EE 451: Communications Systems

Homework 5 - Probability, Noise, and SNR

Topics: Probability theory, Gaussian distribution, thermal noise, SNR calculations **Textbook Reference:** Haykin & Moher, Chapters 8, 9, 11.1-11.3

Total Points: 100

Instructions

- Show all work for full credit
- Include units in all final answers
- For probability problems, clearly state assumptions
- Q-function table and standard normal table will be provided on exam

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Problem 1: Basic Probability (10 points)

A binary communication system transmits bits "0" and "1" with probabilities: - P(0) = 0.6 - P(1) = 0.4

(a) What is the probability of receiving "1" followed by "0" if bits are independent? (3 points)

(b) What is the probability of receiving at least one "1" in three consecutive bits? (4 points)

(c) If 1000 bits are transmitted, approximately how many "1" bits are sent? (3 points)

Problem 2: Conditional Probability and Bayes' Theorem (12 points)

A receiver detects bits with the following error probabilities: - P(detect 1 | sent 0) = 0.01 (false alarm) - P(detect 0 | sent 1) = 0.02 (miss) - P(sent 0) = 0.7 - P(sent 1) = 0.3

(a) What is P(detect 0 | sent 0)? (2 points)

(b) What is the probability of detecting a "1"? (4 points)

(c) Given that a "1" was detected, what is the probability that a "1" was actually sent? Use Bayes' theorem. (6 points)

Problem 3: Gaussian Distribution (15 points)

A received signal has amplitude that is Gaussian distributed with mean $\mu = 5$ V and standard deviation $\sigma = 2$ V.

(a) Write the probability density function (PDF) for this signal. (3 points)

(b) What is the probability that the amplitude is between 3 V and 7 V? (4 points)

(c) What is the probability that the amplitude exceeds 9 V? Express in terms of the Q-function. (4 points)

(d) If the threshold for detection is set at 4 V, what fraction of signals will be below this threshold? (4 points)

Given: $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2}\, dt$

Problem 4: Q-Function Calculations (12 points)

Use the Q-function for the following calculations:

(a) Calculate Q(1.5). You may use the approximation $Q(x) \approx \frac{1}{\sqrt{2\pi}x} \cdot e^{-x^2/2}$ for $x > 3$, or look up the value. (3 points)

(b) A bit error occurs if noise exceeds 3σ. What is the probability of this event? (3 points)

(c) The complementary error function is $\text{erfc}(x) = 2 \cdot Q(x\sqrt{2})$. Express Q(x) in terms of erfc. (3 points)

(d) For BER $= 10^{-6}$, what value of SNR (as a ratio, not dB) is required if BER $= Q(\sqrt{\text{SNR}})$? (3 points)

Problem 5: Thermal Noise Power (15 points)

(a) Calculate the thermal noise power in a 1 MHz bandwidth at room temperature (T = 290 K). Use $k = 1.38 \times 10^{-23}$ J/K. (4 points)

(b) Express this noise power in dBm. (Recall: dBm $= 10 \cdot \log_{10}(P/1 \text{ mW})$) (3 points)

(c) If the bandwidth increases to 20 MHz (WiFi channel), what is the new noise power in dBm? (3 points)

(d) A receiver has a noise figure NF = 6 dB. What is the total noise power in 20 MHz? (5 points)

Problem 6: Signal-to-Noise Ratio (15 points)

A communication system has the following parameters: - Received signal power: $P_s = -80$ dBm - System bandwidth: B = 10 MHz - Noise temperature: T = 290 K - Noise figure: NF = 5 dB

(a) Calculate the noise power in dBm. (5 points)

(b) Calculate the SNR in dB. (3 points)

(c) Convert the SNR to a linear ratio. (2 points)

(d) If the signal power increases by 3 dB, what is the new SNR? (3 points)

(e) What signal power is required to achieve SNR = 20 dB? (2 points)

Problem 7: Noise in AM Systems (12 points)

An AM receiver has the following specifications: - Carrier power: $P_c = 10$ W - Modulation index: $\mu = 0.8$ - Noise power in the message bandwidth: $P_n = 0.1$ W

