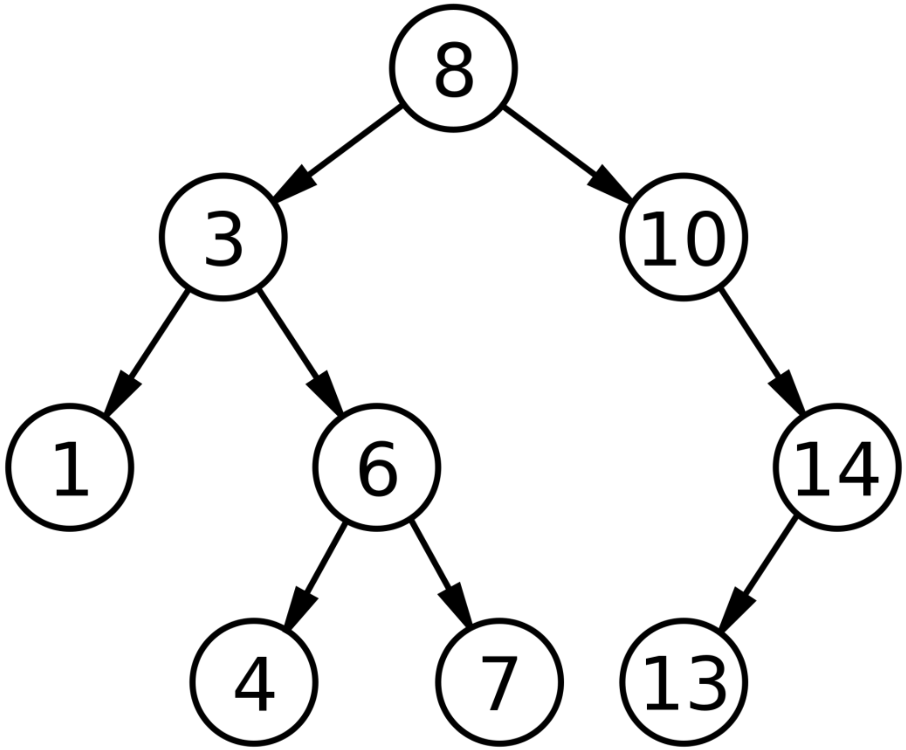
Binary Search Tree

Binary Search Tree

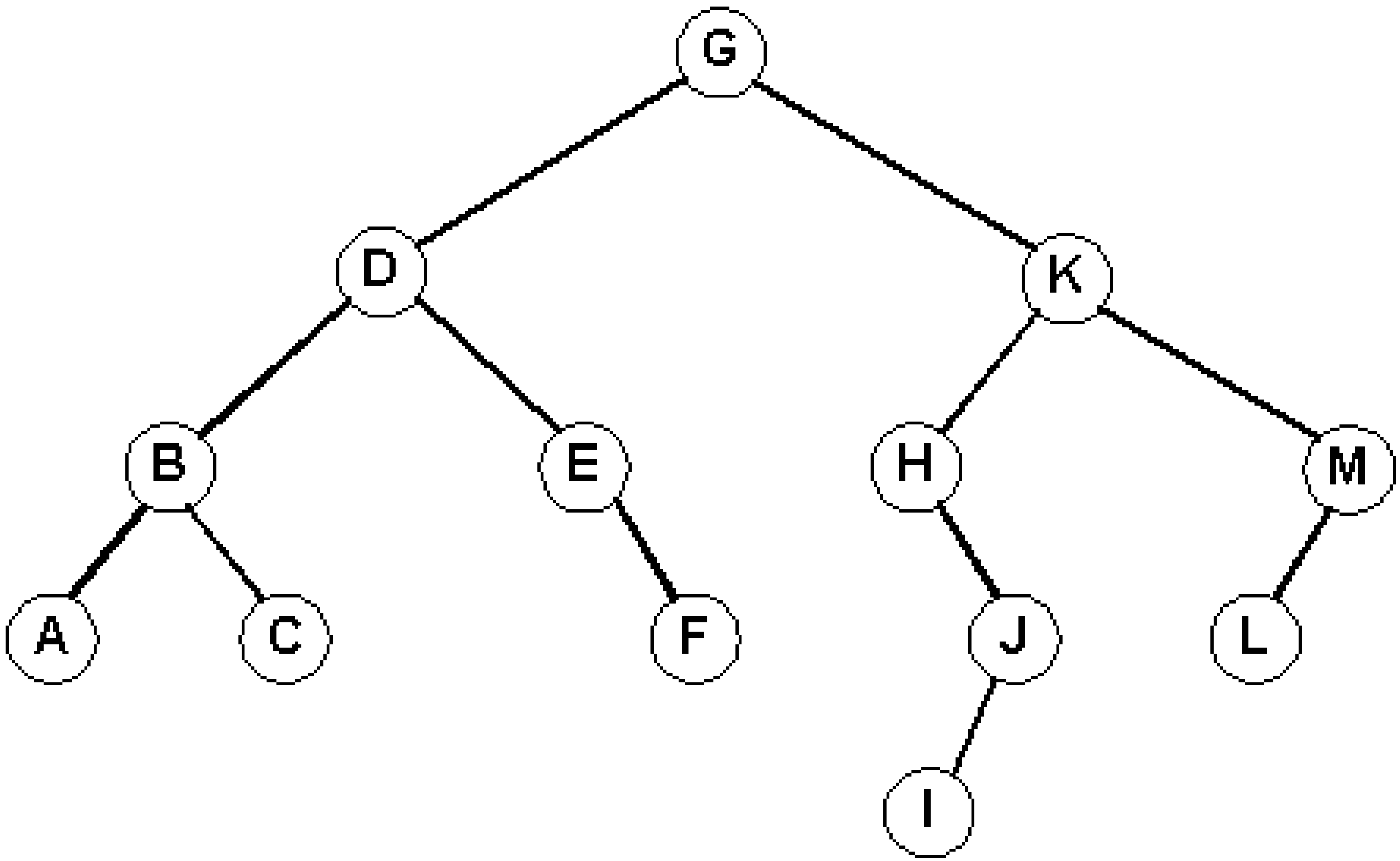
- A binary search tree (BST) is a binary tree in which
  - The values in the left subtree of a node are all less than the value in the node
  - The values in the right subtree of a node are all greater than the value of the node.
  - The subtrees of a binary search tree must themselves be binary search trees.

