The problem of Landau pole in abelian gauge theory and the possible quantum gravity corrections

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Introduction

1 Standard model ...

The Standard Model of particle physics is a unified description of all quantum fields observed in physics. It strives to predict all the phenomena observed at microscopic scales while maintaining theoretical self-consistency and certain mathematical aesthetics. The predictive power of Standard Model, with its most famous examples like precision tests of electron anomalous magnetic moment or the discovery of Higgs boson, makes it ungrounded to postulate a fundamental physical theory that would not reduce to SM in the suitable limit, at least as an effective field theory. Standard Model, however, certainly is not a complete theory of physical reality. It does not include a description of gravity and above the Planck scale $E_p \approx 10^{19}$ GeV, due to quantum effects of gravity, predictions of both the SM and Einstein's theory of gravity are not expected to apply. One should then not be surprised, if above E_p Standard Model exhibits certain inconsistencies. One of the big issues of the SM is the quantum triviality problem for the electroweak $U(1)$ gauge coupling and the scalar higgs boson quartic coupling. In the pure electroweak theory, one loop β-function of the abelian gauge coupling is: [4]

$$\beta_{g_Y}^{\mathrm{SM}} = \beta_{g_Y}^{(3)} g_Y^3 = \frac{41}{6}\frac{g_Y^3}{16\pi^2} \tag{1}$$

This predicts a running of gauge coupling that would diverge at a momentum scale $\mu = \left(2g_{Y\mathrm{obs}}^2\, \beta_{g_Y}^{(3)}\right)^{-1}$. Conversely, taking any arbitrarily high value of the bare coupling, the only possible value of $g_{Y\mathrm{obs}}$ should be 0, making the theory trivial, i.e. non-interacting.

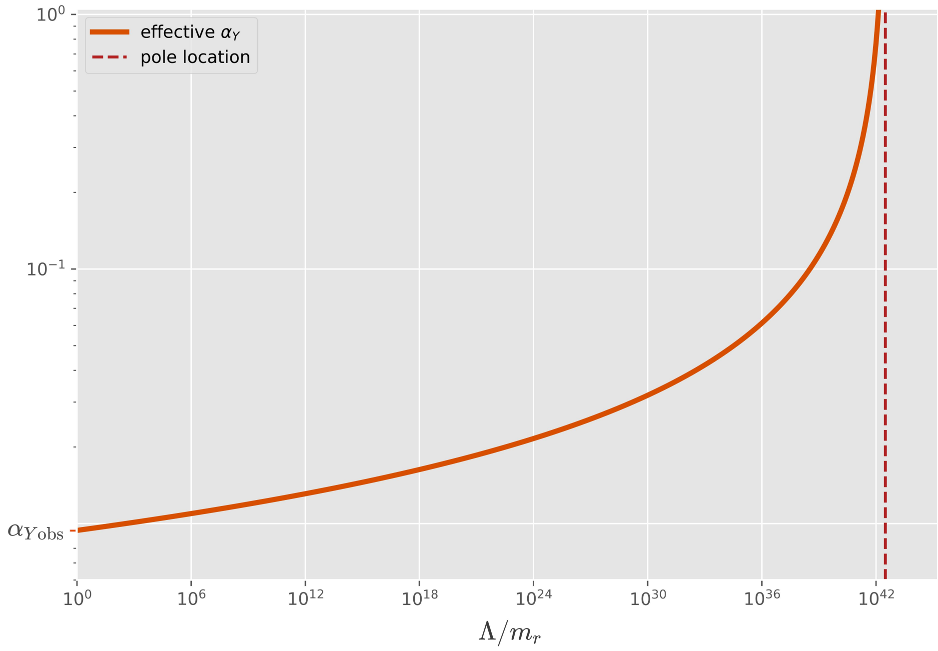


Figure 1: Running of the abelian gauge parameter $\alpha_Y = \frac{g_Y^2}{4\pi}$ in the Standard Model. Observed value $\alpha_{Y\text{obs}}$ is taken from [1]. Landau Pole marked at $\mu \approx 10^{42.5}$

Taking an electron mass as a reference scale, the location of abelian coupling landau pole will be of order $\Lambda \sim 10^{48}$ eV, well above the Planck scale. Similarly, quartic self-interaction of higgs field theoretically may exhibit a pole at a finite momentum scale. Existence of both poles remains an open question, but a theory containing any such singularity cannot be fundamental. One solution to this problem could be a grand unified theory (GUT), which proposes a larger symmetry group, spontaneously broken to the known $SU(3) \times SU(2) \times U(1)$ of standard model at around the scale where running values of electroweak and strong couplings coincide. This scale is predicted to be below planck energy [ref]. Above it, the theory could introduce a running of unified coupling that does not exhibit a triviality issue. The existence of grand unification, however has not been yet confirmed in any way, while the experimental bounds on the proton lifetime heavily constraints or even rules out many of the GUTs[ref]. This constrasts sharply with the other potential completion of the SM, which is quantum gravity. Although no quantum theory of gravity has been widely accepted, the existence of gravity and the quantum nature of physical reality most certainly are. Should quantum gravitational solution to the triviality issues of SM exist, it could provide an argument of consistency between the particular theory of quantum gravity and the SM, as well as eliminate one of the arguments for the need of GUT.

