(Example)

T.U. Microsyllabus

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5.1 The energy gap E_g in silicon is 1.1 eV. The average electron effective mass is $0.31\, m_e$, where m is the free electron mass. Calculate the electron concentration in the conduction band of Si at room temp^r, $T = 300K$. Assume $E_F = \frac{E_g}{2}$.

Soln:

We have,

$$N_e = 2\left(\frac{2\pi m_e^* K_B T}{h^2}\right)^{\frac{3}{2}} e^{\frac{E_F - E_g}{K_B T}}$$

Here, $m_e^* = 0.31\, m_e = 0.31 \times 9.1 \times 10^{-31}$ kg.

$K_B = 1.38 \times 10^{-23}\ JK^{-1}$

$h = 6.62 \times 10^{-34}\ Js^{-1}$.

$$\therefore N_e = 4.36 \times 10^{-3}\ e^{\left(-\frac{0.55eV \times 1.6\times 10^{-19}}{1.38\times 10^{-23} \times 300}\right)}$$

$\because E_F = E_g/2$

$$= 2.6 \times 10^{15} / m^3 \quad \#$$

25.2 (worked example)

A sample of Si is dopped with phosphorus. The donor impurity level lies 0.045 eV below the bottom of the conduction band. At T = 300 K, E_F is 0.010 eV above the donor level. Calculate
(a) the impurity concentration,
(b) the no. of ionised impurities,
(c) the free electron concentration and
(d) the hole concentration.
(For Si, E_g = 1.1 eV, $m_e^* = 0.31 m$, $m_h^* = 0.38 m$)

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Solution:

We have,

(a) $$N_c e^{E_F - E_g/k_BT} = N_v e^{-E_F/k_BT} + N_D\left[1 - \frac{1}{e^{\frac{E_D - E_F}{k_BT}} + 1}\right] \quad \text{—— (i)}$$

Here, $$N_c = 2\left(\frac{2\pi m_e^* k_BT}{h^2}\right)^{3/2}$$

$$= 2\left(\frac{2\pi \times 0.31 \times 9.1 \times 10^{-31} \times 1.38 \times 10^{-23} \times 300}{(6.62 \times 10^{-34})^2}\right)^{3/2}$$

$$= 4.39 \times 10^{24}\, m^{-3}.$$

$$N_v = 2\left(\frac{2\pi m_h^* k_BT}{h^2}\right)^{3/2}$$

$$= 2\left(\frac{2\pi \times 0.38 \times 9.1 \times 10^{-31} \times 1.38 \times 10^{-23} \times 300}{(6.62 \times 10^{-34})^2}\right)$$

$$= 5.95 \times 10^{24}\, m^{-3}.$$

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Substituting these values alongwith E_F and E_D in eq^n ① above, one gets

$$4.39\times10^{24}\, e^{\left[-\frac{(1.1-1.065)}{0.025}\right]} - 5.95\times10^{24}\, e^{\left(-\frac{1.065}{0.025}\right)} + N_D\left[1-\frac{1}{e^{\left(\frac{-0.010}{0.025}\right)}+1}\right]$$

or, $1.08\times10^{24} = 1.88\times10^{6} + N_D\,(0.40)$

$\therefore\ N_D = 2.7\times10^{24}\ m^{-3}.$

(b) For the ionized no. of impurities,

$$N_D^{+} = N_D\left[1-\frac{1}{e^{\frac{E_D-E_F}{K_BT}}+1}\right]$$

$$= 2.7\times10^{24}\left[1-\frac{1}{e^{\left(\frac{-0.010}{0.025}\right)}+1}\right]$$

$$= 1.08\times10^{24}\ m^{-3}.$$

(c) The free electron concentration,

$$N_e = 2\left(\frac{2\pi m_e^{*} K_BT}{h^2}\right)^{3/2} e^{\frac{E_F-E_g}{K_BT}}$$

$$= 4.39\times10^{24}\, e^{\frac{1.065-1.1}{0.025}}$$

$$= 1.08\times10^{24}$$

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(d) $$n_h = 2\left(\frac{2\pi m_h^* k_B T}{h^2}\right)^{3/2} e^{-E_F/k_B T}$$

$$= 5.95 \times 10^{24}\, e^{-1.065/0.025}$$

$$= 2.88 \times 10^{6}\ m^{-3}.$$

Problem 25.1.

