Tribhuvan University

Institute of Science and Technology

2070

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Bachelor Level/First Year/ Second Semester/ Science
Computer Science and Information Technology
(MTH.155 – Linear Algebra)

Full Marks: 80
Pass Marks: 32
Time: 3hours

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.

Attempt all questions:

Group A **(10 x 2 = 20)**

1. Why the system $x_1 - 3x_2 = 4; -3x_1 + 9x_2 = 8$ is inconsistent? Give the graphical representation?

2. Define linear combination of vectors. If v_1, v_2, v_3 are vectors, writhe the linear combination of $3v_1 - 5v_2 + 7v_3$ as a matrix times a vector.

3. Is $\begin{bmatrix} 2 & 3 & 4 \\ 2 & 3 & 4 \\ 2 & 3 & 4 \end{bmatrix}$ invertible matrix?

4. Define invertible linear transformation.

5. Let S be the parallelogram determined by the vectors $b_1 = (1, 3)$ and $b_2 = (5, 1)$ and let $A = \begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix}$. Compute the area of the image S under the mapping $x \to Ax$.

6. Define vector space.

7. Show that the entries in the vector $x = (1, 6)$ are the coordinates of x relative to the standard basis (e_1, e_2).

8. Is $\lambda_1 = -2$ an Eigen value of $\begin{pmatrix} 7 & 3 \\ 3 & -1 \end{pmatrix}$?

9. Find the inner product of (1, 2, 3) and (2, 3, 4).

10. Compute the norm between the vectors 4 = (7, 1) and v = (3, 2).

Group B (5 x 4 = 20)

11. A linear transformation $T: R^2 \to R^2$ is defined by

$$T(x) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -x_2 \\ x_1 \end{bmatrix}$$

Find the image of T of $u = \begin{bmatrix} 4 \\ 1 \end{bmatrix}, y = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $u + v = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$.

12. If $A = \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix}$ and $x = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$ compute $(Ax)^T, x^T A^T$ and xx^T. Can you compute $x^T A^T$?

13. If $b_1 = (2, 1), b_2 = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$ and $B = \{b_1, b_2\}$, find the co-ordinate vector $[x]_B$ of x relative to B.

14. Find the eigen values of $A = \begin{pmatrix} 2 & 3 \\ 3 & -6 \end{pmatrix}$.

15. Show that $\{v_1, v_2, v_3\}$ is an orthogonal set, where $v_1 = (3, 1, 1), v_2 = (-1, 2, 1), v_3 = \left(-\frac{1}{2}, -2, \frac{7}{2}\right)$.

Group C (5 x 8 = 40)

16. Let $a_1 = (1, 2, -5), a_2 = (2, 5, -3)$ and $b = (7, 4, -3)$. Determine whether b can be generated as a linear combination of a_1 and a_2. That is, determine whether x_1 and x_2 exists such that $x_1 a_1 + x_2 a_2 = b$ has the solution, find it.

OR

Determine if the following system is consistent

$$x_2 - 4x_3 = 8$$

$$2x_1 - 3x_2 + 2x_3 = 1$$

$$5x_1 - 8x_2 + 7x_3 = 1$$

17. Compute the multiplication of partitioned matrices for

$$A = \begin{bmatrix} 1 & -3 & 2 & 0 & -4 \\ 1 & 5 & -2 & 3 & -1 \\ 0 & 4 & -2 & 7 & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 6 & 4 \\ -2 & 1 \\ -3 & 7 \\ -1 & 3 \\ 5 & 2 \end{bmatrix}$$

18. Let $b_1 = (1, 0, 3), b_2 = (2, 1, 8), b_3 = (1, -1, 2) and\ x = (3, -5, 4)$. Does $B = \{b_1, b_2, b_3\}$ form a basis? Find $[x]_B$, for x.

19. Diagonalize the matrix, if possible

$$A = \begin{bmatrix} -1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}.$$

20. When two vectors *u* and *v* are orthogonal? If *u* and *v* are vectors, prove that $[dist(u, -v)]^2 = [dist(u, v)]^2$ iff *u, v = 0*.

OR

Find a least square solution of Ax = b for

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}, b = \begin{bmatrix} -3 \\ -2 \\ 0 \\ 2 \\ 5 \\ 1 \end{bmatrix}.$$