Binary Search Tree



Left <= Node <= Right

Binary Search Tree



Properties and Operations

- Note that under this definition, <u>a BST never contains duplicate nodes.</u>

- Some operations for BSTs:
  - InsertItem
  - DeleteItem
  - ItemExists
  - Traverse(Pre, In, Post)
  - Count
  - Height

Binary Tree Algorithms

```
struct Node{
    Node *left;
    Node *right;
    int data;
};

Node *MakeNode(int Data)
{
    Node *node = new Node;
    node->data = Data;
    node->left = 0;
    node->right = 0;
    return node;
}
```

```
void FreeNode(Node *node){
    delete node;
}

typedef Node* Tree;
```

Finding an Item in a BST

- State the recursive algorithm in English for finding an item.

```
bool ItemExists(Tree tree, int Data){
    if (tree == 0)
        return false;
    else if (Data == tree->data)
        return true;
    else if (Data < tree->data)
        return ItemExists(tree->left, Data);
    else
        return ItemExists(tree->right, Data);
}
```

Complexity

Best case?

Worst case?

Insert an Item in a BST

- State the recursive algorithm in English for inserting an item.

```
void InsertItem(Tree &tree, int Data){
    if (tree == 0)
        tree = MakeNode(Data);
    else if (Data < tree->data)
        InsertItem(tree->left, Data);
    else if (Data > tree->data)
        InsertItem(tree->right, Data);
    else
        cout << "Error, duplicate item" << endl;
}
```

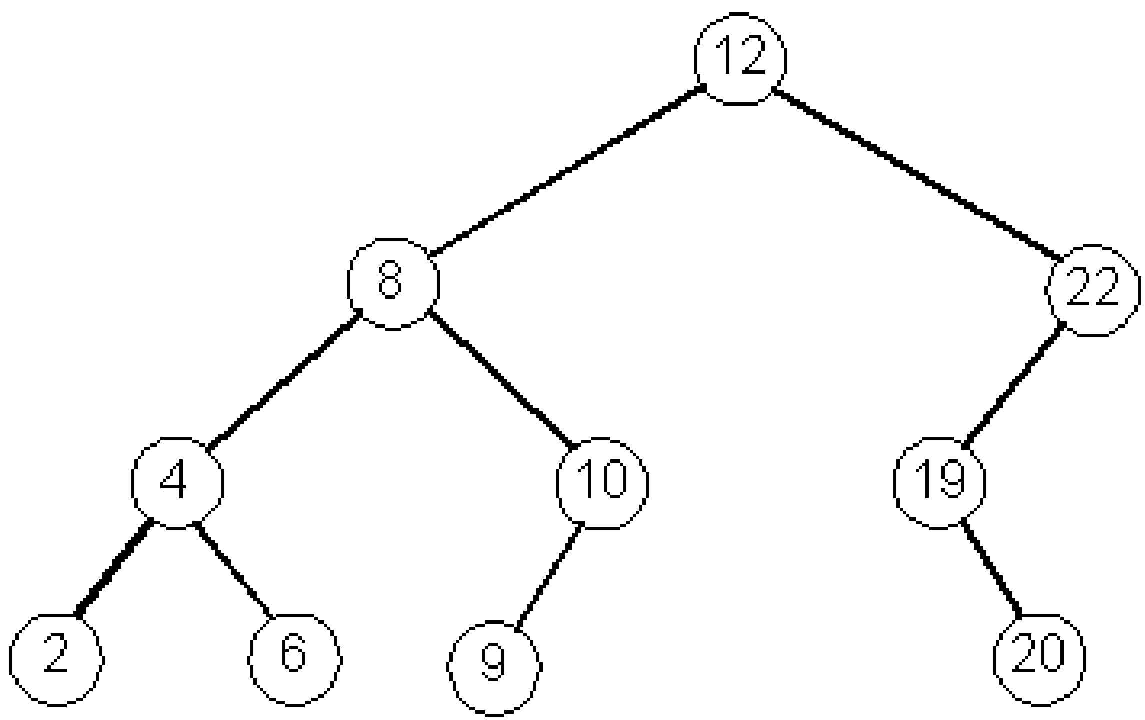
Complexity

Best case?