The band gap in pure Ge is 0.67 eV.

(a) Calculate the no. of electrons per unit volume in the conduction band at 250 K, 300 K, and at 350 K.

(b) Do the same for Si assuming $E_g = 1.1$ eV. The effective mass of the electrons in germanium is 0.12 m and in Silicon 0.31 m, where m is the free electron mass.

Sol^n: Here,

Fermi energy for Ge, ~~E_g~~ $E_F = \frac{E_g}{2} = \frac{0.67}{2}$

$= 0.33$ eV.

For $T_1 = 250$ K,

$$N_e = 2\left(\frac{2\pi m_e^* k_B T}{h^2}\right)^{3/2} e^{\frac{E_F - E_g}{k_B T}}$$

$$= 2\left(\frac{2\pi \times 0.12 \times 9.1\times10^{-31} \times 1.38\times10^{-23} \times 250}{(6.62\times10^{-34})^2}\right)^{3/2} e^{\frac{0.33-0.67}{1.38\times10^{-23}\times 250}}$$

$$= 1.52 \times 10^{17}\ m^{-3}.$$

For $T_2 = 300K$,

$$n_e = 2\left\{\frac{2\pi \times 0.12 \times 9.1\times10^{-31} \times 1.38\times10^{-23}\times 300}{(6.62\times10^{-34})^2}\right\}^{3/2} e^{\frac{0.33-0.}{1.38\times10}}$$

$$= 2.63\times10^{18}\ m^{-3}.$$

And $T_3 = 350\ K$,

$$n_e = 2\left\{\frac{2\pi\times0.12\times9.1\times10^{-31}\times1.38\times10^{-23}\times350}{(6.62\times10^{-34})^2}\right\}^{3/2} e^{\frac{0.33-0.67}{1.38\times10^{-23}\times350}}$$

$$= 2.09\times10^{20}\ m^{-3}.$$

For Silicon,
(Do urself)

Problem 25.2

Suppose that the effective mass of hole in a material is four times that of electrons. At what temp^r would the Fermi level shifted by 10% from the middle of the forbidden energy gap? $E_g = 1\ eV$.

Sol^n:

Here, $m_h^* = 4 m_e^*$

Let E_g be the forbidden energy gap and E_F, the Fermi energy

The new Fermi energy,

$$E_F' = E_F + \frac{E_g}{2} \times 10\%$$

$$= \frac{E_g}{2} + \frac{E_g}{20} = \frac{11\,E_g}{20}$$

Now, $E_F' = \frac{E_g}{2} + \frac{3}{4} kT \ln\left(\frac{m_h^*}{m_e^*}\right)$

or, $\frac{11\,E_g}{20} = \frac{E_g}{2} + 3/4\, kT \ln 4.$

$\Rightarrow T = 567.70\,K.$

Problem 25.3

The energy gap in Ge is 0.67 eV. The electron and the hole effective masses are 0.12 m and 0.23 m respectively, m is the free electron mass.
Calculate the (a) Fermi energy, (b) the electron density and (c) the hole density at T = 300K.

Soln: Here,

$E_g = 0.67\,eV = 0.67 \times 1.6 \times 10^{-19}$ J.

$m_e^* = 0.12\,m = \cancel{0.12 \times 1.6 \times 10^{-19}}\ \cancel{9.1 \times 10^{-31}\,kg}.$

$m_h^* = 0.23\,m = \cancel{0.23 \times 9.1 \times 10^{-31}\,kg}.$

Fermi energy $(E_F) = ?$

Electron density $(N_e) = ?$

Hole density $(N_h) = ?$

$T = 300K.$

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We have,

(a) $$E_F = \frac{E_g}{2} + \frac{3}{4} kT \ln\left(\frac{m_h^*}{m_e^*}\right)$$

$$= \frac{0.67 \times 1.6 \times 10^{-19}}{2} + \frac{3}{4} \times 1.38 \times 10^{-23} \times 300 \ln\left(\frac{0.23m}{0.12m}\right)$$

$$= 5.36 \times 10^{-20} + 2.02 \times 10^{-21}$$

$$= 5.58 \times 10^{-20} \, J$$

$$= \frac{5.58 \times 10^{-20}}{1.6 \times 10^{-19}} \, eV = 0.348 \, eV.$$

Now, (b) $$N_e = 2\left(\frac{2\pi m_e^* k_B T}{h^2}\right)^{3/2} e^{\frac{E_F - E_g}{k_B T}}$$

