# EE 451: Communications Systems

# Final Exam - Comprehensive Study Guide

**Exam Date:** Thursday, May 21, 2026 (12:45-2:45 PM) **Duration:** 180 minutes (3 hours) **Coverage:** All course material (Chapters 1-11) with emphasis on Chapters 8-11 **Format:** Closed book, one 8.5" × 11" formula sheet (both sides) allowed

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## Overview

The final exam is **comprehensive** but with **emphasis on later material** (probability, noise, BER, link budgets). Structure: - **Part I (30 pts):** Fundamentals review (Fourier, AM, FM/PM) - **Part II (35 pts):** Digital modulation (BPSK, QPSK, FSK, QAM) - **Part III (40 pts):** Probability and noise (Gaussian, SNR, BER) - **Part IV (35 pts):** Link budgets and system performance - **Part V (10 pts):** Advanced topics (OFDM, spread spectrum) - **Bonus (10 pts):** Satellite link analysis

**Key insight:** 85 of 150 points (57%) come from material AFTER Midterm 2!

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## Study Strategy

### Priority Levels

**HIGHEST PRIORITY** (Study first - 60% of exam) - Link budget calculations (Problems 10, 11) - Gaussian distribution and Q-function (Problem 7) - Thermal noise and SNR calculations (Problem 8) - BER analysis for BPSK (Problem 9) - Shannon capacity (Problem 11)

**MEDIUM PRIORITY** (Important - 25% of exam) - BPSK vs QPSK comparison (Problem 4) - 16-QAM and Gray coding (Problem 5) - FSK system design (Problem 6) - Adaptive modulation (Problem 12)

**LOWER PRIORITY** (Review quickly - 15% of exam) - Fourier transforms (Problem 1) - AM power calculations (Problem 2) - FM vs PM (Problem 3) - OFDM and spread spectrum concepts (Problem 13)

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## Part I: Fundamentals Review (30 points)

### Quick Review Topics

**Fourier Transforms (10 pts)** - Sinc   rect transform pair - Time-frequency duality - Modulation property (DSB-SC spectrum)

**AM (10 pts)** - Modulation index $\mu = A_m/A_c$ - Power: $P_{total} = \frac{A_c^2}{2R}(1 + \mu^2/2)$ - Efficiency: $\eta = \frac{\mu^2/2}{1+\mu^2/2}$

**FM/PM (10 pts)** - FM: $\Delta f = k_f A_m$ (independent of $f_m$) - PM: $\Delta f = k_p A_m f_m$ (proportional to $f_m$) - Carson's rule: $B_{FM} = 2(\Delta f + f_m)$

**Study Tip:** These are REVIEW topics. Don't spend too much time here if you understand Midterms 1 & 2 material. Focus on new material!

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## Part II: Digital Modulation (35 points)

### BPSK vs QPSK (12 pts - Problem 4)

**Key Comparisons:**

| Feature | BPSK | QPSK |
|---|---|---|
| Bits/symbol | 1 | 2 |
| Symbol rate | $R_b$ | $R_b/2$ |
| Bandwidth | $2R_b$ | $R_b$ |
| Spectral efficiency | 0.5 bits/s/Hz | 1.0 bits/s/Hz |
| BER (same $E_b/N_0$) | Same! | Same! |

**Critical insight:** QPSK is twice as bandwidth efficient with no BER penalty!

### QAM and Gray Coding (12 pts - Problem 5)

**16-QAM Basics:** - M = 16 symbols, k = 4 bits/symbol - Rectangular constellation: 4×4 grid - Amplitude levels: typically {±1, ±3}

**Gray Coding Rules:** - Adjacent symbols differ by 1 bit - Minimizes BER (most errors → adjacent symbols) - Mandatory in modern systems!

