EE 451: Communications Systems

Final Exam - Comprehensive Study Guide

Exam Date: Week 16 (May 18-22, 2026) **Duration:** 180 minutes (3 hours) **Coverage:** All course material (Chapters 1-11) with emphasis on Chapters 8-11 **Format:** Closed book, one 8.5” × 11” formula sheet (both sides) allowed

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Overview

The final exam is **comprehensive** but with **emphasis on later material** (probability, noise, BER, link budgets). Structure: - **Part I (30 pts):** Fundamentals review (Fourier, AM, FM/PM) - **Part II (35 pts):** Digital modulation (BPSK, QPSK, FSK, QAM) - **Part III (40 pts):** Probability and noise (Gaussian, SNR, BER) - **Part IV (35 pts):** Link budgets and system performance - **Part V (10 pts):** Advanced topics (OFDM, spread spectrum) - **Bonus (10 pts):** Satellite link analysis

Key insight: 85 of 150 points (57%) come from material AFTER Midterm 2!

Study Strategy

Priority Levels

HIGHEST PRIORITY (Study first - 60% of exam) - Link budget calculations (Problems 10, 11) - Gaussian distribution and Q-function (Problem 7) - Thermal noise and SNR calculations (Problem 8) - BER analysis for BPSK (Problem 9) - Shannon capacity (Problem 11)

MEDIUM PRIORITY (Important - 25% of exam) - BPSK vs QPSK comparison (Problem 4) - 16-QAM and Gray coding (Problem 5) - FSK system design (Problem 6) - Adaptive modulation (Problem 12)

LOWER PRIORITY (Review quickly - 15% of exam) - Fourier transforms (Problem 1) - AM power calculations (Problem 2) - FM vs PM (Problem 3) - OFDM and spread spectrum concepts (Problem 13)

Part I: Fundamentals Review (30 points)

Quick Review Topics

Fourier Transforms (10 pts) - Sinc rect transform pair - Time-frequency duality - Modulation property (DSB-SC spectrum)

AM (10 pts) - Modulation index $\mu = A_m/A_c$ - Power: $P_{total} = \frac{A_c^2}{2R}(1 + \mu^2/2)$ - Efficiency: $\eta = \frac{\mu^2/2}{1+\mu^2/2}$

FM/PM (10 pts) - FM: $\Delta f = k_f A_m$ (independent of f_m) - PM: $\Delta f = k_p A_m f_m$ (proportional to f_m) - Carson’s rule: $B_{FM} = 2(\Delta f + f_m)$

Study Tip: These are REVIEW topics. Don't spend too much time here if you understand Midterms 1 & 2 material. Focus on new material!

Part II: Digital Modulation (35 points)

BPSK vs QPSK (12 pts - Problem 4)

Key Comparisons:

Feature	BPSK	QPSK
Bits/symbol	1	2
Symbol rate	R_b	$R_b/2$
Bandwidth	$2R_b$	R_b
Spectral efficiency	0.5 bits/s/Hz	1.0 bits/s/Hz
BER (same E_b/N_0)	Same!	Same!

Critical insight: QPSK is twice as bandwidth efficient with no BER penalty!

QAM and Gray Coding (12 pts - Problem 5)

16-QAM Basics: - M = 16 symbols, k = 4 bits/symbol - Rectangular constellation: 4×4 grid - Amplitude levels: typically {±1, ±3}

Gray Coding Rules: - Adjacent symbols differ by 1 bit - Minimizes BER (most errors → adjacent symbols) - Mandatory in modern systems!

EVM (Error Vector Magnitude): - Measures constellation quality - EVM% = (Error distance / Reference distance) × 100% - WiFi/LTE have strict EVM limits (e.g., 256-QAM < 3%)

FSK System Design (11 pts - Problem 6)

Orthogonal FSK: - Minimum separation: $\Delta f_{min} = \frac{1}{2T_b}$ - Modulation index: $h = \Delta f \times T_b$ - MSK: $h = 0.5$ exactly

Quick formulas:

```
Tb = 1/Rb
Δf = |f  - f |
h = Δf·Tb
Orthogonal: Δf   1/(2Tb)
MSK: h = 0.5
```

Part III: Probability and Noise (40 points)

THIS IS THE MOST IMPORTANT SECTION!

