

# EE 451: Communications Systems

## Midterm Exam 2 - Study Guide

**Exam Date:** Week 11 (April 15, 2026) **Duration:** 90 minutes **Coverage:** Chapters 4-7 (FM, PM, FSK, PSK, M-ary Modulation, QAM) **Format:** Closed book, one 8.5" × 11" formula sheet (both sides) allowed

---

### Overview

Midterm 2 focuses on angle modulation (FM/PM) and digital modulation schemes. You should be comfortable with: - FM and PM fundamentals and relationships - Carson's rule for FM bandwidth - Binary digital modulation (FSK, BPSK) - M-ary modulation (QPSK, QAM) - Gray coding and constellation diagrams - Spectral efficiency concepts

---

### Topic 1: Frequency Modulation (FM) Fundamentals

#### Key Concepts

- 1. FM Signal Characteristics** - Frequency varies with message signal - Amplitude remains constant (noise immunity!) - Frequency deviation:  $\Delta f = k_f \max |m(t)|$  - Modulation index:  $\beta = \Delta f / f_m$
- 2. Narrowband vs. Wideband FM - NBFM:**  $\beta \ll 1$  (typically  $\beta < 0.3$ ) - **WBFM:**  $\beta \gg 1$  (broadcast FM:  $\beta = 5$ ) - Different bandwidth requirements!
- 3. FM vs. AM** - AM: Constant frequency, varying amplitude - FM: Constant amplitude, varying frequency - FM: Better noise performance (but more bandwidth)

#### Essential Formulas

FM Signal:

$$s(t) = A \cos[2\pi f_c t + 2\pi k_f \int m(t) dt]$$

Instantaneous Frequency:

$$f_i(t) = f_c + k_f \cdot m(t)$$

where  $k_f$  = frequency sensitivity (Hz/V)

Frequency Deviation:

$$\Delta f = k_f \cdot \max |m(t)|$$

For  $m(t) = A_m \cdot \cos(2\pi f_m t)$ :

$$\Delta f = k_f \cdot A_m$$

Modulation Index:

$$= \Delta f / f_m$$

Carson's Rule (FM Bandwidth):

$$\text{BFM} = 2(\Delta f + f_m) = 2f_m(1 + \beta)$$

NBFM Criterion:

$\beta \ll 1$  (typically  $\beta < 0.3$ )

BFM  $\approx 2f_m$  (like AM!)

WBFM:

$\beta \gg 1$

BFM  $\approx 2\Delta f$  (dominated by deviation)

## Common Mistakes to Avoid

Confusing  $\Delta f$  (deviation) with  $f_m$  (message frequency) Forgetting that  $\Delta f$  depends on  $k_f$  AND message amplitude Using wrong Carson's rule form (two equivalent forms!) Mixing up NBFM and WBFM criteria

## Practice Problems

From **Homework 3**: - Problem 1: FM fundamentals - Problem 2: Carson's rule application - Problem 3: NBFM vs. WBFM classification

**Strategy:** 1. Identify  $k_f$ ,  $A_m$ ,  $f_m$  from problem 2. Calculate  $\Delta f = k_f \times A_m$  3. Calculate  $\beta = \Delta f / f_m$  4. Check if NBFM ( $\beta < 0.3$ ) or WBFM ( $\beta > 1$ ) 5. Apply Carson's rule:  $B = 2(\Delta f + f_m)$

---

## Topic 2: Phase Modulation (PM)

### Key Concepts

**1. PM Signal Characteristics** - Phase varies directly with message -  $\theta_i(t) = 2\pi f_c t + k_p m(t)$  - Phase sensitivity:  $k_p$  (rad/V)

**2. FM vs. PM Relationship - Key insight:** PM = FM with derivative - FM can be generated from PM by integrating message - PM can be generated from FM by differentiating message

**3. Frequency Deviation in PM** - PM has frequency deviation that **increases with  $f_m$** ! - FM has frequency deviation **independent of  $f_m$**  - This is the KEY difference!

