2. Find MAP solution to Λ for $N = 10, 100, 500$

$\because p(\Lambda | X) = \mathcal{W}(\Lambda | W_\Lambda, \nu_\Lambda)$

$\therefore \Lambda_{MAP}$ happened when $p(\Lambda | X)$ achieve maximum

$$\Rightarrow \frac{d\, \mathcal{W}(\Lambda | W_\Lambda, \nu_\Lambda)}{d\Lambda} = 0$$

$$\Rightarrow \frac{d\, B \cdot |\Lambda|^{\nu_\Lambda - D - 1/2} \cdot e^{-\frac{1}{2} Tr(W_\Lambda^{-1} \cdot \Lambda)}}{d\Lambda} = 0$$

$$\Rightarrow B \frac{\nu_\Lambda - D - 1}{2} |\Lambda|^{\nu_\Lambda - D - 1/2} \cdot (\Lambda^{-1})^T \cdot e^{-\frac{1}{2} Tr(W_\Lambda^{-1} \Lambda)} + B \cdot |\Lambda|^{\nu_\Lambda - D - 1/2} \cdot \frac{-1}{2} (W_\Lambda^{-1})^T e^{-\frac{1}{2} Tr(W_\Lambda^{-1} \Lambda)} = 0$$

$$\Rightarrow |\Lambda|^{\nu_\Lambda - D - 1/2} e^{-\frac{1}{2} Tr(W_\Lambda^{-1} \Lambda)} \cdot B \cdot \left(\frac{\nu_\Lambda - D - 1}{2} (\Lambda^{-1})^T + \frac{-1}{2} (W_\Lambda^{-1})^T \right) = 0$$

$$\Rightarrow (\Lambda^{-1})^T = \frac{1}{\cancel{2}} (W_\Lambda^{-1})^T \times \frac{\cancel{2}}{(\nu_\Lambda - D - 1)}$$

$$\Rightarrow \Lambda^{-1} = W_\Lambda^{-1} \times \frac{1}{(\nu_\Lambda - D - 1)}$$

$$\Rightarrow \Lambda = (\nu_\Lambda - D - 1) \cdot W_\Lambda$$

$$= ((N + \nu_0) - D - 1) \cdot (W_0^{-1} + \mathbb{K})$$, which $\mathbb{K} = \sum_{n=1}^{N} K$, $K = \begin{bmatrix} (x_{n_1} - \mu_1)^2 & \\ & (x_{n_2} - \mu_2)^2 \end{bmatrix}$

(在這題 : $K = \begin{bmatrix} (x_{n_1} - \mu_1)^2 & (x_{n_1} - \mu_1)(x_{n_2} - \mu_2) \\ (x_{n_2} - \mu_2) \cdot (x_{n_1} - \mu_1) & (x_{n_2} - \mu_2)^2 \end{bmatrix}$,

$\mu = [\mu_1, \mu_2]^T = [1, -1]^T$)

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