Worst case?

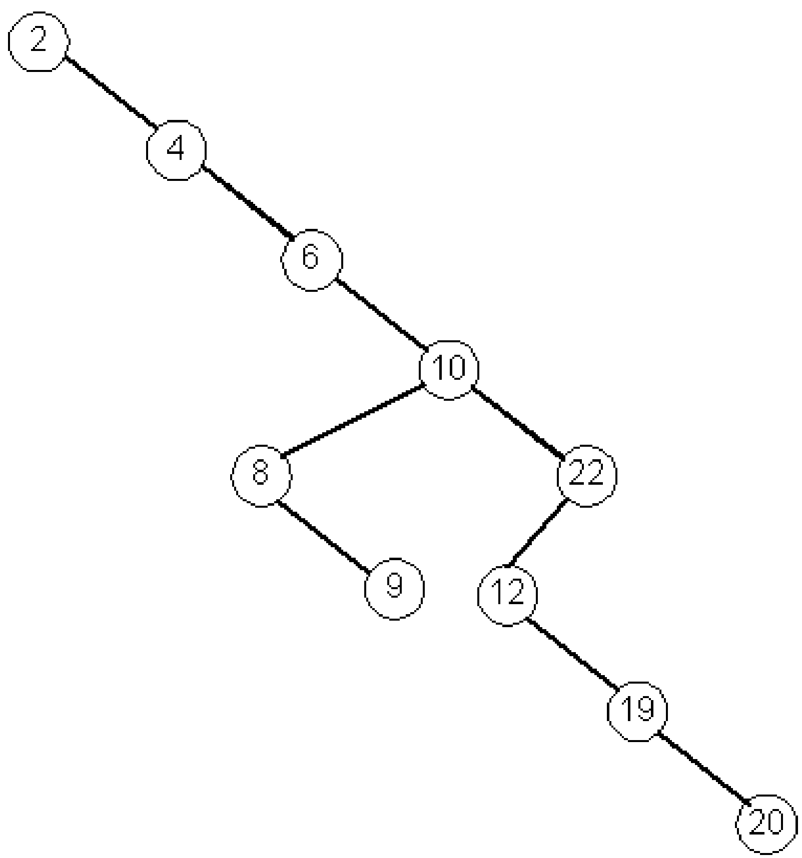
Insert an Item in a BST

- Create a tree using these values (in this order):
  - 12, 22, 8, 19, 10, 9, 20, 4, 2, 6



