EE 451: Communications Systems

Midterm Exam 1 - Study Guide

Exam Date: Week 5 (February 25, 2026) **Duration:** 90 minutes **Coverage:** Chapters 2-3, 7.1-7.2 (Fourier Analysis & Amplitude Modulation) **Format:** Closed book, one 8.5” × 11” formula sheet (both sides) allowed

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Overview

Midterm 1 focuses on foundational concepts in signal analysis and amplitude modulation. You should be comfortable with: - Fourier transform analysis and properties - Time-frequency duality and bandwidth concepts - AM modulation and demodulation - Power calculations in communication systems - Binary ASK as digital extension of AM

Topic 1: Fourier Transform Fundamentals

Key Concepts

1. Rectangular Pulse and Sinc Function - Rectangular pulse in time → Sinc function in frequency - Time-frequency duality: Narrow pulse → Wide spectrum - First zero crossing determines bandwidth

2. Bandwidth Definition - Null-to-null bandwidth: Distance between first zeros - 3-dB bandwidth: Half-power points - Relationship: $B_{3dB} \approx 0.88 \times B_{null}$

3. Signal Energy - Time domain: $E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$ - Use Parseval’s theorem to verify in frequency domain

Essential Formulas

```
Fourier Transform Pair:
  rect(t/T)  T·sinc(fT)

Sinc Function:
  sinc(x) = sin( x)/( x)
  First zero at x = ±1

Rectangular Pulse:
  x(t) = A·rect(t/T)
  X(f) = AT·sinc(fT)
  First zero at f = ±1/T

Energy:
  E =  |x(t)|² dt = A²T (for rectangular pulse)
```

Practice Problems

From **Homework 1:** - Problem 1: Rectangular pulse Fourier transform - Problem 2: Sinc function properties - Problem 3: Energy calculations

Strategy: - Draw time and frequency domain sketches - Identify key parameters: amplitude, duration, bandwidth - Check units (time frequency reciprocity)

Topic 2: Fourier Transform Properties

Key Concepts

1. Time-Shift Property - Delay in time $\rightarrow$ Phase shift in frequency - Magnitude spectrum unchanged - $x(t-t_0) \leftrightarrow X(f)e^{-j2\pi f t_0}$

2. Modulation Property (Critical for AM!) - Multiplication by cosine $\rightarrow$ Frequency translation - $x(t)\cos(2\pi f_c t) \leftrightarrow \frac{1}{2}[X(f-f_c)+X(f+f_c)]$ - Creates upper and lower sidebands

3. Bandwidth in Modulated Signals - DSB-SC bandwidth = $2B$ (where B is message bandwidth) - Spectrum centered at $\pm f_c$

Essential Formulas

```
Time-Shift:
  x(t - t )   X(f)e^(-j2 ft )

Modulation (DSB-SC):
  x(t)cos(2 f t)   ½[X(f - f ) + X(f + f )]

Bandwidth:
  Message bandwidth: B
  DSB-SC bandwidth: 2B
  AM bandwidth: 2B
  SSB bandwidth: B
```

Practice Problems

From **Homework 1:** - Problem 4: Modulation property - Problem 5: DSB-SC spectrum sketching

Strategy: - Always sketch BOTH time and frequency domains - Label carrier frequency f_c and sidebands clearly - Remember: modulation creates TWO copies (at $\pm f_c$)

Topic 3: Parseval's Theorem

Key Concepts

1. Energy Conservation - Energy in time = Energy in frequency - Useful for verifying calculations - Different domains, same physical quantity

2. Application - Calculate energy in whichever domain is easier - Time domain for simple pulses - Frequency domain for sinc functions

Essential Formulas

```
Parseval's Theorem:
   |x(t)|² dt =  |X(f)|² df

For exponential decay x(t) = Ae^(-at)u(t):
  E = A²/(2a)
```

Practice Problems

From **Homework 1:** - Problem 6: Verify energy using both domains

Strategy: - Calculate in time domain first (usually easier) - Use Parseval to verify in frequency domain - Check units: energy has units of $V^2 \cdot s$ or J

Topic 4: Amplitude Modulation (AM)

Key Concepts

1. AM Signal Structure - $s(t) = A_c[1+\mu m_n(t)]\cos(2\pi f_c t)$ - Carrier amplitude: A_c - Modulation index: $\mu = A_m/A_c$ - Normalized message: $|m_n(t)| \leq 1$

2. Modulation Index - $\mu < 1$: Normal modulation - $\mu = 1$: 100% modulation - $\mu > 1$: Overmodulation (envelope distortion!)

