

# EE 451: Communications Systems

## Final Exam - Comprehensive Study Guide

**Exam Date:** Week 16 (May 18-22, 2026) **Duration:** 180 minutes (3 hours) **Coverage:** All course material (Chapters 1-11) with emphasis on Chapters 8-11 **Format:** Closed book, one 8.5" × 11" formula sheet (both sides) allowed

---

### Overview

The final exam is **comprehensive** but with **emphasis on later material** (probability, noise, BER, link budgets). Structure: - **Part I (30 pts):** Fundamentals review (Fourier, AM, FM/PM) - **Part II (35 pts):** Digital modulation (BPSK, QPSK, FSK, QAM) - **Part III (40 pts):** Probability and noise (Gaussian, SNR, BER) - **Part IV (35 pts):** Link budgets and system performance - **Part V (10 pts):** Advanced topics (OFDM, spread spectrum) - **Bonus (10 pts):** Satellite link analysis

**Key insight:** 85 of 150 points (57%) come from material AFTER Midterm 2!

---

### Study Strategy

#### Priority Levels

**HIGHEST PRIORITY** (Study first - 60% of exam) - Link budget calculations (Problems 10, 11) - Gaussian distribution and Q-function (Problem 7) - Thermal noise and SNR calculations (Problem 8) - BER analysis for BPSK (Problem 9) - Shannon capacity (Problem 11)

**MEDIUM PRIORITY** (Important - 25% of exam) - BPSK vs QPSK comparison (Problem 4) - 16-QAM and Gray coding (Problem 5) - FSK system design (Problem 6) - Adaptive modulation (Problem 12)

**LOWER PRIORITY** (Review quickly - 15% of exam) - Fourier transforms (Problem 1) - AM power calculations (Problem 2) - FM vs PM (Problem 3) - OFDM and spread spectrum concepts (Problem 13)

---

### Part I: Fundamentals Review (30 points)

#### Quick Review Topics

**Fourier Transforms (10 pts)** - Sinc-rect transform pair - Time-frequency duality - Modulation property (DSB-SC spectrum)

**AM (10 pts)** - Modulation index  $\mu = A_m/A_c$  - Power:  $P_{total} = \frac{A_c^2}{2R}(1 + \mu^2/2)$  - Efficiency:  $\eta = \frac{\mu^2/2}{1 + \mu^2/2}$

**FM/PM (10 pts)** - FM:  $\Delta f = k_f A_m$  (independent of  $f_m$ ) - PM:  $\Delta f = k_p A_m f_m$  (proportional to  $f_m$ ) - Carson's rule:  $B_{FM} = 2(\Delta f + f_m)$

**Study Tip:** These are REVIEW topics. Don't spend too much time here if you understand Midterms 1 & 2 material. Focus on new material!

---

## Part II: Digital Modulation (35 points)

### BPSK vs QPSK (12 pts - Problem 4)

**Key Comparisons:**

Feature	BPSK	QPSK
Bits/symbol	1	2
Symbol rate	$R_b$	$R_b/2$
Bandwidth	$2R_b$	$R_b$
Spectral efficiency	0.5 bits/s/Hz	1.0 bits/s/Hz
BER (same $E_b/N_0$ )	Same!	Same!

**Critical insight:** QPSK is twice as bandwidth efficient with no BER penalty!

### QAM and Gray Coding (12 pts - Problem 5)

**16-QAM Basics:** - M = 16 symbols, k = 4 bits/symbol - Rectangular constellation: 4×4 grid - Amplitude levels: typically  $\{\pm 1, \pm 3\}$

**Gray Coding Rules:** - Adjacent symbols differ by 1 bit - Minimizes BER (most errors → adjacent symbols) - Mandatory in modern systems!

**EVM (Error Vector Magnitude):** - Measures constellation quality -  $\text{EVM}\% = (\text{Error distance} / \text{Reference distance}) \times 100\%$  - WiFi/LTE have strict EVM limits (e.g., 256-QAM < 3%)

### FSK System Design (11 pts - Problem 6)

**Orthogonal FSK:** - Minimum separation:  $\Delta f_{min} = \frac{1}{2T_b}$  - Modulation index:  $h = \Delta f \times T_b$  - MSK:  $h = 0.5$  exactly

