

EE 451: Communications Systems

Midterm Exam 2 - Study Guide

Exam Date: Week 12 (Tuesday, April 21, 2026) **Duration:** 75 minutes **Coverage:** Chapters 4-7 (FM, PM, FSK, PSK, M-ary Modulation, QAM) **Format:** Closed book, one 8.5" × 11" formula sheet (both sides) allowed

Overview

Midterm 2 focuses on angle modulation (FM/PM) and digital modulation schemes. You should be comfortable with:
- FM and PM fundamentals and relationships
- Carson's rule for FM bandwidth
- Binary digital modulation (FSK, BPSK)
- M-ary modulation (QPSK, QAM)
- Gray coding and constellation diagrams
- Spectral efficiency concepts

Topic 1: Frequency Modulation (FM) Fundamentals

Key Concepts

1. **FM Signal Characteristics** - Frequency varies with message signal - Amplitude remains constant (noise immunity!) - Frequency deviation: $\Delta f = k_f \max |m(t)|$ - Modulation index: $\beta = \Delta f / f_m$
2. **Narrowband vs. Wideband FM - NBFM:** $\beta \ll 1$ (typically $\beta < 0.3$) - **WBFM:** $\beta \gg 1$ (broadcast FM: $\beta = 5$) - Different bandwidth requirements!
3. **FM vs. AM** - AM: Constant frequency, varying amplitude - FM: Constant amplitude, varying frequency - FM: Better noise performance (but more bandwidth)

Essential Formulas

FM Signal:

$$s(t) = A \cos[2\pi f_c t + 2\pi k_f m(t)dt]$$

Instantaneous Frequency:

$$f_i(t) = f_c + k_f \cdot m(t)$$

where k_f = frequency sensitivity (Hz/V)

Frequency Deviation:

$$\Delta f = k_f \cdot \max|m(t)|$$

For $m(t) = A_m \cos(2\pi f_m t)$:

$$\Delta f = k_f \cdot A_m$$

Modulation Index:

$$= \Delta f / f_m$$

Carson's Rule (FM Bandwidth):

$$BFM = 2(\Delta f + f_m) = 2f_m(1 + \beta)$$

NBFM Criterion:

$$\beta < 1 \text{ (typically } \beta < 0.3\text{)} \\ BFM = 2f_m \text{ (like AM!)}$$

WBFM:

$$\beta \gg 1 \\ BFM = 2\Delta f \text{ (dominated by deviation)}$$

Common Mistakes to Avoid

Confusing Δf (deviation) with f_m (message frequency) Forgetting that Δf depends on k_f AND message amplitude Using wrong Carson's rule form (two equivalent forms!) Mixing up NBFM and WBFM criteria

Practice Problems

From **Homework 3**: - Problem 1: FM fundamentals - Problem 2: Carson's rule application - Problem 3: NBFM vs. WBFM classification

Strategy: 1. Identify k_f , A_m , f_m from problem 2. Calculate $\Delta f = k_f \times A_m$ 3. Calculate $\beta = \Delta f / f_m$ 4. Check if NBFM ($\beta < 0.3$) or WBFM ($\beta > 1$) 5. Apply Carson's rule: $B = 2(\Delta f + f_m)$

Topic 2: Phase Modulation (PM)

Key Concepts

1. **PM Signal Characteristics** - Phase varies directly with message - $\theta_i(t) = 2\pi f_c t + k_p m(t)$ - Phase sensitivity: k_p (rad/V)
2. **FM vs. PM Relationship - Key insight:** PM = FM with derivative - FM can be generated from PM by integrating message - PM can be generated from FM by differentiating message
3. **Frequency Deviation in PM** - PM has frequency deviation that **increases with f_m !** - FM has frequency deviation **independent of f_m** - This is the KEY difference!

