The Focimeter— Measuring Eyeglass Lenses

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ABSTRACT

The focimeter (also called lensmeter, Lensometer, and Vertometer, the last two being trademarks) is an instrument for measuring the optical parameters ("prescription") of an eyeglass lens. Although there exist today digital readout, and wholly automatic, focimeters, in this article we concentrate on the classical manual focimeter. After the instrument is introduced, background is given on various topics in lens optics and human vision correction. Then the operation of a typical focimeter is described. Appendixes give background in the scheme of measurement used for vision correction lenses, describe the actual optical principles of the focimeter, and give some history of the instrument.

CAVEAT

I am not an eye care professional, nor do I have any formal training in the practice in that field nor in its own unique branch of optical science. The information in this article is my own interpretation of the results of extensive (mostly quite recent) research into the available literature, through the prism of my own scientific and engineering background and outlook.

INTRODUCTION

The *focimeter* is an instrument for measuring the optical parameters ("prescription") of an eyeglass lens. It is also called "lensmeter", "Lensometer", and "Vertometer", the last two being trademarks, originally of American Optical Company and Bausch & Lomb, Incorporated, respectively. It is useful in a number of situations, notably:

- Allowing an ophthalmologist to become acquainted with the current corrective lens context of a new patient.
- Confirming that a new pair of eyeglasses has been correctly made.
- Making certain determinations of an eyeglass lens "blank", or a partially-finished lens, to allow it to be properly mounted for completion.

There exist today focimeters with digital readouts, and computer-driven fully-automated focimeters, but here we will concentrate on the classical manual focimeter.

In figure 1, we see my favorite manual focimeter, a Bausch and Lomb "Model 70" (formally, model 21-65-70), considered by many to be the *sine qua non* of manual focimeters. The photo is of our personal machine.



Figure 1. B&L Model 70 Vertometer

This particular specimen was evidently manufactured in 1963 (based on its serial number). It is in fine working condition (although its calibration is a tad out of date).

Before we take this beauty out for a test spin, I'll provide some background in pertinent areas.

LENSES

Lens refractive power

The *refractive power* ("power") of a lens is the degree to which it will converge (or diverge) rays of light emanating from the same point on an object and entering the lens at different points on its face.

Quantitatively, the power of a lens is the reciprocal of its *focal length*. The traditional unit of power is the diopter (symbol D). A lens with a power of one diopter has a focal length of one meter.

In connection with ophthalmic (vision correction) lenses, a different definition of power is used from the one most often found in general

optical work. It is called the *vertex power*. An explanation of this, and why it is used, is covered in Appendix A.

Spherical lenses

In optometric work, a *spherical lens* is any lens that has rotational symmetry, whether or not its surface is actually a portion of the sphere. Thus we can (and often do) have aspherical "spherical" lenses.

A spherical lens exhibits the same power along any direction.

A converging lens (which has a positive focal length) has a positive power. A diverging lens (which has a negative focal length) has a negative power.

We can present the variation (if any) in the refractive power of a lens with direction on a polar chart. In figure 2, panel A, we see a plot of a spherical lens with refractive power +1.0 D (a converging lens). This is a trivial case, and hardly requires a chart to explain. But we show the plot here to establish the format and notation.

The radius to the curve in a certain direction indicates the refractive power (in diopters) for that direction. Recall that a "direction" here means both ways: either way along the line at a certain angle ("a meridian"). Because of that symmetry, we only need to plot half the curve. But I show the curve for a full 360° for aesthetic completeness.

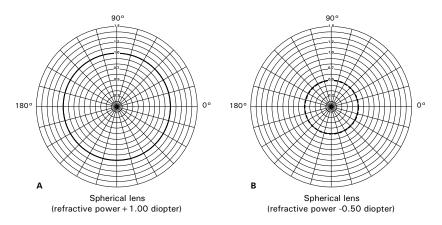


Figure 2. Spherical lens—power plot

In this field, the usual scientific convention is followed, with the angle reference (0°) being to the right.¹

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 $^{^{1}}$ In optometric practice, the angle can vary over only the range 0° to 180° , and by custom, exactly 0° is called 180° .

It is difficult to express negative values on a chart in polar coordinates—a "negative" radius would put the point on the opposite side of the chart, where it would just look like the (positive) value for an angle 180° from the actual angle.

To escape this difficulty, here I will plot negative values of the refractive power as a dotted line. And we see that in figure 2, panel B, the plot for a spherical lens with a refractive power of -0.5 D (a diverging lens).

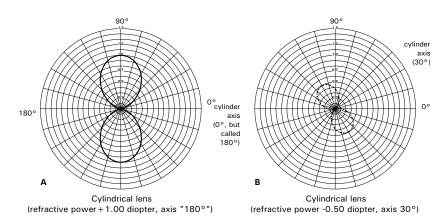


Figure 3. Cylindrical lens—power plot

Cylindrical lenses

A cylindrical lens has a surface that is a portion of a cylinder (which may or may not be exactly a circular cylinder). A cylindrical lens exhibits a certain power (its "rated" power) in one direction (perpendicular to its axis). Along its axis, it exhibits zero power. At intermediate angles, it exhibits intermediate values of power.

We see this illustrated in figure 3 for two cylindrical lenses, one with a positive power and one with a negative power.

Imagine that we combine a spherical lens and a cylindrical lens (and we assume here the convenience of the fanciful "thin lens" conceit, which, although impossible to have in practice, makes all the math work out in a very simple way).

In the direction of the cylinder lens axis, where the cylinder lens has zero power, there is no effect of the cylindrical lens on the overall result. In the direction at right angles to that, the power of the cylindrical lens combines with that of the spherical lens (taking into account the applicable algebraic signs) and so we have a power different from that of the spherical lens alone (perhaps even of the opposite sign).

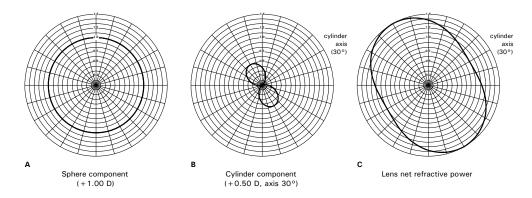


Figure 4. Composite lens—power plot

In figure 4, we see one example of this.