3. Power Distribution - Carrier power: Constant, carries NO information - Sideband power: Varies with μ, contains information - Efficiency: Low for small μ

Essential Formulas

```
AM Signal:
  s(t) = A [1 +  ·cos(2 f t)]cos(2 f t)
  A  = carrier amplitude
    = modulation index = A /A

Total Power:
  P_total = (A ²/2R)(1 +  ²/2)

Carrier Power:
  P_carrier = A ²/(2R)

Sideband Power:
  P_sideband = (A ²/2R)( ²/2)

Efficiency:
    = P_sideband/P_total =  ²/(2 +  ²)
```

```
Bandwidth:
  B_AM = 2f  (same as DSB-SC)
```

Common Mistakes to Avoid

Using A_c+A_m instead of $A_c(1+\mu)$ Forgetting factor of 1/2 in power formulas (RMS!) Confusing μ with $m(t)$ Not checking for overmodulation ($\mu > 1$)

Practice Problems

From **Homework 2:** - Problem 1: Basic AM analysis - Problem 2: Power calculations - Problem 3: Modulation index and efficiency - Problem 4: Overmodulation detection

Strategy: 1. Identify A_c, A_m, f_c, f_m from equation 2. Calculate $\mu = A_m/A_c$ 3. Check if $\mu \leq 1$ (overmodulated?) 4. Use power formulas (don't forget the R!) 5. Calculate efficiency as percentage

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Topic 5: DSB-SC vs. Full AM

Key Concepts

1. DSB-SC (Double Sideband Suppressed Carrier) - $s(t) = A_c m(t)\cos(2\pi f_c t)$ - No carrier component → more efficient - Requires coherent detection (complex receiver)

2. Comparison Table

Feature	DSB-SC	Full AM
Carrier	Suppressed	Transmitted
Power efficiency	100% in sidebands	$\mu^2/(2+\mu^2)$
Detection	Coherent required	Envelope detector
Bandwidth	2B	2B
Complexity	High (receiver)	Low (receiver)

3. When to Use - DSB-SC: Power-limited systems, don't mind receiver complexity - AM: Simple receivers (AM radio), power not critical

Essential Formulas

```
DSB-SC Signal:
  s(t) = A ·m(t)·cos(2 f t)

DSB-SC Power:
  P_DSB = (A ²/2R)·P_m
  where P_m = mean square of m(t)

For sinusoidal m(t) = A cos(2 f t):
  P_DSB = A ²A ²/(4R)
```

```
Comparison:
  P_DSB_sideband = P_AM_sideband (same sidebands)
  But P_AM_total = P_AM_sideband + P_carrier
```

Practice Problems

From **Homework 2:** - Problem 5: DSB-SC power calculation - Problem 6: Efficiency comparison

Strategy: - For DSB-SC: All power is in sidebands - For AM: Must calculate carrier + sideband power - Compare efficiencies as percentages

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Topic 6: Single Sideband (SSB)

Key Concepts

1. Motivation - DSB-SC wastes bandwidth (redundant sidebands) - SSB transmits only one sideband - Half the bandwidth of DSB-SC or AM!

