EE 451: Communications Systems

Midterm Exam 2 - Study Guide

Exam Date: Week 11 (April 15, 2026) **Duration:** 90 minutes **Coverage:** Chapters 4-7 (FM, PM, FSK, PSK, M-ary Modulation, QAM) **Format:** Closed book, one 8.5” × 11” formula sheet (both sides) allowed

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Overview

Midterm 2 focuses on angle modulation (FM/PM) and digital modulation schemes. You should be comfortable with: - FM and PM fundamentals and relationships - Carson’s rule for FM bandwidth - Binary digital modulation (FSK, BPSK) - M-ary modulation (QPSK, QAM) - Gray coding and constellation diagrams - Spectral efficiency concepts

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Topic 1: Frequency Modulation (FM) Fundamentals

Key Concepts

1. FM Signal Characteristics - Frequency varies with message signal - Amplitude remains constant (noise immunity!) - Frequency deviation: $\Delta f = k_f \max|m(t)|$ - Modulation index: $\beta = \Delta f / f_m$

2. Narrowband vs. Wideband FM - **NBFM:** $\beta \ll 1$ (typically $\beta < 0.3$) - **WBFM:** $\beta \gg 1$ (broadcast FM: $\beta = 5$) - Different bandwidth requirements!

3. FM vs. AM - AM: Constant frequency, varying amplitude - FM: Constant amplitude, varying frequency - FM: Better noise performance (but more bandwidth)

Essential Formulas

```
FM Signal:
  s(t) = A cos[2 f t + 2 kf m(t)dt]

Instantaneous Frequency:
  fi(t) = fc + kf·m(t)
  where kf = frequency sensitivity (Hz/V)

Frequency Deviation:
  Δf = kf·max|m(t)|
  For m(t) = Am·cos(2 fmt):
    Δf = kf·Am

Modulation Index:
    = Δf/fm

Carson's Rule (FM Bandwidth):
```

```
  BFM = 2(Δf + fm) = 2fm(1 +  )

NBFM Criterion:
    << 1 (typically  < 0.3)
  BFM  2fm (like AM!)

WBFM:
    >> 1
  BFM  2Δf (dominated by deviation)
```

Common Mistakes to Avoid

Confusing Δf (deviation) with f_m (message frequency) Forgetting that Δf depends on k_f AND message amplitude Using wrong Carson's rule form (two equivalent forms!) Mixing up NBFM and WBFM criteria

Practice Problems

From **Homework 3:** - Problem 1: FM fundamentals - Problem 2: Carson's rule application - Problem 3: NBFM vs. WBFM classification

Strategy: 1. Identify k_f, A_m, f_m from problem 2. Calculate $\Delta f = k_f \times A_m$ 3. Calculate $\beta = \Delta f/f_m$ 4. Check if NBFM ($\beta < 0.3$) or WBFM ($\beta > 1$) 5. Apply Carson's rule: $B = 2(\Delta f + f_m)$

Topic 2: Phase Modulation (PM)

Key Concepts

1. PM Signal Characteristics - Phase varies directly with message - $\theta_i(t) = 2\pi f_c t + k_p m(t)$ - Phase sensitivity: k_p (rad/V)

2. FM vs. PM Relationship - **Key insight:** PM = FM with derivative - FM can be generated from PM by integrating message - PM can be generated from FM by differentiating message

3. Frequency Deviation in PM - PM has frequency deviation that **increases with** f_m! - FM has frequency deviation **independent of** f_m - This is the KEY difference!

Essential Formulas

```
PM Signal:
  s(t) = A cos[2 f t + kp·m(t)]
  where kp = phase sensitivity (rad/V)

Instantaneous Phase:
   i(t) = 2 f t + kp·m(t)

Phase Deviation:
  Δ  = kp·max|m(t)|
```

```
Instantaneous Frequency:
  fi(t) = (1/2 )·d i/dt = fc + (kp/2 )·dm/dt

For m(t) = Am·cos(2 fmt):
  Maximum Frequency Deviation:
    Δf_PM = kp·Am·fm
    (Note: proportional to fm!)

Comparison:
  FM: Δf = kf·Am (independent of fm)
  PM: Δf = kp·Am·fm (proportional to fm)
```

Common Mistakes to Avoid

Thinking PM and FM have the same Δf behavior   Forgetting to take derivative when finding $f_i(t)$ from PM   Confusing $\Delta\phi$ (phase deviation) with Δf (frequency deviation)

