Optimization Solutions Final Project

Manufacturing Optimization

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Goal: Maximize profits and minimize production & labor costs

Our Client: *Car Engine Manufacturer*

- Operates in one production plant and produces one type of engine

Business Problem: *Maximize Profit*

1. Several parts, raw materials, and laborers are required to produce one engine
2. The client wants to determine the profit maximizing **quantities of parts and engines** to produce and the resulting **levels of raw materials and labor required**

Opportunity: Determine how to maximize profits and minimize cost/labor across a number of environmental changes the client is facing

Mixed Integer programming problem with multiple variations

Optimization Problem:

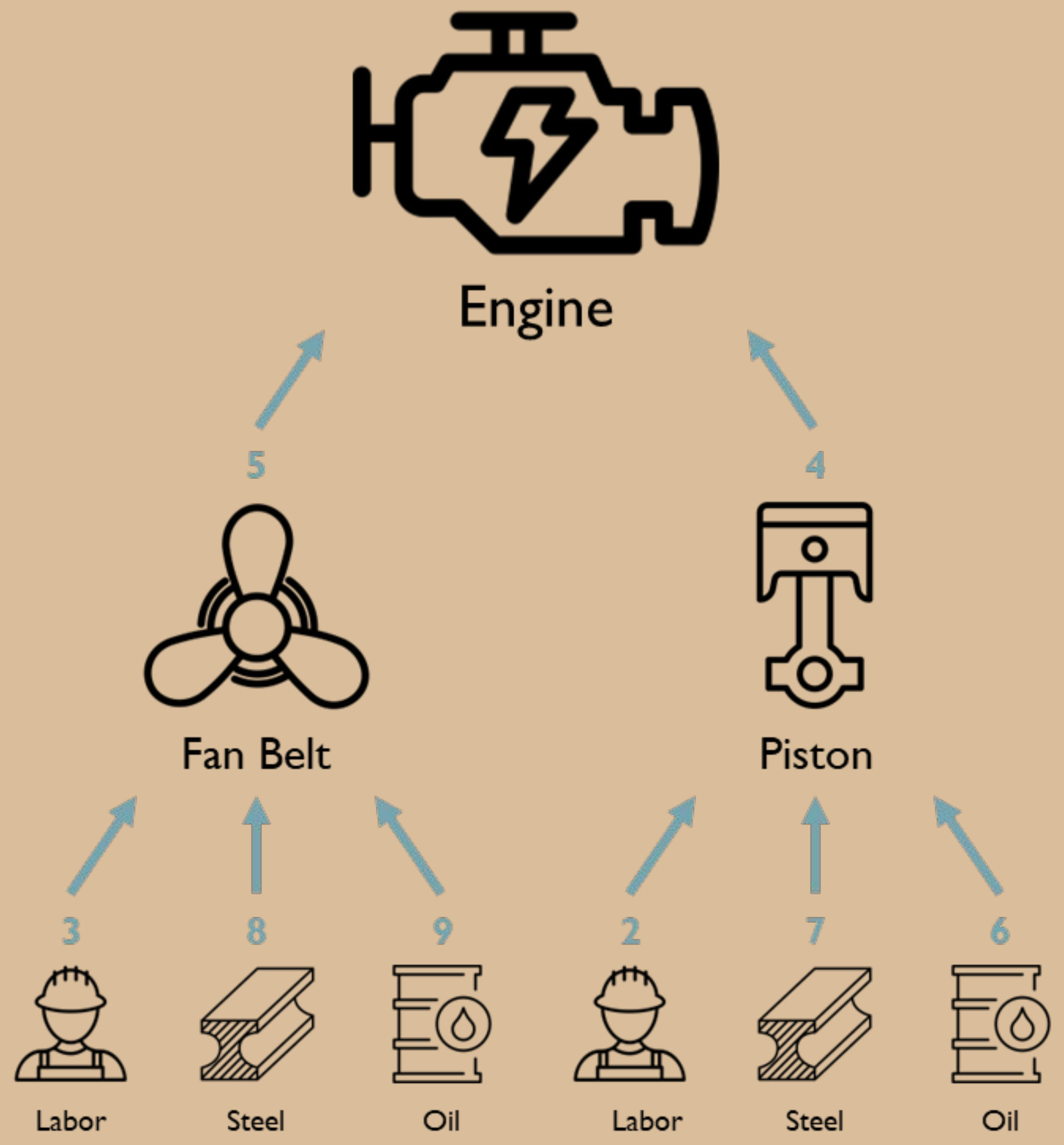
1. **Mixed Integer programming**
   - Objective function is linear in the arguments
   - Constraints include linear equalities and linear inequalities
   - Arguments can only take on integer values
2. **Test multiple variations of the base problem to understand how outputs are impacted**
   - Supply chain shortages
   - COVID restrictions
   - Liquidity constraints
3. **Required packages and solver**
   - **JuMP**: allows for a linear class of optimization problems with easy-to-read code
   - **GLPK**: wrapper for the GNU Linear Programming Kit library (solver)