2. Bandwidth Efficiency - Message bandwidth: B - AM/DSB-SC bandwidth: 2B - SSB bandwidth: B (50% savings!)

3. Generation Methods - **Filter method:** Generate DSB-SC, then filter - **Phasing method:** Use Hilbert transform (90° phase shift)

4. Applications - Amateur radio (voice communications) - Telephone systems (frequency division multiplexing) - Anywhere bandwidth is precious

Essential Formulas

```
Bandwidth Comparison:
  Message: B (e.g., 3 kHz for voice)
  DSB-SC: 2B = 6 kHz
  SSB: B = 3 kHz
  AM: 2B = 6 kHz

Power:
  P_SSB = ½ P_DSB (only one sideband)
```

Practice Problems

From **Homework 2:** - Problem 7: Bandwidth calculations - Problem 8: SSB generation methods

Strategy: - Identify message bandwidth B - SSB always uses exactly B (one sideband) - AM and DSB-SC use 2B (two sidebands)

Topic 7: Binary ASK (Amplitude Shift Keying)

Key Concepts

1. ASK as Digital AM - Bit "0" $\rightarrow$ Low amplitude (often 0) - Bit "1" $\rightarrow$ High amplitude - On-Off Keying (OOK): Special case where "0" = 0V

2. Bit Duration and Bandwidth - Bit rate: R_b bits/s - Bit duration: $T_b = 1/R_b$ - Null-to-null bandwidth: $B \approx 2R_b$

3. Relationship to AM - ASK is AM with digital message signal - Rectangular pulses $\rightarrow$ sinc spectrum - Bandwidth determined by bit rate, not carrier frequency

Essential Formulas

```
ASK Parameters:
  Bit rate: R  (bits/s)
  Bit duration: T  = 1/R
  Null-to-null BW: B   2/T  = 2R

On-Off Keying (OOK):
  s (t) = 0              (bit "0")
  s (t) = A·cos(2 f t)   (bit "1")

Carson's Rule (estimate):
  B_ASK   2R
```

Practice Problems

From **Homework 2:** - Problem 9: ASK bandwidth calculations - Problem 10: Bit rate vs. bandwidth

Strategy: 1. Find bit duration $T_b = 1/R_b$ 2. Null-to-null BW = $2/T_b = 2R_b$ 3. Compare to carrier frequency ($B \ll f_c$ for RF)

Formula Sheet Recommendations

Must-Have Formulas

Fourier Transform:

```
rect(t/T)   T·sinc(fT)
x(t - t )   X(f)e^(-j2 ft )
x(t)cos(2 f t)   ½[X(f-f ) + X(f+f )]
```

AM:

```
s(t) = A [1 +  m (t)]cos(2 f t)
  = A /A
P_total = (A ²/2R)(1 +  ²/2)
  =  ²/(2 +  ²)
```

DSB-SC:

```
s(t) = A m(t)cos(2 f t)
P_DSB = (A ²/2R)·P_m
```

Bandwidth:

```
B_AM = B_DSB = 2B
B_SSB = B
B_ASK   2R
```

Energy/Power:

```
Parseval:  |x(t)|² dt =  |X(f)|² df
P_avg = V²_rms/R = V²_peak/(2R) for sinusoid
```

Useful Constants

```
Room temperature: T = 290 K
Boltzmann constant: k = 1.38 × 10 ²³ J/K (probably not needed for Midterm 1)
```

Exam Strategy

Time Management (90 minutes total)

- **5 min:** Read entire exam, mark easy problems
- **10 min:** Problem 1 (Fourier fundamentals - typically straightforward)
- **10 min:** Problem 2 (Fourier properties)
- **10 min:** Problem 3 (Parseval's theorem)
- **20 min:** Problem 4 (AM analysis - usually longest)
- **10 min:** Problem 5 (DSB-SC comparison)
- **12 min:** Problem 6 (SSB and bandwidth)
- **8 min:** Problem 7 (ASK bonus - if time permits)
- **5 min:** Review and check answers

Problem-Solving Approach

For Fourier Transform Problems: 1. Identify signal type (rect, sinc, exponential) 2. Draw time domain 3. Apply transform pair or properties 4. Draw frequency domain with labels 5. Calculate bandwidth from first zero crossing

