

# EE 451: Communications Systems

## Final Exam - Comprehensive Study Guide

**Exam Date:** Week 16 (May 18-22, 2026) **Duration:** 180 minutes (3 hours) **Coverage:** All course material (Chapters 1-11) with emphasis on Chapters 8-11 **Format:** Closed book, one 8.5" × 11" formula sheet (both sides) allowed

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### Overview

The final exam is **comprehensive** but with **emphasis on later material** (probability, noise, BER, link budgets). Structure: - **Part I (30 pts):** Fundamentals review (Fourier, AM, FM/PM) - **Part II (35 pts):** Digital modulation (BPSK, QPSK, FSK, QAM) - **Part III (40 pts):** Probability and noise (Gaussian, SNR, BER) - **Part IV (35 pts):** Link budgets and system performance - **Part V (10 pts):** Advanced topics (OFDM, spread spectrum) - **Bonus (10 pts):** Satellite link analysis

**Key insight:** 85 of 150 points (57%) come from material AFTER Midterm 2!

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### Study Strategy

#### Priority Levels

**HIGHEST PRIORITY** (Study first - 60% of exam) - Link budget calculations (Problems 10, 11) - Gaussian distribution and Q-function (Problem 7) - Thermal noise and SNR calculations (Problem 8) - BER analysis for BPSK (Problem 9) - Shannon capacity (Problem 11)

**MEDIUM PRIORITY** (Important - 25% of exam) - BPSK vs QPSK comparison (Problem 4) - 16-QAM and Gray coding (Problem 5) - FSK system design (Problem 6) - Adaptive modulation (Problem 12)

**LOWER PRIORITY** (Review quickly - 15% of exam) - Fourier transforms (Problem 1) - AM power calculations (Problem 2) - FM vs PM (Problem 3) - OFDM and spread spectrum concepts (Problem 13)

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### Part I: Fundamentals Review (30 points)

#### Quick Review Topics

**Fourier Transforms (10 pts)** - Sinc rect transform pair - Time-frequency duality - Modulation property (DSB-SC spectrum)

**AM (10 pts)** - Modulation index  $\mu = A_m/A_c$  - Power:  $P_{total} = \frac{A_c^2}{2R}(1 + \mu^2/2)$  - Efficiency:  $\eta = \frac{\mu^2/2}{1+\mu^2/2}$

**FM/PM (10 pts)** - FM:  $\Delta f = k_f A_m$  (independent of  $f_m$ ) - PM:  $\Delta f = k_p A_m f_m$  (proportional to  $f_m$ ) - Carson's rule:  $B_{FM} = 2(\Delta f + f_m)$

**Study Tip:** These are REVIEW topics. Don't spend too much time here if you understand Midterms 1 & 2 material. Focus on new material!

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## Part II: Digital Modulation (35 points)

### BPSK vs QPSK (12 pts - Problem 4)

**Key Comparisons:**

Feature	BPSK	QPSK
Bits/symbol	1	2
Symbol rate	$R_b$	$R_b/2$
Bandwidth	$2R_b$	$R_b$
Spectral efficiency	0.5 bits/s/Hz	1.0 bits/s/Hz
BER (same $E_b/N_0$ )	Same!	Same!

**Critical insight:** QPSK is twice as bandwidth efficient with no BER penalty!

### QAM and Gray Coding (12 pts - Problem 5)

**16-QAM Basics:** - M = 16 symbols, k = 4 bits/symbol - Rectangular constellation: 4×4 grid - Amplitude levels: typically  $\{\pm 1, \pm 3\}$

**Gray Coding Rules:** - Adjacent symbols differ by 1 bit - Minimizes BER (most errors → adjacent symbols) - Mandatory in modern systems!

**EVM (Error Vector Magnitude):** - Measures constellation quality -  $EVM\% = (\text{Error distance} / \text{Reference distance}) \times 100\%$  - WiFi/LTE have strict EVM limits (e.g., 256-QAM < 3%)

### FSK System Design (11 pts - Problem 6)

**Orthogonal FSK:** - Minimum separation:  $\Delta f_{min} = \frac{1}{2T_b}$  - Modulation index:  $h = \Delta f \times T_b$  - MSK:  $h = 0.5$  exactly

**Quick formulas:**

$$T_b = 1/R_b$$

$$\Delta f = |f_+ - f_-|$$

$$h = \Delta f \cdot T_b$$

$$\text{Orthogonal: } \Delta f = 1/(2T_b)$$

$$\text{MSK: } h = 0.5$$

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## Part III: Probability and Noise (40 points)

