Documents for simple typed lambda calculus

Syntax

$t ::= x \mid \lambda x{:}\, T.\, t \mid t\ t \mid True \mid False \mid if\ t_1\ then\ t_2\ else\ t_3$

$v ::= \lambda x{:}\, T.\, t$

$T ::= Bool \mid T \to T$

Evaluation

$$\frac{t_1 \to t_1'}{t_1\ t_2 \to t_1'\ t_2}\ (E-APP1)$$

$$\frac{t_2 \to t_2'}{v_1\ t_2 \to v_1\ t_2'}\ (E-APP2)$$

$$(\lambda x{:}\, T_{11}.\, t_{12})\ v_2 \to [x \mapsto v_2]t_{12}\ \ (E-APP\ ABS)$$

$$\frac{t_1 \to t_1'}{if\ t_1\ then\ t_2\ else\ t_3 \to if\ t_1'\ then\ t_2\ else\ t_3}\ (E-IF)$$

$$if\ true\ then\ t_2\ else\ t_3 \to t_2\ \ (E-IFTRUE)$$

$$if\ false\ then\ t_2\ else\ t_3 \to t_3\ \ (E-IFFALSE)$$

Typing

$$\frac{x{:}\, T \in \Gamma}{\Gamma \vdash x{:}\, T}\ (T-VAR)$$

$$\frac{\Gamma,\ x{:}\, T_1 \vdash t_2{:}\, T_2}{\Gamma \vdash \lambda x{:}\, T_1.\, t_2{:}\, T_1 \to T_2}\ (T-ABS)$$

$$\frac{\Gamma \vdash t_1{:}\, T_1 \to T_2\ \ \Gamma \vdash t_2{:}\, T_1}{\Gamma \vdash t_1\ t_2{:}\, T_2}\ (T-APP)$$

$\vdash true: Bool \ \ (\mathrm{T}-\mathrm{TRUE})$

$\vdash false: Bool \ (\mathrm{T}-\mathrm{FALSE})$

$$\frac{\Gamma \vdash t_1: Bool \ \ \Gamma \vdash t_2: T \ \Gamma \vdash t_3: T}{\Gamma \vdash if \ t_1 \ then \ t_2 \ else \ t_3: T} \ \ (\mathrm{T}-\mathrm{IF})$$

Substitution

$[j \mapsto s]k = \ s \ (if \ k = j)$

$[j \mapsto s]k = k \ (otherwise)$

$[j \mapsto s](\lambda. t_1) = \lambda. [j + 1 \mapsto \uparrow^1 (s)]t_1$

$[j \mapsto s](t_1 \ t_2) = ([j \mapsto s]t_1 \ [j \mapsto s]t_2)$

$[j \mapsto s]true = true$

$[j \mapsto s]false = false$

$[j \mapsto s]if \ t_1 \ then \ t_2 \ else \ t_3$

$$= if \ ([j \mapsto s]t_1) \ then \ ([j \mapsto s]t_2) \ else \ ([j \mapsto s]t_3)$$