Manufacturing
Base Problem Setup

Manufacturing Requirements

One <u>engine</u> requires

- two unique subcomponents
  (fan belt and piston)
- two different raw materials (steel and oil)
- five employees



Variables

INPUTS

Raw Material

STEEL
raw material 1
a_1: quantity
r_1: cost/unit

OIL
raw material 2
a_2: quantity
r_2: cost/unit

Production

FAN BELT
part A
w_A: machinery cost/unit

PISTON
part B
w_B: machinery cost/unit

LABORER
person
w_L: wages/shift per person

OUTPUTS

Selling Price

FAN BELT
part A
p_A: market price

PISTON
part B
p_B: market price

ENGINE
final product
p_E: market price

DECISIONS

Production & Allocation of Fan Belts and Pistons

FAN BELT
part A
x_1: quantity in final engine production

FAN BELT
part A
x_2: quantity sold individually

PISTON
part B
x_3: quantity in final engine production

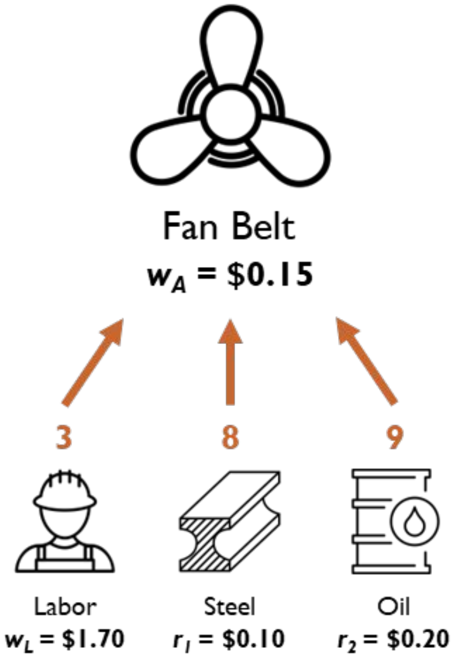
PISTON
part B
x_4: quantity sold individually