The straightforward approach to quantum gravity - a metric fluctuation quantum field governed by Einstein - Hilbert action is non-renormalizable by the perturbative methods[2]. However, for energy scales small compared to Planck energy, the infinite number of divergent terms in lagrangian affects predictions in a negligible way and can be effectively ignored. In this framework, called the effective field theory of gravity (EFT), there has been a number of works investigating the gravitational contribution to the running of gauge couplings. The predictions of EFT does not apply above the Planck scale, but an assesment of the quantitative nature of gravitational corrections can still give valuable information. Since gravitons carry no gauge charge, the contribution to beta function of gauge coupling from SM particles and gravitons are separate and in the one-loop approximation takes the general form

$$\beta_g(k) = a \cdot Gk^2 g + b \cdot g^3 = a \cdot Gk^2 g + \beta_g^{\text{SM}} \tag{2}$$

The solution to the quantum triviality problem therefore depends on the values of factor a. Calculations in the cutoff scheme, employing the background field method, performed by Wilczek and Robinson [5] have indicated $a = -\frac{3}{\pi}$, which implied the asymptotic freedom of any Yang-Mills theory in the presence of gravity and thus a solution to the triviality problem. Later work by Pietrykowski [6] argued, that a result obtained by Wilczek and Robinson is specific to the choice of graviton gauge fixing. There, the calculations of [5] was reproduced and no dependence on gauge parameter was confirmed, but when using a different class of gauge fixing terms, no gravitational contribution was found. In [7], a Vilkovsky-DeWitt method was used and the calculations also indicated no gravitational contribution to β_g in dimensional regularization. In [8] calculations in both cutoff and dimensional regularization shemes shown no gravitational contribution. The application of Loop Regularization method, which can appropriately treat the quadratic divergences and preserve non-abelian gauge symmetry, has confirmed the conclusions of [5] and found a non-vanishing contribution $a = -\frac{1}{\pi}$ [9]. The cited works all worked in the one-loop approximation. Conflicting results leave open the question of existence and value of factor a from (2) in the EFT. The conceptual problem of EFT in this setting is the use of perturbative methods for scales approaching Planck energy. Also, the use of some renormalization shemes for extracting beta functions is problematic, as e.g. dimensional regularization is insensitive to quadratic divergences that are claimed to give contribution to β_g [8]. Adressing the problem properly would require the use of non-perturbative methods and mathematical tools that are better behaved. A framework which we will now discuss and conduct our own calculations in, is the quantum einstein gravity treated with functional renormalization group, aka the asymptotic safety programme. This theory is potentially a fundamental theory, valid at all energy scales [11] and is governed by a well-defined diferential equation governing the RG flow.

2 Asymptotic safety ...

2.1 The Functional Renormalization Group

The central object of the functional renormalization group is the effective average action. To introduce it, let us start from the scalar field theory. The definition for other theories come as a straight-forward generalization. From now on, we will assume a Wick rotation and all scalar products should be understood as euclidean. The partition function and the generating functional of connected Green's functions reads

$$Z[j] = \int \mathcal{D}\phi \; e^{-S[\phi]+\int dx j\phi} \tag{3}$$

$$W[j] = \log Z[j] \tag{4}$$

The effective action functional is then defined using the Legendre transform of $W[j]$.

$$\Gamma[\phi_c] = W[j_\phi] - \int d^4x j_\phi(x)\phi(x) \tag{5}$$

The two fields ϕ_c and j_ϕ are inverses of each other, defined as the solution to

$$\phi_c(x) = \langle\hat{\phi}(x)\rangle_j = \frac{\delta W[j]}{\delta j(x)} \tag{6}$$

The argument of the effective action is a classical field and there is no functional integral to be performed over it. Rather, in Γ all of the fluctuations are integrated out, but only one-particle irreducible diagrams are included. Γ acts as a generating functional of 1PI Green's functions. Moreover, extremizing effective, rather than the clasical action, yields the equations of motion for vacuum expectation values of the quantum fields.

In its bare form, effective action is ill-defined. One option is to introduce a UV cutoff Λ and study the RG flow through divergences proportional to Λ. The modification we will employ, however, involves an IR cutoff inserted through adding a regulator term $\Delta S_k[\phi]$ to the bare action $S[\phi]$ in the definition of partition function and subtracted from the final form of effective action. Explicitly, this new object called the effective average action (EAA) is defined as

$$\Gamma_k[\phi_c] = W_k[j_\phi] - \int d^4x j_\phi(x)\phi(x) - \Delta S_k[\phi] \tag{7}$$

$$W_k[j] = \log \int \mathcal{D}\phi \; e^{-S[\phi]-\Delta S_k[\phi]+\int dx j\phi} \qquad \Delta S_k[\phi] = \frac{1}{2}\int d^4x \phi \hat{R}_k(-i\partial_\mu\partial^\mu)\phi \tag{8}$$

The operator $\hat{R}_k(-i\partial_\mu\partial^\mu)$ is what introduces a momentum scale dependence to the action. Its purpose is to act as an IR regulator and extinguish the propagation of modes of the field ϕ with laplacian operator eigenvalues below a certain scale k.