### Essential Formulas

PM Signal:

$$s(t) = A \cos[2\pi f_c t + k_p m(t)]$$

where  $k_p$  = phase sensitivity (rad/V)

Instantaneous Phase:

$$\phi(t) = 2\pi f_c t + k_p m(t)$$

Phase Deviation:

$$\Delta\phi = k_p \cdot \max|m(t)|$$

Instantaneous Frequency:

$$f_i(t) = (1/2) \cdot d i / dt = f_c + (k_f/2) \cdot dm/dt$$

For  $m(t) = A_m \cdot \cos(2\pi f_m t)$ :

Maximum Frequency Deviation:

$$\Delta f_{PM} = k_f \cdot A_m \cdot f_m$$

(Note: proportional to  $f_m$ !)

Comparison:

FM:  $\Delta f = k_f \cdot A_m$  (independent of  $f_m$ )

PM:  $\Delta f = k_f \cdot A_m \cdot f_m$  (proportional to  $f_m$ )

### Common Mistakes to Avoid

Thinking PM and FM have the same  $\Delta f$  behavior Forgetting to take derivative when finding  $f_i(t)$  from PM Confusing  $\Delta\phi$  (phase deviation) with  $\Delta f$  (frequency deviation)

### Practice Problems

From **Homework 3**: - Problem 4: PM analysis - Problem 5: FM vs. PM relationship

**Strategy:** 1. For PM: Write  $\theta_i(t) = 2\pi f_c t + k_p m(t)$  2. Find  $\Delta\phi = k_p \times \max|m(t)|$  3. Find  $f_i(t) = \frac{1}{2\pi} \frac{d\theta_i}{dt}$  4. For sinusoidal message:  $\Delta f = k_p \times A_m \times f_m$

---

## Topic 3: Binary FSK (Frequency Shift Keying)

### Key Concepts

- FSK Basics** - Bit "0"  $\rightarrow$  Frequency  $f_0$  - Bit "1"  $\rightarrow$  Frequency  $f_1$  - Frequency separation:  $\Delta f = |f_1 - f_0|$
- Modulation Index for FSK** -  $h = \Delta f \times T_b$  - Critical parameter for FSK design - Determines orthogonality and spectral efficiency
- Orthogonality for Coherent Detection** - Minimum separation:  $\Delta f_{min} = \frac{1}{2T_b}$  - This gives  $h = 0.5$  (MSK!) - Larger separation OK, but wastes bandwidth
- Coherent vs. Non-Coherent Detection** - **Coherent:** Requires phase lock, better performance - **Non-coherent:** Simpler, no phase tracking needed - FSK can use either (unlike PSK which requires coherent)

### Essential Formulas

FSK Signal Expressions:

$$s(t) = A \cdot \cos(2\pi f_0 t) \quad (\text{bit "0"})$$

$$s(t) = A \cdot \cos(2\pi f_1 t) \quad (\text{bit "1"})$$

Frequency Separation:

$$\Delta f = |f_1 - f_0|$$

Bit Duration:

$$T_b = 1/R_b \quad (\text{where } R_b = \text{bit rate})$$

Modulation Index:

$$h = \Delta f \cdot T_b$$

Orthogonality Condition (Coherent FSK):

$$\Delta f_{\min} = 1/(2T_b) = R_b/2$$

Gives  $h = 0.5$  (Minimum Shift Keying!)

Bandwidth (Carson's Rule):

$$B_{\text{FSK}} = 2(\Delta f + R_b)$$

### Common Mistakes to Avoid

Forgetting to calculate  $T_b = 1/R_b$  first   Confusing  $\Delta f$  (separation) with  $f_0$  or  $f_1$  (individual frequencies)   Using wrong orthogonality condition ( $1/T_b$  instead of  $1/(2T_b)$ )

### Practice Problems

From **Homework 3**: - Problem 6: Binary FSK fundamentals - Problem 7: MSK analysis - Problem 9: Coherent vs. non-coherent detection

**Strategy:** 1. Find  $T_b = 1/R_b$  2. Calculate  $\Delta f = |f_1 - f_0|$  3. Calculate  $h = \Delta f \times T_b$  4. Check orthogonality: Is  $\Delta f \geq 1/(2T_b)$ ? 5. If  $h = 0.5$ , it's MSK!

