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Setting the Renormalization Scale in QFT

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Abstract. In this work a setting for the “scale-field” is proposed at the level of effective action, which is consistent with the conservation of the stress-energy tensor. The mechanism and its potential is exemplified for scalar ϕ^4 theory and for Einstein-Hilbert-Maxwell theory.

1. Introduction

These proceedings on “Setting the Renormalization Scale in QFT” are based on a more complete article on the same topic [1], where also complementary discussions can be found. The effective action approach [2] can be seen as an elegant way of defining a generating functional for one-particle-irreducible Green’s functions. Following Wilson’s idea [3] one can study the effect of integrated quantum degrees of freedom at different scales k . The scale dependent effective action Γ_k is to be understood as interpolation between the ultraviolet (UV) bare action Γ_∞ and the fully integrated action in the IR Γ_0 . Γ_k contains scale dependent couplings g_k^a which are obtained from a suitable flow equation $k\partial_k\Gamma_k$. The space of solutions g_k^a is called the “coupling flow” [4]. A specific trajectory is selected out of this flow by imposing conditions for the couplings at an initial scale k_0 . The evaluation of the effective action Γ_k is typically hampered by various technical difficulties such as singularities, anomalies, and non-localities. However, in many cases those difficulties can be overcome by the “regularization - renormalization” technique, where infinities are absorbed in the initial conditions at a scale k_0 .

Minimizing a given effective action with respect to variations of its (average) field content ϕ_a gives the equations of motion of the effective action

$$\frac{\delta\Gamma_k}{\delta\phi_a} = 0 \quad . \quad (1)$$

Those equations are typically non-linear and sometimes non-local differential equations and are frequently referred to as “gap equations” [5]. Therefore, finding solutions for the “gap equations” is highly relevant for defining a self-consistent background in quantum field theory.

However, even if it is technically possible to solve the “gap equations”, the physical interpretation of such a solution is still biased by the way the scale k is related to the quantities x_i, Q_i, \dots (for example positions and charges) that are used to describe the physical system. Choosing a relation $k = k(x_i, Q_i, \dots)$ is called “scale setting”. The main focus of this article is on the role of scale setting in the quest of finding self-consistent solutions of (1). The method



of improving solutions has been successfully applied in many different contexts (see refs.[6] to [22]).

2. Scale-field setting at the level of effective action

In this section a general scale setting at the level of effective action will be proposed. Lets assume that within the quantum field theoretical model it was possible, to evaluate the corresponding coupling flow and to select a particular trajectory due to the choice of initial conditions ($g_i(k_0) = g_{i,0}$). Thus, one can start with the effective quantum action [2]

$$\Gamma_k(\phi_a(x), \partial\phi_a(x), g_k^a) = \int d^4x \sqrt{-g} \mathcal{L}(\phi_a(x), \partial\phi_a(x), g_k^a) \quad , \quad (2)$$

where ϕ_a are actually the expectation values of the quantum fields and g_k^a are the scale dependent couplings, including the coupling multiplying the kinetic term.

Note that doing this, one frequently has to truncate higher order and nonlocal couplings [23, 24, 25] from the model, that might appear due to the quantum integration procedure. Then, it will be assumed that all relevant couplings are taken into account and a local expansion are consider.

Now, one can derive the equations of motion for the average quantum fields ϕ_a from

$$\frac{\delta\Gamma_k}{\delta\phi_a} = 0 \quad . \quad (3)$$

As mentioned in the introduction, the solutions $\bar{\phi}_a(x, k)$ of those “gap equations” will also be functions of the arbitrary scale k . From a physical point of view this is however not yet satisfactory since no possible observable can be a function of an a priori arbitrary scale. In order to obtain a physical quantity one has to define some kind of scale setting procedure, that establishes a relation between the physical quantities (charges Q_i and positions x_j) of a given problem and the scale k . When doing this one can borrow an idea from the calculation of observables $\langle T\phi(x_i) \dots \rangle_k$ in standard quantum field theory. Also there, the observables turn out to be scale “ k ” dependent quantities¹. Subsequently, the scale setting for those observables in terms of initial conditions and kinematical variables $k = k(x_i; Q_i \dots)$ is chosen such that any k dependence of the time ordered correlation function for observable is minimized

$$\left. \frac{d}{dk} \langle T\phi_1(x_1)\phi_2(x_2) \dots \rangle_k \right|_{k=k_{opt}} \equiv 0 \quad . \quad (4)$$

This is the key philosophy that is used when deriving the “Callan-Symanzik” equations [26, 27], the “principal of minimal sensitivity [28], or the “principle of maximal conformality” [29, 30].

