Solution	Marks	Remarks
1. (a) O, A and B are collinear.	1M	
(b) $\lvert\overrightarrow{OA}\rvert = \sqrt{3^2+4^2} = 5$		
$\lvert\overrightarrow{OB}\rvert = \lvert\overrightarrow{OA}\rvert + \lvert\overrightarrow{AB}\rvert$		
$20 = 5 + \lvert\overrightarrow{AB}\rvert$		
$\lvert\overrightarrow{AB}\rvert = 15$		
$\overrightarrow{AB}$		
$= \lvert\overrightarrow{AB}\rvert\left(\frac{1}{5}(3\mathbf{i}+4\mathbf{j})\right)$	1M	
$= 9\mathbf{i}+12\mathbf{j}$	1A	
	----------(3)	
2. (a) $\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}$		
$= \lim_{h\to 0}\frac{(\sqrt{x+h}-\sqrt{x})(\sqrt{x+h}+\sqrt{x})}{h(\sqrt{x+h}+\sqrt{x})}$	1M	
$= \lim_{h\to 0}\frac{h}{h(\sqrt{x+h}+\sqrt{x})}$		
$= \lim_{h\to 0}\frac{1}{\sqrt{x+h}+\sqrt{x}}$		
$= \frac{1}{2\sqrt{x}}$	1A	
(b) $\frac{\mathrm{d}}{\mathrm{d}x}e^{\sqrt{x}}$		
$= \lim_{h\to 0}\frac{e^{\sqrt{x+h}}-e^{\sqrt{x}}}{h}$	1M	
$= \lim_{h\to 0}\frac{e^{\sqrt{x}}(e^{\sqrt{x+h}-\sqrt{x}}-1)}{h}$		
$= e^{\sqrt{x}}\lim_{h\to 0}\frac{\left(\frac{e^{\sqrt{x+h}-\sqrt{x}}-1}{\sqrt{x+h}-\sqrt{x}}\right)(\sqrt{x+h}-\sqrt{x})}{h}$	1M	
$= e^{\sqrt{x}}\left(\lim_{h\to 0}\frac{e^{\sqrt{x+h}-\sqrt{x}}-1}{\sqrt{x+h}-\sqrt{x}}\right)\left(\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}\right)$		
$= e^{\sqrt{x}}(1)\left(\frac{1}{2\sqrt{x}}\right)$ (by (a))		
$= \frac{e^{\sqrt{x}}}{2\sqrt{x}}$	1A	f.t.
	----------(5)	

Solution	Marks	Remarks
3. (a) $\left(x^m - \dfrac{2}{x}\right)^{24}$		
$= (x^m)^{24} + C_1^{24}(x^m)^{23}\left(\dfrac{-2}{x}\right)^1 + C_2^{24}(x^m)^{22}\left(\dfrac{-2}{x}\right)^2 + \cdots + \left(\dfrac{-2}{x}\right)^{24}$	1M	
$= x^{24m} - 48x^{23m-1} + 1104x^{22m-2} + \cdots + 2^{24}x^{-24}$		
Thus, the first three terms are x^{24m} , $-48x^{23m-1}$ and $1104x^{22m-2}$.	1A	
(b) The 19th term in the expansion		
$= C_{18}^{24}(x^m)^{24-18}\left(\dfrac{-2}{x}\right)^{18}$	1M	
$= 2^{18}C_{18}^{24}x^{6m-18}$		
$6m - 18 = 0$		
$m = 3$	1A	
The $(r+1)$th term in the expansion		
$= C_r^{24}(x^3)^{24-r}\left(\dfrac{-2}{x}\right)^r$		
$= (-2)^r C_r^{24} x^{72-4r}$		
$72 - 4r = 60$		
$r = 3$		
The required coefficient		
$= (-2)^3 C_3^{24}$		
$= -16\,192$	1A	
	----------(5)	

