An asymptotically safe solution to the U(1) triviality problem

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A B S T R A C T

We explore whether quantum gravity effects within the asymptotic safety paradigm can provide a predictive ultraviolet completion for Abelian gauge theories. We evaluate the effect of quantum gravity fluctuations on the running couplings in the gauge sector and discover an asymptotically safe fixed point of the Renormalization Group. In particular, if the strength of gravitational interactions remains below a critical strength, the minimal gauge coupling becomes asymptotically free. Further, we point out that a completely asymptotically free dynamics for the gauge field is impossible to achieve, as asymptotically safe quantum gravity necessarily induces non-vanishing higher-order interactions for the gauge field in the ultraviolet.

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1. The asymptotic safety paradigm for gauge-gravity systems

Here, we explore the scenario that the coupling of a U(1) gauge theory to asymptotically safe quantum gravity [1] induces a predictive ultraviolet completion for the gauge theory. In particular, this could provide a solution to the triviality problem in the Abelian sector of the Standard Model [2–5], and would thus constitute an intriguing alternative to models of grand unification. Specifically, we find evidence suggesting that under the impact of quantum gravitational fluctuations, Abelian gauge theories develop an ultraviolet (UV) fixed point, as first discussed in [6]. In principle, one scenario for such a fixed point could be that the gauge theory becomes completely asymptotically free, and all gauge interactions vanish in the ultraviolet. Here, we will show that the UV completion for gauge theories induced by asymptotically safe quantum gravity is a little more intricate. In particular, higher-order interactions play an important role: Higher-order field-strength invariants such as $(F_{\mu\nu} F^{\mu\nu})^2$ are necessarily present in the ultraviolet, i.e., quantum gravity induces interactions for Abelian gauge fields even in the absence of charged fields. This has a crucial impact on the Renormalization Group (RG) flow: While the beta-function for a pure U(1) gauge theory without charged matter is of course zero in the absence of gravity, the presence of gravity-induced higher-order operators means that the gauge couplings run as a function of the momentum scale. Moreover, an interacting RG fixed point

for gravity necessarily requires the coupling of the higher-order operator $(F_{\mu\nu} F^{\mu\nu})^2$ to be non-zero. These couplings contribute to rendering the U(1) gauge coupling e either asymptotically free or safe in the ultraviolet. In both cases, complete asymptotic freedom for the gauge sector is impossible to achieve, as even in the case of an asymptotically free gauge coupling e, the couplings of higher-order gauge interactions must remain non-zero, and thus must become asymptotically safe.

As has been explored for the case of scalars and fermions in the context of asymptotic safety before, quantum gravity necessarily induces new interactions [7–13]. These interactions are those that are compatible with the symmetries of the kinetic terms of the respective fields, see [12,13]. For the case of gauge fields, the whole tower of gauge-invariant interactions can be induced by quantum gravity. For the minimal gauge coupling e with vanishing canonical dimension, the Ward–Takahashi identities imply a relation between the β-function of the coupling and the anomalous dimension η_A of the gauge field

$$\beta_{e^2} = e^2 \, \eta_A. \tag{1}$$

Quantum-gravity corrections enter η_A and imply that this coupling features a fixed point at zero in the presence of gravity. In the presence of charged matter, the free fixed point is ultraviolet repulsive, leading to the triviality problem. Quantum gravity can alter that scaling behavior, and thus provide a solution.

In this paper, we have three main goals

- We will show that higher-order gauge interactions are non-vanishing in a quantum-gravity induced ultraviolet completion for an Abelian gauge theory.

- We will point out that the requirement that the fixed point lies at real values of the couplings imposes bounds on the viable gravitational parameter space.
- We will study the impact of induced interactions on the canonical gauge coupling to determine whether it is rendered relevant or irrelevant in the ultraviolet. In the latter case, the Standard Model value for the U(1) hypercharge coupling at the Planck scale would be difficult to reconcile with an asymptotically safe UV completion.