Practice Problems

From **Homework 3:** - Problem 4: PM analysis - Problem 5: FM vs. PM relationship

Strategy: 1. For PM: Write $\theta_i(t) = 2\pi f_c t + k_p m(t)$ 2. Find $\Delta\phi = k_p \times \max|m(t)|$ 3. Find $f_i(t) = \frac{1}{2\pi}\frac{d\theta_i}{dt}$ 4. For sinusoidal message: $\Delta f = k_p \times A_m \times f_m$

Topic 3: Binary FSK (Frequency Shift Keying)

Key Concepts

1. FSK Basics - Bit "0" → Frequency f_0 - Bit "1" → Frequency f_1 - Frequency separation: $\Delta f = |f_1 - f_0|$

2. Modulation Index for FSK - $h = \Delta f \times T_b$ - Critical parameter for FSK design - Determines orthogonality and spectral efficiency

3. Orthogonality for Coherent Detection - Minimum separation: $\Delta f_{min} = \frac{1}{2T_b}$ - This gives $h = 0.5$ (MSK!) - Larger separation OK, but wastes bandwidth

4. Coherent vs. Non-Coherent Detection - **Coherent:** Requires phase lock, better performance - **Non-coherent:** Simpler, no phase tracking needed - FSK can use either (unlike PSK which requires coherent)

Essential Formulas

```
FSK Signal Expressions:
  s (t) = A·cos(2 f t)    (bit "0")
  s (t) = A·cos(2 f t)    (bit "1")

Frequency Separation:
  Δf = |f  - f |
```

```
Bit Duration:
  Tb = 1/Rb  (where Rb = bit rate)

Modulation Index:
  h = Δf·Tb

Orthogonality Condition (Coherent FSK):
  Δf_min = 1/(2Tb) = Rb/2
  Gives h = 0.5 (Minimum Shift Keying!)

Bandwidth (Carson's Rule):
  B_FSK   2(Δf + Rb)
```

Common Mistakes to Avoid

Forgetting to calculate $T_b = 1/R_b$ first   Confusing Δf (separation) with f_0 or f_1 (individual frequencies)   Using wrong orthogonality condition ($1/T_b$ instead of $1/(2T_b)$)

Practice Problems

From **Homework 3:** - Problem 6: Binary FSK fundamentals - Problem 7: MSK analysis - Problem 9: Coherent vs. non-coherent detection

Strategy: 1. Find $T_b = 1/R_b$ 2. Calculate $\Delta f = |f_1 - f_0|$ 3. Calculate $h = \Delta f \times T_b$ 4. Check orthogonality: Is $\Delta f \geq 1/(2T_b)$? 5. If $h = 0.5$, it's MSK!

Topic 4: Minimum Shift Keying (MSK)

Key Concepts

1. MSK Definition - Special case of CPFSK (Continuous Phase FSK) - Modulation index $h = 0.5$ exactly - **Minimum** frequency separation for orthogonal FSK

2. Advantages of MSK - Continuous phase → no discontinuities - Better spectral efficiency than standard FSK - Faster sidelobe decay (less interference) - Constant envelope (power amplifier friendly)

3. Applications - GSM cellular (GMSK variant) - Satellite communications - Anywhere spectral efficiency matters

Essential Formulas

```
MSK Characteristics:
  h = 0.5 (defining property!)
  Δf = Rb/2 = 1/(2Tb)

For MSK with bit rate Rb:
  Δf = Rb/2
```

```
If center frequency is fc:
  f  = fc - Rb/4
  f  = fc + Rb/4

MSK Advantages:
  - Continuous phase
  - Minimum bandwidth for orthogonal FSK
  - Constant envelope
```

Practice Problems

From **Homework 3:** - Problem 7: Complete MSK analysis

Strategy: 1. If $h = 0.5$, it's MSK 2. From $h = 0.5$ and bit rate, find $\Delta f = 0.5R_b$ 3. Frequencies: $f_0 = f_c - \Delta f/2$, $f_1 = f_c + \Delta f/2$

Topic 5: Binary Phase Shift Keying (BPSK)

Key Concepts

1. BPSK Basics - Bit "0" → Phase 0° (or +A) - Bit "1" → Phase 180° (or -A) - Information in PHASE, not amplitude - Constant envelope (like FM/MSK)

