Chapter 3

Solutions to Try it

MohamadAli Khajeian*

Faculty of Engineering Sciences, University of Tehran, Iran

November 7, 2024

Abstract

This document presents the solution of "Quantum Computing Explained by David McMAHON" exercises.

Try it - (page 42)

Let an arbitrary qubit state be given by

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle,$$

now, apply the operator $\hat{A} = |0\rangle\langle 0| + |1\rangle\langle 1|$ to this state

$$\hat{A}|\psi\rangle = (|0\rangle\langle 0| + |1\rangle\langle 1|)(\alpha|0\rangle + \beta|1\rangle).$$

$$\hat{A}|\psi\rangle = |0\rangle\langle 0|(\alpha|0\rangle + \beta|1\rangle) + |1\rangle\langle 1|(\alpha|0\rangle + \beta|1\rangle).$$

we get

$$\hat{A}|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = |\psi\rangle.$$

Try it - (page 44)

Let

$$|\psi\rangle = a\,|1\rangle + b\,|2\rangle + c\,|3\rangle\,, \quad |\phi\rangle = e\,|1\rangle + f\,|2\rangle + g\,|3\rangle$$

we can calculate outer product

$$\begin{aligned} |\psi\rangle\langle\phi| &= (a\,|1\rangle + b\,|2\rangle + c\,|3\rangle)(e^*\,\langle 1| + f^*\,\langle 2| + g^*\,\langle 3|) \\ &= ae^*|1\rangle\langle 1| + af^*|1\rangle\langle 2| + ag^*|1\rangle\langle 3| + be^*|2\rangle\langle 1| + bf^*|2\rangle\langle 2| \\ &+ bg^*|2\rangle\langle 3| + ce^*|3\rangle\langle 1| + cf^*|3\rangle\langle 2| + cg^*|3\rangle\langle 3| \end{aligned}$$

*khajeian@ut.ac.ir

Try it - (page 45)

since

$$\sigma_0 \left|0\right\rangle = \left|0\right\rangle, \quad \sigma_0 \left|1\right\rangle = \left|1\right\rangle$$

we have

$$\sigma_0 = \begin{pmatrix} \langle 0|\sigma_0|0\rangle & \langle 0|\sigma_0|1\rangle \\ \langle 1|\sigma_0|0\rangle & \langle 1|\sigma_0|1\rangle \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Try it - (page 47)

Let

$$\hat{B} = 3i|0\rangle\langle 0| + 2i|0\rangle\langle 1|$$

we can compute the adjoint of each term

$$\begin{aligned} \hat{B}^\dagger &= (3i|0\rangle\langle 0|)^\dagger + (2i|0\rangle\langle 1|)^\dagger \\ &= -3i|0\rangle\langle 0| - 2i|1\rangle\langle 0| \end{aligned}$$

Try it - (page 50)

According to

$$\hat{\mathrm{A}} \left|\gamma\right\rangle = \lambda \left|\gamma\right\rangle \tag{1}$$

We assume $\left|\gamma\right\rangle = \alpha \left|0\right\rangle + \beta \left|1\right\rangle$.

$$\sigma_{\mathrm{Z}} = \mathrm{Z} = 1 \left|0\right\rangle \left\langle 0\right| - 1 \left|1\right\rangle \left\langle 1\right| \tag{2}$$

Since 2, we can find eigenvalues through

$$\det\left(\sigma_{\mathrm{Z}} - \lambda \mathrm{I}\right) = 0 \tag{3}$$

$$\det\Big((1-\lambda)\left|0\right\rangle\left\langle 0\right| + (-1-\lambda)\left|1\right\rangle\left\langle 1\right|\Big) = 0$$

$$(1-\lambda)(-1-\lambda) = -1 - \lambda + \lambda + \lambda^2 = \lambda^2 - 1 = 0 \Longrightarrow \lambda = \pm 1$$

For $\lambda_1 = 1$, from 1 and 2 we have

$$\sigma_{\mathrm{Z}} \left|\gamma\right\rangle = \left|\gamma\right\rangle \Longrightarrow (\left|0\right\rangle\left\langle 0\right| - \left|1\right\rangle\left\langle 1\right|)(\alpha\left|0\right\rangle + \beta\left|1\right\rangle) = \alpha\left|0\right\rangle - \beta\left|1\right\rangle \tag{4}$$

From 4 we have

$$\alpha = \alpha, \quad \beta = -\beta \Longrightarrow \beta = 0 \tag{5}$$

To find α and β

$$\|\alpha\|^2 + \|\beta\|^2 = \|\alpha\|^2 + 0 = \|\alpha\|^2 = 1 \Longrightarrow \alpha = 1 \tag{6}$$

So, $\left|\gamma_1\right\rangle$ is

$$\left|\gamma_1\right\rangle = \left|0\right\rangle \tag{7}$$

For $\lambda_2 = -1$, from 1 and 2 we have

$$\sigma_{\mathrm{Z}} \left|\gamma\right\rangle = \left|\gamma\right\rangle \Longrightarrow (\left|0\right\rangle\left\langle 0\right| - \left|1\right\rangle\left\langle 1\right|)(\alpha\left|0\right\rangle + \beta\left|1\right\rangle) = \alpha\left|0\right\rangle - \beta\left|1\right\rangle \tag{8}$$

From 8 we have

$$\alpha = -\alpha \Longrightarrow \alpha = 0, \quad \beta = \beta \tag{9}$$

To find α and β

$$\|\beta\|^2 = \|\beta\|^2 + 0 = \|\beta\|^2 = 1 \Longrightarrow \beta = 1 \tag{10}$$

So, $|\gamma_2\rangle$ is

$$|\gamma_2\rangle = |1\rangle \tag{11}$$

Try it - (page 63)

to show that

$$P_+ + P_- = |0\rangle\langle 0| + |1\rangle\langle 1| = I$$

we know

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

we need to compute $P_+ = |+\rangle\langle +|$ and $P_- = |-\rangle\langle -|$

$$P_+ = \left(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\right)\left(\frac{1}{\sqrt{2}}(\langle 0| + \langle 1|)\right) = \frac{1}{2}(|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| + |1\rangle\langle 1|).$$

$$P_- = \left(\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\right)\left(\frac{1}{\sqrt{2}}(\langle 0| - \langle 1|)\right) = \frac{1}{2}(|0\rangle\langle 0| - |0\rangle\langle 1| - |1\rangle\langle 0| + |1\rangle\langle 1|).$$

now, let's calculate $P_+ - P_-$

$$P_+ + P_- = \frac{1}{2}(|0\rangle\langle 0| + |0\rangle\langle 0| + |1\rangle\langle 1| + |1\rangle\langle 1|) = |0\rangle\langle 0| + |1\rangle\langle 1|.$$

thus, we find that

$$P_+ = P_- = |0\rangle\langle 0| + |1\rangle\langle 1| = I.$$