Chapter 9

Solutions to Odd-Numbered Exercises

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Abstract

This document presents the solution of "Quantum Computing Explained by David McMAHON" exercises.

Exercise 9.1

we know

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

since we need to compute $(H \otimes H)(|0\rangle \otimes |1\rangle)$

$$\begin{aligned}(H \otimes H)(|0\rangle \otimes |1\rangle) &= H|0\rangle \otimes H|1\rangle = |+\rangle \otimes |-\rangle \\ &= \left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right) \otimes \left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)\end{aligned}$$

Exercise 9.3

to compute $HP(\theta)HP(\phi)$ we have

$$HP(\theta) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & e^{i\theta} \\ 1 & -e^{i\theta} \end{pmatrix}$$

$$HP(\phi) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & e^{i\phi} \\ 1 & -e^{i\phi} \end{pmatrix}$$

so

$$\begin{aligned}HP(\theta)HP(\phi) &= \frac{1}{2}\begin{pmatrix} 1 & e^{i\theta} \\ 1 & -e^{i\theta} \end{pmatrix}\begin{pmatrix} 1 & e^{i\phi} \\ 1 & -e^{i\phi} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} e^{i\theta}+1 & e^{i\phi} - e^{i\phi}e^{i\theta} \\ -e^{i\theta}+1 & e^{i\phi} + e^{i\phi}e^{i\theta} \end{pmatrix} \\ &= \frac{1}{2}\begin{pmatrix} e^{i\theta}+1 & e^{i\phi}(-e^{i\theta}+1) \\ -e^{i\theta}+1 & e^{i\phi}(e^{i\theta}+1) \end{pmatrix} \qquad (1)\end{aligned}$$

now for $e^{i\theta}+1$

$$\begin{aligned}e^{i\theta}+1 = \cos\theta + i\sin\theta + 1 = 2\cos\frac{\theta}{2}\cos\frac{\theta}{2} - 1 + i\sin\theta + 1 &= 2\cos\frac{\theta}{2}\cos\frac{\theta}{2} + i2\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ &= 2\cos\frac{\theta}{2}(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}) \\ &= 2\cos\frac{\theta}{2}e^{i\theta/2}\end{aligned}$$

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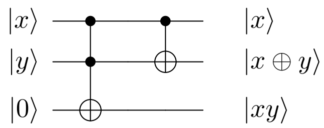
and for $-e^{i\theta}+1$

$$\begin{aligned}
-e^{i\theta}+1 = -\cos\theta - i\sin\theta + 1 = 2\sin\frac{\theta}{2}\sin\frac{\theta}{2} - 1 - i\sin\theta + 1 &= 2\sin\frac{\theta}{2}\sin\frac{\theta}{2} - i2\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\
&= -2i\sin\frac{\theta}{2}(i\sin\frac{\theta}{2}+\cos\frac{\theta}{2}) \\
&= -2i\sin\frac{\theta}{2}e^{i\theta/2}
\end{aligned}$$

we can replace them into 1

$$HP(\theta)HP(\phi) = \frac{1}{2}\begin{pmatrix} 2\cos\frac{\theta}{2}e^{i\theta/2} & e^{i\phi}(-2i\sin\frac{\theta}{2}e^{i\theta/2}) \\ -2i\sin\frac{\theta}{2}e^{i\theta/2} & e^{i\phi}(2\cos\frac{\theta}{2}e^{i\theta/2}) \end{pmatrix} = e^{i\theta/2}\begin{pmatrix} \cos\frac{\theta}{2} & -ie^{i\phi}\sin\frac{\theta}{2} \\ -i\sin\frac{\theta}{2} & e^{i\phi}\cos\frac{\theta}{2} \end{pmatrix}$$

Exercise 9.5



Exercise 9.7

to derive relation (9.67) we have

$$W = 2|\psi\rangle\langle\psi| - I$$

$$|\psi'\rangle = \frac{1}{\sqrt{2^n-1}}\sum_{x\in\{0,1\}^n, x\neq x'}|x\rangle$$

and we have

$$|x'\rangle = \sqrt{2^n}\,|\psi\rangle - \sqrt{2^n-1}\,|\psi'\rangle$$

to get $W\,|\psi'\rangle$

$$\begin{aligned}
W\,|\psi'\rangle &= \Big(2|\psi\rangle\langle\psi| - I\Big)\Big(\frac{\sqrt{2^n}}{\sqrt{2^n-1}}\,|\psi\rangle - \frac{1}{\sqrt{2^n-1}}\,|x'\rangle\Big) \\
&= \frac{2\sqrt{2^n}}{\sqrt{2^n-1}}\,|\psi\rangle - \frac{2}{\sqrt{2^n-1}}|\psi\rangle\langle\psi|x'\rangle - \frac{\sqrt{2^n}}{\sqrt{2^n-1}}\,|\psi\rangle + \frac{1}{\sqrt{2^n-1}}\,|x'\rangle \\
&= \frac{\sqrt{2^n}}{\sqrt{2^n-1}}\,|\psi\rangle - \frac{2}{\sqrt{2^n}\sqrt{2^n-1}}\,|\psi\rangle + \frac{1}{\sqrt{2^n-1}}\,|x'\rangle \\
&= \frac{2^n-2}{\sqrt{2^n}\sqrt{2^n-1}}\,|\psi\rangle + \frac{1}{\sqrt{2^n-1}}\,|x'\rangle \\
&= \frac{2^n-2}{\sqrt{2^n}\sqrt{2^n-1}}\Big[\frac{\sqrt{2^n-1}}{\sqrt{2^n}}\,|\psi'\rangle + \frac{1}{\sqrt{2^n}}\,|x'\rangle\Big] + \frac{1}{\sqrt{2^n-1}}\,|x'\rangle \\
&= \frac{2^n-2}{2^n}\,|\psi'\rangle + \frac{2^n-2}{2^n\sqrt{2^n-1}}\,|x'\rangle + \frac{2^n}{2^n\sqrt{2^n-1}}\,|x'\rangle \\
&= -(\frac{2}{2^n}-1)\,|\psi'\rangle + \frac{2(2^n-1)}{2^n\sqrt{2^n-1}}\,|x'\rangle \\
&= -(\frac{2}{2^n}-1)\,|\psi'\rangle + \frac{2\sqrt{2^n-1}\sqrt{2^n-1}}{2^n\sqrt{2^n-1}}\,|x'\rangle \\
&= -(\frac{2}{2^n}-1)\,|\psi'\rangle + \frac{2\sqrt{2^n-1}}{2^n}\,|x'\rangle
\end{aligned}$$