# EE 451: Communications Systems

# Homework 6 - BER, Link Budgets, and System Analysis

**Topics:** Bit error rate, matched filtering, link budgets, WiFi/cellular systems **Textbook Reference:** Haykin & Moher, Chapters 10, 11.4-11.7

**Total Points:** 100

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## Instructions

- Show all work for full credit
- Include units in all final answers
- For link budgets, show the complete calculation chain
- State all assumptions clearly

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## Problem 1: BER for BPSK (12 points)

A BPSK system operates with $E_b/N_0 = 10$ dB.

**(a)** Convert $E_b/N_0$ to a linear ratio. (2 points)

**(b)** The BER for BPSK is given by: $\text{BER} = Q(\sqrt{2E_b/N_0})$. Calculate the BER. (4 points)

**(c)** If the bit rate is 1 Mbps, approximately how many bit errors occur per second? (3 points)

**(d)** What $E_b/N_0$ is required to achieve $\text{BER} = 10^{-6}$? (3 points)

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## Problem 2: BER Comparison (15 points)

Compare the BER performance of BPSK, QPSK, and 8-PSK.

**(a)** For BPSK: $\text{BER} = Q(\sqrt{2E_b/N_0})$ For QPSK: $\text{BER} \approx Q(\sqrt{2E_b/N_0})$ (approximately same as BPSK) For 8-PSK: $\text{BER} \approx \frac{2}{3} \cdot Q(\sqrt{6E_b/N_0} \cdot \sin(\pi/8))$

`Calculate the BER for each modulation at $E_b/N_0 = 12$ dB. (6 points)`

**(b)** Which modulation has the best BER performance? Why? (3 points)

**(c)** Which modulation has the highest spectral efficiency? (3 points)

**(d)** If you need to transmit 10 Mbps in 5 MHz bandwidth, which modulation should you choose? Consider both BER and spectral efficiency. (3 points)

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## Problem 3: Matched Filtering (12 points)

**(a)** Explain what a matched filter is and why it's optimal for detecting signals in AWGN. (4 points)

**(b)** For a rectangular pulse of duration T and amplitude A, what is the output SNR of a matched filter? Express in terms of signal energy $E_s$ and noise power spectral density $N_0$. (4 points)

**(c)** A BPSK system uses matched filtering. If the bit energy is $E_b = 10^{-13}$ J and $N_0 = 10^{-20}$ W/Hz, calculate the output SNR. (4 points)

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## Problem 4: QAM Bit Error Rate (12 points)

For M-QAM, the symbol error rate (SER) is approximately: $\text{SER} \approx 4 \cdot \left(1 - \frac{1}{\sqrt{M}}\right) \cdot Q\left(\sqrt{\frac{3E_s}{N_0(M-1)}}\right)$

For 16-QAM at $E_s/N_0 = 18$ dB:

**(a)** Convert $E_s/N_0$ to linear ratio. (2 points)

**(b)** Calculate the symbol error rate (SER). (4 points)

**(c)** For Gray coding, approximate $\text{BER} \approx \text{SER}/\log_2(M)$. Calculate the BER. (3 points)

**(d)** If the symbol rate is 10 Msymbols/s, what is the bit rate and how many bit errors per second occur? (3 points)

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## Problem 5: Complete Link Budget - WiFi (18 points)

A WiFi 6 system operates at 5 GHz with the following parameters: - Transmit power: $P_t = 23$ dBm (200 mW, FCC limit for 5 GHz) - Transmit antenna gain: $G_t = 3$ dBi - Receive antenna gain: $G_r = 0$ dBi (laptop internal antenna) - Frequency: 5.0 GHz - Distance: 30 m - System bandwidth: 80 MHz - Noise figure: NF = 6 dB - Temperature: T = 290 K - Implementation loss: 3 dB - Fade margin: 10 dB

**(a)** Calculate free-space path loss using: $\text{FSPL(dB)} = 20 \cdot \log_{10}(4\pi d/\lambda)$, where $\lambda = c/f$. (4 points)

**(b)** Calculate received signal power $P_r$ including all gains and losses. (4 points)

**(c)** Calculate noise power $P_n$ in the 80 MHz bandwidth. (3 points)

**(d)** Calculate the SNR. (2 points)

**(e)** Based on the SNR, what is the highest modulation that can be used? Use these thresholds: - BPSK/QPSK: 5 dB - 16-QAM: 12 dB - 64-QAM: 20 dB - 256-QAM: 28 dB - 1024-QAM: 35 dB (3 points)

**(f)** Calculate the maximum data rate (assuming ideal spectral efficiency and no overhead). (2 points)

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## Problem 6: Link Budget - Cellular 5G (15 points)

A 5G base station communicates with a smartphone at 3.5 GHz: - Base station transmit power: $P_t = 43$ dBm (20 W) - Base station antenna gain: $G_t = 15$ dBi (directional) - Smartphone antenna gain: $G_r = -2$ dBi - Distance: 500 m - Path loss exponent: n = 3.5 (urban environment) - Shadow fading margin: 8 dB - Bandwidth: 100 MHz - Noise figure: NF = 7 dB