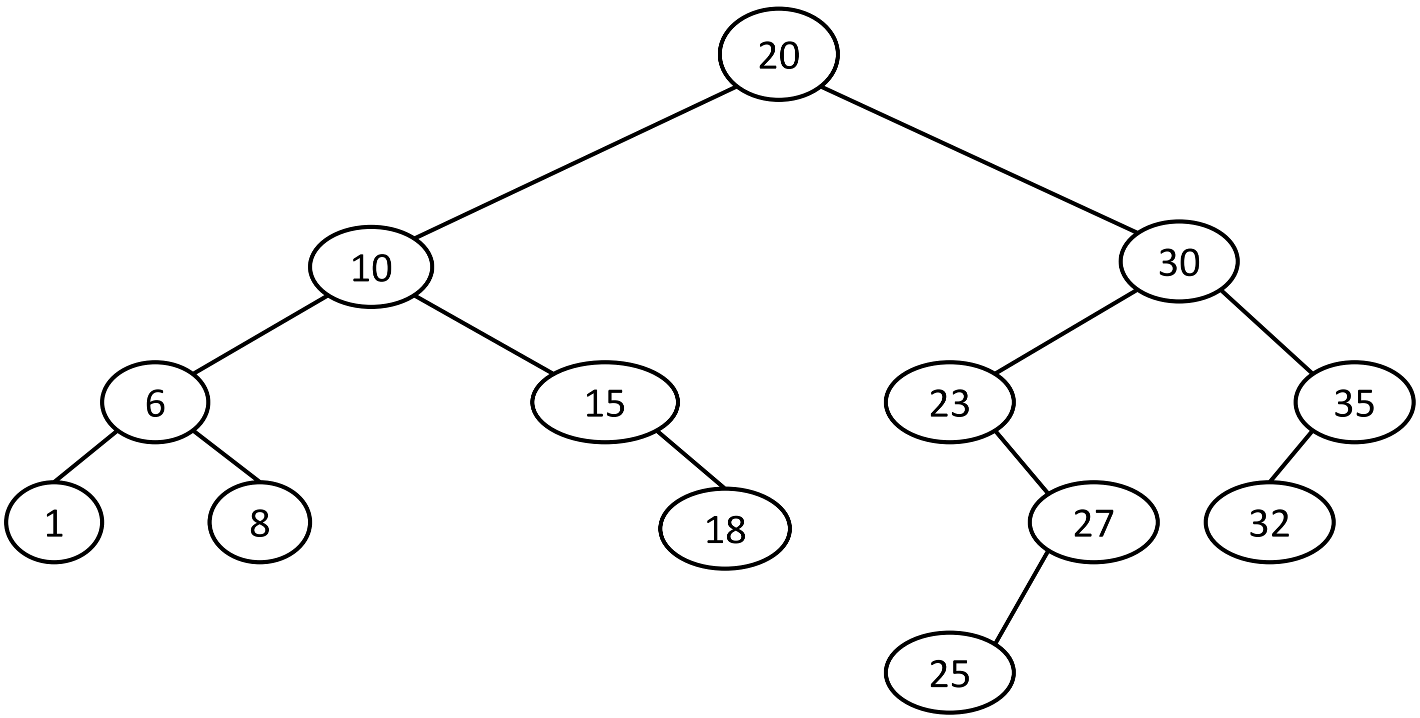
Insert an Item in a BST

- Create a tree using these values (in this order):
  - 2, 4, 6, 10, 8, 22, 12, 9, 19, 20



Deleting a Node

- The caveat of deleting a node is that, after deletion, the tree must still be a BST.



Deleting a Node

- Case 1: The node to be deleted is a leaf node.
- Case 2: The node to be deleted has an empty left child but non-empty right child.
- Case 3: The node to be deleted has an empty right child but non-empty left child.
- Case 4: The node has both children non-empty.

Case 1: Leaf Node

- Set the parent's pointer to this node to NULL.
- Release the memory of the leaf node.

Case 2: Empty Left Child

- Replace the deleted node with its right child.

Case 3: Empty Right Child

- Replace the deleted node with its left child.

Case 4: Non-empty Left and Right Child

- Replace the data in the deleted node with its predecessor under in-order traversal.
- Delete the node that holds the predecessor.

Deleting a Node

```
void DeleteItem(Tree &tree, int Data){
     if (tree == 0) return;
     else if (Data < tree->data)
          DeleteItem(tree->left, Data);
     else if (Data > tree->data)
          DeleteItem(tree->right, Data);
     else { // (Data == tree->data)
          if (tree->left == 0){
               Tree temp = tree;
               tree = tree->right;
               FreeNode(temp);
          }
          else if (tree->right == 0){
               Tree temp = tree;
               tree = tree->left;
               FreeNode(temp);
          }
          else{
               Tree pred = 0;
               FindPredecessor(tree, pred);
               tree->data = pred->data;
               DeleteItem(tree->left, tree-
               >data);
          }
     }
}
```

```
void FindPredecessor(Tree tree, Tree
&predecessor){
     predecessor = tree->left;
     while (predecessor->right != 0)
          predecessor = predecessor->right;
}
```

- We are replacing the data in the node, not the node itself
- Because the predecessor comes from the left subtree:
  - It must be ***less*** than everything in the right subtree.
  - It must be ***greater*** than everything in the left subtree.
- The recursive deletion of the predecessor's node will lead to one of the simpler cases.

Summary

- Binary tree
- Binary search tree
  - Finding an item
  - Insertion
  - Deletion