**EVM (Error Vector Magnitude):** - Measures constellation quality - EVM% = (Error distance / Reference distance) × 100% - WiFi/LTE have strict EVM limits (e.g., 256-QAM < 3%)

### FSK System Design (11 pts - Problem 6)

**Orthogonal FSK:** - Minimum separation: $\Delta f_{min} = \frac{1}{2T_b}$ - Modulation index: $h = \Delta f \times T_b$ - MSK: $h = 0.5$ exactly

**Quick formulas:**

```
Tb = 1/Rb
Δf = |f  - f |
h = Δf·Tb
Orthogonal: Δf   1/(2Tb)
MSK: h = 0.5
```

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## Part III: Probability and Noise (40 points)

### THIS IS THE MOST IMPORTANT SECTION!

**Gaussian Distribution (15 pts - Problem 7)**

**PDF of Gaussian:**

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)}$$

**Standardization:**

$$Z = \frac{X-\mu}{\sigma} \sim N(0,1)$$

**Q-Function:**

$$Q(x) = P(Z > x) \text{ for } Z \sim N(0,1)$$

**For general Gaussian X ~ N( , ²):**

$$P(X > a) = Q\left(\frac{a-\mu}{\sigma}\right)$$

**Key Q-function values (memorize these!):**

| | | | |
|---|---|---|---|
| Q(1.0) | 0.159 | Q(2.0) | 0.023 |
| Q(1.5) | 0.067 | Q(3.0) | 0.0013 |
| Q(4.0) | 3e-5 | Q(4.75) | 1e-6 |

**Property:** $Q(-x) = 1 - Q(x)$

**Practice:** 1. Identify and from problem 2. Standardize: $z = (x-\mu)/\sigma$ 3. Express using Q-function 4. Look up or calculate

**Thermal Noise and SNR (12 pts - Problem 8)**

**Thermal Noise Power:**

$$P_n = kTB$$

where: - $k = 1.38 \times 10^{-23}$ J/K (Boltzmann constant) - $T$ = temperature (K) - $B$ = bandwidth (Hz)

**Noise Figure:**

$$P_{n,total} = P_n \times F = P_n \times 10^{NF_{dB}/10}$$

**In dBm:**

$$P_n(dBm) = 10\log_{10}\left(\frac{kTB}{10^{-3}}\right)$$

**Quick calculation:** At T = 290 K:

$$P_n(dBm) \approx -174 + 10\log_{10}(B_{Hz})$$

**SNR:**

$$SNR(dB) = P_s(dBm) - P_n(dBm)$$

**Critical conversions:**

```
dBm to W: P_W = 10^((P_dBm - 30)/10)
W to dBm: P_dBm = 10·log (P_W) + 30
dB addition: Ptotal(dB) = P1(dB) + P2(dB)
Linear addition: Ptotal_W = P1_W + P2_W
```

**Common values to know:**

```
0 dBm = 1 mW
-30 dBm = 1  W
-60 dBm = 1 nW
-90 dBm = 1 pW

+3 dB = ×2
+10 dB = ×10
+20 dB = ×100
```

**BER Performance (13 pts - Problem 9)**

**BPSK BER:**

$$BER_{BPSK} = Q(\sqrt{2E_b/N_0})$$

**Conversions:**

$$\frac{E_b}{N_0}(dB) = 10\log_{10}(E_b/N_0)$$

**To find linear from dB:**

$$\frac{E_b}{N_0} = 10^{(E_b/N_0)_{dB}/10}$$

**Bit errors per second:**

$$N_{errors/s} = BER \times R_b$$

**Common $E_b/N_0$ values for BER targets:**

```
BER = 10 ³: Eb/N0   6.8 dB
BER = 10 : Eb/N0   8.4 dB
BER = 10 : Eb/N0   9.6 dB
BER = 10 : Eb/N0   10.5 dB
```

**Step-by-step for BER problems:** 1. Convert $E_b/N_0$ from dB to linear 2. Calculate $\sqrt{2E_b/N_0}$ 3. Find $Q(\sqrt{2E_b/N_0})$ using table 4. This is your BER 5. Errors/sec = BER $\times$ bit rate

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## Part IV: Link Budgets (35 points)

**CRITICAL SECTION - WORTH 35 POINTS!**

**Free-Space Path Loss (FSPL)**

**Two equivalent forms:**

**Form 1 (dB, easier for calculations):**

$$FSPL(dB) = 20\log_{10}(d_{km}) + 20\log_{10}(f_{MHz}) + 32.45$$

**Form 2 (from wavelength):**

$$FSPL(dB) = 20\log_{10}\left(\frac{4\pi d}{\lambda}\right)$$

where $\lambda = c/f$

**Example calculation:**

```
f = 5.8 GHz = 5800 MHz
d = 50 m = 0.05 km

FSPL = 20·log (0.05) + 20·log (5800) + 32.45
     = 20·(-1.301) + 20·(3.763) + 32.45
     = -26.02 + 75.26 + 32.45
     = 81.69 dB
```