Essential Formulas

PM Signal:

$$s(t) = A \cos[2\pi f_c t + k_p \cdot m(t)] \\ \text{where } k_p = \text{phase sensitivity (rad/V)}$$

Instantaneous Phase:

$$\theta_i(t) = 2\pi f_c t + k_p \cdot m(t)$$

Phase Deviation:

$$\Delta = k_p \cdot \max|m(t)|$$

Instantaneous Frequency:

$$f_i(t) = (1/2) \cdot d i/dt = f_c + (k_p/2) \cdot dm/dt$$

For $m(t) = A_m \cos(2\pi f_m t)$:

Maximum Frequency Deviation:

$$\Delta f_{PM} = k_p \cdot A_m \cdot f_m$$

(Note: proportional to f_m !)

Comparison:

FM: $\Delta f = k_f \cdot A_m$ (independent of f_m)

PM: $\Delta f = k_p \cdot A_m \cdot f_m$ (proportional to f_m)

Common Mistakes to Avoid

Thinking PM and FM have the same Δf behavior
Forgetting to take derivative when finding $f_i(t)$ from PM
Confusing $\Delta\phi$ (phase deviation) with Δf (frequency deviation)

Practice Problems

From **Homework 3**: - Problem 4: PM analysis - Problem 5: FM vs. PM relationship

Strategy: 1. For PM: Write $\theta_i(t) = 2\pi f_c t + k_p m(t)$ 2. Find $\Delta\phi = k_p \times \max |m(t)|$ 3. Find $f_i(t) = \frac{1}{2\pi} \frac{d\theta_i}{dt}$ 4. For sinusoidal message: $\Delta f = k_p \times A_m \times f_m$

Topic 3: Binary FSK (Frequency Shift Keying)

Key Concepts

1. FSK Basics - Bit "0" → Frequency f_0 - Bit "1" → Frequency f_1 - Frequency separation: $\Delta f = |f_1 - f_0|$

2. Modulation Index for FSK - $h = \Delta f \times T_b$ - Critical parameter for FSK design - Determines orthogonality and spectral efficiency

3. Orthogonality for Coherent Detection - Minimum separation: $\Delta f_{min} = \frac{1}{2T_b}$ - This gives $h = 0.5$ (MSK!) - Larger separation OK, but wastes bandwidth

4. Coherent vs. Non-Coherent Detection - **Coherent:** Requires phase lock, better performance - **Non-coherent:** Simpler, no phase tracking needed - FSK can use either (unlike PSK which requires coherent)

Essential Formulas

FSK Signal Expressions:

$$s(t) = A \cdot \cos(2\pi f_0 t) \quad (\text{bit "0"})$$

$$s(t) = A \cdot \cos(2\pi f_1 t) \quad (\text{bit "1"})$$

Frequency Separation:

$$\Delta f = |f_1 - f_0|$$

Bit Duration:

$$T_b = 1/R_b \quad (\text{where } R_b = \text{bit rate})$$

Modulation Index:

$$h = \Delta f \cdot T_b$$

Orthogonality Condition (Coherent FSK):

$$\Delta f_{\min} = 1/(2T_b) = R_b/2$$

Gives $h = 0.5$ (Minimum Shift Keying!)

Bandwidth (Carson's Rule):

$$B_{FSK} = 2(\Delta f + R_b)$$

Common Mistakes to Avoid

Forgetting to calculate $T_b = 1/R_b$ first Confusing Δf (separation) with f_0 or f_1 (individual frequencies) Using wrong orthogonality condition ($1/T_b$ instead of $1/(2T_b)$)

Practice Problems

From **Homework 3:** - Problem 6: Binary FSK fundamentals - Problem 7: MSK analysis - Problem 9: Coherent vs. non-coherent detection

Strategy: 1. Find $T_b = 1/R_b$ 2. Calculate $\Delta f = |f_1 - f_0|$ 3. Calculate $h = \Delta f \times T_b$ 4. Check orthogonality: Is $\Delta f \geq 1/(2T_b)$? 5. If $h = 0.5$, it's MSK!