Production Costs

Fan Belt

Labor + Steel + Oil + Machinery

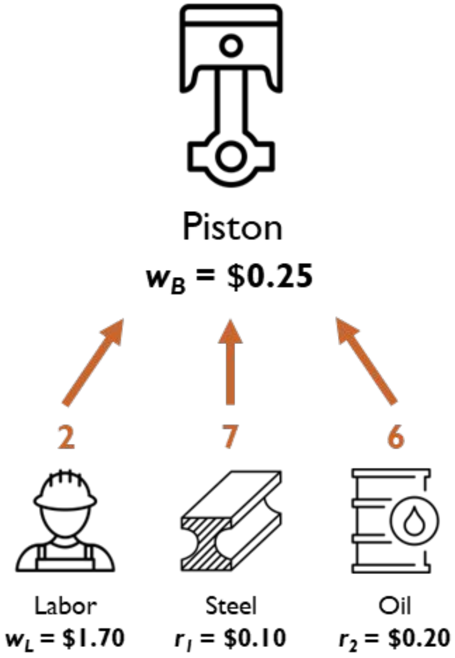
c_A = 7.85



Piston

Labor + Steel + Oil + Machinery

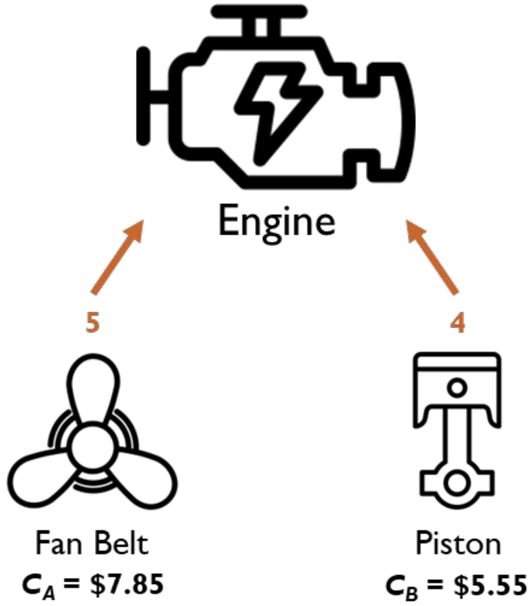
c_B = 5.55



Engine

*5 * Fan Belt Cost + 4 * Piston Cost*

c_E = 61.45



MAX PROFIT

Objective Function

Objective: Maximize Profit

```
@objective(model,Max,(p_E-c_E)*(((x_1/5)+(x_2/4))/2)+(p_A-c_A)*x_3+(p_B-c_B)*x_4)
```

$$\pi = \pi_{\text{Engine}} + \pi_{\text{Fan Belt}} + \pi_{\text{Piston}}$$

<u>Engine</u>

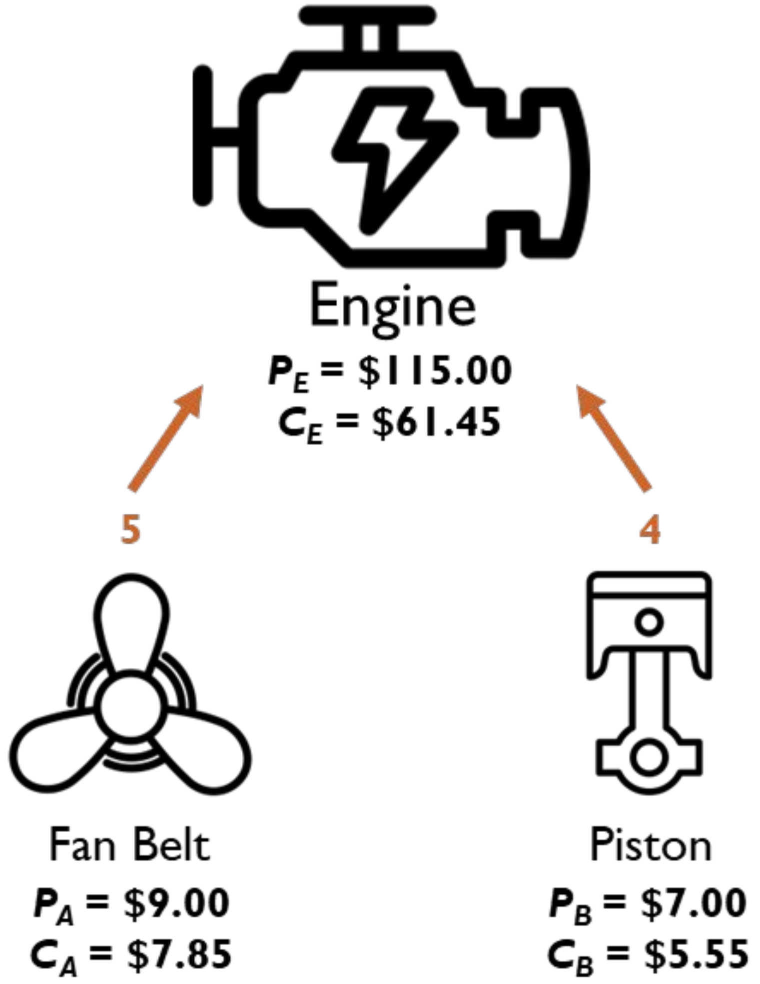
$$\pi_{\textbf{Engine}} = (p_E - c_E) * ((x_1/5)+(x_2/4))/2$$