---

## Topic 4: Minimum Shift Keying (MSK)

### Key Concepts

**1. MSK Definition** - Special case of CPFSK (Continuous Phase FSK) - Modulation index  $h = 0.5$  exactly - **Minimum** frequency separation for orthogonal FSK

**2. Advantages of MSK** - Continuous phase  $\rightarrow$  no discontinuities - Better spectral efficiency than standard FSK - Faster sidelobe decay (less interference) - Constant envelope (power amplifier friendly)

**3. Applications** - GSM cellular (GMSK variant) - Satellite communications - Anywhere spectral efficiency matters

### Essential Formulas

MSK Characteristics:

$$h = 0.5 \quad (\text{defining property!})$$

$$\Delta f = R_b/2 = 1/(2T_b)$$

For MSK with bit rate  $R_b$ :

$$\Delta f = R_b/2$$

If center frequency is  $f_c$ :

$$f = f_c - R_b/4$$

$$f = f_c + R_b/4$$

MSK Advantages:

- Continuous phase
- Minimum bandwidth for orthogonal FSK
- Constant envelope

## Practice Problems

From **Homework 3**: - Problem 7: Complete MSK analysis

**Strategy:** 1. If  $h = 0.5$ , it's MSK 2. From  $h = 0.5$  and bit rate, find  $\Delta f = 0.5R_b$  3. Frequencies:  
 $f_0 = f_c - \Delta f/2$ ,  $f_1 = f_c + \Delta f/2$

---

## Topic 5: Binary Phase Shift Keying (BPSK)

### Key Concepts

- 1. BPSK Basics** - Bit "0"  $\rightarrow$  Phase  $0^\circ$  (or  $+A$ ) - Bit "1"  $\rightarrow$  Phase  $180^\circ$  (or  $-A$ ) - Information in PHASE, not amplitude - Constant envelope (like FM/MSK)
- 2. Constellation Diagram** - Two points on I (real) axis -  $180^\circ$  apart - Maximum separation = best noise immunity
- 3. Detection** - **Must use coherent detection!** - Envelope detector won't work (constant envelope) - Requires carrier phase synchronization
- 4. Bandwidth** - Null-to-null:  $B = 2R_b$  - Similar to ASK (same symbol rate)

### Essential Formulas

BPSK Signals:

$$s(t) = A \cdot \cos(2\pi f t) \quad (\text{bit "0", } 0^\circ)$$

$$s(t) = -A \cdot \cos(2\pi f t) \quad (\text{bit "1", } 180^\circ)$$

$$\text{Or: } s(t) = A \cdot \cos(2\pi f t + \pi)$$

Constellation:

$$\text{Bit "0": } I = +A, Q = 0$$

$$\text{Bit "1": } I = -A, Q = 0$$

$$\text{Phase difference: } 180^\circ$$

Bandwidth:

$$B_{\text{null-null}} = 2/T_b = 2R_b$$

$$B_{\text{3dB}} \approx R_b$$

Cannot use envelope detection!

(Envelope is constant =  $A$  for both symbols)

## Common Mistakes to Avoid

Saying BPSK can use envelope detection (NO!)    Confusing BPSK bandwidth with carrier frequency    Drawing constellation in wrong locations

## Practice Problems

From **Homework 3**: - Problem 8: BPSK fundamentals

**Strategy:** 1. Write two signal expressions ( $180^\circ$  apart) 2. Draw constellation: two points on I-axis  
3. Calculate bandwidth:  $B = 2R_b$  4. Explain why coherent detection is required

---

## Topic 6: QPSK (Quadrature Phase Shift Keying)

### Key Concepts

- 1. QPSK Basics** - 4 phase states ( $45^\circ$ ,  $135^\circ$ ,  $225^\circ$ ,  $315^\circ$ ) - Encodes **2 bits per symbol** - Twice as efficient as BPSK! - Symbol rate = Bit rate / 2
- 2. I/Q Representation** - QPSK uses both I (In-phase) and Q (Quadrature) channels - Each symbol: combination of I and Q amplitudes - 4 constellation points equally spaced
- 3. BER Performance** - QPSK has same BER as BPSK for same  $E_b/N_0$  - But uses half the bandwidth! - Best of both worlds for spectral efficiency

### Essential Formulas

QPSK Parameters:

M = 4 symbols  
k = 2 bits/symbol  
Symbol rate:  $R_s = R_b/2$

Four Signals (phases  $45^\circ$ ,  $135^\circ$ ,  $225^\circ$ ,  $315^\circ$ ):

