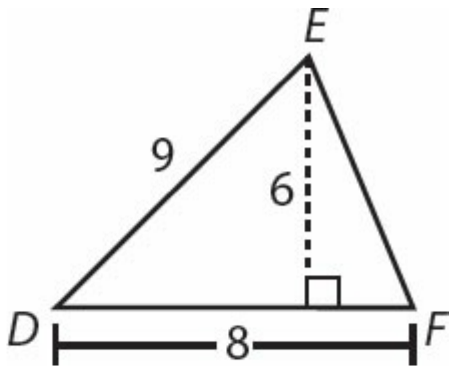
Triangles

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

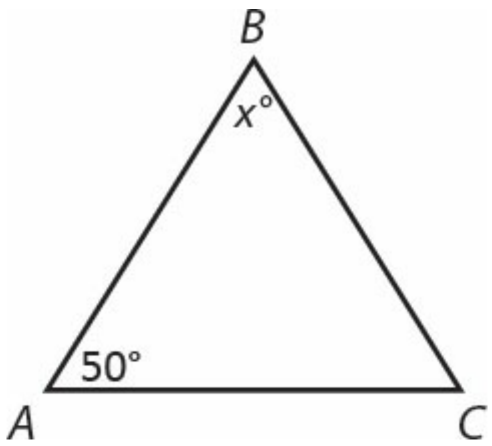
For questions followed by a numeric entry box , you are to enter your own answer in the box. For questions followed by a fraction-style numeric entry box , you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is $\frac{1}{4}$, you may enter $\frac{25}{100}$ or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations, such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

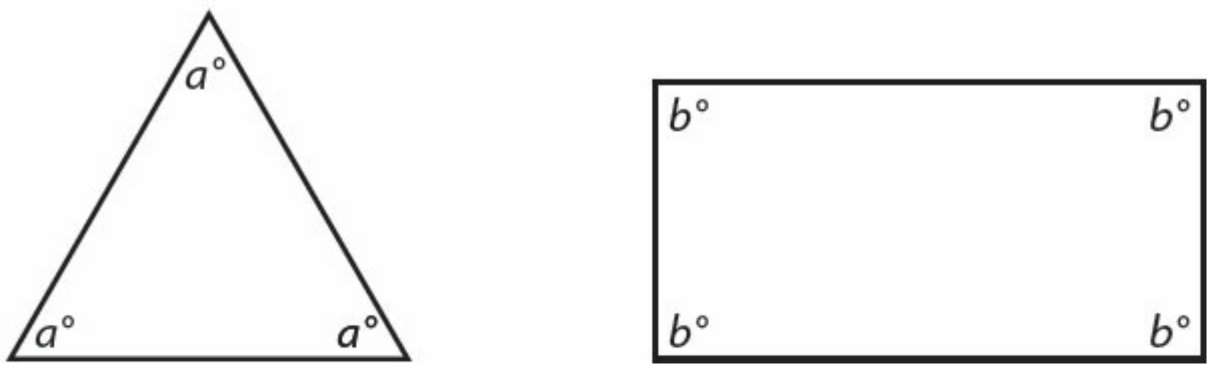


1. What is the area of triangle *DEF*?

   (A) 23
   (B) 24
   (C) 48
   (D) 56
   (E) 81

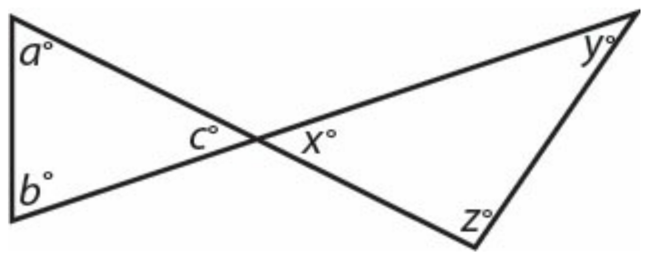


2. If *AB* and *BC* have equal lengths, what is the value of *x*?



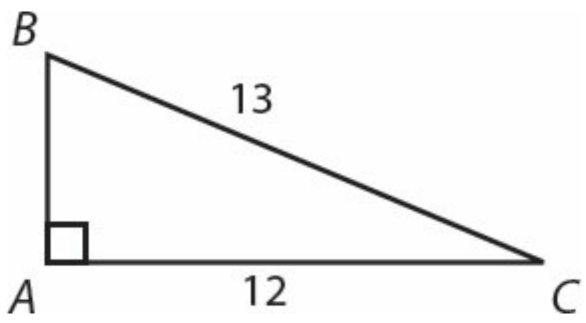
3.

Quantity A	**Quantity B**
$2a + b$	$3a + \frac{b}{3}$

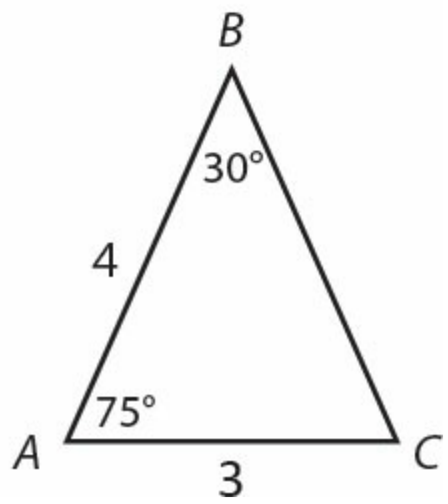


4.

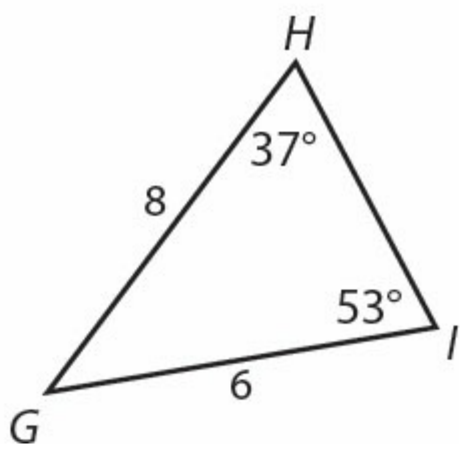
Quantity A	**Quantity B**
$a + b + x$	$c + y + z$



5. What is the area of right triangle *ABC*?



6. What is the perimeter of triangle *ABC*?

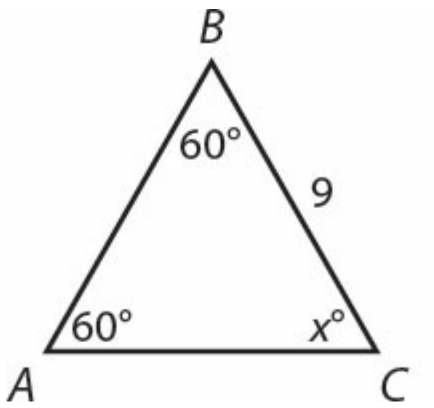


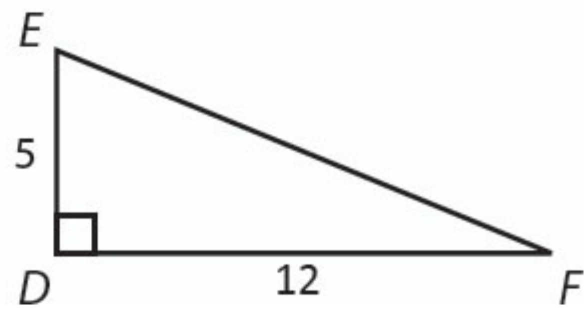
7. What is the length of side *HI*?

$\boxed{}$

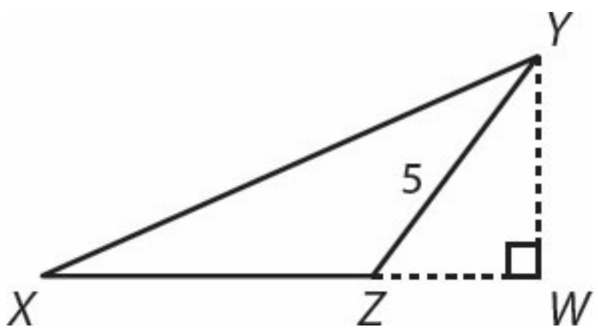
8. If the hypotenuse of an isosceles right triangle is $8\sqrt{2}$, what is the area of the triangle?

(A) 18
(B) 24
(C) 32
(D) 48
(E) 64

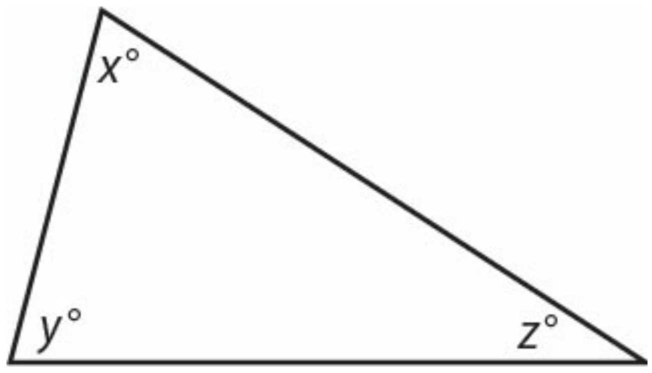




	Quantity A	**Quantity B**
9.	Perimeter of triangle *ABC*	Perimeter of triangle *DEF*



10. *ZW* has a length of 3 and *XZ* has a length of 6. What is the area of triangle *XYZ*?



In the figure above, $x + z$ equals 110.

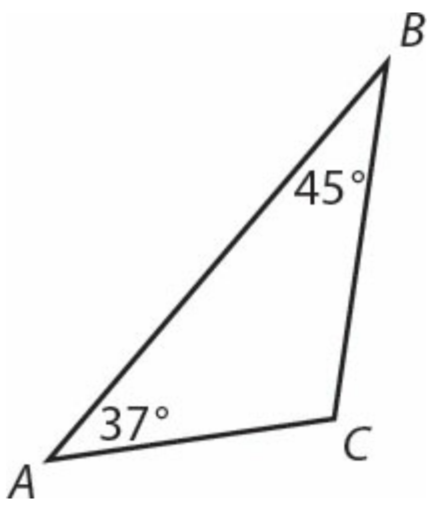
	Quantity A	**Quantity B**
11.	x	y

Two sides of an isosceles triangle are 8 and 5 in length, respectively.

