Chapter 6

Solutions to Try it

MohamadAli Khajeian*

Faculty of Engineering Sciences, University of Tehran, Iran

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Abstract

This document presents the solution of "Quantum Computing Explained by David McMAHON" exercises.

Try it - (page 103)

we have

$$\mathrm{Y}\,|u_i\rangle = \lambda_i\,|u_i\rangle \tag{1}$$

where

$$|u_i\rangle = \alpha\,|0\rangle + \beta\,|1\rangle \tag{2}$$

let's consider

$$\sigma_y = \mathrm{Y} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = -i|0\rangle\langle 1| + i|1\rangle\langle 0| \tag{3}$$

to get EigenVectors and EigenValues of Y matrix, we have

$$\begin{aligned}
\det|\mathrm{Y}-\lambda\mathrm{I}| &= \det\Big| -\lambda|0\rangle\langle 0| - i|0\rangle\langle 1| + i|1\rangle\langle 0| - \lambda|1\rangle\langle 1|\Big| \\
&= (-\lambda)(-\lambda) - (-i)(i) \\
&= \lambda^2 - 1 = 0
\end{aligned}$$

so

$$\lambda_{1,2} = \pm 1$$

now to calculate $|u_1\rangle$, using eqs. (1) to (3)

$$\begin{aligned}
\mathrm{Y}\,|u_1\rangle &= \lambda_i\,|u_1\rangle \\
&= (-i\,|0\rangle\,\langle 1| + i\,|1\rangle\,\langle 0|)(\alpha\,|0\rangle + \beta\,|1\rangle) \\
&= i(-\beta\,|0\rangle + \alpha\,|1\rangle) = -\,|u_1\rangle
\end{aligned}$$

from 2 we have

$$\alpha = -i\beta, \quad \beta = i\alpha$$

to find α and β

$$\|\alpha\|^2 + \|\beta\|^2 = \|\alpha\|^2 + (-i\alpha^*)(i\alpha) = 2\|\alpha\|^2 = 1 \Longrightarrow \alpha = \frac{1}{\sqrt{2}}$$

*khajeian@ut.ac.ir

so, $|u_1\rangle$ is

$$\boxed{|u_1\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i\,|1\rangle)}$$

to calculate $|u_2\rangle$, using eqs. (1) to (3)

$$\begin{aligned}
\mathrm{Y}\,|u_2\rangle &= \lambda_i\,|u_2\rangle \\
&= (-i\,|0\rangle\,\langle 1| + i\,|1\rangle\,\langle 0|)(\alpha\,|0\rangle + \beta\,|1\rangle) \\
&= i(-\beta\,|0\rangle + \alpha\,|1\rangle) = -\,|u_2\rangle
\end{aligned}$$

from 2 we have

$$\alpha = i\beta, \quad \beta = -i\alpha$$

To find α and β

$$\|\alpha\|^2 + \|\beta\|^2 = \|\alpha\|^2 + (-i\alpha^*)(i\alpha) = 2\|\alpha\|^2 = 1 \Longrightarrow \alpha = \frac{1}{\sqrt{2}}$$

So, $|u_2\rangle$ is

$$\boxed{|u_2\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i\,|1\rangle)}$$

Try it - (page 136)

if we have

$$|\psi^{'}\rangle = \left(\frac{\sqrt{2}+i}{\sqrt{15}}\right)|000\rangle + \sqrt{\frac{2}{3}}\,|001\rangle + \sqrt{\frac{2}{15}}\,|011\rangle$$

to determine if the state is normalized, we compute the sum of the squares of the coefficients

$$\begin{aligned}
\sum_i \|c_i\|^2 &= \left(\frac{\sqrt{2}-i}{\sqrt{15}}\right)\left(\frac{\sqrt{2}+i}{\sqrt{15}}\right) + \left(\sqrt{\frac{2}{3}}\right)\left(\sqrt{\frac{2}{3}}\right) + \left(\sqrt{\frac{2}{15}}\right)\left(\sqrt{\frac{2}{15}}\right) \\
&= \frac{3}{15} + \frac{2}{3} + \frac{2}{15} = \frac{15}{15} = 1
\end{aligned}$$

therefore the state is normalized.