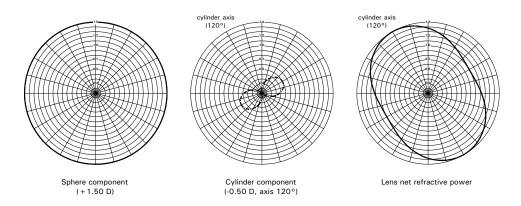


Figure 5. Composite lens—power plot

In figure 5 we see a different example.

Note that the result here is identical to the previous case.² This is reminiscent of the two ways we might make an ellipse. We might start with a circle of small diameter, and stretch it in the direction of the ellipse's major axis. Or we might start with a circle of large diameter, and shrink it in the direction of the ellipse's minor axis.³

We can of course make a single lens that will exhibit this overall behavior. A simple version (not actually used in modern practice) would have a front surface that is a portion of a sphere and a rear surface that is a portion of a cylinder. Note that this would have the result shown regardless of which of the two "recipes" we thought of as describing it.

² The specific mathematical variation of the power of a cylindrical lens with angle— cos²— makes this equivalence exact.

³ This is only a metaphor; the plot of the power of such a composite lens is not an ellipse.

HUMAN VISION

Accommodation

Ideally, the human eye can focus on objects at a wide range of distance, from very near to "infinity". This is called *accommodation*.

Typically, with advancing age, the eye's accommodation ability can become compromised (and the same may be true of young people as a result of congenital malformation of the eye or of various ailments). Several types of deficiency are common.

<u>Hyperopia</u> (also called <u>hypermetropia</u>, and often, but somewhat-misleadingly, called "far-sightedness") is the deficiency in which the total range of accommodation is "offset out", such that the person cannot focus on near objects, and in some cases not even on far objects.

Myopia ("near-sightedness") is the deficiency in which the total range of accommodation is "offset in", such that close objects can be focused on but the far limit is not to infinity.

<u>Presbyopia</u> (the term means "old person's seeing") is the deficiency in which the total range of accommodation (the accommodation amplitude) is decreased. The remaining limited range may be in the far, intermediate, or near regimes, in the individual case.

Astigmatism

Astigmatism is the deficiency in which the refractive power of the eye's lens is not the same in different directions. An illustrative result is that if we have astigmatism and look at a cross of thin lines on a card, we can focus so that the vertical line is sharp, or the horizontal line is sharp, but not both at the same time.

Correction with lenses

We can overcome basic deficiencies in accommodation with the use of a corrective lens. For farsightedness, we can use a (spherical) corrective lens with a positive power; this will shift the range of focus in the "nearer" direction. For nearsightedness, we can use a (spherical) corrective lens with a negative power; this will shift the range of focus in the "farther" direction.

We can overcome astigmatism with the use of a cylindrical lens.

Not surprisingly, in typical cases, both "spherical" and "cylindrical" components are combined to deal with the overall visual syndrome.

Two notations for the prescription

Recall that, as we saw in figures 4 and 5, the identical lens result can be conceptually implemented with either of two conceptual "recipes". For that particular example, we could combine:

- A spherical lens with power + 1.00 D
- A cylindrical lens with power +0.50 D and axis 30°

or

- A spherical lens with power + 1.50 D
- A cylindrical lens with power –0.50 D and axis 120°

Recall that in reality the way the lens is actually made may not directly follow either of those "recipes".

Either model could be used as the premise for defining the desired result in written form: the lens *prescription*. It turns out that, when the prescription is written by an <u>ophthalmologist</u> (a physician and surgeon specializing in the eyes), it would be in the first form (the cylinder component always being with a positive power), called the "plus cylinder" form. When the prescription is written by an <u>optometrist</u> (a Doctor of Optometry, qualified and certified to examine eyes and issue eyeglass prescriptions), it would be in the second form, (the cylinder component always being with a negative power), called the "minus cylinder" form.



Figure 6. Focimeter parts and controls

To get a little ahead of the story, we might wonder, when we ask a focimeter to tell us the nature of an eyeglass lens submitted to it, how it knows whether we are an optometrist or an ophthalmologist. It

doesn't know; we manipulate it following one of two procedures to force the answer to be delivered in the desired form.⁴

OPERATING THE FOCIMETER

Refer to figure 6. We will assume that the lenses being measured are part of a complete pair of eyeglasses (as seen in the figure).

We place the glasses on the spectacle table with the lens to be measured in front of the measuring aperture, with its optical center (as best we can guess where that is) aligned with the aperture. (We will refine that shortly). The spectacle table can be raised or lowered as needed for this. A "lens chuck", an articuating spider-like assembly with four plastic-tipped prongs, under spring pressure, presses on the face of the lens to hold it against the rim ("nose") of the aperture. This assures that the rear vertex of the lens is at exactly the proper position along the instrument axis, and that the lens surface at the aperture is perpendicular to the instrument axis.

When we look through the eyepiece, we see a pattern of green lines, illuminated by a lamp at the far end of the instrument shining through a reticle bearing the pattern.

The basic pattern is seen, all in focus, in figure 7. There are three thin, closely-spaced lines in one direction, and three wider, more widely-spaced lines in the direction exactly 90° to that.

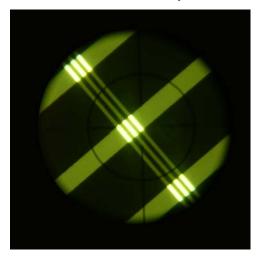


Figure 7. All lines in focus

(The field of view in this photo is less than the operator would see; more of the length of the fat lines would be visible.)

⁴ This is quite different from the situation with the refractor, or phoropter, the instrument used to determine the optimal prescription for a patient's eye. There, the practitioner orders one of two versions of the instrument, one which always delivers the result in "plus cylinder" form and one in "minus cylinder form".

The location of the reticle carrying these lines can be shifted along the axis of the instrument by rotation of the large wheel on the right side (called the *power drum*). It carries markings in diopters, from -20 through zero and on to +20, and there is a fiducial pointer against which that scale can be read.

The reticle can also be rotated about the instrument axis. In this instrument, this is done with a drum on the left side of the instrument, seen in figure 8. It has a scale (0°-180°), read against a fiducial line, showing the orientation of the recticle. In an instrument of the American Optical Company design, there is a different arrangement (we will see it later).



Figure 8. Cylinder axis drum

If the lens has no cylindrical power component, then the lens does not exhibit astigmatism: its power is the same in all directions. When the power drum is turned to the optimum position, the entire pattern (both sets of lines) will be seen in sharp focus (just as we saw in figure 7). The overall optical arrangement is such that, at this point, the reading of the power drum will be the power of the lens.