**Complete Link Budget (Problem 10 - 18 pts)**

**Link Budget Equation:**

$$P_r(dBm) = P_t(dBm) + G_t(dBi) + G_r(dBi) - FSPL(dB) - L_{misc}(dB)$$

**Step-by-step procedure:**

**1. Calculate FSPL** - Use formula above - Make sure units match (km and MHz, OR meters and Hz)

**2. Calculate received power**

$$P_r = P_t + G_t + G_r - FSPL - L_{other}$$

**3. Calculate noise power**

$$P_n = kTB \text{ (in W)}$$

$$P_n(dBm) = 10\log_{10}(P_n/0.001)$$

OR use quick formula:

$$P_n(dBm) \approx -174 + 10\log_{10}(B_{Hz}) + NF(dB)$$

**4. Calculate SNR**

$$SNR(dB) = P_r(dBm) - P_n(dBm) - \text{fade margin}$$

**5. Determine supported modulation** Compare SNR to thresholds:

```
QPSK:    8 dB
16-QAM:  15 dB
64-QAM:  22 dB
256-QAM: 28 dB
```

**Common mistakes:** Forgetting to convert distance to km or frequency to MHz Mixing up + and - signs in link budget Forgetting fade margin Not adding noise figure to thermal noise

### Shannon Capacity (Problem 11 - 10 pts)

**Shannon-Hartley Theorem:**

$$C = B \log_2(1 + SNR)$$

bits/s

where: - $C$ = channel capacity (bits/s) - $B$ = bandwidth (Hz) - $SNR$ = linear ratio (NOT dB!)

**Convert SNR from dB:**

$$SNR = 10^{SNR(dB)/10}$$

**Spectral Efficiency:**

$$\eta = \frac{C}{B} = \log_2(1 + SNR)$$

bits/s/Hz

**Example:**

```
B = 10 MHz, SNR = 20 dB

SNR_linear = 10^(20/10) = 100

C = 10×10  × log (1 + 100)
  = 10×10  × log (101)
  = 10×10  × 6.658
  = 66.58 Mbps

 = 6.658 bits/s/Hz
```

**Common mistake:** Using SNR in dB directly (must convert to linear!)

### Adaptive Modulation (Problem 12 - 7 pts)

**Concept:** - Choose modulation based on SNR - Higher SNR → higher-order modulation → more bits/s - Lower SNR → lower-order modulation → more reliable

**SNR vs. distance model:**

$$SNR(dB) = SNR_0 - n \times 10 \log_{10}(d/d_0)$$

where $n$ is path loss exponent (typical: 2-4)

**Procedure:** 1. Calculate SNR at each distance 2. Compare to modulation thresholds 3. Select highest modulation with SNR threshold

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## Part V: Advanced Topics (10 points)

### OFDM (4 pts)

**Why OFDM?** - Solves frequency-selective fading problem - Divides wideband channel into many narrowband subchannels - Each subchannel experiences flat fading - Simpler equalization than single-carrier

**Applications:** - WiFi (802.11a/g/n/ac/ax) - LTE, 5G NR - DVB-T (digital TV)

**Key concept:** Convert frequency-selective fading → flat fading per subcarrier

### Spread Spectrum (6 pts)

**DSSS (Direct Sequence Spread Spectrum):** - Data rate: $R_b$ - Chip rate: $R_c$ (much higher) - Processing gain: $G_p = \frac{R_c}{R_b}$ (ratio) - In dB: $G_p(dB) = 10\log_{10}(R_c/R_b)$

**Advantages:** - Jamming resistance (processing gain) - Multiple access (CDMA) - Low probability of intercept

**Example:**

```
Rb = 1 Mbps, Rc = 10 Mchips/s
Gp = 10 Mchips/s / 1 Mbps = 10
Gp(dB) = 10·log (10) = 10 dB
```

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## Bonus: Satellite Link (10 points)