$$= 2\left(\frac{2\pi \times 0.12 \times 9.1 \times 10^{-31} \times 1.38 \times 10^{-23} \times 300}{h^2}\right)^{3/2} e^{\frac{5.58 \times 10^{-20} - 0.67 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23} \times 300}}$$

$$= 3.96 \times 10^{18} \, m^{-3}.$$

and (c) $$N_h = 2\left(\frac{2\pi m_h^* k_B T}{h^2}\right)^{3/2} e^{-\frac{E_F}{k_B T}}$$

$$= 2\left(\frac{2\pi \times 0.23 \times 9.1 \times 10^{-31} \times 1.38 \times 10^{-23} \times 300}{(6.62 \times 10^{-34})^2}\right)^{3/2} e^{-\frac{5.58 \times 10^{-20}}{1.38 \times 10^{-23} \times 300}}$$

$$= 1.05 \times 10^{19} \, m^{-3}.$$

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Problem 25.13

A certain intrinsic semiconductor has a band gap E_g is 0.2 eV. Measurement shows that it has a resistivity at room tempr. 300K of 0.3 Ωm. What would you predict its resistivity to be at 350 K?

(T.U 2074)

Soln: Here,

$E_g = 0.2 eV = 0.2 \times 1.6 \times 10^{-19} J$

ρ at 300K $= 0.3 \Omega m \Rightarrow \rho_1 = ?$

ρ at 350 K $= ? \Rightarrow \rho_2 = ?$

For intrinsic semiconductor, $N_e = N_h = 2\left(\frac{2\pi k_B T}{h^2}\right)^{3/2} (m_e^* m_h^*)^{3/4} e^{-E_g/2k_BT}$

From $\sigma = ne\mu$,

$$\sigma = |e| N_e (\mu_e + \mu_h)$$

$$\therefore \rho = \frac{1}{\sigma} = \frac{1}{|e| N_e (\mu_e + \mu_h)}$$

$$\Rightarrow \frac{\rho_1}{\rho_2} = \frac{|e| (N_e)_2 (\mu_e + \mu_h)}{|e| (N_e)_1 (\mu_e + \mu_h)}$$

$$= \frac{2\left\{\left(\frac{2\pi k_B T_2}{h^2}\right)\right\}^{3/2}}{2\left\{\left(\frac{2\pi k_B T_1}{h^2}\right)\right\}^{3/2}} \frac{(m_e^* m_h^*)^{3/4}}{(m_e^* m_h^*)^{3/4}} \frac{e^{-E_g/2kT_2}}{e^{-E_g/2kT_1}}$$

$$= \left(\frac{T_2}{T_1}\right)^{3/2} e^{-\frac{E_g (T_1 - T_2)}{2kT_1T_2}}$$

$$\text{or, } \frac{0.3}{\rho_2} = \left(\frac{310}{300}\right)^{3/2} e^{\frac{0.2\times1.6\times10^{-19}(300-310)}{2\times1.38\times10^{-23}\times300\times310}}$$

$$= 2.19$$

$$\therefore \rho_2 = 0.136\ \Omega m.$$

Problem 25.16

The energy gap in Si is 1.1 eV, whereas in diamond it is 6 eV. What conclusion can you draw about the transparency of the two materials to visible light (4000 Å to 7000 Å)

Soln:

Here,

$E_g = 1.1\ eV = 1.1\times1.6\times10^{-19}$ J. (Si)

$E_g = 6\ eV = 6\times1.6\times10^{-19}$ J (diamond)

We have, $E_g = h\nu = h\frac{c}{\lambda}$

$$\therefore \lambda = \frac{hc}{E_g}$$

~~For silic~~ For Si, $\lambda_c = \frac{6.62\times10^{-34}\times3\times10^{8}}{1.1\times1.6\times10^{-19}}$

$= 1.128\times10^{-6}$ m $> (4\times10^{-7}$ m)

Opaque.

For diamond,

$$\lambda_c = \frac{6.62\times10^{-34}\times3\times10^{8}}{6\times1.6\times10^{-19}}$$

$= 2.068\times10^{-7}$ m $< 4\times10^{-7}$ m. $\Rightarrow$ Transparent.