**THIS IS THE MOST IMPORTANT SECTION!**

### Gaussian Distribution (15 pts - Problem 7)

**PDF of Gaussian:**

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)}$$

**Standardization:**

$$Z = \frac{X - \mu}{\sigma} \sim N(0, 1)$$

**Q-Function:**

$$Q(x) = P(Z > x) \text{ for } Z \sim N(0, 1)$$

**For general Gaussian  $\mathbf{X} \sim \mathbf{N}(\mu, \Sigma)$ :**

$$P(X > a) = Q\left(\frac{a - \mu}{\sigma}\right)$$

**Key Q-function values (memorize these!):**

Q(1.0)	0.159	Q(2.0)	0.023
Q(1.5)	0.067	Q(3.0)	0.0013
Q(4.0)	3e-5	Q(4.75)	1e-6

**Property:**  $Q(-x) = 1 - Q(x)$

**Practice:** 1. Identify  $\mu$  and  $\Sigma$  from problem 2. Standardize:  $z = (x - \mu)/\sigma$  3. Express using Q-function 4. Look up or calculate

### Thermal Noise and SNR (12 pts - Problem 8)

**Thermal Noise Power:**

$$P_n = kTB$$

where: -  $k = 1.38 \times 10^{-23}$  J/K (Boltzmann constant) -  $T$  = temperature (K) -  $B$  = bandwidth (Hz)

**Noise Figure:**

$$P_{n,total} = P_n \times F = P_n \times 10^{NF_{dB}/10}$$

**In dBm:**

$$P_n(dBm) = 10 \log_{10} \left( \frac{kTB}{10^{-3}} \right)$$

**Quick calculation:** At  $T = 290$  K:

$$P_n(dBm) \approx -174 + 10 \log_{10}(B_{Hz})$$

**SNR:**

$$SNR(dB) = P_s(dBm) - P_n(dBm)$$

**Critical conversions:**

dBm to W:  $P_W = 10^{((P_{dBm} - 30)/10)}$   
 W to dBm:  $P_{dBm} = 10 \cdot \log(P_W) + 30$   
 dB addition:  $P_{total}(dB) = P1(dB) + P2(dB)$   
 Linear addition:  $P_{total\_W} = P1\_W + P2\_W$

**Common values to know:**

0 dBm = 1 mW  
 -30 dBm = 1 W  
 -60 dBm = 1 nW  
 -90 dBm = 1 pW

+3 dB =  $\times 2$   
 +10 dB =  $\times 10$   
 +20 dB =  $\times 100$

### BER Performance (13 pts - Problem 9)

**BPSK BER:**

$$BER_{BPSK} = Q(\sqrt{2E_b/N_0})$$

**Conversions:**

$$\frac{E_b}{N_0}(dB) = 10 \log_{10}(E_b/N_0)$$

**To find linear from dB:**

$$\frac{E_b}{N_0} = 10^{(E_b/N_0)_{dB}/10}$$

**Bit errors per second:**

$$N_{errors/s} = BER \times R_b$$

**Common  $E_b/N_0$  values for BER targets:**

BER =  $10^{-3}$ : Eb/N0 6.8 dB  
 BER =  $10^{-2}$ : Eb/N0 8.4 dB  
 BER =  $10^{-1}$ : Eb/N0 9.6 dB  
 BER =  $10^{-6}$ : Eb/N0 10.5 dB

**Step-by-step for BER problems:** 1. Convert  $E_b/N_0$  from dB to linear 2. Calculate  $\sqrt{2E_b/N_0}$  3. Find  $Q(\sqrt{2E_b/N_0})$  using table 4. This is your BER 5. Errors/sec = BER  $\times$  bit rate

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### Part IV: Link Budgets (35 points)

**CRITICAL SECTION - WORTH 35 POINTS!**

## Free-Space Path Loss (FSPL)

Two equivalent forms:

**Form 1 (dB, easier for calculations):**

$$FSPL(dB) = 20 \log_{10}(d_{km}) + 20 \log_{10}(f_{MHz}) + 32.45$$

**Form 2 (from wavelength):**

$$FSPL(dB) = 20 \log_{10} \left( \frac{4\pi d}{\lambda} \right)$$

where  $\lambda = c/f$

**Example calculation:**

f = 5.8 GHz = 5800 MHz

d = 50 m = 0.05 km

$$\begin{aligned} FSPL &= 20 \cdot \log(0.05) + 20 \cdot \log(5800) + 32.45 \\ &= 20 \cdot (-1.301) + 20 \cdot (3.763) + 32.45 \\ &= -26.02 + 75.26 + 32.45 \\ &= 81.69 \text{ dB} \end{aligned}$$

