EE 451: Communications Systems

Homework 4 - M-ary Modulation, QAM, and EVM

Topics: QPSK, M-ary PSK, QAM, spectral efficiency, EVM, WiFi/cellular modulation **Textbook Reference:** Haykin & Moher, Chapter 7.5-7.7

Total Points: 100

Instructions

- Show all work for full credit
- Include units in all final answers
- Constellation diagrams should be clearly labeled with I/Q axes
- For system design problems, state all assumptions

Problem 1: QPSK Fundamentals (15 points)

A QPSK system transmits at 10 Mbps with carrier frequency $f_c = 1.8$ GHz.

(a) How many bits does each symbol represent? What is the symbol rate? (4 points)

(b) Write the four possible signal expressions for QPSK. Use phases 45°, 135°, 225°, and 315°. (6 points)

(c) Draw the constellation diagram showing all four symbols in the I/Q plane. Label each point with its corresponding bit pattern using Gray coding. (5 points)

Problem 2: QPSK Gray Coding (12 points)

(a) Explain what Gray coding is and why it's used in digital modulation. (4 points)

(b) For the QPSK constellation in Problem 1, assign 2-bit patterns to each symbol using Gray coding such that adjacent symbols differ by only one bit. (4 points)

(c) If a symbol error causes the receiver to select an adjacent symbol, how many bit errors occur with Gray coding? Without Gray coding? (4 points)

Problem 3: M-ary PSK Performance (12 points)

Consider 8-PSK modulation with symbol rate $R_s = 5$ Msymbols/s.

(a) What is the bit rate R_b? (3 points)

(b) What is the phase separation between adjacent symbols? (3 points)

(c) Draw the 8-PSK constellation diagram with Gray coding. (4 points)

(d) What is the null-to-null RF bandwidth? (2 points)

Problem 4: QAM Constellation Design (15 points)

A 16-QAM system uses a rectangular constellation.

(a) How many bits per symbol does 16-QAM transmit? (2 points)

(b) Sketch a 16-QAM constellation diagram. Use amplitude levels $\{\pm 1, \pm 3\}$ for both I and Q. Label all 16 points with 4-bit Gray-coded patterns. (6 points)

(c) If the symbol rate is 10 Msymbols/s, what is the bit rate? (3 points)

(d) What is the average symbol energy if the unit amplitude is 1 V and the system resistance is $50\,\Omega$? (4 points)

Problem 5: Spectral Efficiency Comparison (15 points)

Compare the spectral efficiency (bits/s/Hz) of the following modulation schemes, all operating at 20 MHz RF bandwidth:

(a) BPSK (3 points)

(b) QPSK (3 points)

(c) 16-QAM (3 points)

(d) 64-QAM (3 points)

(e) Which modulation scheme provides the highest data rate in 20 MHz? What is that data rate? (3 points)

Problem 6: Error Vector Magnitude (EVM) (12 points)

An ideal QPSK symbol is located at $I = 1.0$ V, $Q = 0$ V. A received symbol is measured at $I = 0.92$ V, $Q = 0.15$ V.

(a) Plot both the ideal and received symbol positions on an I/Q diagram. Draw the error vector. (4 points)

(b) Calculate the error vector magnitude (EVM) in volts. (4 points)

(c) Calculate the EVM as a percentage: $\text{EVM\%} = \frac{\text{EVM}}{\text{Reference}} \times 100\%$, where the reference is the distance from the origin to the ideal symbol. (2 points)

(d) What does EVM measure in a communication system? (2 points)

Problem 7: WiFi Modulation Schemes (12 points)

WiFi 6 (802.11ax) can use BPSK, QPSK, 16-QAM, 64-QAM, 256-QAM, and 1024-QAM depending on channel conditions.