Topic 4: Minimum Shift Keying (MSK)

Key Concepts

1. MSK Definition - Special case of CPFSK (Continuous Phase FSK) - Modulation index $h = 0.5$ exactly - Minimum frequency separation for orthogonal FSK

2. Advantages of MSK - Continuous phase \rightarrow no discontinuities - Better spectral efficiency than standard FSK - Faster sidelobe decay (less interference) - Constant envelope (power amplifier friendly)

3. Applications - GSM cellular (GMSK variant) - Satellite communications - Anywhere spectral efficiency matters

Essential Formulas

MSK Characteristics:

$$h = 0.5 \quad (\text{defining property!})$$

$$\Delta f = R_b/2 = 1/(2T_b)$$

For MSK with bit rate R_b :

$$\Delta f = R_b/2$$

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If center frequency is fc:  

f = fc - Rb/4  

f = fc + Rb/4
```

MSK Advantages:

- Continuous phase
- Minimum bandwidth for orthogonal FSK
- Constant envelope

Practice Problems

From **Homework 3**: - Problem 7: Complete MSK analysis

Strategy: 1. If $h = 0.5$, it's MSK 2. From $h = 0.5$ and bit rate, find $\Delta f = 0.5R_b$ 3. Frequencies: $f_0 = f_c - \Delta f/2$, $f_1 = f_c + \Delta f/2$

Topic 5: Binary Phase Shift Keying (BPSK)

Key Concepts

1. **BPSK Basics** - Bit "0" → Phase 0° (or $+A$) - Bit "1" → Phase 180° (or $-A$) - Information in PHASE, not amplitude - Constant envelope (like FM/MSK)
2. **Constellation Diagram** - Two points on I (real) axis - 180° apart - Maximum separation = best noise immunity
3. **Detection** - Must use coherent detection! - Envelope detector won't work (constant envelope) - Requires carrier phase synchronization
4. **Bandwidth** - Null-to-null: $B = 2R_b$ - Similar to ASK (same symbol rate)

Essential Formulas

BPSK Signals:

$$\begin{aligned}s(t) &= A \cdot \cos(2\pi f t) && (\text{bit "0", } 0^\circ) \\ s(t) &= -A \cdot \cos(2\pi f t) && (\text{bit "1", } 180^\circ) \\ \text{Or: } s(t) &= A \cdot \cos(2\pi f t + \phi)\end{aligned}$$

Constellation:

Bit "0": $I = +A$, $Q = 0$

Bit "1": $I = -A$, $Q = 0$

Phase difference: 180°

Bandwidth:

$$B_{\text{null-null}} = 2/T_b = 2R_b$$

$$B_{\text{3dB}} = R_b$$

Cannot use envelope detection!

(Envelope is constant = A for both symbols)

Common Mistakes to Avoid

Saying BPSK can use envelope detection (NO!) Confusing BPSK bandwidth with carrier frequency Drawing constellation in wrong locations

Practice Problems

From **Homework 3:** - Problem 8: BPSK fundamentals

Strategy: 1. Write two signal expressions (180° apart) 2. Draw constellation: two points on I-axis
3. Calculate bandwidth: $B = 2R_b$ 4. Explain why coherent detection is required

Topic 6: QPSK (Quadrature Phase Shift Keying)

Key Concepts

1. **QPSK Basics** - 4 phase states ($45^\circ, 135^\circ, 225^\circ, 315^\circ$) - Encodes **2 bits per symbol** - Twice as efficient as BPSK! - Symbol rate = Bit rate / 2
2. **I/Q Representation** - QPSK uses both I (In-phase) and Q (Quadrature) channels - Each symbol: combination of I and Q amplitudes - 4 constellation points equally spaced