(a) Calculate the total transmitted power. (3 points)

(b) Calculate the signal power in the sidebands. (3 points)

(c) For DSB-SC (suppressed carrier), what is the output SNR? Assume the output SNR equals (sideband power)/(noise power). (3 points)

(d) For full carrier AM, the output SNR is reduced by the factor $\frac{\mu^2}{2+\mu^2}$. Calculate the output SNR for this AM system. (3 points)

Problem 8: FM Noise Performance (12 points)

An FM system has: - Carrier power: $P_c = 20$ W - Noise power: $P_n = 0.5$ W - Modulation index: $\beta = 5$ - Message bandwidth: $B_m = 15$ kHz

(a) Calculate the input SNR (carrier power to noise power ratio). (3 points)

(b) The output SNR for FM is given by: $(\text{SNR})_{\text{out}} = 3\beta^2(\beta+1) \cdot (\text{SNR})_{\text{in}}$ for $\beta \gg 1$. Calculate the output SNR. (4 points)

(c) What is the FM improvement factor (output SNR / input SNR)? (3 points)

(d) Express the output SNR in dB. (2 points)

Problem 9: Additive White Gaussian Noise (AWGN) (10 points)

(a) What does "white" mean in the context of white Gaussian noise? (3 points)

(b) What does "Gaussian" refer to? (2 points)

(c) The power spectral density of AWGN is $N_0/2$ W/Hz. If $N_0/2 = 10^{-9}$ W/Hz and the bandwidth is 5 MHz, what is the total noise power? (3 points)

(d) Why is AWGN a good model for thermal noise in communication systems? (2 points)

Problem 10: Link Budget Basics (12 points - Challenge)

A WiFi system operates at 2.4 GHz with the following parameters: - Transmit power: $P_t = 20$ dBm - Transmit antenna gain: $G_t = 2$ dBi - Receive antenna gain: $G_r = 2$ dBi - Distance: d = 10 m - System bandwidth: 20 MHz - Noise figure: NF = 7 dB - Temperature: T = 290 K

(a) Calculate the free-space path loss at 2.4 GHz and 10 m using: $\text{FSPL(dB)} = 20 \cdot \log_{10}(d) + 20 \cdot \log_{10}(f) + 32.45$, where d is in km and f is in MHz. (4 points)

(b) Calculate the received signal power P_r in dBm. (3 points)

(c) Calculate the noise power in dBm. (3 points)

(d) Calculate the SNR in dB. (2 points)

Bonus Problem 1: Shannon's Theorem (5 points extra credit)

Shannon's theorem states that the channel capacity is: $C = B \cdot \log_2(1 + \text{SNR})$

where C is in bits/s, B is bandwidth in Hz, and SNR is a linear ratio.

(a) Calculate the channel capacity for a system with B = 20 MHz and SNR = 30 dB. (3 points)

(b) If you want to transmit at 100 Mbps with B = 20 MHz, what minimum SNR (in dB) is required? (2 points)

Bonus Problem 2: Noise Temperature (5 points extra credit)

(a) A receiver has a noise figure NF = 3 dB. Calculate the equivalent noise temperature T_e. Use the relationship: $\text{NF} = 1 + \frac{T_e}{T_0}$ where $T_0 = 290$ K. (3 points)

(b) If two amplifiers are cascaded with noise figures $\text{NF}_1 = 3$ dB and $\text{NF}_2 = 6$ dB, and gains $G_1 = 20$ dB and $G_2 = 10$ dB, calculate the overall noise figure using Friis formula: $F_{\text{total}} = F_1 + \frac{F_2 - 1}{G_1}$ where F is the noise factor (linear, not dB). (2 points)

Submission Instructions: - Submit a single PDF via Brightspace - Show all conversions between dB and linear units - For probability problems, show the complete calculation - Q-function values can be looked up or approximated

Academic Integrity: - Individual work required - Show all work for full credit - Cite any formula references

Useful Constants: - Boltzmann constant: $k = 1.38 \times 10^{-23}$ J/K - Room temperature: $T_0 = 290$ K - Speed of light: $c = 3 \times 10^8$ m/s