EE 451: Communications Systems

Homework 3 - FM, PM, FSK, and BPSK

Topics: Frequency and phase modulation, FSK, BPSK, Carson's rule **Textbook Reference:** Haykin & Moher, Chapters 4.1-4.5, 7.2-7.4

Total Points: 100

Instructions

- Show all work for full credit
- Include units in all final answers
- For bandwidth calculations, clearly state which rule or method you're using
- Sketches should be clearly labeled

Problem 1: FM Fundamentals (12 points)

A message signal $m(t) = 2\cos(2\pi \cdot 1000 \cdot t)$ V modulates an FM carrier with $f_c = 100$ MHz and frequency sensitivity $k_f = 10$ kHz/V.

(a) Write the expression for the instantaneous frequency $f_i(t)$. (3 points)

(b) What is the maximum frequency deviation Δf? (3 points)

(c) Write the complete FM signal expression $s(t)$. You may leave the answer in terms of the integral of $m(t)$. (4 points)

(d) What is the modulation index β for FM? (2 points)

Problem 2: Carson's Rule Application (15 points)

An FM broadcast station transmits audio signals with maximum frequency $f_m = 15$ kHz and maximum frequency deviation $\Delta f = 75$ kHz (standard FM broadcasting parameters).

(a) Calculate the modulation index β. (3 points)

(b) Use Carson's rule to estimate the FM bandwidth. (4 points)

(c) Is this narrowband FM (NBFM) or wideband FM (WBFM)? Justify your answer. (3 points)

(d) How does this compare to the bandwidth required for AM broadcasting of the same audio signal? (3 points)

(e) What is the advantage of using such a large bandwidth for FM? (2 points)

Problem 3: Narrowband vs. Wideband FM (12 points)

(a) For NBFM, the criterion is $\beta \ll 1$. If $f_m = 3$ kHz, what is the maximum Δf that qualifies as NBFM? Use $\beta < 0.3$ as the threshold. (3 points)

(b) Estimate the bandwidth of NBFM using the small-angle approximation. (3 points)

(c) For WBFM with the same $f_m = 3$ kHz but $\Delta f = 30$ kHz, calculate β and the bandwidth using Carson's rule. (4 points)

(d) Compare the bandwidth efficiency (in terms of β) between NBFM and WBFM. (2 points)

Problem 4: Phase Modulation (12 points)

A PM signal is given by $s(t) = A_c \cos[2\pi f_c t + k_p m(t)]$ where: - $A_c = 10$ V - $f_c = 200$ MHz - $k_p = 2$ rad/V (phase sensitivity) - $m(t) = 3\cos(2\pi \cdot 5000 \cdot t)$ V

(a) What is the maximum phase deviation $\Delta\phi$? (3 points)

(b) Write the expression for the instantaneous phase $\theta_i(t)$. (3 points)

(c) Find the instantaneous frequency $f_i(t)$. (4 points)

(d) What is the maximum frequency deviation? (2 points)

Problem 5: FM vs. PM Relationship (10 points)

(a) Explain the fundamental difference between FM and PM. How are they related? (4 points)

(b) If you integrate a message signal before applying it to a frequency modulator, what type of modulation do you effectively create? (3 points)

(c) For the message $m(t) = \cos(2\pi f_m t)$, how does the frequency deviation in FM compare to the frequency deviation in PM as f_m varies? (3 points)

Problem 6: Binary FSK Fundamentals (15 points)

A binary FSK system transmits data at 9600 bits/s using: - "0" bit: $f_0 = 1200$ Hz - "1" bit: $f_1 = 2200$ Hz - Carrier amplitude: $A = 5$ V

(a) Write the signal expressions $s_0(t)$ and $s_1(t)$ for the two binary symbols. (4 points)

(b) What is the frequency separation $\Delta f = |f_1 - f_0|$? (2 points)

(c) Calculate the modulation index $h = \Delta f \cdot T_b$ where T_b is the bit duration. (4 points)

(d) For coherent FSK detection, the minimum frequency separation for orthogonality is $\Delta f = \frac{1}{2T_b}$. Is this FSK system using orthogonal tones? (3 points)

(e) Estimate the bandwidth using Carson's rule, treating FSK as FM with a rectangular baseband signal. (2 points)

Problem 7: Minimum Shift Keying (MSK) (12 points)

MSK is a special case of continuous-phase FSK (CPFSK) with modulation index $h = 0.5$.