3. **BER Performance** - QPSK has same BER as BPSK for same E_b/N_0 - But uses half the bandwidth! - Best of both worlds for spectral efficiency

Essential Formulas

QPSK Parameters:

$M = 4$ symbols

$k = 2$ bits/symbol

Symbol rate: $R_s = R_b/2$

Four Signals (phases $45^\circ, 135^\circ, 225^\circ, 315^\circ$):

$$s(t) = A \cdot \cos(2\pi f t + \pi/4)$$

$$s(t) = A \cdot \cos(2\pi f t + 3\pi/4)$$

$$s(t) = A \cdot \cos(2\pi f t + 5\pi/4)$$

$$s(t) = A \cdot \cos(2\pi f t + 7\pi/4)$$

Constellation Points:

All at distance A from origin

90° apart

On circle of radius A

Bandwidth:

$$B_{QPSK} = 2R_s = 2(R_b/2) = R_b$$

Half the bandwidth of BPSK for same bit rate!

Spectral Efficiency:

BPSK: 0.5 bits/s/Hz (in null-null BW)

QPSK: 1.0 bits/s/Hz

Common Mistakes to Avoid

Forgetting symbol rate = bit rate / 2 Drawing constellation points at wrong phases Not using Gray coding (see next topic!) Thinking QPSK has worse BER than BPSK

Practice Problems

From **Homework 4:** - Problem 1: QPSK fundamentals - Problem 2: Gray coding for QPSK

Strategy: 1. Calculate symbol rate: $R_s = R_b/2$ 2. Write 4 signal expressions ($45^\circ, 135^\circ, 225^\circ, 315^\circ$) 3. Draw constellation with 4 points 4. Bandwidth: $B = 2R_s = R_b$

Topic 7: Gray Coding

Key Concepts

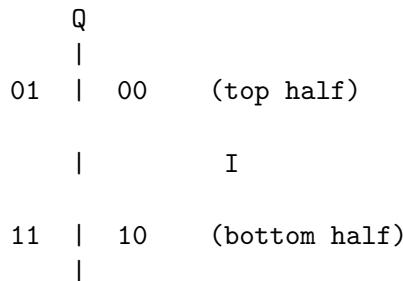
1. **Why Gray Coding?** - Adjacent symbols differ by only **1 bit** - Most errors go to adjacent symbols (AWGN) - Symbol error → typically 1 bit error (not 2!) - Reduces bit error rate (BER)
2. **Gray Code Properties** - Used in ALL modern systems (WiFi, LTE, 5G) - Essential for M-ary modulation - Simple mapping rule: adjacent symbols differ by 1 bit
3. **Example for QPSK**

Natural Binary: 00, 01, 10, 11 (can differ by 2 bits!)

Gray Coded: 00, 01, 11, 10 (adjacent differ by 1 bit)

QPSK Gray Coding Example

Constellation (typical Gray mapping):



Key: Moving horizontally or vertically changes only 1 bit!

Common Mistakes to Avoid

Using natural binary instead of Gray code Assigning codes randomly without checking adjacency

Practice Problems

From **Homework 4:** - Problem 2: Gray coding for QPSK - Problem 3: 8-PSK Gray coding

Strategy: 1. Draw constellation 2. Assign codes so adjacent symbols differ by 1 bit 3. Verify: moving one step (any direction) changes 1 bit

Topic 8: M-ary Modulation and QAM

Key Concepts

1. **M-ary PSK** - M symbols, $k = \log_2(M)$ bits/symbol - Better spectral efficiency as M increases
- But requires higher SNR for same BER
2. **QAM (Quadrature Amplitude Modulation)** - Combines amplitude AND phase modulation
- Uses I and Q channels independently - Rectangular constellation (e.g., 16-QAM: 4×4 grid) - Even better spectral efficiency than M-PSK
3. **Spectral Efficiency** - BPSK: 1 bit/symbol - QPSK: 2 bits/symbol - 16-QAM: 4 bits/symbol - 64-QAM: 6 bits/symbol - Higher M = more bits/symbol but needs more SNR

Essential Formulas

M-ary Modulation:

$M = \text{number of symbols}$

$k = \log_2(M) = \text{bits per symbol}$

$R_s = R_b/k = \text{symbol rate}$

Examples:

BPSK: $M=2$, $k=1$

QPSK: $M=4$, $k=2$

8-PSK: $M=8$, $k=3$