For AM Power Problems: 1. Write out given signal equation 2. Identify A_c, A_m, f_c, f_m 3. Calculate $\mu = A_m/A_c$ 4. **Check for overmodulation** ($\mu > 1$?) 5. Calculate carrier power: $P_c = A_c^2/(2R)$ 6. Calculate total power: $P_{tot} = P_c(1 + \mu^2/2)$ 7. Calculate sideband power: $P_{sb} = P_c(\mu^2/2)$ 8. Efficiency: $\eta = P_{sb}/P_{tot}$

For Bandwidth Problems: 1. Identify message bandwidth B or frequency f_m 2. Choose correct formula: - AM/DSB-SC: 2B - SSB: B - ASK: $2R_b$ 3. Label spectrum sketch clearly

Common Mistakes to Avoid

Forgetting the factor of 1/2 in power formulas (sinusoid RMS = peak/√2) **Mixing up time and frequency domains** (check units!) **Not checking for overmodulation** before calculating AM power **Confusing bandwidth with carrier frequency** **Using wrong sideband formula** for SSB (B, not 2B) **Forgetting to include resistance R** in power calculations **Not showing units** in final answers

Tips for Success

Draw diagrams! Time and frequency sketches help visualize **Write out formulas** before substituting numbers **Check dimensions** (frequency in Hz, power in W, etc.) **Show all work** - partial credit is generous **Box final answers** - makes grading easier **Use your formula sheet** - don't waste time memorizing **Sanity check** - Does $\mu > 1$? Is power positive? Is BW $< f_c$?

Practice Resources

Homework Problems (Strongly Recommended)

Homework 1 (Fourier Analysis): - All problems 1-10 - Focus on: Problems 1, 4, 5, 6 (most exam-relevant)

Homework 2 (AM): - All problems 1-10 - Focus on: Problems 1, 2, 3, 4, 8 (core AM concepts)

Lecture Notebooks

- **Lecture 2:** Fourier Transform fundamentals
- **Lecture 3:** Fourier properties and bandwidth
- **Lecture 5:** AM modulation and demodulation

Textbook Sections

- **Chapter 2.1:** Fourier Transform
- **Chapter 2.2-2.3:** Filtering and bandwidth
- **Chapter 3:** Amplitude Modulation
- **Chapter 7.1-7.2:** ASK (digital AM)

Self-Assessment Checklist

Before the exam, make sure you can:

Fourier Transform: - [ ] Calculate Fourier transform of rectangular pulse - [ ] Identify first zero crossing and bandwidth - [ ] Apply time-shift property - [ ] Apply modulation property (create sidebands) - [ ] Sketch magnitude spectrum with labeled axes - [ ] Calculate signal energy in time domain - [ ] Verify energy using Parseval's theorem

Amplitude Modulation: - [ ] Identify A_c, A_m, f_c, f_m from AM equation - [ ] Calculate modulation index μ - [ ] Detect overmodulation ($\mu > 1$) - [ ] Calculate carrier power - [ ] Calculate total transmitted power - [ ] Calculate sideband power - [ ] Calculate power efficiency - [ ] Determine bandwidth (always 2B for AM)

DSB-SC: - [ ] Write DSB-SC signal equation - [ ] Calculate transmitted power - [ ] Compare efficiency to full AM - [ ] Explain advantage (power) vs. disadvantage (receiver complexity)

SSB: - [ ] Calculate bandwidth (always B, not 2B!) - [ ] Explain bandwidth advantage - [ ] Describe filter method or phasing method - [ ] List applications (amateur radio, telephony)

ASK: - [ ] Relate bit rate to bandwidth - [ ] Calculate null-to-null bandwidth ($2R_b$) - [ ] Explain relationship to AM

Final Thoughts

This exam tests fundamentals! If you understand: 1. Fourier transforms create sidebands when you multiply by a carrier 2. AM puts information in sidebands while wasting power in carrier 3. Bandwidth = 2 × (message bandwidth) for AM/DSB-SC 4. Power = $V^2/(2R)$ for sinusoids

…you'll do well!

Focus your study time on: - Homework 1 & 2 problems (they mirror exam problems) - Drawing accurate spectrum diagrams - AM power calculations (most common exam mistakes here) - Checking for overmodulation

Good luck!