## Complete Link Budget (Problem 10 - 18 pts)

**Link Budget Equation:**

$$P_r(dBm) = P_t(dBm) + G_t(dBi) + G_r(dBi) - FSPL(dB) - L_{misc}(dB)$$

**Step-by-step procedure:**

1. Calculate FSPL - Use formula above - Make sure units match (km and MHz, OR meters and Hz)
2. Calculate received power

$$P_r = P_t + G_t + G_r - FSPL - L_{other}$$

3. Calculate noise power

$$P_n = kTB \text{ (in W)}$$

$$P_n(dBm) = 10 \log_{10}(P_n/0.001)$$

OR use quick formula:

$$P_n(dBm) \approx -174 + 10 \log_{10}(B_{Hz}) + NF(dB)$$

4. Calculate SNR

$$SNR(dB) = P_r(dBm) - P_n(dBm) - \text{fade margin}$$

5. Determine supported modulation Compare SNR to thresholds:

QPSK: 8 dB  
 16-QAM: 15 dB  
 64-QAM: 22 dB  
 256-QAM: 28 dB

**Common mistakes:** Forgetting to convert distance to km or frequency to MHz Mixing up + and - signs in link budget Forgetting fade margin Not adding noise figure to thermal noise

### Shannon Capacity (Problem 11 - 10 pts)

#### Shannon-Hartley Theorem:

$$C = B \log_2(1 + SNR)$$

bits/s

where: -  $C$  = channel capacity (bits/s) -  $B$  = bandwidth (Hz) -  $SNR$  = linear ratio (NOT dB!)

#### Convert SNR from dB:

$$SNR = 10^{SNR(dB)/10}$$

#### Spectral Efficiency:

$$\eta = \frac{C}{B} = \log_2(1 + SNR)$$

bits/s/Hz

#### Example:

$B = 10$  MHz,  $SNR = 20$  dB

$SNR_{linear} = 10^{(20/10)} = 100$

$$\begin{aligned}
 C &= 10 \times 10 \times \log(1 + 100) \\
 &= 10 \times 10 \times \log(101) \\
 &= 10 \times 10 \times 6.658 \\
 &= 66.58 \text{ Mbps}
 \end{aligned}$$

$$= 6.658 \text{ bits/s/Hz}$$

**Common mistake:** Using SNR in dB directly (must convert to linear!)

### Adaptive Modulation (Problem 12 - 7 pts)

**Concept:** - Choose modulation based on SNR - Higher SNR → higher-order modulation → more bits/s - Lower SNR → lower-order modulation → more reliable

#### SNR vs. distance model:

$$SNR(dB) = SNR_0 - n \times 10 \log_{10}(d/d_0)$$

where  $n$  is path loss exponent (typical: 2-4)

**Procedure:** 1. Calculate SNR at each distance 2. Compare to modulation thresholds 3. Select highest modulation with SNR threshold

## Part V: Advanced Topics (10 points)

### OFDM (4 pts)

**Why OFDM?** - Solves frequency-selective fading problem - Divides wideband channel into many narrowband subchannels - Each subchannel experiences flat fading - Simpler equalization than single-carrier

**Applications:** - WiFi (802.11a/g/n/ac/ax) - LTE, 5G NR - DVB-T (digital TV)

**Key concept:** Convert frequency-selective fading → flat fading per subcarrier

### Spread Spectrum (6 pts)

**DSSS (Direct Sequence Spread Spectrum):** - Data rate:  $R_b$  - Chip rate:  $R_c$  (much higher) - Processing gain:  $G_p = \frac{R_c}{R_b}$  (ratio) - In dB:  $G_p(dB) = 10 \log_{10}(R_c/R_b)$

**Advantages:** - Jamming resistance (processing gain) - Multiple access (CDMA) - Low probability of intercept

**Example:**

$$R_b = 1 \text{ Mbps}, R_c = 10 \text{ Mchips/s}$$

$$G_p = 10 \text{ Mchips/s} / 1 \text{ Mbps} = 10$$

$$G_p(dB) = 10 \cdot \log(10) = 10 \text{ dB}$$

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### Bonus: Satellite Link (10 points)

**Typical satellite parameters:** - High transmit power: 40 dBW (10 kW) - High antenna gains: 30-40 dBi - Long distance: 36,000 km (GEO) - Very high FSPL: ~200 dB! - Low noise temperature: 150 K (LNA)

