Chapter 8

Universality of the NAND Gate

MohamadAli Khajeian*

Faculty of Engineering Sciences, University of Tehran, Iran

December 21, 2024

Abstract

This document presents the solution of "Quantum Computing Explained by David McMAHON" exercises.

Proof

The universality of the NAND gate is demonstrated by showing it can reproduce all basic Boolean operations (AND, OR, NOT), which are sufficient to construct any Boolean function.

NOT Gate

The NOT operation can be derived using a single NAND gate by tying both inputs together:

$$\text{NAND}(A, A) = \neg A$$

AND Gate

The AND operation can be obtained by negating the output of a NAND gate:

$$\text{AND}(A, B) = \neg(\text{NAND}(A, B)) = \text{NAND}(\text{NAND}(A, B), \text{NAND}(A, B))$$

OR Gate

Using De Morgan's laws, the OR operation is constructed as follows:

$$\text{OR}(A, B) = \neg(\neg A \wedge \neg B) = \text{NAND}(\text{NAND}(A, A), \text{NAND}(B, B))$$

Since any Boolean function can be expressed in terms of AND, OR, and NOT, the NAND gate can replicate all Boolean logic operations, proving its universality.

*khajeian@ut.ac.ir