<u>Fan Belt</u>

$$\pi_{\textbf{Fan Belt}} = (p_A - c_A) * x_3$$

<u>Piston</u>

$$\pi_{\textbf{Piston}} = (p_B - c_B) * x_4$$



x_1 = quantity of fan belts (part A) used in engines
x_2 = quantity of pistons (part B) used in engines
x_3 = quantity of fan belts (part A) sold individually
x_4 = quantity of pistons (part B) sold individually

Constraints

Raw Material:

- Required units of **steel** (a1)
```
@constraint(model, 8*x1 + 7*x2 + 8*x3 + 7*x4 <= a1)
```
- Required units of **oil** (a2)
```
@constraint(model, 9*x1 + 6*x2 + 9*x3 + 6*x4 <= a2)
```
- ***Unlimited*** supply of steel (a1) and oil (a2) available for purchase
```
@variable(model, a1 ≥ 0.0)
@variable(model, a2 ≥ 0.0)
```

Factory Personale Capacity:

- **250 laborers** can work at a given time
```
@constraint(model, 3*x1 + 2*x2 + 3*x3 + 2*x4 <= 250)
```

Production Requirements:

- Final engine requires **5 Fan Belts** (x1) and **4 Pistons** (x2)
```
@constraint(model, (x1/5) - (x2/4) == 0.0)
```
- No manufacturing cost constraint

Basic Solution

Model

Objective Function:

- Maximize $5.4\,\boldsymbol{x1} + 6.7\,\boldsymbol{x2} + 1.15\,\boldsymbol{x3} + 1.4\,\boldsymbol{x4}$

Subject to:

- $0.2\,\boldsymbol{x1} - 0.25\,\boldsymbol{x2} = 0.0$
- $8\,\boldsymbol{x1} + 7\,\boldsymbol{x2} + 8\,\boldsymbol{x3} + 7\,\boldsymbol{x4} - \boldsymbol{a1} \leq 0.0$
- $9\,\boldsymbol{x1} + 6\,\boldsymbol{x2} + 9\,\boldsymbol{x3} + 6\,\boldsymbol{x4} - \boldsymbol{a2} \leq 0.0$
- $3\,\boldsymbol{x1} + 2\,\boldsymbol{x2} + 3\,\boldsymbol{x3} + 2\,\boldsymbol{x4} \leq 250.0$
- $\mathbf{x1, x2, x3, x4, a1, a2} \geq 0.0$
- $x1 \in \mathbb{Z},\ x2 \in \mathbb{Z},\ x3 \in \mathbb{Z},\ x4 \in \mathbb{Z}$

Results

Maximal Profit: **$550**

Engines Produced: 10

Fan Belts Produced: 50

Pistons Produced: 50

Basic Solution Outputs

Raw Material

Steel
750.0 units for $75

Oil
750.0 units for $150

Fan Belt Production

50 units produced

- Used in engines: **50**
- Sold individually: **0**

Allocated Costs

- Wages: $255 for 150 laborers
- Raw material: $130
- Total variable cost: $392.50

Piston Production

50 units produced

- Used in engines: **40**
- Sold individually: **10**

Allocated Costs

- Wages: $170 for 100 laborers
- Raw material: $95
- Total variable cost: $277.50

