EE 451: Communications Systems

Homework 2 - Amplitude Modulation and ASK

Topics: DSB-SC, full carrier AM, envelope detection, ASK/OOK **Textbook Reference:** Haykin & Moher, Chapters 3.1-3.3, 7.1-7.2

Total Points: 100

Instructions

- Show all work for full credit
- Include units in all final answers
- Sketch waveforms where requested
- You may use Python/MATLAB for verification, but show analytical work

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Problem 1: DSB-SC Fundamentals (12 points)

A message signal $m(t) = 5\cos(2\pi \cdot 1000 \cdot t)$ V modulates a carrier $c(t) = 10\cos(2\pi \cdot 100 \times 10^3 \cdot t)$ V using DSB-SC modulation.

(a) Write the expression for the DSB-SC modulated signal $s(t)$. (3 points)

(b) Use trigonometric identities to express $s(t)$ as a sum of two sinusoids and identify the upper and lower sideband frequencies. (5 points)

(c) What is the bandwidth of the DSB-SC signal? (2 points)

(d) What is the power in each sideband if the system has $50\,\Omega$ resistance? (2 points)

Problem 2: Full Carrier AM (15 points)

An AM broadcast station transmits with a carrier frequency of 1.2 MHz. The carrier amplitude is $A_c = 100$ V and the message signal is $m(t) = 0.6\cos(2\pi \cdot 5000 \cdot t)$ V.

(a) Write the complete expression for the AM signal $s(t)$. (3 points)

(b) What is the modulation index μ? (3 points)

(c) Sketch one complete cycle of the modulating signal showing the envelope of the AM waveform. Label key amplitudes. (4 points)

(d) Calculate the total transmitted power and the percentage of power in the sidebands vs. the carrier (assume $50\,\Omega$). (5 points)

Problem 3: Modulation Index and Overmodulation (12 points)

An AM transmitter has carrier amplitude $A_c = 50$ V.

(a) What is the maximum message amplitude A_m that can be used without overmodulation? (3 points)

(b) If $m(t) = 40\cos(2\pi \cdot 3000 \cdot t)$ V, calculate the modulation index and determine if the signal is overmodulated. (3 points)

(c) Sketch the AM envelope for $\mu = 0.5$, $\mu = 1.0$, and $\mu = 1.5$. Explain what happens during overmodulation. (6 points)

Problem 4: Envelope Detection (10 points)

An envelope detector is used to demodulate an AM signal. The AM signal has: - Carrier frequency: $f_c = 900$ kHz - Message signal: $m(t) = \cos(2\pi \cdot 5000 \cdot t)$ - Modulation index: $\mu = 0.8$

(a) Sketch a simple envelope detector circuit (diode, RC components). (3 points)

(b) What is the requirement on the RC time constant to avoid distortion? Express in terms of the carrier frequency f_c and the maximum message frequency f_m. (4 points)

(c) What happens if the RC time constant is too large? Too small? (3 points)

Problem 5: AM Bandwidth and Spectral Efficiency (12 points)

An AM radio station broadcasts audio from 50 Hz to 5 kHz.

(a) What is the RF bandwidth required for DSB-SC? For full carrier AM? (4 points)

(b) If the station is assigned a carrier frequency of 1050 kHz, what frequencies appear in the transmitted spectrum? (4 points)

(c) Calculate the spectral efficiency (bits/s/Hz) if the audio is digitized at 8 bits per sample with Nyquist rate sampling, then transmitted using AM. (4 points)

Problem 6: On-Off Keying (ASK) (15 points)

A binary data stream "1 0 1 1 0" is transmitted using OOK with: - Bit rate: $R_b = 1000$ bits/s - Carrier frequency: $f_c = 10$ kHz - Carrier amplitude for "1": $A = 5$ V - Carrier amplitude for "0": 0 V

(a) Write the expression for the OOK signal during transmission of a "1" bit. (3 points)

(b) Sketch the transmitted waveform $s(t)$ for the entire data sequence. Show at least 3 carrier cycles per bit. (5 points)

(c) What is the null-to-null bandwidth of the OOK signal? (4 points)

(d) How does this compare to the bandwidth of the unmodulated baseband data stream? (3 points)

Problem 7: Binary ASK Spectral Analysis (12 points)

A binary ASK system transmits rectangular pulses at a bit rate of 2400 bits/s using a carrier frequency of 2.4 GHz (WiFi band, simplified).

(a) What is the bit duration T_b? (2 points)

(b) The baseband rectangular pulse has a sinc-shaped spectrum. What is the first null bandwidth of the baseband signal? (4 points)

(c) After modulation to $f_c = 2.4$ GHz, what frequencies define the main lobe of the ASK spectrum? (4 points)

(d) Compare the bandwidth efficiency of ASK to DSB-SC AM for the same baseband signal. (2 points)

Problem 8: AM vs. ASK Comparison (10 points)

(a) Explain the fundamental difference between analog AM and digital ASK. What do they have in common? (4 points)

(b) Both AM and ASK can use envelope detection. Why is this possible? (3 points)

(c) Why might ASK be preferred over AM in a digital communication system? (3 points)

Problem 9: Superheterodyne Receiver (12 points - Challenge)

An AM broadcast receiver uses the superheterodyne architecture with an intermediate frequency (IF) of 455 kHz.

(a) If you want to receive a station at 1200 kHz, to what frequency should the local oscillator (LO) be tuned? (3 points)

(b) What is the image frequency for this receiver? (4 points)

(c) If an interfering signal exists at the image frequency, explain what problem this causes. How do receivers mitigate this? (5 points)

Problem 10: Real-World AM System Design (10 points - Challenge)

You are designing an AM transmitter for a campus radio station: - Carrier frequency: f_c = 88.5 MHz (FM band, but using AM for this problem) - Audio bandwidth: 20 Hz to 15 kHz - Maximum transmit power: 100 W - Carrier amplitude: $A_c = 30$ V - Load resistance: $50\,\Omega$

(a) What is the maximum modulation index you can use to transmit at 100 W total power? (5 points)

(b) What is the RF bandwidth of your transmission? (2 points)

(c) If you want to maximize power efficiency (power in sidebands), would you choose DSB-SC or full carrier AM? Justify your answer. (3 points)

Bonus Problem 1: SSB (5 points extra credit)

Single Sideband (SSB) modulation transmits only one sideband, saving bandwidth and power.

(a) For a message signal with bandwidth 3 kHz, what is the bandwidth of USB (Upper Sideband) SSB? (2 points)

(b) If DSB-SC requires 50 W to transmit both sidebands, how much power is required for SSB (transmitting one sideband)? (3 points)

Bonus Problem 2: Mathematical Derivation (5 points extra credit)

Starting from $s(t) = A_c[1 + \mu \cdot m_n(t)]\cos(2\pi f_c t)$ where $m_n(t)$ is normalized to $|m_n(t)| \leq 1$:

Show that the carrier power is $P_c = \frac{A_c^2}{2}$ and the total sideband power is $P_{sb} = \frac{\mu^2 A_c^2}{4} \cdot P_m$, where P_m is the normalized message power.

For a single tone $m_n(t) = \cos(2\pi f_m t)$, verify that the efficiency (sideband power / total power) is $\frac{\mu^2}{2+\mu^2}$.

Submission Instructions: - Submit a single PDF via Brightspace - Include all sketches and waveform drawings - Label all graphs with frequencies, amplitudes, and time scales - Show all intermediate steps for calculations

Academic Integrity: - Individual work required - Cite any external references used - Discussion of concepts is allowed; copying solutions is not