Gaussian Distribution (15 pts - Problem 7)

PDF of Gaussian:

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)}$$

Standardization:

$$Z = \frac{X - \mu}{\sigma} \sim N(0, 1)$$

Q-Function:

$$Q(x) = P(Z > x) \text{ for } Z \sim N(0, 1)$$

For general Gaussian X ~ N(, ²):

$$P(X > a) = Q\left(\frac{a - \mu}{\sigma}\right)$$

Key Q-function values (memorize these!):

Q(1.0)	0.159	Q(2.0)	0.023
Q(1.5)	0.067	Q(3.0)	0.0013
Q(4.0)	3e-5	Q(4.75)	1e-6

Property: $Q(-x) = 1 - Q(x)$

Practice: 1. Identify and from problem 2. Standardize: $z = (x - \mu)/\sigma$ 3. Express using Q-function 4. Look up or calculate

Thermal Noise and SNR (12 pts - Problem 8)

Thermal Noise Power:

$$P_n = kTB$$

where: - $k = 1.38 \times 10^{-23}$ J/K (Boltzmann constant) - T = temperature (K) - B = bandwidth (Hz)

Noise Figure:

$$P_{n,total} = P_n \times F = P_n \times 10^{NF_{dB}/10}$$

In dBm:

$$P_n(dBm) = 10\log_{10}\left(\frac{kTB}{10^{-3}}\right)$$

Quick calculation: At T = 290 K:

$$P_n(dBm) \approx -174 + 10\log_{10}(B_{Hz})$$

SNR:

$$SNR(dB) = P_s(dBm) - P_n(dBm)$$

Critical conversions:

```
dBm to W: P_W = 10^((P_dBm - 30)/10)
W to dBm: P_dBm = 10·log (P_W) + 30
dB addition: Ptotal(dB) = P1(dB) + P2(dB)
Linear addition: Ptotal_W = P1_W + P2_W
```

Common values to know:

```
0 dBm = 1 mW
-30 dBm = 1  W
-60 dBm = 1 nW
-90 dBm = 1 pW

+3 dB = ×2
+10 dB = ×10
+20 dB = ×100
```

BER Performance (13 pts - Problem 9)

BPSK BER:

$$BER_{BPSK} = Q(\sqrt{2E_b/N_0})$$

Conversions:

$$\frac{E_b}{N_0}(dB) = 10\log_{10}(E_b/N_0)$$

To find linear from dB:

$$\frac{E_b}{N_0} = 10^{(E_b/N_0)_{dB}/10}$$

Bit errors per second:

$$N_{errors/s} = BER \times R_b$$

Common E_b/N_0 values for BER targets:

```
BER = 10 ³: Eb/N0   6.8 dB
BER = 10 : Eb/N0   8.4 dB
BER = 10 : Eb/N0   9.6 dB
BER = 10 : Eb/N0   10.5 dB
```

Step-by-step for BER problems: 1. Convert E_b/N_0 from dB to linear 2. Calculate $\sqrt{2E_b/N_0}$ 3. Find $Q(\sqrt{2E_b/N_0})$ using table 4. This is your BER 5. Errors/sec = BER × bit rate

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Part IV: Link Budgets (35 points)

CRITICAL SECTION - WORTH 35 POINTS!