If the lens has a cylindrical power component, then (just as for a human eye with astigmatism) the two sets of lines cannot both be in sharp focus at the same time.

If we have the orientation of the thin lines of the reticle at 90° to the axis of the cylindrical component, and the power drum is set to the value corresponding to the power of the spherical component (which is the power of the entire lens in the direction along the cylindrical lens axis, where the cylinder power has no effect), the set of three thin lines will be in perfect focus. And the fat ones won't.

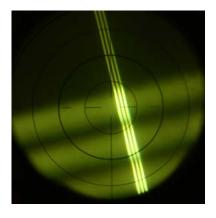


Figure 9. Fine lines in focus

Figure 9 shows that situation (we have a slightly larger field of view in this, and the following, photos).

Incidentally, the pattern of black lines we see here is the eyepiece reticle, used for various purposes (more about it in Appendix B).

With that same orientation of the reticle, but the power drum set to the power of the lens along the direction perpendicular to the cylinder lens axis (which is affected by the cylinder power of the lens), the set of three fat lines will be in perfect focus.

Figure 10 shows what that looks like.

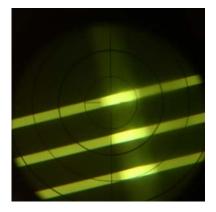


Figure 10. Fat lines in focus

The difference between these two readings of the power drum (the second minus the first, observing the sign) will be the power of the cylindrical component of the lens behavior. And the orientation of the reticle (read from the axis knob) is the value of the cylindrical component axis.

Use for plus vs. minus cylinder notation

That description of the test regimen didn't seem to offer the operator any choice as to whether the cylinder power (the second power drum reading minus the first) comes out positive or negative. How to we cater to the two different conventions?

If we want the cylinder power to come out positive, we start this process by moving the power drum to a large negative power. Then we rotate it in the positive direction, with the other hand on the axis knob, manipulating it as required, until we **first**⁵ have the three fine lines in focus. We note the first power drum reading (the spherical power for this case). Then we leave the axis knob alone and continue the positive-ward motion of the power drum until the three fat lines come into exact focus. We note this as the second power drum reading.

Clearly, when we subtract the first power drum reading from the second, the sign of the difference will be positive. Thus we have a positive value of the cylinder power, as needed for the plus-cylinder notation we wanted to use.

If we want the cylinder power to come out negative (for minus cylinder notation), we just start with the power drum at a large positive power setting and then move it in the negative-ward direction. In this case, the cylinder power will come out negative, as desired.

Note that the first drum setting at which we can get the three fine lines in focus is not the same setting as it was in the first case—it will be at a more positive setting of the power drum. And the axis setting for that event will be 90° from the axis setting for that event in the first case. Thus all the ingredients of the prescription, the spherical power (first power drum reading), the cylinder axis (axis knob reading at that time) and the sign of the cylinder power, will differ between the cases, as we expect between a positive-cylinder and negative-cylinder "prescription" for the same lens.

Help with the axis setting

We have said that in the first phase of the procedure we move the power drum and the cylinder axis drum until the three thin lines are in best focus. Small discrepancies in axis setting do not make a prominent change in the apparent degree of focus of the thin lines, so it may be difficult to find the precisely correct axis setting. The instrument includes an ingenious scheme to help with this. It is very reminiscent of the "split-prism" focus aid found on some single-lens reflex (SLR) cameras (although it occurs in a completely different way—See Appendix B).

We see it in figure 11. On the left, we have the axis setting not quite at the proper point. Notice that, although the thin lines might not be

is is important. If we keep going, we will find anot

⁵ This is important. If we keep going, we will find another power drum setting (with a different axis setting) where the fine lines will also be in focus. This event will in fact come into play in the next portion of the description, but is not what we want in the present scenario.

noticeably out of focus, there are breaks in the lines, certain segments of them seem to be "rotated" out of alignment with the other segments. (It is actually only their end portions in which this effect occurs.) This clearly reveals that we are not quite at the proper axis setting.

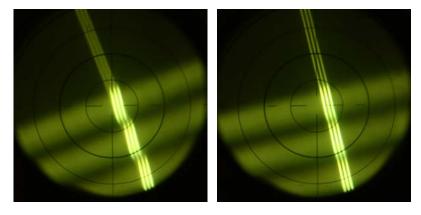


Figure 11. Axis adjustment help

On the right, we have the axis setting "right on the money". Not only is the degree of focus better (maybe not noticeably so), but (more prominently) all portions of the lines are now aligned.

MEASURING BIFOCAL LENSES

Bifocal lenses of the traditional type have a "near vision segment" located in the lower part of the lens. This region has a more positive cylinder component than the rest of the lens, intended to make it possible for a subject with *presbyopia* (limited range of focus change) to focus on nearer objects.

This is ordinarily specified in the prescription as an "add" clause: "Add +2.00". This means that the spherical power in the near vision segment is to be 2.00 D more positive than in the lens generally.

There are three methods of determining this "incremental" spherical power with a focimeter:

Method 1

In the basic determination of the lens prescription, with the "eye side" of the lens toward the nose, and the optical center of the main part of the lens centered in the aperture, we set the power drum and the cylinder axis control until the fine lines are in focus. We note that drum reading (reading A). Then, leaving the cylinder axis unchanged, we move the power drum until the fat lines are in focus. We note that drum reading (reading B).

The spherical power of the lens is judged to be A. The cylinder power is judged to be B-A (observing the algebraic signs).

We then move the lens so the optical center of the near vision segment is centered in the measuring aperture. We leave the cylinder axis unchaged, and move the power drum (always in the positive-ward direction) until the fat lines are again in focus, and record the drum indication (reading C).

The near vision add power is judged to be C-B (observing the algebraic signs).

For reasons associated with the working of the vertex power convention in the case of near vision segments this basic approach does not give a "correct" indication of the power of the segment as it affects near vision. The next two methods overcome this problem. I will describe them briefly here. A more complete description of these processes, and their theoretical underpinnings, is given in the companion paper, "The Vertex Power of Ophthalmic Lenses", by this same author.

Note that the near vision effective vertex power of the lens must be stated in the light of the specific distance of near vision that is contemplated.

Method 2

In this method, to measure the power in the segment, an auxiliary lens is placed on the near side of the lens under test. This lens must have a negative vertex power corresponding to the distance of near vision contemplated for the lens under test. For example, if distant vision at 500 mm is the premise for the lens, the auxiliary lens should have a vertex power of -2.00 D.