Total Production

10 engines produced

Revenues

Fan Belt sales: **$0**
Piston sales: **$70**
Engine sales: **$1150**

Total Costs

- Total variable cost: $670

Maximal profit: $550

Key Insight: With no constraint on raw material availability and production cost, we see a profit of $550, with 10 engines produced

Variation I.
Supply Chain Disruption

Variation 1

Constraints: Supply Chain Impact

Business Problem:

Supply chain shortages have caused a **reduction** in the **availability of raw materials** *(steel & oil)*

Raw Material:

- Required units of steel (a1)
  `@constraint(model, 8*x1 + 7*x2 + 8*x3 + 7*x4 <= a1)`
- Required units of oil (a2)
  `@constraint(model, 9*x1 + 6*x2 + 9*x3 + 6*x4 <= a2)`
- Availability of steel (a1) is **56 units**
  `@variable(model, 0.0 <= a1 <= 56)`
- Availability of oil (a2) is **54 units**
  `@variable(model, 0.0 <= a1 <= 54)`

Factory Personale Capacity:

- 250 laborers can work at a given time
  `@constraint(model, 3*x1 + 2*x2 + 3*x3 + 2*x4 <= 250)`

Production Requirements:

- Final engine requires 5 Fan Belts (x1) and 4 Pistons (x2)
  `@constraint(model, (x1/5) - (x2/4) == 0.0)`
- No manufacturing cost constraint

Variation 1

Model: Supply Chain Impact

Business Problem:

Supply chain shortages have caused a reduction in the availability of raw materials

Problem Adjustments

Limited Raw Materials

- Availability of steel (a1) is 56 units
  `@variable(model, 0.0 <= a1 <= 56)`
- Availability of oil (a2) is 54 units
  `@variable(model, 0.0 <= a2 <= 54)`

Model

Objective Function:

- Maximize $5.4\boldsymbol{x1}+6.7\boldsymbol{x2}+1.15\boldsymbol{x3}+1.4\boldsymbol{x4}$

Subject to:

- $0.2\boldsymbol{x1}-0.25\boldsymbol{x2}=0.0$
- $8\boldsymbol{x1}+7\boldsymbol{x2}+8\boldsymbol{x3}+7\boldsymbol{x4}-\boldsymbol{a1}\leq 0.0$
- $9\boldsymbol{x1}+6\boldsymbol{x2}+9\boldsymbol{x3}+6\boldsymbol{x4}-\boldsymbol{a2}\leq 0.0$
- $3\boldsymbol{x1}+2\boldsymbol{x2}+3\boldsymbol{x3}+2\boldsymbol{x4}\leq 250.0$
- $\mathbf{x1,x2,x3,x4,a1,a2} \geq 0.0$
- $x1\in\mathbb{Z},\ x2\in\mathbb{Z},\ x3\in\mathbb{Z},\ x4\in\mathbb{Z}$
- $\boldsymbol{a1}\leq 56.0$
- $\boldsymbol{a2}\leq 54.0$

Results

Maximal Profit: $11.60

Engines Produced: 0

Fan Belts Produced: 0

Pistons Produced: 8

Variation 1

Solution Outputs: Supply Chain Impact

Raw Material

Steel
56 units for $5.6

Oil
48 units for $9.6

+

Fan Belt Production

0 units produced

- Used in engines: **0**
- Sold individually: **0**

Allocated Costs

- Wages: $0 for 0 laborers
- Raw material: $0
- Total variable cost: $0

+

Piston Production

8 units produced

- Used in engines: **0**
- Sold individually: **8**

Allocated Costs

- Wages: $27.20 for 16 laborers
- Raw material: $15.20
- Total variable cost: $44.40

Total Production

0 engines produced

Revenues

Fan Belt sales: **$0**
Piston sales: **$56**
Engine sales: **$0**

Total Costs

- Total variable cost: $44.40

Maximal profit: $11.60

Key Insight: Supply chain constraints led to the optimizer focussing on production of pistons to maximize profit, producing 8 pistons and no engines

Variation II.
COVID Restrictions

Variation 2

Constraints: COVID Restrictions Impact

Business Problem:

