Bare inverse propagator = two-point function

$$\frac{\delta^2 S}{\delta\varphi\delta\varphi} = \frac{\delta^2}{\delta\varphi\delta\varphi}\int d^4x\ \mathcal{L}_x$$

$$\mathcal{L} = \sqrt{g}\left(\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2\alpha_A}(\partial_\mu A^\mu)^2\right)$$

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wartości w punktach

$$\mathcal{L}(A^\mu(x), \partial A^\mu(x)) = \frac{1}{2}\sqrt{g}\left(\partial_\nu A_\mu \partial^\nu A^\mu - \partial_\mu A_\nu \partial^\nu A^\mu + \frac{1}{\alpha_A}\partial_\mu A^\mu \partial_\nu A^\nu\right)$$

$$S[A^\mu] = \int d^4x\ \mathcal{L} \qquad \frac{\delta}{\delta A^\mu}S = \frac{\partial \mathcal{L}}{\partial A^\mu} - \partial_\mu\frac{\partial\mathcal{L}}{\partial(\partial_\mu A)}$$

pole tensorowe jako cały objekt

Is it what I think! $\rightarrow$ $\frac{\partial S}{\partial A^\mu} = \frac{\partial \mathcal{L}}{\partial A^\mu} - \partial_a\frac{\partial\mathcal{L}}{\partial(\partial_a A^\mu)}$

$$\frac{1}{2}\sqrt{g}\left(\partial_\nu A_\mu \partial^\nu A^\mu - \partial_\mu A_\nu \partial^\nu A^\mu + \frac{1}{\alpha_A}\partial_\mu A^\mu \partial_\nu A^\nu\right)$$

$$0 - \frac{1}{2}\sqrt{g}\,\partial_a\Bigg(\frac{1}{\alpha_A}2\cdot\partial_c A^c\cdot\delta_{a\mu} + \underbrace{\partial_b A_c \cdot \underbrace{\frac{\partial(\partial^b A^c)}{\partial(\partial_a A^\mu)}}_{\pm\delta_{ba}\delta_{c\mu}}}_{\partial^a A_\mu} + \underbrace{\partial^b A^c \frac{\partial(\partial_b A_c)}{\partial(\partial_a A^\mu)}}_{\partial_a A^\mu} +$$

$$- \partial_b A_c \underbrace{\frac{\partial(\partial^c A^b)}{\partial(\partial_a A^\mu)}}_{(\pm\delta_{ac})\cdot\delta_{b\mu}} - \partial^c A^b \underbrace{\frac{\partial(\partial_b A_c)}{\partial(\partial_a A^\mu)}}_{\delta_{ab}(\pm\delta_{c\mu})}\Bigg) = -\sqrt{g}\left(\frac{1}{\alpha_A}\partial_\mu\partial_a A^a + \partial_a\partial^a A_\mu - \partial_a\partial_\mu A^a\right)$$

$$\partial_\mu A^a - \partial_\mu A^a$$

Dobrze – w mathematice jest $\frac{\delta S}{\delta A_\mu}$ więc jest μ na dole – tutaj na górze więc ok

Wynik : $\frac{\delta S}{\delta A^\mu} = -\sqrt{g}\left(\partial_a \partial^a A^\mu - \partial_a \partial^\mu A^a + \frac{1}{\alpha_1} \partial^\mu \partial_a A^a\right)$

Dla pochodnej funkcyjnej mamy:

$$\frac{\delta F[G[\varsigma]]}{\delta \varsigma(y)} = \int dx \left.\frac{\delta F[G]}{\delta G(x)}\right|_{G=G[\varsigma]} \cdot \frac{\delta G[\varsigma](x)}{\delta \varsigma(y)}$$

a jeśli G to lokalny funkcjonał, czyli różniczkowalna funkcja, to

$$\frac{\delta F[g(\varsigma)]}{\delta \varsigma(y)} = \frac{\delta F[g(\varsigma)]}{\delta g[\varsigma(y)]} \cdot \frac{d g(\varsigma)}{d \varsigma(y)}$$