2. Constellation Diagram - Two points on I (real) axis - 180° apart - Maximum separation = best noise immunity

3. Detection - **Must use coherent detection!** - Envelope detector won't work (constant envelope) - Requires carrier phase synchronization

4. Bandwidth - Null-to-null: $B = 2R_b$ - Similar to ASK (same symbol rate)

Essential Formulas

```
BPSK Signals:
  s (t) = A·cos(2 f t)         (bit "0", 0°)
  s (t) = -A·cos(2 f t)        (bit "1", 180°)
  Or: s (t) = A·cos(2 f t +  )

Constellation:
  Bit "0": I = +A, Q = 0
  Bit "1": I = -A, Q = 0
  Phase difference: 180°

Bandwidth:
  B_null-null = 2/Tb = 2Rb
  B_3dB   Rb

Cannot use envelope detection!
  (Envelope is constant = A for both symbols)
```

Common Mistakes to Avoid

Saying BPSK can use envelope detection (NO!) Confusing BPSK bandwidth with carrier frequency Drawing constellation in wrong locations

Practice Problems

From **Homework 3:** - Problem 8: BPSK fundamentals

Strategy: 1. Write two signal expressions (180° apart) 2. Draw constellation: two points on I-axis 3. Calculate bandwidth: $B = 2R_b$ 4. Explain why coherent detection is required

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Topic 6: QPSK (Quadrature Phase Shift Keying)

Key Concepts

1. QPSK Basics - 4 phase states (45°, 135°, 225°, 315°) - Encodes **2 bits per symbol** - Twice as efficient as BPSK! - Symbol rate = Bit rate / 2

2. I/Q Representation - QPSK uses both I (In-phase) and Q (Quadrature) channels - Each symbol: combination of I and Q amplitudes - 4 constellation points equally spaced

3. BER Performance - QPSK has same BER as BPSK for same E_b/N_0 - But uses half the bandwidth! - Best of both worlds for spectral efficiency

Essential Formulas

```
QPSK Parameters:
  M = 4 symbols
  k = 2 bits/symbol
  Symbol rate: Rs = Rb/2

Four Signals (phases 45°, 135°, 225°, 315°):
  s (t) = A·cos(2 f t +  /4)
  s (t) = A·cos(2 f t + 3 /4)
  s (t) = A·cos(2 f t + 5 /4)
  s (t) = A·cos(2 f t + 7 /4)

Constellation Points:
  All at distance A from origin
  90° apart
  On circle of radius A

Bandwidth:
  B_QPSK = 2Rs = 2(Rb/2) = Rb
  Half the bandwidth of BPSK for same bit rate!

Spectral Efficiency:
  BPSK: 0.5 bits/s/Hz (in null-null BW)
  QPSK: 1.0 bits/s/Hz
```

Common Mistakes to Avoid

Forgetting symbol rate = bit rate / 2 Drawing constellation points at wrong phases Not using Gray coding (see next topic!) Thinking QPSK has worse BER than BPSK

Practice Problems

From **Homework 4:** - Problem 1: QPSK fundamentals - Problem 2: Gray coding for QPSK

Strategy: 1. Calculate symbol rate: $R_s = R_b/2$ 2. Write 4 signal expressions (45°, 135°, 225°, 315°) 3. Draw constellation with 4 points 4. Bandwidth: $B = 2R_s = R_b$

Topic 7: Gray Coding

Key Concepts

1. Why Gray Coding? - Adjacent symbols differ by only **1 bit** - Most errors go to adjacent symbols (AWGN) - Symbol error → typically 1 bit error (not 2!) - Reduces bit error rate (BER)

2. Gray Code Properties - Used in ALL modern systems (WiFi, LTE, 5G) - Essential for M-ary modulation - Simple mapping rule: adjacent symbols differ by 1 bit

3. Example for QPSK

```
Natural Binary:  00, 01, 10, 11 (can differ by 2 bits!)
Gray Coded:      00, 01, 11, 10 (adjacent differ by 1 bit)
```

QPSK Gray Coding Example

```
Constellation (typical Gray mapping):
     Q
     |
 01  |  00    (top half)

     |         I

 11  |  10    (bottom half)
     |
```

Key: Moving horizontally or vertically changes only 1 bit!

