$

$\therefore$ δ = 0.02

In this special case we got δright away because we started with $0 < |x - 2| < \delta$ and got $|x - 2| < 0.02$ there for δ = 0.02.

Similarly, $\delta = \frac{0.05}{5} = 0.01$ and $\delta = \frac{0.01}{5} = 0.002$

Exercise 2.2

1. Since sinx is continuous in its domain. The function 2 + cosx is the sum of two continuous functions is therefore continuous. Note that 2 + cosx is never zero because cosx ≥ –1 and ≤ 1 for all x so 2 + cosx > 0 everywhere. Then the ratio the quotient of two continuous functions is out continuous at r = 0.

Hence,

$$\lim_{x \to \pi} \frac{\sin x}{2 + \cos x} = f(0) = \frac{0}{2 + 1} = 0$$

2. If f is continuous over (–∞, ∞), the graph will have no vertical asymptotes or holes. You can draw the complete graph without lifting the pen of the paper.

3.

(a) (i) f has irremovable discontinuing with f(–u) is not defined.
(ii) If x = –2 and x = 2 both are jump discontinuity.
(iii) x = 4 infinite discontinuity.

(b) f(x) has a point discontinuity at x = –4 (neither), a jump discontinuity at x = –2 (continuous from the left), x = 2 (continuous from the right), an infinite discontinuity at n = 4 (from the right).

Compare the value of f(a) to the limit of f(x) as x approaches a.

4. At x = –4, graph is continuous from the left.
At x = –2, graph is not continuous from both left and right.
At x = 2, graph is not continuous from left but continuous from right.
At x = 4, graph is not continuous from both left and right.
At x = 6, graph is not continuous from both left and right.
At x = 8, graph is not continuous from left.
So the interval of continuity is

5. Use the definitions of continuity and the properties of limits to show that the function is continuous at the given number a

i. $f(x) = 3x^4 - 5x + 3\sqrt{x^2 + 4}$, a = 2

Here, D = (– ∞, ∞), because the function is polynomial with a cubic root, the domain of the function is all the real numbers. Therefore, a is part of the domain

Now, $\lim_{x \to a} f(x) = \lim_{x \to 2} 3x^2 - 5x + \sqrt[3]{x^2+4} = 3(2)^4 - 5(2) + \sqrt[3]{2^2+4} = 40$

$f(2) = 3x^2 - 5x + \sqrt[3]{x^2+4} = 40$

Therefore, $\lim_{x \to a} f(x)$ exists and $\lim_{x \to a} f(x) = f(a)$.

Because the function when a = 2 meet every requirements the function is said to be continuous at a = 2.

f(x) is continuous at a = 2.

ii. Here, $f(x) = (x + 2x^3)^4$, $a = -1$

$\lim_{x \to -1} f(x) = \lim_{x \to -1} (x + 2x^3)^4$

$\lim_{x \to -1} (x + 2x^3)^4 = \lim_{x \to -1} (x + 2x^3)^4$.

Power law of limits.

$\lim_{x \to -1} (x + 2x^3)^4 = \left(\lim_{x \to -1} x + \lim_{x \to -1} 2x^3\right)^4$ sum of limits.

$\left(\lim_{x \to -1} x + \lim_{x \to -1} 2x^3\right)^4 = \left(\lim_{x \to -1} x + 2 + \lim_{x \to -1} x^3\right)^4$ constant law for limits.

$\left(\lim_{x \to -1} x + 2 + \lim_{x \to -1} x^3\right)^4 = ((-1) + 2 . (-1)^3)^4$

$= f(-1)$

Hence prove that f(x) is continuous at %.

iii. $h(t) = \frac{2t - 3t^2}{1 + t^3}$, $a = 1$

Here, $\lim_{t \to 1} h(t) = \frac{2 \times 1 - 3 \times 1^2}{1 + 1^3} = \frac{2-3}{2} = -\frac{1}{2}$

and $h(1) = -\frac{1}{2}$

Replacing t with 1 will allow you to find the value of h as t = 1. It's equal to the limit so the function is continuous.