2. Functional Renormalization group for gauge-gravity systems

We will employ the functional Renormalization Group which allows us to probe the scale dependence of a quantum field theory of gravity and matter. Specifically, the Wetterich equation encodes the response of the theory to the change of a momentum scale k: As k is changed, quantum fluctuations with momenta $p^2 \approx k^2$ yield the main contribution to the change of the effective dynamics, which is encoded in scale dependent running couplings. Technically, this coarse-graining procedure is implemented with the help of a cutoff function $R_k(p^2)$. Specifically, we will choose a Litim cutoff [14]

$$R_k = Z_k \left(k^2 - p^2 \right) \theta(k^2 - p^2). \tag{2}$$

The full, regularized and scale-dependent propagator of quantum fluctuations is given by $\left(\Gamma_k^{(2)} + R_k \right)^{-1}$, where $\Gamma_k^{(2)}$ denotes the second functional derivative of the scale-dependent effective action with respect to the fields and R_k is the shape function for the infrared cutoff term. With these ingredients, the Wetterich equation [15], see also [16,17] is given by

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{Tr} \left(\Gamma_k^{(2)} + R_k \right)^{-1} \partial_t R_k, \tag{3}$$

where $\partial_t = k \, \partial_k$. For reviews see [18–24]. For the gravity fluctuations, we will employ a linear split

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}, \tag{4}$$

which allows us to write a background-field gauge fixing as well as a regulator term that depend on the background metric $\bar{g}_{\mu\nu}$. We thereby arrive at a regularized propagator for the fluctuation field $h_{\mu\nu}$. The beta functions for the gauge field are independent of the choice of background metric $\bar{g}_{\mu\nu}$, and we employ the minimal choice $\bar{g}_{\mu\nu} = \delta_{\mu\nu}$. Following Reuter's groundbreaking work [25], substantial evidence for the viability of the asymptotic safety paradigm in quantum gravity has been collected in [26–63], also in the case with matter [6–13,64–71], see, e.g., [72] for reviews. Consequences in astrophysics and cosmology have been explored, e.g., in [73].

As quantum fluctuations generate all field monomials compatible with the symmetries, the Wetterich equation provides a flow in the infinite dimensional space of all couplings. For practical calculations, a truncation of the effective dynamics is necessary. Here, we follow the assumption that canonical scaling provides a good guideline for the setup of truncations in asymptotically safe gravity with matter. This assumption is justified if the interacting fixed point underlying asymptotic safety is not strongly non-perturbative, as then canonical scaling determines the main contribution to the critical exponents. Within the asymptotic safety paradigm for gravity as well as matter, the assumption appears to be justified [12,45,48,74–76].

We will focus on a minimal truncation in the gauge sector that exhibits how asymptotically safe quantum gravity induces an interacting fixed point in the gauge sector. We discuss how this might provide a mechanism to solve the triviality problem. To that end, it is sufficient to include the first higher-order operator in the gauge sector. A more extended truncation will be studied elsewhere. Accordingly, our truncation is

$$\begin{aligned} \Gamma_k = \frac{Z_A}{4} &\int d^4x \sqrt{g} \, g^{\mu\nu} g^{\kappa\lambda} F_{\mu\kappa} F_{\nu\lambda} \\ &+ \frac{\bar{w}_2}{8} \int d^4x \sqrt{g} \left(g^{\mu\nu} g^{\kappa\lambda} F_{\mu\kappa} F_{\nu\lambda} \right)^2 \\ &+ \Gamma_{k\,\mathrm{EH}} + S_{\mathrm{gf},\,h} + S_{\mathrm{gf},\,A}. \end{aligned} \tag{5}$$

Note that all couplings are k-dependent, which we do not indicate explicitly for notational simplicity. In the above, the Einstein–Hilbert truncation for the metric fluctuations is obtained from

$$\Gamma_{k\,\mathrm{EH}} = \frac{-1}{16\pi \bar{G}_N} \int d^4x \sqrt{g} \left(R - 2\bar{\Lambda} \right), \tag{6}$$

by expanding to second order in $h_{\mu\nu}$ and then redefining the field according to

$$h_{\mu\nu} \to \sqrt{Z_h \bar{G}_N} h_{\mu\nu}. \tag{7}$$

The two-gauge fixing terms are given by

$$S_{\mathrm{gf},\,h} = \frac{Z_h}{\alpha_h 32\pi} \int d^4x \sqrt{\bar{g}} \, \bar{g}^{\mu\nu} \mathcal{F}_\mu \mathcal{F}_\nu, \tag{8}$$

$$\mathcal{F}_\mu = \bar{D}^\nu h_{\mu\nu} - \frac{1+\beta}{4} \bar{D}_\mu h, \tag{9}$$

$$S_{\mathrm{gf},\,A} = \frac{Z_A}{2\alpha_A} \int d^4x \sqrt{\bar{g}} \left(\bar{D}^\mu A_\mu \right)^2. \tag{10}$$