The Γ_k is IR-regulated, but the UV divergences still cause it to be ill-defined. However, our central object in the functional renormalization group will not be the full EAA, but its derivative with respect to $t = \log k$. We assume the theory space in which Γ_k takes the form

$$\Gamma_k = \sum_i g_i(k)\ \mathcal{O}_i[\phi] \tag{9}$$

Where $\mathcal{O}_i(\phi)$ are integrals of monomials of fields or positive powers of field derivatives and $g_i(k)$ are scale-dependent couplings. The coefficients in the derivative $\partial_t \Gamma_k$ are therefore simply the beta functions of corresponding operators

$$\frac{d\Gamma_k}{dt} = \sum_i \frac{dg_i}{dt}\ \mathcal{O}_i[\phi] = \beta_i(g,k)\ \mathcal{O}_i[\phi] \tag{10}$$

The beta functions may depend on all the couplings, as well as the renormalization scale k and they can later be extracted from $\frac{d\Gamma_k}{dt}$ via a suitable projection operator. The $\frac{d\Gamma_k}{dt}$ is therefore called the beta functional and it is finite and well-defined. This is because it can be viewed as a difference between effective actions with infinitesimally different cutoffs, where the UV divergences in the difference cancel and what remains is the finite rest dependent on the degrees of freedom with momenta close to the scale k. At the one loop level, the derivative of effective average action can be expressed as

$$\frac{d\Gamma_k^{(1)}}{dt} = \frac{1}{2}\,\mathrm{STr}\left[\left(\frac{\delta^2 S}{\delta\phi\delta\phi} + R_k\right)^{-1} \cdot \frac{dR_k}{dt}\right] \tag{11}$$

Where the second functional derivative $F^(2)[\phi]$ of some functional F and the supertrace operator STr are defined as

Which follows from the expression for the one-loop effective action [ref]. One may guess, that the "renormalization group improvement" of this equation - the replacement of ordinary action by EAA, will lead to a more accurate description of physics:

$$\frac{d\Gamma_k}{dt} = \frac{1}{2}\,\mathrm{STr}\left[\left(\frac{\delta^2 \Gamma_k}{\delta\phi\delta\phi} + R_k\right)^{-1} \cdot \frac{dR_k}{dt}\right] \tag{12}$$

This turns out to be an exactly correct equation, that does not rely on any perturbative expansion [3]. It is called the functional renormalization group equation (FRGE) or the Wetterich equation and it's a simple, first order differential equation that governs the renormalization group flow of Γ_k functional.

In its original form, FRGE is not well suited for performing specific calculations. One very intuitive method, which we will use, is called the $\mathcal{PF}$-expansion. The term inside the trace including Second derivative of EAA will in general be, for spinor or tensor fields, a functional hessian matrix. We can decompose this term into a regulated inverse propagator matrix $\mathcal{P}$ and a rest, which will include the derivatives of terms non quadratic in fields.

$$\frac{\delta^2 \Gamma_k}{\delta\phi\delta\phi} + R_k = \mathcal{P} + \mathcal{F} \tag{13}$$

First, let us notice that the entire expression inside trace can be expressed as a $\log\left(\mathcal{P}+\mathcal{F}\right)$, upon which acts a t-derivative sensitive only on the t dependence in R_k. Explicitly, we can write:

$$\left(\mathcal{P}+\mathcal{F}\right)^{-1}\cdot\partial_t R_k = \left(\mathcal{P}+\mathcal{F}\right)^{-1}\cdot\widetilde{\partial}_t\left(\mathcal{P}+\mathcal{F}\right) = \widetilde{\partial}_t\log\left(\mathcal{P}+\mathcal{F}\right); \quad \widetilde{\partial}_t = \int \partial_t R_k \frac{\delta}{\delta R_k} \tag{14}$$

Now, we can recall the series expansion of $\log\left(1+x\right)$ around $x=0$ and after some simple manipulations, obtain an expansion of functional trace in (12) in the number of $\mathcal{F}$-terms

$$\frac{d\Gamma_k}{dt} = \frac{1}{2}\,\mathrm{STr}\left[\widetilde{\partial}_t\log\left(\mathcal{P}+\mathcal{F}\right)\right] = \frac{1}{2}\,\mathrm{STr}\left[\widetilde{\partial}_t\left(\log\mathcal{P}+\log\left(1+\mathcal{P}^{-1}\mathcal{F}\right)\right)\right] \tag{15}$$

$$= \frac{1}{2}\,\mathrm{STr}\left[\widetilde{\partial}_t\log\mathcal{P}\right] + \frac{1}{2}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}\,\mathrm{STr}\left[\widetilde{\partial}_t\left(\mathcal{P}^{-1}\mathcal{F}\right)^n\right] \tag{16}$$

Using this expression, we can reduce the problem of finding $\partial_t\Gamma_k$ to computing a set of feynman diagrams.

