Investigating the Change of Capacitor's Voltage Against Time in RC Charging Circuit

> Word count: 2790 words
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> ***All files including .csv-s, .txt-s and photos of the experiment can be found at https://github.com/w1Tz3R-LW/Physics-IA-Data-and-Code**

Introduction

Nearly every modern electrical appliance utilizes the capacitor, a simple component consisting of two closely-spaced surfaces insulated from each other. The two surfaces accumulate opposite charge, enabling the storage of electrical energy. The properties of the capacitor are widely studied using a resistor-capacitor (RC) charging circuit, as depicted in Figure 1 below.

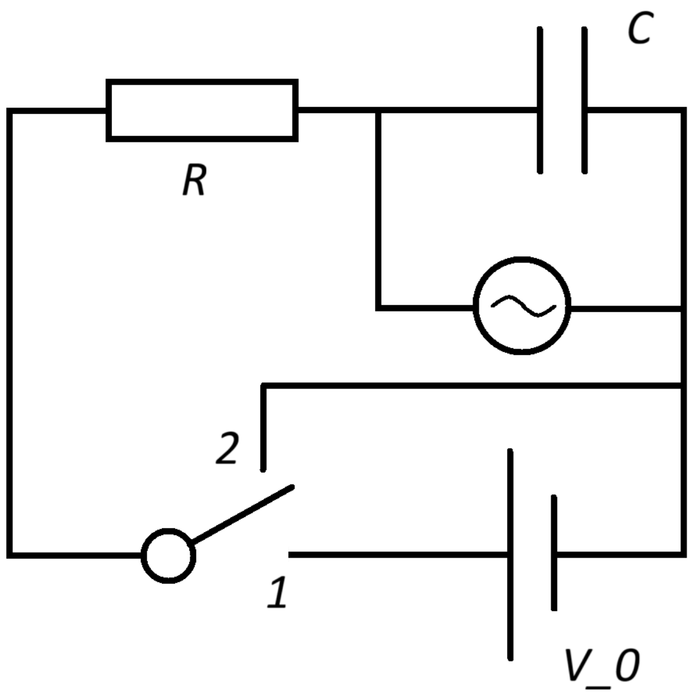


Figure 1. RC charging circuit

It is found that in the RC charging circuit, with the same capacitance, the time taken for the potential difference across the capacitor to increase by a fixed proportion of the remaining voltage difference is constant. Specifically, every time interval equal to the time constant RC, the voltage increases by $1 - \frac{1}{e}$, approximately 63.2%, of the difference between the source voltage and the current capacitor voltage, indicating **exponential asymptotic growth**. This relationship can be derived with basic knowledge of electrical science.

Let the p.d. across the capacitor be V, the charge accumulated by the capacitor be Q, and the current in the circuit be I. Using symbols denoted in Figure 1, the following equations must hold true:

$$V_0 = IR + V \tag{1}$$

$$I = \frac{\mathrm{d}Q}{\mathrm{d}t} \text{ and } Q = C \times V \Rightarrow I = \frac{\mathrm{d}(CV)}{\mathrm{d}t} = C \times \frac{\mathrm{d}V}{\mathrm{d}t} \tag{2}$$

Subsituting equation (2) into the equation (1), we get:

$$V_0 = RC \times \frac{\mathrm{d}V}{\mathrm{d}t} + V \Leftrightarrow \frac{\mathrm{d}V}{V_0 - V} = \frac{\mathrm{d}t}{RC}$$

Integrating both sides, we get:

$$-\ln|V_0 - V| = \frac{t}{RC} + c \tag{3}$$

where c is the integration constant. To solve it, we suppose at $t = 0$ we have $V = 0$, thus substituting both values into equation (3) we get:

$$-\ln V_0 = c$$

Substitute $c = -\ln V_0$ into equation (2) we get:

$$-\ln|V_0 - V| = \frac{t}{RC} - \ln V_0$$

$$\Leftrightarrow \ln \frac{V_0}{V_0 - V} = \frac{t}{RC} \Leftrightarrow \frac{V_0 - V}{V_0} = e^{-t/RC} \tag{4}$$

From equation (4), we can express V explicitly:

$$V = V_0\,(1 - e^{-t/RC}) \tag{5}$$

which suggests that when t increases by RC, V increases so that the remaining voltage $V_0 - V$ is multipiled by $\frac{1}{e}$, demonstrating **exponential asymtotic growth**. As time progresses, voltage V increases but at a decreasing rate and infinitely approaches source voltage V_0.