6. Explain. Using theorem of continuous of every number in it's domain. State the domain.

a. $G(x) = \frac{x^2 + 1}{2x^2 - x - 1}$

This function is rational so it is continuous on its domain. The domain of G(x) is every value of x for which the denominator $2x^2 - x - 1$ is non zero. Let the denominator equal to 0; by factoring we find that this happens when $x = -\frac{1}{2}$ and x = 1. The domain of G(x) is all values of x expect for $-\frac{1}{2}$ and 1.

Domain: $\left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, 1\right) \cup (1, \infty)$

b. $f(x) = \frac{\sqrt[3]{x-2}}{x^3 - 2}$

Here, set the denominator equal to 0 and solve for x,

$x - \sqrt[3]{2}$

This value of x would make the denominator equal 0 and the function would not be defined a this point and therefore not continuously this point.

$x \neq \sqrt[3]{2}$

This is the domain and the function will be continuous everywhere on the domain because the denominator is not 0. You can take the cube root of any number so the numerator does not affect the domain.

Exercise 2.3

1. (a) $\lim_{x \to \infty} f(x) = 5$ (b) $\lim_{x \to \infty} f(x) = 3$

a. As x increases toward positive infinity, the value of f(x) becomes very close to 5 yet never reaches it. In fact, y = 5 is an horizontal asymptote of the curve f(x).

b. As x increases toward positive infinity, the value of f(x) becomes very close to 3 yet never reaches it. In fact y = 3 is an horizontal asymptote of the curve of (x).

2. For the function f whose graph is given, state the following

a. Here we can see that the function moves to the right, the curve seems to level at y = -2. This is the limit as x approaches to infinity.

Hence the line y = -2 is horizontal asymptote to the curve f(x)

$\lim_{x \to \infty} f(x) = -2$

b. Here, we see that, as the function moves to the left, the curve seems to level out at y = 2. This is the limit as x approaches negative infinity. The line y = 2 is also horizontal asymptote of the curve

$\lim_{x \to -\infty} f(x) = 2$

c. Here, we notice from the graph, as the function approaches x = 1 from the left side, it diverges towards infinity. Notice that it also does this as the function approaches x = 1 from the right side. It seems that as x approaches 1, the function becomes large and approaches infinity. Hence x = 1 is a vertical asymptote.

$\lim_{x \to 1} f(x) = \infty$

d. Notice that on the graph, as the function approaches x = 3 from the left side, it diverges towards negative infinity. Notice that it also does this as the function approaches x = 3 from the right side. It seems that as x approaches 3, the function drops and approaches negative infinity.

$\lim_{x \to 3} f(x) = -\infty$

e. The limits help you find the equation. The equation asked for the equation of the asymptotes

x = 1, x = 3, y = 2, y = -2

3. For the function of whose graph is given, state the following.

a. $\lim_{x\to\infty} g(x)$

From the observation of the graph we can see that as we go along the x-axis, the oscillations decrease in amplitude and frequency, becoming a straight line. The straight line it is approaching is y = 2.

$\therefore \lim_{x\to\infty} g(x) = 2$

b. Observe the graph, notice that, as you go along the x-axis in the negative direction, the oscillations decrease in amplitude and frequency, becoming a straight line. The straight line it is approaching is y = –1.

$\therefore \lim_{x\to -\infty} g(x) = -1$

c. $\lim_{x\to 0} g(x)$

Here from the graph, as the function approaches x = 0. From the lest side, if diverges towards negative infinity. Notice that it also does this as the function approaches x = 0. From the right side. It seems that as x as approaches 0, the function drops and approaches negative infinity.

$\lim_{x\to 0} g(x) = -\infty$

d. $\lim_{x\to 2^-} g(x)$

In the graph, as the function approaches x = 2 from the left side (negative side), it drops towards negative infinity.

i.e. $\lim_{x\to 2^-} g(x) = -\infty$

e. $\lim_{x\to 2^+} g(x)$

Notice that on the graph, as the function approaches x = 2 from the right side (from the positive side) it rises towards infinity.

