Chapter 8

Proof of Gate Decomposition of Figure 8.6

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December 22, 2024

Abstract

This document presents the solution of "Quantum Computing Explained by David McMAHON" exercises.

Proof

Let

$$A = R_z(\alpha)R_y\left(\frac{\beta}{2}\right), \quad B = R_y\left(-\frac{\beta}{2}\right)R_z\left(-\frac{\alpha+\gamma}{2}\right), \quad C = R_z\left(-\frac{\alpha-\gamma}{2}\right).$$

Then

$$\begin{aligned} AXBXC &= R_z(\alpha)R_y\left(\frac{\beta}{2}\right)XR_y\left(-\frac{\beta}{2}\right)R_z\left(-\frac{\alpha+\gamma}{2}\right)XR_z\left(-\frac{\alpha-\gamma}{2}\right) \\ &= R_z(\alpha)R_y\left(\frac{\beta}{2}\right)\left[XR_y\left(-\frac{\beta}{2}\right)X\right]\left[XR_z\left(-\frac{\alpha+\gamma}{2}\right)X\right]R_z\left(-\frac{\alpha-\gamma}{2}\right) \\ &= R_z(\alpha)R_y\left(\frac{\beta}{2}\right)R_y\left(\frac{\beta}{2}\right)R_z\left(\frac{\alpha+\gamma}{2}\right)R_z\left(-\frac{\alpha-\gamma}{2}\right) \\ &= R_z(\alpha)R_y(\beta)R_z(\gamma) = U, \end{aligned}$$

as required. where use has been made of the identities $X^2 = I$ and $X\sigma_{y,z}X = -\sigma_{y,z}$. It is also verified that

$$\begin{aligned} ABC &= R_z(\alpha)R_y\left(\frac{\beta}{2}\right)R_y\left(-\frac{\beta}{2}\right)R_z\left(-\frac{\alpha+\gamma}{2}\right)R_z\left(-\frac{\alpha-\gamma}{2}\right) \\ &= R_z(\alpha)R_y(0)R_z(-\alpha) = I. \end{aligned}$$

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