The RC charging circuit and its variants, including high/low-pass filters and signal amplifiers, are widely adapted in real life (https://www.geeksforgeeks.org/). The circuit itself is commonly studied in physics, which relates to the topic of this IA report.

Aim of Investigation

The aim of this study is to experimentally validate the relationship shown in equation (4), that in a RC charging circuit, the voltage V experiences exponential asymtotic growth as time progresses.

$$V = V_0 \left(1 - e^{-t/RC}\right) \tag{5}$$

Variables

The independent variable is time t and the grouping variable is resistance R.

The dependent variable is the voltage V of the capacitor.

The controlled variables are listed below:

- Capacitance of capacitor C: Capacitors, as laboratory consumables, are not stable, so the same capacitor is used throughout the experiment to prevent introducing unwanted unstabilities. The capacitor is also never connected to a voltage beyond its rated voltage to prevent laboratory accidents. In the end, a capacitor of around $1000\ \mu\mathrm{F}$ was used.
- Supply voltage V_0: The supply voltage V_0 is kept constant to obtain more accurate and comparable results. Considering that a dry-cell battery cannot maintain stable voltage output as its energy is drained and multiple supply voltages are needed in trial experiments to find the most appropriate value, a laboratory DC power supply which delivers consistent, adjustable voltage is used instead.

Step-by-step Process

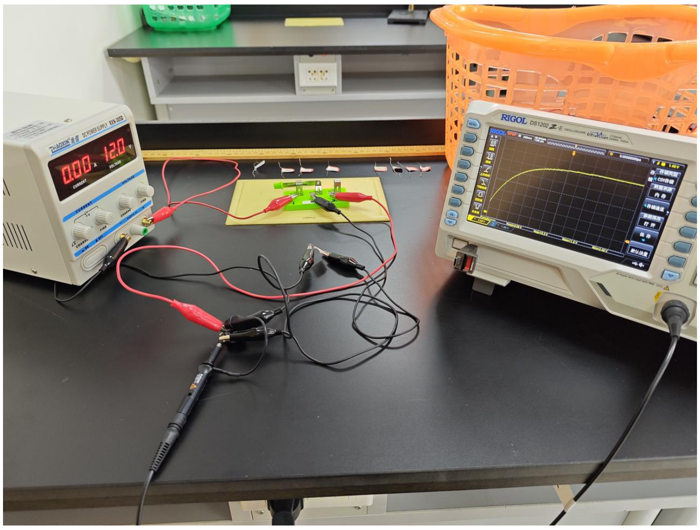
Pre-experiment

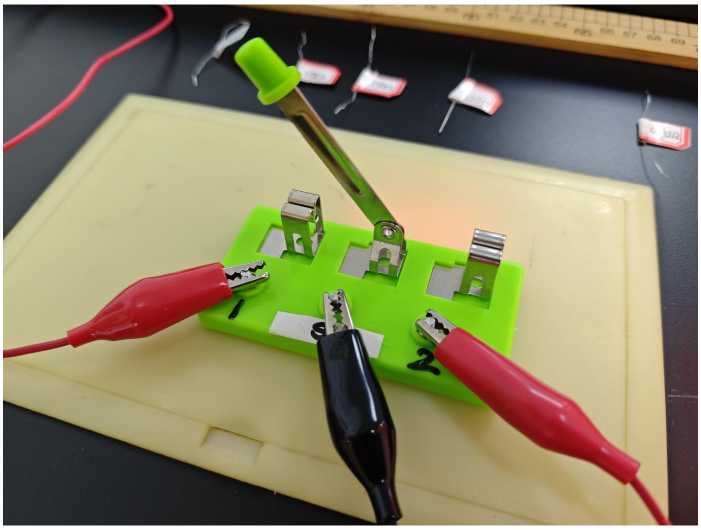
1. Use a multimeter to measure the resistance of every resistor that would be used in the experiment. The measured values are recorded as the actual resistance R.
2. Adjust the oscilloscope parameters to the experimental settings.
3. Assemble the circuit as described in Figure 1. A laboratory DC power supply is used and the oscilloscope is used to measure the voltage V across the capacitor. Unless otherwise mentioned, the DC power supply is switched on at all times and the circuit is turned on/off using the switch.