Along with the supply chain shortages, COVID restrictions have caused **factory capacity to shrink**

Raw Material:

- Required units of steel (a1)
  `@constraint(model, 8*x1 + 7*x2 + 8*x3 + 7*x4 <= a1)`
- Required units of oil (a2)
  `@constraint(model, 9*x1 + 6*x2 + 9*x3 + 6*x4 <= a2)`
- Availability of steel (a1) is **56 units**
  `@variable(model, 0.0 <= a1 <= 56)`
- Availability of oil (a2) is **54 units**
  `@variable(model, 0.0 <= a1 <= 54)`

Factory Personale Capacity:

- **47 laborers** can work at a given time
  `@constraint(model, 3*x1 + 2*x2 + 3*x3 + 2*x4 <= 47)`

Production Requirements:

- Final engine requires 5 Fan Belts (x1) and 4 Pistons (x2)
  `@constraint(model, (x1/5) - (x2/4) == 0.0)`
- No manufacturing cost constraint

Variation 2

Model: COVID Restrictions Impact

Business Problem:

Along with the supply chain shortages, COVID restrictions have caused factory capacity to shrink

Problem Adjustments

Limited Raw Materials

- Availability of steel (a1) is 56 units
  `@variable(model, 0.0 <= a1 <= 56)`
- Availability of oil (a2) is 54 units
  `@variable(model, 0.0 <= a2 <= 54)`

COVID Restrictions

- Only 47 laborers can be in the factory at a given time
  `@constraint(model, 3*x1+2*x2+3*x3+2*x4<=47)`

Model

Objective Function:

- Maximize $5.4x1+6.7x2+1.15x3+1.4x4$

Subject to:

- $0.2x1-0.25x2=0.0$
- $8x1+7x2+8x3+7x4-a1\leq 0.0$
- $9x1+6x2+9x3+6x4-a2\leq 0.0$
- $3x1+2x2+3x3+2x4\leq 47.0$
- $x1,x2,x3,x4,a1,a2 \geq 0.0$
- $x1\in\mathbb{Z},\ x2\in\mathbb{Z},\ x3\in\mathbb{Z},\ x4\in\mathbb{Z}$
- $a1\leq 56.0$
- $a2\leq 54.0$

Results

Maximal Profit: $11.60

Engines Produced: 0

Fan Belts Produced: 0

Pistons Produced: 8

Variation 2

Solution Outputs: COVID Restrictions Impact

Raw Material

Steel
56 units for $5.6

Oil
48 units for $9.6

+

Fan Belt Production

0 units produced

- Used in engines: **0**
- Sold individually: **0**

Allocated Costs

- Wages: $0 for 0 laborers
- Raw material: $0
- Total variable cost: $0

+

Piston Production

8 units produced

- Used in engines: **0**
- Sold individually: **8**

Allocated Costs

- Wages: $27.20 for 16 laborers
- Raw material: $15.20
- Total variable cost: $44.40

Total Production

0 engines produced

Revenues

Fan Belt sales: **$0**
Piston sales: **$56**
Engine sales: **$0**

Total Costs

- Total variable cost: $44.40

Maximal profit: $11.60

Key Insight: The supply chain shortages limited the number of laborers needed for production, meaning labor is already in compliance with COVID restrictions

Variation III.
Liquidity Limits

Variation 3

Constraints: Liquidity Limits

Business Problem:

Supply chain shortages and COVID labor restrictions have led to liquidity constraints. The company must **limit total production costs**

Raw Material:

- Required units of steel (a1)
  `@constraint(model, 8*x1 + 7*x2 + 8*x3 + 7*x4 <= a1)`
- Required units of oil (a2)
  `@constraint(model, 9*x1 + 6*x2 + 9*x3 + 6*x4 <= a2)`
- Availability of steel (a1) is **56 units**
  `@variable(model, 0.0 <= a1 <= 56)`
- Availability of oil (a2) is **54 units**
  `@variable(model, 0.0 <= a1 <= 54)`

