# Chapter 2

Solutions to Even-Numbered Exercises

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October 17, 2024

**Abstract**

This document presents the solution of "Quantum Computing Explained by David McMAHON" exercises.

## Exercise 2.2

Two quantum states are given by

$$|a\rangle = \begin{pmatrix} -4i \\ 2 \end{pmatrix}, \quad |b\rangle = \begin{pmatrix} 1 \\ -1+i \end{pmatrix}$$

### (A)

$$|a+b\rangle = -4i|0\rangle + 2|1\rangle + 1|0\rangle + (i-1)|1\rangle = (-4i+1)|0\rangle + (i+1)|1\rangle$$

### (B)

$$|a\rangle = \begin{pmatrix} -4i \\ 2 \end{pmatrix}, \quad |b\rangle = \begin{pmatrix} 1 \\ -1+i \end{pmatrix}$$

First, we calculate:

$$3|a\rangle = -12i|0\rangle + 6|1\rangle, \quad 2|b\rangle = 2|0\rangle + (2i-2)|1\rangle$$

Now, subtract the two vectors:

$$3|a\rangle - 2|b\rangle = (-12i-2)|0\rangle + (8-2i)|1\rangle$$

### (C)

$$\|a\| = \sqrt{(4i\langle 0| + 2\langle 1|)(-4i|0\rangle + 2|1\rangle)}$$

$$= \sqrt{16\langle 0|0\rangle + 8i\langle 0|1\rangle - 8i\langle 1|0\rangle + 4\langle 1|1\rangle} = \sqrt{16+4} = \sqrt{20} \Longrightarrow |a\rangle = \frac{1}{\sqrt{20}}(-4i|0\rangle + 2|1\rangle)$$

$$\|b\| = \sqrt{(1\langle 0| + (-1-i)\langle 1|)(1|0\rangle + (-1+i)|1\rangle)}$$

$$= \sqrt{1\langle 0|0\rangle + (-1+i)\langle 0|1\rangle + (-1-i)\langle 1|0\rangle + 2\langle 1|1\rangle} = \sqrt{2+1} = \sqrt{3} \Longrightarrow |b\rangle = \frac{1}{\sqrt{3}}(1|0\rangle + (-1+i)|1\rangle)$$

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# Exercise 2.4

A quantum system is in the state

$$|\psi\rangle = \frac{3i\,|0\rangle + 4\,|1\rangle}{5}$$

### (A)

Yes, because we have

$$|\psi\rangle = \frac{1}{5}(3i\,|0\rangle + 4\,|1\rangle)$$

$$\|\psi\| = \sqrt{\frac{1}{25}(-3i\,\langle 0| + 4\,\langle 1|)(3i\,|0\rangle + 4\,|1\rangle)} = \frac{1}{5}\sqrt{(-3i\,\langle 0| + 4\,\langle 1|)(3i\,|0\rangle + 4\,|1\rangle)}$$

$$= \frac{1}{5}\sqrt{9\,\langle 0|0\rangle - 12i\,\langle 0|1\rangle + 12i\,\langle 1|0\rangle + 16\,\langle 1|1\rangle} = \frac{1}{5}\sqrt{9+16} = \frac{1}{5}\sqrt{25} = 1$$

### (B)

Now, we know

$$|\psi\rangle = (\frac{3i}{5}\,|0\rangle + \frac{4}{5}\,|1\rangle) \tag{1}$$

We need to find $\alpha$ and $\beta$ such that satisfy

$$|\phi\rangle = \alpha\,|+\rangle + \beta\,|-\rangle$$

Where

$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}, \quad |-\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

So

$$|\phi\rangle = \alpha(\frac{|0\rangle + |1\rangle}{\sqrt{2}}) + \beta(\frac{|0\rangle - |1\rangle}{\sqrt{2}}) = (\frac{\alpha+\beta}{\sqrt{2}})\,|0\rangle + (\frac{\alpha-\beta}{\sqrt{2}})\,|1\rangle \tag{2}$$