2.2 Background field method and gauge invariance

The gauge theories require a more careful definition of functional integral. A method, which is very useful for introducing gauge fixing and defining covariant derivative nad almost indispensable in the case of gravity as a quantum field theory is the background field method. For the Yang-Mills and the metric field we split the full field into a classical constant background and a quantum fluctuation field

$$g^{\mu\nu} = \bar{g}^{\mu\nu} + h^{\mu\nu} \tag{17}$$

$$A^{\mu} = \bar{A}^{\mu} + a^{\mu} \tag{18}$$

It is possible to work in the curved background $\bar{g}^{\mu\nu}$, and all the covariant derivatives and integration measures could be then defined based on this background metric. The most convenient choice for a background metric, that we adopt in this work, is the metric of a flat space. In the euclidean space, this will simply be a unit matrix.

$$\bar{g}^{\mu\nu} = \delta^{\mu\nu} \tag{19}$$

...

(discussion of results in asymptotic safety, results obtained in this work and the general conclusion) A discussion of the gravitational solution to the abelian gauge triviality problem, prompted by the work of Wilczek and Robinson, has also been analysed in the framework of asymptotic safety. Numerous works, employing different types of truncations, cutoff profiles and the treatment of symmetries, agree on the fact of existence and negativity of gravitational contribution in this setting. The ambiguities caused by the use of background field method and treatment of diffeomorphism symmetry in the perturbative approach, make it important to carefully construct gauge fixing and ghost sectors that also enter EAA in the non-perturbative approach. In one of the first works on this topic employing the functional renormalization group, Daum, Harst and Reuter [12] analyzed the problem of background field dependence of action and presented a construction of Einstein-Hilbert-Yang-Mills action with gauge fixing and ghost sectors that break symmetry of gauge transformations of full physical fields, but are invariant under the gauge transformations of background fields. In the truncation involving Einstein-Hilbert action and the Yang-Mills kinetic term, a universal gravitational contribution to the beta function of $SU(N)$ gauge theory coupling was found. In the one loop FRGE approximation, for the arbitrary form of cutoff profile $r(y)$ the result found was

$$\beta_g = -\frac{3}{\pi} G k^2 \; g \; \Phi[r(y)] + \beta_g^0(N) \qquad \Phi[r(y)] = \int_0^\infty dy \frac{r(y) - y r'(y)}{y + r(y)} \tag{20}$$

Where $\beta_g^0(N)$ is generated by gauge bosons interactions and independent of gravitational coupling. For the abelian theory, $\beta_{g_Y}^0 = 0$. The result is very similar to the one originally found in [5]. For the simplest regulator profile $r^{\text{litim}}(y)$, as defined in (**??**), results from the two works exactly coincide. Actually, this general conclusion about existence, negativity and the general form of gravitational contribution to gauge coupling beta function in FRG has been confirmed by almost all subsequent works on that topic. In the works by Christiansen, Eichhorn [13] and Eichhorn, Kwapisz, Schiffer [14] an extended truncations involving the higher order effective gauge interaction was investigated, with the following effective gauge sector:

$$\Gamma_k^{U(1)} = \frac{1}{4} \int d^4x \sqrt{g} F^{\mu\nu} F_{\mu\nu} + \frac{w_2}{8k^4} \int d^4x \sqrt{g} \left(F^{\mu\nu} F_{\mu\nu}\right)^2 + \frac{v_2}{8k^4} \int d^4x \sqrt{g} \left(\frac{1}{2} \epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma} F^{\mu\nu}\right)^2 \tag{21}$$

With the totally antisymmetric tensor $\epsilon_{\mu\nu\rho\sigma}$. Both terms was investigated in [14], while in [13] $v_2 = 0$. An interesting result was found there, that an effective four-photon interactions does not affect the asymptotic freedom of effective gauge coupling in the UV, however in itself they possess a non-Gaussian UV fixed point. This implies that the asymptotic safety, instead of asymptotic freedom of the abelian gauge sector is a solution to the triviality problem. Also, this result shows the necessity of going beyond canonical power counting to fully understand the effects of quantum gravity on abelian gauge interaction. In the perturbative approximation, where the anomalous dimension coming from the regulator insertion is neglected, the abelian gauge beta function found in [14] was

$$\beta_{g_Y} = -G k^2 \frac{1}{4\pi} g_Y - w_2 \frac{1}{6\pi^2} - v_2 \frac{1}{24\pi^2} \tag{22}$$

The corresponding result in [13] is equivalent, when $v_2 \to 0$. Cutoff profile $r^{\text{litim}}(y)$ was used in both of these calculations.