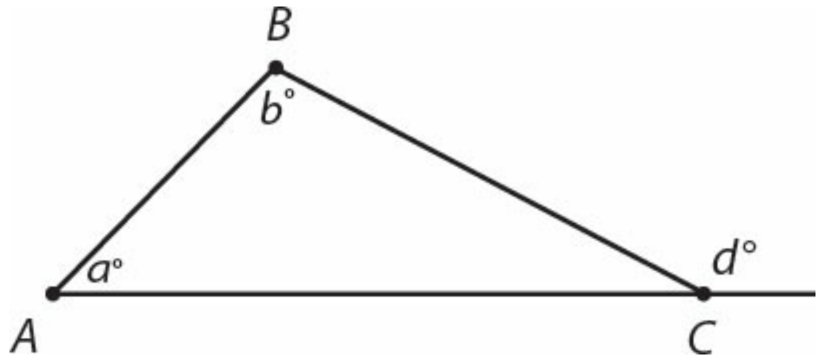
	Quantity A	**Quantity B**
12.	The length of the third side	8

Two sides of an isosceles triangle are 2 and 11 in length, respectively.

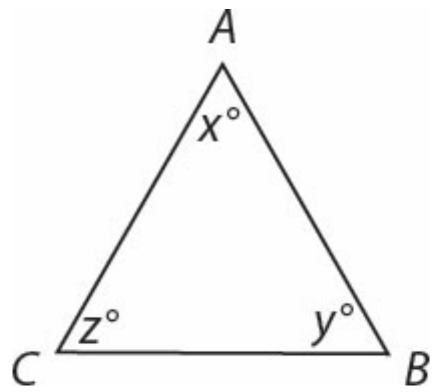
	Quantity A	**Quantity B**
13.	The length of the third side	11



	Quantity A	**Quantity B**
14.	AC	BC

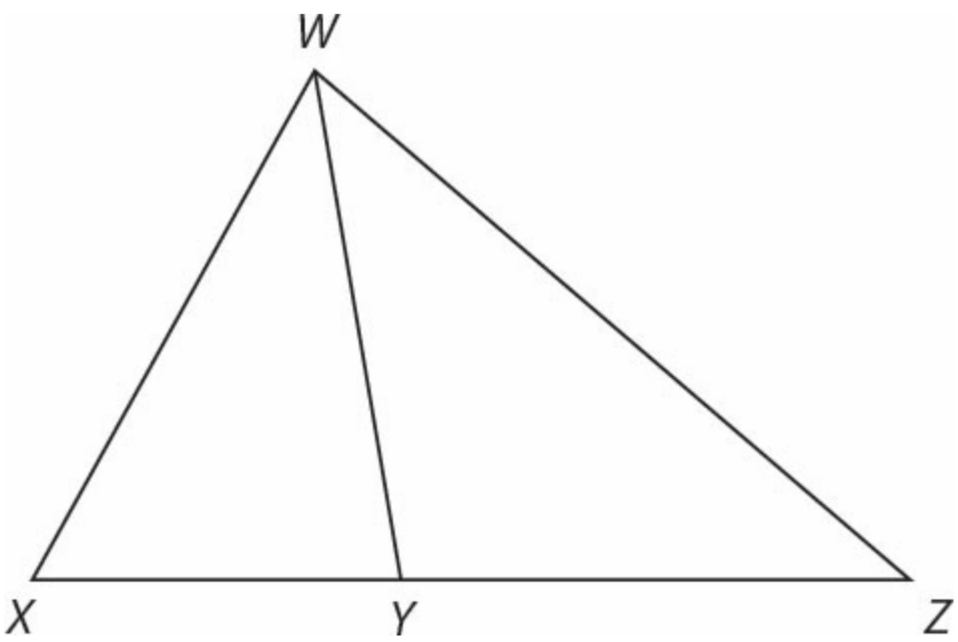


	Quantity A	**Quantity B**
15.	$a + b$	d



$y < z$
$y > 60$

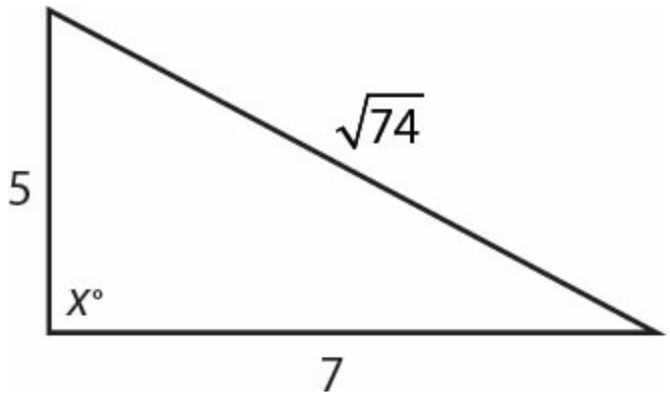
	Quantity A	**Quantity B**
16.	The length of side AC	The length of side BC



$XY = YZ$

Quantity A	**Quantity B**

17. The area of XWY The area of YWZ



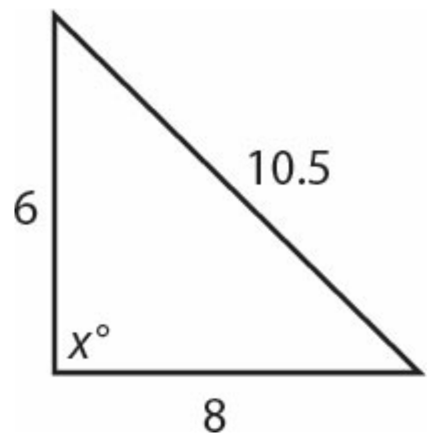
	Quantity A	**Quantity B**
18.	x	90

19. If p is the perimeter of a triangle with one side of 7 and another side of 9, what is the range of possible values for p?

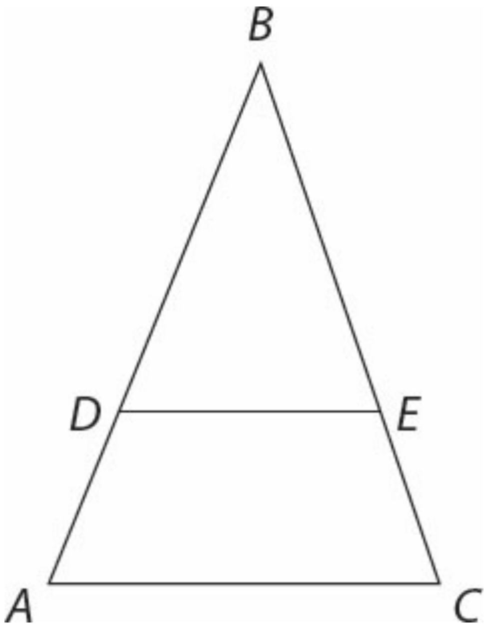
(A) $2 < p < 16$
(B) $3 < p < 17$
(C) $18 < p < 32$
(D) $18 < p < 33$
(E) $17 < p < 63$

A right triangle has a hypotenuse of 12 and legs of 9 and y.

	Quantity A	**Quantity B**
20.	y	15



	Quantity A	**Quantity B**
21.	x	90



In the figure above, DE is parallel to AC.

$BE = 2EC$

$DE = 12$

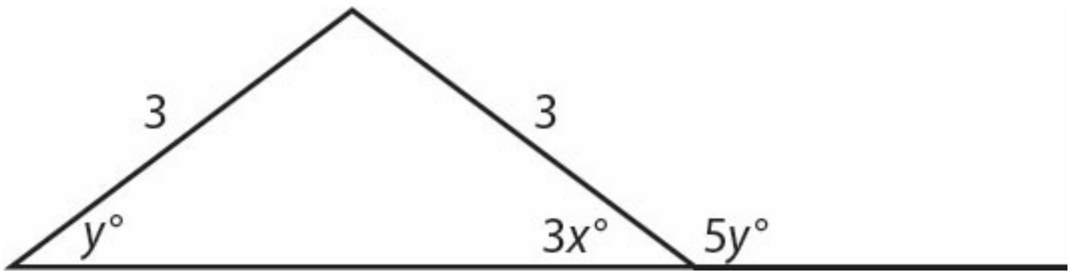
	Quantity A	**Quantity B**
22.	AC	18

Two sides of a triangle are 8 and 9 long.