In the following, we will focus on Landau gauge for the photons, where $\alpha_A = 0$, and on $\beta = 1$, $\alpha = 0$ for the metric. We introduce dimensionless couplings as follows:

$$w_2 = \frac{\bar{w}_2}{Z_A^2} k^4, \quad g = \frac{\bar{G}_N}{Z_h} k^2, \quad \mu_h = -2\bar{\Lambda} Z_h k^{-2}. \tag{11}$$

Moreover the anomalous dimensions are given by

$$\eta_h = -\partial_t \ln Z_h, \quad \eta_A = -\partial_t \ln Z_A. \tag{12}$$

Diagrammatically, the flow of the coupling w_2 is encoded in the diagrams in Fig. 1, and that of the anomalous dimension in the diagrams in Fig. 2.

To project the RG flow obtained with the help of the Wetterich equation onto $\bar{w}_2$, we employ the following rule:
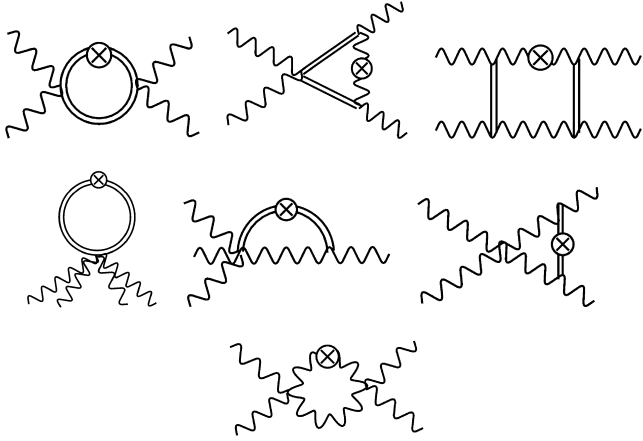
$$\begin{aligned} \partial_t \bar{w}_2 = \lim_{p^2 \to 0} &\frac{1}{720 \, (p^2)^2} P_{\mu\nu\kappa\lambda}(p, p, -p, -p) \\ &\times \left(\frac{\delta}{\delta A_\mu(p_1)} \frac{\delta}{\delta A_\nu(p_2)} \frac{\delta}{\delta A_\kappa(p_3)} \frac{\delta}{\delta A_\lambda(p_4)} \right. \\ &\times \left. \partial_t \Gamma_k \right) \bigg|_{p_1 = p_2 = -p_3 = -p_4 = p, A=0, h=0} \end{aligned} \tag{13}$$

Herein, the tensor $P_{\mu\nu\kappa\lambda}(p)$ is defined as

$$\begin{aligned} &P_{\mu\nu\kappa\lambda}(p_1, p_2, p_3, p_4) \\ &= \left(\frac{\delta}{\delta A_\mu(p_1)} \frac{\delta}{\delta A_\nu(p_2)} \frac{\delta}{\delta A_\kappa(p_3)} \frac{\delta}{\delta A_\lambda(p_4)} \frac{1}{8} \left(F^2 \right)^2 \right). \end{aligned} \tag{14}$$

Similarly, we project onto the anomalous dimension for the photon as follows

$$\eta_A = -\lim_{p^2 \to 0} \frac{1}{Z_A p^2} P_{\mu\nu}(p) \left(\frac{\delta}{\delta A_\mu(p)} \frac{\delta}{\delta A_\nu(-p)} \partial_t \Gamma_k \right) \bigg|_{A=0, h=0}, \tag{15}$$

Fig. 1. These diagrams contribute to the flow of w_2. Curly lines denote photons and double lines metric fluctuations. The regulator insertion $\partial_t R_k$ is denoted by a crossed circle, and can in turn be found on any of the internal lines of a diagram. We only show one representative of each class. The first line contains all diagrams $\sim g^2$, the second line diagrams $\sim g w_2$ and the third line the diagram $\sim w_2^2$.
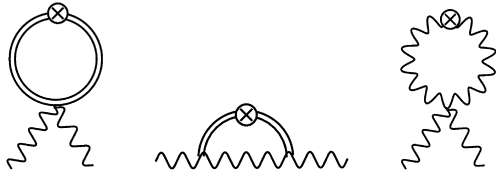
Fig. 2. These diagrams contribute to the flow of η_A. The diagram to the left and in the middle are the gravitational contribution to η_A, whereas the diagram to the right is the pure-gauge contribution $\sim w_2$.

where

$$P_{\mu\nu}(p) = \frac{1}{3}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right). \tag{16}$$

We will not evaluate the running of the gravitational couplings, instead treating them as free parameters. Thereby we test how the system will respond to extensions of the truncation in the gravitational sector, which could shift the fixed-point values of the gravitational couplings.