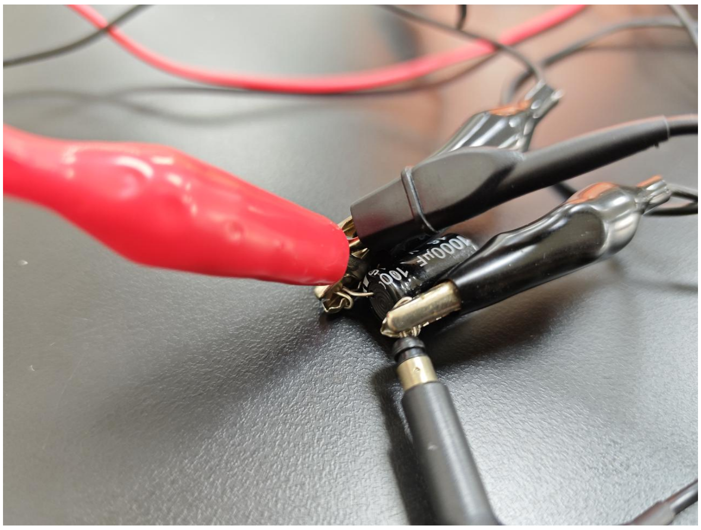
Process for any experiment

1. Connect the switch to contact 2 for a period of time no less than 5τ to discharge the capacitor. The value 5τ is chosen because discharge is usually considered to be complete at 5 times the time constant, at which point the voltage drops to lower than 0.7% the original voltage.
2. Replace the resistor in the circuit with the desired resistor.
3. Adjust the time base of the oscilloscope so that the sampling time of 12 times the time base is around 3τ, or 3 times the time constant. The value 3τ is chosen so that the most distinctive section of the V-t curve can be recorded, which means that most of the full $t=[0,5\tau]$ curve can be recorded, while the smoother section, in which voltage V approaches V_0 and is almost constant as t approaches 5τ, could be left out to save system memory.
4. Connect the switch to contact 1 to begin the experiment; immediately begin recording with the oscilloscope. Before the capacitor is fully charged, press the control key so that the recording will automatically stop as the sampling time runs out.
5. Connect the switch to contact 2 to discharge the capacitor for no less than 5τ; download the recorded V-t curve with a flash drive.

Photos of Experiment





Raw Data from Experiment

For every chosen value for the grouping value, resistance R, 3 sets of data (V-t curve) is obtained. The source voltage V_0 was set to $0.7\mathrm{V}$ throughout the experiment. The following chart shows the data obtained. All the experiments are done in a short period of time without unnecessary changes of equipment, thus experiment conditions could be assumed to be constant.

Group	No.	Resistance R (Ω)	Time Base (s/div)	Sampling Rate (kSa/s)	Data
1	1	319	0.1	50	6F11.csv 6F11_csv.txt
	2	319	0.1	50	6F12.csv 6F12_csv.txt
	3	319	0.1	50	6F13.csv 6F13_csv.txt
2	1	466	0.1	50	6G11.csv 6G11_csv.txt
	2	466	0.1	50	6G12.csv 6G12_csv.txt
	3	466	0.1	50	6G13.csv 6G13_csv.txt
3	1	559	0.2	25	6H11.csv

					6H11_csv.txt
	2	559	0.2	25	6H12.csv 6H12_csv.txt
	3	559	0.2	25	6H13.csv 6H13_csv.txt
	1	665	0.2	25	6I11.csv 6I11_csv.txt
4	2	665	0.2	25	6I12.csv 6I12_csv.txt
	3	665	0.2	25	6I13.csv 6I13_csv.txt
	1	815	0.2	25	6J11.csv 6J11_csv.txt
5	2	815	0.2	25	6J12.csv 6J12_csv.txt
	3	815	0.2	25	6J13.csv 6J13_csv.txt

The .txt files contain related information about the oscilloscope and its setting. The .csv files are downloaded from the oscilloscope, in which, beginning from the third row, the second column records the voltage value and the first column records the timestamp, plotting which would give a graph similar to Figure 2 below.

