EE 451: Communications Systems

Homework 6 - BER, Link Budgets, and System Analysis

Topics: Bit error rate, matched filtering, link budgets, WiFi/cellular systems **Textbook Reference:** Haykin & Moher, Chapters 10, 11.4-11.7

Total Points: 100

Instructions

- Show all work for full credit
- Include units in all final answers
- For link budgets, show the complete calculation chain
- State all assumptions clearly

Problem 1: BER for BPSK (12 points)

A BPSK system operates with $E_b/N_0 = 10$ dB.

(a) Convert E_b/N_0 to a linear ratio. (2 points)

(b) The BER for BPSK is given by: BER $= Q(\sqrt{2E_b/N_0})$. Calculate the BER. (4 points)

(c) If the bit rate is 1 Mbps, approximately how many bit errors occur per second? (3 points)

(d) What E_b/N_0 is required to achieve BER $= 10^{-6}$? (3 points)

Problem 2: BER Comparison (15 points)

Compare the BER performance of BPSK, QPSK, and 8-PSK.

(a) For BPSK: BER $= Q(\sqrt{2E_b/N_0})$ For QPSK: BER $\approx Q(\sqrt{2E_b/N_0})$ (approximately same as BPSK) For 8-PSK: BER $\approx \frac{2}{3} \cdot Q(\sqrt{6E_b/N_0} \cdot \sin(\pi/8))$

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Calculate the BER for each modulation at $E_b/N_0 = 12$ dB. (6 points)
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(b) Which modulation has the best BER performance? Why? (3 points)

(c) Which modulation has the highest spectral efficiency? (3 points)

(d) If you need to transmit 10 Mbps in 5 MHz bandwidth, which modulation should you choose? Consider both BER and spectral efficiency. (3 points)

Problem 3: Matched Filtering (12 points)

(a) Explain what a matched filter is and why it's optimal for detecting signals in AWGN. (4 points)

(b) For a rectangular pulse of duration T and amplitude A, what is the output SNR of a matched filter? Express in terms of signal energy E_s and noise power spectral density N_0. (4 points)

(c) A BPSK system uses matched filtering. If the bit energy is $E_b = 10^{-13}$ J and $N_0 = 10^{-20}$ W/Hz, calculate the output SNR. (4 points)

Problem 4: QAM Bit Error Rate (12 points)

For M-QAM, the symbol error rate (SER) is approximately: $\text{SER} \approx 4 \cdot \left(1 - \frac{1}{\sqrt{M}}\right) \cdot Q\left(\sqrt{\frac{3E_s}{N_0(M-1)}}\right)$

For 16-QAM at $E_s/N_0 = 18$ dB:

(a) Convert E_s/N_0 to linear ratio. (2 points)

(b) Calculate the symbol error rate (SER). (4 points)

(c) For Gray coding, approximate $\text{BER} \approx \text{SER}/\log_2(M)$. Calculate the BER. (3 points)

(d) If the symbol rate is 10 Msymbols/s, what is the bit rate and how many bit errors per second occur? (3 points)

Problem 5: Complete Link Budget - WiFi (18 points)

A WiFi 6 system operates at 5 GHz with the following parameters: - Transmit power: $P_t = 23$ dBm (200 mW, FCC limit for 5 GHz) - Transmit antenna gain: $G_t = 3$ dBi - Receive antenna gain: $G_r = 0$ dBi (laptop internal antenna) - Frequency: 5.0 GHz - Distance: 30 m - System bandwidth: 80 MHz - Noise figure: NF = 6 dB - Temperature: T = 290 K - Implementation loss: 3 dB - Fade margin: 10 dB

(a) Calculate free-space path loss using: $\text{FSPL(dB)} = 20 \cdot \log_{10}(4\pi d/\lambda)$, where $\lambda = c/f$. (4 points)

(b) Calculate received signal power P_r including all gains and losses. (4 points)

(c) Calculate noise power P_n in the 80 MHz bandwidth. (3 points)

(d) Calculate the SNR. (2 points)

(e) Based on the SNR, what is the highest modulation that can be used? Use these thresholds: - BPSK/QPSK: 5 dB - 16-QAM: 12 dB - 64-QAM: 20 dB - 256-QAM: 28 dB - 1024-QAM: 35 dB (3 points)

(f) Calculate the maximum data rate (assuming ideal spectral efficiency and no overhead). (2 points)

Problem 6: Link Budget - Cellular 5G (15 points)

A 5G base station communicates with a smartphone at 3.5 GHz: - Base station transmit power: $P_t = 43$ dBm (20 W) - Base station antenna gain: $G_t = 15$ dBi (directional) - Smartphone antenna gain: $G_r = -2$ dBi - Distance: 500 m - Path loss exponent: n = 3.5 (urban environment) - Shadow fading margin: 8 dB - Bandwidth: 100 MHz - Noise figure: NF = 7 dB