On the lens under test, the segment is aligned with the focimeter nose. The spherical power of that situation is read. Then the power of the auxiliary lens is subtracted from the reading (observing the algebraic sign, to give the near vision effective vertex power of the segment. For example, if the auxiliary lens has a vertex power of -2.00 D, and the drum reading for the spherical power is +2.50 D, then the near vision effective vertex power of the segment is +4.50 D.

We then subtract from that the measured spherical power of the distant vision part of the lens (done earlier, without the auxiliary lens). The difference is the "near vision effective add" of the lens (which should match the "add" of the prescription.

The theory and technical details of this are given in appendix C.

Method 3

I'll "take it from the top" for completeness. I'll indent the unchanged part.

In the basic determination of the lens prescription, with "eye side" of the lens toward the measuring aperture, and the optical center of the main part of the lens centered in the aperture, we set the power drum and the cylinder axis control until the fine lines are in focus. We note that drum reading (reading A). Then, leaving the cylinder axis unchanged, we move the power drum until the fat lines are in focus. We note that drum reading (reading B).

The spherical power of the lens is judged to be A. The cylinder power is judged to be B-A (observing the algebraic signs).

We then reverse the glasses so the eye side is away from the measuring aperture. Again, we center the optical axis of the main part of the lens in the aperture.

We adjust the power drum and the cylinder axis control until the thin lines are in focus, and note that drum reading (reading D). With the glasses still "eye side out", we move the lens so an appropriate point in the near vision segment is centered in the measuring aperture. We leave the cylinder axis unchaged, and move the power drum (always in the positive-ward direction) until the thin lines are again in focus, and record the drum indication (reading E).

The near vision add power is judged to be E-D (observing the algebraic signs).

Where did we tell the process the assumed near vision distance? We didn't. This method only gives the near vision effective add power precisely if in fact that value corresponds to the near vision distance assumed for the lens. That is, if the assumed near vision distance is 500 mm, and the technique gives us a near vision add of +2.00 D, that is accurate. This will generally be the case in the case of lenses for a patient with severe presbyopia.

If the near vision effective add power does not correspond to the assumed distance of near vision, then the result is not quite accurate.

Again, the theory and technical details of this are given in the companion article cited above.

ODDS AND ENDS

Why is everything green?

What we see in the eyepiece is mostly green owing to a green filter in the upper part of the system. This is to minimize "spreading" of the lines due to chromatic aberration (the differing behavior of the optical system at different light wavelengths).

The American Optical approach to axis setting

The classical focimeter made by American Optical Company (AO) (under the name "Lensometer") uses a different scheme of adjusting the orientation of the recticle. Rather than using a drum on the left side of the machine (as in the Bausch and Lomb unit illustrated before), these units have an axial handwheel that directly rotates a barrel containing the reticle.

We see it in figure 12, a lovely advertising illustration (*ca.* 1920) for the first commercial American Optical Lensometer, the "Wellsworth" ⁶ Lensometer.



Figure 12. American Optical Wellsworth Lensometer (1920)

The operator's left hand is on the axis handwheel, or "protractor".

You may note the two pins projecting from the rear of the unit at the right. This is where the power cord for the lamp is to be attached. Evidently the photographer (or director) decided that having the cord in place would mess up the beauty of the scene.

Many operators find the side-mounted axis drum of the B&L design (adopted on modern AO units) to be more convenient.

The marking feature

Especially when making measurements of partially-finished lenses, it is desirable to mark the location of the optical center and the reference axis that should be horizontal when the lens is mounted in the frame (often called "spotting" the lens). Once the lens is positioned

⁶ The name "Wellsworth" was a common AO trademark at the time, honoring the name of George Washington Wells, who founded the company that was AO's principal ancestor.

appropriately, a marking device will place three small ink dots in a line along the horizontal axis, the centermost being at the optical center. We see this in figure 13.







Figure 13. Marking device

On the left, we see the marking head (the shiny bar with the three prongs) in its rest position. Using the knob just below the eyepiece, the operator moves the marking head forward so the prongs contact an ink roller in the small black rectangular box (this unit did not have an ink roller when that shot was taken). In the center, we see that the operator has allowed the head to retract from the ink box and rotated it to its upper position (in front of the lens). On the right, we see the head pressed forward so the three inked prongs contact the lens.

Centering the lens

I referred to this earlier, but delayed discussing it until now so as to not disrupt the overall story.

For the measurement process to work properly, the optical center of the lens must be centered in the measuring aperture. When it is, the pattern of lines will be centered on the pattern of the eyepiece reticle, as we see in figure 7 (but not in figure 9 and later, since I had neglected to make that adjustment before taking those shots).

Reference axis

On completed eyeglasses, the horizontal reference direction for the specification of cylindrical axis⁷ is a line tangent to the bottom of both frame sides. When measuring completed glasses, this is automatically set as the reference since the bottoms of both frame sides sit on the spectacle table.

In some work involving lens blanks (which are round), we must also orient horizontally a reference direction on the lens, which is often marked on the blank as a line in temporary ink.

⁷ Scientists would call this 0°; in optometry, it is called 180°.



Figure 14. B&L Model 70 Vertometer—Reference axis locating pins

The B&L Model 70 Vertometer has provision for dealing with this (see figure 14).

We see, on either side of the nose, two brass pins with blunt points, which are spring-loaded and removable. The lens blank is put in place so that the tips of these pins fall on the marked reference line.

In the photo, the lens chuck, has been "retracted" to allow a clear view of the pins.⁹

CONTACT LENSES

We have concentrated here on conventional eyeglass lenses. Contact lenses use essentially the same principles for vision correction, and their refractive properties are specified by a prescription following generally the same conventions we discussed here (with certain additional parameters, primarily relating to the lens dimensions, included).

The refractive behavior of a contact lens can be measured by a focimeter. An issue, though, is properly holding the lens in place. The arrangement described above for "classical" focimeters is not workable.

If the axis of the focimeter can be placed in the vertical position (true for most "modern" focimeters, but not the B&L Model 70), it is

⁸ On our machine, they had indeed been removed, and not replaced before we got it. The picture is of another specimen of the same model.