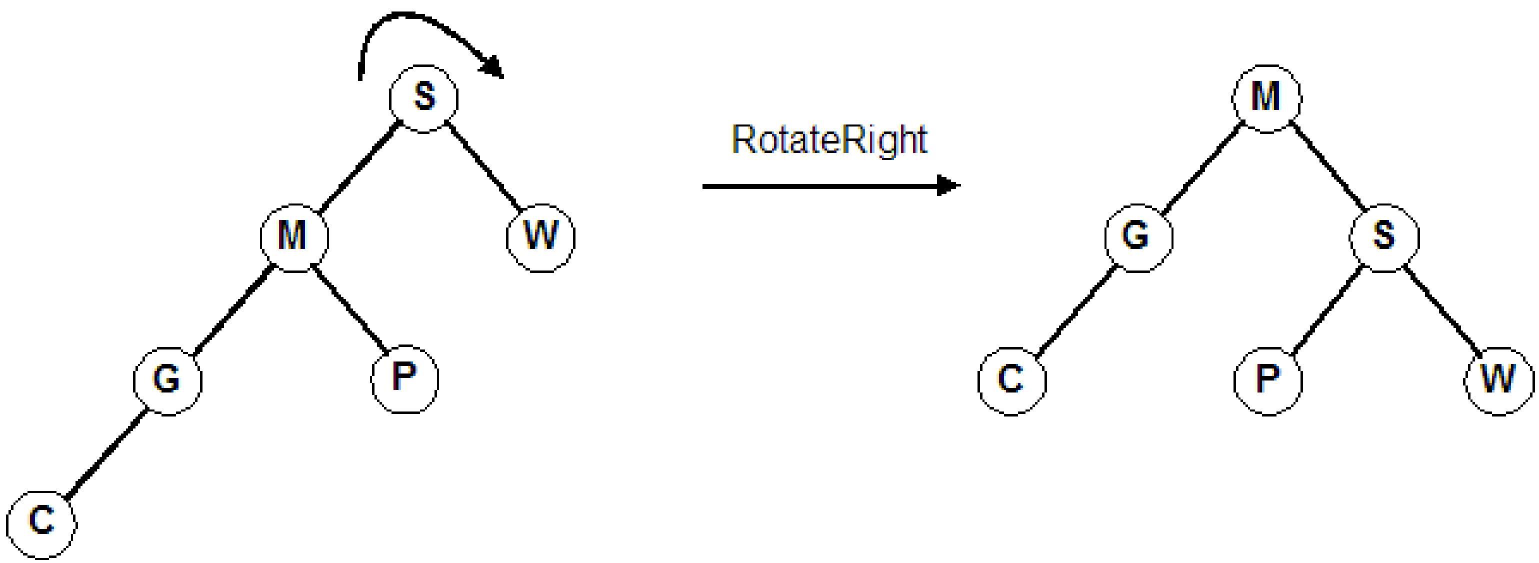
Rotating Nodes

Rotating Nodes

- Rotation is a fundamental technique performed on BSTs.
- Two types of rotations:
  - Left
  - Right
- Promoting a node is the same as rotating around the node's parent.
- There is no direction in promotion.
- You can rotate about any node that has children.
- After the rotation, the sort order is preserved.
  - The resulting is STILL a BST

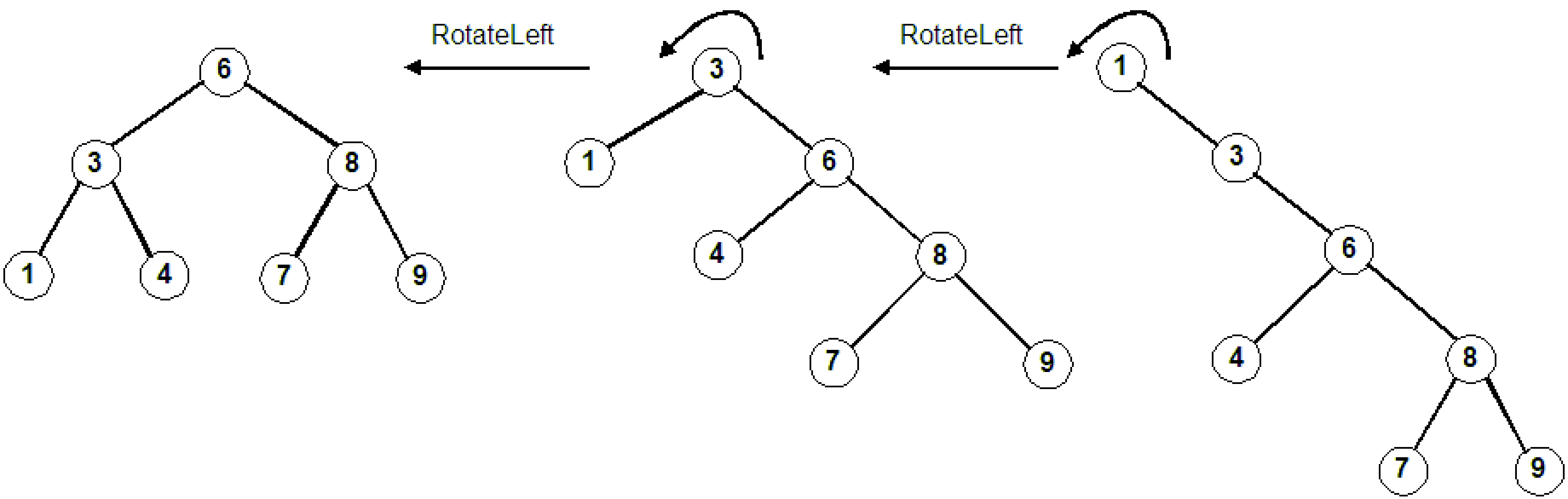
Right Rotation

- Rotate right around the root, S (Same as promoting M)



Left Rotation

- Rotate left twice around the root. First around 1, then around 3. (Same as promoting 3 then 6)