$s(t) = A \cdot \cos(2\pi f t + \pi/4)$   
 $s(t) = A \cdot \cos(2\pi f t + 3\pi/4)$   
 $s(t) = A \cdot \cos(2\pi f t + 5\pi/4)$   
 $s(t) = A \cdot \cos(2\pi f t + 7\pi/4)$

Constellation Points:

All at distance A from origin  
 $90^\circ$  apart  
On circle of radius A

Bandwidth:

$B_{\text{QPSK}} = 2R_s = 2(R_b/2) = R_b$   
Half the bandwidth of BPSK for same bit rate!

Spectral Efficiency:

BPSK: 0.5 bits/s/Hz (in null-null BW)  
QPSK: 1.0 bits/s/Hz

## Common Mistakes to Avoid

Forgetting symbol rate = bit rate / 2   Drawing constellation points at wrong phases   Not using Gray coding (see next topic!)   Thinking QPSK has worse BER than BPSK

## Practice Problems

From **Homework 4**: - Problem 1: QPSK fundamentals - Problem 2: Gray coding for QPSK

**Strategy:** 1. Calculate symbol rate:  $R_s = R_b/2$  2. Write 4 signal expressions ( $45^\circ$ ,  $135^\circ$ ,  $225^\circ$ ,  $315^\circ$ ) 3. Draw constellation with 4 points 4. Bandwidth:  $B = 2R_s = R_b$

---

## Topic 7: Gray Coding

### Key Concepts

**1. Why Gray Coding?** - Adjacent symbols differ by only **1 bit** - Most errors go to adjacent symbols (AWGN) - Symbol error  $\rightarrow$  typically 1 bit error (not 2!) - Reduces bit error rate (BER)

**2. Gray Code Properties** - Used in ALL modern systems (WiFi, LTE, 5G) - Essential for M-ary modulation - Simple mapping rule: adjacent symbols differ by 1 bit

### 3. Example for QPSK

Natural Binary: 00, 01, 10, 11 (can differ by 2 bits!)

Gray Coded: 00, 01, 11, 10 (adjacent differ by 1 bit)

### QPSK Gray Coding Example

Constellation (typical Gray mapping):

	Q	
01		00 (top half)
		I
11		10 (bottom half)

**Key:** Moving horizontally or vertically changes only 1 bit!

## Common Mistakes to Avoid

Using natural binary instead of Gray code   Assigning codes randomly without checking adjacency

## Practice Problems

From **Homework 4**: - Problem 2: Gray coding for QPSK - Problem 3: 8-PSK Gray coding

**Strategy:** 1. Draw constellation 2. Assign codes so adjacent symbols differ by 1 bit 3. Verify: moving one step (any direction) changes 1 bit

---

## Topic 8: M-ary Modulation and QAM

### Key Concepts

- 1. M-ary PSK** - M symbols,  $k = \log_2(M)$  bits/symbol - Better spectral efficiency as M increases  
- But requires higher SNR for same BER
- 2. QAM (Quadrature Amplitude Modulation)** - Combines amplitude AND phase modulation  
- Uses I and Q channels independently - Rectangular constellation (e.g., 16-QAM: 4×4 grid) - Even better spectral efficiency than M-PSK
- 3. Spectral Efficiency** - BPSK: 1 bit/symbol - QPSK: 2 bits/symbol - 16-QAM: 4 bits/symbol - 64-QAM: 6 bits/symbol - Higher M = more bits/symbol but needs more SNR

### Essential Formulas

M-ary Modulation:

M = number of symbols

k =  $\log_2(M)$  = bits per symbol

R<sub>s</sub> = R<sub>b</sub>/k = symbol rate

Examples:

BPSK: M=2, k=1

QPSK: M=4, k=2

8-PSK: M=8, k=3

16-QAM: M=16, k=4

64-QAM: M=64, k=6

Spectral Efficiency:

=  $R_b/B = k \cdot R_s/B$  bits/s/Hz

Higher M → higher

Trade-off:

Higher M: Better spectral efficiency

Higher M: Needs higher SNR (worse BER)

### 16-QAM Constellation

4×4 rectangular grid:

Amplitude levels: {±1, ±3} for I and Q

Q  
3  
1  
I  
-1  
-3  
-3 -1 1 3

## Common Mistakes to Avoid

Confusing M (symbols) with k (bits/symbol) Forgetting symbol rate:  $R_s = R_b/k$  Not using Gray coding for QAM Thinking more bits/symbol is always better (ignores SNR!)