It is proposed to implement an analogous philosophy at the level of the effective action Γ_k . This means that one should choose an optimal scale setting prescription for which a variation of k has a minimal impact on the self-consistent background $\bar{\phi}_i$. This principle can be implemented by promoting the a priori arbitrary scale to a physical scale-field in the effective quantum action

$$\Gamma_k(\phi_a(x), \partial\phi_a(x), g_k^a) \rightarrow \Gamma(\phi_a(x), \partial\phi_a(x), k(x), g_k^a) \quad . \quad (5)$$

This leads to the coupled equations of motion

$$\frac{\delta\Gamma}{\delta\phi_a} = 0 \quad , \quad \left. \frac{d}{dk} \mathcal{L}(\phi_a(x), \partial\phi_a(x), k(x), g_k^a) \right|_{k=k_{opt}} = 0 \quad . \quad (6)$$

¹ It is, argued that this k dependence is an artifact of the truncation in the loop expansion

Clearly it is not guaranteed that a solution for (6) can be found, but such a prescription is not limited to be a variation of a classical solution or to a saddle point approximation. The procedure (6) has already been applied for some particular gravitational actions [31, 32, 33, 34] but in this work it is discussed in a broader context. A nice feature of such a procedure is that any solution of the equations (6) is automatically independent of k , which is actually the fundamental precondition for a physical observable in the language of the renormalization group approach.

Promoting the scale k to a scale-field $k(x)$ raises the question whether this new field only appears in the couplings g_k^a , or whether it has to be equipped with other additional couplings, for instance a proper kinetic term. A standard procedure when introducing new fields into a Lagrangian is to incorporate actually all couplings that are in agreement with the symmetry of the Lagrangian. This abundant freedom is then restricted by imposing some other additional conditions such as renormalizability, simplicity, and/or agreement with experimental constraints. However, in the presented approach the philosophy is different. The scale-field k is understood to have its origin in the process of renormalization and throughout this process, no such extra couplings are taken into account. In particular, the beta functions of the couplings g_k^a are calculated without any additional couplings. Therefore, the presented version of scale-field setting is chosen in a sense “minimal”, since it contemplates the appearance of $k(x)$ only as dictated by the running couplings g_k^a .

Finally, there is one other consistency condition one would like to impose, the conservation of the stress-energy tensor, even at the quantum-improved level:

$$\nabla^\mu T_{\mu\nu} \equiv 0 \quad . \quad (7)$$

Therefore, the idea will be studied for some examples, where self-consistency of the approach can be shown explicitly.

3. Scale-field setting for scalar ϕ^4 theory

As most simple example without any further complications due to gauge symmetry lets study the scale-field setting procedure for scalar ϕ^4 theory. There are various ways of writing the effective action for ϕ^4 theory. One of them is in terms of a scale dependent wave function renormalization Z_k , running mass m_k , and running quartic coupling g_k . The other way of writing this action is in terms of separate couplings for every term appearing in the Lagrangian, which are a coupling for the kinetic term α_k , a coupling for the ϕ^2 term \tilde{m}_k^2 , and a coupling for the quartic term \tilde{g}_k . As long as the scale k is assumed to be fixed, the formalism for both is exactly equivalent. However, in the context of scale-field setting $k \rightarrow k(x)$, derivatives do not necessarily commute with $k(x)$ and both formulations could to be treated differently. This subtlety will be exemplified in the following subsection, before applying the scale setting to ϕ^4 theory at the one loop level.