**Quick formulas:**

$$T_b = 1/R_b$$

$$\Delta f = |f - f|$$

$$h = \Delta f \cdot T_b$$

$$\text{Orthogonal: } \Delta f = 1/(2T_b)$$

$$\text{MSK: } h = 0.5$$

---

## Part III: Probability and Noise (40 points)

**THIS IS THE MOST IMPORTANT SECTION!**

## Gaussian Distribution (15 pts - Problem 7)

**PDF of Gaussian:**

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)}$$

**Standardization:**

$$Z = \frac{X - \mu}{\sigma} \sim N(0, 1)$$

**Q-Function:**

$$Q(x) = P(Z > x) \text{ for } Z \sim N(0, 1)$$

**For general Gaussian  $X \sim N(\mu, \sigma^2)$ :**

$$P(X > a) = Q\left(\frac{a - \mu}{\sigma}\right)$$

**Key Q-function values (memorize these!):**

Q(1.0)	0.159	Q(2.0)	0.023
Q(1.5)	0.067	Q(3.0)	0.0013
Q(4.0)	3e-5	Q(4.75)	1e-6

**Property:**  $Q(-x) = 1 - Q(x)$

**Practice:** 1. Identify  $\mu$  and  $\sigma$  from problem 2. Standardize:  $z = (x - \mu)/\sigma$  3. Express using Q-function 4. Look up or calculate

## Thermal Noise and SNR (12 pts - Problem 8)

**Thermal Noise Power:**

$$P_n = kTB$$

where: -  $k = 1.38 \times 10^{-23}$  J/K (Boltzmann constant) -  $T$  = temperature (K) -  $B$  = bandwidth (Hz)

**Noise Figure:**

$$P_{n,total} = P_n \times F = P_n \times 10^{NF_{dB}/10}$$

**In dBm:**

$$P_n(dBm) = 10 \log_{10} \left( \frac{kTB}{10^{-3}} \right)$$

**Quick calculation:** At  $T = 290$  K:

$$P_n(dBm) \approx -174 + 10 \log_{10}(B_{Hz})$$

**SNR:**

$$SNR(dB) = P_s(dBm) - P_n(dBm)$$

**Critical conversions:**

dBm to W:  $P_W = 10^{((P_{dBm} - 30)/10)}$   
W to dBm:  $P_{dBm} = 10 \cdot \log(P_W) + 30$   
dB addition:  $P_{total}(dB) = P1(dB) + P2(dB)$   
Linear addition:  $P_{total\_W} = P1\_W + P2\_W$

**Common values to know:**

0 dBm = 1 mW  
-30 dBm = 1 W  
-60 dBm = 1 nW  
-90 dBm = 1 pW

+3 dB =  $\times 2$   
+10 dB =  $\times 10$   
+20 dB =  $\times 100$

## BER Performance (13 pts - Problem 9)

**BPSK BER:**

$$BER_{BPSK} = Q(\sqrt{2E_b/N_0})$$

**Conversions:**

$$\frac{E_b}{N_0}(dB) = 10 \log_{10}(E_b/N_0)$$

**To find linear from dB:**

$$\frac{E_b}{N_0} = 10^{(E_b/N_0)_{dB}/10}$$

**Bit errors per second:**

$$N_{errors/s} = BER \times R_b$$

**Common  $E_b/N_0$  values for BER targets:**

BER =  $10^{-3}$ :  $E_b/N_0$  6.8 dB  
BER =  $10^{-4}$ :  $E_b/N_0$  8.4 dB  
BER =  $10^{-5}$ :  $E_b/N_0$  9.6 dB  
BER =  $10^{-6}$ :  $E_b/N_0$  10.5 dB

**Step-by-step for BER problems:** 1. Convert  $E_b/N_0$  from dB to linear 2. Calculate  $\sqrt{2E_b/N_0}$   
3. Find  $Q(\sqrt{2E_b/N_0})$  using table 4. This is your BER 5. Errors/sec = BER  $\times$  bit rate

## Part IV: Link Budgets (35 points)

**CRITICAL SECTION - WORTH 35 POINTS!**

## Free-Space Path Loss (FSPL)

Two equivalent forms:

Form 1 (dB, easier for calculations):

$$FSPL(dB) = 20 \log_{10}(d_{km}) + 20 \log_{10}(f_{MHz}) + 32.45$$

Form 2 (from wavelength):

$$FSPL(dB) = 20 \log_{10} \left( \frac{4\pi d}{\lambda} \right)$$

where  $\lambda = c/f$

**Example calculation:**

$$f = 5.8 \text{ GHz} = 5800 \text{ MHz}$$

$$d = 50 \text{ m} = 0.05 \text{ km}$$

$$\begin{aligned} FSPL &= 20 \cdot \log(0.05) + 20 \cdot \log(5800) + 32.45 \\ &= 20 \cdot (-1.301) + 20 \cdot (3.763) + 32.45 \\ &= -26.02 + 75.26 + 32.45 \\ &= 81.69 \text{ dB} \end{aligned}$$