	Quantity A	**Quantity B**
23.	The length of the third side of the triangle	$\sqrt{290}$

24. What is the area of an equilateral triangle with side length 6?

(A) $4\sqrt{3}$
(B) $6\sqrt{2}$
(C) $6\sqrt{3}$
(D) $9\sqrt{2}$
(E) $9\sqrt{3}$

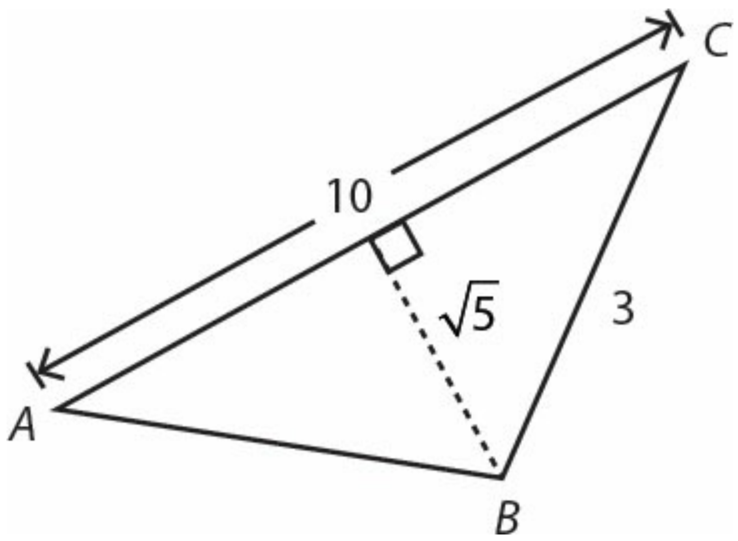


25. What is the value of x in the figure above?

(A) 5
(B) 10
(C) 18
(D) 30
(E) 54

26. An isosceles right triangle has an area of 50. What is the length of the hypotenuse?

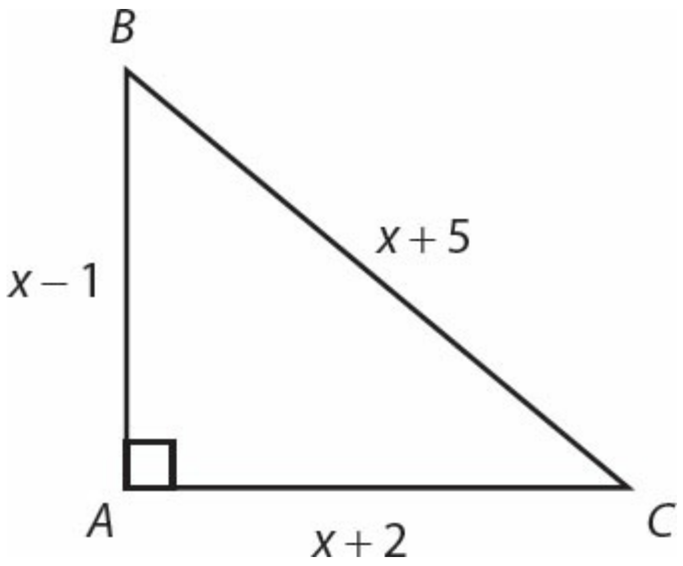
(A) 5
(B) $5\sqrt{2}$
(C) $5\sqrt{3}$
(D) 10
(E) $10\sqrt{2}$



27. In the figure above, what is the length of side *AB*?

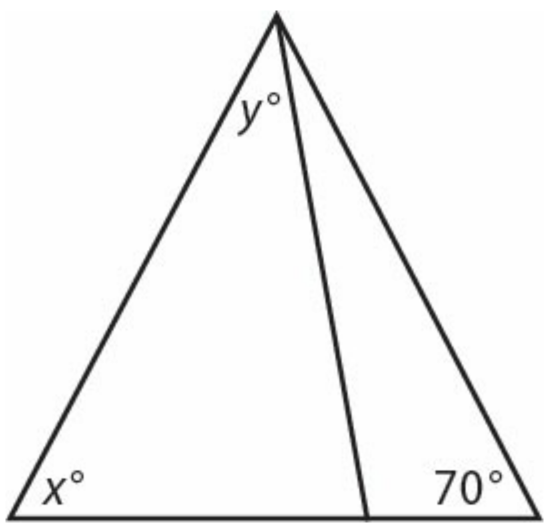
(A) 5
(B) $\sqrt{30}$
(C) $5\sqrt{2}$
(D) 8

(E) $\sqrt{69}$

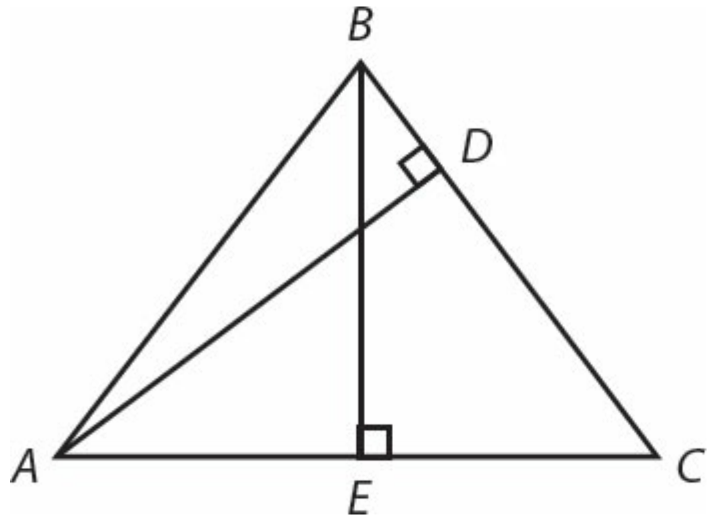


28. In the right triangle above, what is the length of AC?

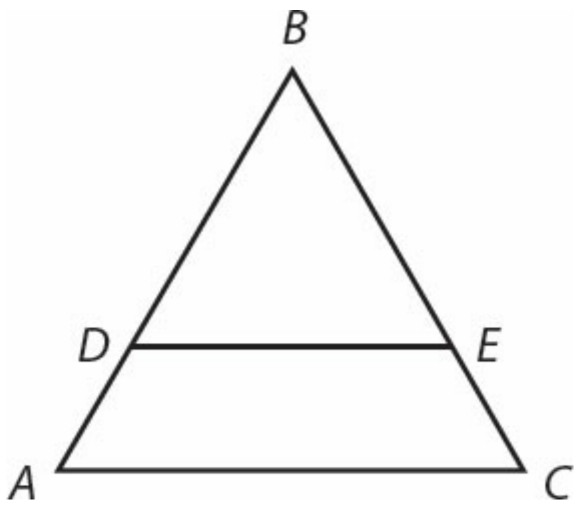
(A) 9
(B) 10
(C) 12
(D) 13
(E) 15



	Quantity A	Quantity B
29.	$x + y$	110

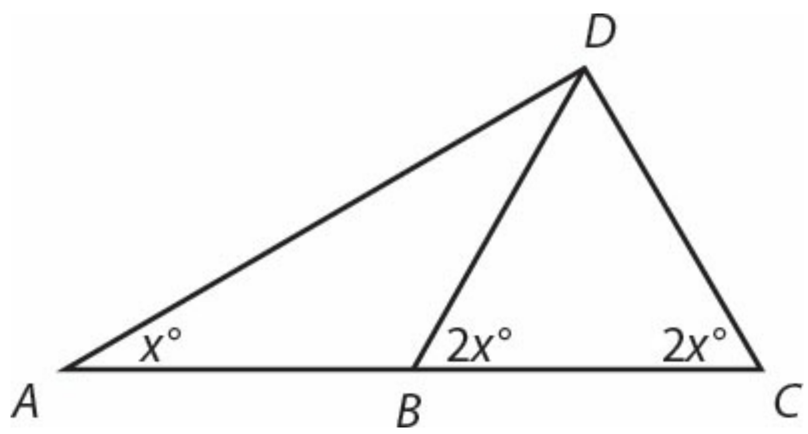


	Quantity A	Quantity B
30.	The product of BE and AC	The product of BC and AD

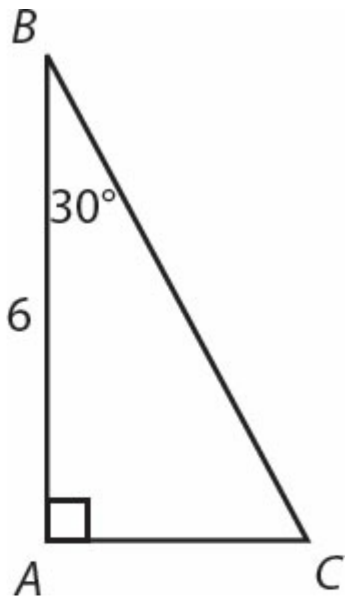


31. In the figure above, DE and AC are parallel lines. If $AC = 12$, $DE = 8$, and $AD = 2$, what is the length of AB?

    (A) 2
    (B) 3
    (C) 4
    (D) 5
    (E) 6



	Quantity A	**Quantity B**
32.	*DC*	*AB*

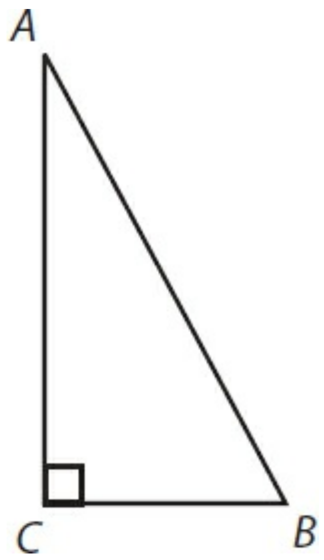


33. What is the perimeter of right triangle *ABC* above?

(A) $6 + 4\sqrt{3}$
(B) $6 + 6\sqrt{3}$
(C) $6 + 8\sqrt{3}$
(D) $9 + 6\sqrt{3}$
(E) $18 + 6\sqrt{3}$

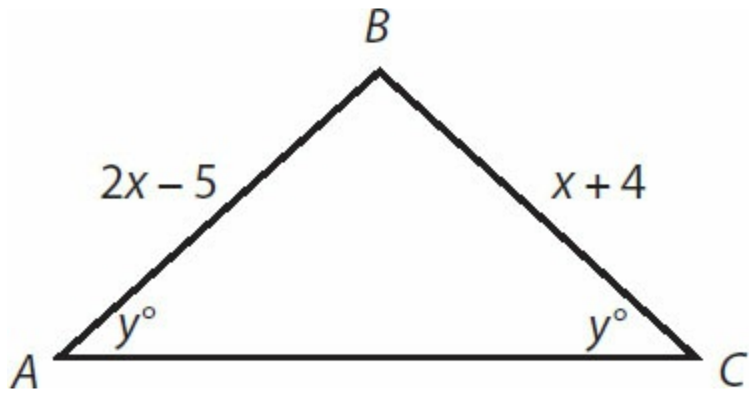
34. A 10-foot ladder leans against a vertical wall and forms a 60-degree angle with the floor. If the ground below the ladder is horizontal, how far above the ground is the top of the ladder?

