

# EE 451: Communications Systems

## Homework 5 - Probability, Noise, and SNR

**Topics:** Probability theory, Gaussian distribution, thermal noise, SNR calculations **Textbook**

**Reference:** Haykin & Moher, Chapters 8, 9, 11.1-11.3

**Total Points:** 100

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### Instructions

- Show all work for full credit
  - Include units in all final answers
  - For probability problems, clearly state assumptions
  - Q-function table and standard normal table will be provided on exam
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### Problem 1: Basic Probability (10 points)

A binary communication system transmits bits “0” and “1” with probabilities: -  $P(0) = 0.6$  -  $P(1) = 0.4$

- (a) What is the probability of receiving “1” followed by “0” if bits are independent? (3 points)
  - (b) What is the probability of receiving at least one “1” in three consecutive bits? (4 points)
  - (c) If 1000 bits are transmitted, approximately how many “1” bits are sent? (3 points)
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### Problem 2: Conditional Probability and Bayes’ Theorem (12 points)

A receiver detects bits with the following error probabilities: -  $P(\text{detect } 1 \mid \text{sent } 0) = 0.01$  (false alarm) -  $P(\text{detect } 0 \mid \text{sent } 1) = 0.02$  (miss) -  $P(\text{sent } 0) = 0.7$  -  $P(\text{sent } 1) = 0.3$

- (a) What is  $P(\text{detect } 0 \mid \text{sent } 0)$ ? (2 points)
  - (b) What is the probability of detecting a “1”? (4 points)
  - (c) Given that a “1” was detected, what is the probability that a “1” was actually sent? Use Bayes’ theorem. (6 points)
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### Problem 3: Gaussian Distribution (15 points)

A received signal has amplitude that is Gaussian distributed with mean  $\mu = 5$  V and standard deviation  $\sigma = 2$  V.

- (a) Write the probability density function (PDF) for this signal. (3 points)
- (b) What is the probability that the amplitude is between 3 V and 7 V? (4 points)

(c) What is the probability that the amplitude exceeds 9 V? Express in terms of the Q-function. (4 points)

(d) If the threshold for detection is set at 4 V, what fraction of signals will be below this threshold? (4 points)

**Given:**  $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2} dt$

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#### Problem 4: Q-Function Calculations (12 points)

Use the Q-function for the following calculations:

(a) Calculate  $Q(1.5)$ . You may use the approximation  $Q(x) \approx \frac{1}{\sqrt{2\pi}x} \cdot e^{-x^2/2}$  for  $x > 3$ , or look up the value. (3 points)

(b) A bit error occurs if noise exceeds  $3\sigma$ . What is the probability of this event? (3 points)

(c) The complementary error function is  $\text{erfc}(x) = 2 \cdot Q(x\sqrt{2})$ . Express  $Q(x)$  in terms of  $\text{erfc}$ . (3 points)

(d) For  $\text{BER} = 10^{-6}$ , what value of SNR (as a ratio, not dB) is required if  $\text{BER} = Q(\sqrt{\text{SNR}})$ ? (3 points)

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#### Problem 5: Thermal Noise Power (15 points)

(a) Calculate the thermal noise power in a 1 MHz bandwidth at room temperature ( $T = 290$  K). Use  $k = 1.38 \times 10^{-23}$  J/K. (4 points)

(b) Express this noise power in dBm. (Recall:  $\text{dBm} = 10 \cdot \log_{10}(P/1 \text{ mW})$ ) (3 points)

(c) If the bandwidth increases to 20 MHz (WiFi channel), what is the new noise power in dBm? (3 points)

(d) A receiver has a noise figure  $\text{NF} = 6$  dB. What is the total noise power in 20 MHz? (5 points)

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#### Problem 6: Signal-to-Noise Ratio (15 points)

A communication system has the following parameters: - Received signal power:  $P_s = -80$  dBm - System bandwidth:  $B = 10$  MHz - Noise temperature:  $T = 290$  K - Noise figure:  $\text{NF} = 5$  dB

(a) Calculate the noise power in dBm. (5 points)

(b) Calculate the SNR in dB. (3 points)

(c) Convert the SNR to a linear ratio. (2 points)

(d) If the signal power increases by 3 dB, what is the new SNR? (3 points)

(e) What signal power is required to achieve  $\text{SNR} = 20$  dB? (2 points)

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### Problem 7: Noise in AM Systems (12 points)

An AM receiver has the following specifications: - Carrier power:  $P_c = 10$  W - Modulation index:  $\mu = 0.8$  - Noise power in the message bandwidth:  $P_n = 0.1$  W