3.1. Consistency in scalar ϕ^4 theory

Let us consider the effective action

$$\Gamma_k = \int d^4x \left(\frac{\alpha_k}{2} (\partial\phi)^2 - \frac{\tilde{m}_k^2}{2} \phi^2 - \frac{\tilde{g}_k}{4!} \phi^4 \right) \quad (8)$$

with two fields ϕ and k . The couplings α_k , \tilde{m}_k^2 , and \tilde{g}_k are functions of the field k . This implies an equation of motion for $\delta\phi$:

$$\partial_\mu (\alpha_k \partial^\mu \phi) + \tilde{m}_k^2 \phi + \frac{\tilde{g}_k}{6} \phi^3 = 0 \quad , \quad (9)$$

an other equation of motion for k :

$$\alpha'_k(\partial\phi)^2 - (\tilde{m}_k^2)'\phi^2 - \frac{1}{12}\tilde{g}'_k\phi^4 = 0 \quad , \quad (10)$$

where $\alpha' = \partial_k \alpha = (\partial_x \alpha) dx/dk$, and we will consider the conserved energy momentum tensor, which is obtained as variation with respect to the metric tensor

$$T_{\mu\nu} = \alpha_k(\partial_\mu\phi)(\partial_\nu\phi) - g_{\mu\nu} \left(\frac{\alpha_k}{2}(\partial\phi)^2 - \frac{\tilde{m}_k^2}{2}\phi^2 - \frac{\tilde{g}_k}{4!}\phi^4 \right) \quad . \quad (11)$$

and the corresponding conservation law reads

$$0 = \partial^\mu T_{\mu\nu} = -\frac{1}{2} \left(\alpha'_k(\partial\phi)^2 - (\tilde{m}_k^2)'\phi^2 - \frac{1}{12}\tilde{g}'_k\phi^4 \right) \cdot \partial_\nu k \quad , \quad (12)$$

and the equation (12) is identical to the equation of motion for k (10), which shows that the approach at the level of effective action (6) is on the one hand implementing the idea of minimal scale dependence and on the other hand maintaining the validity of improved equations of motion and the fundamental conservation law.

Instead of writing a coupling for the kinetic term one frequently works with wave function renormalization where the bare field is $\phi_B = \sqrt{Z_k}\phi$. In this case one could simply identify $\alpha_k = Z_k$, $\tilde{m}_k^2 = Z_k m_k^2$, and $\tilde{g}_k = g_k Z_k^2$ and observe that the corresponding effective action is completely equivalent to (8). However, if one allows for field valued scales $k = k(x)$, this identification is not the only possibility, since the derivatives of the kinetic term, acting on the scale field might contribute to the action. Still, even in this case it can be shown that the scale setting procedure is consistent with the conservation law, just like in (12).

In order to demonstrate the functionality of the procedure, an approximated self-consistent scale setting for spherically symmetric backgrounds in ϕ^4 theory is calculated at the one loop level. The loop expansion of ϕ^4 theory has been calculated up to high order in perturbation theory [35], and the one-loop beta functions γ_Z, β_g and β_{m^2} can be found in [36]. Integrating those flow equations with initial conditions at $k = k_0$, for $Z_{k_0} \equiv 1$, $g_{k_0} = g_0$ and $m_{k_0}^2 = m_0^2$, one can find the flow trajectory g_k and m_k^2 .

Considering the equation of motion for ϕ , the scale setting equation (10), the flow trajectories, and the specific case of static spherical symmetry the only allowed coordinate dependence of k is with respect to the radial distance $k = k(r)$. In this case one can find a scale-field setting which actually reproduces the standard $k \sim 1/r$ behavior for very small radii, but it has more complex behavior.