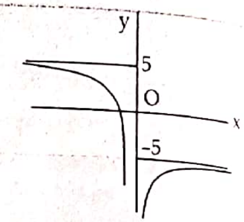
i.e. $\lim_{x\to 2^+} g(x) = \infty$

f. y = –1, y = 2. As you follow the function line from the right, the left curve seems to level out as y = –1, this means that it is a horizontal, asymptote. It does the same for the right curve, are you follow the function line from the left it levels our as y = 2, which is the second horizontal asymptote and x = 0, x = 2 are vertical asymptotes starting from the left of the graph. The two curves infinitely approaches 0, which makes it a vertical asymptote, going farther to the right, the right curve infinity approaches 2 as the left curve negative infinitely approaches 2, making it a vertical asymptote.

$\therefore$ x = 0, x = 2, y = –1, y = 2

4. Sketch the graph of an example of a function f that satisfies all of the given conditions.

a. $\lim_{x\to 0} f(x) = -\infty$ (b) $\lim_{x\to -\infty} f(x) = 5$, $\lim_{x\to \infty} f(x) = -5$



5. Guess the value of limit $\lim_{x\to\infty} \frac{x^2}{2^x}$ by evaluating the function $f(x) = \frac{x^2}{2^x}$, for x = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 20, 50, 100. Then use the graph to support your P guess.

x	$\frac{x^2}{2^x}$
0	0
1	0.5
2	1
3	1.125
4	1
5	0.78125
6	0.5625
7	0.3828125
8	0.25
9	0.15820313
10	0.09765625
20	3.814697×10^{-4}
50	2.220446×10^{-12}
100	0

from the table, above we conclude that $\lim_{x\to\infty} \frac{x^2}{2^x} = 0$

6. Evaluate the limit and justify each step by indicating the properties of limits.

a. Divide both numerator and denominator by x^2.

$$= \lim_{x\to\infty} \frac{\frac{3x^2}{x^2} - \frac{x}{x^2} + \frac{14}{x^2}}{\frac{2x^2}{x^2} + \frac{5x}{x^2} - \frac{8}{x^2}}$$

$$= \lim_{x\to\infty} \frac{3 - \frac{1}{x} + \frac{14}{x^2}}{2 + \frac{5}{x} - \frac{8}{x^2}}$$

$$= \frac{\lim_{x\to\infty} 3 - \frac{1}{x} + \frac{14}{x^2}}{\lim_{x\to\infty} 2 + \frac{5}{x} - \frac{8}{x^2}} = \frac{\lim_{x\to\infty} 3 - \lim_{x\to\infty}\frac{1}{x} + \lim_{x\to\infty}\frac{14}{x^2}}{\lim_{x\to\infty} 2 + \lim_{x\to\infty}\frac{5}{x} - \lim_{x\to\infty}\frac{8}{x^2}} = \frac{3-0+0}{2+0-0} = \frac{3}{2}$$

b. Divide both numerator and denominator by x^3.

$$= \lim_{x\to\infty} \sqrt{\frac{\frac{12x^3 - 5x + 2}{x^3}}{\frac{1 + 4x^2 + 3x^3}{x^3}}}$$

$$= \lim_{x\to\infty}\sqrt{\frac{12-\frac{5}{x^2}+\frac{2}{x^3}}{\frac{1}{x^3}+\frac{4}{x}+3}}$$

$$= \sqrt{\lim_{x\to\infty}\frac{12-\frac{5}{x^2}+\frac{2}{x^3}}{\frac{1}{x^3}+\frac{4}{x}+3}} \qquad \left[\text{Using } \lim_{x\to\infty} f(g(x)) = f\left(\lim_{x\to\infty} g/m\right)\right]$$

$$= \sqrt{\frac{\lim_{x\to\infty}12-\lim_{x\to\infty}\frac{5}{x^2}+\lim_{x\to\infty}\frac{2}{x^3}}{\lim_{x\to\infty}\frac{1}{x^3}+\lim_{x\to\infty}\frac{4}{x}+\lim_{x\to\infty}3}} = \sqrt{\frac{12-0+0}{0+0+3}} = \sqrt{4} = 2$$

7. Find the limit or show that it does not exists.

a. $\lim_{x\to\infty}\frac{3x-2}{2x+1}$

Solution.

