

EE 451: Communications Systems

Homework 3 - FM, PM, FSK, and BPSK

Topics: Frequency and phase modulation, FSK, BPSK, Carson's rule **Textbook Reference:** Haykin & Moher, Chapters 4.1-4.5, 7.2-7.4

Total Points: 100

Instructions

- Show all work for full credit
 - Include units in all final answers
 - For bandwidth calculations, clearly state which rule or method you're using
 - Sketches should be clearly labeled
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Problem 1: FM Fundamentals (12 points)

A message signal $m(t) = 2 \cos(2\pi \cdot 1000 \cdot t)$ V modulates an FM carrier with $f_c = 100$ MHz and frequency sensitivity $k_f = 10$ kHz/V.

- (a) Write the expression for the instantaneous frequency $f_i(t)$. (3 points)
 - (b) What is the maximum frequency deviation Δf ? (3 points)
 - (c) Write the complete FM signal expression $s(t)$. You may leave the answer in terms of the integral of $m(t)$. (4 points)
 - (d) What is the modulation index β for FM? (2 points)
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Problem 2: Carson's Rule Application (15 points)

An FM broadcast station transmits audio signals with maximum frequency $f_m = 15$ kHz and maximum frequency deviation $\Delta f = 75$ kHz (standard FM broadcasting parameters).

- (a) Calculate the modulation index β . (3 points)
 - (b) Use Carson's rule to estimate the FM bandwidth. (4 points)
 - (c) Is this narrowband FM (NBFM) or wideband FM (WBFM)? Justify your answer. (3 points)
 - (d) How does this compare to the bandwidth required for AM broadcasting of the same audio signal? (3 points)
 - (e) What is the advantage of using such a large bandwidth for FM? (2 points)
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Problem 3: Narrowband vs. Wideband FM (12 points)

- (a) For NBFM, the criterion is $\beta \ll 1$. If $f_m = 3$ kHz, what is the maximum Δf that qualifies as NBFM? Use $\beta < 0.3$ as the threshold. (3 points)
 - (b) Estimate the bandwidth of NBFM using the small-angle approximation. (3 points)
 - (c) For WBFM with the same $f_m = 3$ kHz but $\Delta f = 30$ kHz, calculate β and the bandwidth using Carson's rule. (4 points)
 - (d) Compare the bandwidth efficiency (in terms of β) between NBFM and WBFM. (2 points)
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Problem 4: Phase Modulation (12 points)

A PM signal is given by $s(t) = A_c \cos[2\pi f_c t + k_p m(t)]$ where: - $A_c = 10$ V - $f_c = 200$ MHz - $k_p = 2$ rad/V (phase sensitivity) - $m(t) = 3 \cos(2\pi \cdot 5000 \cdot t)$ V

- (a) What is the maximum phase deviation $\Delta\phi$? (3 points)
 - (b) Write the expression for the instantaneous phase $\theta_i(t)$. (3 points)
 - (c) Find the instantaneous frequency $f_i(t)$. (4 points)
 - (d) What is the maximum frequency deviation? (2 points)
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Problem 5: FM vs. PM Relationship (10 points)

- (a) Explain the fundamental difference between FM and PM. How are they related? (4 points)
 - (b) If you integrate a message signal before applying it to a frequency modulator, what type of modulation do you effectively create? (3 points)
 - (c) For the message $m(t) = \cos(2\pi f_m t)$, how does the frequency deviation in FM compare to the frequency deviation in PM as f_m varies? (3 points)
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Problem 6: Binary FSK Fundamentals (15 points)

A binary FSK system transmits data at 9600 bits/s using: - "0" bit: $f_0 = 1200$ Hz - "1" bit: $f_1 = 2200$ Hz - Carrier amplitude: $A = 5$ V

- (a) Write the signal expressions $s_0(t)$ and $s_1(t)$ for the two binary symbols. (4 points)
- (b) What is the frequency separation $\Delta f = |f_1 - f_0|$? (2 points)
- (c) Calculate the modulation index $h = \Delta f \cdot T_b$ where T_b is the bit duration. (4 points)
- (d) For coherent FSK detection, the minimum frequency separation for orthogonality is $\Delta f = \frac{1}{2T_b}$. Is this FSK system using orthogonal tones? (3 points)
- (e) Estimate the bandwidth using Carson's rule, treating FSK as FM with a rectangular baseband signal. (2 points)

Problem 7: Minimum Shift Keying (MSK) (12 points)

MSK is a special case of continuous-phase FSK (CPFSK) with modulation index $h = 0.5$.