**$E_b/N_0$  and SNR relationship:**

$$\frac{E_b}{N_0} = SNR \times \frac{B}{R_b}$$

**Use this to find maximum bit rate:**

$$R_b = \frac{SNR \times B}{E_b/N_0}$$

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## Master Formula Sheet

### Probability & Noise

**Gaussian PDF:**

$$f(x) = (1/\sqrt{2\pi}) \exp(-(x-\mu)^2/(2\sigma^2))$$

**Q-function:**

$$P(X > a) = Q((a-\mu)/\sigma) \text{ for } X \sim N(\mu, \sigma^2)$$

$$Q(-x) = 1 - Q(x)$$

Thermal Noise:

$$P_n = kTB \quad (k = 1.38 \times 10^{-23} \text{ J/K})$$

$$P_n(\text{dBm}) = -174 + 10\log(B_{\text{Hz}}) + \text{NF(dB)} \quad [\text{at } T=290\text{K}]$$

SNR:

$$\text{SNR(dB)} = P_s(\text{dBm}) - P_n(\text{dBm})$$

BER:

$$\text{BER_BPSK} = Q(\sqrt{2E_b/N_0})$$

## Link Budget

FSPL:

$$\text{FSPL(dB)} = 20\log(d_{\text{km}}) + 20\log(f_{\text{MHz}}) + 32.45$$

Link Budget:

$$P_r(\text{dBm}) = P_t(\text{dBm}) + G_t + G_r - \text{FSPL} - \text{Losses}$$

Shannon Capacity:

$$C = B \cdot \log(1 + \text{SNR}) \quad [\text{SNR must be linear!}]$$

Eb/No vs SNR:

$$\text{Eb/No} = \text{SNR} \times (B/R_b)$$

## Digital Modulation (Review)

BPSK:  $k=1$ ,  $R_s=R_b$ ,  $B=2R_b$

QPSK:  $k=2$ ,  $R_s=R_b/2$ ,  $B=R_b$

M-ary:  $k=\log(M)$ ,  $R_s=R_b/k$

FSK:  $h = \Delta f \cdot T_b$

Orthogonal:  $\Delta f = 1/(2T_b)$

MSK:  $h = 0.5$

## Analog Modulation (Review)

AM:

$$s(t) = A_c [1 + m(t)] \cos(2\pi f_c t)$$

$$P_{\text{total}} = (A_c^2/2R)(1 + m^2/2)$$

$$= m^2/(2 + m^2)$$

FM:

$$\Delta f = kf \cdot \max|m(t)|$$

$$= \Delta f/f_m$$

$$B_{\text{FM}} = 2(\Delta f + f_m)$$

PM:

$$\Delta \phi = kp \cdot \max|m(t)|$$

$$\Delta f_{PM} = k_p \cdot A_m \cdot f_m \quad (\text{depends on } f_m!)$$

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## Exam Strategy for 180 Minutes

### Time Allocation

**Part I (30 pts) - 25 minutes** - Problem 1: 7 min (Fourier - straightforward) - Problem 2: 10 min (AM power) - Problem 3: 8 min (FM vs PM)

**Part II (35 pts) - 30 minutes** - Problem 4: 10 min (BPSK vs QPSK) - Problem 5: 12 min (16-QAM, Gray coding) - Problem 6: 8 min (FSK design)

**Part III (40 pts) - 50 minutes** - Problem 7: 18 min (Gaussian & Q-function - 5 parts!) - Problem 8: 15 min (Thermal noise & SNR) - Problem 9: 17 min (BER analysis)

**Part IV (35 pts) - 45 minutes** - Problem 10: 25 min (WiFi link budget - longest problem!) - Problem 11: 12 min (Shannon capacity) - Problem 12: 8 min (Adaptive modulation)

**Part V (10 pts) - 10 minutes** - Problem 13: 10 min (OFDM & spread spectrum concepts)

**Bonus (10 pts) - 10 minutes** (if time!) - Satellite link analysis

**Review: 10 minutes**

### Priority Order (if running low on time)

1. **Do first:** Problems 7, 8, 9, 10 (new material, high points)
2. **Do second:** Problems 11, 12 (new material)
3. **Do third:** Problems 4, 5, 6 (review, but still good points)
4. **Do last:** Problems 1, 2, 3 (pure review)
5. **If time:** Problems 13 and Bonus

