

# EE 451: Communications Systems

## Homework 6 - BER, Link Budgets, and System Analysis

**Topics:** Bit error rate, matched filtering, link budgets, WiFi/cellular systems **Textbook Reference:** Haykin & Moher, Chapters 10, 11.4-11.7

**Total Points:** 100

---

### Instructions

- Show all work for full credit
  - Include units in all final answers
  - For link budgets, show the complete calculation chain
  - State all assumptions clearly
- 

### Problem 1: BER for BPSK (12 points)

A BPSK system operates with  $E_b/N_0 = 10$  dB.

- (a) Convert  $E_b/N_0$  to a linear ratio. (2 points)
  - (b) The BER for BPSK is given by:  $\text{BER} = Q(\sqrt{2E_b/N_0})$ . Calculate the BER. (4 points)
  - (c) If the bit rate is 1 Mbps, approximately how many bit errors occur per second? (3 points)
  - (d) What  $E_b/N_0$  is required to achieve  $\text{BER} = 10^{-6}$ ? (3 points)
- 

### Problem 2: BER Comparison (15 points)

Compare the BER performance of BPSK, QPSK, and 8-PSK.

(a) For BPSK:  $\text{BER} = Q(\sqrt{2E_b/N_0})$  For QPSK:  $\text{BER} \approx Q(\sqrt{2E_b/N_0})$  (approximately same as BPSK) For 8-PSK:  $\text{BER} \approx \frac{2}{3} \cdot Q(\sqrt{6E_b/N_0} \cdot \sin(\pi/8))$

Calculate the BER for each modulation at  $E_b/N_0 = 12$  dB. (6 points)

- (b) Which modulation has the best BER performance? Why? (3 points)
  - (c) Which modulation has the highest spectral efficiency? (3 points)
  - (d) If you need to transmit 10 Mbps in 5 MHz bandwidth, which modulation should you choose? Consider both BER and spectral efficiency. (3 points)
- 

### Problem 3: Matched Filtering (12 points)

- (a) Explain what a matched filter is and why it's optimal for detecting signals in AWGN. (4 points)

(b) For a rectangular pulse of duration  $T$  and amplitude  $A$ , what is the output SNR of a matched filter? Express in terms of signal energy  $E_s$  and noise power spectral density  $N_0$ . (4 points)

(c) A BPSK system uses matched filtering. If the bit energy is  $E_b = 10^{-13}$  J and  $N_0 = 10^{-20}$  W/Hz, calculate the output SNR. (4 points)

---

#### Problem 4: QAM Bit Error Rate (12 points)

For M-QAM, the symbol error rate (SER) is approximately:  $SER \approx 4 \cdot \left(1 - \frac{1}{\sqrt{M}}\right) \cdot Q\left(\sqrt{\frac{3E_s}{N_0(M-1)}}\right)$

For 16-QAM at  $E_s/N_0 = 18$  dB:

(a) Convert  $E_s/N_0$  to linear ratio. (2 points)

(b) Calculate the symbol error rate (SER). (4 points)

(c) For Gray coding, approximate  $BER \approx SER/\log_2(M)$ . Calculate the BER. (3 points)

(d) If the symbol rate is 10 Msymbols/s, what is the bit rate and how many bit errors per second occur? (3 points)

---

#### Problem 5: Complete Link Budget - WiFi (18 points)

A WiFi 6 system operates at 5 GHz with the following parameters: - Transmit power:  $P_t = 23$  dBm (200 mW, FCC limit for 5 GHz) - Transmit antenna gain:  $G_t = 3$  dBi - Receive antenna gain:  $G_r = 0$  dBi (laptop internal antenna) - Frequency: 5.0 GHz - Distance: 30 m - System bandwidth: 80 MHz - Noise figure:  $NF = 6$  dB - Temperature:  $T = 290$  K - Implementation loss: 3 dB - Fade margin: 10 dB

(a) Calculate free-space path loss using:  $FSPL(\text{dB}) = 20 \cdot \log_{10}(4\pi d/\lambda)$ , where  $\lambda = c/f$ . (4 points)

(b) Calculate received signal power  $P_r$  including all gains and losses. (4 points)

(c) Calculate noise power  $P_n$  in the 80 MHz bandwidth. (3 points)

(d) Calculate the SNR. (2 points)

(e) Based on the SNR, what is the highest modulation that can be used? Use these thresholds: - BPSK/QPSK: 5 dB - 16-QAM: 12 dB - 64-QAM: 20 dB - 256-QAM: 28 dB - 1024-QAM: 35 dB (3 points)

(f) Calculate the maximum data rate (assuming ideal spectral efficiency and no overhead). (2 points)

---

#### Problem 6: Link Budget - Cellular 5G (15 points)

A 5G base station communicates with a smartphone at 3.5 GHz: - Base station transmit power:  $P_t = 43$  dBm (20 W) - Base station antenna gain:  $G_t = 15$  dBi (directional) - Smartphone antenna gain:  $G_r = -2$  dBi - Distance: 500 m - Path loss exponent:  $n = 3.5$  (urban environment) - Shadow fading margin: 8 dB - Bandwidth: 100 MHz - Noise figure:  $NF = 7$  dB