- (a) If the bit rate is $R_b = 10$ kbps, what is the bit duration T_b ? (2 points)
 - (b) Given $h = 0.5 = \Delta f \cdot T_b$, calculate the required frequency separation Δf . (3 points)
 - (c) What are the two frequencies f_0 and f_1 if the center frequency is 1800 Hz? (4 points)
 - (d) Why is MSK preferred over standard FSK in many applications? (3 points)
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Problem 8: Binary Phase Shift Keying (BPSK) (15 points)

A BPSK system transmits at 1 Mbps with carrier frequency $f_c = 2.4$ GHz.

- (a) Write the two possible signal expressions for BPSK representing bits “0” and “1”. (4 points)
 - (b) Sketch the constellation diagram for BPSK in the I/Q plane. Label the two signal points. (4 points)
 - (c) What is the phase difference between the two BPSK symbols? (2 points)
 - (d) Calculate the null-to-null bandwidth of the BPSK signal. (3 points)
 - (e) Can BPSK be demodulated using an envelope detector? Why or why not? (2 points)
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Problem 9: Coherent vs. Non-Coherent Detection (10 points)

- (a) Explain the difference between coherent and non-coherent detection. (3 points)
 - (b) Which modulation schemes can use non-coherent detection: ASK, FSK, BPSK? Explain. (4 points)
 - (c) What is the advantage of coherent detection? What is the disadvantage? (3 points)
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Problem 10: Real-World Application - FM Deviation (12 points - Challenge)

Commercial FM radio in the US uses: - Maximum frequency deviation: $\Delta f = 75$ kHz (audio) - Pilot tone for stereo: 19 kHz - Subcarrier for RDS (Radio Data System): 57 kHz - Maximum audio frequency: 15 kHz

- (a) Calculate the modulation index for maximum audio frequency. (3 points)
- (b) Use Carson’s rule to calculate the bandwidth occupied by the FM station. (3 points)
- (c) FM stations are allocated 200 kHz of bandwidth. Does your calculated bandwidth fit? If not, what accounts for the difference? (3 points)

(d) An FM station also has a subcarrier at 67 kHz for SCA (Subsidiary Communications Authorization) services. How does this affect the total deviation and bandwidth? (3 points)

Bonus Problem 1: Bessel Functions in FM (5 points extra credit)

The FM spectrum consists of infinite sidebands with amplitudes given by Bessel functions $J_n(\beta)$.

For single-tone FM with $\beta = 2$: - $J_0(2) = 0.224$ - $J_1(2) = 0.577$ - $J_2(2) = 0.353$ - $J_3(2) = 0.129$ - $J_4(2) = 0.034$

(a) Sketch the frequency spectrum showing the carrier and first four sidebands on each side. Label amplitudes relative to carrier (assume $A_c = 10$ V). (3 points)

(b) Notice that $J_0(2)$ is small. What does this mean about the carrier power in FM compared to AM? (2 points)

Bonus Problem 2: Frequency Deviation Calculation (5 points extra credit)

For the FM signal $s(t) = 10 \cos[2\pi \cdot 100 \times 10^6 \cdot t + 50 \sin(2\pi \cdot 1000 \cdot t)]$:

(a) Identify the carrier frequency f_c , message frequency f_m , and the modulation index β . (3 points)

(b) Calculate the frequency deviation Δf . (2 points)

Hint: The general form is $s(t) = A_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)]$ for single-tone FM.

Submission Instructions: - Submit a single PDF via Brightspace - Show all derivations and calculations - Include clear sketches with labeled axes - State all assumptions

Academic Integrity: - Individual work required - You may use textbook formulas and course notes - Cite any external references