(A) 5 feet
(B) $5\sqrt{3}$ feet
(C) 7.5 feet
(D) 10 feet
(E) $10\sqrt{3}$ feet

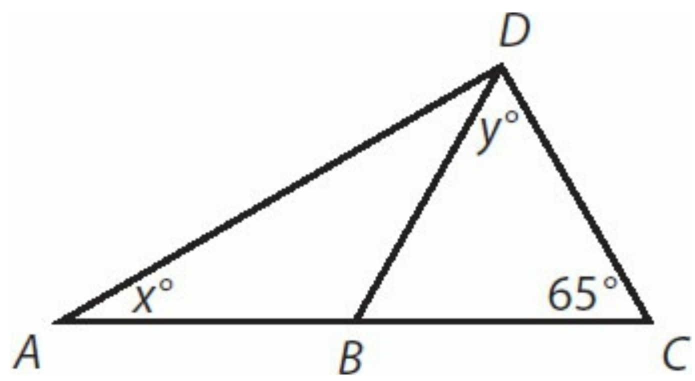


35. Triangle *ABC* has an area of 9. If *AC* is three times as long as *CB*, what is the length of *AB*?

(A) 6
(B) $3\sqrt{6}$
(C) $2\sqrt{15}$
(D) $4\sqrt{15}$
(E) 15

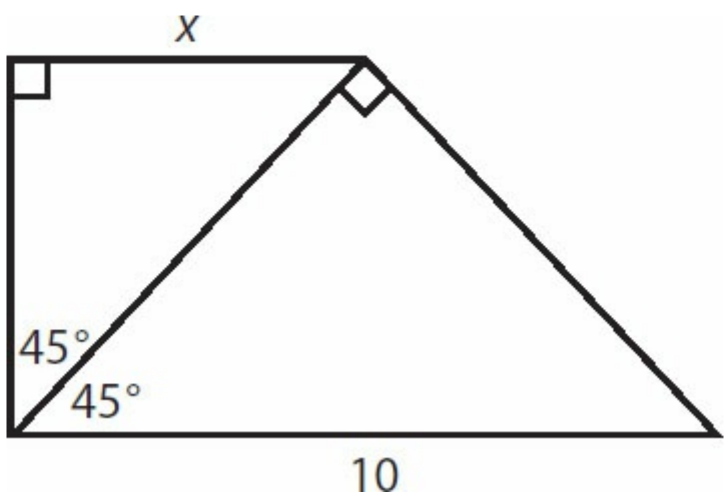


	Quantity A	Quantity B
36.	*CB*	7



In the figure above, side lengths *AB, BD*, and *DC* are all equal.

	Quantity A	Quantity B
37.	x	y



38. In the figure above, what is the value of x?

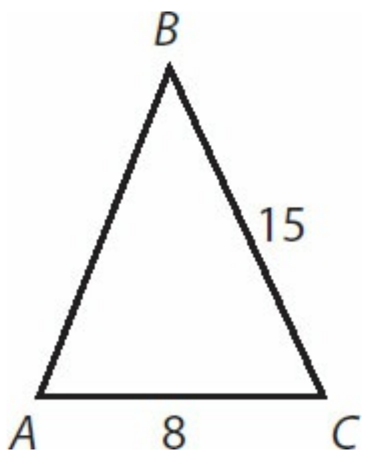
(A) 2.5

(B) $\dfrac{5}{\sqrt{2}}$

(C) 5

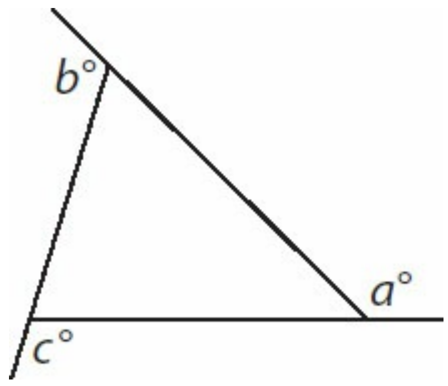
(D) $5\sqrt{2}$

(E) $\dfrac{10}{\sqrt{2}}$

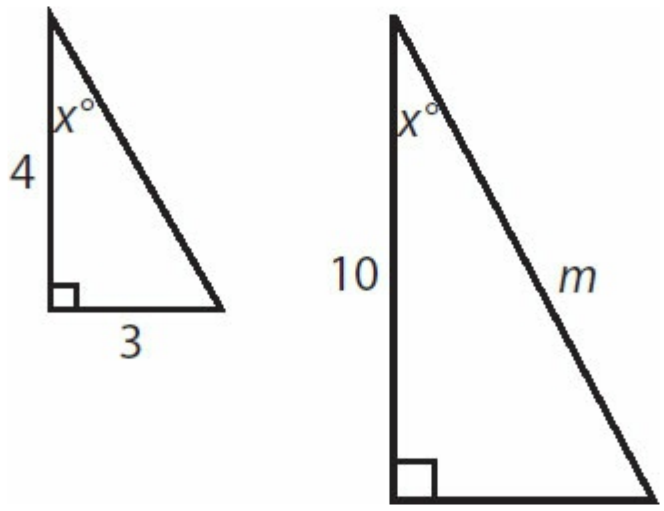


39. Which of the following statements, considered independently, provide sufficient information to calculate the area of triangle *ABC*?

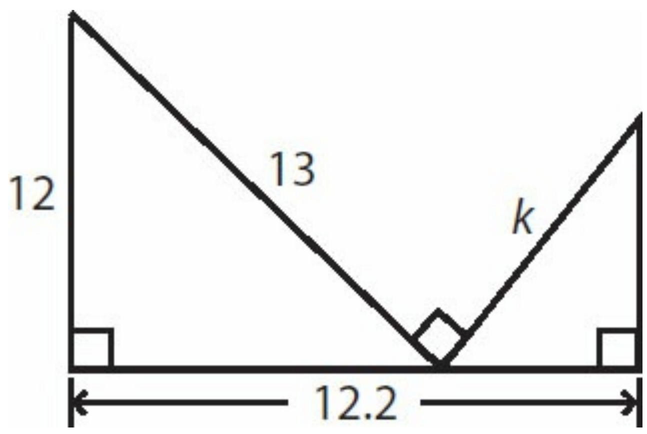
- ☐ Angle *ACB* equals 90°
- ☐ *AB* = 17
- ☐ *ABC* is a right triangle



	Quantity A	Quantity B
40.	$a + b + c$	180



	Quantity A	Quantity B
41.	m	15

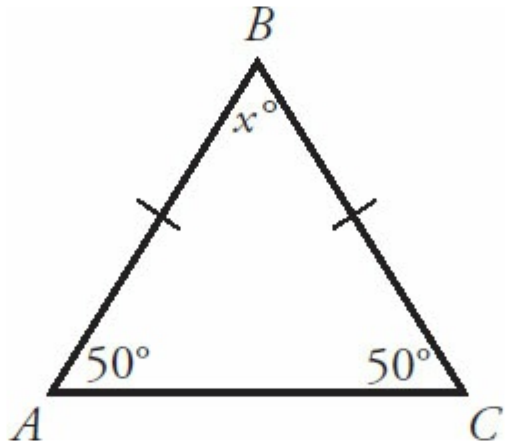


42. What is the length of hypotenuse k?

Triangles Answers

1. **(B).** The area of a triangle is equal to $\frac{bh}{2}$. Base and height must always be perpendicular. Use 8 as the base and 6 as the height: $A = \frac{(8)(6)}{2} = 24$.

2. **80.** If two of the angles in a triangle are known, the third can be found because all three angles must sum to 180°. In triangle *ABC*, sides *AB* and *BC* are equal. That means their opposite angles are also equal, so angle *ACB* is 50°.



Because $50 + 50 + x = 180$, $x = 80$.

3. **(C).** The three angles in a triangle must sum to 180°, so $3a = 180$ and $a = 60$ (the triangle is equilateral). The four angles in a quadrilateral must sum to 360°, so $4b = 360$ and $b = 90$ (the angles are right angles, so the figure is a rectangle).

Substitute the values of *a* and *b* into Quantity A to get $2(60) + 90 = 120 + 90 = 210$. Likewise, substitute into Quantity B to get $3(60) + \frac{90}{3} = 180 + 30 = 210$. The two quantities are equal.

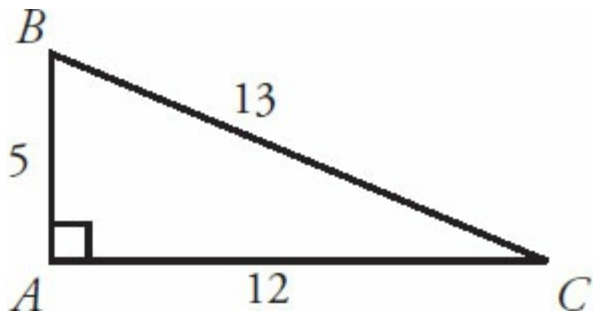
4. **(C).** Since *c* and *x* are vertical angles, they are equal. So their positions in the quantities can be switched, to put all the angles in the same triangle together:

Quantity A	Quantity B
$a + b + c$	$x + y + z$

The three angles inside a triangle sum to 180°, so the two quantities are equal.

5. **30.** To find the area of a triangle, a base and height are needed. If the length of *AB* can be determined, then *AB* can be the height and *AC* can be the base, because the two sides are perpendicular to each other.

Use the Pythagorean theorem to find the length of side *AB*: $(a)^2 + (12)^2 = (13)^2$, so $a^2 + 144 = 169$, which means that $a^2 = 25$, and finally $a = 5$. Alternatively, recognize that the triangle is a Pythagorean triple 5–12–13.