3. Results: quantum-gravity induced asymptotic safety in the gauge sector

3.1. Shifting the Gaußian fixed point

At vanishing gravitational coupling $g = 0$, the system exhibits a non-interacting, i.e., Gaußian fixed point, at which $w_2 = 0$. Moreover, w_2 corresponds to an UV-irrelevant direction, according to its canonical dimension -4. In the presence of quantum gravity, there are several diagrams that shift the Gaußian fixed point to an interacting one. In other words, these diagrams are independent of the coupling w_2, and are non-zero as soon as the canonical field-strength-squared interaction for photons is minimally coupled to gravity. Thus, higher-order gauge interactions are automatically induced in the ultraviolet. Depending on the sign and magnitude of the gravitational contribution, the fixed point might even get shifted into the complex plane. In that case, asymptotically safe gravity cannot be reconciled with the existence of a fundamental Abelian gauge sector in the Standard Model.

Specifically, these are the diagrams in the first line of Fig. 1. Together, we call this contribution to β_{w_2}, which is independent of w_2 itself, but proportional to g^2, the induced contribution, as it

induces an interacting fixed point. It contains the following contributions in the order in which they are shown in Fig. 1.

$$\beta_{w_2}\Big|_{\text{induced}} = \frac{8}{3\,(1+\mu_h)^3}g^2(6-\eta_h) \tag{17}$$

$$- g^2\left(2\frac{8-\eta_h}{(1+\mu_h)^3} + \frac{8-\eta_A}{(1+\mu_h)^2}\right)$$

$$+ g^2 4\left(\frac{10-\eta_h}{5(1+\mu_h)^3} + \frac{10-\eta_A}{5(1+\mu_h)^2}\right).$$

These contributions are responsible for the main effect that we highlight in this work: All terms in eq. (17) are non-zero even if we set $w_2 = 0$. Therefore this value of the coupling is *not* a fixed point, i.e., its beta-function is non-zero at this point. Accordingly the only possible fixed point for the gauge sector in the presence of quantum gravity, i.e., for $g \neq 0$, is an interacting one. Thus, as g is increased starting from zero, the Gaußian fixed point at $w_2 = 0$ is shifted to become an interacting one.

3.2. Asymptotic safety for U(1) gauge theories

Further, there are contributions to the beta-function which are $\sim w_2\, g$. We call them mixed, as they only exist for non-zero g and w_2. These can be understood as a quantum-gravity correction to the scaling dimension of the coupling w_2. In the order in which they appear in the second line in Fig. 1, they are given by

$$\beta_{w_2}\Big|_{\text{mixed}} = -\frac{2}{3\pi\,(1+\mu_h)^2}g\,w_2\,(6-\eta_h) \tag{18}$$

$$+ g\,w_2\left(\frac{8-\eta_h}{2\pi(1+\mu_h)^2} + \frac{8-\eta_A}{2\pi(1+\mu_h)}\right)$$

$$- \frac{13g\,w_2}{60\pi}\left(\frac{10-\eta_h}{(1+\mu_h)^2} + 2\frac{10-\eta_A}{1+\mu_h}\right).$$

The pure-photon-diagram in the 3rd line of Fig. 1 yields

$$\beta_{w_2}\Big|_{\text{pure-photon}} = \frac{19}{240\pi^2}(10-\eta_A)w_2^2. \tag{19}$$

If the theory is coupled to charged matter fields, there will be contributions of box diagrams, which are all of order e^4. In particular, these will not contribute to the fixed-point value nor the critical exponents at the shifted Gaussian fixed point characterized by $e_* = 0$. The complete beta function for w_2 is given by

$$\beta_{w_2} = 4w_2 + 2\eta_A\,w_2 + e^4\beta_{w_2}\Big|_{\text{matter}}$$

$$+ \beta_{w_2}\Big|_{\text{induced}} + \beta_{w_2}\Big|_{\text{mixed}} + \beta_{w_2}\Big|_{\text{pure-photon}}. \tag{20}$$