16-QAM: $M=16$, $k=4$

64-QAM: $M=64$, $k=6$

Spectral Efficiency:

$= R_b/B = k \cdot R_s/B = k \cdot R_b/M \cdot B = k \cdot B/M = k \cdot B \cdot 10^{-6} \text{ bits/s/Hz}$

Higher $M \rightarrow$ higher

Trade-off:

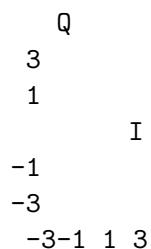
Higher M : Better spectral efficiency

Higher M : Needs higher SNR (worse BER)

16-QAM Constellation

4×4 rectangular grid:

Amplitude levels: $\{\pm 1, \pm 3\}$ for I and Q



Common Mistakes to Avoid

Confusing M (symbols) with k (bits/symbol) Forgetting symbol rate: $R_s = R_b/k$ Not using Gray coding for QAM Thinking more bits/symbol is always better (ignores SNR!)

Practice Problems

From **Homework 4:** - Problem 3: M-ary PSK - Problem 4: QAM constellation design - Problem 5: Spectral efficiency comparison

Strategy: 1. Find k: $k = \log_2(M)$ 2. Find symbol rate: $R_s = R_b/k$ 3. For QAM: Draw rectangular constellation 4. Assign Gray codes to minimize bit errors

Topic 9: EVM (Error Vector Magnitude)

Key Concepts

1. **EVM Definition** - Measures constellation quality - EVM = error distance / reference distance - Expressed as percentage
2. **Why EVM Matters** - WiFi/LTE specs have strict EVM limits - Higher-order modulation needs lower EVM - Indicates transmitter/receiver impairments
3. **EVM Requirements** - 256-QAM: EVM < 3% (very tight!) - 64-QAM: EVM < 8% - 16-QAM: EVM < 12% - QPSK: EVM < 17%

Essential Formulas

EVM Calculation:

$$\begin{aligned}\text{Error vector} &= \text{Received} - \text{Ideal} \\ |\text{Error}| &= \sqrt{(\Delta I)^2 + (\Delta Q)^2}\end{aligned}$$

$$\text{EVM (\%)} = (|\text{Error}| / |\text{Reference}|) \times 100\%$$

Example:

$$\begin{aligned}\text{Ideal: } (I, Q) &= (1.0, 0.0) \\ \text{Received: } (I, Q) &= (0.92, 0.15) \\ \text{Error} &= \sqrt{(1.0 - 0.92)^2 + (0.0 - 0.15)^2} \\ &= \sqrt{[0.08^2 + 0.15^2]} \\ &= \sqrt{[0.0064 + 0.0225]} \\ &= \sqrt{0.0289} = 0.17 \\ \text{EVM} &= (0.17/1.0) \times 100\% = 17\%\end{aligned}$$

Practice Problems

From **Homework 4:** - Problem 6: EVM calculation - Bonus 2: EVM limits in standards

Strategy: 1. Plot ideal and received points 2. Calculate error vector 3. Find magnitude of error 4. Divide by reference (usually distance to ideal point) 5. Convert to percentage

Formula Sheet Recommendations

FM/PM Essentials

FM:

$$\begin{aligned} f_i(t) &= f_c + k_f \cdot m(t) \\ \Delta f &= k_f \cdot \max|m(t)| \\ &= \Delta f/f_m \\ BFM &= 2(\Delta f + f_m) = 2f_m(1 +) \\ NBFM: &<< 1 \\ WBFM: &>> 1 \end{aligned}$$

PM:

$$\begin{aligned} i(t) &= 2f_{ct} + k_p \cdot m(t) \\ \Delta &= k_p \cdot \max|m(t)| \\ f_i(t) &= f_c + (k_p/2) \cdot dm/dt \\ \text{For sinusoidal } m(t): \\ \Delta f_{PM} &= k_p \cdot A_m \cdot f_m \end{aligned}$$

Digital Modulation

FSK:

$$\begin{aligned} h &= \Delta f \cdot T_b \\ \text{Orthogonal: } \Delta f &= 1/(2T_b) \\ \text{MSK: } h &= 0.5 \end{aligned}$$

BPSK:

$$\begin{aligned} 2 \text{ symbols, } k &= 1 \text{ bit/symbol} \\ B &= 2R_b \\ \text{Coherent detection required} \end{aligned}$$

QPSK:

