Chapter 7

Proof of Relation 7.39, 7.40, 7.41, 7.42

MohamadAli Khajeian*

Faculty of Engineering Sciences, University of Tehran, Iran

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Abstract

This document presents the solution of "Quantum Computing Explained by David McMAHON" exercises.

Proof

$$|\beta_{00}\rangle\langle\beta_{00}| = \frac{1}{4}\Big(\mathrm{I}\otimes\mathrm{I} + \mathrm{X}\otimes\mathrm{X} - \mathrm{Y}\otimes\mathrm{Y} + \mathrm{Z}\otimes\mathrm{Z}\Big) \tag{1}$$

$$|\beta_{01}\rangle\langle\beta_{01}| = \frac{1}{4}\Big(\mathrm{I}\otimes\mathrm{I} + \mathrm{X}\otimes\mathrm{X} + \mathrm{Y}\otimes\mathrm{Y} - \mathrm{Z}\otimes\mathrm{Z}\Big) \tag{2}$$

$$|\beta_{10}\rangle\langle\beta_{10}| = \frac{1}{4}\Big(\mathrm{I}\otimes\mathrm{I} - \mathrm{X}\otimes\mathrm{X} + \mathrm{Y}\otimes\mathrm{Y} + \mathrm{Z}\otimes\mathrm{Z}\Big) \tag{3}$$

$$|\beta_{11}\rangle\langle\beta_{11}| = \frac{1}{4}\Big(\mathrm{I}\otimes\mathrm{I} - \mathrm{X}\otimes\mathrm{X} - \mathrm{Y}\otimes\mathrm{Y} - \mathrm{Z}\otimes\mathrm{Z}\Big) \tag{4}$$

7.39

to proof 1 we have

$$\begin{aligned}
&\frac{1}{4}\Bigg(\Bigg(\big(|0\rangle\langle 0| + |1\rangle\langle 1|\big)\otimes\big(|0\rangle\langle 0| + |1\rangle\langle 1|\big)\Bigg) + \Bigg(\big(|0\rangle\langle 1| + |1\rangle\langle 0|\big)\otimes\big(|0\rangle\langle 1| + |1\rangle\langle 0|\big)\Bigg)\\
&-\Bigg(\big(-i|0\rangle\langle 1| + i|1\rangle\langle 0|\big)\otimes\big(-i|0\rangle\langle 1| + i|1\rangle\langle 0|\big)\Bigg) + \Bigg(\big(|0\rangle\langle 0| - |1\rangle\langle 1|\big)\otimes\big(|0\rangle\langle 0| - |1\rangle\langle 1|\big)\Bigg)\Bigg)\\
&= \frac{1}{4}\Bigg(\big(|00\rangle\langle 00| + |01\rangle\langle 01| + |10\rangle\langle 10| + |11\rangle\langle 11|\big) + \big(|00\rangle\langle 11| + |01\rangle\langle 10| + |10\rangle\langle 01| + |11\rangle\langle 00|\big)\\
&-\big(-|00\rangle\langle 11| + |01\rangle\langle 10| + |10\rangle\langle 01| - |11\rangle\langle 00|\big) + \big(|00\rangle\langle 00| - |01\rangle\langle 01| - |10\rangle\langle 10| + |11\rangle\langle 11|\big)\Bigg)\\
&= \frac{1}{4}\Bigg(2|00\rangle\langle 00| + 2|00\rangle\langle 11| + 2|11\rangle\langle 00| + 2|11\rangle\langle 11|\Bigg)\\
&= \frac{1}{2}\Bigg(|00\rangle\langle 00| + |00\rangle\langle 11| + |11\rangle\langle 00| + |11\rangle\langle 11|\Bigg) = \Bigg(\frac{|00\rangle + |11\rangle}{\sqrt{2}}\Bigg)\Bigg(\frac{\langle 00| + \langle 11|}{\sqrt{2}}\Bigg) = |\beta_{00}\rangle\langle\beta_{00}|
\end{aligned}$$

*khajeian@ut.ac.ir

7.40

to proof 2 we have

$$
\begin{aligned}
&\frac{1}{4}\Bigg(\Big(\big(|0\rangle\langle 0|+|1\rangle\langle 1|\big)\otimes\big(|0\rangle\langle 0|+|1\rangle\langle 1|\big)\Big)+\Big(\big(|0\rangle\langle 1|+|1\rangle\langle 0|\big)\otimes\big(|0\rangle\langle 1|+|1\rangle\langle 0|\big)\Big)\\
&+\Big(\big(-i|0\rangle\langle 1|+i|1\rangle\langle 0|\big)\otimes\big(-i|0\rangle\langle 1|+i|1\rangle\langle 0|\big)\Big)-\Big(\big(|0\rangle\langle 0|-|1\rangle\langle 1|\big)\otimes\big(|0\rangle\langle 0|-|1\rangle\langle 1|\big)\Big)\Bigg)\\
&=\frac{1}{4}\Big(\big(|00\rangle\langle 00|+|01\rangle\langle 01|+|10\rangle\langle 10|+|11\rangle\langle 11|\big)+\big(|00\rangle\langle 11|+|01\rangle\langle 10|+|10\rangle\langle 01|+|11\rangle\langle 00|\big)\\
&\big(-|00\rangle\langle 11|+|01\rangle\langle 10|+|10\rangle\langle 01|-|11\rangle\langle 00|\big)-\big(|00\rangle\langle 00|-|01\rangle\langle 01|-|10\rangle\langle 10|+|11\rangle\langle 11|\big)\Big)\\
&=\frac{1}{4}\Big(2|01\rangle\langle 01|+2|10\rangle\langle 10|+2|01\rangle\langle 10|+2|10\rangle\langle 01|\Big)\\
&=\frac{1}{2}\Big(|01\rangle\langle 01|+|10\rangle\langle 10|+|01\rangle\langle 10|+|10\rangle\langle 01|\Big)=\Big(\frac{|01\rangle+|10\rangle}{\sqrt{2}}\Big)\Big(\frac{\langle 01|+\langle 10|}{\sqrt{2}}\Big)=|\beta_{01}\rangle\langle\beta_{01}|
\end{aligned}
$$