Free-Space Path Loss (FSPL)

Two equivalent forms:

Form 1 (dB, easier for calculations):

$$FSPL(dB) = 20\log_{10}(d_{km}) + 20\log_{10}(f_{MHz}) + 32.45$$

Form 2 (from wavelength):

$$FSPL(dB) = 20\log_{10}\left(\frac{4\pi d}{\lambda}\right)$$

where $\lambda = c/f$

Example calculation:

```
f = 5.8 GHz = 5800 MHz
d = 50 m = 0.05 km

FSPL = 20·log (0.05) + 20·log (5800) + 32.45
     = 20·(-1.301) + 20·(3.763) + 32.45
     = -26.02 + 75.26 + 32.45
     = 81.69 dB
```

Complete Link Budget (Problem 10 - 18 pts)

Link Budget Equation:

$$P_r(dBm) = P_t(dBm) + G_t(dBi) + G_r(dBi) - FSPL(dB) - L_{misc}(dB)$$

Step-by-step procedure:

1. Calculate FSPL - Use formula above - Make sure units match (km and MHz, OR meters and Hz)

2. Calculate received power

$$P_r = P_t + G_t + G_r - FSPL - L_{other}$$

3. Calculate noise power

$$P_n = kTB \text{ (in W)}$$

$$P_n(dBm) = 10\log_{10}(P_n/0.001)$$

OR use quick formula:

$$P_n(dBm) \approx -174 + 10\log_{10}(B_{Hz}) + NF(dB)$$

4. Calculate SNR

$$SNR(dB) = P_r(dBm) - P_n(dBm) - \text{fade margin}$$

5. Determine supported modulation Compare SNR to thresholds:

```
QPSK:    8 dB
16-QAM:  15 dB
64-QAM:  22 dB
256-QAM: 28 dB
```

Common mistakes: Forgetting to convert distance to km or frequency to MHz Mixing up + and - signs in link budget Forgetting fade margin Not adding noise figure to thermal noise

Shannon Capacity (Problem 11 - 10 pts)

Shannon-Hartley Theorem:

$$C = B\log_2(1 + SNR)$$

bits/s

where: - C = channel capacity (bits/s) - B = bandwidth (Hz) - SNR = linear ratio (NOT dB!)

Convert SNR from dB:

$$SNR = 10^{SNR(dB)/10}$$

Spectral Efficiency:

$$\eta = \frac{C}{B} = \log_2(1 + SNR)$$

bits/s/Hz

Example:

```
B = 10 MHz, SNR = 20 dB

SNR_linear = 10^(20/10) = 100

C = 10×10  × log (1 + 100)
  = 10×10  × log (101)
  = 10×10  × 6.658
  = 66.58 Mbps

  = 6.658 bits/s/Hz
```

Common mistake: Using SNR in dB directly (must convert to linear!)

Adaptive Modulation (Problem 12 - 7 pts)

Concept: - Choose modulation based on SNR - Higher SNR → higher-order modulation → more bits/s - Lower SNR → lower-order modulation → more reliable

SNR vs. distance model:

$$SNR(dB) = SNR_0 - n \times 10\log_{10}(d/d_0)$$

where n is path loss exponent (typical: 2-4)

Procedure: 1. Calculate SNR at each distance 2. Compare to modulation thresholds 3. Select highest modulation with SNR threshold

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Part V: Advanced Topics (10 points)

OFDM (4 pts)

Why OFDM? - Solves frequency-selective fading problem - Divides wideband channel into many narrowband subchannels - Each subchannel experiences flat fading - Simpler equalization than single-carrier

Applications: - WiFi (802.11a/g/n/ac/ax) - LTE, 5G NR - DVB-T (digital TV)

Key concept: Convert frequency-selective fading → flat fading per subcarrier

Spread Spectrum (6 pts)

DSSS (Direct Sequence Spread Spectrum): - Data rate: R_b - Chip rate: R_c (much higher) - Processing gain: $G_p = \frac{R_c}{R_b}$ (ratio) - In dB: $G_p(dB) = 10\log_{10}(R_c/R_b)$

Advantages: - Jamming resistance (processing gain) - Multiple access (CDMA) - Low probability of intercept

Example:

```
Rb = 1 Mbps, Rc = 10 Mchips/s
Gp = 10 Mchips/s / 1 Mbps = 10
Gp(dB) = 10·log (10) = 10 dB
```

Bonus: Satellite Link (10 points)

Typical satellite parameters: - High transmit power: 40 dBW (10 kW) - High antenna gains: 30-40 dBi - Long distance: 36,000 km (GEO) - Very high FSPL: ~200 dB! - Low noise temperature: 150 K (LNA)

