

# EE 451: Communications Systems

## Midterm Exam 1 - Study Guide

**Exam Date:** Week 5 (Thursday, February 26, 2026) **Duration:** 75 minutes **Coverage:** Chapters 2-3, 7.1-7.2 (Fourier Analysis & Amplitude Modulation) **Format:** Closed book, one 8.5" × 11" formula sheet (both sides) allowed

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### Overview

Midterm 1 focuses on foundational concepts in signal analysis and amplitude modulation. You should be comfortable with: - Fourier transform analysis and properties - Time-frequency duality and bandwidth concepts - AM modulation and demodulation - Power calculations in communication systems - Binary ASK as digital extension of AM

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### Topic 1: Fourier Transform Fundamentals

#### Key Concepts

1. **Rectangular Pulse and Sinc Function** - Rectangular pulse in time → Sinc function in frequency - Time-frequency duality: Narrow pulse → Wide spectrum - First zero crossing determines bandwidth
2. **Bandwidth Definition** - Null-to-null bandwidth: Distance between first zeros - 3-dB bandwidth: Half-power points - Relationship:  $B_{3dB} \approx 0.88 \times B_{null}$
3. **Signal Energy** - Time domain:  $E = \int_{-\infty}^{\infty} |x(t)|^2 dt$  - Use Parseval's theorem to verify in frequency domain

#### Essential Formulas

Fourier Transform Pair:

$$\text{rect}(t/T) \quad T \cdot \text{sinc}(fT)$$

Sinc Function:

$$\text{sinc}(x) = \sin(x)/(x)$$

First zero at  $x = \pm 1$

Rectangular Pulse:

$$x(t) = A \cdot \text{rect}(t/T)$$

$$X(f) = AT \cdot \text{sinc}(fT)$$

First zero at  $f = \pm 1/T$

Energy:

$$E = |x(t)|^2 dt = A^2 T \text{ (for rectangular pulse)}$$

## Practice Problems

From **Homework 1:** - Problem 1: Rectangular pulse Fourier transform - Problem 2: Sinc function properties - Problem 3: Energy calculations

**Strategy:** - Draw time and frequency domain sketches - Identify key parameters: amplitude, duration, bandwidth - Check units (time frequency reciprocity)

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## Topic 2: Fourier Transform Properties

### Key Concepts

1. **Time-Shift Property** - Delay in time  $\rightarrow$  Phase shift in frequency - Magnitude spectrum unchanged -  $x(t - t_0) \leftrightarrow X(f)e^{-j2\pi f t_0}$
2. **Modulation Property (Critical for AM!)** - Multiplication by cosine  $\rightarrow$  Frequency translation -  $x(t) \cos(2\pi f_c t) \leftrightarrow \frac{1}{2}[X(f - f_c) + X(f + f_c)]$  - Creates upper and lower sidebands
3. **Bandwidth in Modulated Signals** - DSB-SC bandwidth =  $2B$  (where  $B$  is message bandwidth) - Spectrum centered at  $\pm f_c$

### Essential Formulas

Time-Shift:

$$x(t - t_0) \leftrightarrow X(f)e^{-j2\pi f t_0}$$

Modulation (DSB-SC):

$$x(t) \cos(2\pi f_c t) \leftrightarrow \frac{1}{2}[X(f - f_c) + X(f + f_c)]$$

Bandwidth:

Message bandwidth:  $B$

DSB-SC bandwidth:  $2B$

AM bandwidth:  $2B$

SSB bandwidth:  $B$

## Practice Problems

From **Homework 1:** - Problem 4: Modulation property - Problem 5: DSB-SC spectrum sketching

**Strategy:** - Always sketch BOTH time and frequency domains - Label carrier frequency  $f_c$  and sidebands clearly - Remember: modulation creates TWO copies (at  $\pm f_c$ )

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## Topic 3: Parseval's Theorem

### Key Concepts

1. **Energy Conservation** - Energy in time = Energy in frequency - Useful for verifying calculations - Different domains, same physical quantity

- 2. Application** - Calculate energy in whichever domain is easier - Time domain for simple pulses  
 - Frequency domain for sinc functions

### Essential Formulas

Parseval's Theorem:

$$|x(t)|^2 dt = |X(f)|^2 df$$

For exponential decay  $x(t) = Ae^{-at}u(t)$ :

$$E = A^2/(2a)$$

### Practice Problems

From **Homework 1**: - Problem 6: Verify energy using both domains

**Strategy:** - Calculate in time domain first (usually easier) - Use Parseval to verify in frequency domain - Check units: energy has units of  $V^2 \cdot s$  or  $J$