(a) Calculate path loss using: $\text{PL(dB)} = \text{FSPL}(d_0) + 10n \cdot \log_{10}(d/d_0)$, where $d_0 = 1$ m. (5 points)

(b) Calculate received power. (3 points)

(c) Calculate noise power. (2 points)

(d) Calculate SNR. (2 points)

(e) Using Shannon's theorem $C = B \cdot \log_2(1 + \text{SNR})$, calculate the theoretical channel capacity. (3 points)

Problem 7: WiFi vs. Cellular Comparison (10 points)

(a) Compare the link budgets from Problems 5 and 6. Which system can communicate over longer distances? Why? (4 points)

(b) Which system provides higher data rates? Explain the trade-offs. (3 points)

(c) Explain why cellular systems use higher transmit power and higher antenna gain compared to WiFi. (3 points)

Problem 8: Adaptive Modulation System (12 points)

A wireless link uses adaptive modulation with the following SNR thresholds: - QPSK: 8 dB, data rate = 20 Mbps - 16-QAM: 15 dB, data rate = 40 Mbps - 64-QAM: 22 dB, data rate = 60 Mbps - 256-QAM: 30 dB, data rate = 80 Mbps

The received SNR varies with distance as: $\text{SNR(dB)} = 45 - 30 \cdot \log_{10}(d/10)$, where d is in meters.

(a) Calculate the SNR at distances of 10 m, 20 m, 50 m, and 100 m. (4 points)

(b) For each distance, determine which modulation scheme is used. (4 points)

(c) Calculate the average data rate if the user spends equal time at each of these four distances. (2 points)

(d) What is the maximum distance at which communication is possible ($\text{SNR} \geq 8$ dB)? (2 points)

Problem 9: Multipath and Fading (10 points)

(a) Explain how multipath propagation affects wireless communication systems. (3 points)

(b) What is frequency-selective fading and why does it occur? (3 points)

(c) How do OFDM-based systems (WiFi, LTE, 5G) mitigate frequency-selective fading? (4 points)

Problem 10: Real-World System Design (14 points - Challenge)

You are designing a long-range WiFi link for a rural area: - Frequency: 2.4 GHz (better propagation than 5 GHz) - Required distance: 1 km - Required data rate: 50 Mbps - Available transmit power: 30 dBm (1 W) - Available antennas: $G_t = G_r = 10$ dBi (directional) - Bandwidth: 20 MHz - Temperature: T = 290 K - Noise figure: NF = 5 dB - Required SNR for 64-QAM: 22 dB - Fade margin: 15 dB (for reliability)

(a) Calculate the free-space path loss at 1 km and 2.4 GHz. (3 points)

(b) Calculate the link budget and determine the received SNR. (5 points)

(c) Can the system achieve the required SNR of 22 dB with 15 dB fade margin? (2 points)

(d) If not, suggest three modifications to improve the link budget and calculate the improvement from each. (4 points)

Bonus Problem 1: Friis Transmission Equation (5 points extra credit)

The Friis transmission equation relates received power to transmitted power: $P_r = P_t \cdot G_t \cdot G_r \cdot \left(\frac{\lambda}{4\pi d}\right)^2$

(a) Derive the free-space path loss formula FSPL(dB) $= 20 \cdot \log_{10}(d) + 20 \cdot \log_{10}(f) + 32.45$ (with d in km and f in MHz) from the Friis equation. (3 points)

(b) Verify your derivation by calculating FSPL at d = 1 km and f = 1 GHz. (2 points)

Bonus Problem 2: Eb/N0 vs. SNR Relationship (5 points extra credit)

For a digital communication system: - Bit rate: R_b - Bandwidth: B - SNR = signal power / noise power - E_b/N_0 = bit energy / noise power spectral density

(a) Show that $E_b/N_0 = \text{SNR} \cdot (B/R_b)$. (3 points)

(b) For QPSK with $R_b = 10$ Mbps in B = 5 MHz, if SNR = 12 dB, calculate E_b/N_0. (2 points)

Submission Instructions: - Submit a single PDF via Brightspace - Show complete link budget calculations - Include all unit conversions (dB $\leftrightarrow$ linear) - Clearly label each step

Academic Integrity: - Individual work required - Show all work for partial credit - Cite any references to standards or specifications

Useful Formulas: - FSPL(dB) $= 20 \cdot \log_{10}(d) + 20 \cdot \log_{10}(f) + 32.45$ (d in km, f in MHz) - FSPL(dB) $= 20 \cdot \log_{10}(4\pi d/\lambda)$ (d and λ in same units) - $P_n = k \cdot T \cdot B$ (thermal noise) - $k = 1.38 \times 10^{-23}$ J/K - $c = 3 \times 10^8$ m/s - Shannon: $C = B \cdot \log_2(1 + \text{SNR})$