### Calculator Tips

#### Pre-calculate common values:

$$\log(2) = 0.301$$

$$\log(10) = 1$$

$$\ln(2) = 0.693$$

$$10^{-3} = 0.001$$

$$10^{-2} = 0.01$$

$$10^{-1} = 0.1$$

$$\text{For } \log(x): \log(x) = \log(x)/\log(2) = 3.32 \cdot \log(x)$$

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## Common Mistakes to AVOID

### Part III (Probability/Noise)

Forgetting to standardize Gaussian:  $z = (x - \mu)/\sigma$  Using SNR in dB directly in Shannon formula (must convert to linear!) Forgetting to add noise figure to thermal noise Confusing Q(x) with 1-Q(x) Using wrong Boltzmann constant or temperature

### Part IV (Link Budgets)

Wrong units in FSPL: using meters instead of km, or Hz instead of MHz Wrong signs in link budget (it's  $P_t + G_t + G_r - FSPL$ , not all positive!) Forgetting fade margin when calculating SNR Comparing to wrong modulation thresholds Not converting  $E_b/N_0$  from dB when using formulas

### General

Not showing units in answers Not checking if answer makes physical sense Spending too long on early problems (save time for Parts III & IV!) Forgetting to box/circle final answers

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## Self-Assessment Checklist

### Can you:

**Probability & Noise:** - [ ] Write Gaussian PDF - [ ] Standardize and use Q-function - [ ] Calculate thermal noise power ( $kTB$ ) - [ ] Convert between dBm and Watts - [ ] Add noise figure correctly - [ ] Calculate SNR in dB - [ ] Convert  $E_b/N_0$  between dB and linear - [ ] Calculate BER for BPSK - [ ] Find bit errors per second

**Link Budgets:** - [ ] Calculate FSPL with correct units - [ ] Perform complete link budget ( $Pr = Pt + gains - losses$ ) - [ ] Calculate noise power in dBm - [ ] Calculate SNR accounting for fade margin - [ ] Select modulation based on SNR - [ ] Calculate Shannon capacity (with SNR in linear!) - [ ] Apply adaptive modulation rules

**Digital Modulation:** - [ ] Compare BPSK vs QPSK (bandwidth, spectral efficiency) - [ ] Draw 16-QAM constellation - [ ] Apply Gray coding - [ ] Calculate EVM - [ ] Design orthogonal FSK system - [ ] Identify MSK

**Review Topics:** - [ ] Fourier transform of rect/sinc - [ ] AM power and efficiency - [ ] FM vs PM frequency deviation

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## Final Study Plan (1 Week Before Exam)

### Days 7-6: New Material (Priority )

- **Study:** Homework 5 (all 10 problems) - Probability & Noise
- **Study:** Homework 6 (all 10 problems) - BER & Link Budgets
- **Practice:** Midterm 1 solutions (Problems 7-9)
- **Focus:** Link budget procedure, Q-function, thermal noise

## Days 5-4: Integration

- **Practice:** Complete a full link budget from scratch
- **Practice:** Gaussian/Q-function problems
- **Practice:** BER calculations
- **Review:** Shannon capacity problems
- **Make:** Your formula sheet!

## Days 3-2: Review Material

- **Quick review:** Homework 3 & 4 (digital modulation)
- **Quick review:** Homework 1 & 2 (Fourier, AM)
- **Practice:** Midterm 2 (all problems quickly)
- **Refine:** Formula sheet

## Day 1: Final Prep

- **Take:** Practice final (under time pressure!)
  - **Review:** Common mistakes list
  - **Check:** Formula sheet is complete
  - **Sleep:** Well!
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## What to Bring

Calculator (scientific, not graphing)   Formula sheet (both sides, handwritten recommended)  
Pencils/pens   Student ID

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## During the Exam

1. **Read all problems first** (5 min)
  2. **Mark easy problems** with a star
  3. **Do high-value problems first** (Parts III & IV)
  4. **Show ALL work** - partial credit is generous!
  5. **Box final answers**
  6. **Check units** on every answer
  7. **Sanity check:** Does the answer make sense?
  8. **If stuck:** Move on, come back later
  9. **Last 10 min:** Review, check signs, check units
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## Final Thoughts

**The exam rewards:** - Systematic problem-solving (link budgets!) - Careful unit management (km vs m, dB vs linear) - Understanding relationships (Shannon, BER, SNR)

**Focus your final study on:** 1. Link budget procedure (practice 3-5 complete examples) 2. Q-function and BER calculations 3. SNR and noise calculations 4. Digital modulation comparisons

You've got this! The comprehensive nature means you can show your understanding across the whole course. Focus on the new material (Parts III-IV) and you'll do great!

Good luck on your final exam!