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## Topic 4: Amplitude Modulation (AM)

### Key Concepts

1. **AM Signal Structure** -  $s(t) = A_c[1 + \mu m_n(t)] \cos(2\pi f_c t)$  - Carrier amplitude:  $A_c$  - Modulation index:  $\mu = A_m/A_c$  - Normalized message:  $|m_n(t)| \leq 1$
2. **Modulation Index** -  $\mu < 1$ : Normal modulation -  $\mu = 1$ : 100% modulation -  $\mu > 1$ : Overmodulation (envelope distortion!)
3. **Power Distribution** - Carrier power: Constant, carries NO information - Sideband power: Varies with  $\mu$ , contains information - Efficiency: Low for small  $\mu$

### Essential Formulas

AM Signal:

$$\begin{aligned}s(t) &= A [1 + \mu \cos(2\pi f_m t)] \cos(2\pi f_c t) \\A &= \text{carrier amplitude} \\&= \text{modulation index} = A_m / A_c\end{aligned}$$

Total Power:

$$P_{\text{total}} = (A^2/2R)(1 + \mu^2/2)$$

Carrier Power:

$$P_{\text{carrier}} = A^2/(2R)$$

Sideband Power:

$$P_{\text{sideband}} = (A^2/2R)(\mu^2/2)$$

Efficiency:

$$= P_{\text{sideband}}/P_{\text{total}} = \mu^2/(2 + \mu^2)$$

**Bandwidth:**

$$B_{AM} = 2f \text{ (same as DSB-SC)}$$

### Common Mistakes to Avoid

Using  $A_c + A_m$  instead of  $A_c(1+\mu)$  Forgetting factor of 1/2 in power formulas (RMS!) Confusing  $\mu$  with  $m(t)$  Not checking for overmodulation ( $\mu > 1$ )

### Practice Problems

From **Homework 2:** - Problem 1: Basic AM analysis - Problem 2: Power calculations - Problem 3: Modulation index and efficiency - Problem 4: Overmodulation detection

**Strategy:** 1. Identify  $A_c$ ,  $A_m$ ,  $f_c$ ,  $f_m$  from equation 2. Calculate  $\mu = A_m/A_c$  3. Check if  $\mu \leq 1$  (overmodulated?) 4. Use power formulas (don't forget the R!) 5. Calculate efficiency as percentage

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## Topic 5: DSB-SC vs. Full AM

### Key Concepts

**1. DSB-SC (Double Sideband Suppressed Carrier)** -  $s(t) = A_c m(t) \cos(2\pi f_c t)$  - No carrier component → more efficient - Requires coherent detection (complex receiver)

### 2. Comparison Table

Feature	DSB-SC	Full AM
Carrier	Suppressed	Transmitted
Power efficiency	100% in sidebands	$\mu^2/(2 + \mu^2)$
Detection	Coherent required	Envelope detector
Bandwidth	2B	2B
Complexity	High (receiver)	Low (receiver)

**3. When to Use** - DSB-SC: Power-limited systems, don't mind receiver complexity - AM: Simple receivers (AM radio), power not critical

### Essential Formulas

**DSB-SC Signal:**

$$s(t) = A \cdot m(t) \cdot \cos(2 f t)$$

**DSB-SC Power:**

$$P_{DSB} = (A^2/2R) \cdot P_m$$

where  $P_m$  = mean square of  $m(t)$

For sinusoidal  $m(t) = A \cos(2 f t)$ :

$$P_{DSB} = A^2 A^2 / (4R)$$

**Comparison:**

$P_{DSB\_sideband} = P_{AM\_sideband}$  (same sidebands)  
But  $P_{AM\_total} = P_{AM\_sideband} + P_{carrier}$

### Practice Problems

From **Homework 2:** - Problem 5: DSB-SC power calculation - Problem 6: Efficiency comparison

**Strategy:** - For DSB-SC: All power is in sidebands - For AM: Must calculate carrier + sideband power - Compare efficiencies as percentages

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## Topic 6: Single Sideband (SSB)

### Key Concepts

1. **Motivation** - DSB-SC wastes bandwidth (redundant sidebands) - SSB transmits only one sideband - Half the bandwidth of DSB-SC or AM!
2. **Bandwidth Efficiency** - Message bandwidth:  $B$  - AM/DSB-SC bandwidth:  $2B$  - SSB bandwidth:  $B$  (50% savings!)
3. **Generation Methods - Filter method:** Generate DSB-SC, then filter - **Phasing method:** Use Hilbert transform ( $90^\circ$  phase shift)
4. **Applications** - Amateur radio (voice communications) - Telephone systems (frequency division multiplexing) - Anywhere bandwidth is precious