$$\begin{aligned} M=4, \ k &= 2 \text{ bits/symbol} \\ R_s &= R_b/2 \\ B &= R_b \text{ (half of BPSK!)} \end{aligned}$$

M-ary:

$$\begin{aligned} k &= \log(M) \\ R_s &= R_b/k \end{aligned}$$

Spectral Efficiency

Bit rate: R_b

Symbol rate: $R_s = R_b/k$

Spectral efficiency: $= R_b/B$ bits/s/Hz

Examples:

$$\begin{aligned} \text{BPSK: } &0.5 \text{ bits/s/Hz} \\ \text{QPSK: } &1.0 \text{ bits/s/Hz} \end{aligned}$$

16-QAM:	2.0 bits/s/Hz
64-QAM:	3.0 bits/s/Hz

Exam Strategy

Time Management (75 minutes)

- **4 min:** Read entire exam
- **12 min:** Problem 1 (FM fundamentals with Carson's rule)
- **10 min:** Problem 2 (PM analysis)
- **8 min:** Problem 3 (FM vs PM comparison)
- **12 min:** Problem 4 (FSK system design)
- **10 min:** Problem 5 (BPSK fundamentals)
- **13 min:** Problem 6 (QPSK and Gray coding)
- **2 min:** Problem 7 (MSK bonus if time)
- **4 min:** Review

Problem-Solving Approach

For FM Problems: 1. Find $\Delta f = k_f \times A_m$ 2. Find $\beta = \Delta f / f_m$ 3. Classify: NBFM ($\beta < 0.3$) or WBFM ($\beta > 1$)? 4. Apply Carson's rule: $B = 2(\Delta f + f_m)$

For PM Problems: 1. Find $\Delta\phi = k_p \times A_m$ 2. Find $f_i(t)$ by taking derivative 3. For sinusoidal: $\Delta f = k_p \times A_m \times f_m$ 4. Compare to FM (key difference: Δf depends on f_m in PM!)

For FSK Problems: 1. Calculate $T_b = 1/R_b$ 2. Calculate $\Delta f = |f_1 - f_0|$ 3. Calculate $h = \Delta f \times T_b$ 4. Check orthogonality: $\Delta f \geq 1/(2T_b)$? 5. If $h = 0.5$: MSK!

For QPSK Problems: 1. Symbol rate: $R_s = R_b/2$ 2. Write 4 signal expressions 3. Draw constellation (4 points equally spaced) 4. Apply Gray coding 5. Bandwidth: $B = 2R_s = R_b$

Common Mistakes

FM: Confusing Δf with f_m , forgetting Carson's rule has TWO forms **PM:** Thinking Δf is independent of f_m (it's not!) **FSK:** Using $1/T_b$ for orthogonality instead of $1/(2T_b)$ **BPSK:** Saying envelope detection works (it doesn't!) **QPSK:** Forgetting symbol rate = bit rate / 2

Gray coding: Not checking that adjacent symbols differ by 1 bit

Self-Assessment Checklist

Before the exam, can you:

FM/PM: - [] Calculate frequency deviation Δf for FM - [] Calculate modulation index β - [] Apply Carson's rule correctly - [] Classify NBFM vs. WBFM - [] Find instantaneous frequency for PM - [] Explain why PM has Δf proportional to f_m

FSK: - [] Calculate bit duration from bit rate - [] Find frequency separation Δf - [] Calculate modulation index h - [] Check orthogonality condition - [] Identify MSK ($h = 0.5$)

BPSK/QPSK: - [] Write signal expressions - [] Draw constellation diagrams - [] Calculate bandwidth - [] Explain why BPSK needs coherent detection - [] Find symbol rate for QPSK - [] Apply Gray coding

M-ary/QAM: - [] Calculate bits per symbol ($k = \log_2 M$) - [] Find symbol rate from bit rate - [] Draw QAM constellation - [] Compare spectral efficiencies - [] Calculate EVM

Practice Resources

Homework 3 (FM/PM/FSK/BPSK): All problems 1-10 **Homework 4 (M-ary/QAM/EVM):** All problems 1-10

Focus especially on: - HW3: Problems 1, 2, 5, 6, 7, 8 - HW4: Problems 1, 2, 4, 6

Good luck!