From (1) and (2)

$$\frac{\alpha+\beta}{\sqrt{2}} = \frac{3i}{5} \Longrightarrow \alpha+\beta = \frac{3\sqrt{2}i}{5} \tag{3}$$

$$\frac{\alpha-\beta}{\sqrt{2}} = \frac{4}{5} \Longrightarrow \alpha-\beta = \frac{4\sqrt{2}}{5} \tag{4}$$

If we calculate (3) + (4)

$$2\alpha = \frac{3\sqrt{2}i}{5} + \frac{4\sqrt{2}}{5} = \frac{\sqrt{2}}{5}(3i+4) \Longrightarrow \alpha = \frac{\sqrt{2}}{10}(3i+4)$$

Also we can get $\beta$ using 3,

$$\beta = \frac{\sqrt{2}}{5}(3i) - \frac{\sqrt{2}}{10}(3i+4) = \frac{\sqrt{2}}{10}(6i - 3i - 4) \Longrightarrow \beta = \frac{\sqrt{2}}{10}(3i-4)$$

Finally we have

$$|\phi\rangle = (\frac{\sqrt{2}}{10}(3i+4))\,|+\rangle + (\frac{\sqrt{2}}{10}(3i-4))\,|-\rangle$$

# Exercise 2.6

Photon horizontal and vertical polarization states are written as $|h\rangle$ and $|v\rangle$, respectively. Suppose

$$|\psi_1\rangle = \frac{1}{2}|h\rangle + \frac{\sqrt{3}}{2}|v\rangle$$
$$|\psi_2\rangle = \frac{1}{2}|h\rangle - \frac{\sqrt{3}}{2}|v\rangle$$
$$|\psi_3\rangle = |h\rangle$$

We can get $\langle\psi_1|$ and $\langle\psi_3|$

$$\langle\psi_1| = \frac{1}{2}\langle h| + \frac{\sqrt{3}}{2}\langle v|$$
$$\langle\psi_3| = \langle h|$$

Now, since $\langle v|h\rangle = \langle h|v\rangle = 0$ and $\langle v|v\rangle = \langle h|h\rangle = 1$, We can calculate $|\langle\psi_1|\psi_2\rangle|^2$, $|\langle\psi_1|\psi_3\rangle|^2$ and $|\langle\psi_3|\psi_2\rangle|^2$,

$$\langle\psi_1|\psi_2\rangle = (\frac{1}{2}\langle h|+\frac{\sqrt{3}}{2}\langle v|)(\frac{1}{2}|h\rangle-\frac{\sqrt{3}}{2}|v\rangle) = \frac{1}{4}\langle h|h\rangle-\frac{\sqrt{3}}{4}\langle h|v\rangle+\frac{\sqrt{3}}{4}\langle v|h\rangle-\frac{3}{4}\langle v|v\rangle = -\frac{1}{2}$$

$$|\langle\psi_1|\psi_2\rangle|^2 = \frac{1}{4},$$
$$\langle\psi_1|\psi_3\rangle = (\frac{1}{2}\langle h| + \frac{\sqrt{3}}{2}\langle v|)(|h\rangle) = \frac{1}{2}\langle h|h\rangle + \frac{\sqrt{3}}{2}\langle v|h\rangle = \frac{1}{2}$$
$$|\langle\psi_1|\psi_3\rangle|^2 = \frac{1}{4},$$
$$\langle\psi_3|\psi_2\rangle = (\langle h|)(\frac{1}{2}|h\rangle - \frac{\sqrt{3}}{2}|v\rangle) = \frac{1}{2}\langle h|h\rangle - \frac{\sqrt{3}}{2}\langle h|v\rangle = \frac{1}{2}$$
$$|\langle\psi_3|\psi_2\rangle|^2 = \frac{1}{4}.$$