- (a) Calculate path loss using:  $PL(\text{dB}) = FSPL(d_0) + 10n \cdot \log_{10}(d/d_0)$ , where  $d_0 = 1$  m. (5 points)
  - (b) Calculate received power. (3 points)
  - (c) Calculate noise power. (2 points)
  - (d) Calculate SNR. (2 points)
  - (e) Using Shannon's theorem  $C = B \cdot \log_2(1 + \text{SNR})$ , calculate the theoretical channel capacity. (3 points)
- 

### Problem 7: WiFi vs. Cellular Comparison (10 points)

- (a) Compare the link budgets from Problems 5 and 6. Which system can communicate over longer distances? Why? (4 points)
  - (b) Which system provides higher data rates? Explain the trade-offs. (3 points)
  - (c) Explain why cellular systems use higher transmit power and higher antenna gain compared to WiFi. (3 points)
- 

### Problem 8: Adaptive Modulation System (12 points)

A wireless link uses adaptive modulation with the following SNR thresholds: - QPSK: 8 dB, data rate = 20 Mbps - 16-QAM: 15 dB, data rate = 40 Mbps - 64-QAM: 22 dB, data rate = 60 Mbps - 256-QAM: 30 dB, data rate = 80 Mbps

The received SNR varies with distance as:  $\text{SNR}(\text{dB}) = 45 - 30 \cdot \log_{10}(d/10)$ , where  $d$  is in meters.

- (a) Calculate the SNR at distances of 10 m, 20 m, 50 m, and 100 m. (4 points)
  - (b) For each distance, determine which modulation scheme is used. (4 points)
  - (c) Calculate the average data rate if the user spends equal time at each of these four distances. (2 points)
  - (d) What is the maximum distance at which communication is possible ( $\text{SNR} \geq 8$  dB)? (2 points)
- 

### Problem 9: Multipath and Fading (10 points)

- (a) Explain how multipath propagation affects wireless communication systems. (3 points)
  - (b) What is frequency-selective fading and why does it occur? (3 points)
  - (c) How do OFDM-based systems (WiFi, LTE, 5G) mitigate frequency-selective fading? (4 points)
-

### Problem 10: Real-World System Design (14 points - Challenge)

You are designing a long-range WiFi link for a rural area: - Frequency: 2.4 GHz (better propagation than 5 GHz) - Required distance: 1 km - Required data rate: 50 Mbps - Available transmit power: 30 dBm (1 W) - Available antennas:  $G_t = G_r = 10$  dBi (directional) - Bandwidth: 20 MHz - Temperature:  $T = 290$  K - Noise figure:  $NF = 5$  dB - Required SNR for 64-QAM: 22 dB - Fade margin: 15 dB (for reliability)

- (a) Calculate the free-space path loss at 1 km and 2.4 GHz. (3 points)
  - (b) Calculate the link budget and determine the received SNR. (5 points)
  - (c) Can the system achieve the required SNR of 22 dB with 15 dB fade margin? (2 points)
  - (d) If not, suggest three modifications to improve the link budget and calculate the improvement from each. (4 points)
- 

### Bonus Problem 1: Friis Transmission Equation (5 points extra credit)

The Friis transmission equation relates received power to transmitted power:  $P_r = P_t \cdot G_t \cdot G_r \cdot \left(\frac{\lambda}{4\pi d}\right)^2$

- (a) Derive the free-space path loss formula  $FSPL(dB) = 20 \cdot \log_{10}(d) + 20 \cdot \log_{10}(f) + 32.45$  (with  $d$  in km and  $f$  in MHz) from the Friis equation. (3 points)
  - (b) Verify your derivation by calculating FSPL at  $d = 1$  km and  $f = 1$  GHz. (2 points)
- 

### Bonus Problem 2: $E_b/N_0$ vs. SNR Relationship (5 points extra credit)

For a digital communication system: - Bit rate:  $R_b$  - Bandwidth:  $B$  -  $SNR = \text{signal power} / \text{noise power}$  -  $E_b/N_0 = \text{bit energy} / \text{noise power spectral density}$

- (a) Show that  $E_b/N_0 = SNR \cdot (B/R_b)$ . (3 points)
  - (b) For QPSK with  $R_b = 10$  Mbps in  $B = 5$  MHz, if  $SNR = 12$  dB, calculate  $E_b/N_0$ . (2 points)
- 

**Submission Instructions:** - Submit a single PDF via Brightspace - Show complete link budget calculations - Include all unit conversions (dB  $\leftrightarrow$  linear) - Clearly label each step

**Academic Integrity:** - Individual work required - Show all work for partial credit - Cite any references to standards or specifications

**Useful Formulas:** -  $FSPL(dB) = 20 \cdot \log_{10}(d) + 20 \cdot \log_{10}(f) + 32.45$  ( $d$  in km,  $f$  in MHz) -  $FSPL(dB) = 20 \cdot \log_{10}(4\pi d/\lambda)$  ( $d$  and  $\lambda$  in same units) -  $P_n = k \cdot T \cdot B$  (thermal noise) -  $k = 1.38 \times 10^{-23}$  J/K -  $c = 3 \times 10^8$  m/s - Shannon:  $C = B \cdot \log_2(1 + SNR)$