Try it - (page 138)

A system is in the GHZ state where

$$|\psi\rangle = \frac{1}{\sqrt{2}}\big(\,|000\rangle + |111\rangle\,\big) \tag{4}$$

we know

$$|0\rangle = \frac{1}{\sqrt{2}}\big(\,|+\rangle + |-\rangle\,\big) \tag{5}$$

$$|1\rangle = \frac{1}{\sqrt{2}}\big(\,|+\rangle - |-\rangle\,\big) \tag{6}$$

so using eqs. (5) and (6)

$$\begin{aligned}
|000\rangle &= \left(\frac{1}{\sqrt{2}}\big(\,|+\rangle + |-\rangle\,\big)\right) \otimes \left(\frac{1}{\sqrt{2}}\big(\,|+\rangle + |-\rangle\,\big)\right) \otimes \left(\frac{1}{\sqrt{2}}\big(\,|+\rangle + |-\rangle\,\big)\right) \\
&= \left(\frac{1}{\sqrt{2}}\big(\,|+\rangle + |-\rangle\,\big)\right) \otimes \frac{1}{2}\Big(\,|{++}\rangle + |{+-}\rangle + |{-+}\rangle + |{--}\rangle\,\Big) \\
&= \frac{1}{2\sqrt{2}}\Big(\,|{+++}\rangle + |{++-}\rangle + |{+-+}\rangle + |{+--}\rangle + |{-++}\rangle + |{-+-}\rangle + |{--+}\rangle + |{---}\rangle\,\Big)
\end{aligned}$$

$$
\begin{aligned}
|111\rangle &= \left(\frac{1}{\sqrt{2}}\big(\,|+\rangle - |-\rangle\,\big)\right) \otimes \left(\frac{1}{\sqrt{2}}\big(\,|+\rangle - |-\rangle\,\big)\right) \otimes \left(\frac{1}{\sqrt{2}}\big(\,|+\rangle - |-\rangle\,\big)\right) \\
&= \left(\frac{1}{\sqrt{2}}\big(\,|+\rangle - |-\rangle\,\big)\right) \otimes \frac{1}{2}\Big(\,|{+}{+}\rangle - |{+}{-}\rangle - |{-}{+}\rangle + |{-}{-}\rangle\,\Big) \\
&= \frac{1}{2\sqrt{2}}\Big(\,|+++\rangle - |++-\rangle - |+-+\rangle + |+--\rangle - |-++\rangle + |-+-\rangle + |--+\rangle - |---\rangle\,\Big)
\end{aligned}
$$

now using eq. (4) we have

$$
\begin{aligned}
|\psi\rangle &= \frac{1}{\sqrt{2}}\big(\,|000\rangle + |111\rangle\,\big) \\
&= \frac{1}{\sqrt{2}}\Bigg(\frac{1}{2\sqrt{2}}\Big(\,|+++\rangle + |++-\rangle + |+-+\rangle + |+--\rangle + |-++\rangle + |-+-\rangle + |--+\rangle + |---\rangle\,\Big) \\
&+ \frac{1}{2\sqrt{2}}\Big(\,|+++\rangle - |++-\rangle - |+-+\rangle + |+--\rangle - |-++\rangle + |-+-\rangle + |--+\rangle - |---\rangle\,\Big)\Bigg) \\
&= \frac{1}{4}\Big(2\,|+++\rangle + 2\,|+--\rangle + 2\,|-+-\rangle + 2\,|--+\rangle\Big) \\
&= \frac{1}{2}\Big(\,|+++\rangle + |+--\rangle + |-+-\rangle + |--+\rangle\,\Big).
\end{aligned}
$$