

# EE 451: Communications Systems

## Homework 2 - Amplitude Modulation and ASK

**Topics:** DSB-SC, full carrier AM, envelope detection, ASK/OOK **Textbook Reference:** Haykin & Moher, Chapters 3.1-3.3, 7.1-7.2

**Total Points:** 100

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### Instructions

- Show all work for full credit
  - Include units in all final answers
  - Sketch waveforms where requested
  - You may use Python/MATLAB for verification, but show analytical work
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### Problem 1: DSB-SC Fundamentals (12 points)

A message signal  $m(t) = 5 \cos(2\pi \cdot 1000 \cdot t)$  V modulates a carrier  $c(t) = 10 \cos(2\pi \cdot 100 \times 10^3 \cdot t)$  V using DSB-SC modulation.

- (a) Write the expression for the DSB-SC modulated signal  $s(t)$ . (3 points)
  - (b) Use trigonometric identities to express  $s(t)$  as a sum of two sinusoids and identify the upper and lower sideband frequencies. (5 points)
  - (c) What is the bandwidth of the DSB-SC signal? (2 points)
  - (d) What is the power in each sideband if the system has  $50 \Omega$  resistance? (2 points)
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### Problem 2: Full Carrier AM (15 points)

An AM broadcast station transmits with a carrier frequency of 1.2 MHz. The carrier amplitude is  $A_c = 100$  V and the message signal is  $m(t) = 0.6 \cos(2\pi \cdot 5000 \cdot t)$  V.

- (a) Write the complete expression for the AM signal  $s(t)$ . (3 points)
  - (b) What is the modulation index  $\mu$ ? (3 points)
  - (c) Sketch one complete cycle of the modulating signal showing the envelope of the AM waveform. Label key amplitudes. (4 points)
  - (d) Calculate the total transmitted power and the percentage of power in the sidebands vs. the carrier (assume  $50 \Omega$ ). (5 points)
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### Problem 3: Modulation Index and Overmodulation (12 points)

An AM transmitter has carrier amplitude  $A_c = 50$  V.

- (a) What is the maximum message amplitude  $A_m$  that can be used without overmodulation? (3 points)
  - (b) If  $m(t) = 40 \cos(2\pi \cdot 3000 \cdot t)$  V, calculate the modulation index and determine if the signal is overmodulated. (3 points)
  - (c) Sketch the AM envelope for  $\mu = 0.5$ ,  $\mu = 1.0$ , and  $\mu = 1.5$ . Explain what happens during overmodulation. (6 points)
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### Problem 4: Envelope Detection (10 points)

An envelope detector is used to demodulate an AM signal. The AM signal has: - Carrier frequency:  $f_c = 900$  kHz - Message signal:  $m(t) = \cos(2\pi \cdot 5000 \cdot t)$  - Modulation index:  $\mu = 0.8$

- (a) Sketch a simple envelope detector circuit (diode, RC components). (3 points)
  - (b) What is the requirement on the RC time constant to avoid distortion? Express in terms of the carrier frequency  $f_c$  and the maximum message frequency  $f_m$ . (4 points)
  - (c) What happens if the RC time constant is too large? Too small? (3 points)
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### Problem 5: AM Bandwidth and Spectral Efficiency (12 points)

An AM radio station broadcasts audio from 50 Hz to 5 kHz.

- (a) What is the RF bandwidth required for DSB-SC? For full carrier AM? (4 points)
  - (b) If the station is assigned a carrier frequency of 1050 kHz, what frequencies appear in the transmitted spectrum? (4 points)
  - (c) Calculate the spectral efficiency (bits/s/Hz) if the audio is digitized at 8 bits per sample with Nyquist rate sampling, then transmitted using AM. (4 points)
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### Problem 6: On-Off Keying (ASK) (15 points)

A binary data stream “1 0 1 1 0” is transmitted using OOK with: - Bit rate:  $R_b = 1000$  bits/s - Carrier frequency:  $f_c = 10$  kHz - Carrier amplitude for “1”:  $A = 5$  V - Carrier amplitude for “0”: 0 V