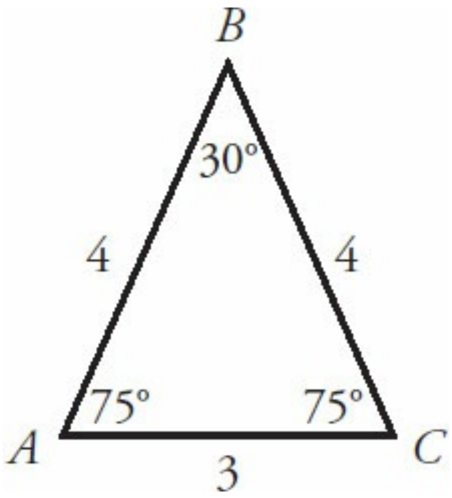


Base and height are known, so Area $= \dfrac{(12)(5)}{2} = 30$.

6. **11.** To find the perimeter of triangle *ABC*, sum the lengths of all three sides. Side *BC* is not labeled, so inferences must be made from the given in the question.

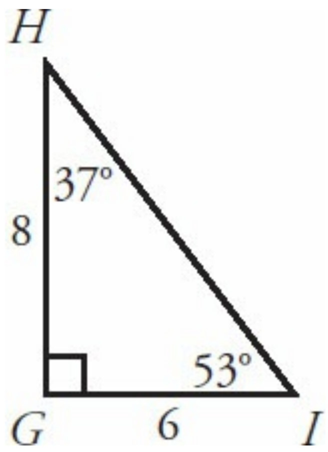
Given the degree measures of two of the angles in triangle *ABC,* the degree measure of the third can be determined. If the third angle is $x°$, then $30 + 75 + x = 180$ and therefore $x = 75$.

Angle *BAC* and angle *BCA* are both 75°, which means triangle *ABC* is an isosceles triangle. If those two angles are equal, their opposite sides are also equal. Side *AB* has a length of 4, so *BC* also has a length of 4:



To find the perimeter, sum the lengths of the three sides: $4 + 4 + 3 = 11$.

7. **10.** Side *HI* is not labeled, so inferences will have to be drawn from other information provided in the figure. Two of the angles of triangle *GHI* are labeled, so if the third angle is $x°$, then $37 + 53 + x = 180$. That means $x = 90$, and the triangle really looks like this:



You should definitely redraw once you discover the triangle is a right triangle!

Now you can use the Pythagorean theorem to find the length of *HI*. *HI* is the hypotenuse, so $(6)^2 + (8)^2 = c^2$, which means $36 + 64 = 100 = c^2$, so $c = 10$. The length of *HI* is 10.

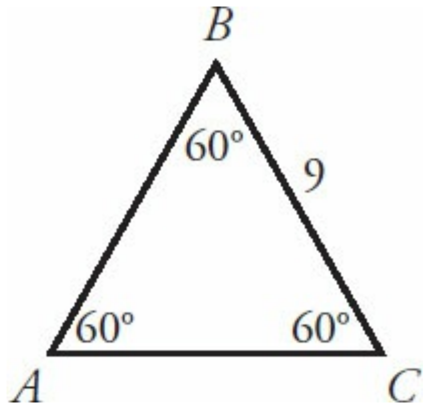
Alternatively, recognize the Pythagorean triple: triangle *GHI* is a 6–8–10 triangle.

8. **(C).** All isosceles right triangles (or 45–45–90 triangles) have sides in the ratio of $1 : 1 : \sqrt{2}$. Thus, an isosceles right triangle with hypotenuse $8\sqrt{2}$ has sides of 8, 8, and $8\sqrt{2}$. Use the two legs

of 8 as base and height of the triangle in the formula for area:

$$A = \frac{bh}{2} = \frac{(8)(8)}{2} = 32$$

9. **(B).** To determine which triangle has the greater perimeter, find all three side lengths of both triangles. Begin with triangle *ABC*, in which two of the angles are labeled, so the third can be calculated. If the unknown angle is $x°$, then $60 + 60 + x = 180$ and, therefore, $x = 60$.



All three angles in triangle *ABC* are 60°. If all three angles are equal, all three sides are equal and every side of triangle *ABC* has a length of 9. The perimeter of *ABC* is $9 + 9 + 9 = 27$.

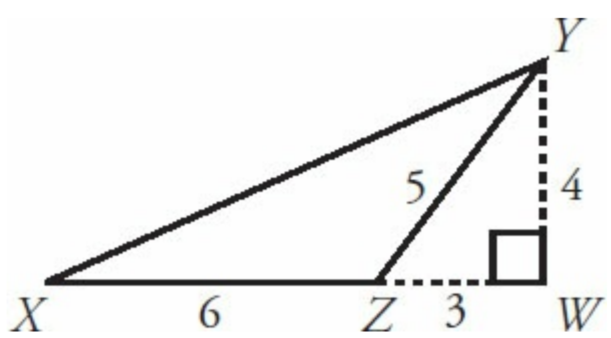
Now look at triangle *DEF,* which is a right triangle. Use the Pythagorean theorem to find the length of side *EF,* which is the hypotenuse, so $(5)^2 + (12)^2 = c^2$, which means $25 + 144 = 169 = c^2$ and, therefore, $c = 13$. The perimeter of *DEF* is $5 + 12 + 13 = 30$. Alternatively, 5–12–13 is a Pythagorean triple.

Because $30 > 27$, triangle *DEF* has a greater perimeter than triangle *ABC*. Quantity B is greater.

10. **12.** Start by redrawing the figure, filling in all the information given in the text. To find the area of triangle *XYZ*, a base and a height are required. If side *XZ* is a base, then *YW* can act as a height, as these two are perpendicular.

Because triangle *ZYW* is a right triangle with two known sides, the third can be determined using the Pythagorean theorem: *ZY* is the hypotenuse, so $(a)^2 + (3)^2 = (5)^2$, meaning that $a^2 + 9 = 25$ and $a^2 = 16$, so $a = 4$.

Alternatively, recognize the Pythagorean triple: *ZYW* is a 3–4–5 triangle:



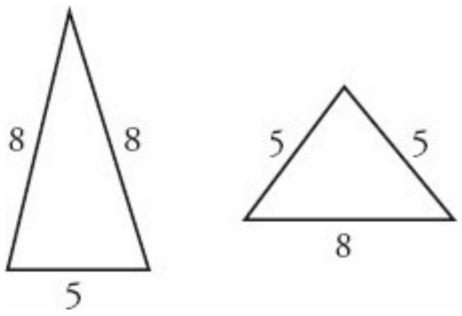
The area of triangle *XYZ* is $\frac{bh}{2} = \frac{(6)(4)}{2} = 12$.

11. **(D).** The problem indicates that $x + z = 110$. Since the angles of a triangle must sum to 180°, $x + y + z = 180$. Substitute 110 for $x + z$ on the left side:

$y + 110 = 180$
$y = 70$

The problem compares x and y. Although y is known, the exact value of x is not known, only that it must be greater than 0 and less than 110. The relationship cannot be determined from the information given.

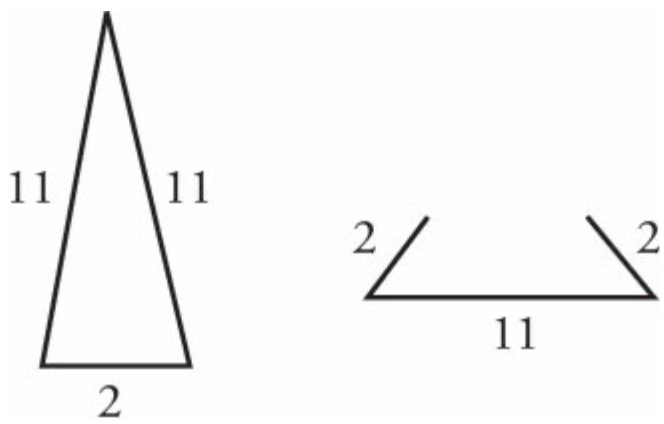
12. **(D).** An isosceles triangle has two equal sides, so this triangle must have a third side of either 8 or 5. Use the Third Side Rule (any side of a triangle must be greater than the difference of the other two sides and less than their sum) to check whether both options are actually possible.



Since 8 – 5 = 3 and 8 + 5 = 13, the third side has to be greater than 3 and less than 13. Therefore, that side could indeed be either 5 or 8. The two quantities could be equal, or Quantity A could be less than Quantity B, so the relationship cannot be determined from the information given.

13. **(C).** An isosceles triangle has two equal sides, so this triangle must have a third side of either 2 or 11. Because one side is so long and the other so short, it is worth testing via the Third Side Rule (any side of a triangle must be greater than the difference of the other two sides and less than their sum) to see whether both possibilities are really possible.

From the Third Side Rule, a triangle with sides of 2 and 11 must have a third side greater than 11 – 2 = 9 and less than 11 + 2 = 13. Since 2 is not between 9 and 13, it is just not possible to have a triangle with sides of length 2, 2, and 11. However, a 2–11–11 triangle is possible. So the third side must be 11.



The two quantities are equal.

14. **(A).** Within any triangle, the following is true: the larger the angle, the longer the side opposite that angle.

The side opposite the 45° angle (*AC*) must be longer than the side opposite the 37° angle (*BC*): $AC > BC$. Quantity A is greater.

15. **(C).** By definition, the exterior angle d is equal to the sum of the two opposite interior angles. Thus, $d = a + b$.

Alternatively, label the interior angle at vertex C as $c°$. The sum of the angles in a triangle is 180°, so $a + b + c = 180$. The sum of angles that form a line is also 180, so $c + d = 180$, or $c = 180 - d$. Substitute into the first equation:

$$a + b + c = 180$$
$$a + b + (180 - d) = 180$$
$$a + b - d = 0$$
$$a + b = d$$

(This, incidentally, is the proof of the rule stated in the first line of this explanation.)

16. **(A).** Putting the constraints together, $60 < y < z$. That means that $y + z > 120$, leaving less than 60° for the remaining angle x. The angles can now be ordered by size: $x < y < z$.

The shortest side is across from the smallest angle, which is x, so the shortest side must be BC. The median length side is across from the median angle, which is y, so the median length side must be AC. Since none of the angles are equal, none of the sides are equal, and the length of AC is greater than the length of BC. Quantity A is greater.