For the gauge coupling e, we obtain

$$\beta_{e^2} = e^2\,\eta_A$$

$$= e^2\Big(\frac{1}{4\pi(1+\mu_h)^2}g(6-\eta_h)$$

$$- \frac{g}{8\pi}\left(\frac{8-\eta_h}{(1+\mu_h)^2} + \frac{8-\eta_A}{1+\mu_h}\right)$$

$$- \frac{w_2}{24\pi^2}(8-\eta_A) + \Delta\eta_{A,\text{matter}}e^2\Big), \tag{21}$$

where we have parameterized the contributions of charged matter as $\Delta\eta_{A,\text{matter}}e^4$, with $\Delta\eta_{A,\text{matter}} > 0$.

From (21) we infer that the gravity contributions dominate for small gauge coupling e, as they couple linearly in e^2. In comparison, the contributions from matter are of order e^4 and therefore
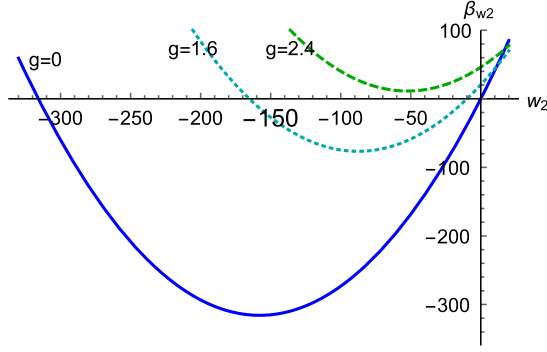
Fig. 3. We show β_{w_2} for $\eta_h = 0 = \eta_A$ as well as $\mu_h = 0$ for different values of g. Any finite value $g > 0$ shifts the Gaußian to an interacting fixed point. Beyond a critical value of g, no real fixed point exists.

do not contribute to the scaling exponent at the fixed point at $e_* = 0$. In the absence of gravity, these contributions induce a triviality problem. Interestingly, they become subdominant in the presence of gravity, and can thus be neglected when exploring whether quantum gravity renders Abelian gauge theories asymptotically free in the gauge coupling e. The main quantum-gravity effects on the Abelian gauge coupling can be understood within a simplified setting, where we set the anomalous dimensions to zero in the loop integrals and also neglect the cosmological constant. In that case, we have

$$\beta_{e^2}|_{\text{loops: }\eta=0} = \left(-\frac{g}{2\pi} - \frac{w_2}{3\pi^2}\right)e^2, \tag{22}$$

$$\beta_{w_2}|_{\text{loops: }\eta=0} = 4w_2 + 8g^2 - \frac{7}{2\pi}g\,w_2 + \frac{1}{8\pi^2}w_2^2. \tag{23}$$

From eq. (22), it is clear that quantum gravity effects can render the U(1) gauge coupling asymptotically free, providing a solution to the triviality problem, as discussed in [6]. Here, we will highlight that this scenario hinges on the weakness of the gravitational coupling, due to an intriguing interplay of the direct contribution ($\sim g$) and the mediated one ($\sim w_2$) in eq. (22). Note that the addition of charged matter does not change the mechanism for gravity-induced asymptotic freedom, as charged matter contributes to the beta function for the gauge coupling at higher order in e.

Without gravity, w_2 features a non-interacting fixed point, at which it is irrelevant according to canonical power counting. With gravity, the situation changes: The term $\sim 8g^2$ prevents the possibility of a free fixed point for the coupling w_2. Instead, the fixed point is shifted towards increasingly negative values as a function of g,

$$w_{2* \text{sGFP}} = 2\pi \left(7g - 8\pi + \sqrt{64\pi^2 - 112g\pi + 33g^2}\right). \tag{24}$$

Note that our expansion of the full "potential" of the gauge field, $V(F^2)$ to second order is insufficient to determine global stability properties. Thus, a non-perturbative phenomenon like the formation of a nontrivial minimum in $V(F^2)$ in the UV, as well as a destabilization in the sense of a microscopic potential that is not bounded from below, are both in principle possible, and might seem indicated by the negative fixed-point value for w_2. To settle this question, studies of $V(F^2)$ are necessary. These are feasible along the lines of, e.g., [77,78].