Solution	Marks	Remarks
4. (a) $\csc 2x - \cot 2x$		
$= \dfrac{1}{\sin 2x} - \dfrac{\cos 2x}{\sin 2x}$		
$= \dfrac{1-\cos 2x}{\sin 2x}$		
$= \dfrac{2\sin^2 x}{2\sin x \cos x}$	1M	
$= \dfrac{\sin x}{\cos x}$		
$= \tan x$	1	
(b) $(\csc 3\theta - \cot 3\theta)(\csc\theta - \cot\theta) = 1$		
$\tan\dfrac{3\theta}{2}\tan\dfrac{\theta}{2} = 1$	1M	for using (a)
$\dfrac{\sin\frac{3\theta}{2}\sin\frac{\theta}{2}}{\cos\frac{3\theta}{2}\cos\frac{\theta}{2}} = 1$		
$\dfrac{\frac{1}{2}(\cos\theta - \cos 2\theta)}{\frac{1}{2}(\cos\theta + \cos 2\theta)} = 1$	1M	
$\cos\theta - \cos 2\theta = \cos\theta + \cos 2\theta$		
$\cos 2\theta = 0$		
$2\theta = \dfrac{\pi}{2}$		
$\theta = \dfrac{\pi}{4}$	1A	f.t.
	----------(5)	

Solution	Marks	Remarks
5. (a) $\int \cos(k\ln x)\mathrm{d}x$		
$= x\cos(k\ln x) - \int x\left(-\sin(k\ln x)\right)\left(\frac{k}{x}\right)\mathrm{d}x$	1M	
$= x\cos(k\ln x) + k\int \sin(k\ln x)\mathrm{d}x$		
$= x\cos(k\ln x) + kx\sin(k\ln x) - k\int x\cos(k\ln x)\left(\frac{k}{x}\right)\mathrm{d}x$	1M	
$= x\cos(k\ln x) + kx\sin(k\ln x) - k^2\int \cos(k\ln x)\mathrm{d}x$		
$(1+k^2)\int \cos(k\ln x)\mathrm{d}x = x\cos(k\ln x) + kx\sin(k\ln x)$ + constant		
$\int \cos(k\ln x)\mathrm{d}x = \frac{x}{1+k^2}\left(\cos(k\ln x) + k\sin(k\ln x)\right)$ + constant	1	
(b) $\int_1^e \sin^2(\pi\ln x)\mathrm{d}x$		
$= \frac{1}{2}\int_1^e \left(1-\cos(2\pi\ln x)\right)\mathrm{d}x$	1M	
$= \frac{e-1}{2} - \frac{1}{2}\int_1^e \cos(2\pi\ln x)\mathrm{d}x$		
$= \frac{e-1}{2} - \frac{1}{2}\left[\frac{x}{1+4\pi^2}\left(\cos(2\pi\ln x) + 2\pi\sin(2\pi\ln x)\right)\right]_1^e$	1M	for using (a)
$= \frac{2(e-1)\pi^2}{1+4\pi^2}$	1A	
	----------(6)	
6. (a) The augmented matrix of (E) is		
$\left(\begin{array}{ccc\|c} 3 & 1 & -9 & 0 \\ 2 & 1 & -7 & 0 \end{array}\right) \sim \left(\begin{array}{ccc\|c} 1 & 0 & -2 & 0 \\ 2 & 1 & -7 & 0 \end{array}\right) \sim \left(\begin{array}{ccc\|c} 1 & 0 & -2 & 0 \\ 0 & 1 & -3 & 0 \end{array}\right)$	1M	
Thus, the solution set is $\{(2t, 3t, t) : t \in \mathbf{R}\}$.	1A	
(b) Putting $x = 2t$, $y = 3t$ and $z = t$, we have		
$\sin 2t + \cos 3t - \cos t = 0$	1M	for using the result of (a)
$\sin 2t - 2\sin 2t \sin t = 0$	1M	
$(\sin 2t)(1 - 2\sin t) = 0$		
$\sin 2t = 0$ or $\sin t = \frac{1}{2}$	1M	
Since $0 < t < \frac{\pi}{2}$, $\sin 2t > 0$.		
$\therefore t = \frac{\pi}{6}$		
Thus, the claim is agreed.	1A	f.t.
	----------(6)	