4. Scale-field setting for Einstein Hilbert Maxwell action

In order to show the consistency of the proposed scale setting, with conservation laws in a less-trivial example, one can study the approach for gravity coupled to a $U(1)$ gauge field and to a cosmological constant. Gravity is exemplary for a non-trivial field theory that is notoriously perturbatively not renormalizable and the situation becomes even less favorable when it is coupled to matter. Using the non-perturbative methods: ERG and the Asymptotic Safety approach one can calculate effective actions and scale dependent couplings for this (see e.g. refs. [37] to [53]). Therefore, it is reasonable to investigate the proposed scale setting procedure in the context of a gravitational action coupled to matter. Let us consider to the discussion the Einstein-Hilbert-Maxwell action:

$$\Gamma_k[g_{\mu\nu}, A_\alpha] = \int_M d^4x \sqrt{-g} \left(\frac{R - 2\Lambda_k}{16\pi G_k} - \frac{1}{4e_k^2} F_{\mu\nu} F^{\mu\nu} \right) \quad , \quad (13)$$

where R is the Ricci scalar and $F_{\mu\nu} = D_\mu A_\nu - D_\nu A_\mu$ is the electromagnetic field strength tensor and the scale dependent couplings are: the Newtons coupling G_k , the cosmological coupling Λ_k , and the electromagnetic coupling e_k . The flow of those couplings has been derived non-perturbatively in [51]. Let us start our study considering the equations of motion for the metric field in (13) which are

$$G_{\mu\nu} = -g_{\mu\nu}\Lambda_k - \Delta t_{\mu\nu} + 8\frac{\pi G_k}{e_k^2}T_{\mu\nu} \quad , \quad (14)$$

where $\Delta t_{\mu\nu} = G_k (g_{\mu\nu} - \nabla_\mu \nabla_\nu) \frac{1}{G_k}$ is an additional contribution to the stress-energy tensor [42] coming from the k -dependence of G_k and $T_{\mu\nu} = F_\nu^\alpha F_{\mu\alpha} - \frac{1}{4}g_{\mu\nu}F_{\mu\nu}F^{\mu\nu}$. The equations of motion (Maxwell equations) for this $U(1)$ gauge field are

$$D_\mu \left(\frac{1}{e_k^2} F^{\mu\nu} \right) = 0 \quad , \quad (15)$$

and finally the equations of motion for the scale-field k are

$$\left[R \nabla_\mu \left(\frac{1}{G_k} \right) - 2 \nabla_\mu \left(\frac{\Lambda_k}{G_k} \right) - \nabla_\mu \left(\frac{4\pi}{e_k^2} \right) F_{\alpha\beta} F^{\alpha\beta} \right] \cdot (\partial^\mu k) = 0 \quad . \quad (16)$$

The above equations of motion are complemented by the relations corresponding to gauge invariance of the system: For the diffeomorphism invariance one has $\nabla^\mu G_{\mu\nu} = 0$ and for the internal $U(1)$ symmetry $\nabla_{[\mu} F_{\alpha\beta]} = 0$.

The new ingredient due to the scale-field is the equation (16), therefore it is important to check whether this equation is actually non-trivial and consistent with the equations (14 and 15). The consistency can be shown by explicitly deriving (16) from (14) and (15) and further imposing the gauge symmetries conditions. This confirms that, the approach at the level of effective action (6) is on the one hand an elegant way of minimizing scale setting ambiguities and on the other hand maintaining the validity of improved equations of motion and the fundamental conservation laws of the effective action.

4.1. Scale-field setting for Einstein Hilbert Maxwell action in UV - Asymptotic Safety

Lets exemplify our proposed method with the Einstein Hilbert Maxwell action (13) at the fixed points. In the deep UV limit ($k \rightarrow \infty$) there is strong evidence [38, 39, 41] for the existence of a non-Gaussian fixed point for the two gravitational couplings

$$G_k \approx \frac{g^*}{k^2} \quad , \quad \Lambda_k \approx \lambda^* k^2 \quad , \quad (17)$$

and there exists further evidence for one UV fixed point for the electromagnetic couplings [51]

$$\lim_{k \rightarrow \infty} \frac{1}{e_{k,2}^2} \approx \frac{1}{e_2^{*2}} \quad . \quad (18)$$