**Typical satellite parameters:** - High transmit power: 40 dBW (10 kW) - High antenna gains: 30-40 dBi - Long distance: 36,000 km (GEO) - Very high FSPL: ~200 dB! - Low noise temperature: 150 K (LNA)

**$E_b/N_0$ and SNR relationship:**

$$\frac{E_b}{N_0} = SNR \times \frac{B}{R_b}$$

**Use this to find maximum bit rate:**

$$R_b = \frac{SNR \times B}{E_b/N_0}$$

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## Master Formula Sheet

### Probability & Noise

```
Gaussian PDF:
  f(x) = (1/( √(2 ))) exp(-(x- )²/(2 ²))

Q-function:
  P(X > a) = Q((a- )/ ) for X ~ N( ,  ²)
  Q(-x) = 1 - Q(x)
```

```
Thermal Noise:
  Pn = kTB  (k = 1.38×10^23 J/K)
  Pn(dBm)   -174 + 10log (B_Hz) + NF(dB)  [at T=290K]

SNR:
  SNR(dB) = Ps(dBm) - Pn(dBm)

BER:
  BER_BPSK = Q(√(2Eb/N0))
```

### Link Budget

```
FSPL:
  FSPL(dB) = 20log (d_km) + 20log (f_MHz) + 32.45

Link Budget:
  Pr(dBm) = Pt(dBm) + Gt + Gr - FSPL - Losses

Shannon Capacity:
  C = B·log (1 + SNR)  [SNR must be linear!]

Eb/N0 vs SNR:
  Eb/N0 = SNR × (B/Rb)
```

### Digital Modulation (Review)

```
BPSK: k=1, Rs=Rb, B=2Rb
QPSK: k=2, Rs=Rb/2, B=Rb
M-ary: k=log (M), Rs=Rb/k

FSK: h = Δf·Tb
     Orthogonal: Δf = 1/(2Tb)
     MSK: h = 0.5
```

### Analog Modulation (Review)

```
AM:
  s(t) = Ac[1 +  ·mn(t)]cos(2 fct)
  Ptotal = (Ac^2/2R)(1 +  ^2/2)
    =  ^2/(2 +  ^2)

FM:
  Δf = kf·max|m(t)|
    = Δf/fm
  BFM = 2(Δf + fm)

PM:
  Δ  = kp·max|m(t)|
```

```
Δf_PM = kp·Am·fm  (depends on fm!)
```

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### Exam Strategy for 180 Minutes

#### Time Allocation

**Part I (30 pts) - 25 minutes** - Problem 1: 7 min (Fourier - straightforward) - Problem 2: 10 min (AM power) - Problem 3: 8 min (FM vs PM)

**Part II (35 pts) - 30 minutes** - Problem 4: 10 min (BPSK vs QPSK) - Problem 5: 12 min (16-QAM, Gray coding) - Problem 6: 8 min (FSK design)

**Part III (40 pts) - 50 minutes** - Problem 7: 18 min (Gaussian & Q-function - 5 parts!) - Problem 8: 15 min (Thermal noise & SNR) - Problem 9: 17 min (BER analysis)

**Part IV (35 pts) - 45 minutes** - Problem 10: 25 min (WiFi link budget - longest problem!) - Problem 11: 12 min (Shannon capacity) - Problem 12: 8 min (Adaptive modulation)

**Part V (10 pts) - 10 minutes** - Problem 13: 10 min (OFDM & spread spectrum concepts)

**Bonus (10 pts) - 10 minutes** (if time!) - Satellite link analysis

**Review: 10 minutes**

#### Priority Order (if running low on time)

1. **Do first:** Problems 7, 8, 9, 10 (new material, high points)
2. **Do second:** Problems 11, 12 (new material)
3. **Do third:** Problems 4, 5, 6 (review, but still good points)
4. **Do last:** Problems 1, 2, 3 (pure review)
5. **If time:** Problems 13 and Bonus

#### Calculator Tips

**Pre-calculate common values:**

```
log (2)   0.301
log (10) = 1
ln(2)   0.693

10 · ³   2
10 ·   3.16
10¹ ·  = 10

For log (x): log (x) = log (x)/log (2)   3.32·log (x)
```