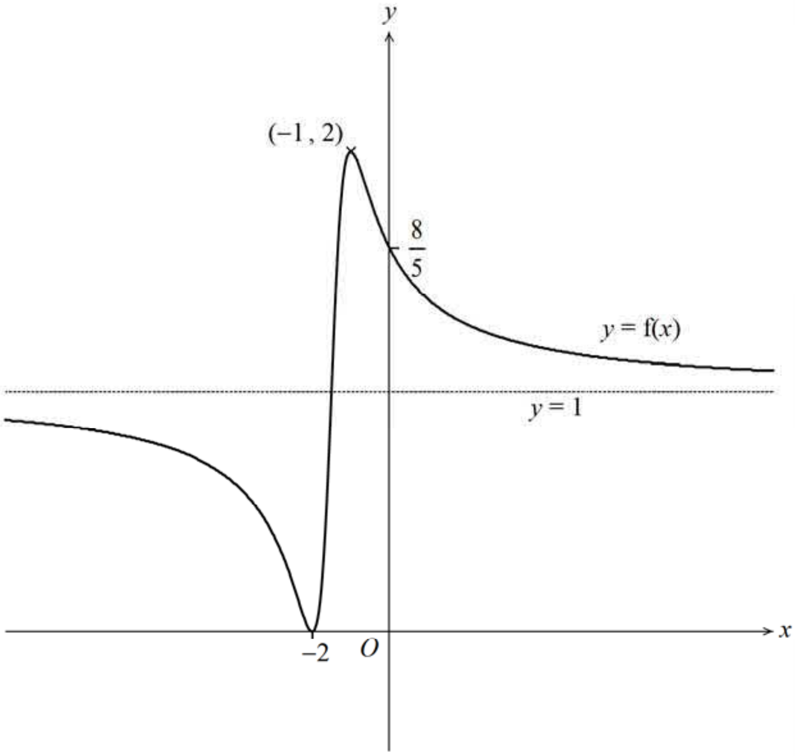
Solution	Marks	Remarks
7. (a) $x^2y+2xy^2+8=0$		
$2xy+x^2\dfrac{dy}{dx}+2y^2+(2x)(2y)\dfrac{dy}{dx}=0$	1M	
$\dfrac{dy}{dx}=\dfrac{-2y(x+y)}{x(x+4y)}$	1A	
(b) The slope of the straight line is $\dfrac{-1}{2}$.		
Let (h,k) be the coordinates of the point(s) of contact of the required tangent(s).		
$\dfrac{-2k(h+k)}{h(h+4k)}=\dfrac{-1}{2}$	1M	
$k^2=\dfrac{h^2}{4}$		
$k=\dfrac{h}{2}$ or $k=\dfrac{-h}{2}$	1M	
When $k=\dfrac{h}{2}$,		
$h^2\left(\dfrac{h}{2}\right)+2h\left(\dfrac{h}{2}\right)^2+8=0$		
$h^3=-8$		
$h=-2$	1M	either one
So, the tangent to C at the point $(-2,-1)$ is parallel to the straight line $x+2y+1=0$.		
When $k=\dfrac{-h}{2}$,		
$h^2\left(\dfrac{-h}{2}\right)+2h\left(\dfrac{-h}{2}\right)^2+8$		
$=8$		
$\neq 0$		
Therefore, there is only one tangent to C which is parallel to the straight line $x+2y+1=0$.		
Thus, the claim is disagreed.	1A ----------(6)	f.t.

Solution	Marks	Remarks
8. (a) Note that $(1)(2^{-1})=2-(1+2)(2^{-1})=\frac{1}{2}$.		
Therefore, the statement is true for $n=1$.	1	
Assume that $\sum_{r=1}^{m} r(2^{-r})=2-(m+2)(2^{-m})$, where m is a positive integer.	1M	
$\sum_{r=1}^{m+1} r(2^{-r})$ $=\sum_{r=1}^{m} r(2^{-r})+(m+1)(2^{-(m+1)})$		
$=2-(m+2)(2^{-m})+(m+1)(2^{-(m+1)})$ (by induction assumption)	1M	for using induction assumption
$=2-(2^{-(m+1)})(2(m+2)-(m+1))$ $=2-((m+1)+2)(2^{-(m+1)})$		
So, the statement is true for $n=m+1$ if it is true for $n=m$. By mathematical induction, the statement is true for all positive integers n .	1	
(b) (i) $\sum_{r=1\,000}^{1\,999} r(2^{-r})$ $=\sum_{r=1}^{1\,999} r(2^{-r})-\sum_{r=1}^{999} r(2^{-r})$ $=\left(2-(2\,001)(2^{-1\,999})\right)-\left(2-(1\,001)(2^{-999})\right)$		
$=1\,001(2^{-999})-2\,001(2^{-1\,999})$	1A	
(ii) $\sum_{r=1}^{1\,000}(2\,000-r)2^{r}$ $=1\,999(2^{1})+1\,998(2^{2})+1\,997(2^{3})+\cdots+1\,000(2^{1\,000})$ $=1\,000(2^{1\,000})+1\,001(2^{999})+1\,002(2^{998})+\cdots+1\,999(2^{1})$ $=(2^{2\,000})\left(1\,000(2^{-1\,000})+1\,001(2^{-1\,001})+1\,002(2^{-1\,002})+\cdots+1\,999(2^{-1\,999})\right)$	1M	
$=(2^{2\,000})\left(\sum_{r=1\,000}^{1\,999} r(2^{-r})\right)$ $=(2^{2\,000})\left(1\,001(2^{-999})-2\,001(2^{-1\,999})\right)$		
$=1\,001(2^{1\,001})-4\,002$	1A	
	----------(7)	