Since the fixed point in Eq. (24) emerges from the non-interacting, Gaußian fixed point, as we increase g, we call it the shifted Gaußian fixed point. In particular, its UV-attractivity properties remain similar to that of the Gaußian fixed point, i.e., w_2 remains irrelevant at that fixed point. Accordingly the critical exponent remains negative
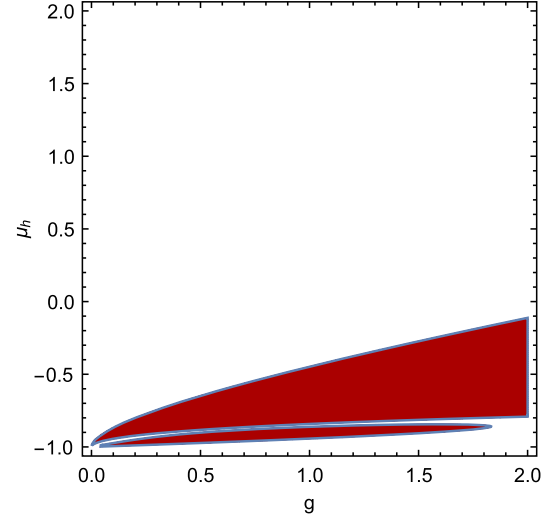
Fig. 4. No real fixed point exists in the red region in the gravitational parameter space for $\eta_h = 0$, i.e., the white region is allowed. The second allowed region at negative μ_h is a new fixed point that appears to be induced by a pole in the beta function. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

$$\theta_{w_2} = -\frac{\partial \beta_{w_2}}{\partial w_2}\bigg|_{w_2 = w_2\,\text{sGFP}} = -\frac{1}{2\pi}\sqrt{64\pi^2 - 112\pi\,g + 33g^2}. \tag{25}$$

It is obvious that once g exceeds a critical value g_{crit}, the critical exponent vanishes and $w_{2* \text{sGFP}}$ becomes complex. At this point, the shifted Gaußian fixed point annihilates with the second zero of β_{w_2} and the two move off into the complex plane, cf. Fig. 3. For rather large values $g \approx 8.4$, real fixed points reappear. For these, the back-coupling of w_2 into β_{e^2} induces a huge departure from canonical scaling, invalidating the rationale behind our truncation. Moreover, fixed-point values for gravity typically do not lie in that regime, and we thus focus on the significantly more important bound on g that arises from the shift of the fixed-point values into the complex plane.

Analyzing the full beta functions eq. (20) instead of the simplification in eq. (23), requires us to use the full anomalous dimension, see eq. (21)

$$\eta_A = -\frac{8w_2(1 + \mu_h)^2 + 3\pi\,g\,(4 + 8\mu_h + \eta_h)}{(1 + \mu_h)((1 + \mu_h)(24\pi^2 - w_2) - 3\pi\,g)}. \tag{26}$$

Confirming our analysis of the simplified beta functions Eq. (23) and Eq. (22) no real fixed point exists for a certain region of the gravitational parameter space, cf. Fig. 4; excluding this region from the viable fixed-point parameter space. The bounds in Fig. 4 only show a very mild dependence on η_h, e.g., for $g = 1$ and $|\eta_h| = 0.5$ there is only a 5% deviation in the critical value of μ_h. In general, for values of $\eta_h > 0$ the red region in Fig. 4 shrinks, while the opposite is true for $\eta_h < 0$.

As a next step, we evaluate the quantum-gravity effects on the gauge coupling e^2. There is firstly a direct effect, i.e., loop diagrams with internal metric fluctuations induce a nontrivial running of the coupling. These contribute to render the gauge coupling asymptotically free, thus providing a solution to the triviality problem. Secondly, the quantum-gravity induced coupling w_2 also contributes to the running of e^2. The second effect is an *indirect* quantum-gravity effect, i.e., it is mediated by gauge interactions. These interactions are induced by quantum-gravity fluctuations and then in turn affect the scaling dimension of the gauge coupling. Thus we call these the mediated contribution. The contributions to the critical exponent of the gauge coupling can be split accordingly,
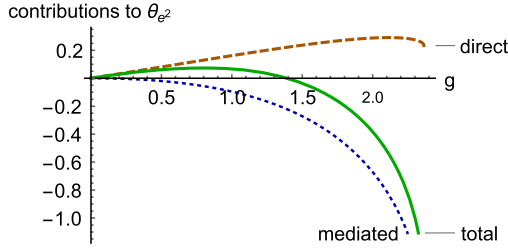
Fig. 5. We show the two competing contributions to θ_{e^2} from eq. (28) and eq. (29) as well as the full result, both for $\eta_h = 0$, with eq. (26). At $g = g_{zero} \approx 1.5$, the gauge coupling e becomes irrelevant, making a connection of the fixed point to phenomenologically acceptable values difficult. g_{zero} gets smaller for $\mu_h < 0$, while the influence of η_h is negligible.