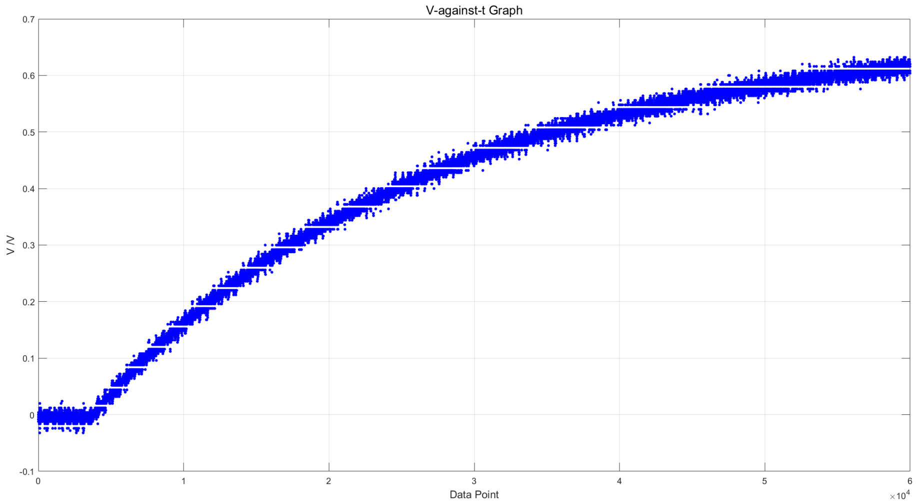


Figure 2: Plotting a raw .csv file

Data Processing

Taking Average

The functioning principle of modern digital oscilloscopes determines that error is unavoidably induced. According to the datasheet, the analog-to-digital (ADC) process induces quantimization error, a significant random error, in the measured voltage values V . Specifically, we can observe the quantimization error in the following figure 3.

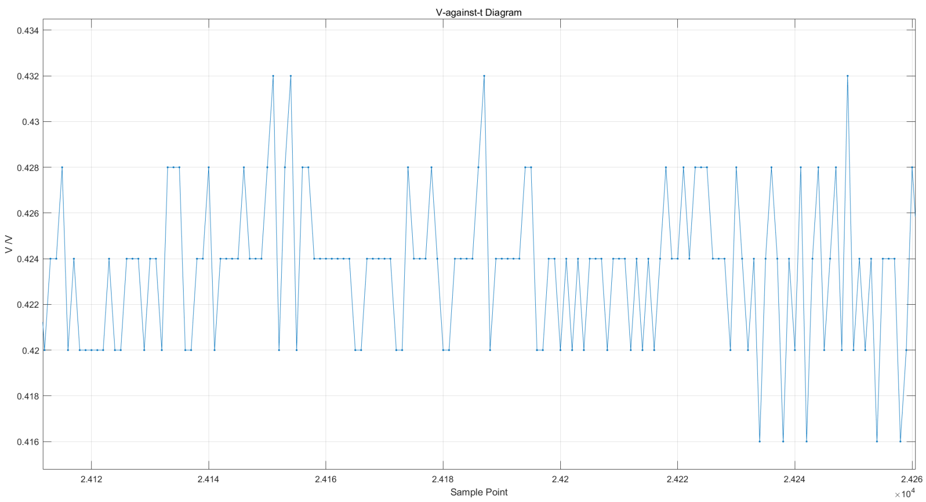


Figure 3: a very short section of a charging curve

In this figure, multiple values 0.416, 0.42, 0.424, 0.428, and 0.432 are recorded in a few milliseconds, which is too short for any significant change in voltage V . Yet measured values fluctuate between these fixed values, located 0.004 apart. This is due to random error induced when taking measurements and the ADC process rounding the actual value to discrete values to save system memory.

It can be observed that the random error is quite significant. However, the oscilloscope is capable of sampling at an astonishing high rate of tens of thousands of datapoints per second. Taking advantage of that, this study samples 1 datapoint from every 128 data points by taking the average of the voltage and timestamp values. The following figures 2 and 4 show the same curve before and after this process, visualizing the effect of reducing random error, and figure 5 shows a segment of the curve in figure 4 for better visibility.