Solution	Marks	Remarks
9. (a) $\tan\theta$		
$=\tan(\angle ACO-\angle BCO)$		
$=\dfrac{\tan\angle ACO-\tan\angle BCO}{1+(\tan\angle ACO)(\tan\angle BCO)}$	1M	
$=\dfrac{\frac{4t}{10}-\frac{t}{10}}{1+\left(\frac{4t}{10}\right)\left(\frac{t}{10}\right)}$		
$=\dfrac{15t}{2(t^2+25)}$	1	
(b) (i) When $t=T$,		
$\tan\angle BAC=\tan\angle ACB$		
$\dfrac{10}{4T}=\dfrac{15T}{2(T^2+25)}$	1M	
$T^2+25=3T^2$		
$T^2=\dfrac{25}{2}$		
$T=\dfrac{5\sqrt{2}}{2}$	1A	
(ii) $\tan\theta=\dfrac{15t}{2(t^2+25)}$		
$\sec^2\theta\dfrac{\mathrm{d}\theta}{\mathrm{d}t}=\dfrac{15}{2}\left(\dfrac{t^2+25-(t)(2t)}{(t^2+25)^2}\right)$	1M + 1M	
$(1+\tan^2\theta)\dfrac{\mathrm{d}\theta}{\mathrm{d}t}=\dfrac{15(25-t^2)}{2(t^2+25)^2}$		
$\left(1+\left(\dfrac{15t}{2(t^2+25)}\right)^2\right)\dfrac{\mathrm{d}\theta}{\mathrm{d}t}=\dfrac{15(25-t^2)}{2(t^2+25)^2}$		
$\dfrac{\mathrm{d}\theta}{\mathrm{d}t}=\dfrac{30(25-t^2)}{4(t^2+25)^2+225t^2}$		
$\left.\dfrac{\mathrm{d}\theta}{\mathrm{d}t}\right\|_{t=\frac{5\sqrt{2}}{2}}=\dfrac{30\left(25-\left(\frac{5\sqrt{2}}{2}\right)^2\right)}{4\left(\left(\frac{5\sqrt{2}}{2}\right)^2+25\right)^2+225\left(\frac{5\sqrt{2}}{2}\right)^2}=\dfrac{2}{45}$		
The required rate of change of θ is $\dfrac{2}{45}$ radians per second.	1A	
	----------(7)	

Solution	Marks	Remarks
10. (a) Note that there are no vertical asymptotes of G.		
$\lim_{x\to\pm\infty}\frac{2(x+2)^2}{2x^2+6x+5}$ $=\lim_{x\to\pm\infty}\frac{2\left(1+\frac{2}{x}\right)^2}{2+\frac{6}{x}+\frac{5}{x^2}}$ $=1$	1M	
Thus, the equation of the horizontal asymptote of G is $y=1$.	1A ----------(2)	
(b) $f(x)=\frac{2(x+2)^2}{2x^2+6x+5}=1+\frac{2x+3}{2x^2+6x+5}$		
$f'(x)$ $=\frac{(2x^2+6x+5)(2)-(2x+3)(4x+6)}{(2x^2+6x+5)^2}$	1M	
$=\frac{-4(x^2+3x+2)}{(2x^2+6x+5)^2}$	1A ----------(2)	
(c) $f'(x)=0$ $x^2+3x+2=0$ $(x+2)(x+1)=0$ $x=-1$ or $x=-2$		
(table below)	1M	for testing
Thus, the maximum point and the minimum point of G are $(-1, 2)$ and $(-2, 0)$ respectively.	1A+1A ----------(3)	

x	$(-\infty, -2)$	-2	$(-2, -1)$	-1	$(-1, \infty)$
$f'(x)$	$-$	0	$+$	0	$-$
$f(x)$	↘	0	↗	2	↘