Since this fixed point in the electromagnetic coupling is not an attractor it is only approached by particular trajectories in the corresponding flow. Other trajectories either run into a Landau pole type of divergence at finite k , or they run to vanishing values of $e_{k,1}$ at infinite k [51]

$$\lim_{k \rightarrow \infty} \frac{1}{e_{k,1}^2} \approx \frac{1}{e_1^{*2}} \cdot (k^2)^B \quad , \quad (19)$$

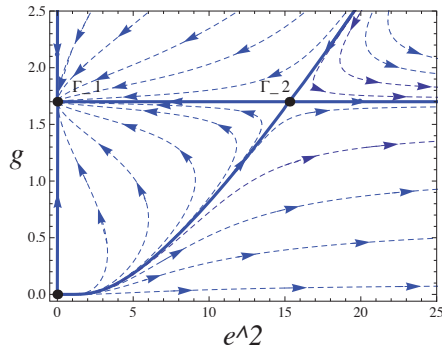


Figure 1. Coupling flow projected to the dimensionless coupling g, e^2 where $B \equiv 1$ was selected. One observes three fixed points: the Gaussian fixed point at vanishing couplings, one fixed point with finite g^* but vanishing e^2 (labeled Γ_1), which corresponds to the limit (19), and an other with finite g^* and e^2 (labeled Γ_2), which corresponds to the limit (18). The arrows point in direction of increasing k .

where the value of B depends on the method of calculation and ranges from 0.8 to 1.6. The behavior of this ER flow using the functions of [51] is shown in figure 1.

In order to integrate out the scale-field from the effective action (13) one has to solve the corresponding equation of motion (16) for k^2 . In the UV limit (17) one finds for the fixed point in (18)

$$k_2^2|_{UV} = \frac{R}{4\lambda^*} \quad (20)$$

and for the asymptotic behavior (19) one finds in the same limit

$$k_1^2|_{UV} = \frac{R - \frac{4\pi g^*}{e_1^{*2}} F^2}{4\lambda^*} \quad (21)$$

Those field-scale settings relate the UV scale k^2 proportional to the curvature scalar R , in agreement with the literature [54, 55, 56, 57, 58], but they go beyond and determine the constant proportionality factor and modifications due to the electromagnetic field strength.

After the “integrating out” is applied, the effective actions valid in the deep UV and at the fixed point in (18) and with the corresponding scale setting (20) is described by:

$$\tilde{\Gamma}_{UV,2} = \int d^4x \sqrt{-g} \left[\frac{R^2}{128 \pi g^* \lambda^*} - \frac{F^2}{4e_2^{*2}} \right] \quad (22)$$

and for the behavior (19) and the scale setting (21) the UV effective action results to be

$$\tilde{\Gamma}_{UV,1} = \int d^4x \sqrt{-g} \left[\frac{\left(R - \frac{4\pi g^*}{e_1^{*2}} F^2 \right)^2}{128 \pi g^* \lambda^*} \right] \quad (23)$$

For the case $F^2 \sim 0$, the R^2 dependence of the UV effective action in Asymptotic Safe gravity is indeed renormalizable [59], in agreement with the literature [55, 56, 58]. Those UV results confirm that the fixed points (18) and (19) correspond to different physical systems with different effective equations of motion for the background.

5. Summary and conclusion

In this proceeding we show a procedure for the scale setting, by promoting the scale k to a scale-field $k(x)$ at the level of effective action Γ_k . We have showed that one might define a scale setting that keeps consistency at the level of improved equations of motion and that the conservation of the stress-energy tensor can be guaranteed throughout the improving solutions procedure.

In order to show the functionality of the procedure, we discussed initially the approach for scalar ϕ^4 theory with spherically symmetric backgrounds at the one loop level.

Finally, this scale-field method is used to study the scale setting prescription in gravity coupled to an electromagnetic stress-energy tensor, represented by Einstein-Hilbert-Maxwell theory. It is explicitly shown that also in this example the conservation of a generalized stress-energy tensor is guaranteed by the scale-field setting. As application, the UV scale-field setting of Asymptotically Safe gravity coupled to an electromagnetic field strength is calculated and the scale independent effective action valid in the UV of this theory is derived by integrating the scale-field out.

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