$$\theta_{e^2} = -\frac{\partial \beta_{e^2}}{\partial e^2}\Big|_{e=0} = \theta_{e^2,\,\text{direct}} + \theta_{e^2,\,\text{mediated}}, \tag{27}$$

in the approximation given by (22) and (23) one obtains

$$\theta_{e^2,\,\text{direct}} = \theta_{e^2,\,g} = \frac{g}{2\pi} > 0, \tag{28}$$

$$\theta_{e^2,\,\text{mediated}} = \theta_{e^2,\,w_2} = \frac{w_{2\,\text{sGFP}}}{3\pi^2} < 0. \tag{29}$$

Intriguingly, the two contributions have the opposite sign, which has important consequences. The shifted Gaußian fixed point in w_2 evolves towards increasingly negative values with growing gravitational coupling g, growing quadratically in g for small g. Thus, the contribution to θ_{e^2} from the induced gauge coupling w_2 ultimately wins over the direct quantum gravity contribution, cf. Fig. 5.

For small gravitational coupling, we thus have a fixed point at which the dimensionless gauge coupling e^2 is asymptotically free, while w_2 becomes asymptotically safe and corresponds to an irrelevant direction of the fixed point. This is the phenomenologically preferred regime for g, as it provides a UV completion for the Abelian gauge sector – thus solving the triviality problem – that can be matched onto the high-energy behavior of the hypercharge gauge coupling in the Standard Model. Inserting fixed-point values for g, μ_h and η_h from the truncation explored in [66], we find $\theta_{e^2} \approx 0.07$, i.e., indications point towards a phenomenologically viable quantum-gravity induced solution of the triviality problem.

Towards larger g, the gauge coupling e is then rendered irrelevant at its Gaußian fixed point, before the full fixed point for the system is destroyed at $g = g_{\text{crit}}$. In the intermediate regime, i.e., between $g = g_{zero} \approx 1.5$ and $g < g_{\text{crit}} \approx 2.4$ (all values for $\mu_h = 0$, $\eta_h = 0$), all couplings in the gauge sector feature fixed points and are irrelevant, i.e., the fixed point is completely UV-repulsive, i.e., completely IR-attractive. This means that quantum-gravity fluctuations force those couplings to remain at their fixed-point values all the way down to the Planck scale and $e(k \approx M_{\text{Pl}}) = 0$. At the Planck scale, quantum-gravity effects switch off, and the further running of the couplings down to the electroweak scale is determined purely in terms of Standard Model beta functions. In that regime, the low-energy values of those couplings therefore correspond to a prediction of the asymptotic safety paradigm, see [79] for a similar argument in the Higgs sector. In a setting where no new physics appears between the electroweak and the Planck scale, the U(1) hypercharge coupling is significantly larger than 0 at the Planck scale. However, this value cannot be reached starting from a fixed point at $e_* = 0$, if that coupling is irrelevant and thus forced to remain zero until quantum gravity fluctuations switch off.

If these results persist beyond our truncation, and the assumption of no new physics up to the Planck scale holds, we would

tentatively conclude that the gravitational coupling should not become too large in the UV, such that e can in fact remain asymptotically free. Interestingly, the excluded region in Fig. 4 is similar to that found in [12], where quantum-gravity fluctuations push an interacting fixed point in the Yukawa sector into the complex plane. We thus observe hints from two separate sectors of the Standard Model, that quantum-gravity fluctuations are restricted to not exceed a critical strength, if the asymptotic safety paradigm is to provide a viable UV completion for gravity and matter.

Broadening our scope beyond the asymptotic-safety scenario, we observe that the generation of higher-order operators by quantum gravity appears to be a generic feature within the effective field theory setting [80]: Any non-zero value of g provides a contribution $\sim g^2$ to the beta function for w_2, i.e., even if w_2 is set to zero at some scale, quantum gravity fluctuations generate it. Our results suggest that going beyond canonical power counting might also be important to understand the full effect of quantum gravity on gauge couplings in the effective field theory framework [81].

