

EE 451: Communications Systems

Homework 5 - Probability, Noise, and SNR

Topics: Probability theory, Gaussian distribution, thermal noise, SNR calculations **Textbook**

Reference: Haykin & Moher, Chapters 8, 9, 11.1-11.3

Total Points: 100

Instructions

- Show all work for full credit
 - Include units in all final answers
 - For probability problems, clearly state assumptions
 - Q-function table and standard normal table will be provided on exam
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Problem 1: Basic Probability (10 points)

A binary communication system transmits bits “0” and “1” with probabilities: - $P(0) = 0.6$ - $P(1) = 0.4$

- (a) What is the probability of receiving “1” followed by “0” if bits are independent? (3 points)
 - (b) What is the probability of receiving at least one “1” in three consecutive bits? (4 points)
 - (c) If 1000 bits are transmitted, approximately how many “1” bits are sent? (3 points)
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Problem 2: Conditional Probability and Bayes’ Theorem (12 points)

A receiver detects bits with the following error probabilities: - $P(\text{detect } 1 \mid \text{sent } 0) = 0.01$ (false alarm) - $P(\text{detect } 0 \mid \text{sent } 1) = 0.02$ (miss) - $P(\text{sent } 0) = 0.7$ - $P(\text{sent } 1) = 0.3$

- (a) What is $P(\text{detect } 0 \mid \text{sent } 0)$? (2 points)
 - (b) What is the probability of detecting a “1”? (4 points)
 - (c) Given that a “1” was detected, what is the probability that a “1” was actually sent? Use Bayes’ theorem. (6 points)
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Problem 3: Gaussian Distribution (15 points)

A received signal has amplitude that is Gaussian distributed with mean $\mu = 5$ V and standard deviation $\sigma = 2$ V.

- (a) Write the probability density function (PDF) for this signal. (3 points)
- (b) What is the probability that the amplitude is between 3 V and 7 V? (4 points)

(c) What is the probability that the amplitude exceeds 9 V? Express in terms of the Q-function. (4 points)

(d) If the threshold for detection is set at 4 V, what fraction of signals will be below this threshold? (4 points)

Given: $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2} dt$

Problem 4: Q-Function Calculations (12 points)

Use the Q-function for the following calculations:

(a) Calculate $Q(1.5)$. You may use the approximation $Q(x) \approx \frac{1}{\sqrt{2\pi}x} \cdot e^{-x^2/2}$ for $x > 3$, or look up the value. (3 points)

(b) A bit error occurs if noise exceeds 3σ . What is the probability of this event? (3 points)

(c) The complementary error function is $\text{erfc}(x) = 2 \cdot Q(x\sqrt{2})$. Express $Q(x)$ in terms of erfc . (3 points)

(d) For $\text{BER} = 10^{-6}$, what value of SNR (as a ratio, not dB) is required if $\text{BER} = Q(\sqrt{\text{SNR}})$? (3 points)

Problem 5: Thermal Noise Power (15 points)

(a) Calculate the thermal noise power in a 1 MHz bandwidth at room temperature ($T = 290$ K). Use $k = 1.38 \times 10^{-23}$ J/K. (4 points)

(b) Express this noise power in dBm. (Recall: $\text{dBm} = 10 \cdot \log_{10}(P/1 \text{ mW})$) (3 points)

(c) If the bandwidth increases to 20 MHz (WiFi channel), what is the new noise power in dBm? (3 points)

(d) A receiver has a noise figure $\text{NF} = 6$ dB. What is the total noise power in 20 MHz? (5 points)

Problem 6: Signal-to-Noise Ratio (15 points)

A communication system has the following parameters: - Received signal power: $P_s = -80$ dBm - System bandwidth: $B = 10$ MHz - Noise temperature: $T = 290$ K - Noise figure: $\text{NF} = 5$ dB

(a) Calculate the noise power in dBm. (5 points)

(b) Calculate the SNR in dB. (3 points)

(c) Convert the SNR to a linear ratio. (2 points)

(d) If the signal power increases by 3 dB, what is the new SNR? (3 points)

(e) What signal power is required to achieve $\text{SNR} = 20$ dB? (2 points)

Problem 7: Noise in AM Systems (12 points)

An AM receiver has the following specifications: - Carrier power: $P_c = 10$ W - Modulation index: $\mu = 0.8$ - Noise power in the message bandwidth: $P_n = 0.1$ W