Solution	Marks	Remarks
(d)		
		
	1M	for shape
	1M	for asymptote
	1A	all correct
	----------(3)	
(e) The required area		
$= \int_{-2}^{0}\left(1+\frac{2x+3}{2x^2+6x+5}\right)\mathrm{d}x$	1M	
$= [x]_{-2}^{0} + \frac{1}{2}\int_{1}^{5}\frac{1}{u}\,\mathrm{d}u$ (by letting $u = 2x^2+6x+5$)	1M	
$= 2+\frac{1}{2}[\ln u]_{1}^{5}$		
$= \frac{1}{2}\ln 5 + 2$	1A	
	----------(3)	

Solution	Marks	Remarks
11. (a) Let $x = a\tan u$.	1M	
$\dfrac{dx}{du} = a\sec^2 u$		
$\displaystyle\int \frac{1}{x^2+a^2}\,dx$		
$\displaystyle= \int \frac{1}{a^2\tan^2 u + a^2}\left(a\sec^2 u\right)du$	1M	
$\displaystyle= \int \frac{1}{a}\,du$		
$= \dfrac{u}{a} + C$		
$= \dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right) + C$, where C is a constant	1A	f.t.
	----------(3)	
(b) Let $x = -t$.	1M	
$\dfrac{dx}{dt} = -1$		
$\displaystyle\int_{-c}^{c} \frac{g(x)}{1+e^{h(x)}}\,dx$		
$\displaystyle= -\int_{c}^{-c} \frac{g(-t)}{1+e^{h(-t)}}\,dt$		
$\displaystyle= \int_{-c}^{c} \frac{g(t)}{1+e^{-h(t)}}\,dt$ ($\because$ $g(x)$ is even and $h(x)$ is odd.)	1M	
$\displaystyle= \int_{-c}^{c} \frac{g(x)}{1+e^{-h(x)}}\,dx$		
$\displaystyle 2\int_{-c}^{c} \frac{g(x)}{1+e^{h(x)}}\,dx$		
$\displaystyle= \int_{-c}^{c} \frac{g(x)}{1+e^{h(x)}}\,dx + \int_{-c}^{c} \frac{g(x)}{1+e^{-h(x)}}\,dx$	1M	
$\displaystyle= \int_{-c}^{c} \frac{g(x)\left((1+e^{-h(x)})+(1+e^{h(x)})\right)}{(1+e^{h(x)})(1+e^{-h(x)})}\,dx$		
$\displaystyle= \int_{-c}^{c} \frac{g(x)(2+e^{-h(x)}+e^{h(x)})}{1+e^{h(x)}+e^{-h(x)}+1}\,dx$		
$\displaystyle= \int_{-c}^{c} g(x)\,dx$		
$\displaystyle= 2\int_{0}^{c} g(x)\,dx$ ($\because$ $g(x)$ is even.)		
$\displaystyle\therefore \int_{-c}^{c} \frac{g(x)}{1+e^{h(x)}}\,dx = \int_{0}^{c} g(x)\,dx$	1	

Solution	Marks	Remarks
$\displaystyle\int_{-c}^{c}\frac{\mathrm{g}(x)}{1+e^{\mathrm{h}(x)}}\mathrm{d}x=\int_{-c}^{0}\frac{\mathrm{g}(x)}{1+e^{\mathrm{h}(x)}}\mathrm{d}x+\int_{0}^{c}\frac{\mathrm{g}(x)}{1+e^{\mathrm{h}(x)}}\mathrm{d}x$	1M	
Let $x=-t$. $\dfrac{\mathrm{d}x}{\mathrm{d}t}=-1$	1M	
$\displaystyle\int_{-c}^{0}\frac{\mathrm{g}(x)}{1+e^{\mathrm{h}(x)}}\mathrm{d}x$ $\displaystyle=-\int_{c}^{0}\frac{\mathrm{g}(-t)}{1+e^{\mathrm{h}(-t)}}\mathrm{d}t$		
$\displaystyle=\int_{0}^{c}\frac{\mathrm{g}(t)}{1+e^{-\mathrm{h}(t)}}\mathrm{d}t$ $(\because$ $\mathrm{g}(x)$ is even and $\mathrm{h}(x)$ is odd.)	1M	
$\displaystyle=\int_{0}^{c}\frac{e^{\mathrm{h}(t)}\mathrm{g}(t)}{e^{\mathrm{h}(t)}+1}\mathrm{d}t$ $\displaystyle=\int_{0}^{c}\frac{e^{\mathrm{h}(x)}\mathrm{g}(x)}{1+e^{\mathrm{h}(x)}}\mathrm{d}x$		
$\displaystyle\int_{-c}^{c}\frac{\mathrm{g}(x)}{1+e^{\mathrm{h}(x)}}\mathrm{d}x$ $\displaystyle=\int_{0}^{c}\frac{e^{\mathrm{h}(x)}\mathrm{g}(x)}{1+e^{\mathrm{h}(x)}}\mathrm{d}x+\int_{0}^{c}\frac{\mathrm{g}(x)}{1+e^{\mathrm{h}(x)}}\mathrm{d}x$ $\displaystyle=\int_{0}^{c}\frac{e^{\mathrm{h}(x)}\mathrm{g}(x)+\mathrm{g}(x)}{1+e^{\mathrm{h}(x)}}\mathrm{d}x$		
$\displaystyle=\int_{0}^{c}\mathrm{g}(x)\mathrm{d}x$	1	
	----------(4)	