Factory Personale Capacity:

- **47 laborers** can work at a given time
  `@constraint(model, 3*x1 + 2*x2 + 3*x3 + 2*x4 <= 47)`

Production Requirements:

- Final engine requires 5 Fan Belts (x1) and 4 Pistons (x2)
  `@constraint(model, (x1/5) - (x2/4) == 0.0)`
- Total production cost must not exceed **$45**
  `@constraint(model, cA*(x1+x3) + cB*(x2+x4) <= 45)`

Variation 3

Model: Liquidity Limits

Business Problem:

Supply chain shortages and COVID labor restrictions have led to liquidity constraints. The company must limit total production costs

Problem Adjustments

Limited Raw Materials

- Availability of steel (a1) is 56 units
  ```
  @variable(model, 0.0 <= a1 <= 56)
  ```
- Availability of oil (a2) is 54 units
  ```
  @variable(model, 0.0 <= a2 <= 54)
  ```

COVID Restrictions

- Only 47 laborers can be in the factory at a given time
  ```
  @constraint(model, 3*x1+2*x2+3*x3+2*x4<=47)
  ```

Liquidity Limit:

- Production costs cannot exceed $45
  ```
  @constraint(model, cA*(x1+x3)+cB*(x2+x4)<=45)
  ```

Model

Objective Function:

- Maximize $5.4\mathbf{x1}+6.7\mathbf{x2}+1.15\mathbf{x3}+1.4\mathbf{x4}$

Subject to:

- $0.2\mathbf{x1}-0.25\mathbf{x2}=0.0$
- $8\mathbf{x1}+7\mathbf{x2}+8\mathbf{x3}+7\mathbf{x4}-\mathbf{a1}\leq 0.0$
- $9\mathbf{x1}+6\mathbf{x2}+9\mathbf{x3}+6\mathbf{x4}-\mathbf{a2}\leq 0.0$
- $3\mathbf{x1}+2\mathbf{x2}+3\mathbf{x3}+2\mathbf{x4}\leq 47.0$
- $\mathbf{x1},\mathbf{x2},\mathbf{x3},\mathbf{x4},\mathbf{a1},\mathbf{a2}\geq 0.0$
- $x1\in\mathbb{Z},\ x2\in\mathbb{Z},\ x3\in\mathbb{Z},\ x4\in\mathbb{Z}$
- $7.9x1+5.5x2+7.9x3+5.5x4\leq 45.0$
- $\mathbf{a1}\leq 56.0$
- $\mathbf{a2}\leq 54.0$

Results

Maximal Profit: $11.60

Engines Produced: 0

Fan Belts Produced: 0

Pistons Produced: 8

Variation 3

Solution Outputs: Liquidity Limits

Raw Material

Steel
56 units for $5.60

Oil
48 units for $9.60

+

Fan Belt Production

0 units produced

- Used in engines: **0**
- Sold individually: **0**

Allocated Costs

- Wages: $0 for 0 laborers
- Raw material: $0
- Total variable cost: $0

+

Piston Production

8 units produced

- Used in engines: **0**
- Sold individually: **8**

Allocated Costs

- Wages: $27.20 for 16 laborers
- Raw material: $15.20
- Total variable cost: $44.40

Total Production

0 engines produced

Revenues

Fan Belt sales: **$0**
Piston sales: **$56**
Engine sales: **$0**

Total Costs

- Total variable cost: $44.40

Maximal profit: $11.60

Key Insight: Liquidity constraints can ensure there is a cap on associated cost. It does not impact production and profit in this instance given material constraints