⁹ Actually, we believe that on that specimen, the chuck arm was maladjusted so that, at the "6 o'clock" position, as seen, it could **only** be retracted (that is, couldn't really be used). Normally, when at "6 o'clock", the spring would always push the arm toward the lens, but it can be "parked" retracted at the "12 o'clock", "3 o'clock", and "9 o'clock" positions.

possible to just have the contact lens lie on the measuring aperture nose. There are other ploys for placing contact lenses in a focimeter. These techniques are beyond the scope of this article.

"NEUTRALIZING" A LENS WITH THE FOCIMETER

We will often see determining the prescription of a lens with a focimeter called "neutralizing" this lens. This is a nostalgic nod to the earlier (pre-focimeter) way of determining the prescription of a lens.

In this technique, standard "trial" lenses drawn from a large set, with known powers (both spherical and cylindrical), were placed in contact with the lens under test until its refractive power was entirely canceled out ("neutralized) along both meridians. This situation was determined by visual observation: if we look through a positive lens (of modest power, such as a typical vision correction lens), held a short distance from our eye, and move the lens from side to side, we will see a shift in the apparent location of the "scene" beyond. This motion, compared to the motion of the lens, will be in one direction (contrary to the motion of the lens) for a positive power in the direction of movement, in the other direction (consistent with the motion of the lens) for a negative power, and absent in the case of zero power in the direction of movement.

Thus, when the stack comprising the lens under test, a spherical trial lens, and a cylindrical trial lens, exhibits no such scene shift, when the lens is moved in either of two directions at right angles, we accept the negative of the trial lens powers as the prescription of the lens under test (noting the cylinder axis from the cylinder axis of the trial cylinder lens).

FOR MORE DETAIL

Appendix B discusses the internal arrangement of the optical system of a typical focimeter, and shows why the position of the power drum is able to run linearly with the power of the lens under test, allowing its scale to be a linear one in terms of power.

ISSUE NOTE

This article is reissued (as issue 5) principally in order to delete two appendixes related to the "auxiliary lens" and "revered" measurement techniques for the near vision segment of bifocal lenses and replace them with a reference to the extensive coverage of the topic in a companion article. Some minor editorial improvements were also made.

Appendix A

Vertex Power

Effective focal length

In most optical work, when we mention the focal length of a lens, we mean what is called formally the "effective focal length" of the lens. That term suggests that this is not the "real" focal length of the lens, but instead is the "real" value adjusted in some way to suit some situation.

But that's not so—it is the "real" focal length of the lens. It gets that misleading name from a matter of historical evolution.

When lens behavior was first studied, it became apparent that if we had a well-behaved lens regard a very distant object ("at infinity"), an image of that object was formed behind the lens. The distance to that image varied with the surface curvature of the lens, and it was clearly an important parameter of the lens.

The location of this image was said to be the *rear focal point* of the lens, and the distance to it was measured from the rearmost point of the lens (on the axis)—the *rear vertex* of the lens. This distance was called the "back focal distance", or *back focal length*, of the lens.

But soon it was realized that the number that affected many properties of the lens (and thus that was needed in many equations about lens behavior) was not the back focal length, but rather a slightly greater (in most cases) distance. Not surprisingly, given the history, this came to be called the "effective focal length".

This was in fact the distance to the rear focal point from a place (usually) inside the lens called the *rear principal plane*. That's not something we can see, nor locate in any simple way, so it was understandable that the effective focal length remained an ethereal, if theoretically important, distance. In formal writing, the designation *effective focal length* continued in use to denote it. In other than formal writing, it is just called the *focal length* of the lens.

And this distance is the "real" focal length of the lens—the only number that is of importance in such things as focus equations, photographic magnification and field of view reckoning, and so forth. It is a constant for the lens, not dependent on any "circumstance" of its deployment.

The power of a lens

The refractive power of a lens (often called just its power) tells us the degree to which it converges, or diverges, arriving rays of light. In regular optical science, we quantify the power of a lens as the

reciprocal of its effective focal length (to again use the full formal name to avoid any misunderstanding).

The modern scientific unit of power is the inverse meter (m⁻¹), but in traditional work, and always in ophthalmic work, the same unit is called the diopter (D). A lens with an effective focal length of one meter has a power of one diopter.

Vertex power

A different convention is used in connection with ophthalmic lenses. Their power "rating" is not the reciprocal of the effective focal length but rather the reciprocal of their **back focal length**. The rationale for doing so is rarely clearly explained in the literature. Here is the short story.

The effect of a lens on the correction or near- or farsightedness depends both on the power of the lens and its distance in front of the eye. In conventional optical theory work, we would define the power as the reciprocal of the effective focal length (to be formal), and the distance that matters is from the second principal point of the lens to the first principal point of the eye's lens system.

To facilitate the whole process of prescribing, making, and fitting eyeglass lenses, wherever possible we place the lens at a fixed standard distance from the eye. But this is not measured to the second principal point of the lens. If we did that, then lenses of differing shape, in which the second principal point falls at different physical locations, would not have a consistent "overall" location—"meniscus" lenses, which have an overall curvature, would be much closer to the face than lenses that are flat on the rear.

Accordingly, the practice emerged of placing the lenses so that the rear vertex of the lens is at a consistent distance from the eye.¹⁰

Having adopted that practice, of course for lenses of differing shape the distance from the eye to the second principal point will vary. And thus, for lenses of differing shape, the power (in the normal optical sense) needed for proper vision correction will vary. Not handy at all—for one thing, the prescriber of the lens has no idea what shape will be used for the actual glasses to be made.

But it turns out that for lenses placed with their rear vertex a consistent distance from the eye, the effect on vision correction is consistently given by the back focal length of the lens.

¹⁰ And that's from the front of the cornea, not the eye's first principal point. It is of course hard to measure the latter, and the distance between them is very consistent. Thus measuring from the front of the cornea is much more practical.

But of course we would rather speak in terms of power than focal length. So we define the *vertex power* of a lens as the reciprocal of its back focal length. And we describe, or prescribe, the power of an ophthalmic lens in terms of its vertex power.

Because this is the value that describes the "effect" of the lens on vision correction (assuming that the rear vertex is at a standard distance from the eye), regardless of the shape of the lens, in ophthalmic work it is called the *effective power of the lens*.

Sometimes is it even spoken of, in an ophthalmic lens context, as the "true power" of the lens! (That's hard to justify. Guys, you should have quit when you were ahead with "effective power"!)

The focimeters we discuss here inherently measure the vertex power of the lens, and in fact that is the premise for the tradename, "Vertometer", used by Bausch & Lomb for their instruments.