Solution	Marks	Remarks
(c) Let $\mathrm{g}(x)=\dfrac{3^x+3^{-x}}{9^x+9^{-x}+7}$.		
$\mathrm{g}(-x)=\dfrac{3^{-x}+3^x}{9^{-x}+9^x+7}=\mathrm{g}(x)$		
Therefore, $\mathrm{g}(x)$ is an even function.		for both
Let $\mathrm{h}(x)=\sin^3 x$.		
$\mathrm{h}(-x)=\sin^3(-x)=-\sin^3 x=-\mathrm{h}(x)$		
Therefore, $\mathrm{h}(x)$ is an odd function.	1M	withhold 1M if this step is skipped
$\displaystyle\int_{-1}^{1}\frac{3^x+3^{-x}}{(1+e^{\sin^3 x})(9^x+9^{-x}+7)}\mathrm{d}x$		
$\displaystyle=\int_0^1\frac{3^x+3^{-x}}{9^x+9^{-x}+7}\mathrm{d}x$	1M	for using (b)
$\displaystyle=\int_0^1\frac{3^x+3^{-x}}{3^{2x}+3^{-2x}+7}\mathrm{d}x$		
$\displaystyle=\int_0^1\frac{3^x+3^{-x}}{(3^x-3^{-x})^2+9}\mathrm{d}x$		
$\displaystyle=\frac{1}{\ln 3}\int_0^{\frac{8}{3}}\frac{1}{u^2+3^2}\mathrm{d}u$ (by letting $u=3^x-3^{-x}$)	1M	
$\displaystyle=\frac{1}{3\ln 3}\left[\tan^{-1}\frac{u}{3}\right]_0^{\frac{8}{3}}$	1M	for using the result of (a)
$\displaystyle=\frac{1}{3\ln 3}\tan^{-1}\frac{8}{9}$	1A	
	----------(5)	

Solution	Marks	Remarks
12. (a) (i) Note that $\begin{vmatrix} 2 & 0 & 1 \\ -(2\lambda+5) & \lambda & 0 \\ \lambda+2 & 1 & -1 \end{vmatrix}$		
$=-2\lambda-(2\lambda+5)-\lambda(\lambda+2)$	1M	
$=-\lambda^2-6\lambda-5$ $=-(\lambda+5)(\lambda+1)$		
If (E) has a unique solution, then $\begin{vmatrix} 2 & 0 & 1 \\ -(2\lambda+5) & \lambda & 0 \\ \lambda+2 & 1 & -1 \end{vmatrix} \neq 0$.	1M	
So, we have $-(\lambda+5)(\lambda+1)\neq 0$.		
Solving, we have $\lambda \neq -5$ and $\lambda \neq -1$. Thus, we have $\lambda < -5$, $-5<\lambda<-1$ or $\lambda > -1$.	1A	
(ii) $x = \dfrac{\begin{vmatrix} -2 & 0 & 1 \\ -1 & \lambda & 0 \\ 0 & 1 & -1 \end{vmatrix}}{-(\lambda+5)(\lambda+1)}$	1M	for Cramer's rule
$=\dfrac{1-2\lambda}{(\lambda+5)(\lambda+1)}$		any one
$y = \dfrac{\begin{vmatrix} 2 & -2 & 1 \\ -(2\lambda+5) & -1 & 0 \\ \lambda+2 & 0 & -1 \end{vmatrix}}{-(\lambda+5)(\lambda+1)}$		
$=\dfrac{-5\lambda-14}{(\lambda+5)(\lambda+1)}$		
$z = \dfrac{\begin{vmatrix} 2 & 0 & -2 \\ -(2\lambda+5) & \lambda & -1 \\ \lambda+2 & 1 & 0 \end{vmatrix}}{-(\lambda+5)(\lambda+1)}$		
$=\dfrac{-2(\lambda^2+4\lambda+6)}{(\lambda+5)(\lambda+1)}$	1A+1A	1A for any one +1A for all
	----------(6)	