(a) If the bit rate is $R_b = 10$ kbps, what is the bit duration T_b? (2 points)

(b) Given $h = 0.5 = \Delta f \cdot T_b$, calculate the required frequency separation Δf. (3 points)

(c) What are the two frequencies f_0 and f_1 if the center frequency is 1800 Hz? (4 points)

(d) Why is MSK preferred over standard FSK in many applications? (3 points)

Problem 8: Binary Phase Shift Keying (BPSK) (15 points)

A BPSK system transmits at 1 Mbps with carrier frequency $f_c = 2.4$ GHz.

(a) Write the two possible signal expressions for BPSK representing bits "0" and "1". (4 points)

(b) Sketch the constellation diagram for BPSK in the I/Q plane. Label the two signal points. (4 points)

(c) What is the phase difference between the two BPSK symbols? (2 points)

(d) Calculate the null-to-null bandwidth of the BPSK signal. (3 points)

(e) Can BPSK be demodulated using an envelope detector? Why or why not? (2 points)

Problem 9: Coherent vs. Non-Coherent Detection (10 points)

(a) Explain the difference between coherent and non-coherent detection. (3 points)

(b) Which modulation schemes can use non-coherent detection: ASK, FSK, BPSK? Explain. (4 points)

(c) What is the advantage of coherent detection? What is the disadvantage? (3 points)

Problem 10: Real-World Application - FM Deviation (12 points - Challenge)

Commercial FM radio in the US uses: - Maximum frequency deviation: $\Delta f = 75$ kHz (audio) - Pilot tone for stereo: 19 kHz - Subcarrier for RDS (Radio Data System): 57 kHz - Maximum audio frequency: 15 kHz

(a) Calculate the modulation index for maximum audio frequency. (3 points)

(b) Use Carson's rule to calculate the bandwidth occupied by the FM station. (3 points)

(c) FM stations are allocated 200 kHz of bandwidth. Does your calculated bandwidth fit? If not, what accounts for the difference? (3 points)

(d) An FM station also has a subcarrier at 67 kHz for SCA (Subsidiary Communications Authorization) services. How does this affect the total deviation and bandwidth? (3 points)

Bonus Problem 1: Bessel Functions in FM (5 points extra credit)

The FM spectrum consists of infinite sidebands with amplitudes given by Bessel functions $J_n(\beta)$.

For single-tone FM with $\beta = 2$: - $J_0(2) = 0.224$ - $J_1(2) = 0.577$ - $J_2(2) = 0.353$ - $J_3(2) = 0.129$ - $J_4(2) = 0.034$

(a) Sketch the frequency spectrum showing the carrier and first four sidebands on each side. Label amplitudes relative to carrier (assume $A_c = 10$ V). (3 points)

(b) Notice that $J_0(2)$ is small. What does this mean about the carrier power in FM compared to AM? (2 points)

Bonus Problem 2: Frequency Deviation Calculation (5 points extra credit)

For the FM signal $s(t) = 10\cos[2\pi \cdot 100 \times 10^6 \cdot t + 50\sin(2\pi \cdot 1000 \cdot t)]$:

(a) Identify the carrier frequency f_c, message frequency f_m, and the modulation index β. (3 points)

(b) Calculate the frequency deviation Δf. (2 points)

Hint: The general form is $s(t) = A_c\cos[2\pi f_c t + \beta\sin(2\pi f_m t)]$ for single-tone FM.

Submission Instructions: - Submit a single PDF via Brightspace - Show all derivations and calculations - Include clear sketches with labeled axes - State all assumptions

Academic Integrity: - Individual work required - You may use textbook formulas and course notes - Cite any external references