- (a) Write the expression for the OOK signal during transmission of a “1” bit. (3 points)
- (b) Sketch the transmitted waveform  $s(t)$  for the entire data sequence. Show at least 3 carrier cycles per bit. (5 points)
- (c) What is the null-to-null bandwidth of the OOK signal? (4 points)
- (d) How does this compare to the bandwidth of the unmodulated baseband data stream? (3 points)

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**Problem 7: Binary ASK Spectral Analysis (12 points)**

A binary ASK system transmits rectangular pulses at a bit rate of 2400 bits/s using a carrier frequency of 2.4 GHz (WiFi band, simplified).

- (a) What is the bit duration  $T_b$ ? (2 points)
- (b) The baseband rectangular pulse has a sinc-shaped spectrum. What is the first null bandwidth of the baseband signal? (4 points)
- (c) After modulation to  $f_c = 2.4$  GHz, what frequencies define the main lobe of the ASK spectrum? (4 points)
- (d) Compare the bandwidth efficiency of ASK to DSB-SC AM for the same baseband signal. (2 points)

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**Problem 8: AM vs. ASK Comparison (10 points)**

- (a) Explain the fundamental difference between analog AM and digital ASK. What do they have in common? (4 points)
- (b) Both AM and ASK can use envelope detection. Why is this possible? (3 points)
- (c) Why might ASK be preferred over AM in a digital communication system? (3 points)

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**Problem 9: Superheterodyne Receiver (12 points - Challenge)**

An AM broadcast receiver uses the superheterodyne architecture with an intermediate frequency (IF) of 455 kHz.

- (a) If you want to receive a station at 1200 kHz, to what frequency should the local oscillator (LO) be tuned? (3 points)
- (b) What is the image frequency for this receiver? (4 points)
- (c) If an interfering signal exists at the image frequency, explain what problem this causes. How do receivers mitigate this? (5 points)

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**Problem 10: Real-World AM System Design (10 points - Challenge)**

You are designing an AM transmitter for a campus radio station: - Carrier frequency:  $f_c = 88.5$  MHz (FM band, but using AM for this problem) - Audio bandwidth: 20 Hz to 15 kHz - Maximum transmit power: 100 W - Carrier amplitude:  $A_c = 30$  V - Load resistance:  $50\ \Omega$

- (a) What is the maximum modulation index you can use to transmit at 100 W total power? (5 points)
- (b) What is the RF bandwidth of your transmission? (2 points)

(c) If you want to maximize power efficiency (power in sidebands), would you choose DSB-SC or full carrier AM? Justify your answer. (3 points)

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**Bonus Problem 1: SSB (5 points extra credit)**

Single Sideband (SSB) modulation transmits only one sideband, saving bandwidth and power.

(a) For a message signal with bandwidth 3 kHz, what is the bandwidth of USB (Upper Sideband) SSB? (2 points)

(b) If DSB-SC requires 50 W to transmit both sidebands, how much power is required for SSB (transmitting one sideband)? (3 points)

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**Bonus Problem 2: Mathematical Derivation (5 points extra credit)**

Starting from  $s(t) = A_c[1 + \mu \cdot m_n(t)] \cos(2\pi f_c t)$  where  $m_n(t)$  is normalized to  $|m_n(t)| \leq 1$ :

Show that the carrier power is  $P_c = \frac{A_c^2}{2}$  and the total sideband power is  $P_{sb} = \frac{\mu^2 A_c^2}{4} \cdot P_m$ , where  $P_m$  is the normalized message power.

For a single tone  $m_n(t) = \cos(2\pi f_m t)$ , verify that the efficiency (sideband power / total power) is  $\frac{\mu^2}{2 + \mu^2}$ .

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**Submission Instructions:** - Submit a single PDF via Brightspace - Include all sketches and waveform drawings - Label all graphs with frequencies, amplitudes, and time scales - Show all intermediate steps for calculations

**Academic Integrity:** - Individual work required - Cite any external references used - Discussion of concepts is allowed; copying solutions is not