- (a) Calculate the total transmitted power. (3 points)
  - (b) Calculate the signal power in the sidebands. (3 points)
  - (c) For DSB-SC (suppressed carrier), what is the output SNR? Assume the output SNR equals (sideband power)/(noise power). (3 points)
  - (d) For full carrier AM, the output SNR is reduced by the factor  $\frac{\mu^2}{2+\mu^2}$ . Calculate the output SNR for this AM system. (3 points)
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### Problem 8: FM Noise Performance (12 points)

An FM system has: - Carrier power:  $P_c = 20$  W - Noise power:  $P_n = 0.5$  W - Modulation index:  $\beta = 5$  - Message bandwidth:  $B_m = 15$  kHz

- (a) Calculate the input SNR (carrier power to noise power ratio). (3 points)
  - (b) The output SNR for FM is given by:  $(\text{SNR})_{\text{out}} = 3\beta^2(\beta + 1) \cdot (\text{SNR})_{\text{in}}$  for  $\beta \gg 1$ . Calculate the output SNR. (4 points)
  - (c) What is the FM improvement factor (output SNR / input SNR)? (3 points)
  - (d) Express the output SNR in dB. (2 points)
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### Problem 9: Additive White Gaussian Noise (AWGN) (10 points)

- (a) What does “white” mean in the context of white Gaussian noise? (3 points)
  - (b) What does “Gaussian” refer to? (2 points)
  - (c) The power spectral density of AWGN is  $N_0/2$  W/Hz. If  $N_0/2 = 10^{-9}$  W/Hz and the bandwidth is 5 MHz, what is the total noise power? (3 points)
  - (d) Why is AWGN a good model for thermal noise in communication systems? (2 points)
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### Problem 10: Link Budget Basics (12 points - Challenge)

A WiFi system operates at 2.4 GHz with the following parameters: - Transmit power:  $P_t = 20$  dBm - Transmit antenna gain:  $G_t = 2$  dBi - Receive antenna gain:  $G_r = 2$  dBi - Distance:  $d = 10$  m - System bandwidth: 20 MHz - Noise figure:  $NF = 7$  dB - Temperature:  $T = 290$  K

- (a) Calculate the free-space path loss at 2.4 GHz and 10 m using:  $\text{FSPL}(\text{dB}) = 20 \cdot \log_{10}(d) + 20 \cdot \log_{10}(f) + 32.45$ , where  $d$  is in km and  $f$  is in MHz. (4 points)
- (b) Calculate the received signal power  $P_r$  in dBm. (3 points)

- (c) Calculate the noise power in dBm. (3 points)
- (d) Calculate the SNR in dB. (2 points)
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**Bonus Problem 1: Shannon's Theorem (5 points extra credit)**

Shannon's theorem states that the channel capacity is:  $C = B \cdot \log_2(1 + \text{SNR})$

where C is in bits/s, B is bandwidth in Hz, and SNR is a linear ratio.

- (a) Calculate the channel capacity for a system with  $B = 20$  MHz and  $\text{SNR} = 30$  dB. (3 points)
- (b) If you want to transmit at 100 Mbps with  $B = 20$  MHz, what minimum SNR (in dB) is required? (2 points)
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**Bonus Problem 2: Noise Temperature (5 points extra credit)**

(a) A receiver has a noise figure  $\text{NF} = 3$  dB. Calculate the equivalent noise temperature  $T_e$ . Use the relationship:  $\text{NF} = 1 + \frac{T_e}{T_0}$  where  $T_0 = 290$  K. (3 points)

(b) If two amplifiers are cascaded with noise figures  $\text{NF}_1 = 3$  dB and  $\text{NF}_2 = 6$  dB, and gains  $G_1 = 20$  dB and  $G_2 = 10$  dB, calculate the overall noise figure using Friis formula:  $F_{\text{total}} = F_1 + \frac{F_2 - 1}{G_1}$  where F is the noise factor (linear, not dB). (2 points)

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**Submission Instructions:** - Submit a single PDF via Brightspace - Show all conversions between dB and linear units - For probability problems, show the complete calculation - Q-function values can be looked up or approximated

**Academic Integrity:** - Individual work required - Show all work for full credit - Cite any formula references

**Useful Constants:** - Boltzmann constant:  $k = 1.38 \times 10^{-23}$  J/K - Room temperature:  $T_0 = 290$  K - Speed of light:  $c = 3 \times 10^8$  m/s