# Homework 7

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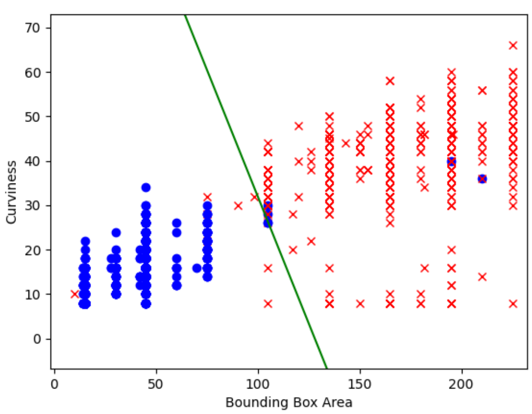
## 1

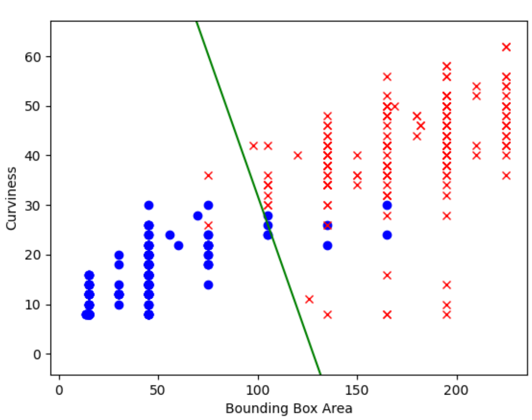
Classification Algorithm: Linear Regression for classification followed by pocket for improvement.

### (a)

Training:




Testing:



### (b)

$$E_{\text{in}} \approx 0.00833$$
$$E_{\text{test}} \approx 0.0165$$

### (c)

From $E_{\text{in}}$:

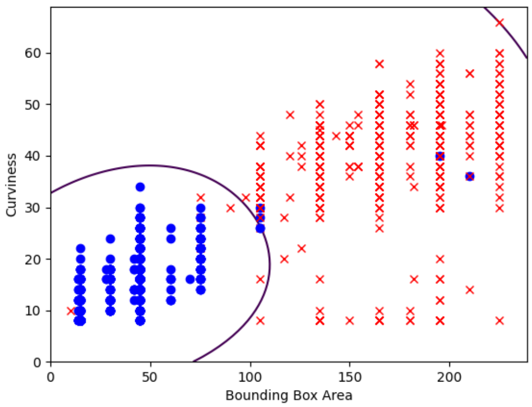
$$\begin{aligned}
E_{\text{out}} &\le E_{\text{in}} + \sqrt{\frac{8}{N} \ln \frac{4(2N)^3}{\delta}} \\
&\approx 0.00833 + \sqrt{\frac{8}{1561} \ln \frac{4(2 \times 1561)^3}{0.05}} \\
&\approx 0.00833 + 0.3823 \\
&= \boxed{0.3906}
\end{aligned}$$

From $E_{\text{test}}$:

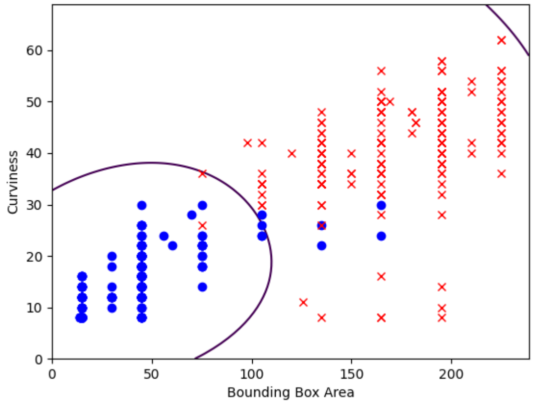
$$\begin{aligned}
E_{\text{out}} &\le E_{\text{test}} + \sqrt{\frac{1}{2K} \ln \frac{2}{\delta}} \\
&\approx 0.0165 + \sqrt{\frac{1}{424} \ln \frac{2}{0.05}} \\
&\approx 0.0165 + 0.0933 \\
&= \boxed{0.1098} \leftarrow \text{The Better Bound}
\end{aligned}$$

## (d)

Training:




$$
\begin{aligned}
E_{\text{in}} &\approx 0.00769 \\
E_{\text{out}} &\leq E_{\text{in}} + \sqrt{\frac{8}{N} \ln \frac{4(2N)^{10}}{\delta}} \\
&\approx 0.00769 + \sqrt{\frac{8}{1561} \ln \frac{4(2 \times 1561)^{10}}{0.05}} \\
&\approx 0.00833 + 0.6589 \\
&= \boxed{0.6671}
\end{aligned}
$$




Testing:

$$
\begin{aligned}
E_{\text{test}} &\approx 0.0165 \\
E_{\text{out}} &\leq E_{\text{test}} + \sqrt{\frac{1}{2K} \ln \frac{2}{\delta}} \\
&\approx 0.0165 + \sqrt{\frac{1}{424} \ln \frac{2}{0.05}} \\
&\approx 0.0165 + 0.0933 \\
&= \boxed{0.1098} \leftarrow \text{The Better Bound}
\end{aligned}
$$