A small wrinkle deserves mention. This handy situation in which the effect of a lens on vision correction, regardless of lens shape, is (assuming that the lens is a certain distance from the eye) given by the vertex power of the lens does not hold when the lens is employed for correction of near vision (as in the near vision segment of a bifocal lens).

There are various ramifications of this. These are notes when they appear in the body of this article and in the other appendixes.

Appendix B

Optical system of the focimeter

Introduction

First, we will first examine the conceptual principle for the determination of lens power upon which the focimeter is based, and we see why it would not be practical to directly implement that in an actual instrument.

Then, we will examine (in somewhat simplified way) the actual optical system of the focimeter, and see how the power is indicated on a linear scale on the power drum.

Finally, we will look into an elaboration in the basic mechanism found in the first commercial American Optical Company focimeter, the Wellsworth Lensometer.

The concept of measurement in a focimeter

Figure 15 shows a conceptual setup for measuring the power of a lens.

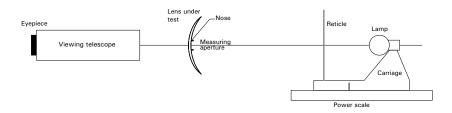


Figure 15. Principle of power measurement

On the left, we have the viewing system, a telescope focused at infinity. The lens being measured is placed in a controlled axial position by being held against the fixed "nose" of a measuring aperture. Thus its rear apex is in a fixed location, as by now we might suspect would be needed to fit in with the apex power concept.

Behind the aperture is a reticle, carrying a transilluminated pattern of crossed lines. It is mounted on a carriage allowing its axial position to be changed. A fiducial on the carriage is read against a scale to indicate the carriage position.

If the reticle is placed so that it lies at the back focal point of the lens under test, then from the front of the lens we could see a "virtual image" of the reticle apparently at infinity. Since the viewing telescope is focused at infinity, then in this situation, the image of the reticle will appear in perfect focus.

Thus, in general, if we move the recticle until the image appears perfectly focused in the telescope, then the distance from the nose to

the reticle is the back focal length of the lens, and the reciprocal of that is its vertex power, the parameter in which we are interested.

But there are several flies in the ointment, particularly:

- This will only work for converging lenses (with positive powers).
 The nearsighted need not apply.
- For the range of powers we are likely to encounter, the distance to the reticle will be quite large (making it impractical for it to remain inside an instrument of any reasonable size). For example, for a lens with a power of 1.00 D, the reticle would have to be one meter behind the aperture nose; for a power of +0.125 D (typically the smallest value accommodated on a focimeter), it would have to be eight meters behind the nose (over 26 feet).
- The reading of the scale on the carriage would be linear with back focal length, but would therefore not be linear with power, its inverse (the parameter of interest, in terms of which we would like to mark the scale). This would not be suave in such an instrument (even once we get over the scale being over 26 feet long).

Thus, a clever optical trick is employed. Rather than trying to place a physical "target" (a pattern on the recticle) at the back focal point of the lens under test, we can create (using a lens in the instrument) a virtual image of the pattern on the reticle, which will itself be the target for the lens under test. This virtual image can be placed at any distance necessary (to accommodate the location of the back focal point of the lens under test). Having done this, we find that the scale on our "new" reticle carriage will in fact be linear with the power of the lens under test. (Sometimes a designer just gets lucky!)

The actual optical system

Figure 16 shows the optical system of a typical focimeter, taking advantage of the ploy just mentioned, in schematic form. We will describe it from scratch, as if we had not heard of the unworkable scheme.

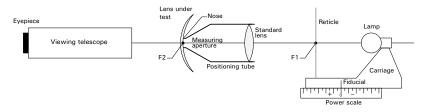


Figure 16. Focimeter optical system schematic

The lens under test is held by the chuck against the nose of the measuring aperture, at the end of the positioning tube.

The reticle and the illuminating lamp are on a carriage that can move axially under control of the power drum (not shown). The drum carries markings, on which we read the power of the lens under test. In the figure, we suggest this with a straight scale for the carriage itself.

The eyepiece through which we observe is part of a Keplerian telescope, focused at infinity. We will not look into its internal details just now.

As a result of the telescope being focused at infinity, if we place a target object at the back focal point of the lens under test, it will appear to be in perfect focus looking through the telescope at the lens. Instead of a physical target, we can have a source image—in this case, an image of the pattern on the reticle, created by the standard lens. (Any image can serve as the object of a lens.)

If we know where that source image is located when it is seen in focus through the telescope (through the lens under test), we know where the back focal point of the lens is, and thus know its (back) focal length, and thus its (vertex) power.

The standard lens has its focal length carefully controlled (it is actually a compound lens, and its focal length can be tweaked by adjusting the spacing between certain of its elements), and the plane of the nose of the measuring aperture is precisely at the rear focal point (F2) of the standard lens (shown as a black dot). This relationship, once set, is precisely maintained by the positioning tube.

The standard lens forms an image of the reticle pattern. Where it falls along the axis of the instrument depends on the position of the reticle (following the customary focus equations). If the power drum is set to zero (the situation illustrated), the reticle will fall at the front focal point (F1) of the standard lens, and the image falls at an infinite distance forward of the measuring aperture, or behind it if we want to think of it that way (this is a "singularity" in the focus equation—one of those "divide by zero" things).

If we move the power drum a little in the direction of positive powers, the image is now a substantial but finite distance toward the rear of the instrument; with the drum set to +0.25 D, it will be four meters to the rear of the measuring aperture, wholly out of the instrument (thus it must be a "virtual image").

If instead we move the power drum a little in the direction of negative powers, the image is now a substantial but finite distance toward the front of the instrument; with the drum set to -0.25 D, it will be four meters to the front of the measuring aperture, behind the operator (again it is a "virtual image").

In fact, for all drum settings in the range of the instrument, the image will be a virtual one—it will never, for example, fall between the lens under test and the standard lens. But never mind—the telescope, looking thorough the lens under test, will be able to focus on it. (In a camera, when we look through the viewfinder, we see—and focus on—a virtual image, typically about one meter in front of us. A telescope can do the same thing.)

A little algebra

Now, it's time for some optical algebra.