E_b/N_0 and SNR relationship:

$$\frac{E_b}{N_0} = SNR \times \frac{B}{R_b}$$

Use this to find maximum bit rate:

$$R_b = \frac{SNR \times B}{E_b/N_0}$$

Master Formula Sheet

Probability & Noise

```
Gaussian PDF:
  f(x) = (1/( √(2 ))) exp(-(x- )²/(2 ²))

Q-function:
  P(X > a) = Q((a- )/ ) for X ~ N( ,  ²)
  Q(-x) = 1 - Q(x)
```

```
Thermal Noise:
  Pn = kTB  (k = 1.38×10 ^23 J/K)
  Pn(dBm)   -174 + 10log (B_Hz) + NF(dB)  [at T=290K]

SNR:
  SNR(dB) = Ps(dBm) - Pn(dBm)

BER:
  BER_BPSK = Q(√(2Eb/N0))
```

Link Budget

```
FSPL:
  FSPL(dB) = 20log (d_km) + 20log (f_MHz) + 32.45

Link Budget:
  Pr(dBm) = Pt(dBm) + Gt + Gr - FSPL - Losses

Shannon Capacity:
  C = B·log (1 + SNR)  [SNR must be linear!]

Eb/N0 vs SNR:
  Eb/N0 = SNR × (B/Rb)
```

Digital Modulation (Review)

```
BPSK: k=1, Rs=Rb, B=2Rb
QPSK: k=2, Rs=Rb/2, B=Rb
M-ary: k=log (M), Rs=Rb/k

FSK: h = Δf·Tb
     Orthogonal: Δf = 1/(2Tb)
     MSK: h = 0.5
```

Analog Modulation (Review)

```
AM:
  s(t) = Ac[1 +  ·mn(t)]cos(2 fct)
  Ptotal = (Ac^2/2R)(1 +  ^2/2)
    =  ^2/(2 +  ^2)

FM:
  Δf = kf·max|m(t)|
    = Δf/fm
  BFM = 2(Δf + fm)

PM:
  Δ  = kp·max|m(t)|
```

```
Δf_PM = kp·Am·fm  (depends on fm!)
```

Exam Strategy for 180 Minutes

Time Allocation

Part I (30 pts) - 25 minutes - Problem 1: 7 min (Fourier - straightforward) - Problem 2: 10 min (AM power) - Problem 3: 8 min (FM vs PM)

Part II (35 pts) - 30 minutes - Problem 4: 10 min (BPSK vs QPSK) - Problem 5: 12 min (16-QAM, Gray coding) - Problem 6: 8 min (FSK design)

Part III (40 pts) - 50 minutes - Problem 7: 18 min (Gaussian & Q-function - 5 parts!) - Problem 8: 15 min (Thermal noise & SNR) - Problem 9: 17 min (BER analysis)

Part IV (35 pts) - 45 minutes - Problem 10: 25 min (WiFi link budget - longest problem!) - Problem 11: 12 min (Shannon capacity) - Problem 12: 8 min (Adaptive modulation)

Part V (10 pts) - 10 minutes - Problem 13: 10 min (OFDM & spread spectrum concepts)

Bonus (10 pts) - 10 minutes (if time!) - Satellite link analysis

Review: 10 minutes

Priority Order (if running low on time)

1. **Do first:** Problems 7, 8, 9, 10 (new material, high points)
2. **Do second:** Problems 11, 12 (new material)
3. **Do third:** Problems 4, 5, 6 (review, but still good points)
4. **Do last:** Problems 1, 2, 3 (pure review)
5. **If time:** Problems 13 and Bonus

Calculator Tips

Pre-calculate common values:

```
log (2)   0.301
log (10) = 1
ln(2)   0.693

10·³   2
10·   3.16
10¹· = 10

For log (x): log (x) = log (x)/log (2)   3.32·log (x)
```

Common Mistakes to AVOID

Part III (Probability/Noise)