Common Mistakes to Avoid

Using natural binary instead of Gray code Assigning codes randomly without checking adjacency

Practice Problems

From **Homework 4:** - Problem 2: Gray coding for QPSK - Problem 3: 8-PSK Gray coding

Strategy: 1. Draw constellation 2. Assign codes so adjacent symbols differ by 1 bit 3. Verify: moving one step (any direction) changes 1 bit

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Topic 8: M-ary Modulation and QAM

Key Concepts

1. M-ary PSK - M symbols, $k = \log_2(M)$ bits/symbol - Better spectral efficiency as M increases - But requires higher SNR for same BER

2. QAM (Quadrature Amplitude Modulation) - Combines amplitude AND phase modulation - Uses I and Q channels independently - Rectangular constellation (e.g., 16-QAM: 4×4 grid) - Even better spectral efficiency than M-PSK

3. Spectral Efficiency - BPSK: 1 bit/symbol - QPSK: 2 bits/symbol - 16-QAM: 4 bits/symbol - 64-QAM: 6 bits/symbol - Higher M = more bits/symbol but needs more SNR

Essential Formulas

```
M-ary Modulation:
  M = number of symbols
  k = log (M) = bits per symbol
  Rs = Rb/k = symbol rate

Examples:
  BPSK: M=2, k=1
  QPSK: M=4, k=2
  8-PSK: M=8, k=3
  16-QAM: M=16, k=4
  64-QAM: M=64, k=6

Spectral Efficiency:
    = Rb/B = k·Rs/B bits/s/Hz
  Higher M → higher

Trade-off:
  Higher M: Better spectral efficiency
  Higher M: Needs higher SNR (worse BER)
```

16-QAM Constellation

```
4×4 rectangular grid:
Amplitude levels: {±1, ±3} for I and Q

     Q
   3
   1
          I
  -1
  -3
   -3-1 1 3
```

Common Mistakes to Avoid

Confusing M (symbols) with k (bits/symbol) Forgetting symbol rate: $R_s = R_b/k$ Not using Gray coding for QAM Thinking more bits/symbol is always better (ignores SNR!)

Practice Problems

From **Homework 4:** - Problem 3: M-ary PSK - Problem 4: QAM constellation design - Problem 5: Spectral efficiency comparison

Strategy: 1. Find k: $k = \log_2(M)$ 2. Find symbol rate: $R_s = R_b/k$ 3. For QAM: Draw rectangular constellation 4. Assign Gray codes to minimize bit errors

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Topic 9: EVM (Error Vector Magnitude)

Key Concepts

1. EVM Definition - Measures constellation quality - EVM = error distance / reference distance - Expressed as percentage

2. Why EVM Matters - WiFi/LTE specs have strict EVM limits - Higher-order modulation needs lower EVM - Indicates transmitter/receiver impairments

3. EVM Requirements - 256-QAM: EVM < 3% (very tight!) - 64-QAM: EVM < 8% - 16-QAM: EVM < 12% - QPSK: EVM < 17%

Essential Formulas

```
EVM Calculation:
  Error vector = Received - Ideal
  |Error| = √[(ΔI)² + (ΔQ)²]

EVM (%) = (|Error| / |Reference|) × 100%

Example:
  Ideal: (I, Q) = (1.0, 0.0)
  Received: (I, Q) = (0.92, 0.15)
  Error = √[(1-0.92)² + (0-0.15)²]
        = √[0.08² + 0.15²]
        = √[0.0064 + 0.0225]
        = √0.0289   0.17
  EVM = (0.17/1.0) × 100% = 17%
```

Practice Problems

From **Homework 4:** - Problem 6: EVM calculation - Bonus 2: EVM limits in standards

Strategy: 1. Plot ideal and received points 2. Calculate error vector 3. Find magnitude of error 4. Divide by reference (usually distance to ideal point) 5. Convert to percentage

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Formula Sheet Recommendations

FM/PM Essentials

```
FM:
  fi(t) = fc + kf·m(t)
  Δf = kf·max|m(t)|
    = Δf/fm
  BFM = 2(Δf + fm) = 2fm(1 +  )
  NBFM:   << 1
  WBFM:   >> 1

PM:
   i(t) = 2 fct + kp·m(t)
  Δ  = kp·max|m(t)|
  fi(t) = fc + (kp/2 )·dm/dt
  For sinusoidal m(t):
    Δf_PM = kp·Am·fm
```