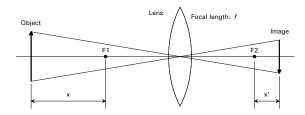


Figure 17. Reference for the Newtonian focus equation (not to scale)

In figure 17, the points F1 and F2 are the front and rear focal points, respectively, of a lens. (The figure is not drawn to scale.) We then consider the "Newtonian" form of the focus equation:

$$x\dot{x} = f^2 \tag{1}$$

where x is the distance of the object from the front focal point of the lens, x' is the distance of the image from the rear focal point of the lens, and f is the focal length of the lens. (We have to be careful about the algebraic signs in all this!)

Now, if we consider our standard lens (figure 16), we note that its rear focal point is located at the measuring aperture (at the rear vertex of the lens under test). Thus, x' also is the distance of the image from the rear focal point of the standard lens—which is thus the distance of the image from the rear vertex of the lens under test, which is the value we want to know, since it indicates the power of the lens.¹¹ In particular:

$$\Phi = \frac{1}{x'} \tag{2}$$

¹¹ Note again that, since the image of the reticle pattern is a virtual image, and may not even lie inside the instrument (and in no case between the lens under test and the standard lens), we cannot think of figure 17 as actually illustrating the situation. The algebra still holds, however.

where Φ (upper-case Greek *phi*) is the power of the lens under test.

Now, solving equation 1 for x', we get:

$$\lambda = \frac{f^2}{x} \tag{3}$$

Substituting for x' from equation 2, we get:

$$\frac{1}{\Phi} = \frac{f^2}{x} \tag{4}$$

Inverting, we get:

$$\Phi = \frac{x}{f^2} \tag{5}$$

which we can rearrange as:

$$\Phi = \left(\frac{1}{f^2}\right) x \tag{6}$$

Now, $1/f^2$ is a constant (the square of the power of the standard lens, in fact). Thus, we see that the power of the lens under test (Φ) is proportional to the position of the reticle (given by x). Accordingly, the scale of the power drum, which linearly moves the reticle, can be directly marked to indicate Φ , the power of the lens under test.

How nicely this worked out for the designers of the instrument!

Telescope focus at infinity

We note that the theory of all this is predicated on our observing telescope being focused at infinity. Changes in the refractive situation of the observer can disrupt this, albeit very slightly, leading to a very slight error in measurement. Nevertheless, the designers of the focimeter were very fastidious, 12 and made provisions to avert any such slight discrepancy.

Inside the telescope is an *eyepiece reticle* (we see it as the pattern of black lines and circles in figure 9). One of its roles is to make measurements beyond what I have described here (such as the orientation and magnitude of a *prism* component in the lens, used to

¹² A principal developer of the AO focimeter shown in figure 12 described it in an article in the Journal of the Optical Society of America (December, 1922) as being equivalent in precision to the company's standard reference lenses, traceable to the National Bureau of Standards.

correct misconvergence of the eyes). In fact, in connection with such measurements, it can be rotated with a ring behind the eyepiece, marked to show the angular orientation of this reticle.

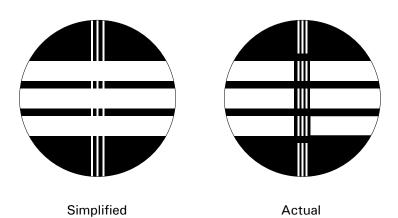
But this reticle has a second purpose. It is at the front focal point of the telescope objective lens. The eyepiece lens proper, in this kind of telescope, working in conjunction with the viewer's eye, should be focused on the point where the eyepiece reticle is located.

This focus situation can be changed by turning the eyepiece, moving it axially. It is adjusted so the eyepiece reticle is in perfect focus as seen by the operator. Then, the intended focus of the "operator plus telescope" at infinity is perfected.

Aid in making the axis setting

In the body of the article, we saw that making the proper setting of the axis was helped by an interesting display phenomenon (seen in figure 11). I drew the parallel with the split-prism focusing aid familiar in SLR cameras. But in fact, in the focimeter, this effect is produced without benefit of any such special optical components. It is inherent in the behavior of the image of the target.

We will now examine the form of that target more carefully than before. In Figure 18, on the left, we see the way that the target is ordinarily visualized. But in fact, its actual arrangement more nearly as shown on the right.



The little segments of the thin lines are sometimes called "boxes".

Figure 18. Focimeter target pattern

When the lens has correction for astigmatism, by way of a cylinder component of its power, the lens itself exhibits astigmatism, which is why, in such a case, we can selectively perfect the **apparent** focus of either the thin lines or the fat lines. I say *apparent* because it is only across the width of the chosen lines that focus is perfected, not along its length. At the ends of a line that is "in focus", we will see a blurring along the length of the line (as if the line had been cut with a "fuzzy cut").

If the axis of the cylindrical component of the lens power is oblique to both sets of lines, that blurring at the ends "goes off to one side" of the length of the line, as if the line were cut off at an angle to its length with a fuzzy cut.

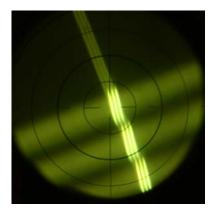


Figure 19. Fine lines in focus but axis not proper

The visual appearance of this is seen again in figure 19. The visual impression given is that the line segments are rotated, and do not form an unbroken line.

When the axis is set ideally (as it is almost ¹³ in figure 9), the line segments appear to have the same azimuth and thus appear as an unbroken line.

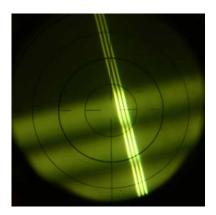


Figure 20. Fine lines in focus and axis (almost) proper

Here, we can clearly see the "fuzzy" extension of the short thin line segments (although the ones from adjacent segments "overlap" a little.

In fact, in the case illustrated by these figures, where there is a substantial difference between the focus in the two medians (the lens having a substantial cylinder component), the "fuzzy angular cut" at the ends of the short segments (seen in figure 19) really has an effect

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¹³ Well, I was a little sloppy while doing this.

from about the middle of the segment on. Thus the "rotation" of the segment is not just an optical illusion resulting from the fuzzy angular cut at the ends—the whole "shaft" of the segment really is rotated in azimuth.

In cases where the cylindrical component is smaller, and thus the degree of blurring is less when one or the other set of lines is brought to the best apparent focus, the phenomenon is more limited to the end regions of the line segments. Still, it is compelling visually.