Solution	Marks	Remarks
(b) (i) Let $\mathbf{v} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, where x , y , $z \in \mathbf{R}$.		
Since $\mathbf{v}$ is perpendicular to $(h+2)\mathbf{i}+\mathbf{j}-\mathbf{k}$, we have $(h+2)x+y-z=0$.	1M	
$\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ h & 2h+5 & -2h \\ x & y & z \end{vmatrix}$	1M	
$=\left(2hy+(2h+5)z\right)\mathbf{i}-(2hx+hz)\mathbf{j}+\left(-(2h+5)x+hy\right)\mathbf{k}$		
So, we have $-(2hx+hz)=2h$ and $-(2h+5)x+hy=-1$. Therefore, we have $2x+z=-2$ and $-(2h+5)x+hy=-1$.		
Hence, x , y and z satisfies (E) where $\lambda = h$.		
By (a)(ii), we have $x=\dfrac{1-2h}{(h+5)(h+1)}$, $y=\dfrac{-5h-14}{(h+5)(h+1)}$ and $z=\dfrac{-2(h^2+4h+6)}{(h+5)(h+1)}$.		
Thus, $\mathbf{v}=\dfrac{1-2h}{(h+5)(h+1)}\mathbf{i}+\dfrac{-5h-14}{(h+5)(h+1)}\mathbf{j}+\dfrac{-2(h^2+4h+6)}{(h+5)(h+1)}\mathbf{k}$.	1M	for using the result of (a)(ii)
$\mu = 2hy+(2h+5)z$		
$=(2h)\left(\dfrac{-5h-14}{(h+5)(h+1)}\right)+(2h+5)\left(\dfrac{-2(h^2+4h+6)}{(h+5)(h+1)}\right)$	1M	
$=\dfrac{-4h^3-36h^2-92h-60}{(h+5)(h+1)}$		
$=\dfrac{-4(h+1)(h+3)(h+5)}{(h+5)(h+1)}$		
$=-4(h+3)$	1A	
(ii) $\mathbf{u}\times\mathbf{v}=-4(h+3)\mathbf{i}+2h\,\mathbf{j}-\mathbf{k}$		
When the area of the parallelogram with adjacent sides $\mathbf{u}$ and $\mathbf{v}$ is 9 , $\lvert\mathbf{u}\times\mathbf{v}\rvert=9$		
$\sqrt{\left(-4(h+3)\right)^2+(2h)^2+1}=9$	1M	
$5h^2+24h+16=0$		
Since $5h^2+24h+16>0$ for all $h>0$, there does not exist a value of h such that the area of the parallelogram with adjacent sides $\mathbf{u}$ and $\mathbf{v}$ is 9 .	1A ----------(7)	f.t.

Solution	Marks	Remarks
13. (a) A^{-1}		
$=\dfrac{1}{\begin{vmatrix}-2 & 2\\ 1 & 2\end{vmatrix}}\begin{pmatrix}2 & -2\\ -1 & -2\end{pmatrix}$	1M	
$=\dfrac{-1}{6}\begin{pmatrix}2 & -2\\ -1 & -2\end{pmatrix}$		
$=\begin{pmatrix}\dfrac{-1}{3} & \dfrac{1}{3}\\ \dfrac{1}{6} & \dfrac{1}{3}\end{pmatrix}$		
$A-6A^{-1}$		
$=\begin{pmatrix}-2 & 2\\ 1 & 2\end{pmatrix}-6\begin{pmatrix}\dfrac{-1}{3} & \dfrac{1}{3}\\ \dfrac{1}{6} & \dfrac{1}{3}\end{pmatrix}$		
$=\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}$	1A	
ABA^{-1}		
$=\begin{pmatrix}-2 & 2\\ 1 & 2\end{pmatrix}\begin{pmatrix}4 & 2\\ 1 & 5\end{pmatrix}\begin{pmatrix}\dfrac{-1}{3} & \dfrac{1}{3}\\ \dfrac{1}{6} & \dfrac{1}{3}\end{pmatrix}$		
$=\begin{pmatrix}-6 & 6\\ 6 & 12\end{pmatrix}\begin{pmatrix}\dfrac{-1}{3} & \dfrac{1}{3}\\ \dfrac{1}{6} & \dfrac{1}{3}\end{pmatrix}$	1M	
$=\begin{pmatrix}3 & 0\\ 0 & 6\end{pmatrix}$	1A	
	----------(4)	