17. **(C).** The area of a triangle is equal to $\frac{bh}{2}$. The two triangles have equal bases, since $XY = YZ$. They also have the same height, since they both have the same height as the larger triangle XWZ. The two quantities are equal.

18. **(C).** Do not *assume* that $x = 90$. Instead, since all three side lengths are labeled, *test* whether the triangle is a right triangle by plugging into the Pythagorean theorem and seeing whether the result is a true statement:

$$5^2 + 7^2 = (\sqrt{74})^2$$
$$25 + 49 = 74$$
$$74 = 74$$

Since 74 equals 74, the Pythagorean theorem does apply to this triangle. So the triangle is a right triangle. Notice also that the side across from x was used as the hypotenuse. It must be that $x = 90$. The two quantities are equal.

19. **(C).** From the Third Side Rule, any side of a triangle must be greater than the difference of the other two sides and less than their sum. Since $9 - 7 = 2$ and $9 + 7 = 16$, the unknown third side must be between 2 and 16, not inclusive. To get the lower boundary for the perimeter, add the lower boundary of the third side to the other two sides: $2 + 7 + 9 = 18$. To get the upper boundary for the perimeter, add the upper boundary for the third side to the other two sides: $16 + 7 + 9 = 32$. Thus, p must be between 18 and 32, not inclusive—in other words, $18 < p < 32$.

20. **(B).** You may have memorized the 3–4–5 Pythagorean triple, of which 9–12–15 is a multiple. This question is trying to exploit this—don't be tricked into thinking that $y = 15$. In a 9–12–15 triangle, 15 would have to be the hypotenuse. In any right triangle, the hypotenuse must be the longest side.

Since the given triangle has 12 as the hypotenuse, the leg of length y must be less than 12, and thus less than 15. At this point, it is safe to choose (B). Although unnecessary, to get the actual value of y, apply the Pythagorean theorem:

$$9^2 + y^2 = 12^2$$
$$81 + y^2 = 144$$
$$y^2 = 63$$

So y is a little less than 8, which is definitely less than 15. Quantity B is greater.

21. **(A).** One good approach here is to test the value in Quantity B. If angle x equals 90°, then this is a right triangle. Use the legs of 6 and 8 to find the hypotenuse using the Pythagorean theorem: $6^2 + 8^2 = c^2$ will tell you that c equals 10 in this case. (Or, memorize the 6–8–10 multiple of the 3–4–5 Pythagorean triple, since it appears often on the GRE.) Since the hypotenuse is slightly longer than 10, the angle across from the "hypotenuse" must actually be slightly larger than 90°. Therefore, x is greater than 90.

22. **(C).** If AC is parallel to DE, then triangles DBE and ABC are similar. If $BE = 2EC$ then if EC is set to equal x, BE would equal $2x$ and BC would equal $x + 2x$, or $3x$. That means that the big triangle is in a 3 : 2 ratio with the small triangle (since $BC : BE = 3x : 2x$).

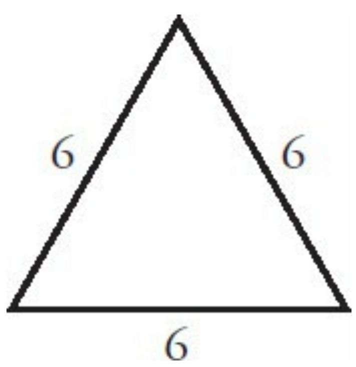
Set up the proportion for the bottom sides of these triangles, both of which are opposite the shared vertex at B:

$$\begin{aligned}\frac{2}{3} &= \frac{DE}{AC}\\ \frac{2}{3} &= \frac{12}{AC}\\ 2(AC) &= 36\\ AC &= 18\end{aligned}$$

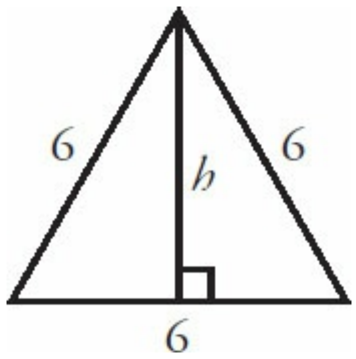
The two quantities are equal.

23. **(B).** From the Third Side Rule, a triangle with sides of 8 and 9 must a have a third side greater than $9 - 8 = 1$ and less than $8 + 9 = 17$. Since 17^2 is 289, which is less than 290, the measure of the third side is definitely less than $\sqrt{290}$. Quantity B is greater.

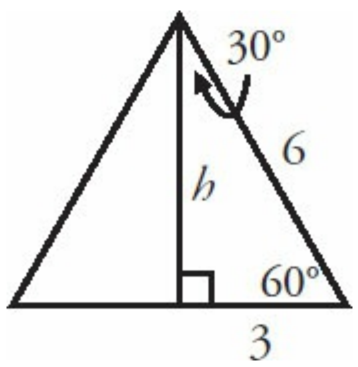
24. **(E).** An equilateral triangle with side length 6 can be drawn as:



In order to find the area, recall that the area of a triangle is $A = \frac{bh}{2}$. The base of the triangle is already known to be 6, so find the height in order to solve for area. The height is the straight line from the highest point on the triangle dropped down perpendicular to the base:


6
6
h
6

The angle opposite h must be 60°, since it is one of the three angles of the original equilateral triangle. Thus, the triangle formed by h is a 30–60–90 triangle as shown below.



Using the properties of 30–60–90 triangles, h is equal to the shortest side multiplied by $\sqrt{3}$. Thus $h = 3\sqrt{3}$ and the area is

$$A = \frac{bh}{2} = \frac{6 \times 3\sqrt{3}}{2} = 9\sqrt{3}$$

25. **(B).** Since there are two unknowns, look for two equations to solve. The first equation comes from the fact that $3x$ and $5y$ make a straight line, so they must sum to 180:

$$3x + 5y = 180$$

A triangle with at least two sides of equal length is called an isosceles triangle. In such a triangle, the angles opposite the two equal sides are themselves equal in measure. In this case, the two sides with length 3 are equal, so the angles opposite them (y and $3x$) must also be equal:

$$y = 3x$$

Substitute for y in the first equation:

$$\begin{aligned} 3x + 5(3x) &= 180 \\ 3x + 15x &= 180 \\ 18x &= 180 \\ x &= 10 \end{aligned}$$

26. **(E).** If the area of the triangle is 50, then $\frac{bh}{2} = 50$. In an isosceles right triangle, the base and height are the two perpendicular legs, which have equal length. Since base = height, substitute another b in for h:

$$\frac{b^2}{2} = 50$$

$$b^2 = 100$$

$$b = 10$$

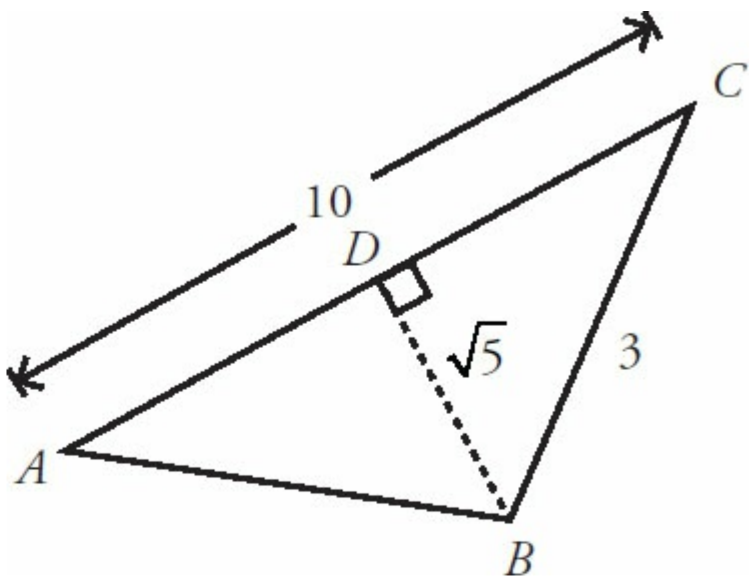
An isosceles right triangle follows the 45–45–90 triangle formula, so the hypotenuse is $10\sqrt{2}$.

Alternatively, use the Pythagorean theorem to find the hypotenuse:

$$\begin{aligned} 10^2 + 10^2 &= c^2 \\ 200 &= c^2 \end{aligned}$$

Thus, $c = \sqrt{200} = \sqrt{100 \times 2} = 10\sqrt{2}$.

27. **(E).** For convenience, put the letter *D* on the point at the right angle between *A* and *C*, as shown:



Solve this multi-step problem by working backwards from the goal. To find the length of *AB*, use the Pythagorean theorem on triangle *ADB*, since angle *ADB* must be a right angle. In order to use the Pythagorean theorem, find the lengths of the two legs. *BD* is known, so *AD* needs to be determined. Since *AD* and *DC* sum to a line segment of length 10, $AD = 10 - DC$.