## Complete Link Budget (Problem 10 - 18 pts)

Link Budget Equation:

$$P_r(dBm) = P_t(dBm) + G_t(dBi) + G_r(dBi) - FSPL(dB) - L_{misc}(dB)$$

Step-by-step procedure:

1. Calculate FSPL - Use formula above - Make sure units match (km and MHz, OR meters and Hz)

2. Calculate received power

$$P_r = P_t + G_t + G_r - FSPL - L_{other}$$

3. Calculate noise power

$$P_n = kTB \text{ (in W)}$$

$$P_n(dBm) = 10 \log_{10}(P_n/0.001)$$

OR use quick formula:

$$P_n(dBm) \approx -174 + 10 \log_{10}(B_{Hz}) + NF(dB)$$

4. Calculate SNR

$$SNR(dB) = P_r(dBm) - P_n(dBm) - \text{fade margin}$$

5. Determine supported modulation Compare SNR to thresholds:

QPSK: 8 dB  
 16-QAM: 15 dB  
 64-QAM: 22 dB  
 256-QAM: 28 dB

**Common mistakes:** Forgetting to convert distance to km or frequency to MHz Mixing up + and - signs in link budget Forgetting fade margin Not adding noise figure to thermal noise

### Shannon Capacity (Problem 11 - 10 pts)

**Shannon-Hartley Theorem:**

$$C = B \log_2(1 + SNR)$$

bits/s

where: -  $C$  = channel capacity (bits/s) -  $B$  = bandwidth (Hz) -  $SNR$  = linear ratio (NOT dB!)

**Convert SNR from dB:**

$$SNR = 10^{SNR(dB)/10}$$

**Spectral Efficiency:**

$$\eta = \frac{C}{B} = \log_2(1 + SNR)$$

bits/s/Hz

**Example:**

$B = 10 \text{ MHz}$ ,  $SNR = 20 \text{ dB}$

$SNR_{\text{linear}} = 10^{(20/10)} = 100$

$C = 10 \times 10^6 \times \log_2(1 + 100)$   
 $= 10 \times 10^6 \times \log_2(101)$   
 $= 10 \times 10^6 \times 6.658$   
 $= 66.58 \text{ Mbps}$

$= 6.658 \text{ bits/s/Hz}$

**Common mistake:** Using SNR in dB directly (must convert to linear!)

### Adaptive Modulation (Problem 12 - 7 pts)

**Concept:** - Choose modulation based on SNR - Higher SNR  $\rightarrow$  higher-order modulation  $\rightarrow$  more bits/s - Lower SNR  $\rightarrow$  lower-order modulation  $\rightarrow$  more reliable

**SNR vs. distance model:**

$$SNR(dB) = SNR_0 - n \times 10 \log_{10}(d/d_0)$$

where  $n$  is path loss exponent (typical: 2-4)

**Procedure:** 1. Calculate SNR at each distance 2. Compare to modulation thresholds 3. Select highest modulation with SNR threshold

## Part V: Advanced Topics (10 points)

### OFDM (4 pts)

**Why OFDM?** - Solves frequency-selective fading problem - Divides wideband channel into many narrowband subchannels - Each subchannel experiences flat fading - Simpler equalization than single-carrier

**Applications:** - WiFi (802.11a/g/n/ac/ax) - LTE, 5G NR - DVB-T (digital TV)

**Key concept:** Convert frequency-selective fading  $\rightarrow$  flat fading per subcarrier

### Spread Spectrum (6 pts)

**DSSS (Direct Sequence Spread Spectrum):** - Data rate:  $R_b$  - Chip rate:  $R_c$  (much higher) - Processing gain:  $G_p = \frac{R_c}{R_b}$  (ratio) - In dB:  $G_p(dB) = 10 \log_{10}(R_c/R_b)$

**Advantages:** - Jamming resistance (processing gain) - Multiple access (CDMA) - Low probability of intercept

#### Example:

$R_b = 1 \text{ Mbps}$ ,  $R_c = 10 \text{ Mchips/s}$   
 $G_p = 10 \text{ Mchips/s} / 1 \text{ Mbps} = 10$   
 $G_p(dB) = 10 \cdot \log(10) = 10 \text{ dB}$

---

### Bonus: Satellite Link (10 points)

**Typical satellite parameters:** - High transmit power: 40 dBW (10 kW) - High antenna gains: 30-40 dBi - Long distance: 36,000 km (GEO) - Very high FSPL: ~200 dB! - Low noise temperature: 150 K (LNA)

