Chapter 4

Solutions to Even-Numbered Exercises

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Abstract

This document presents the solution of "Quantum Computing Explained by David McMAHON" exercises.

Exercise 4.2

The basis states for $H \equiv \mathbb{C}^4$ can be constructed by using $|+\rangle, |-\rangle$ as the basis for H_1 and H_2.

$$\begin{aligned}
|w_1\rangle &= |+\rangle|+\rangle \\
|w_2\rangle &= |+\rangle|-\rangle \\
|w_3\rangle &= |-\rangle|+\rangle \\
|w_4\rangle &= |-\rangle|-\rangle
\end{aligned}$$

we have

$$\begin{aligned}
\langle w_3|w_4\rangle &= (\langle -|\langle +|)(|-\rangle|-\rangle) = \langle -|-\rangle\langle +|-\rangle = (1)(0) = 0 \\
\langle w_4|w_3\rangle &= (\langle -|\langle -|)(|-\rangle|+\rangle) = \langle -|-\rangle\langle -|+\rangle = (1)(0) = 0
\end{aligned}$$

Exercise 4.4

To calculate the tensor product of

$$|\psi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} \quad \text{and} \quad |\phi\rangle = \frac{1}{2}\begin{pmatrix}1\\\sqrt{3}\end{pmatrix}$$

we have

$$|\psi\rangle \otimes |\phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} \otimes \frac{1}{2}\begin{pmatrix}1\\\sqrt{3}\end{pmatrix} = \frac{1}{2\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} \otimes \begin{pmatrix}1\\\sqrt{3}\end{pmatrix}$$

then

$$|\psi\rangle \otimes |\phi\rangle = \frac{1}{2\sqrt{2}}\begin{pmatrix}1\\\sqrt{3}\\1\\\sqrt{3}\end{pmatrix}.$$

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Exercise 4.6

No we can't.

Exercise 4.8

To show that $(A \otimes B)^\dagger = A^\dagger \otimes B^\dagger$, assume

$$|x\rangle = |x_1\rangle \otimes |x_2\rangle \tag{1}$$

$$|y\rangle = |y_1\rangle \otimes |y_2\rangle \tag{2}$$

when you apply the operator $A \otimes B$ to a product state, say $|i\rangle \otimes |j\rangle$, it acts as follows

$$(A \otimes B)(|i\rangle \otimes |j\rangle) = A|i\rangle \otimes B|j\rangle.$$

now using 1 and 2,

$$\langle y|(A \otimes B)|x\rangle = \langle y_1|A|x_1\rangle \cdot \langle y_2|B|x_2\rangle. \tag{3}$$

we know

$$\langle x|(A \otimes B)^\dagger|y\rangle = \langle y|(A \otimes B)|x\rangle^*.$$

using 3,

$$\begin{aligned}\langle x|(A \otimes B)^\dagger|y\rangle &= \langle y|(A \otimes B)|x\rangle^* \\ &= \langle y_1|A|x_1\rangle^* \cdot \langle y_2|B|x_2\rangle^* \\ &= \langle x_1|A^\dagger|y_1\rangle \cdot \langle x_2|B^\dagger|y_2\rangle.\end{aligned}$$

we have

$$\langle x|(A^\dagger \otimes B^\dagger)|y\rangle = \langle x_1|A^\dagger|y_1\rangle \cdot \langle x_2|B^\dagger|y_2\rangle = \langle x|(A \otimes B)^\dagger|y\rangle.$$

since both expressions yield the same result, we conclude that

$$(A \otimes B)^\dagger = A^\dagger \otimes B^\dagger.$$

Exercise 4.10

Let's write down the Pauli matrices

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

now we have

$$X \otimes Y = \begin{pmatrix} (0)Y & (1)Y \\ (1)Y & (0)Y \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}.$$