Octavian's Saga – README

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1 Task 1

In order to prove that $SAT \leq_P$ *Set Cover*, I have decided to use the fact that $SAT \leq_P Vertex\ Cover$ has already been proven.

1.1 Formal definition of the Set Cover problem

Let U be a set of elements and $C = \{S_1, S_2, ..., S_n\}$ be a collection of subsets of U. A *set cover* is a subcollection $C' \subseteq C$ whose union is U, meaning that, if $C' = \{X_1, X_2, \ldots, X_{|C'|}\}$, then $X_1 \cup X_2 \cup \ldots \cup X_{|C'|} = U$.

In the assignment, the example provided states that $U = \{1, 2, 3, 4\}$, $S_1 = \{1, 2\}$, $S_2 = \{2, 3, 4\}$ and $S_3 = \{2, 3\}$. It is then obvious that $C' = \{S_1, S_2\}$ is a set cover, while $C'' = \{S_2, S_3\}$ is not.

1.2 Proving that $Vertex\ Cover \leq_P$ *Set Cover*

Let $G = (V, E)$ be a graph. Then, let U and C be defined such that:

- The edges represent the elements in U, meaning that $E \to U$;
- The vertices correspond to the elements in C, meaning that $V \to C$.

Example 1. Consider the graph $G(V, E)$ where $V = \{1, \ldots, 6\}$ and:

$$E = \{e_1(1,2); e_2(1,5); e_3(2,3); e_4(2,4); e_5(3,4); e_6(3,6); e_7(4,5); e_8(5,6)\}$$

A representation of such a graph can be found bellow: TODO

In such a scenario, $U = E$ would yield $U = \{e_1, \ldots, e_8\}$ and the mapping $V \to C$ would correspond to:

$$S_1 = \{e_1, e_2\}$$
$$S_2 = \{e_1, e_3, e_4\}$$
$$S_3 = \{e_3, e_5, e_6\}$$
$$S_4 = \{e_4, e_5, e_7\}$$
$$S_5 = \{e_2, e_7, e_8\}$$
$$S_6 = \{e_6, e_8\}$$