$E_b/N_0$  and SNR relationship:

$$\frac{E_b}{N_0} = SNR \times \frac{B}{R_b}$$

Use this to find maximum bit rate:

$$R_b = \frac{SNR \times B}{E_b/N_0}$$

---

## Master Formula Sheet

### Probability & Noise

Gaussian PDF:

$$f(x) = (1/(\sqrt{2\pi})) \exp(-(x-\mu)^2/(2\sigma^2))$$

Q-function:

$$P(X > a) = Q(a/\sigma) \text{ for } X \sim N(\mu, \sigma^2)$$

$$Q(-x) = 1 - Q(x)$$

Thermal Noise:

$$P_n = kTB \quad (k = 1.38 \times 10^{-23} \text{ J/K})$$

$$P_n(\text{dBm}) = -174 + 10 \log(B_{\text{Hz}}) + NF(\text{dB}) \quad [\text{at } T=290\text{K}]$$

SNR:

$$SNR(\text{dB}) = P_s(\text{dBm}) - P_n(\text{dBm})$$

BER:

$$BER_{\text{BPSK}} = Q(\sqrt{2E_b/N_0})$$

## Link Budget

FSPL:

$$FSPL(\text{dB}) = 20 \log(d_{\text{km}}) + 20 \log(f_{\text{MHz}}) + 32.45$$

Link Budget:

$$P_r(\text{dBm}) = P_t(\text{dBm}) + G_t + G_r - FSPL - \text{Losses}$$

Shannon Capacity:

$$C = B \cdot \log(1 + SNR) \quad [SNR \text{ must be linear!}]$$

$E_b/N_0$  vs SNR:

$$E_b/N_0 = SNR \times (B/R_b)$$

## Digital Modulation (Review)

BPSK:  $k=1$ ,  $R_s=R_b$ ,  $B=2R_b$

QPSK:  $k=2$ ,  $R_s=R_b/2$ ,  $B=R_b$

M-ary:  $k=\log(M)$ ,  $R_s=R_b/k$

FSK:  $h = \Delta f \cdot T_b$

Orthogonal:  $\Delta f = 1/(2T_b)$

MSK:  $h = 0.5$

## Analog Modulation (Review)

AM:

$$s(t) = A_c[1 + \mu m(t)] \cos(2\pi f_c t)$$

$$P_{\text{total}} = (A_c^2/2R)(1 + \mu^2/2)$$

$$= P_c(1 + \mu^2/2)$$

FM:

$$\Delta f = k_f \cdot \max|m(t)|$$

$$= \Delta f / f_m$$

$$BFM = 2(\Delta f + f_m)$$

PM:

$$\Delta = k_p \cdot \max|m(t)|$$



$$\Delta f_{PM} = k_p \cdot A_m \cdot f_m \quad (\text{depends on } f_m!)$$


---

## Exam Strategy for 180 Minutes

### Time Allocation

**Part I (30 pts) - 25 minutes** - Problem 1: 7 min (Fourier - straightforward) - Problem 2: 10 min (AM power) - Problem 3: 8 min (FM vs PM)

**Part II (35 pts) - 30 minutes** - Problem 4: 10 min (BPSK vs QPSK) - Problem 5: 12 min (16-QAM, Gray coding) - Problem 6: 8 min (FSK design)

**Part III (40 pts) - 50 minutes** - Problem 7: 18 min (Gaussian & Q-function - 5 parts!) - Problem 8: 15 min (Thermal noise & SNR) - Problem 9: 17 min (BER analysis)

**Part IV (35 pts) - 45 minutes** - Problem 10: 25 min (WiFi link budget - longest problem!) - Problem 11: 12 min (Shannon capacity) - Problem 12: 8 min (Adaptive modulation)

**Part V (10 pts) - 10 minutes** - Problem 13: 10 min (OFDM & spread spectrum concepts)

**Bonus (10 pts) - 10 minutes** (if time!) - Satellite link analysis

**Review: 10 minutes**

### Priority Order (if running low on time)

1. **Do first:** Problems 7, 8, 9, 10 (new material, high points)
2. **Do second:** Problems 11, 12 (new material)
3. **Do third:** Problems 4, 5, 6 (review, but still good points)
4. **Do last:** Problems 1, 2, 3 (pure review)
5. **If time:** Problems 13 and Bonus