Although this phenomenon is given glancing mention in Charles J. Troppman's definitive patent on this instrument (U. S. Patent 1,609,895, issued in 1926), and is even represented there in a fanciful portrayal of the development of the blurred image under an "oblique" axis setting, the wonderful advantage given by this phenomenon in facilitating axis adjustment is not claimed. It is almost as if it were discovered by accident. (Or perhaps it was "prior art" and not eligible to be claimed.)

A more elaborate system

The AO "Wellsworth" Lensometer (see figure 12) has a more elaborate design in one respect than the B&L instrument described earlier (and more modern AO instruments). In this instrument, the fine lines and fat lines are on separate reticles, which can be separately moved along the instrument axis, using separate, coaxial knobs on the power drum.

Thus, even for a lens with a cylindrical component, both sets of lines can be brought into focus at the same time. The power implications of both reticles' positions are shown on two separate, adjacent rings on the power drum. Therefore the sphere and cylinder power result can be read simultaneously. (But you still have to subtract two readings to get the cylinder power!)

What if there were no cylindrical component? Then the two reticles would have to be at the same place, obviously not possible.

To avert this, the instrument includes a cylindrical lens immediately in front of the fat line reticle (the frontmost one), oriented so that it affects the focus of the fat lines (upon which we rely in ascertaining the cylinder power). This shifts "forward" the physical range over which the fat line reticle must operate, avoiding the possibility of any collision. The cylinder markings on the power drum for the fat line reticle are offset to recognize this.

There are other, rather subtle advantages of this arrangement, which are beyond the scope of this article.

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¹⁴ If the cylinder power is not too large and negative.

Evidently, the complications of this feature were later found to not be justified by its benefits. In all "modern" instruments, we consecutively set the single power drum to two positions to make the complete measurement, not really any more complicated.

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Appendix C

Some historical background

In 1876, Herman Snellen (whom we mostly honor for the development of the widely-used eye chart, the one with the big "E") developed a technique for determining the power of eyeglass lenses which serves as the principle of today's focimeters.

In 1912, Charles J. Trottman, of F. A. Hardy & Co., developed the first practical instrument exploiting that principle. Successive designs were covered by U. S. patents 1,803,309 (1914), 1,187,579 (1916), and 1,281,717. The name "Lensometer" was evidently first used in connection with instruments made by the Hardy firm.

In 1913, F. A. Hardy & Co. was merged into American Optical Company. Troppman continued his work on lens measurement instruments, evidently in collaboration with Edgar D. Tillyer, American Optical's chief optical wonk for many years. Tillyer presumably helped refine the design, helped to articulate the theoretical basis of the instrument, and contributed to its introduction as a practical commercial product in 1922 (see figure 12 in the body of this article). Tillyer become widely associated with the instrument. ¹⁵

Essentially that commercial instrument configuration was essentially the premise of Troppman's 1926 U.S. Patent (1,609,895), considered the definitive patent in this area.

In December, 1922, Tillyer co-authored (not with Troppman—rather with Tillyer's boss, Charles Sheard) the seminal paper on the instrument for the Journal of the Optical Society of America. Although AO often applied Tillyer's name to commercial products (such as an important series of eyeglass lens blanks), in the case of their focimeter, they declined to honor either of the key figures in its development, but rather called it the "Wellsworth Lensometer".

Later, Bausch & Lomb, Incorporated, AO's chief United States rival in the optical field, introduced their Vertometer (made under license from AO under an early Troppman patent). Figure 21 shows what I believe to be essentially its first commercial version, the "Model 90" (type 21-05-90). It is suspected that this specimen dates from *ca.* 1940.

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¹⁵ The May, 1922 issue of the Rutgers Alumni Monthly (Tillyer got his undergraduate degree at Rutgers in 1902) said, "Mr. Tillyer's latest invention is an instrument called the Lensometer." We might conjecture that there were could have been some ill feelings between Troppman and Tillyer over this state of affairs, although I have never seen this spoken of in the historical record.



Figure 21. Bausch & Lomb "Model 90" Vertometer

Photograph by Maria Felix Used by permission

As in the AO instrument, the axis control here is coaxial, a small knurled knob at the front of the rearmost portion of the instrument (the "projector"), which turns a disk bearing a white fiducial that is read against a fixed circular scale (with the white scale lines).

I conjecture that the pointer on the power drum is intended to allow the first ("sphere") reading to be captured when the drum is used to take the second reading ("cylinder").

The unit seen has a marking attachment (we see its three prongs in the cylindrical ink box). The lens chuck swings from an overhead pivot.

In the next generation, the famous Model 70 (seen again in figure 22), B&L gave their instrument a somewhat more stylish design, and incorporated several improvements in the user interface.



Figure 22. B&L Model 70 Vertometer

Most prominently, the cylinder axis control, which in the AO instruments is a handwheel mounted coaxially with the rear instrument barrel (see for example figure 24), was made a knob on the left side of the instrument. Many operators considered this a more convenient

location, since it needed to be operated with the left hand (the right hand being on the power drum). We see it in figure 23.



Figure 23. B&L Model 70—Cylinder axis drum

However, during that era, AO bragged in their literature about **their** axis control, pointing out that it could, conveniently, be operated with either hand. That argument, although true, is not of much consequence, given that in the most common operations with the instrument the operator must simultaneously manipulate both the axis control and the power control—which in the AO instrument (as in the B&L version) was located (only) on the right side of the instrument.

The first commercial AO Lensometer (see figure 12) had separately movable thin and fat line reticles (as discussed in Appendix B). The next commercial model (ca. 1938) had a simpler mechanism, with a single reticle (as we find on essentially all modern focimeters), and was originally billed as the Lensometer Junior. The "junior" was eventually dropped, and as the M603 (later, just 603) this model became the centerpiece of the AO commercial focimeter line. The more complicated model and its dual reticle design were retired.



Figure 24. American Optical M603 Lensometer

Figure 24 shows an AO M603B Lensometer ("B" means with the optional marking device feature—the marking device has in fact been raised in this photo).

In figure 25 we see a more recent AO model, the 12603 (ca. 1977).



Figure 25. American Optical 12603 Lensometer

In the promotional literature, the arrangement of the power and axis controls is described as providing "simple one-hand operation".

In figure 26, we see a modern B&L Vertometer, the Model 62 (formally, 71-26-62).



Figure 26. B&L "Model 62" Vertometer

We believe this specimen was made in 1965.

The axis control is on the front, and turns a full revolution for the range from 0° -180°. There is a duplicate power knob on the left side.

It's pretty uninteresting compared to our Model 70.