### Essential Formulas

**Bandwidth Comparison:**

Message:  $B$  (e.g., 3 kHz for voice)  
DSB-SC:  $2B = 6$  kHz  
SSB:  $B = 3$  kHz  
AM:  $2B = 6$  kHz

**Power:**

$P_{SSB} = \frac{1}{2} P_{DSB}$  (only one sideband)

### Practice Problems

From **Homework 2:** - Problem 7: Bandwidth calculations - Problem 8: SSB generation methods

**Strategy:** - Identify message bandwidth  $B$  - SSB always uses exactly  $B$  (one sideband) - AM and DSB-SC use  $2B$  (two sidebands)

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## Topic 7: Binary ASK (Amplitude Shift Keying)

### Key Concepts

1. **ASK as Digital AM** - Bit “0” → Low amplitude (often 0) - Bit “1” → High amplitude - On-Off Keying (OOK): Special case where “0” = 0V
2. **Bit Duration and Bandwidth** - Bit rate:  $R_b$  bits/s - Bit duration:  $T_b = 1/R_b$  - Null-to-null bandwidth:  $B \approx 2R_b$
3. **Relationship to AM** - ASK is AM with digital message signal - Rectangular pulses → sinc spectrum - Bandwidth determined by bit rate, not carrier frequency

### Essential Formulas

ASK Parameters:

Bit rate:  $R$  (bits/s)

Bit duration:  $T = 1/R$

Null-to-null BW:  $B = 2/T = 2R$

On-Off Keying (OOK):

$s(t) = 0$  (bit "0")

$s(t) = A \cdot \cos(2\pi f_c t)$  (bit "1")

Carson's Rule (estimate):

$B_{ASK} = 2R$

### Practice Problems

From **Homework 2**: - Problem 9: ASK bandwidth calculations - Problem 10: Bit rate vs. bandwidth

**Strategy:** 1. Find bit duration  $T_b = 1/R_b$  2. Null-to-null BW =  $2/T_b = 2R_b$  3. Compare to carrier frequency ( $B \ll f_c$  for RF)

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### Formula Sheet Recommendations

#### Must-Have Formulas

Fourier Transform:

$$\text{rect}(t/T) = T \cdot \text{sinc}(fT)$$

$$x(t - t_0) = X(f)e^{-j2\pi f t_0}$$

$$x(t)\cos(2\pi f t) = \frac{1}{2}[X(f-f_0) + X(f+f_0)]$$

AM:

$$\begin{aligned}s(t) &= A [1 + m(t)] \cos(2\pi f t) \\&= A / A\end{aligned}$$

$$\begin{aligned}P_{\text{total}} &= (A^2/2R)(1 + m^2/2) \\&= P_0^2/(2 + P_0^2)\end{aligned}$$

### **DSB-SC:**

$$s(t) = A_m(t) \cos(2\pi f_c t)$$

$$P_{DSB} = (A_m^2 / 2R) \cdot P_m$$

### **Bandwidth:**

$$B_{AM} = B_{DSB} = 2B$$

$$B_{SSB} = B$$

$$B_{ASK} = 2R$$

### **Energy/Power:**

Parseval:  $\int |x(t)|^2 dt = \int |X(f)|^2 df$   
 $P_{avg} = V_{rms}^2 / R = V_{peak}^2 / (2R)$  for sinusoid

### **Useful Constants**

Room temperature:  $T = 290$  K

Boltzmann constant:  $k = 1.38 \times 10^{-23}$  J/K (probably not needed for Midterm 1)

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### **Exam Strategy**

#### **Time Management (75 minutes total)**

- **4 min:** Read entire exam, mark easy problems
- **8 min:** Problem 1 (Fourier fundamentals - typically straightforward)
- **8 min:** Problem 2 (Fourier properties)
- **8 min:** Problem 3 (Parseval's theorem)
- **17 min:** Problem 4 (AM analysis - usually longest)
- **8 min:** Problem 5 (DSB-SC comparison)
- **10 min:** Problem 6 (SSB and bandwidth)
- **8 min:** Problem 7 (ASK bonus - if time permits)
- **4 min:** Review and check answers

### **Problem-Solving Approach**

**For Fourier Transform Problems:** 1. Identify signal type (rect, sinc, exponential) 2. Draw time domain 3. Apply transform pair or properties 4. Draw frequency domain with labels 5. Calculate bandwidth from first zero crossing