3 Obtained results ...

3.1 FRGE for the Einstein-Hilbert-Maxwell theory

A great advantage of functional renormalization group is that, if all terms allowed by the symmetries are included in the effective action, one obtains a set of first order differential equations containing full information about the RG flow in the theory, without referring to the perturbative methods. For typical theories, that means an infinite set of coupled differential equations. What allows calculations to be feasible, is considering only a manageable subset of terms in the effective action, the so-called truncation method. For a good choice of truncation, adding subsequent terms beyond this subset will contribute to beta functions in a negligible way and the results will be a good approximation of the actual behaviour of the theory.

In the present calculations, the following truncation of effective action is assumed:

$$\Gamma = \int d^4x \left(\mathcal{L}_{EH} + \mathcal{L}_A + \mathcal{L}_{GGF} + \mathcal{L}_{AGF}\right) \tag{23}$$

The gauge sector contains a kinetic term of the photon and a gravitational gauge fixing term

$$\mathcal{L}_A = \sqrt{\mathbf{g}}\left(Z_A \frac{1}{4} F^{\mu\nu} F_{\mu\nu}\right) \tag{24}$$

$$\mathcal{L}_{AGF} = \sqrt{\mathbf{g}}\, Z_A \frac{1}{2\alpha_A}\left(\partial_\mu A^\mu \, \partial_\nu A^\nu\right) \tag{25}$$

$$\tag{26}$$

Whereas the gravitational sector consists of the Einstein - Hilbert action with a cosmological constant Λ and a gravitational gauge fixing term

$$\mathcal{L}_{EH} = \sqrt{\mathbf{g}}\, \frac{k^2}{\kappa}\left(k^2\Lambda - R(\partial)\right) \tag{27}$$

$$\mathcal{L}_{GGF} = \sqrt{\mathbf{g}}\, Z_h \frac{1}{32\pi\alpha_h}\left(\partial_\mu h^{\mu\nu} - \frac{1+\beta_h}{4}\partial^\nu h^\rho{}_\rho\right)^2 \tag{28}$$

$$\tag{29}$$

An introduction of term fixing the gravitational diffeomorphism symmetry requires an introduction of graviton ghost fields. These would couple only to the graviton field and influence its anomalous dimension. In the present calculation, we are not analysing the rg flow of the entire system, instead focusing on the gauge field anomalous dimension and treating the values of gravitational couplings as free parameters. The diffeomorphism ghosts are therefore neglected. The analysis of the rg flow of pure gravity is one of the main concerns of the asymptotic safety programme and has been investigated in many ways [refs]. In particular, the contribution of ghosts was analysed in [refs].

The effective action can be represented as a functional dependending only on the fourier transform of gauge and graviton fields.

$$\Gamma\left[A^\mu, h^{\mu\nu}, \partial_\mu A^\mu, \partial_\mu \partial_\nu h^{\mu\nu}, \cdots\right] = \Gamma[\tilde{A}^\mu, \tilde{h}^{\mu\nu}] \tag{30}$$

$$\tilde{\phi}(p) = \mathfrak{F}[\phi] = \int d^4x e^{-ip_\mu x^\mu}\phi(x) \tag{31}$$

Due to properties of fourier transform, when we express the fields in effective lagrangian as $\phi(x) = \mathfrak{F}^{-1}[\tilde{\phi}(p)]$, any spacetime derivative $\partial_\mu \phi^\mu(x)$ will be replaced by $ip_\mu \tilde{\phi}^\mu(p)$. By further rearrangements and performing spacetime integral, we can arrive at the momentum-space lagrangian, depending only on $\tilde{\phi}$

$$\int d^4x \prod_i \partial_{\mu_i}\phi^{\mu_i}(x) \prod_j \phi^{\nu_j} = \tag{32}$$

$$= \int d^4x \int \left(\prod_i d^4p_i i p_{i_{\mu_i}} \tilde{\phi}^{\mu_i}(p_i)\right)\left(\prod_j d^4p_j \tilde{\phi}^{\nu_j}(p_j)\right) e^{ix\left(\sum_i p_i + \sum_j p_j\right)} \tag{33}$$

$$= \int \left(\prod_i d^4p_i i p_{i_{\mu_i}} \tilde{\phi}^{\mu_i}(p_i)\right)\left(\prod_j d^4p_j \tilde{\phi}^{\nu_j}(p_j)\right) \delta\left(\sum_i p_i + \sum_j p_j\right) \tag{34}$$

Using the generalized chain rule property, one can check that a functional derivative of action with respect to any field is equal to functional derivative with respect to the fourier transformed field.