### Calculator Tips

#### Pre-calculate common values:

$$\log(2) = 0.301$$

$$\log(10) = 1$$

$$\ln(2) = 0.693$$

$$10^{-3} = 0.001$$

$$10^{-2} = 0.01$$

$$10^{-1} = 0.1$$

$$\text{For } \log(x): \log(x) = \log(x)/\log(2) \approx 3.32 \cdot \log(x)$$


---

## Common Mistakes to AVOID

### Part III (Probability/Noise)

Forgetting to standardize Gaussian:  $z = (x - \mu)/\sigma$  Using SNR in dB directly in Shannon formula (must convert to linear!) Forgetting to add noise figure to thermal noise Confusing  $Q(x)$  with  $1-Q(x)$  Using wrong Boltzmann constant or temperature

### Part IV (Link Budgets)

Wrong units in FSPL: using meters instead of km, or Hz instead of MHz Wrong signs in link budget (it's  $P_t + G_t + G_r$  - FSPL, not all positive!) Forgetting fade margin when calculating SNR Comparing to wrong modulation thresholds Not converting  $E_b/N_0$  from dB when using formulas

### General

Not showing units in answers Not checking if answer makes physical sense Spending too long on early problems (save time for Parts III & IV!) Forgetting to box/circle final answers

---

## Self-Assessment Checklist

### Can you:

**Probability & Noise:** - ☐ Write Gaussian PDF - ☐ Standardize and use Q-function - ☐ Calculate thermal noise power (kTB) - ☐ Convert between dBm and Watts - ☐ Add noise figure correctly - ☐ Calculate SNR in dB - ☐ Convert  $E_b/N_0$  between dB and linear - ☐ Calculate BER for BPSK - ☐ Find bit errors per second

**Link Budgets:** - ☐ Calculate FSPL with correct units - ☐ Perform complete link budget ( $P_r = P_t + \text{gains} - \text{losses}$ ) - ☐ Calculate noise power in dBm - ☐ Calculate SNR accounting for fade margin - ☐ Select modulation based on SNR - ☐ Calculate Shannon capacity (with SNR in linear!) - ☐ Apply adaptive modulation rules

**Digital Modulation:** - ☐ Compare BPSK vs QPSK (bandwidth, spectral efficiency) - ☐ Draw 16-QAM constellation - ☐ Apply Gray coding - ☐ Calculate EVM - ☐ Design orthogonal FSK system - ☐ Identify MSK

**Review Topics:** - ☐ Fourier transform of rect/sinc - ☐ AM power and efficiency - ☐ FM vs PM frequency deviation

---

## Final Study Plan (1 Week Before Exam)

### Days 7-6: New Material (Priority )

- **Study:** Homework 5 (all 10 problems) - Probability & Noise
- **Study:** Homework 6 (all 10 problems) - BER & Link Budgets
- **Practice:** Midterm 1 solutions (Problems 7-9)
- **Focus:** Link budget procedure, Q-function, thermal noise

## Days 5-4: Integration

- **Practice:** Complete a full link budget from scratch
- **Practice:** Gaussian/Q-function problems
- **Practice:** BER calculations
- **Review:** Shannon capacity problems
- **Make:** Your formula sheet!

## Days 3-2: Review Material

- **Quick review:** Homework 3 & 4 (digital modulation)
- **Quick review:** Homework 1 & 2 (Fourier, AM)
- **Practice:** Midterm 2 (all problems quickly)
- **Refine:** Formula sheet

## Day 1: Final Prep

- **Take:** Practice final (under time pressure!)
  - **Review:** Common mistakes list
  - **Check:** Formula sheet is complete
  - **Sleep:** Well!
- 

## What to Bring

Calculator (scientific, not graphing)   Formula sheet (both sides, handwritten recommended)  
Pencils/pens   Student ID

---

## During the Exam

1. **Read all problems first** (5 min)
  2. **Mark easy problems** with a star
  3. **Do high-value problems first** (Parts III & IV)
  4. **Show ALL work** - partial credit is generous!
  5. **Box final answers**
  6. **Check units** on every answer
  7. **Sanity check:** Does the answer make sense?
  8. **If stuck:** Move on, come back later
  9. **Last 10 min:** Review, check signs, check units
- 

## Final Thoughts

**The exam rewards:** - Systematic problem-solving (link budgets!) - Careful unit management (km vs m, dB vs linear) - Understanding relationships (Shannon, BER, SNR)

**Focus your final study on:** 1. Link budget procedure (practice 3-5 complete examples) 2. Q-function and BER calculations 3. SNR and noise calculations 4. Digital modulation comparisons

**You've got this! The comprehensive nature means you can show your understanding across the whole course. Focus on the new material (Parts III-IV) and you'll do great!**

**Good luck on your final exam!**