Digital Modulation

```
FSK:
  h = Δf·Tb
  Orthogonal: Δf = 1/(2Tb)
  MSK: h = 0.5

BPSK:
  2 symbols, k=1 bit/symbol
  B = 2Rb
  Coherent detection required

QPSK:
  M=4, k=2 bits/symbol
  Rs = Rb/2
  B = Rb (half of BPSK!)

M-ary:
  k = log (M)
  Rs = Rb/k
```

Spectral Efficiency

```
Bit rate: Rb
Symbol rate: Rs = Rb/k
Spectral efficiency:   = Rb/B bits/s/Hz

Examples:
  BPSK:    0.5 bits/s/Hz
  QPSK:    1.0 bits/s/Hz
```

```
16-QAM:    2.0 bits/s/Hz
64-QAM:    3.0 bits/s/Hz
```

Exam Strategy

Time Management (90 minutes)

- **5 min:** Read entire exam
- **15 min:** Problem 1 (FM fundamentals with Carson's rule)
- **12 min:** Problem 2 (PM analysis)
- **10 min:** Problem 3 (FM vs PM comparison)
- **15 min:** Problem 4 (FSK system design)
- **12 min:** Problem 5 (BPSK fundamentals)
- **15 min:** Problem 6 (QPSK and Gray coding)
- **6 min:** Problem 7 (MSK bonus if time)
- **5 min:** Review

Problem-Solving Approach

For FM Problems: 1. Find $\Delta f = k_f \times A_m$ 2. Find $\beta = \Delta f / f_m$ 3. Classify: NBFM ($\beta < 0.3$) or WBFM ($\beta > 1$)? 4. Apply Carson's rule: $B = 2(\Delta f + f_m)$

For PM Problems: 1. Find $\Delta \phi = k_p \times A_m$ 2. Find $f_i(t)$ by taking derivative 3. For sinusoidal: $\Delta f = k_p \times A_m \times f_m$ 4. Compare to FM (key difference: Δf depends on f_m in PM!)

For FSK Problems: 1. Calculate $T_b = 1/R_b$ 2. Calculate $\Delta f = |f_1 - f_0|$ 3. Calculate $h = \Delta f \times T_b$ 4. Check orthogonality: $\Delta f \geq 1/(2T_b)$? 5. If $h = 0.5$: MSK!

For QPSK Problems: 1. Symbol rate: $R_s = R_b/2$ 2. Write 4 signal expressions 3. Draw constellation (4 points equally spaced) 4. Apply Gray coding 5. Bandwidth: $B = 2R_s = R_b$

Common Mistakes

FM: Confusing Δf with f_m, forgetting Carson's rule has TWO forms **PM:** Thinking Δf is independent of f_m (it's not!) **FSK:** Using $1/T_b$ for orthogonality instead of $1/(2T_b)$ **BPSK:** Saying envelope detection works (it doesn't!) **QPSK:** Forgetting symbol rate = bit rate / 2 **Gray coding:** Not checking that adjacent symbols differ by 1 bit

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Self-Assessment Checklist

Before the exam, can you:

FM/PM: - [ ] Calculate frequency deviation Δf for FM - [ ] Calculate modulation index β - [ ] Apply Carson's rule correctly - [ ] Classify NBFM vs. WBFM - [ ] Find instantaneous frequency for PM - [ ] Explain why PM has Δf proportional to f_m

FSK: - [ ] Calculate bit duration from bit rate - [ ] Find frequency separation Δf - [ ] Calculate modulation index h - [ ] Check orthogonality condition - [ ] Identify MSK ($h = 0.5$)

BPSK/QPSK: - [ ] Write signal expressions - [ ] Draw constellation diagrams - [ ] Calculate bandwidth - [ ] Explain why BPSK needs coherent detection - [ ] Find symbol rate for QPSK - [ ] Apply Gray coding

M-ary/QAM: - [ ] Calculate bits per symbol ($k = \log_2 M$) - [ ] Find symbol rate from bit rate - [ ] Draw QAM constellation - [ ] Compare spectral efficiencies - [ ] Calculate EVM

Practice Resources

Homework 3 (FM/PM/FSK/BPSK): All problems 1-10 **Homework 4 (M-ary/QAM/EVM):** All problems 1-10

Focus especially on: - HW3: Problems 1, 2, 5, 6, 7, 8 - HW4: Problems 1, 2, 4, 6

Good luck!