## Practice Problems

From **Homework 4**: - Problem 3: M-ary PSK - Problem 4: QAM constellation design - Problem 5: Spectral efficiency comparison

**Strategy:** 1. Find k:  $k = \log_2(M)$  2. Find symbol rate:  $R_s = R_b/k$  3. For QAM: Draw rectangular constellation 4. Assign Gray codes to minimize bit errors

---

## Topic 9: EVM (Error Vector Magnitude)

### Key Concepts

- 1. EVM Definition** - Measures constellation quality -  $\text{EVM} = \text{error distance} / \text{reference distance}$   
- Expressed as percentage
- 2. Why EVM Matters** - WiFi/LTE specs have strict EVM limits - Higher-order modulation needs lower EVM - Indicates transmitter/receiver impairments
- 3. EVM Requirements** - 256-QAM:  $\text{EVM} < 3\%$  (very tight!) - 64-QAM:  $\text{EVM} < 8\%$  - 16-QAM:  $\text{EVM} < 12\%$  - QPSK:  $\text{EVM} < 17\%$

### Essential Formulas

EVM Calculation:

$$\begin{aligned}\text{Error vector} &= \text{Received} - \text{Ideal} \\ |\text{Error}| &= \sqrt{(\Delta I)^2 + (\Delta Q)^2}\end{aligned}$$

$$\text{EVM (\%)} = (|\text{Error}| / |\text{Reference}|) \times 100\%$$

Example:

$$\begin{aligned}\text{Ideal: (I, Q)} &= (1.0, 0.0) \\ \text{Received: (I, Q)} &= (0.92, 0.15) \\ \text{Error} &= \sqrt{(1-0.92)^2 + (0-0.15)^2} \\ &= \sqrt{0.08^2 + 0.15^2} \\ &= \sqrt{0.0064 + 0.0225} \\ &= \sqrt{0.0289} = 0.17 \\ \text{EVM} &= (0.17/1.0) \times 100\% = 17\%\end{aligned}$$

## Practice Problems

From **Homework 4**: - Problem 6: EVM calculation - Bonus 2: EVM limits in standards

**Strategy:** 1. Plot ideal and received points 2. Calculate error vector 3. Find magnitude of error 4. Divide by reference (usually distance to ideal point) 5. Convert to percentage

---

## Formula Sheet Recommendations

### FM/PM Essentials

FM:

$$f_i(t) = f_c + k_f \cdot m(t)$$

$$\Delta f = k_f \cdot \max |m(t)|$$

$$= \Delta f / f_m$$

$$\text{BFM} = 2(\Delta f / f_m) = 2f_m(1 + \beta)$$

$$\text{NBFM: } \beta \ll 1$$

$$\text{WBFM: } \beta \gg 1$$

PM:

$$i(t) = 2\pi f_c t + k_p \cdot m(t)$$

$$\Delta \omega = k_p \cdot \max |m(t)|$$

$$f_i(t) = f_c + (k_p / 2\pi) \cdot dm/dt$$

For sinusoidal  $m(t)$ :

$$\Delta f_{\text{PM}} = k_p \cdot A_m \cdot f_m$$

### Digital Modulation

FSK:

$$h = \Delta f \cdot T_b$$

$$\text{Orthogonal: } \Delta f = 1/(2T_b)$$

$$\text{MSK: } h = 0.5$$

BPSK:

$$2 \text{ symbols, } k=1 \text{ bit/symbol}$$

$$B = 2R_b$$

Coherent detection required

QPSK:

$$M=4, k=2 \text{ bits/symbol}$$

$$R_s = R_b/2$$

$$B = R_b \text{ (half of BPSK!)}$$

M-ary:

$$k = \log_2(M)$$

$$R_s = R_b/k$$

### Spectral Efficiency

Bit rate:  $R_b$

Symbol rate:  $R_s = R_b/k$

Spectral efficiency:  $\eta = R_b/B \text{ bits/s/Hz}$

Examples:

$$\text{BPSK: } 0.5 \text{ bits/s/Hz}$$

$$\text{QPSK: } 1.0 \text{ bits/s/Hz}$$

16-QAM: 2.0 bits/s/Hz  
64-QAM: 3.0 bits/s/Hz

---

## Exam Strategy

### Time Management (90 minutes)

- **5 min:** Read entire exam
- **15 min:** Problem 1 (FM fundamentals with Carson's rule)
- **12 min:** Problem 2 (PM analysis)
- **10 min:** Problem 3 (FM vs PM comparison)
- **15 min:** Problem 4 (FSK system design)
- **12 min:** Problem 5 (BPSK fundamentals)
- **15 min:** Problem 6 (QPSK and Gray coding)
- **6 min:** Problem 7 (MSK bonus if time)
- **5 min:** Review

### Problem-Solving Approach

**For FM Problems:** 1. Find  $\Delta f = k_f \times A_m$  2. Find  $\beta = \Delta f / f_m$  3. Classify: NBFM ( $\beta < 0.3$ ) or WBFM ( $\beta > 1$ )? 4. Apply Carson's rule:  $B = 2(\Delta f + f_m)$

**For PM Problems:** 1. Find  $\Delta\phi = k_p \times A_m$  2. Find  $f_i(t)$  by taking derivative 3. For sinusoidal:  $\Delta f = k_p \times A_m \times f_m$  4. Compare to FM (key difference:  $\Delta f$  depends on  $f_m$  in PM!)

**For FSK Problems:** 1. Calculate  $T_b = 1/R_b$  2. Calculate  $\Delta f = |f_1 - f_0|$  3. Calculate  $h = \Delta f \times T_b$  4. Check orthogonality:  $\Delta f \geq 1/(2T_b)$ ? 5. If  $h = 0.5$ : MSK!

**For QPSK Problems:** 1. Symbol rate:  $R_s = R_b/2$  2. Write 4 signal expressions 3. Draw constellation (4 points equally spaced) 4. Apply Gray coding 5. Bandwidth:  $B = 2R_s = R_b$

### Common Mistakes

**FM:** Confusing  $\Delta f$  with  $f_m$ , forgetting Carson's rule has TWO forms **PM:** Thinking  $\Delta f$  is independent of  $f_m$  (it's not!) **FSK:** Using  $1/T_b$  for orthogonality instead of  $1/(2T_b)$  **BPSK:** Saying envelope detection works (it doesn't!) **QPSK:** Forgetting symbol rate = bit rate / 2 **Gray coding:** Not checking that adjacent symbols differ by 1 bit

---

## Self-Assessment Checklist

Before the exam, can you:

**FM/PM:** - [ ] Calculate frequency deviation  $\Delta f$  for FM - [ ] Calculate modulation index  $\beta$  - [ ] Apply Carson's rule correctly - [ ] Classify NBFM vs. WBFM - [ ] Find instantaneous frequency for PM - [ ] Explain why PM has  $\Delta f$  proportional to  $f_m$

**FSK:** - [ ] Calculate bit duration from bit rate - [ ] Find frequency separation  $\Delta f$  - [ ] Calculate modulation index  $h$  - [ ] Check orthogonality condition - [ ] Identify MSK ( $h = 0.5$ )

**BPSK/QPSK:** - [ ] Write signal expressions - [ ] Draw constellation diagrams - [ ] Calculate bandwidth - [ ] Explain why BPSK needs coherent detection - [ ] Find symbol rate for QPSK - [ ] Apply Gray coding

**M-ary/QAM:** - [ ] Calculate bits per symbol ( $k = \log_2 M$ ) - [ ] Find symbol rate from bit rate - [ ] Draw QAM constellation - [ ] Compare spectral efficiencies - [ ] Calculate EVM

---

## Practice Resources

**Homework 3 (FM/PM/FSK/BPSK):** All problems 1-10 **Homework 4 (M-ary/QAM/EVM):** All problems 1-10

**Focus especially on:** - HW3: Problems 1, 2, 5, 6, 7, 8 - HW4: Problems 1, 2, 4, 6

**Good luck!**