Rotating Left and Right

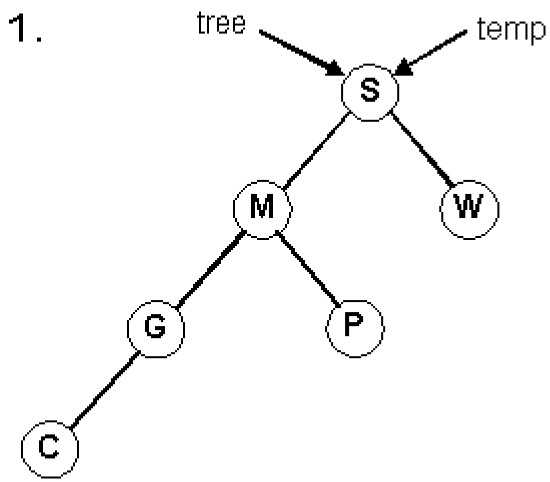
```
void RotateRight(Tree &tree){
    Tree temp = tree;
    tree = tree->left;
    temp->left = tree->right;
    tree->right = temp;
}
```

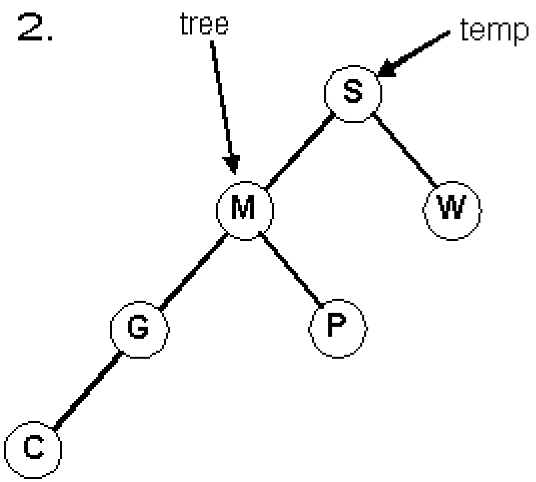
```
void RotateLeft(Tree &tree){
    Tree temp = tree;
    tree = tree->right;
    temp->right = tree->left;
    tree->left = temp;
}
```