(a) Fill in the table below:

Modulation	Bits/Symbol	Symbol Rate (Msym/s) for 40 MHz channel	Data Rate (Mbps)
BPSK		40	
QPSK		40	
16-QAM		40	
64-QAM		40	
256-QAM		40	
1024-QAM		40	

(6 points)

(b) Why doesn't WiFi always use 1024-QAM for maximum throughput? (3 points)

(c) What is adaptive modulation and coding (AMC)? How does WiFi decide which modulation to use? (3 points)

Problem 8: LTE and 5G Modulation (10 points)

LTE and 5G NR use QPSK, 16-QAM, 64-QAM, and 256-QAM.

(a) An LTE system has 20 MHz bandwidth and uses 64-QAM. Assuming ideal spectral efficiency (bits/s/Hz equals bits/symbol), what is the theoretical maximum data rate? (4 points)

(b) In practice, LTE achieves about 75% of the theoretical maximum due to overhead (control channels, guard bands, cyclic prefix). What is the practical data rate? (2 points)

(c) 5G NR can use 100 MHz bandwidth with 256-QAM. What is the theoretical data rate? (2 points)

(d) Why can 5G use wider bandwidths than LTE? (2 points)

Problem 9: Power Efficiency vs. Spectral Efficiency (15 points - Challenge)

(a) Explain the trade-off between power efficiency and spectral efficiency in M-ary modulation schemes. (5 points)

(b) For a fixed bit rate of 100 Mbps, compare the symbol rate required for QPSK vs. 16-QAM. Which requires higher bandwidth? (5 points)

(c) In a power-limited system (e.g., satellite communication), would you prefer QPSK or 16-QAM? In a bandwidth-limited system (e.g., urban cellular), which would you prefer? Justify your answers. (5 points)

Problem 10: Real-World System Design - 5G NR (12 points - Challenge)

You are designing a 5G NR downlink with the following specifications: - Carrier frequency: 3.5 GHz - Channel bandwidth: 100 MHz - Target data rate: 500 Mbps - Available modulation schemes: QPSK, 16-QAM, 64-QAM, 256-QAM

(a) Assuming ideal spectral efficiency (no overhead), which modulation scheme(s) can achieve the 500 Mbps target? (4 points)

(b) If overhead reduces efficiency to 70% of theoretical, which modulation schemes can still meet the target? (4 points)

(c) The link budget analysis shows SNR = 25 dB. Based on typical SNR requirements below, which modulation should you choose?

- QPSK: 5-10 dB
- 16-QAM: 10-15 dB
- 64-QAM: 18-23 dB
- 256-QAM: 25-30 dB

(4 points)

Bonus Problem 1: OFDM and Spectral Efficiency (5 points extra credit)

WiFi and LTE use OFDM (Orthogonal Frequency Division Multiplexing) with QAM on each subcarrier.

(a) If WiFi uses 52 subcarriers (802.11a/g) in a 20 MHz channel, and each subcarrier uses 64-QAM, what is the total data rate assuming ideal conditions? (3 points)

(b) How does OFDM improve spectral efficiency compared to single-carrier modulation? (2 points)

Bonus Problem 2: EVM Limits in Standards (5 points extra credit)

WiFi 6 specifies maximum EVM requirements for different modulations: - BPSK/QPSK: -5 dB (56%) - 16-QAM: -13 dB (22%) - 64-QAM: -19 dB (11%) - 256-QAM: -25 dB (5.6%) - 1024-QAM: -30 dB (3.2%)

(a) Why do higher-order modulations have stricter EVM requirements? (3 points)

(b) Convert the EVM requirement for 64-QAM from dB to percentage. Verify: EVM(dB) = $20\log_{10}$(EVM%). Does it match the given 11%? (2 points)

Submission Instructions: - Submit a single PDF via Brightspace - Constellation diagrams should be neat and clearly labeled - Include all calculations with units - Tables should be complete and readable

Academic Integrity: - Individual work required - Show all intermediate steps - Cite any references to WiFi/LTE specifications