**(a)** Calculate path loss using: $\text{PL(dB)} = \text{FSPL}(d_0) + 10n \cdot \log_{10}(d/d_0)$, where $d_0 = 1$ m. (5 points)

**(b)** Calculate received power. (3 points)

**(c)** Calculate noise power. (2 points)

**(d)** Calculate SNR. (2 points)

**(e)** Using Shannon's theorem $C = B \cdot \log_2(1 + \text{SNR})$, calculate the theoretical channel capacity. (3 points)

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## Problem 7: WiFi vs. Cellular Comparison (10 points)

**(a)** Compare the link budgets from Problems 5 and 6. Which system can communicate over longer distances? Why? (4 points)

**(b)** Which system provides higher data rates? Explain the trade-offs. (3 points)

**(c)** Explain why cellular systems use higher transmit power and higher antenna gain compared to WiFi. (3 points)

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## Problem 8: Adaptive Modulation System (12 points)

A wireless link uses adaptive modulation with the following SNR thresholds: - QPSK: 8 dB, data rate = 20 Mbps - 16-QAM: 15 dB, data rate = 40 Mbps - 64-QAM: 22 dB, data rate = 60 Mbps - 256-QAM: 30 dB, data rate = 80 Mbps

The received SNR varies with distance as: $\text{SNR(dB)} = 45 - 30 \cdot \log_{10}(d/10)$, where d is in meters.

**(a)** Calculate the SNR at distances of 10 m, 20 m, 50 m, and 100 m. (4 points)

**(b)** For each distance, determine which modulation scheme is used. (4 points)

**(c)** Calculate the average data rate if the user spends equal time at each of these four distances. (2 points)

**(d)** What is the maximum distance at which communication is possible ($\text{SNR} \geq 8$ dB)? (2 points)

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## Problem 9: Multipath and Fading (10 points)

**(a)** Explain how multipath propagation affects wireless communication systems. (3 points)

**(b)** What is frequency-selective fading and why does it occur? (3 points)

**(c)** How do OFDM-based systems (WiFi, LTE, 5G) mitigate frequency-selective fading? (4 points)

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### Problem 10: Real-World System Design (14 points - Challenge)

You are designing a long-range WiFi link for a rural area: - Frequency: 2.4 GHz (better propagation than 5 GHz) - Required distance: 1 km - Required data rate: 50 Mbps - Available transmit power: 30 dBm (1 W) - Available antennas: $G_t = G_r = 10$ dBi (directional) - Bandwidth: 20 MHz - Temperature: T = 290 K - Noise figure: NF = 5 dB - Required SNR for 64-QAM: 22 dB - Fade margin: 15 dB (for reliability)

**(a)** Calculate the free-space path loss at 1 km and 2.4 GHz. (3 points)

**(b)** Calculate the link budget and determine the received SNR. (5 points)

**(c)** Can the system achieve the required SNR of 22 dB with 15 dB fade margin? (2 points)

**(d)** If not, suggest three modifications to improve the link budget and calculate the improvement from each. (4 points)

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### Bonus Problem 1: Friis Transmission Equation (5 points extra credit)

The Friis transmission equation relates received power to transmitted power: $P_r = P_t \cdot G_t \cdot G_r \cdot \left(\frac{\lambda}{4\pi d}\right)^2$

**(a)** Derive the free-space path loss formula FSPL(dB) $= 20 \cdot \log_{10}(d) + 20 \cdot \log_{10}(f) + 32.45$ (with d in km and f in MHz) from the Friis equation. (3 points)

**(b)** Verify your derivation by calculating FSPL at d = 1 km and f = 1 GHz. (2 points)

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### Bonus Problem 2: Eb/N0 vs. SNR Relationship (5 points extra credit)

For a digital communication system: - Bit rate: $R_b$ - Bandwidth: B - SNR = signal power / noise power - $E_b/N_0$ = bit energy / noise power spectral density

**(a)** Show that $E_b/N_0 = \text{SNR} \cdot (B/R_b)$. (3 points)

**(b)** For QPSK with $R_b = 10$ Mbps in B = 5 MHz, if SNR = 12 dB, calculate $E_b/N_0$. (2 points)

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**Submission Instructions:** - Submit a single PDF via Brightspace - Show complete link budget calculations - Include all unit conversions (dB $\leftrightarrow$ linear) - Clearly label each step

**Academic Integrity:** - Individual work required - Show all work for partial credit - Cite any references to standards or specifications

**Useful Formulas:** - FSPL(dB) $= 20 \cdot \log_{10}(d) + 20 \cdot \log_{10}(f) + 32.45$ (d in km, f in MHz) - FSPL(dB) $= 20 \cdot \log_{10}(4\pi d/\lambda)$ (d and $\lambda$ in same units) - $P_n = k \cdot T \cdot B$ (thermal noise) - $k = 1.38 \times 10^{-23}$ J/K - $c = 3 \times 10^8$ m/s - Shannon: $C = B \cdot \log_2(1 + \text{SNR})$