NO.

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$$= (\cancel{1-\lambda_k}) \cdot \frac{1}{(\cancel{1-\lambda_k})} \sum_{i=1}^{k-1} \lambda_i f(x_i) + \lambda_k f(x_k)$$

$$= \sum_{i=1}^{k-1} \lambda_i f(x_i) + \lambda_k f(x_k)$$

$$= \sum_{i=1}^{k} \lambda_i f(x_i)$$

\# 得證!

$\forall M$, $f\left(\sum_{i=1}^{M} \lambda_i x_i\right) \leq \sum_{i=1}^{M} \lambda_i f(x_i)$, which $\lambda_i \geq 0$ and $\sum_i \lambda_i = 1$ \#

2. Derive the Entropy of the Universe Gaussian.

Base on definition:

$$H[x] = -\int p(x) \ln(p(x))\, dx$$

$$= -\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \times \left(\ln\left(\frac{1}{\sqrt{2\pi\sigma^2}}\right) + \ln\left(e^{\frac{-(x-\mu)^2}{2\sigma^2}}\right)\right) dx$$

$$= -\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \times \left(-\frac{1}{2} \ln(2\pi\sigma^2) + \frac{-(x-\mu)^2}{2\sigma^2}\right) dx$$

$$= \frac{1}{2} \ln(2\pi\sigma^2) \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} dx - \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \times \frac{-(x-\mu)^2}{2\sigma^2} dx$$

$$= \frac{1}{2} \ln(2\pi\sigma^2) \int_{-\infty}^{\infty} p(x) dx - \int_{-\infty}^{\infty} p(x) \times \frac{-(x-\mu)^2}{2\sigma^2} dx$$

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$$= \frac{1}{2}\ln(2\pi\sigma^2)\int_{-\infty}^{\infty} p(x)\,dx - \frac{-1}{2\sigma^2}\int_{-\infty}^{\infty} p(x)\,(x-\mu)^2\,dx$$

$$= \frac{1}{2}\ln(2\pi\sigma^2)\times 1 - \frac{-1}{2\sigma^2}\times\sigma^2$$

$$= \frac{1}{2}\ln(2\pi\sigma^2) + \frac{1}{2} = \frac{1}{2}\left(\ln(2\pi\sigma^2)+1\right) \#$$

得證!

3、

$\because KL(p\|q) = -\int p(x)\ln\left(\frac{q(x)}{p(x)}\right)$

$\therefore$ 代入 $p(x)$, $q(x)$

$$\Rightarrow KL(p\|q) = -\int \mathcal{N}(x|\mu,\sigma^2)\times\ln\left(\frac{\mathcal{N}(x|m,s^2)}{\mathcal{N}(x|\mu,\sigma^2)}\right)dx$$

$$= -\int \frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{1}{2\sigma^2}(x-\mu)^2}\times\ln\left(\frac{\frac{1}{\sqrt{2\pi s^2}}\times e^{\frac{-(x-m)^2}{2s^2}}}{\frac{1}{\sqrt{2\pi\sigma^2}}\times e^{\frac{-(x-\mu)^2}{2\sigma^2}}}\right)dx$$

$$= -\int \frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}\times\ln\left(\sqrt{\frac{\cancel{2\pi}\sigma^2}{\cancel{2\pi}s^2}}\times e^{\frac{-(x-m)^2}{2s^2}+\frac{(x-\mu)^2}{2\sigma^2}}\right)dx$$

$$= -\int \frac{1}{\sqrt{2\pi\sigma^2}}e^{\frac{-(x-\mu)^2}{2\sigma^2}}\times\left(\ln\left(\frac{\sigma}{s}\right)+\frac{-(x-m)^2}{2s^2}+\frac{(x-\mu)^2}{2\sigma^2}\right)dx$$