$\lim_{x\to\infty}\frac{3x-2}{2x+1}$

Divide both numerator and denominator by x

$$= \lim_{x\to\infty}\frac{\frac{3x-2}{x}}{\frac{2x+1}{x}} = \lim_{x\to\infty}\frac{\frac{3x}{x}-\frac{2}{x}}{\frac{2x}{x}+\frac{1}{x}} = \lim_{x\to\infty}\frac{3-\frac{2}{x}}{2+\frac{1}{x}} = \lim_{x\to\infty}\frac{3-\frac{2}{x}}{2+\frac{1}{x}} = \frac{\lim_{x\to\infty}3-\frac{2}{x}}{\lim_{x\to\infty}2+\frac{1}{x}}$$

$$= \frac{\lim_{x\to\infty}3-\lim_{x\to\infty}\frac{2}{x}}{\lim_{x\to\infty}2+\lim_{x\to\infty}\frac{1}{x}} = \frac{3-0}{2+0} = \frac{3}{2}$$

b. $\lim_{t\to\infty}\frac{\sqrt{t}+t^2}{2t-t^2}$

Solution.

Divide both the numerator and denominator by t^2.

$$= \lim_{t\to\infty}\frac{\frac{\sqrt{t}}{t^2}+\frac{t^2}{t^2}}{\frac{2t}{t^2}-\frac{t^2}{t^2}} = \frac{\lim_{t\to\infty}t^{-3/2}+1}{\lim_{t\to\infty}\frac{2}{t}-1} = \frac{\lim_{t\to\infty}\frac{1}{t^{3/2}}+1}{\lim_{t\to\infty}\frac{2}{t}-1} = \frac{0+1}{0-1} = -1$$

c. $$\lim_{x\to\infty}\frac{\sqrt{9x^6-x}}{x^3+1} = \lim_{x\to\infty}\frac{\sqrt{\frac{9x^6}{x^6}-\frac{x}{x^6}}}{\frac{x^3}{x^3}+\frac{1}{x^3}} = \sqrt{\frac{\lim_{x\to\infty}\left(9-\frac{1}{x^5}\right)}{\lim_{x\to\infty}\left(1+\frac{1}{x^3}\right)}} = \frac{\sqrt{9-0}}{1} = 3$$

d. $\lim_{x\to\infty}\left(\sqrt{9x^2+x}-3x\right)$

$$= \lim_{x\to\infty}\sqrt{9x^2+x}-3x \times \frac{\sqrt{9x^2+x}+3x}{\sqrt{9x^2+x}+3x}$$

$$= \lim_{x\to\infty}\frac{9x^2+x-9x^2}{\sqrt{9x^2+x}+3x}$$

$$= \lim_{x\to\infty}\frac{x}{\sqrt{9x^2+x}+3x}$$

$$= \lim_{x\to\infty}\frac{x/x}{\sqrt{\frac{9x^2}{x^2}+\frac{x}{x^2}}+\frac{3x}{x}}$$

$$= \lim_{x\to\infty}\frac{1}{\sqrt{9+\frac{1}{x}}+3}$$

$$= \frac{1}{\lim_{x\to\infty}\sqrt{9+\frac{1}{x}}+3}$$

$$= \frac{1}{\sqrt{9}+3}$$

$$= \frac{1}{3+3}$$

$$= \frac{1}{6}$$

e. $\lim_{x\to\infty}\arctan(e^x)$

Solution.

Since, $\lim_{x\to\infty} f(g(x)) = f\left(\lim_{x\to\infty} g(x)\right)$

$$= \arctan\left(\lim_{x\to\infty}e^x\right)$$

$$= \arctan(\infty)$$

$$= \frac{\pi}{2}$$

8.

a. $y = \frac{2x+1}{x-2}$

Solution.

Since the rational function $y = \frac{2x+1}{x-2}$ is in lowest terms (It can't be reduced or simplified further), and x = 2 makes the denominator = 0, therefore it must be the vertical asymptote.

Again, since the degree of the numerator is same on the degree of the denominator.