**For AM Power Problems:** 1. Write out given signal equation 2. Identify  $A_c$ ,  $A_m$ ,  $f_c$ ,  $f_m$  3. Calculate  $\mu = A_m/A_c$  4. **Check for overmodulation** ( $\mu > 1$ ?) 5. Calculate carrier power:  $P_c = A_c^2/(2R)$  6. Calculate total power:  $P_{tot} = P_c(1 + \mu^2/2)$  7. Calculate sideband power:  $P_{sb} = P_c(\mu^2/2)$  8. Efficiency:  $\eta = P_{sb}/P_{tot}$

**For Bandwidth Problems:** 1. Identify message bandwidth B or frequency  $f_m$  2. Choose correct formula: - AM/DSB-SC:  $2B$  - SSB:  $B$  - ASK:  $2R_b$  3. Label spectrum sketch clearly

## Common Mistakes to Avoid

**Forgetting the factor of 1/2** in power formulas (sinusoid RMS = peak/ $\sqrt{2}$ ) **Mixing up time and frequency domains** (check units!) **Not checking for overmodulation** before calculating AM power **Confusing bandwidth with carrier frequency** **Using wrong sideband formula** for SSB (B, not 2B) **Forgetting to include resistance R** in power calculations **Not showing units** in final answers

## Tips for Success

**Draw diagrams!** Time and frequency sketches help visualize **Write out formulas** before substituting numbers **Check dimensions** (frequency in Hz, power in W, etc.) **Show all work** - partial credit is generous **Box final answers** - makes grading easier **Use your formula sheet** - don't waste time memorizing **Sanity check** - Does  $\mu > 1$ ? Is power positive? Is BW  $< f_c$ ?

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## Practice Resources

### Homework Problems (Strongly Recommended)

**Homework 1 (Fourier Analysis):** - All problems 1-10 - Focus on: Problems 1, 4, 5, 6 (most exam-relevant)

**Homework 2 (AM):** - All problems 1-10 - Focus on: Problems 1, 2, 3, 4, 8 (core AM concepts)

## Lecture Notebooks

- **Lecture 2:** Fourier Transform fundamentals
- **Lecture 3:** Fourier properties and bandwidth
- **Lecture 5:** AM modulation and demodulation

## Textbook Sections

- **Chapter 2.1:** Fourier Transform
  - **Chapter 2.2-2.3:** Filtering and bandwidth
  - **Chapter 3:** Amplitude Modulation
  - **Chapter 7.1-7.2:** ASK (digital AM)
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## Self-Assessment Checklist

Before the exam, make sure you can:

**Fourier Transform:** - [ ] Calculate Fourier transform of rectangular pulse - [ ] Identify first zero crossing and bandwidth - [ ] Apply time-shift property - [ ] Apply modulation property (create sidebands) - [ ] Sketch magnitude spectrum with labeled axes - [ ] Calculate signal energy in time domain - [ ] Verify energy using Parseval's theorem

**Amplitude Modulation:** - [ ] Identify  $A_c$ ,  $A_m$ ,  $f_c$ ,  $f_m$  from AM equation - [ ] Calculate modulation index  $\mu$  - [ ] Detect overmodulation ( $\mu > 1$ ) - [ ] Calculate carrier power - [ ] Calculate total

transmitted power - [ ] Calculate sideband power - [ ] Calculate power efficiency - [ ] Determine bandwidth (always 2B for AM)

**DSB-SC:** - [ ] Write DSB-SC signal equation - [ ] Calculate transmitted power - [ ] Compare efficiency to full AM - [ ] Explain advantage (power) vs. disadvantage (receiver complexity)

**SSB:** - [ ] Calculate bandwidth (always B, not 2B!) - [ ] Explain bandwidth advantage - [ ] Describe filter method or phasing method - [ ] List applications (amateur radio, telephony)

**ASK:** - [ ] Relate bit rate to bandwidth - [ ] Calculate null-to-null bandwidth ( $2R_b$ ) - [ ] Explain relationship to AM

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## Final Thoughts

**This exam tests fundamentals!** If you understand: 1. Fourier transforms create sidebands when you multiply by a carrier 2. AM puts information in sidebands while wasting power in carrier 3. Bandwidth =  $2 \times$  (message bandwidth) for AM/DSB-SC 4. Power =  $V^2/(2R)$  for sinusoids

...you'll do well!

**Focus your study time on:** - Homework 1 & 2 problems (they mirror exam problems) - Drawing accurate spectrum diagrams - AM power calculations (most common exam mistakes here) - Checking for overmodulation

**Good luck!**