NO.

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$$= \frac{1}{2}\ln(2\pi\sigma^2)\int_{-\infty}^{\infty} p(x)dx - \frac{-1}{2\sigma^2}\int_{-\infty}^{\infty} p(x)(x-\mu)^2 dx$$

$$= \frac{1}{2}\ln(2\pi\sigma^2) \times 1 - \frac{-1}{2\sigma^2} \times \sigma^2$$

$$= \frac{1}{2}\ln(2\pi\sigma^2) + \frac{1}{2} = \frac{1}{2}\left(\ln(2\pi\sigma^2)+1\right) \#$$

得證!

3.

$\because KL(p\|q) = -\int p(x)\ln\left(\frac{q(x)}{p(x)}\right)$

$\therefore$ 代入 $p(x)$, $q(x)$

$$\Rightarrow KL(p\|q) = -\int N(x|u,\sigma^2) \times \ln\left(\frac{N(x|m,s^2)}{N(x|u,\sigma^2)}\right)dx$$

$$= -\int \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(x-u)^2} \times \ln\left(\frac{\frac{1}{\sqrt{2\pi s^2}} \times e^{\frac{-(x-m)^2}{2s^2}}}{\frac{1}{\sqrt{2\pi\sigma^2}} \times e^{\frac{-(x-u)^2}{2\sigma^2}}}\right)dx$$

$$= -\int \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-u)^2}{2\sigma^2}} \times \ln\left(\sqrt{\frac{2\pi\sigma^2}{2\pi s^2}} \times e^{\frac{-(x-m)^2}{2s^2} + \frac{(x-u)^2}{2\sigma^2}}\right)dx$$

$$= -\int \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-u)^2}{2\sigma^2}} \times \left(\ln\left(\frac{\sigma}{s}\right) + \frac{-(x-m)^2}{2s^2} + \frac{(x-u)^2}{2\sigma^2}\right)dx$$

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$$= -\int p(x) \ln \frac{\sigma}{s}\,dx - \int p(x) \times \frac{-(x-m)^2}{2s^2}\,dx - \int p(x) \frac{(x-\mu)^2}{2\sigma^2}\,dx$$

$$= -\ln\left(\frac{\sigma}{s}\right) \times \int p(x)dx + \frac{1}{2s^2}\int \left(p(x)x^2 - 2mp(x)x + p(x)m^2\right)dx$$

$$- \frac{1}{2\sigma^2}\int \left(p(x)x^2 - 2\mu p(x)x + p(x)\mu^2\right)dx$$

$$= -\ln\left(\frac{\sigma}{s}\right)\times 1 + \frac{1}{2s^2}\times\left(\int p(x)x^2dx - 2m\int p(x)x\,dx + m^2\int p(x)dx\right)$$

$$- \frac{1}{2\sigma^2}\times\left(\int p(x)x^2dx - 2\mu\int p(x)x\,dx + \mu^2\int p(x)dx\right)$$

$$= -\ln\left(\frac{\sigma}{s}\right) + \frac{1}{2s^2}\times\left((\sigma^2+\mu^2) - 2m\times\mu + m^2\times 1\right)$$

$$- \frac{1}{2\sigma^2}\times\left((\sigma^2+\mu^2) - 2\mu\times\mu + \mu^2\times 1\right)$$

$$= -\ln\left(\frac{\sigma}{s}\right) + \frac{1}{2s^2}\left(\sigma^2+\mu^2-2m\mu+m^2\right) - \left(\frac{1}{2}\right)$$

$$= -\ln(\sigma) + \ln(s) + \frac{1}{2}\left(\frac{\sigma}{s}\right)^2 + \frac{1}{2}\left(\frac{\mu-m}{s}\right)^2 - \frac{1}{2}$$

$$= -\ln(\sigma) + \ln(s) + \frac{1}{2}\left(\frac{\sigma^2+(\mu-m)^2}{s^2} - 1\right) \quad \#$$