### (e)

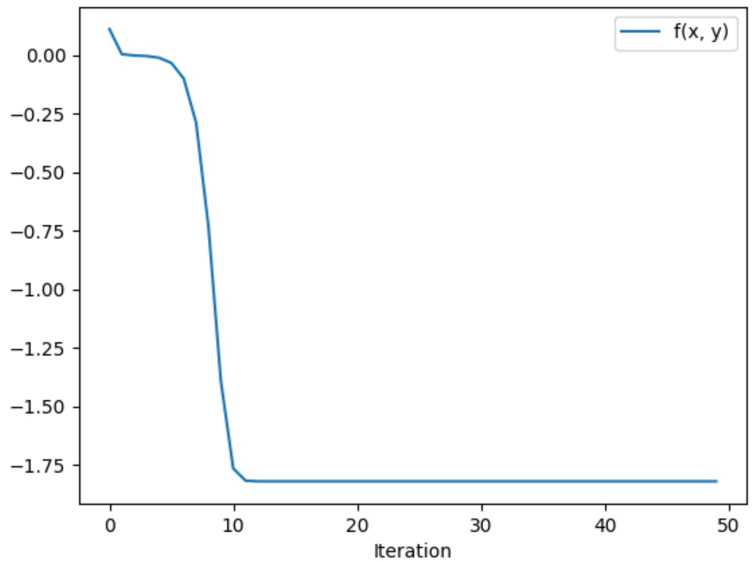
I would use the linear model because even though both models produce similar $E_{\text{test}}$, the visualization of the third order model just does not make sense with the boundary at the upper right corner. There's no way that a digit with even bigger bounding box and more curve is more likely to be a 1.

## 2

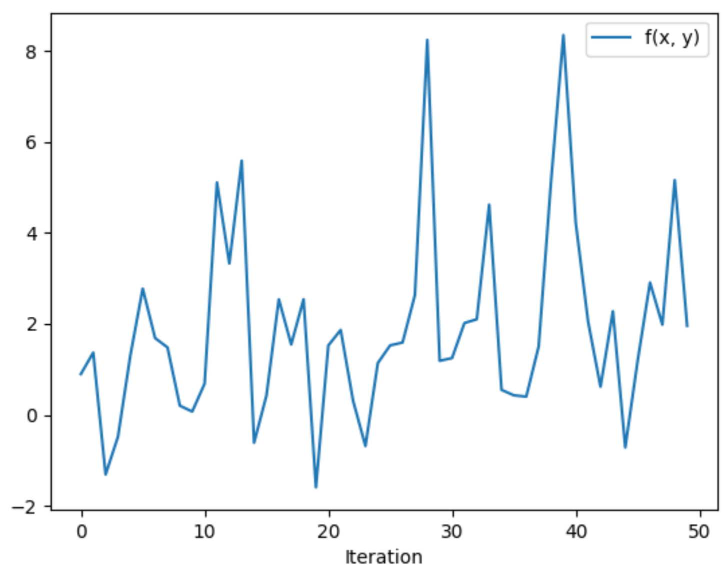
### (a)

$$
\nabla f(x, y) = \begin{pmatrix} 2x + 4\pi \cos(2\pi x) \sin(2\pi y) \\ 4y + 4\pi \sin(2\pi x) \cos(2\pi y) \end{pmatrix}
$$

For $\eta = 0.01$:




For $\eta = 0.1$:




$f(x, y)$ changes unpredictably because the step is too large. $(x, y)$ in each step sometimes moves to the other side of the local minimum, which potentially increases $f(x, y)$.

## (b)

| initial point | (0.1, 0.1) | (1, 1) | (-0.5, -0.5) | (-1, -1) |
|---|---|---|---|---|
| argmin $f(x, y)$ | (0.244, -0.238) | (1.218, 0.713) | (-0.731, -0.238) | (-1.218, -0.713) |
| min $f(x, y)$ | -1.820 | 0.593 | -1.332 | 0.593 |

## Problem 3.16

### (a)

$$\begin{aligned}
\text{cost(accept)} &= P[\text{correct}] \times 0 + P[\text{incorrect}]c_a \\
&= P[\text{incorrect}]c_a \\
&= (1 - g(\mathbf{x}))c_a \qquad \text{because } g(\mathbf{x}) \text{ is the probablilty that this person is correct} \\
\text{cost(reject)} &= P[\text{correct}]c_r + P[\text{incorrect}] \times 0 \\
&= P[\text{correct}]c_r \\
&= g(\mathbf{x})c_r
\end{aligned}$$

### (b)

$$\begin{aligned}
\text{Let } C = & \text{ the total cost of actions taken on each person} \\
&= \sum_{\mathbf{x}:g(\mathbf{x})\geq\kappa} \text{cost(accept)} + \sum_{\mathbf{x}:g(\mathbf{x})<\kappa} \text{cost(reject)} \\
&= \sum_{\mathbf{x}:g(\mathbf{x})\geq\kappa} (1 - g(\mathbf{x}))c_a + \sum_{\mathbf{x}:g(\mathbf{x})<\kappa} g(\mathbf{x})c_r
\end{aligned}$$

Consider $\frac{dC}{d\kappa}$ as the change in $C$ if we increase $\kappa$ a little bit such a single accepted data point $\mathbf{x}^*$ is now rejected. we know that $g(\mathbf{x}^*) = \kappa$.
To minimize $C$ with respect to $\kappa$, we set $\frac{dC}{d\kappa}$ to 0.

$$\begin{aligned}
\frac{dC}{d\kappa} &= -(1 - g(\mathbf{x}^*))c_a + g(\mathbf{x}^*)c_r \\
&= -(1 - \kappa)c_a + \kappa c_r \\
&= \kappa(c_a + c_r) - c_a \\
&= 0 \\
\kappa &= \frac{c_a}{c_r + c_a}
\end{aligned}$$

### (c)

Supermarket: $\kappa = \frac{1}{11}$
Because it costs a lot to reject someone, we want to accept as many as possible, which results in a lower $\kappa$.
CIA: $\kappa = \frac{1000}{1001}$
Because it costs a lot to accept someone, we want to reject as many as possible, which results in a higher $\kappa$.