$$\frac{\delta\Gamma[\phi]}{\delta\phi(x)} = \frac{\delta\Gamma[\tilde{\phi}]}{\delta\phi(x)} = \frac{\delta\Gamma[\tilde{\phi}]}{\delta\tilde{\phi}} \cdot \int d^4p \frac{\mathfrak{F}[\phi](p)}{\delta\phi(x)} = \frac{\delta\Gamma[\tilde{\phi}]}{\delta\tilde{\phi}} \cdot \int d^4p\, \delta(p) \tag{35}$$

$$= \frac{\delta\Gamma[\tilde{\phi}]}{\delta\tilde{\phi}} \tag{36}$$

To extract scale dependence of the couplings, we employ the functional renormalization group equation. The scheme used for the calculation of beta functions amounts to the following steps: finding expressions for effective vertices and regulated propagators, calculating relevant feynman diagrams and projecting the result onto the given coupling, thus obtaining the beta function. Thanks to the decomposition of beta functional in equation (9), any of the beta functions can be extracted from the FRGE using a set of projection operators satisfying the following property:

$$\mathit{\Pi}_i\, \mathcal{O}_j = \delta_{ij} \tag{37}$$

Operator from such set, applied to the right hand side of FRGE, will yield the desired beta function β_i. As a projection operator onto the gauge field kinetic term, we will use

$$\mathit{\Pi}_A = \lim_{p^2\to 0} \frac{1}{3p^2}\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) \frac{\delta}{\delta A_\mu(-p)} \frac{\delta}{\delta A_\nu(p)}\bigg|_{A=0,\ h=0} \tag{38}$$

Any operator containing both the graviton and photon fields or purely graviton field, will be annihilated by the propagator by performing the functional derivative at point $h = 0$. For the remaining operators that enter Γ_k, the property (37) can be checked by a direct computation.

3.1.1 Effective vertices and propagators

The matrices $\mathcal{P}$ and $\mathcal{F}$ from the expansion in equation (13) will take form:

$$\mathcal{P} = \left.\begin{pmatrix} \frac{\delta^2\Gamma_k}{\delta A_\mu \delta A_\nu} & 0 \\ 0 & \frac{\delta^2\Gamma_k}{\delta h_{\rho\sigma}\delta h_{\tau\kappa}} \end{pmatrix}\right|_{A,h=0} \tag{39}$$

$$\mathcal{F} = \begin{pmatrix} \frac{\delta^2\Gamma_k}{\delta A_\mu \delta A_\nu} & \frac{\delta^2\Gamma_k}{\delta A_\mu \delta h_{\rho\sigma}} \\ \frac{\delta^2\Gamma_k}{\delta h_{\rho\sigma}\delta A_\mu} & \frac{\delta^2\Gamma_k}{\delta h_{\rho\sigma}\delta h_{\tau\kappa}} \end{pmatrix} - \mathcal{P} \tag{40}$$

Diagonal elements of $\mathcal{P}$ are the inverses of the regulated photon and graviton propagators. The gauge sector of effective action is bilinear in gauge fields and the nonlinear term $\sqrt{\mathrm{g}}$ is a constant with respect to functional derivative. After the straightforward use of derivative and setting remaining fields equal to zero, we obtain

$$\left(\mathcal{P}^{11}\right)^{\mu\nu} = \left(Z_A + \mathrm{RegA}_k(p^2)\right)\left(g^{\mu\nu}p^2 - \left(1 - \frac{1}{\alpha_A}\right)p^\mu p^\nu\right) \tag{41}$$

The gravitational sector contains nonlinear functions $\sqrt{\mathrm{g}}$ and $R(\partial)$. These can be expanded around the flat minkowskian background as a power series in the metric perturbation field:

$$\sqrt{\mathrm{g}} = 1 + \frac{1}{2}h + \left(\frac{1}{8}h^2 - \frac{1}{4}h_{\mu\nu}h^{\mu\nu}\right) + \mathcal{O}(h^3) \tag{42}$$

$$R(\partial) = \partial_\mu\partial_\nu h^{\mu\nu} - \partial_\mu\partial^\mu h + \Big(h^{\mu\nu}\left(\partial_\mu\partial_\nu h + \partial_\mu\partial^\mu h_{\mu\nu} - 2\partial_\nu\partial_\rho h_\mu^\rho\right) \tag{43}$$

$$+\, \partial^\mu h \partial_\rho h_\nu^\rho + \frac{3}{4}\partial_\rho h_{\mu\nu}\partial^\rho h^{\mu\nu} - \partial_\mu h^{\mu\nu}\partial_\rho h_\nu^\rho - \frac{1}{2}\partial^\rho h^{\mu\nu}\partial_\nu h_{\mu\rho} - \frac{1}{4}\partial_\mu h \partial^\mu h\Big) + \mathcal{O}(h^3) \tag{44}$$

In the calculation of $\mathcal{P}$ matrix, terms that are not quadratic in fields will not contribute, after we set any fields remaining after derivation to zero. These higher terms are responsible for the nonlinear interaction of graviton. The perturbed action, from which we extract expression for the graviton propagator is given in (Apx). The gauge fixing term is not expanded in h field (wyjaśnienie dlaczego) Functional derivative performed on the action (Apx) yields the expression for $\mathcal{P}^{22}$, given in full form in (Apx).