Finally, to find *DC*, apply the Pythagorean theorem to triangle *BDC*:

$$\begin{aligned} (\sqrt{5})^2 + (DC)^2 &= 3^2 \\ 5 + (DC)^2 &= 9 \\ (DC)^2 &= 4 \\ DC &= 2 \end{aligned}$$

Therefore, $AD = 10 - DC = 10 - 2 = 8$. Now apply the Pythagorean theorem to *ADB*:

$$(\sqrt{5})^2 + 8^2 = (AB)^2$$
$$5 + 64 = (AB)^2$$
$$69 = (AB)^2$$
$$AB = \sqrt{69}$$

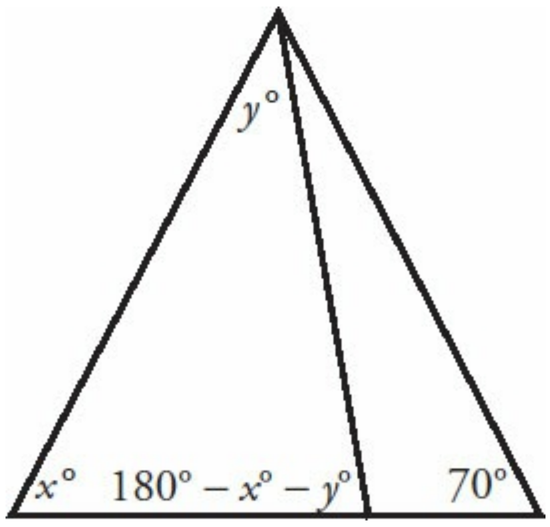
28. **(C).** Because this is a right triangle, the Pythagorean theorem applies to the lengths of the sides. The Pythagorean theorem states that $a^2 + b^2 = c^2$, where c is the hypotenuse and a and b are the legs of a right triangle. Plug the expressions into the theorem and simplify:

$(x - 1)^2 + (x + 2)^2 = (x + 5)^2$
$(x^2 - 2x + 1) + (x^2 + 4x + 4) = x^2 + 10x + 25$
$2x^2 + 2x + 5 = x^2 + 10x + 25$
$x^2 - 8x - 20 = 0$
$(x - 10)(x + 2) = 0$
$x = 10$ or $x = -2$

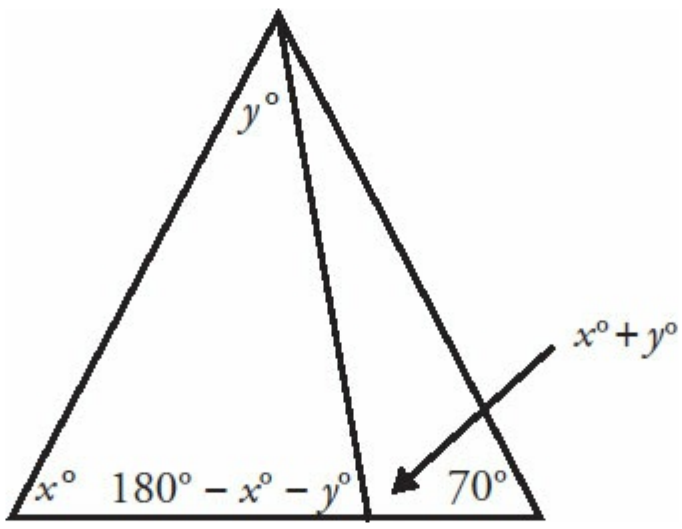
However, $x = -2$ is not an option; side lengths can't be negative. So x must equal 10. This is *not* the final answer, however. The question asks for side length AC:

$AC = x + 2 = 10 + 2 = 12$

29. **(B).** The question compares $x + y$ with 110. To do so, fill in the missing angles on the triangles. In the triangle on the left, all three angles must sum to 180°. Therefore, the missing angle must be $(180 - x - y)$, as shown here:



Now consider the angle next to the one you just solved for. These two angles sum to 180°, forming a straight line. So the adjacent angle must be $x + y$:



Alternatively, notice also that $x + y$ is the exterior angle to the triangle on the left, so it must be the sum of the two opposite interior angles (namely, x and y).

Now, the three angles of a triangle must sum to 180°, and no angle can equal 0. So any *two* angles in a triangle must sum to *less* than 180°. Consider the triangle on the right side, which contains angles of $x + y$ and 70°. Their sum is less than 180°:

$(x + y) + 70 < 180$

Subtract 70 from both sides:

$x + y < 110$

Quantity B is greater.

30. **(C).** First determine how the quantities relate to the triangle. For instance, examine Quantity A, the product of BE and AC. Notice that BE is the height of the largest triangle ABC, while AC is the base. This product should remind you of the formula for area: $A = \frac{bh}{2}$.

Plug in $b = AC$ and $h = BE$, then move the 2:

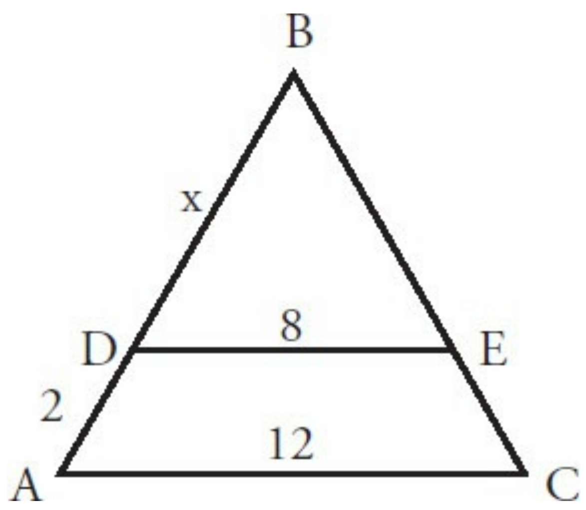
$2 \times \text{Area} = (AC)(BE)$

What about Quantity B, the product ofBC and AD? If you consider BC the base, then AD is the height to that base. So you can put $b = BC$ and $h = AD$ into the area formula, as before:

$2 \times \text{Area} = (BC)(AD)$

Both Quantity A and Quantity B are twice the area of the big triangle ABC. The two quantities are equal.

31. **(E).** If DE and AC are parallel lines, triangles ABC and DBE are similar. That means that there is a fixed ratio between corresponding sides of the two triangles. Since $AC = 12$ and $DE = 8$, that ratio is 12 : 8 or 3 : 2. This means that each side of triangle ABC (the larger triangle) is 1.5 times the corresponding side of triangle DBE (the smaller triangle), as shown below:



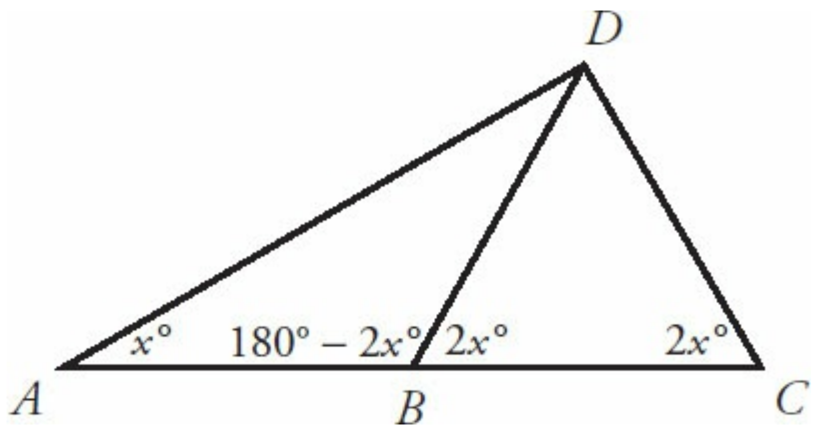
Assign a value of x to line DB, in order to set up the following proportion:

$$\frac{AB}{DB} = \frac{3}{2} = \frac{2+x}{x}$$

$3x = 2(2 + x)$
$3x = 4 + 2x$
$x = 4$

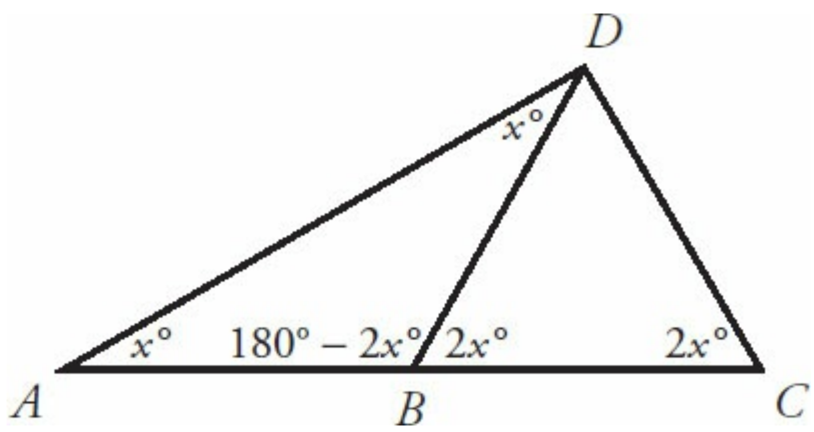
Since the question is asking for the value of AB, the answer is $2 + 4 = 6$.

32. **(C).** To compare *DC* and *AB*, first solve for the unlabeled angles in the diagram. The two angles at point *B* make a straight line, so they sum to 180°, and the unlabeled angle is 180 – 2*x*, as shown:



Now ensure that the angles of triangle *ADB* on the left sum to 180°. The top vertex of triangle *ADB* must measure 180 – *x* – (180 – 2*x*) = 180 – *x* – 180 + 2*x* = *x*.

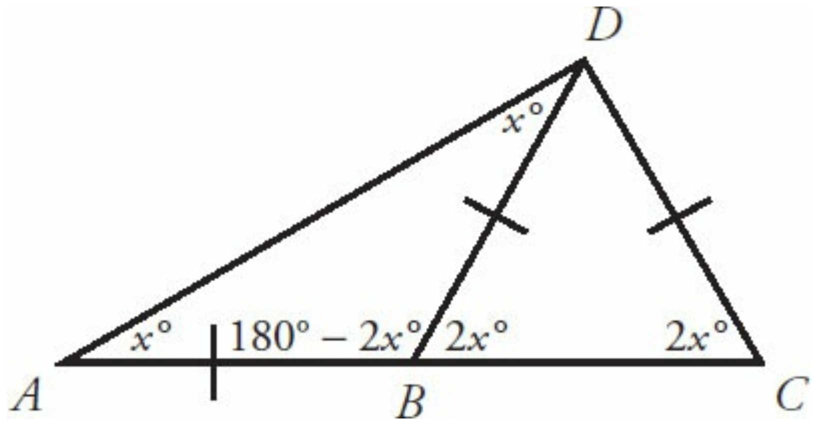
Therefore, the figure becomes:



Alternatively, notice that angle *DBC* (equal to 2*x*) is the exterior angle to the triangle on the left, and so it equals the sum of the two opposite interior angles in that triangle on the left. One of those angles is *x*, so the other one must be *x* as well.