In particular, an analysis within the EFT regime for gravity applies to the case where asymptotically safe matter models [82] are subjected to quantum-gravity corrections. In that case, corrections linear in the gauge coupling as in eq. (22) arise, and contribute towards asymptotic freedom in the gauge coupling, potentially impacting the phase structure in those models, [83].

4. Conclusions and outlook

In this work, we have found evidence that asymptotically safe quantum gravity can generate an interacting fixed point for a U(1) gauge theory, thus providing a solution to the triviality problem. We have further elucidated the fixed-point structure by pointing out that it is necessary to go beyond canonically dimensionless couplings in the gauge sector: Asymptotically safe quantum gravity induces non-vanishing higher order interactions. Therefore, it is impossible for an Abelian gauge theory to become completely asymptotically free under the impact of quantum gravity: While the leading-order gauge coupling e features a fixed point at zero, the higher-order coupling w_2 is non-vanishing. This effect becomes of critical importance for two questions: Firstly, the very existence of a quantum-gravity generated fixed point in the gauge sector looks very different when higher-order couplings are neglected. This is the case since quantum gravity first shifts the Gaußian fixed point for the higher order couplings to non-zero real values and then to complex values once a critical strength of gravitational interactions is exceeded. Secondly, the inclusion of the higher-order coupling w_2 is critical to understand whether e can become asymptotically free under the impact of quantum gravity: Once generated by quantum gravity, w_2 yields a non-zero contribution to the scaling dimension of e at its free fixed point. Beyond a critical strength of the gravitational coupling, the gauge coupling is then rendered irrelevant at its fixed point. This setting is difficult to reconcile with the Planck-scale value of the hypercharge coupling unless additional new physics is invoked.

We tentatively conclude that quantum gravity can provide a UV completion for Abelian gauge theories within the asymptotic-safety paradigm. Crucially, this mechanism relies on the strength of gravitational interactions not exceeding a certain critical value. Beyond the shifted Gaußian fixed point, further fully interacting fixed points could exist, but these would typically feature significantly different sets of relevant operators. Interestingly, quantum fluctuations of minimally coupled gauge fields force the fixed-point value for the Newton coupling towards lower values, see [66]. Therefore, even if gravity destroys the fixed point for one gauge field, the shift towards weaker gravity that is induced by coupling additional gauge fields will ultimately shift the fixed point for w_2 back

onto the real axis. Thus, a scenario could be realized, in which a gauge-gravity system can only be asymptotically safe if a critical lower number of gauge fields is exceeded. With the caveat that these were obtained with a different regularization scheme and pertain to the background field, we can insert the fixed-point values for g, η_h and μ from [66] for one vector field, and we find that the shifted Gaußian fixed point already exists. Studies of (non-minimally) gauge-gravity systems will have to confirm this result.

Our work paves the way for several exciting questions to be explored in the future: Firstly, similar mechanisms will exist in models with additional ("dark") U(1) sectors, and will induce couplings between photons and hidden photons. It will be interesting to explore the properties of an asymptotically safe portal to the dark sector.

Moreover, our results also carry over to the case of non-Abelian gauge theories, where similar higher-order operators will be induced. It is critical to test their impact on the running of the non-Abelian gauge coupling, as quantum gravity could then either destroy asymptotic freedom, strengthen it, or turn it into asymptotic safety [84–86]. In that case, the non-Abelian gauge couplings could even be become irrelevant couplings, and their low-energy values would be a prediction of asymptotic safety.

Further, it is known that non-Abelian gauge theories are asymptotically safe by themselves in $d = 4 + \epsilon$ dimensions [87,88]. Thus, a setting in $d > 4$ could feature an intricate interplay of fixed-point dynamics for the gauge and the gravity sector.

Moreover, our result that quantum gravity induces higher-order interactions in gauge theories, which then affect the question whether the gauge theory can become asymptotically free under the effect of quantum gravity, is not restricted to asymptotically safe gravity, as it does not require a particular value for the Newton coupling. Thus, our work suggests that going beyond canonical power counting could be necessary to obtain a full picture of the effect of quantum gravity also within an effective field theory setting. In particular, it would be interesting to explore how the effects that we have observed here are encoded in other schemes.

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