- (a) Calculate the total transmitted power. (3 points)
 - (b) Calculate the signal power in the sidebands. (3 points)
 - (c) For DSB-SC (suppressed carrier), what is the output SNR? Assume the output SNR equals (sideband power)/(noise power). (3 points)
 - (d) For full carrier AM, the output SNR is reduced by the factor $\frac{\mu^2}{2+\mu^2}$. Calculate the output SNR for this AM system. (3 points)
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Problem 8: FM Noise Performance (12 points)

An FM system has: - Carrier power: $P_c = 20$ W - Noise power: $P_n = 0.5$ W - Modulation index: $\beta = 5$ - Message bandwidth: $B_m = 15$ kHz

- (a) Calculate the input SNR (carrier power to noise power ratio). (3 points)
 - (b) The output SNR for FM is given by: $(\text{SNR})_{\text{out}} = 3\beta^2(\beta + 1) \cdot (\text{SNR})_{\text{in}}$ for $\beta \gg 1$. Calculate the output SNR. (4 points)
 - (c) What is the FM improvement factor (output SNR / input SNR)? (3 points)
 - (d) Express the output SNR in dB. (2 points)
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Problem 9: Additive White Gaussian Noise (AWGN) (10 points)

- (a) What does “white” mean in the context of white Gaussian noise? (3 points)
 - (b) What does “Gaussian” refer to? (2 points)
 - (c) The power spectral density of AWGN is $N_0/2$ W/Hz. If $N_0/2 = 10^{-9}$ W/Hz and the bandwidth is 5 MHz, what is the total noise power? (3 points)
 - (d) Why is AWGN a good model for thermal noise in communication systems? (2 points)
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Problem 10: Link Budget Basics (12 points - Challenge)

A WiFi system operates at 2.4 GHz with the following parameters: - Transmit power: $P_t = 20$ dBm - Transmit antenna gain: $G_t = 2$ dBi - Receive antenna gain: $G_r = 2$ dBi - Distance: $d = 10$ m - System bandwidth: 20 MHz - Noise figure: $NF = 7$ dB - Temperature: $T = 290$ K

- (a) Calculate the free-space path loss at 2.4 GHz and 10 m using: $\text{FSPL}(\text{dB}) = 20 \cdot \log_{10}(d) + 20 \cdot \log_{10}(f) + 32.45$, where d is in km and f is in MHz. (4 points)
- (b) Calculate the received signal power P_r in dBm. (3 points)

- (c) Calculate the noise power in dBm. (3 points)
- (d) Calculate the SNR in dB. (2 points)
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Bonus Problem 1: Shannon's Theorem (5 points extra credit)

Shannon's theorem states that the channel capacity is: $C = B \cdot \log_2(1 + \text{SNR})$

where C is in bits/s, B is bandwidth in Hz, and SNR is a linear ratio.

- (a) Calculate the channel capacity for a system with $B = 20$ MHz and $\text{SNR} = 30$ dB. (3 points)
- (b) If you want to transmit at 100 Mbps with $B = 20$ MHz, what minimum SNR (in dB) is required? (2 points)
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Bonus Problem 2: Noise Temperature (5 points extra credit)

(a) A receiver has a noise figure $\text{NF} = 3$ dB. Calculate the equivalent noise temperature T_e . Use the relationship: $\text{NF} = 1 + \frac{T_e}{T_0}$ where $T_0 = 290$ K. (3 points)

(b) If two amplifiers are cascaded with noise figures $\text{NF}_1 = 3$ dB and $\text{NF}_2 = 6$ dB, and gains $G_1 = 20$ dB and $G_2 = 10$ dB, calculate the overall noise figure using Friis formula: $F_{\text{total}} = F_1 + \frac{F_2 - 1}{G_1}$ where F is the noise factor (linear, not dB). (2 points)

Submission Instructions: - Submit a single PDF via Brightspace - Show all conversions between dB and linear units - For probability problems, show the complete calculation - Q-function values can be looked up or approximated

Academic Integrity: - Individual work required - Show all work for full credit - Cite any formula references

Useful Constants: - Boltzmann constant: $k = 1.38 \times 10^{-23}$ J/K - Room temperature: $T_0 = 290$ K - Speed of light: $c = 3 \times 10^8$ m/s