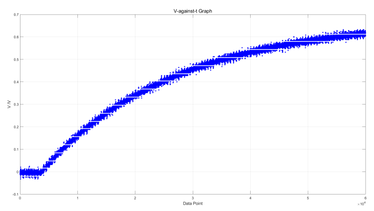


Figure 2: before taking average

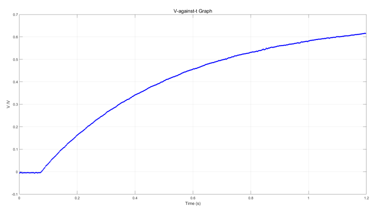


Figure 4: after taking average

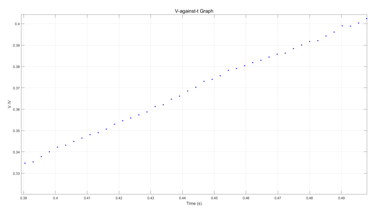


Figure 5: curve in figure 4 enhanced

Extracting Valid Data

The raw data recorded in the experiment typically forms a curve that consists of 3 sections, as described in Figure xxx below. In section A, the switch is yet to be turned on; the capacitor is not charged. In section B, the capacitor charges for around 2 times the time constant; the curve is most characteristic. In section C, the voltage V slowly approaches the source voltage V_0 ; the overall trend is heavily affected by random error. Sections A and C must be removed from the data to achieve high accuracy. In this study, an algorithm identifies and erases section A and C as the third step of data processing, sampling the most distinctive and least twisted section B for later processes. The algorithm utilizes different strategies for identifying these two sections, described in the following.

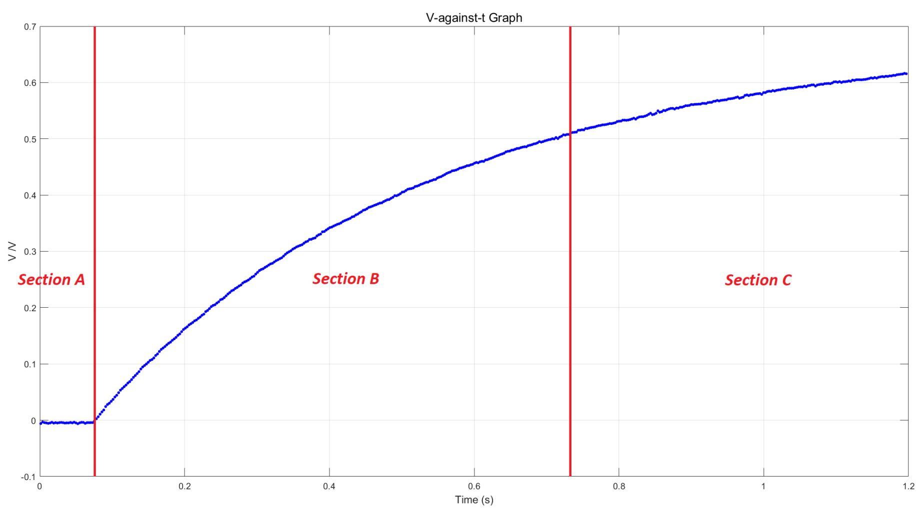


Figure 6: a raw data curve in 3 parts

Sections B and C, in fact, have no clear boundary in between. It's rather a trade-off between having more data or data that's less distorted. The algorithm finds the position of the first appearance of a voltage value larger than $80\% V_0$, and removes every datapoint after that position. The value 80% can be altered, and similar results would be achieved.

The boundary between sections A and B, however, must be drawn to the highest possible accuracy. The difference between the calculated timestamp and actual timestamp of experiment initiation is actually a systematic error on values t, and therefore a special approach is utilized. The algorithm first calculates the difference between every value V and its previous value, or the "difference" of every V value. Then the average difference value $\overline{diff}$ of the 1st to 30th difference values counting from the back to front is calculated, mathematically speaking $\overline{diff} = \frac{1}{30}\sum_{\mathrm{i}=1}^{30} V_{\mathrm{n-i+1}} - V_{\mathrm{n-i}}$ where n is the number of valid datapoints after removing section C. Finally, the smallest index p that satisfies $\prod_{\mathrm{i=p}}^{\mathrm{p+5}}(V_{\mathrm{i}} > \overline{diff}) = 1$, which is the smallest index that the consecutive five indices starting from it all have difference values greater than $\overline{diff}$, is considered the boundary between sections A and B, with V_{p} itself belonging to section B. The segemented section B is translated to the left so that V_{p} is placed under index 0. This segmentation process is displayed below in figure 7.