Production Curve

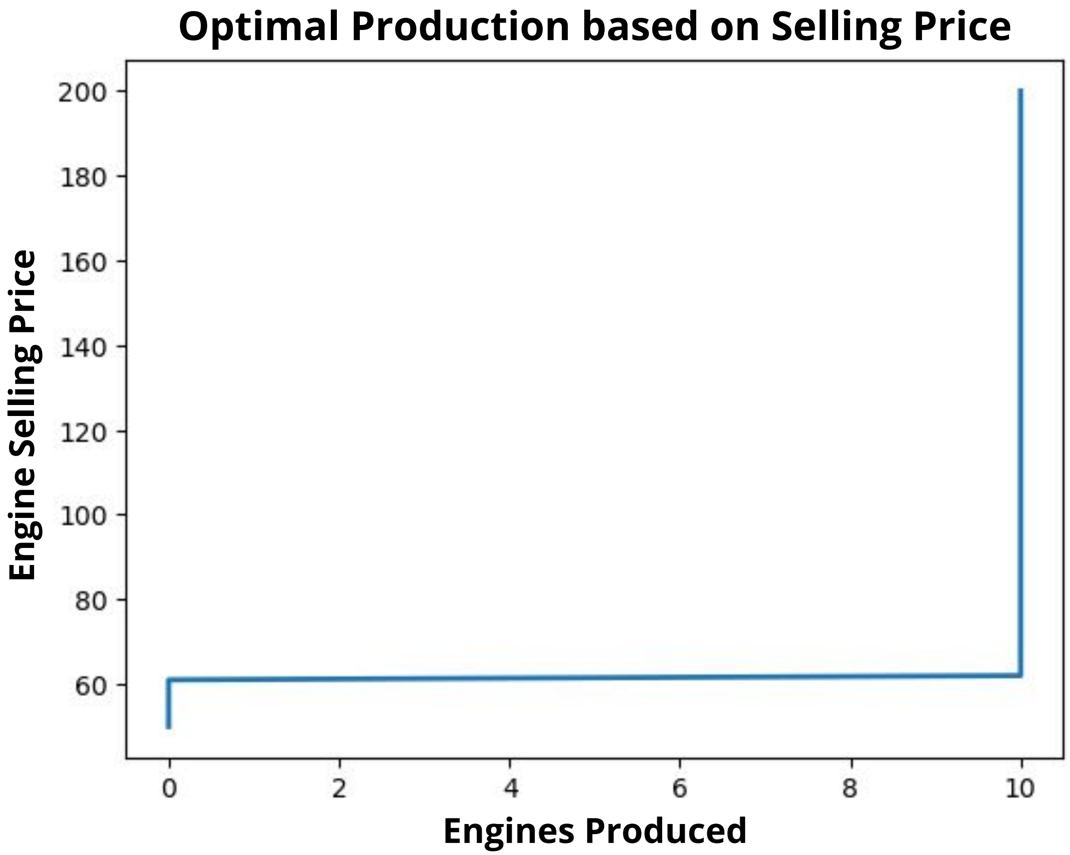
Optimal production quantity

Two optimal levels of production

- Produce zero when price < marginal cost
- Produce at full capacity (*10 engines*) when price ≥ marginal cost

→ *No fixed costs*

→ *Constant marginal cost*



**Simplified by removing option to sell individual fan belts and pistons – all subcomponents go to engine production*

Key Takeaways

Key takeaways

Downstream Impacts:

1. Steel and oil are required in the production of both fan belts and pistons, which are both required to produce engines. **Limitations to steel** and **oil** can have a severe impact on production and profit maximization
2. The impacts of COVID on the factory were downplayed because **labor needs were already reduced** from raw material shortages
3. Limiting production costs is a good way to manage liquidity, but it will put a **cap on profits and production outputs**

Production Takeaways :

1. Profits are maximized when **Fan Belts** and **Pistons** are **used in engines** and not sold individually. But, given material constraints, pistons should be prioritized

Additional Considerations

Future work

Inclusion of …

1. Penalty coefficients for multiple optimization opportunities
2. Additional constraints
3. Stochastic program
4. Fixed costs
5. Changes to production capabilities (opening additional facility)
6. Product line expansion

Appendix

Github and References

- **References**
  - **Ceramics Manufacturing Problem**
    - **https://bibliotekanauki.pl/articles/406208.pdf**