Chapter 8

Solutions to Try it

MohamadAli Khajeian*

Faculty of Engineering Sciences, University of Tehran, Iran

December 21, 2024

Abstract

This document presents the solution of "Quantum Computing Explained by David McMAHON" exercises.

Try it - (page 175)

Using De Morgan's laws, the OR operation is constructed as follows:

$$\mathrm{OR}(A,B) = \neg(\neg A \wedge \neg B) = \mathrm{NAND}(\mathrm{NAND}(A,A),\mathrm{NAND}(B,B))$$

Try it - (page 179)

$$U^H_{\mathrm{NOT}} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = |0\rangle\langle 0| - |1\rangle\langle 1|$$

$$|+\rangle = \frac{1}{\sqrt{2}}\big(\,|0\rangle + |1\rangle\,\big)$$
$$|-\rangle = \frac{1}{\sqrt{2}}\big(\,|0\rangle - |1\rangle\,\big)$$

now we can apply it on $|+\rangle$

$$U^H_{\mathrm{NOT}}\,|+\rangle = \big(|0\rangle\langle 0| - |1\rangle\langle 1|\big)\frac{1}{\sqrt{2}}\big(\,|0\rangle + |1\rangle\,\big) = \frac{1}{\sqrt{2}}\big(\,|0\rangle - |1\rangle\,\big) = |-\rangle$$

also for $|-\rangle$

$$U^H_{\mathrm{NOT}}\,|-\rangle = \big(|0\rangle\langle 0| - |1\rangle\langle 1|\big)\frac{1}{\sqrt{2}}\big(\,|0\rangle - |1\rangle\,\big) = \frac{1}{\sqrt{2}}\big(\,|0\rangle + |1\rangle\,\big) = |+\rangle$$

Try it - (page 181)

$$Y = -i|0\rangle\langle 1| + i|1\rangle\langle 0|$$

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$$

we can apply Y operator on it

$$Y\,|\psi\rangle = \big(-i|0\rangle\langle 1| + i|1\rangle\langle 0|\big)\big(\alpha\,|0\rangle + \beta\,|1\rangle\,\big) = -i\beta\,|0\rangle + i\alpha\,|1\rangle$$

*khajeian@ut.ac.ir

Try it - (page 183)

Hadamard operator is

$$H = \frac{1}{\sqrt{2}}\big(|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| - |1\rangle\langle 1|\big)$$

and our qubit is

$$|\psi\rangle = \cos\theta\,|0\rangle + e^{i\phi}\sin\theta\,|1\rangle$$

we can apply Hadamard on it

$$\begin{aligned}
H\,|\psi\rangle = \cos\theta H\,|0\rangle + e^{i\phi}\sin\theta H\,|1\rangle &= \cos\theta\,|+\rangle + e^{i\phi}\sin\theta\,|-\rangle \\
&= \cos\theta\big(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\big) + e^{i\phi}\sin\theta\big(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\big) \\
&= \big(\frac{\cos\theta\,|0\rangle + e^{i\phi}\sin\theta\,|0\rangle}{\sqrt{2}}\big) + \big(\frac{\cos\theta\,|1\rangle - e^{i\phi}\sin\theta\,|1\rangle}{\sqrt{2}}\big) \\
&= \big(\frac{\cos\theta + e^{i\phi}\sin\theta}{\sqrt{2}}\big)\,|0\rangle + \big(\frac{\cos\theta - e^{i\phi}\sin\theta}{\sqrt{2}}\big)\,|1\rangle = |\psi^{'}\rangle
\end{aligned}$$

the probability that this measurement finds the system in the state $|1\rangle$

$$\begin{aligned}
\langle\psi^{'}|\mathrm{P}_1|\psi^{'}\rangle &= \left(\frac{\cos\theta - e^{i\phi}\sin\theta}{\sqrt{2}}\right)^{*}\left(\frac{\cos\theta - e^{i\phi}\sin\theta}{\sqrt{2}}\right) \\
&= \frac{1}{2}\big(\cos\theta - e^{-i\phi}\sin\theta\big)\big(\cos\theta - e^{i\phi}\sin\theta\big) \\
&= \frac{1}{2}\big(\cos^2\theta + \sin^2\theta\big) - \frac{1}{2}\sin\theta\cos\theta\big(e^{i\phi} + e^{-i\phi}\big) = \frac{1}{2}\big(1 - \sin\theta\cos\theta\cos\phi\big)
\end{aligned}$$

the probability that this measurement finds the system in the state $|0\rangle$

$$\begin{aligned}
\langle\psi^{'}|\mathrm{P}_0|\psi^{'}\rangle &= \left(\frac{\cos\theta + e^{i\phi}\sin\theta}{\sqrt{2}}\right)^{*}\left(\frac{\cos\theta + e^{i\phi}\sin\theta}{\sqrt{2}}\right) \\
&= \frac{1}{2}\big(\cos\theta + e^{-i\phi}\sin\theta\big)\big(\cos\theta + e^{i\phi}\sin\theta\big) \\
&= \frac{1}{2}\big(\cos^2\theta + \sin^2\theta\big) + \frac{1}{2}\sin\theta\cos\theta\big(e^{i\phi} + e^{-i\phi}\big) = \frac{1}{2}\big(1 + \sin\theta\cos\theta\cos\phi\big)
\end{aligned}$$

Try it - (page 184)

$$\exp(-i\theta U) = \cos\theta I - i\sin\theta U$$

we can repalce $\theta = \dfrac{\gamma}{2}$ and $U = Z$,

$$\begin{aligned}
\exp(-i\frac{\gamma}{2}Z) = \cos\frac{\gamma}{2}I - i\sin\frac{\gamma}{2}Z &= \cos\frac{\gamma}{2}\big(|0\rangle\langle 0| + |1\rangle\langle 1|\big) - i\sin\frac{\gamma}{2}\big(|0\rangle\langle 0| - |1\rangle\langle 1|\big) \\
&= \big(\cos\frac{\gamma}{2} - i\sin\frac{\gamma}{2}\big)|0\rangle\langle 0| + \big(\cos\frac{\gamma}{2} + i\sin\frac{\gamma}{2}\big)|1\rangle\langle 1| \\
&= e^{-i\gamma/2}|0\rangle\langle 0| + e^{i\gamma/2}|1\rangle\langle 1| = R_z(\gamma)
\end{aligned}$$