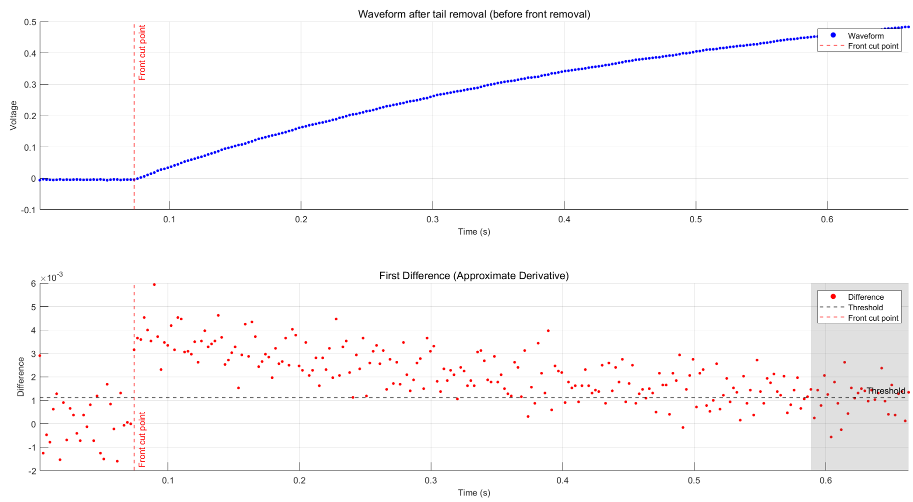


Figure 7: segmentation of sections A and B

The overall effect of these operations on raw data is shown below in figures 6 and 8.

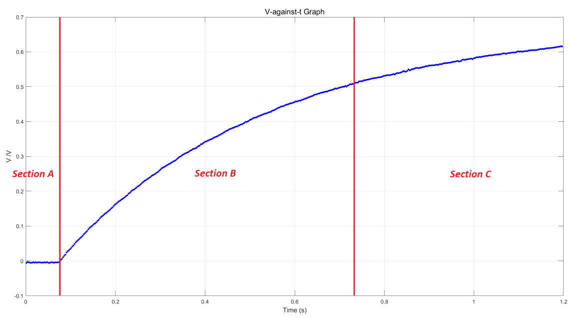


Figure 6: a raw data curve in 3 parts

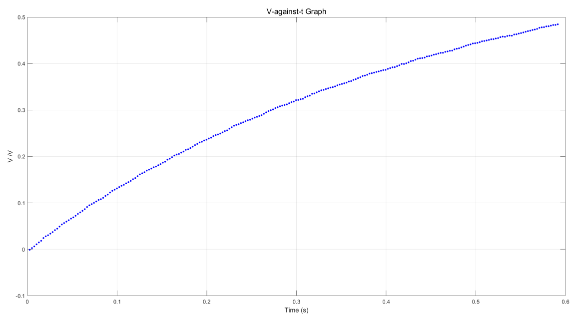


Figure 8: segmented section B of raw data curve

Fitting

The aim of this study is to experimentally validate the relationship that in a RC charging circuit, the voltage V experiences exponential asymtotic growth as time progresses. The curves obtained after previous preprocessing procedures should take the shape of an exponential asymptotic growth curve, though such curves are not ideal for straight-forwardly researching the desired relationship. This study transforms the data so that the curve morphs from exponential to linear.

From equation (4) we have:

$$\ln(\frac{V_0}{V_0 - V}) = t \times \frac{1}{RC} \tag{4}$$

If we map a y -value to each V -value where $y = \ln(\frac{V_0}{V_0 - V})$, we have:

$$y = \ln(\frac{V_0}{V_0 - V}) = t \times \frac{1}{RC} \tag{5}$$

We can see that the y-t graph should be a straight line passing origin, its gradient being constant value $\frac{1}{RC}$.

With that, for every preprocessed curve, the y -value is calculated for every datapoint (V, t) with equation (5) and plotted against t . The transformed curve is fitted to a linear function. The effect of this process on the same curve in Figure 8 is demonstrated in Figure 9 and 10 below, with fitting results.

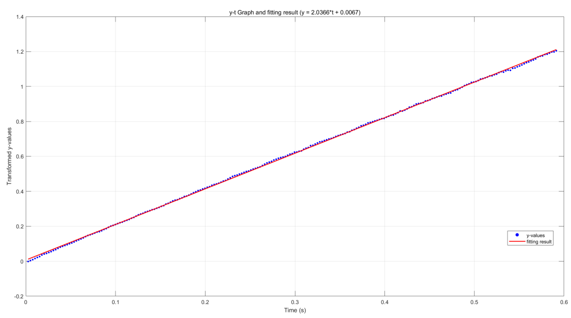


Figure 9: transformed data and fitting result

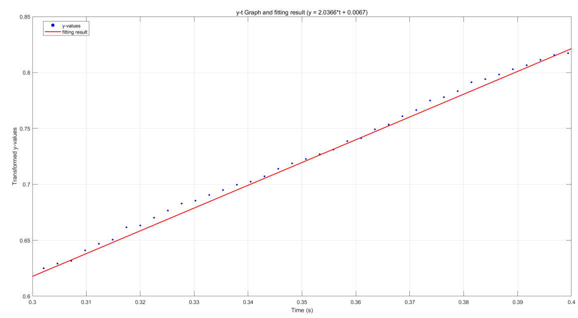


Figure 10: transformed data and fitting result enhanced

The whole process is carried out for every set of data, and results for fitting to a linear function are as follows:

Group	No.	Resistance R (Ω)	Gradient k (s^{-1})	Intercept (V)	$RMSE$ (V)	R^2

1	1	319	2.97952493	-0.00199887	0.00517728	0.999781
	2	319	2.97078336	0.00417882	0.00581313	0.999726
	3	319	2.97703789	0.00225135	0.00558107	0.999737
2	1	466	2.03935620	-0.00152792	0.00508810	0.999790
	2	466	2.03888946	-0.00124561	0.00496692	0.999777
	3	466	2.03656937	0.00673029	0.00499648	0.999798
3	1	559	1.70265612	-0.00713255	0.00491173	0.999808
	2	559	1.69986104	0.00127203	0.00482888	0.999809
	3	559	1.70272162	0.00178517	0.00511268	0.999786
4	1	665	1.42432470	0.00479054	0.00491151	0.999802
	2	665	1.42576452	0.00860237	0.00486418	0.999802
	3	665	1.42126468	0.01007710	0.00458907	0.999825
5	1	815	1.15634130	-0.00193304	0.00481210	0.999813
	2	815	1.15755949	0.00531055	0.00467220	0.999821
	3	815	1.16253833	0.00485817	0.00484884	0.999807

Preliminary Conclusion

In every case, a coefficient of determination R^2 greater than 0.9997 is achieved, indicating that every transformed set of data fits well to a linear function. Considering that the fitted linear functions all have intercepts with absolute values around or less than 0.01, this study concludes that y is directly proportional to t . Thus, this study preliminarily concludes that in a RC charging circuit, the voltage V does experience exponential asymtotic growth as time progresses.

Considering Error

This study contends that the majority of errors are introduced by the oscilloscope, although known information about these errors proved to be insufficient to construct an accurate mathematical model for error (for known information, see in the Evaluation Section). Thus, this study qualitatively analyzes its errors by calculating the capacitance C of the capacitor used in the experiments.

Per equation (5):

$$y = \ln(\frac{V_0}{V_0 - V}) = t \times \frac{1}{RC} \tag{5}$$

The gradient k of the y-t curve equals to $\frac{1}{RC}$, so the desired quantity C can be calculated by $C = \frac{1}{kR}$.

For every group i of experiments done using the same resistance R_i, the average measured capacitance C_i of the group is obtained by taking the average of the measured capacitances of the 3 experiments within that group. The record is as follows:

Group i	Resistance R_i (Ω)	Measured Capacitance C_i (F)
1	319	0.00105344
2	466	0.00105281
3	559	0.00105122
4	665	0.00105617
5	815	0.00105884

The best estimate for the desired capacitance $\overline{C}$ is calculated by taking the average of the average measured capacitance C_i of the 5 groups: $\overline{C} = \frac{1}{5}\sum_{\mathrm{i}=1}^{5} C_\mathrm{i} = 0.00105450\ \mathrm{F}$. The uncertainty of $\overline{C}$ is estimated using the standard sample deviation: $\Delta C = \sqrt{\frac{1}{5-1}\sum_{\mathrm{i}=1}^{5}(C_\mathrm{i} - \overline{C})^2} = 0.00000301\ \mathrm{F}$. Thus, after considering error, this study concludes that capacitance C of the capacitor used in the experiments is $C = \overline{C} \pm \Delta C = (0.001054 \pm 0.000003)\ \mathrm{F} = (1054 \pm 3)\ \mu\mathrm{F}$., with the percentage uncertainty $\%\Delta C = \frac{\Delta C}{\overline{C}} = 0.286\%$.

The actual capacitor used in the experiments was $1000\ \mu\mathrm{F}$ rated, and considering manufacturing imperfections and degradation from usage causing the actual capacitance at the time of the experiment to be off by a small percentage, the measured value is concluded to be accurate. This indicates that the experiment was quite successful and overall errors less than significant.