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## Common Mistakes to AVOID

### Part III (Probability/Noise)

Forgetting to standardize Gaussian: $z = (x-\mu)/\sigma$ Using SNR in dB directly in Shannon formula (must convert to linear!) Forgetting to add noise figure to thermal noise Confusing Q(x) with 1-Q(x) Using wrong Boltzmann constant or temperature

### Part IV (Link Budgets)

Wrong units in FSPL: using meters instead of km, or Hz instead of MHz Wrong signs in link budget (it's Pt + Gt + Gr - FSPL, not all positive!) Forgetting fade margin when calculating SNR Comparing to wrong modulation thresholds Not converting $E_b/N_0$ from dB when using formulas

### General

Not showing units in answers Not checking if answer makes physical sense Spending too long on early problems (save time for Parts III & IV!) Forgetting to box/circle final answers

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## Self-Assessment Checklist

### Can you:

**Probability & Noise:** - [ ] Write Gaussian PDF - [ ] Standardize and use Q-function - [ ] Calculate thermal noise power (kTB) - [ ] Convert between dBm and Watts - [ ] Add noise figure correctly - [ ] Calculate SNR in dB - [ ] Convert $E_b/N_0$ between dB and linear - [ ] Calculate BER for BPSK - [ ] Find bit errors per second

**Link Budgets:** - [ ] Calculate FSPL with correct units - [ ] Perform complete link budget (Pr = Pt + gains - losses) - [ ] Calculate noise power in dBm - [ ] Calculate SNR accounting for fade margin - [ ] Select modulation based on SNR - [ ] Calculate Shannon capacity (with SNR in linear!) - [ ] Apply adaptive modulation rules

**Digital Modulation:** - [ ] Compare BPSK vs QPSK (bandwidth, spectral efficiency) - [ ] Draw 16-QAM constellation - [ ] Apply Gray coding - [ ] Calculate EVM - [ ] Design orthogonal FSK system - [ ] Identify MSK

**Review Topics:** - [ ] Fourier transform of rect/sinc - [ ] AM power and efficiency - [ ] FM vs PM frequency deviation

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## Final Study Plan (1 Week Before Exam)

### Days 7-6: New Material (Priority )

- **Study:** Homework 5 (all problems) - Probability, noise, SNR
- **Practice:** BER and link budget problems from lecture notebooks
- **Practice:** Midterm 1 solutions (Problems 7-9)
- **Focus:** Link budget procedure, Q-function, thermal noise

**Days 5-4: Integration**

- **Practice:** Complete a full link budget from scratch
- **Practice:** Gaussian/Q-function problems
- **Practice:** BER calculations
- **Review:** Shannon capacity problems
- **Make:** Your formula sheet!

**Days 3-2: Review Material**

- **Quick review:** Homework 3 & 4 (digital modulation)
- **Quick review:** Homework 1 & 2 (Fourier, AM)
- **Practice:** Midterm 2 (all problems quickly)
- **Refine:** Formula sheet

**Day 1: Final Prep**

- **Take:** Practice final (under time pressure!)
- **Review:** Common mistakes list
- **Check:** Formula sheet is complete
- **Sleep:** Well!

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## What to Bring

Calculator (scientific, not graphing) Formula sheet (both sides, handwritten recommended) Pencils/pens Student ID

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## During the Exam

1. **Read all problems first** (5 min)
2. **Mark easy problems** with a star
3. **Do high-value problems first** (Parts III & IV)
4. **Show ALL work** - partial credit is generous!
5. **Box final answers**
6. **Check units** on every answer
7. **Sanity check:** Does the answer make sense?
8. **If stuck:** Move on, come back later
9. **Last 10 min:** Review, check signs, check units

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## Final Thoughts

**The exam rewards:** - Systematic problem-solving (link budgets!) - Careful unit management (km vs m, dB vs linear) - Understanding relationships (Shannon, BER, SNR)

**Focus your final study on:** 1. Link budget procedure (practice 3-5 complete examples) 2. Q-function and BER calculations 3. SNR and noise calculations 4. Digital modulation comparisons

**You've got this! The comprehensive nature means you can show your understanding across the whole course. Focus on the new material (Parts III-IV) and you'll do great!**

**Good luck on your final exam!**