Propagators are the tensorial inverses of the expressions on the diagonal of $\mathcal{P}$ matrix, which for the second- and fourth-order tensors are given by the condition

$$\left(\mathcal{P}^{11}\right)^\mu_{\ \rho}\ \mathrm{PropA}^{\rho\nu} = g_{\mu\nu} \tag{45}$$

$$\left(\mathcal{P}^{22}\right)^{\mu\nu}_{\ \ \rho\sigma}\ \mathrm{PropG}^{\rho\sigma\tau\kappa} = \frac{1}{2}\left(g^{\mu\nu}g^{\tau\kappa} + g^{\mu\kappa}g^{\nu\tau}\right) \tag{46}$$

The task of finding tensorial inverse can be greatly simplified by noticing, that only certain combinations of tensor products of four-momentum and metric, allowed by the symmetry and Lorentz invariance, can enter the propagator. This lets us write the ansatz

$$\mathrm{PropA}^{\mu\nu} = b_1 p^\mu p^\nu + b^2 g^{\mu\nu} \tag{47}$$

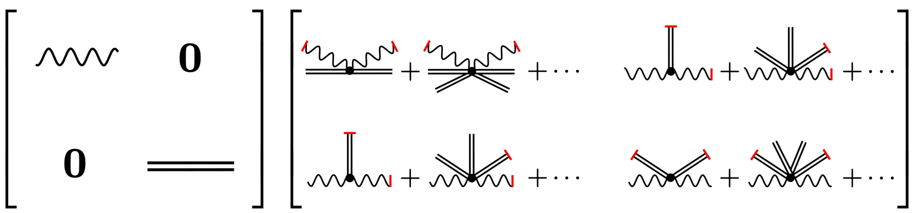
$$\mathrm{PropG}^{\mu\nu\rho\sigma} = c_1 \cdot p^\mu p^\nu p^\rho p^\sigma + c_2 \cdot (g^{\rho\sigma} p^\mu p^\nu + g^{\mu,\nu} p^\rho p^\sigma) + c_3 \cdot (g^{\mu\sigma} g^{\nu\rho} + g^{\mu\rho} g^{\nu\sigma}) + \tag{48}$$

$$c_4 \cdot (g^{\nu\sigma} p^\mu p^\rho + g^{\mu\sigma} p^\nu p^\rho + g^{\nu\rho} p^\mu p^\sigma + g^{\mu\rho} p^\nu p^\sigma) + c_5 \cdot g^{\mu\nu} g^{\rho\sigma} \tag{49}$$

Which reduces (45) and (46) to a set of linear equations for $\{b_i\}$ and $\{c_i\}$. After solving it, a final form of the regulated photon propagator agrees with the well known result, with the additional insertion of the regulator in field renormalization. Expression for the regulated graviton propagator is given in (Apx).

Interaction part of effective action Γ, due to nonlinear functions of metric perturbation field, will be the infinite series of interactions with two photon fields and any number of graviton fields. Yet, our goal is not to compute the entire functional. Let us note, that the fact which functional derivatives δh and δA has acted on the interaction term, determines what elements of $\mathcal{P}^{-1}$ later multiply it. In the diagramatical representation, this is equivalent to the fact that in any effective vertex two of the external legs are opened to form a loop, while the rest is left uncontracted. This can be easily understood by representing the product $\mathcal{P}^{-1}\mathcal{F}$ diagramatically

$\mathcal{P}^{-1}\mathcal{F} =$



Where a red line at the end of a vertex leg denotes the functional derivative with respect to the corresponding field. To extract the gauge field anomalous dimension from the FRGE, we act on it with a projection operator Π_A, which then annihilate all terms not originating from diagrams with two external photon legs and leave only the contribution to photon two-point function. As can be seen, the only vertices that may form such diagrams come from terms of the form $\delta_A \delta_h \int c \cdot AAh$ and $\delta_h \delta_h \int c \cdot AAhh$

Figure 2: Necessary vertices for computing gauge field anomalous dimension. Red line at the end of a vertex leg denotes a functional derivative with respect to the corresponding field applied to the interaction term

We denote the expressions for these vertices, respectively

$$\left.\frac{\delta^2\Gamma}{\delta h^{\mu\nu}(p_1)\delta A^{\rho}(p_2)}\right|_{h=0} = \frac{\delta^2}{\delta\tilde{h}^{\mu\nu}(p_1)\delta\tilde{A}^{\rho}(p_2)}\int\prod_{i=1}^{3} d^4p_i\mathcal{L}_{\text{AAh}}\left(\tilde{h}(p_1),\tilde{A}(p_2),\tilde{A}(p_3)\right) \tag{50}$$

$$= \text{VertAAh}_{\mu\nu\rho}(p_1,p_2,p_3) \tag{51}$$

$$\left.\frac{\delta^2\Gamma}{\delta h^{\mu\nu}(p_1)\delta h^{\rho\sigma}(p_2)}\right|_{h=0} = \frac{\delta^2}{\delta\tilde{h}^{\mu\nu}(p_1)\delta\tilde{h}^{\rho\sigma}(p_2)}\int\prod_{i=1}^{4} d^4p_i\mathcal{L}_{\text{AAhh}}\left(\tilde{h}(p_1),\tilde{h}(p_2),\tilde{A}(p_3),\tilde{A}(p_4)\right) \tag{52}$$

$$= \text{VertAAhh}_{\mu\nu\rho\sigma}(p_1,p_2,p_3,p_4) \tag{53}$$

Where $\mathcal{L}_{\text{AAh}}$, $\mathcal{L}_{\text{AAhh}}$ is the sum of terms from lagrangian proportional to the suitable powers of fields. Full expressions for (..) are given in the appendix.