7.41

to proof 3 we have

$$
\begin{aligned}
&\frac{1}{4}\Bigg(\Big(\big(|0\rangle\langle 0|+|1\rangle\langle 1|\big)\otimes\big(|0\rangle\langle 0|+|1\rangle\langle 1|\big)\Big)-\Big(\big(|0\rangle\langle 1|+|1\rangle\langle 0|\big)\otimes\big(|0\rangle\langle 1|+|1\rangle\langle 0|\big)\Big)\\
&+\Big(\big(-i|0\rangle\langle 1|+i|1\rangle\langle 0|\big)\otimes\big(-i|0\rangle\langle 1|+i|1\rangle\langle 0|\big)\Big)+\Big(\big(|0\rangle\langle 0|-|1\rangle\langle 1|\big)\otimes\big(|0\rangle\langle 0|-|1\rangle\langle 1|\big)\Big)\Bigg)\\
&=\frac{1}{4}\Big(\big(|00\rangle\langle 00|+|01\rangle\langle 01|+|10\rangle\langle 10|+|11\rangle\langle 11|\big)-\big(|00\rangle\langle 11|+|01\rangle\langle 10|+|10\rangle\langle 01|+|11\rangle\langle 00|\big)\\
&\big(-|00\rangle\langle 11|+|01\rangle\langle 10|+|10\rangle\langle 01|-|11\rangle\langle 00|\big)+\big(|00\rangle\langle 00|-|01\rangle\langle 01|-|10\rangle\langle 10|+|11\rangle\langle 11|\big)\Big)\\
&=\frac{1}{4}\Big(2|00\rangle\langle 00|+2|11\rangle\langle 11|-2|00\rangle\langle 11|-2|11\rangle\langle 00|\Big)\\
&=\frac{1}{2}\Big(|00\rangle\langle 00|+|11\rangle\langle 11|-|00\rangle\langle 11|-|11\rangle\langle 00|\Big)=\Big(\frac{|00\rangle-|11\rangle}{\sqrt{2}}\Big)\Big(\frac{\langle 00|-\langle 11|}{\sqrt{2}}\Big)=|\beta_{10}\rangle\langle\beta_{10}|
\end{aligned}
$$

7.42

to proof 4 we have

$$
\begin{aligned}
&\frac{1}{4}\Bigg(\Big(\big(|0\rangle\langle 0|+|1\rangle\langle 1|\big)\otimes\big(|0\rangle\langle 0|+|1\rangle\langle 1|\big)\Big)-\Big(\big(|0\rangle\langle 1|+|1\rangle\langle 0|\big)\otimes\big(|0\rangle\langle 1|+|1\rangle\langle 0|\big)\Big)\\
&-\Big(\big(-i|0\rangle\langle 1|+i|1\rangle\langle 0|\big)\otimes\big(-i|0\rangle\langle 1|+i|1\rangle\langle 0|\big)\Big)-\Big(\big(|0\rangle\langle 0|-|1\rangle\langle 1|\big)\otimes\big(|0\rangle\langle 0|-|1\rangle\langle 1|\big)\Big)\Bigg)\\
&=\frac{1}{4}\Big(\big(|00\rangle\langle 00|+|01\rangle\langle 01|+|10\rangle\langle 10|+|11\rangle\langle 11|\big)-\big(|00\rangle\langle 11|+|01\rangle\langle 10|+|10\rangle\langle 01|+|11\rangle\langle 00|\big)\\
&-\big(-|00\rangle\langle 11|+|01\rangle\langle 10|+|10\rangle\langle 01|-|11\rangle\langle 00|\big)-\big(|00\rangle\langle 00|-|01\rangle\langle 01|-|10\rangle\langle 10|+|11\rangle\langle 11|\big)\Big)\\
&=\frac{1}{4}\Big(2|01\rangle\langle 01|+2|10\rangle\langle 10|-2|01\rangle\langle 10|-2|10\rangle\langle 01|\Big)\\
&=\frac{1}{2}\Big(|01\rangle\langle 01|+|10\rangle\langle 10|-|01\rangle\langle 10|-|10\rangle\langle 01|\Big)=\Big(\frac{|01\rangle-|10\rangle}{\sqrt{2}}\Big)\Big(\frac{\langle 01|-\langle 10|}{\sqrt{2}}\Big)=|\beta_{11}\rangle\langle\beta_{11}|
\end{aligned}
$$