U nas : $S = S[A^\mu] = \int \mathcal{L}(A^\mu, \partial_a A^\mu) = \int \mathcal{L}(\mathcal{F}[A_p^\mu], \partial_a \mathcal{F}[A_p^\mu]) = S[\mathcal{F}[A_p^\mu]]$

więc $\frac{\delta S[\mathcal{F}[A_p^\mu]]}{\delta A^\mu} = \int dp \left(\frac{\delta S[A_p^\mu]}{\delta A_p^\mu} \cdot \frac{\delta \mathcal{F}[A^\mu](p)}{\delta A^\mu}\right)$

$\mathcal{F}[\varphi] = \int dx\, e^{-ipx} \varphi(x) = \int dx\, \mathcal{L}(x, \varphi(x))$ $\quad \frac{\delta \mathcal{F}[\varphi]}{\delta \varphi}(p) = e^{-ipx} - 0 = e^{-ipx}$

$$\Rightarrow \frac{\delta S[\mathcal{F}[A_p^\mu]]}{\delta A^\mu} = \int dp \left(\frac{\delta S[A_p^\mu]}{\delta A_p^\mu}(p) \cdot e^{-ipx}\right) = \int dx\, e^{-ipx} \frac{\delta S[A^\mu]}{\delta A^\mu}(x)$$

$$= \mathcal{F}\left[\frac{\delta S[A^\mu]}{\delta A^\mu}\right]$$

wynik ten sam A_p^μ, A^μ tylko kwestia jak to nazywamy
stąd dostajemy $\partial_\mu \to$ momentum
zakładam że $(2\pi)^n$ z transformaty i z pochodnych się pokoś zjedają

$\Downarrow$

$$\frac{\delta}{\delta A^\mu} \int \mathcal{L}(p, \mathcal{F}[A^\mu](p)) = \frac{\delta S'}{\delta A^\mu}[\mathcal{F}[A^\mu]] \rightsquigarrow \mathcal{F}[\, p^\mu p_\mu + (\tfrac{1}{\alpha}-1)(p^\mu)^2]$$

$$\frac{\delta S[A_p^{\ \mu}]}{\delta A^\mu} = -\sqrt{g}\,\Big(\underset{p^\mu p_a A_p^a}{\tfrac{1}{\alpha_A} \mathcal{F} \partial^\mu \partial_a A^a} + \underset{\underbrace{p_a p^a}_{\text{momentum squared}} \cdot A_p^\mu}{\mathcal{F} \partial_a \partial^a A^\mu} - \underset{p_a p^\mu A_p^a}{\mathcal{F} \partial_a \partial^\mu A^a} \Big) = -\sqrt{g}\,\big(p^2 A_p^\mu + (\tfrac{1}{\alpha_A}-1) p^\mu p_a A_p^a \big)$$

<u>Druga wariacja:</u> $-\sqrt{g} \int dp \; e^{-ipx} \frac{\delta}{\delta A_p^\mu} \big(p^2 A_p^\mu + (\tfrac{1}{\alpha_A}-1) p^\mu p_a A_p^a \big)$

↳ Dlaczego żeby wyszło dobrze, domnożyć trzeba A_p^μ przed δ ?

$$A_p^{\ a}\big(-(\tfrac{1}{\alpha}-1) A_p^{\ b} p_a p_b + A_{p\,a} \cdot p^2 \big) = -(\tfrac{1}{\alpha}-1) A_p^{\ a} p_a A_p^{\ b} p_b + A_p^{\ a} A_{p\,a}\, p^2$$

$$= -(\tfrac{1}{\alpha}-1)(A_p^{\ a} p_a)^2 + \underset{\hookrightarrow A_\mu A^\mu}{A_p^2}\, p^2 = (1-\tfrac{1}{\alpha_A})(A_p^{\ a} p_a)^2 + A_p^2\, p^2$$

$$\frac{\delta}{\delta A^\nu} \int \mathcal{L}(x, \mathcal{F}^{-1}[A_p^\mu]) = \mathcal{F}^{-1}\Big[\frac{\delta}{\delta A_p^\nu} \int \mathcal{L}(p, A_p^\mu) \Big]$$

$\rightsquigarrow$ Dalej zostać chcemy w p-space, nie wykonamy $\mathcal{F}^{-1}$

$$\longrightarrow A_p^\nu \frac{\delta}{\delta A_p^\nu} A_p^\mu \frac{\delta}{\delta A_p^\mu} S =$$

$$= (1-\tfrac{1}{\alpha_A}) A^a A^\nu p_a p_\nu - A_\nu A^\nu p^2$$