Solution	Marks	Remarks
(b) $ABA^{-1}=\begin{pmatrix}3&0\\0&6\end{pmatrix}$		
$(ABA^{-1})^n=\begin{pmatrix}3&0\\0&6\end{pmatrix}^n$		
$AB^nA^{-1}=\begin{pmatrix}3^n&0\\0&6^n\end{pmatrix}$	1M	
$B^n=A^{-1}\begin{pmatrix}3^n&0\\0&6^n\end{pmatrix}A$	1M	
B^n $=\begin{pmatrix}\frac{-1}{3}&\frac{1}{3}\\\frac{1}{6}&\frac{1}{3}\end{pmatrix}\begin{pmatrix}3^n&0\\0&6^n\end{pmatrix}\begin{pmatrix}-2&2\\1&2\end{pmatrix}$ $=\begin{pmatrix}-3^{n-1}&\frac{6^n}{3}\\\frac{3^n}{6}&\frac{6^n}{3}\end{pmatrix}\begin{pmatrix}-2&2\\1&2\end{pmatrix}$ $=\begin{pmatrix}2(3^{n-1})+\frac{6^n}{3}&-2(3^{n-1})+\frac{2(6^n)}{3}\\-3^{n-1}+\frac{6^n}{3}&3^{n-1}+\frac{2(6^n)}{3}\end{pmatrix}$	1A	
	----------(3)	

Solution	Marks	Remarks
(c) By (a), we have		
$A-6A^{-1}=0$		
$A^2-6I=0$		
$A^2=6I$	1M	either one
$A^2=6I$ $(A^2)^{-1}=(6I)^{-1}$ $(A^{-1})^2=\frac{1}{6}I$		
$A^{2k}B^{2k}(A^{-1})^{2k}$ $=(A^2)^kB^{2k}((A^{-1})^2)^k$ $=(6I)^kB^{2k}\left(\frac{1}{6}I\right)^k$		
$=B^{2k}$	1M	
$=\begin{pmatrix} 2(3^{2k-1})+\frac{6^{2k}}{3} & -2(3^{2k-1})+\frac{2(6^{2k})}{3} \\ -3^{2k-1}+\frac{6^{2k}}{3} & 3^{2k-1}+\frac{2(6^{2k})}{3} \end{pmatrix}$		
$(ABA^{-1})^{2k}$ $=\begin{pmatrix} 3 & 0 \\ 0 & 6 \end{pmatrix}^{2k}$ $=\begin{pmatrix} 3^{2k} & 0 \\ 0 & 6^{2k} \end{pmatrix}$		
Note that $-2(3^{2k-1})+\frac{2(6^{2k})}{3}\neq 0$ for all positive integers k.		
Thus, there does not exist a positive integer k such that $A^{2k}B^{2k}(A^{-1})^{2k}=(ABA^{-1})^{2k}$.	1A	f.t.
	----------(3)	
(d) $A^{999}B^{999}(A^{-1})^{999}$ $=(A^{998})AB^{999}A^{-1}(A^{-1})^{998}$ $=(A^2)^{499}AB^{999}A^{-1}((A^{-1})^2)^{499}$ $=(6I)^{499}AB^{999}A^{-1}\left(\frac{1}{6}I\right)^{499}$		
$=AB^{999}A^{-1}$	1M	
$=(ABA^{-1})^{999}$ $=\begin{pmatrix} 3 & 0 \\ 0 & 6 \end{pmatrix}^{999}$		
$=\begin{pmatrix} 3^{999} & 0 \\ 0 & 6^{999} \end{pmatrix}$	1A	f.t.
	----------(2)	