3.1.2 Calculation of feynman diagrams

(...) Then, continuing the argument from previous section, the contribution from second term that will not be annihilated by Π_A comes from products of effective vertices and propagators that involve only the second power of gauge field. There are just two such products. After considering coefficients in the expansion of logarithm from (16) and from performing trace, the expression for gauge field renormalization beta function can be written as

$$\beta_{Z_A} = \Pi_A\cdot\partial_t\Gamma_k = \Pi_A\left(\frac{1}{2}\tilde{\partial}_t\int d^4q\ \text{VertAAhh}_{\mu\nu\rho\sigma}(p,-p,q,-q)\,\text{PropG}^{\mu\nu\rho\sigma}(q)\right. \tag{54}$$

$$-\frac{1}{2}\tilde{\partial}_t\int d^4q\ \text{VertAAh}^{\mu\nu\rho}(p,-q,p+q)\,\text{VertAAh}^{\sigma\tau\kappa}(-p,q,-p-q)\cdot \tag{55}$$

$$\left.\cdot\,\text{PropG}_{\mu\nu\sigma\tau}(p+q)\,\text{PropA}_{\rho\kappa}(q)\right) \tag{56}$$

This reduces the problem of extracting information about RG flow in functional renormalization group to computing amputated one-loop feynman diagrams.

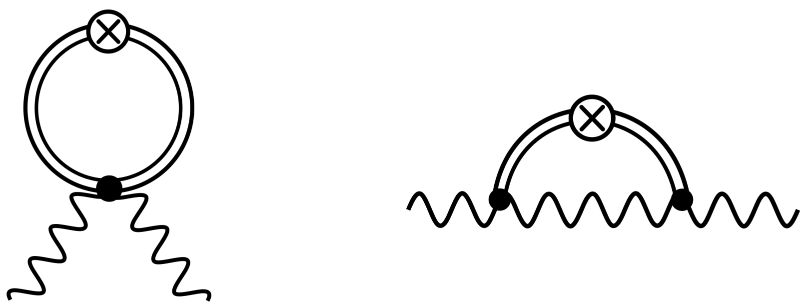


Figure 3: Feynman diagrams contributing to the anomalous dimension of gauge field. Wavy line denotes a photon propagator and the double straight line denotes the graviton propagator. A crossed circle denotes an insertion of the regulator.

(...)

Finally, the resulting beta function of abelian gauge coupling g in the spacetime with four dimensions is

$$\beta_{g_Y} = g_Y \frac{\eta_A}{2} = g_Y \frac{G}{4\pi} \left(\frac{6-\eta_h}{6(1-2\Lambda)^2} + \frac{(\eta_A - 8)(1-2\Lambda) + (\eta_h - 8)}{8(1-2\Lambda)^2} \right) + \beta_{g_Y}^{\mathrm{SM}} \tag{57}$$

The result for an arbitrary spacetime dimension is given in (apx). To study the qualitative aspects of gravitational corrections, we can neglect the cosmological constant and the corrections to gauge and graviton propagators entering via η_A and η_h. This approximations yields a very simple form of beta function, that indicates a negative gravitational contribution to the running of gauge coupling.

$$\beta_{g_Y} = -\frac{G}{4\pi} + \beta_{g_Y}^{\mathrm{SM}} \tag{58}$$

(...)

Beta functions extracted from the full EA cannot depend on the choice of gauge parameters. Introducing truncations, however, means that gauge dependence may not cancel entirely. Another in principle redundant choice is a form of the regulator. For two different regulators, Γ_k will include field modes weighted in a different manner, so this choice simply sets the definition of the object Γ_k. This means, that beta functions computed with different regulators will differ, but if the regulator is picked in such way that it executes a proper IR cutoff, it will always give the same results in the physical limit $k \to 0$. There should be no qualitative differences in the behaviour of RG flow computed with different regulators.

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Background field i techniczne sprawy w rozdziale 2
Summary dyskusji wyników z asymptotic safety, powiedzieć o predictivity np weak gravity bound i higgs mass
Ogarnąć przestrzeń pędową, całki i pochodne funkcyjne jak to działa od A do Z
Jak wchodzi gauge fixing i dlaczego pomijamy duchy
Sekcja 3.1.2
Prezentacja i dyskusja wyników
Systematycznie wypisać wszystkie wykorzystywane przybliżenia i przedyskutować ich wpływ, ważność i możliwą weryfikację czy działają
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