Therefore,

$$y = \frac{\text{Leading coefficient of N}^n\text{ polynomial}}{\text{Leading coefficient of Dr. polynomial}} = \frac{2}{1} = 2$$

$\therefore \quad y = 2$

i.e. $\lim\limits_{x\to\infty} \dfrac{2x+1}{x-2} = \lim\limits_{x\to\infty} \dfrac{2+\frac{1}{x}}{1-\frac{2}{x}} = 2$ i.e. limit at infinity is horizontal asymptote.

b. $y = \dfrac{x^2+1}{2x^2-3x-2}$

Here the function is rational $y = \dfrac{x^2+1}{(2x+1)(x-2)}$ is in lowest terms.

y becomes infinite for $2x + 1 = 0$ i.e. $x = \dfrac{-1}{2}$ and $x - 2 = 0$ i.e. $x = 2$.

∴ $x = 2$ and $x = \dfrac{-1}{2}$ are the vertical asymptotes of the curve.

Again for the horizontal asymptote, we have to evaluate.

$$\lim_{x\to\infty} \frac{x^2+1}{2x^2-3x-2} = \lim_{x\to\infty} \frac{1+\frac{1}{x^2}}{2-\frac{3}{x}-\frac{2}{x^2}} = \frac{1}{2}$$

∴ $y = \dfrac{1}{2}$ is the horizontal asymptote of the curve.

c. $y = \dfrac{2x^2+x-1}{x^2+x-2} = \dfrac{2x^2+x-1}{(x-1)(x+2)}$

Here, f(x) becomes infinite for $x = 1$ and $x = -2$ are vertical asymptotes of curve.

For the horizontal asymptotes.

$$\lim_{x\to\infty} \frac{2x^2+x-1}{x^2+x-2} = \lim_{x\to\infty}\left(\frac{2+\frac{1}{x}-\frac{1}{x^2}}{1+\frac{1}{x}-\frac{2}{x^2}}\right) = \frac{\lim\limits_{x\to\infty} 2+\frac{1}{x}-\frac{1}{x^2}}{\lim\limits_{n\to\infty} 1+\frac{1}{x}-\frac{2}{x^2}} = \frac{2}{1}$$

∴ $y = 2$ is horizontal asymptote.

d. $y = \dfrac{1+x^4}{x^2-x^4} = \dfrac{1+x^4}{x^2(1-x)(1+x)}$

Here, f(x) becomes infinite for $x = 0$, $x = 1$, $x = -1$.

Hence, these are the vertical asymptotes.

For horizontal asymptotes.

$$\lim_{x\to\infty} \frac{1+x^4}{x^2-x^4} = \lim_{x\to\infty} \frac{x^4\left(\frac{1}{x^4}+1\right)}{x^4\left(\frac{x^2}{x^4}-1\right)} = \frac{\lim\limits_{x\to\infty}\left(\frac{1}{x^4}+1\right)}{\lim\limits_{x\to\infty}\left(\frac{1}{x^2}-1\right)} = \frac{1}{-1} = -1$$

∴ $y = -1$ is a horizontal asymptote.

9. Find the horizontal and vertical asymptotes of $y = \dfrac{x^3}{x^2-6x+5}$

Solution.

Here, $g = \dfrac{x^3-x}{(x-5)(x-1)}$

Here, f(x) becomes infinite for $x = 5$, $x = 1$, hence $x = 5$, $x = 1$ are ver[illegible] asymptotes of the curve. Again,

$\lim\limits_{x\to\infty} \dfrac{x^3-x}{(x-5)(x-1)}$, $f(x) \to \infty$, Hence it has no horizontal asymptote.

10.

a. $f(x) = \dfrac{2x^2}{1-x}$

Since degree of N^n > Degree of denominator. So, slant asymptote exists.

$\dfrac{2x^2}{1-x} = -2x - 2 + \dfrac{2}{1-x}$

$$\begin{array}{r|l} -x+1 & 2x^2 \;(-2x-2 \\ & \underline{2x^2-2x} \\ & \quad 2x \\ & \underline{\quad 2x-2} \\ & \qquad 2 \end{array}$$

Which shows that as $x\to\infty$ the function $\dfrac{2x^2}{1-x}$ approach to $-2x - 2$. Hence, the slant asymptote is $y = -2x - 2$ i.e. $y \neq 2x + 2 = 0$

b. $f(x) = \dfrac{x^3-3x^2}{x^2-1}$

here degree of N^r > Degree of d^r so slant asymptote exists.

