Chapter 7

Solutions to Try it

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Abstract

This document presents the solution of "Quantum Computing Explained by David McMAHON" exercises.

Try it - (page 162)

we have

$$\mathrm{Y_A} = -i|0_A\rangle\langle 1_A| + i|1_A\rangle\langle 0_A|$$
$$\mathrm{Y_B} = -i|0_B\rangle\langle 1_B| + i|1_B\rangle\langle 0_B|$$

then

$$\begin{aligned}
\mathrm{Y_A}\otimes\mathrm{Y_B} &= \big(-i|0_A\rangle\langle 1_A| + i|1_A\rangle\langle 0_A|\big)\otimes\big(-i|0_B\rangle\langle 1_B| + i|1_B\rangle\langle 0_B|\big)\\
&= -|0_A0_B\rangle\langle 1_A1_B| + |0_A1_B\rangle\langle 1_A0_B| + |1_A0_B\rangle\langle 0_A1_B| - |1_A1_B\rangle\langle 0_A0_B|\\
&= \begin{pmatrix} 0 & 0 & 0 & -1\\ 0 & 0 & 1 & 0\\ 0 & 1 & 0 & 0\\ -1 & 0 & 0 & 0 \end{pmatrix}
\end{aligned}$$

also

$$\mathrm{Z_A} = |0_A\rangle\langle 0_A| - |1_A\rangle\langle 1_A|$$
$$\mathrm{Z_B} = |0_B\rangle\langle 0_B| - |1_B\rangle\langle 1_B|$$

then

$$\begin{aligned}
\mathrm{Z_A}\otimes\mathrm{Z_B} &= \big(|0_A\rangle\langle 0_A| - |1_A\rangle\langle 1_A|\big)\otimes\big(|0_B\rangle\langle 0_B| - |1_B\rangle\langle 1_B|\big)\\
&= |0_A0_B\rangle\langle 0_A0_B| - |0_A1_B\rangle\langle 0_A1_B| - |1_A0_B\rangle\langle 1_A0_B| + |1_A1_B\rangle\langle 1_A1_B|\\
&= \begin{pmatrix} 1 & 0 & 0 & 0\\ 0 & -1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & 1 \end{pmatrix}
\end{aligned}$$

Try it - (page 162)

we have

$$|\beta_{01}\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}}, \quad |\beta_{00}\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

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$$\mathrm{H}_I = \frac{\mu^2}{r^3}\begin{pmatrix} -2 & 0 & 0 & 0 \\ 0 & 2 & 2 & 0 \\ 0 & 2 & 2 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix} = \frac{2\mu^2}{r^3}\big(-|00\rangle\langle 00| + |01\rangle\langle 01| + |01\rangle\langle 10| + |10\rangle\langle 01| + |10\rangle\langle 10| - |11\rangle\langle 11|\big)$$

first we need to find $\mathrm{H}_I\,|\beta_{01}\rangle$

$$\begin{aligned}
\mathrm{H}_I\,|\beta_{01}\rangle &= \frac{2\mu^2}{r^3}\big(-|00\rangle\langle 00| + |01\rangle\langle 01| + |01\rangle\langle 10| + |10\rangle\langle 01| + |10\rangle\langle 10| - |11\rangle\langle 11|\big)\big(\frac{|01\rangle + |10\rangle}{\sqrt{2}}\big) \\
&= \frac{2\mu^2}{\sqrt{2}r^3}\Bigg(\big(\,|01\rangle + |10\rangle\,\big) + \big(|01\rangle + |10\rangle\,\big)\Bigg) \\
&= \frac{4\mu^2}{\sqrt{2}r^3}\Bigg(\ |01\rangle + |10\rangle\ \Bigg) = \frac{4\mu^2}{r^3}\,|\beta_{01}\rangle\,.
\end{aligned}$$

let's get $\mathrm{H}_I\,|\beta_{11}\rangle$

$$\begin{aligned}
\mathrm{H}_I\,|\beta_{11}\rangle &= \frac{2\mu^2}{r^3}\big(-|00\rangle\langle 00| + |01\rangle\langle 01| + |01\rangle\langle 10| + |10\rangle\langle 01| + |10\rangle\langle 10| - |11\rangle\langle 11|\big)\big(\frac{|01\rangle - |10\rangle}{\sqrt{2}}\big) \\
&= \frac{2\mu^2}{\sqrt{2}r^3}\Bigg(\big(\,|01\rangle + |10\rangle\,\big) - \big(|01\rangle + |10\rangle\,\big)\Bigg) \\
&= \frac{2\mu^2}{\sqrt{2}r^3}\Bigg(0\Bigg) = 0.
\end{aligned}$$