Conclusion and Evaluation

After collecting and processing the data and considering errors, this study concludes that the voltage V does experience exponential asymtotic growth as time progresses, achieving the aim of this study.

However, many inaccuracies have been introduced in this study. Part of which aren't quantifiable, and the majority of the rest are difficult or impossible to be accurately determined. Some significant, identified, and yet untreated errors are listed as follows.

- Offset error. This is caused by oscilloscope's internal component imperfections. It's a systematic error that will cause the measured voltages to deviate by a value that's constant under the same setting and experiment conditions. Under the settings used in this study, the offset error is $\pm 10 \text{ mV}$.
- Gain error. Induced after offset error, this is a systematic error caused by inaccuracies in the oscilloscope's internal amplification of the input voltage. Per the datasheet, it introduces a percentage error of up to 3% the applied voltage, and the exact percentage varies under different settings and experiment conditions.
- Quantimization error. Induced at the very end, this is the random error that occurred during the ADC process of the oscilloscope, when the actual measured voltage value is rounded to save system memory. Under the setting used in this study, the quantimization error is $\pm 2 \text{ mV}$. After taking 1 average out of every 128 measured values, this error is reduced to $\pm \frac{1}{64} \text{mV} \approx \pm 0.015625 \text{ mV}$ (https://www.analog.com).

A more accurate apporach to achieving the aim of this study would require a replacement to the oscilloscope, an instrument that delivers high precision of measurement rather than high sampling rate. In addition, to decrease the impact of performance of degradation caused by the short lifespan of laboratory capacitors, higher-quality capacitors could be used, and the experiment could be repeated using multiple different capacitors.

Unexpected Discovery

It is recorded shortly after the experiments are done that the voltage reading of the capacitor increases slowly but surely as time progresses. Given enough time, the emf of capacitors have managed to exceed 0.2V. This "self-charging" behavior is evident in this 10-minute-long voltage-time graph (figure 11) recorded with the oscilloscope at 1000 samples per second after taking an average every 1000 points.

huh.csv huh_csv.txt

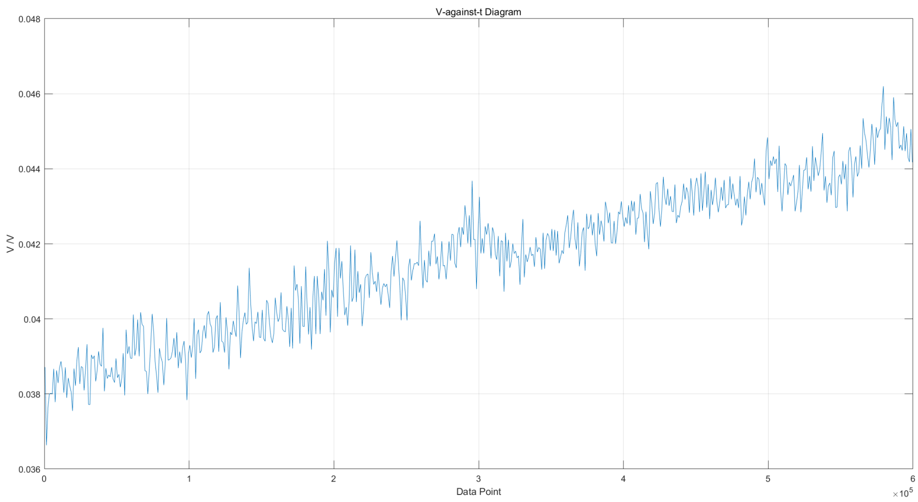


Figure 11: graph displaying "self-charging" behavior

An oscilloscope with input resistance around $1\ \mathrm{M\Omega}$ was used to eliminate the possibility of the measuring device charging the capacitor by any unoticed means. This study preliminarily finds that this phenomenon only occurs when the capacitor has recently been charged, meaning that such behaviors are not found on aged inventory capacitors. This phenomenon may be due to small amounts of charge accumulated somewhere in the internal physical structure of the capacitor, creating an emf insignificant compared to applied voltages, yet sufficient to charge the capacitor when nothing is connected to it on the exterior.

Reference List

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4. El-Tawargy, A. S., & Elshabaan, M. M. (2023). Charging and discharging a capacitor through a DC circuit: “PHET” demonstration and data analysis. Scientific Journal for Damietta Faculty of Science, 13(1), 135-144.