$\dfrac{x^3-3x^2}{x^2-1} = x - 3 + \dfrac{x-3}{x^2-1}$

Similarly as $x \to \infty$, $\dfrac{x^3-3x^2}{x^2-1}$ approaches to $x - 3$. Hence

$y = x - 3$ is oblique asymptote.

c. $\dfrac{4-6x+2x-3x^2}{4x^2+12x+9} = \dfrac{-3x^2-4x+4}{4x^2+12x+9}$

Since degree N^r = Degree of D^r so oblique asymptote does not exists.

Since, $\dfrac{-3x^2-4x+4}{4x^2+12x+9} = \dfrac{-3}{4} + \dfrac{5x-\frac{11}{4}}{4x^2+12+9}$

Since for $x\to\infty$, we have $\dfrac{-3x^2-4x+4}{4x^2+12x+9}$ approach to $\dfrac{-3}{4}$.

Hence $y = \dfrac{-3}{4}$ is horizontal asymptote,

Does not exists.

d. $f(x) = \dfrac{x^3-1}{x^2-x-2}$

$\dfrac{x^3-1}{x^2-x-2} = x + 1\dfrac{3x+1}{x^2-x-2}$

For $x\to\infty$, $\dfrac{x^3-1}{x^2-x-2}$ approach to $x + 1$

Hence, oblique asymptote is $y = x + 1$.

Does not exists.

e. $f(x) = \dfrac{x^2-2x}{x^3+1}$

Since degree of N^r < Degree of D^r. So oblique asymptote. Which does

Here as $x\to\infty$, $f(x)\to 0$ so $y = 0$ is horizontal asymptote. exists.

$$\frac{1-x^3}{x} = -x^2 + \frac{1}{x}$$

Here, oblique asymptote is $y = -x^2$ which is not linear.

$$\frac{x^3-1}{2x^2-2} = \frac{x}{2} + \frac{x-1}{2x^2-2}$$ Which shows $y = \frac{x}{2}$ oblique asymptote.

$$f(x) = \frac{x^4-2x^3+1}{x^2}$$

$$\frac{x^4-2x^3+1}{x^2} = x^2 - 2x + \frac{1}{x^2}$$

Oblique asymptote is: $y = x^2 - 2x.y$

I.

$$\lim_{x\to\infty} \sqrt{x^2+1} - x$$

$$= \lim_{x\to\infty} \frac{\sqrt{x^2+1}-x}{\sqrt{x^2+1}+x} \times \sqrt{x^2+1}+x = \lim_{x\to\infty} \frac{x^2+1-x^2}{\sqrt{x^2+1}+x} = \lim_{x\to\infty} \frac{1}{\sqrt{x^2+1}+x} = 0$$

$$\lim_{x\to 2^+} \text{arc tan}\left(\frac{1}{x-2}\right)$$

$$= \lim_{x\to 2^+} \tan^{-1}\left(\frac{1}{x-2}\right) = \tan^{-1} \lim_{x\to 2^+}\left(\frac{1}{x-2}\right) = \tan^{-1}(\infty) = \frac{\pi}{2}$$

$$\lim_{x\to 0^-} e^{\frac{1}{x}}$$

Here x is very (small in magnitude) number but negative so $\frac{1}{x}$ is becomes very large in magnitude but negative in sign and hence $e^{\frac{1}{x}} \to 0$

$$\lim_{x\to\infty} \sin x.$$

Here, $\lim_{x\to\infty} \sin x = \sin\infty$ = Finite. But we can not declare the figure value. So, limit does not exists.

$$\lim_{x\to\infty} x^3 \text{ and } \lim_{x\to -\infty} x^3$$

Here, $\lim_{x\to\infty} x^3 = \infty$, $\lim_{x\to -\infty} x^3 = -\infty$

$$\lim_{x\to\infty} (x^2 - x) = \lim_{x\to\infty} x(x-1) = \infty \times \infty = \infty$$

Does not exists.

$$\lim_{x\to\infty} \frac{x^2+x}{3-x} = \lim_{x\to\infty} \frac{x^2\left(1+\frac{1}{x}\right)}{x\left(\frac{3}{x}-1\right)} = \lim_{x\to\infty} \frac{x}{-1} = \lim_{x\to\infty} -x = -\infty$$

Hence limit does not exists.

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