Forgetting to standardize Gaussian: $z = (x-\mu)/\sigma$ Using SNR in dB directly in Shannon formula (must convert to linear!) Forgetting to add noise figure to thermal noise Confusing Q(x) with 1-Q(x) Using wrong Boltzmann constant or temperature

Part IV (Link Budgets)

Wrong units in FSPL: using meters instead of km, or Hz instead of MHz Wrong signs in link budget (it's Pt + Gt + Gr - FSPL, not all positive!) Forgetting fade margin when calculating SNR Comparing to wrong modulation thresholds Not converting E_b/N_0 from dB when using formulas

General

Not showing units in answers Not checking if answer makes physical sense Spending too long on early problems (save time for Parts III & IV!) Forgetting to box/circle final answers

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Self-Assessment Checklist

Can you:

Probability & Noise: - [ ] Write Gaussian PDF - [ ] Standardize and use Q-function - [ ] Calculate thermal noise power (kTB) - [ ] Convert between dBm and Watts - [ ] Add noise figure correctly - [ ] Calculate SNR in dB - [ ] Convert E_b/N_0 between dB and linear - [ ] Calculate BER for BPSK - [ ] Find bit errors per second

Link Budgets: - [ ] Calculate FSPL with correct units - [ ] Perform complete link budget (Pr = Pt + gains - losses) - [ ] Calculate noise power in dBm - [ ] Calculate SNR accounting for fade margin - [ ] Select modulation based on SNR - [ ] Calculate Shannon capacity (with SNR in linear!) - [ ] Apply adaptive modulation rules

Digital Modulation: - [ ] Compare BPSK vs QPSK (bandwidth, spectral efficiency) - [ ] Draw 16-QAM constellation - [ ] Apply Gray coding - [ ] Calculate EVM - [ ] Design orthogonal FSK system - [ ] Identify MSK

Review Topics: - [ ] Fourier transform of rect/sinc - [ ] AM power and efficiency - [ ] FM vs PM frequency deviation

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Final Study Plan (1 Week Before Exam)

Days 7-6: New Material (Priority)

- **Study:** Homework 5 (all 10 problems) - Probability & Noise
- **Study:** Homework 6 (all 10 problems) - BER & Link Budgets
- **Practice:** Midterm 1 solutions (Problems 7-9)
- **Focus:** Link budget procedure, Q-function, thermal noise

Days 5-4: Integration

- **Practice:** Complete a full link budget from scratch
- **Practice:** Gaussian/Q-function problems
- **Practice:** BER calculations
- **Review:** Shannon capacity problems
- **Make:** Your formula sheet!

Days 3-2: Review Material

- **Quick review:** Homework 3 & 4 (digital modulation)
- **Quick review:** Homework 1 & 2 (Fourier, AM)
- **Practice:** Midterm 2 (all problems quickly)
- **Refine:** Formula sheet

Day 1: Final Prep

- **Take:** Practice final (under time pressure!)
- **Review:** Common mistakes list
- **Check:** Formula sheet is complete
- **Sleep:** Well!

What to Bring

Calculator (scientific, not graphing) Formula sheet (both sides, handwritten recommended) Pencils/pens Student ID

During the Exam

1. **Read all problems first** (5 min)
2. **Mark easy problems** with a star
3. **Do high-value problems first** (Parts III & IV)
4. **Show ALL work** - partial credit is generous!
5. **Box final answers**
6. **Check units** on every answer
7. **Sanity check:** Does the answer make sense?
8. **If stuck:** Move on, come back later
9. **Last 10 min:** Review, check signs, check units

Final Thoughts

The exam rewards: - Systematic problem-solving (link budgets!) - Careful unit management (km vs m, dB vs linear) - Understanding relationships (Shannon, BER, SNR)

Focus your final study on: 1. Link budget procedure (practice 3-5 complete examples) 2. Q-function and BER calculations 3. SNR and noise calculations 4. Digital modulation comparisons

You've got this! The comprehensive nature means you can show your understanding across the whole course. Focus on the new material (Parts III-IV) and you'll do great!

Good luck on your final exam!