Step by Step

```
1.Tree temp = tree;
```

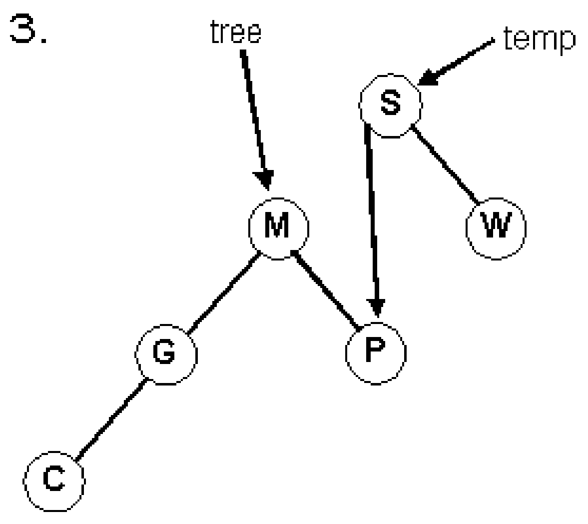
```
2.tree = tree->left;
```



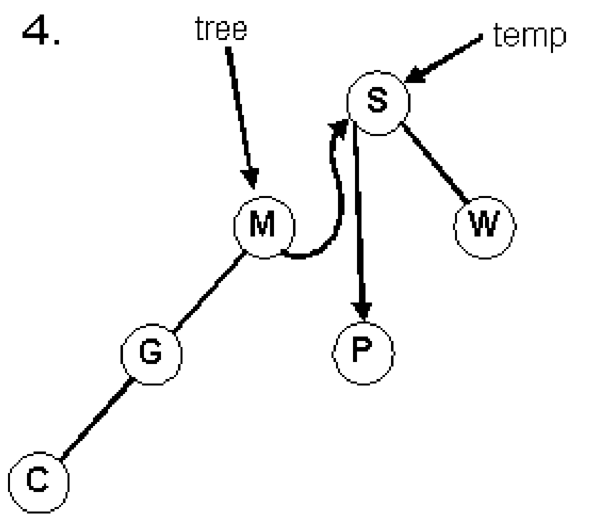


Step by Step

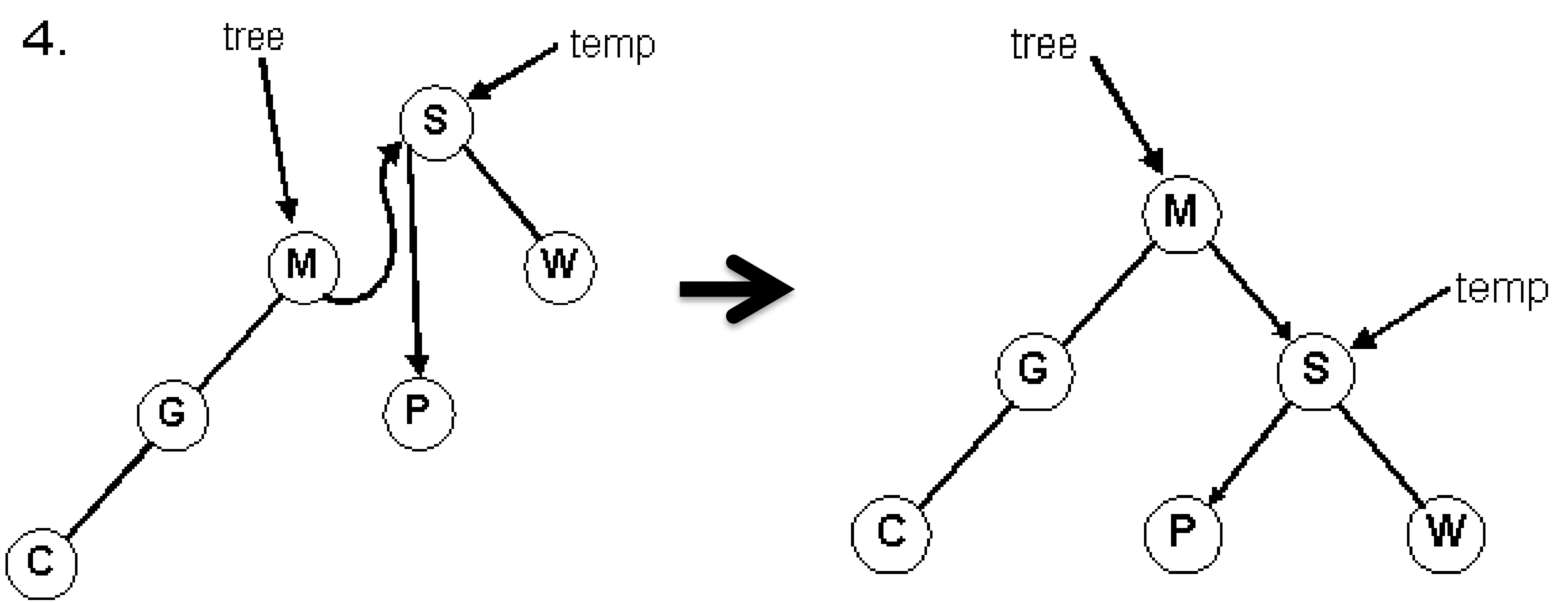
`temp->left = tree->right;`



`tree->right = temp;`



Adjusting the Diagram



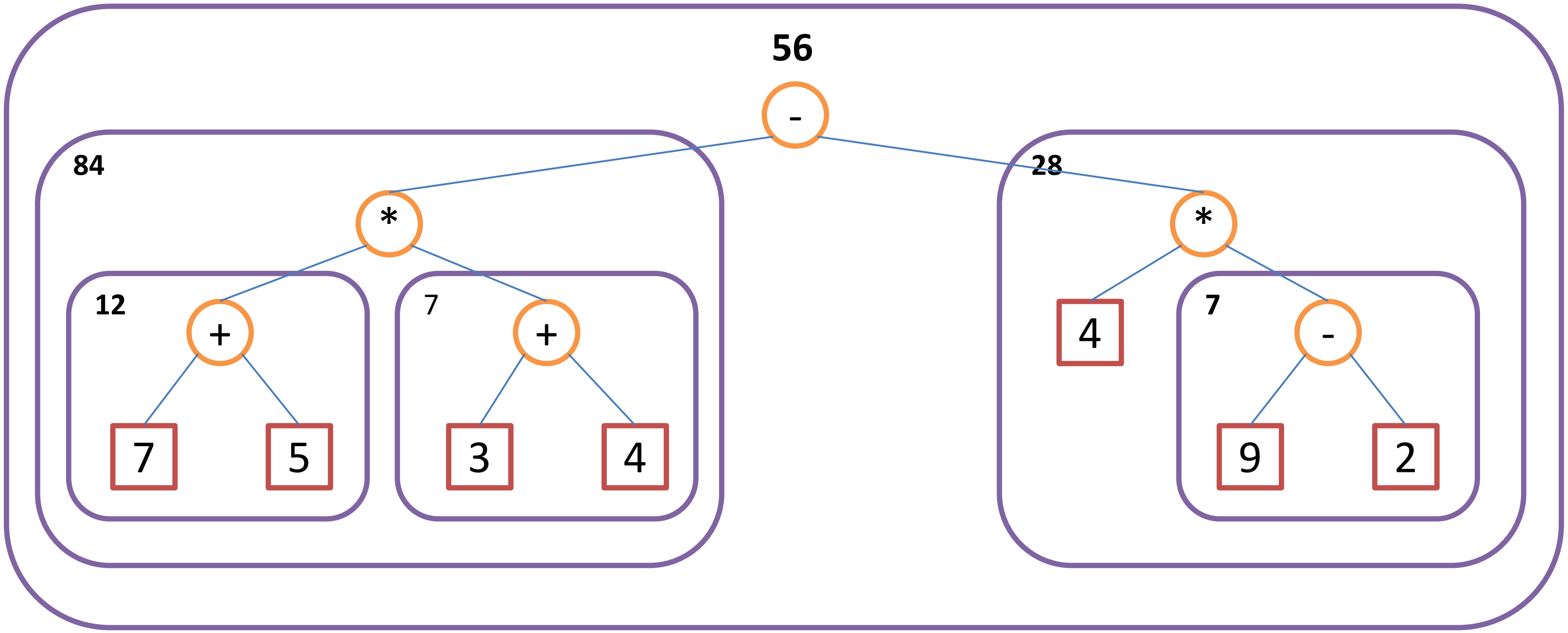
Expression Trees

Expression Trees

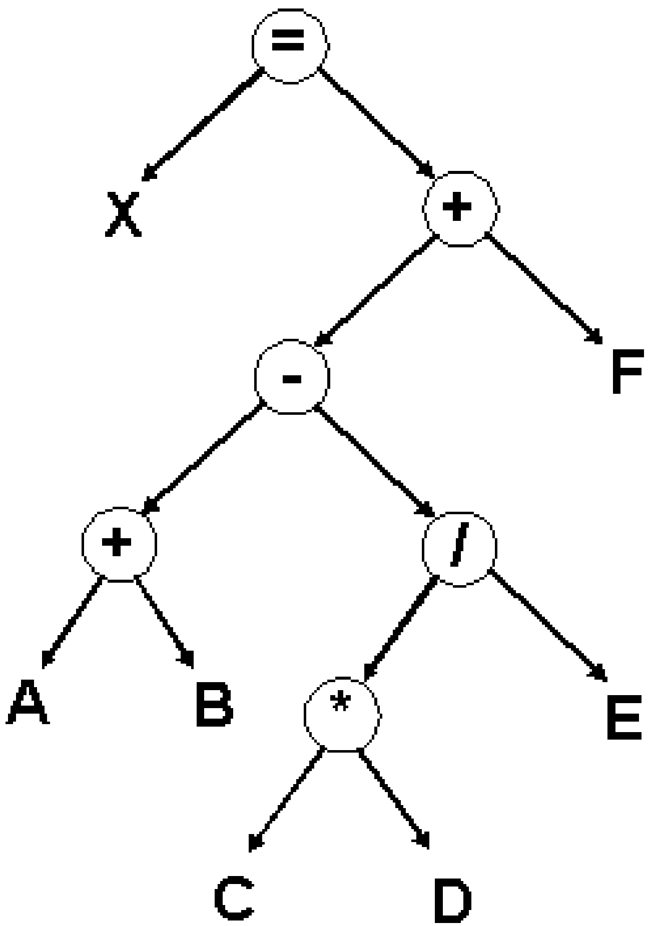
- Expression trees are like binary trees, but they are not “sorted” in the usual way.

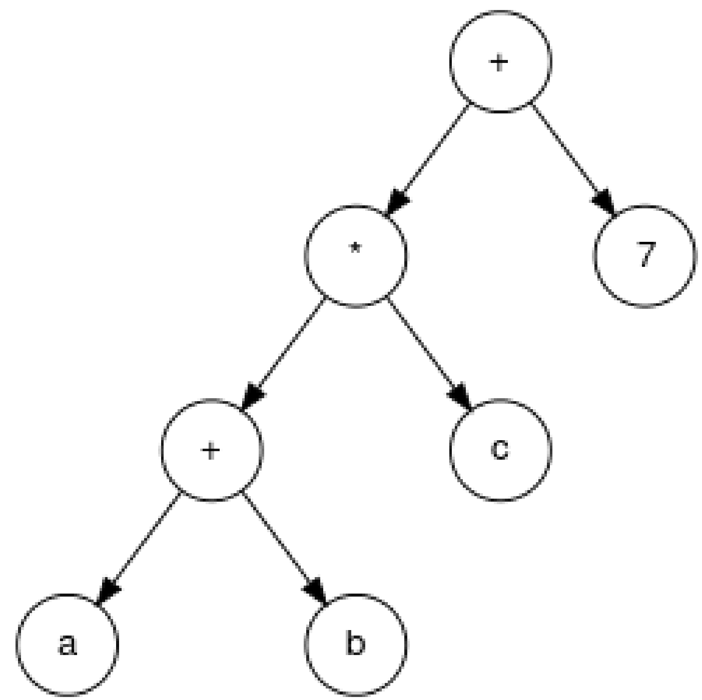
- Expression trees are a way to
  - Solve arithmetic expressions.
  - Compile a language

Expression Tree



Expression Trees





Evaluate an expression tree

```
int evaluate(Tree node){
  if(node->type == NODE_OPERAND)
        return node->opd;
  else {
        int left  =  evaluate(node->left);
        int right = evaluate(node->right);

        switch(node->opt){
                    case '+': return (left+right);
                    case '-': return (left-right);
                    case '*': return (left*right);
                    case '/': return (left/right);
                    default: return -1;
      }
  }
}
```

- We only consider binary operators here and the list of operators you need to consider includes “+”, “-“, “*” and “/”.