

# EE 451: Communications Systems

## Homework 1 - Complex Numbers, Phasors, and Signal Operations

**Topics:** Complex numbers, Euler's formula, phasors, signal energy and power **Textbook Reference:** Haykin & Moher, Chapter 2.1

**Total Points:** 100

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### Instructions

- Show all work for full credit
  - Include units in all final answers
  - Box or clearly indicate your final answers
  - You may use Python/MATLAB for verification, but show analytical work
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### Problem 1: Complex Number Arithmetic (10 points)

Given two complex numbers: -  $z_1 = 3 + 4j$  -  $z_2 = 2 - 5j$

Calculate the following:

- (a)  $z_1 + z_2$  (2 points)
  - (b)  $z_1 \cdot z_2$  (3 points)
  - (c)  $\frac{z_1}{z_2}$  (express in both rectangular and polar form) (5 points)
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### Problem 2: Polar and Rectangular Conversion (12 points)

Convert the following complex numbers between rectangular and polar forms:

- (a) Convert  $z = -6 + 8j$  to polar form (magnitude and phase in degrees) (4 points)
  - (b) Convert  $z = 15\angle -135^\circ$  to rectangular form (4 points)
  - (c) A signal has magnitude 10 V and phase  $-60^\circ$ . Express this in rectangular form and verify that  $|z|^2$  equals the sum of the squares of the real and imaginary parts. (4 points)
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### Problem 3: Euler's Formula Application (12 points)

Using Euler's formula  $e^{j\theta} = \cos(\theta) + j\sin(\theta)$ :

- (a) Express  $\cos(\omega t)$  as the real part of a complex exponential. (4 points)
- (b) Express  $\sin(\omega t + \phi)$  as the imaginary part of a complex exponential. (4 points)

(c) Prove the trigonometric identity:

$$\cos(A) \cos(B) = \frac{1}{2} [\cos(A + B) + \cos(A - B)]$$

using Euler's formula. (4 points)

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#### Problem 4: Phasor Addition (15 points)

Two sinusoidal voltages are added together: -  $v_1(t) = 10 \cos(2\pi \cdot 1000 \cdot t + 30^\circ)$  V -  $v_2(t) = 15 \cos(2\pi \cdot 1000 \cdot t - 45^\circ)$  V

- (a) Convert each voltage to phasor form. (4 points)
  - (b) Add the phasors to find the resultant phasor  $V_{\text{total}}$ . (5 points)
  - (c) Convert the resultant phasor back to a time-domain sinusoid  $v_{\text{total}}(t)$ . (4 points)
  - (d) Verify your answer using Python or MATLAB (optional, 2 points extra credit).
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#### Problem 5: I/Q Representation (15 points)

A communication signal is represented as:

$$s(t) = I(t) \cos(2\pi f_c t) - Q(t) \sin(2\pi f_c t)$$

where  $I(t) = 5$  V and  $Q(t) = 12$  V are constant values, and  $f_c = 900$  MHz.

- (a) Express  $s(t)$  as a single sinusoid in the form  $A \cos(2\pi f_c t + \phi)$ . Find  $A$  and  $\phi$ . (7 points)
  - (b) What is the complex envelope of this signal? (4 points)
  - (c) Sketch the position of this signal in the I/Q plane (complex plane). (4 points)
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#### Problem 6: Signal Energy (12 points)

Consider the rectangular pulse signal:

$$x(t) = \begin{cases} 5 \text{ V}, & 0 \leq t \leq 2 \text{ seconds} \\ 0, & \text{otherwise} \end{cases}$$

- (a) Calculate the energy of this signal. (6 points)
  - (b) Is this an energy signal or a power signal? Justify your answer. (3 points)
  - (c) What would be the energy if the amplitude doubled to 10 V? (3 points)
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**Problem 7: Power Signal Analysis (15 points)**

A periodic signal is given by:

$$x(t) = 10 \cos(2\pi \cdot 60 \cdot t) + 5 \sin(2\pi \cdot 120 \cdot t) \text{ V}$$

- (a) Calculate the average power of each frequency component separately. (6 points)
  - (b) Calculate the total average power of the signal. (4 points)
  - (c) Is this an energy signal or a power signal? Explain. (3 points)
  - (d) What is the period of the composite signal? (2 points)
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**Problem 8: Convolution Fundamentals (9 points)**

The unit step function is defined as:

$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

- (a) Sketch the signal  $x(t) = u(t) - u(t - 3)$ . (3 points)
- (b) What is the relationship between the unit step  $u(t)$  and the unit impulse  $\delta(t)$ ? (3 points)
- (c) Evaluate the integral:

$$\int_{-\infty}^{\infty} x(\tau) \delta(\tau - 2) d\tau$$

where  $x(t)$  is defined in part (a). (3 points)

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**Problem 9: Real-World Application - WiFi Signal (10 points - Challenge)**

A WiFi signal at 2.4 GHz has I and Q components: -  $I(t) = 3 \cos(2\pi \cdot 100 \cdot t)$  V -  $Q(t) = 4 \cos(2\pi \cdot 100 \cdot t)$  V

where the 100 Hz represents a slow modulating signal (in reality, WiFi modulation is much faster, but we're simplifying).

- (a) Express the complete RF signal  $s(t) = I(t) \cos(2\pi f_c t) - Q(t) \sin(2\pi f_c t)$  where  $f_c = 2.4$  GHz. (4 points)
  - (b) What is the instantaneous magnitude of the complex envelope at  $t = 0$ ? (3 points)
  - (c) Explain why modern software-defined radios (like the RTL-SDR) output I and Q samples rather than the full RF signal  $s(t)$ . (3 points)
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### Problem 10: Conceptual Understanding (10 points)

Answer the following short-answer questions (2-3 sentences each):

- (a) Why is phasor representation useful in analyzing communication systems? (3 points)
  - (b) What is the physical meaning of the phase angle in a complex number representing a sinusoid? (3 points)
  - (c) Explain the difference between the complex envelope and the actual RF carrier signal in a communication system. (4 points)
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### Bonus Problem (5 points extra credit)

Prove that for any complex number  $z = a + jb$ :

$$z \cdot z^* = |z|^2$$

where  $z^*$  is the complex conjugate of  $z$ , and  $|z|$  is the magnitude of  $z$ .

Then explain why this property is useful for calculating signal power in communication systems.

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**Submission Instructions:** - Submit a single PDF via Brightspace - Scanned handwritten work is acceptable if legible - Include any code used for verification

**Academic Integrity:** - You may discuss concepts with classmates, but all submitted work must be your own - Show your work - answers without supporting calculations will receive minimal credit