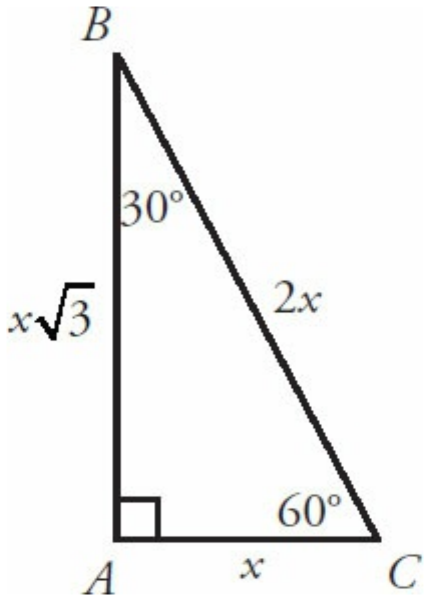
Now apply the properties of isosceles triangles. The two angles labeled *x* are equal, so the triangle that contains them (triangle *ADB*) is isosceles, and the sides opposite those equal angles are also equal. Put a slash through those sides (*AB* and *BD*) to mark them as the same length.

Likewise, the two angles labeled 2*x* are equal, so the triangle that contains them (triangle *BDC*) is isosceles, and the sides opposite those angles (*BD* and *DC*) are equal. Add one more slash through *DC* in the figure:



Thus, sides *AB* and *DC* have the same length. The two quantities are equal.

33. **(B).** To compute the perimeter of this triangle, sum the lengths of all three sides. Because one angle is labeled as a right angle and another as 30°, right triangle *ABC* is a 30–60–90 triangle. For any 30–60–90 triangle, the sides are in these proportions:



Match up this universal 30–60–90 triangle to the given triangle, in order to find x in this particular case. The only labeled side in the given triangle (6) matches the $x\sqrt{3}$ side in the universal triangle (they're both opposite the 60° angle), so set them equal to each other:

$$6 = x\sqrt{3}$$
$$x = \frac{6}{\sqrt{3}}$$

Rationalize the denominator by multiplying by $\frac{\sqrt{3}}{\sqrt{3}}$ (which does not change the value of x, as $\frac{\sqrt{3}}{\sqrt{3}}$ is just a form of 1):

$$x = \frac{6}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right)$$
$$x = \frac{6\sqrt{3}}{3}$$
$$x = 2\sqrt{3}$$

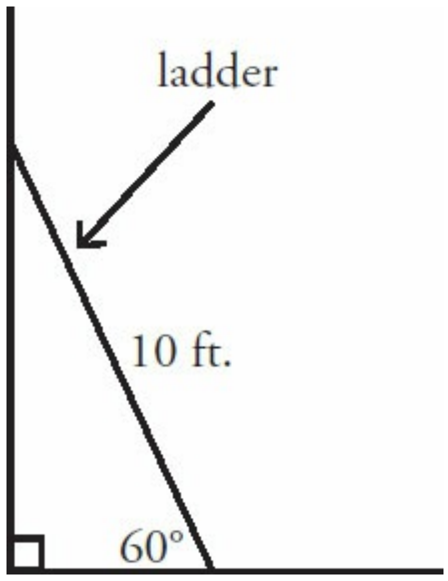
Now figure out all the sides in the given triangle. The length of side *AC* is $x = 2\sqrt{3}$, the length of side *AB* is given as 6, and the length of side *BC* is $2x = 2(2\sqrt{3}) = 4\sqrt{3}$.

Finally, sum all the sides to get the perimeter:

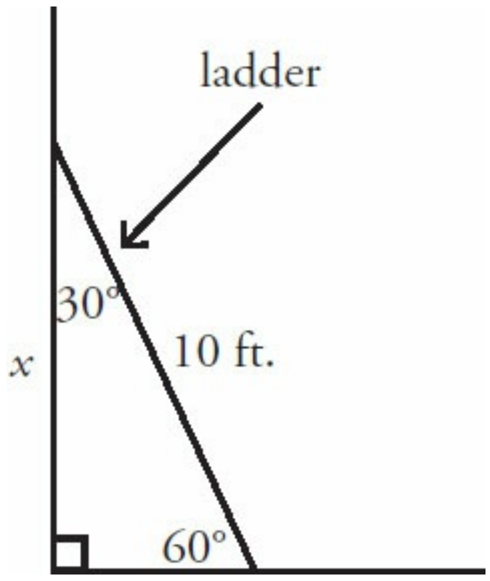
Perimeter = 6 + $2\sqrt{3}$ + $4\sqrt{3}$

Perimeter = 6 + $6\sqrt{3}$

34. **(B).** First, draw a diagram and label all the givens:

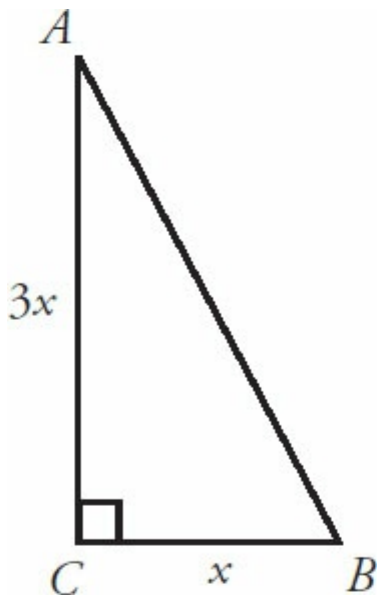


Since the wall is vertical and the floor is horizontal, the angle where they meet is 90°. So the triangle is 30–60–90. The question asks for the vertical distance from the top of the ladder to the floor, so represent this length as x:



In any 30–60–90 triangle, the short leg (opposite the 30° angle) is the hypotenuse divided by 2, making the floor side equal to $10 \div 2 = 5$ feet. The longer leg (opposite the 60° angle) is $\sqrt{3}$ times the short leg. So $x = 5\sqrt{3}$ feet.

35. **(C).** Draw a diagram and label the sides of the triangle with the information given. Since AC is three times as long as CB, label CB as x and AC as $3x$, as shown:



Use the area of the triangle, which is given as 9, to find the base x and the height $3x$. The formula for area is $A = \frac{bh}{2}$. Plug in and solve for x:

$$9 = \frac{x(3x)}{2} = \frac{3x^2}{2}$$

$18 = 3x^2$
$6 = x^2$
$\sqrt{6} = x$

So $CB = \sqrt{6}$ and $AC = 3\sqrt{6}$. Use the Pythagorean theorem to find AB:

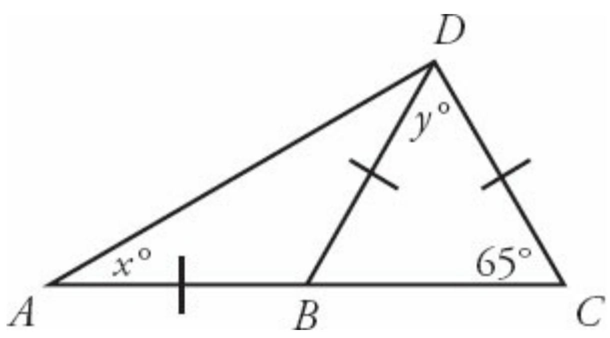
$(AB)^2 = (\sqrt{6})^2 + (3\sqrt{6})^2$
$(AB)^2 = 6 + 54 = 60$
$AB = \sqrt{60} = 2\sqrt{15}$

36. **(A).** Since the bottom left and bottom right angles are both equal to $y°$, the triangle is isosceles, and the sides opposite those angles (AB and BC) must also be equal:

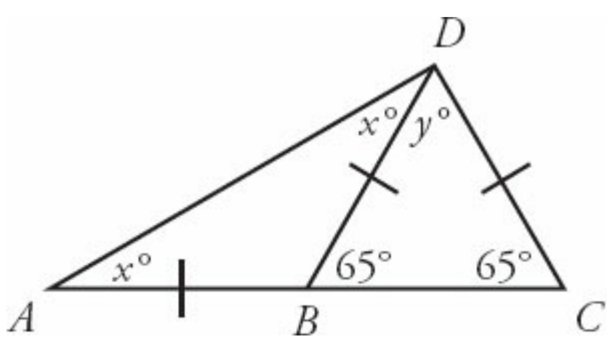
$2x – 5 = x + 4$
$x – 5 = 4$
$x = 9$

BC is therefore equal to 9 + 4 = 13. Quantity A is greater.

37. **(B).** Redraw the figure and label the equal sides:



The small triangle on the left (ADB) is isosceles as it has two sides of equal length. Likewise for the small triangle on the right (BDC) within it. In an isosceles triangle, the angles opposite the two equal sides are themselves equal in measure. Accordingly, label more angles on the figure:



The three angles in the triangle on the right must sum to 180°:

$65 + 65 + y = 180$

$130 + y = 180$
$y = 50$

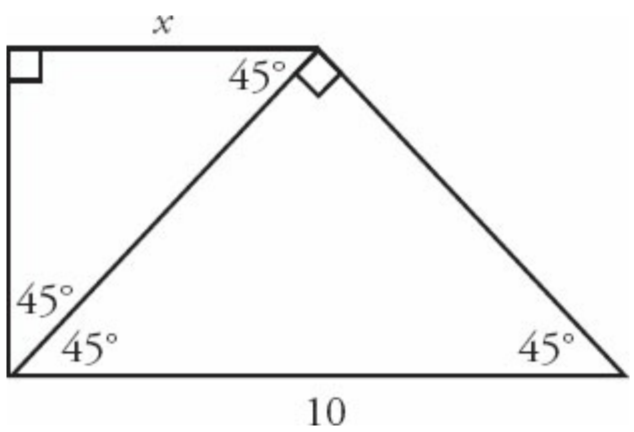
The two angles at point B make a straight line, so they sum to 180°. So the unlabeled angle must be $180 - 65 = 115°$.

Finally, the three angles in the triangle on the left must sum to 180°:

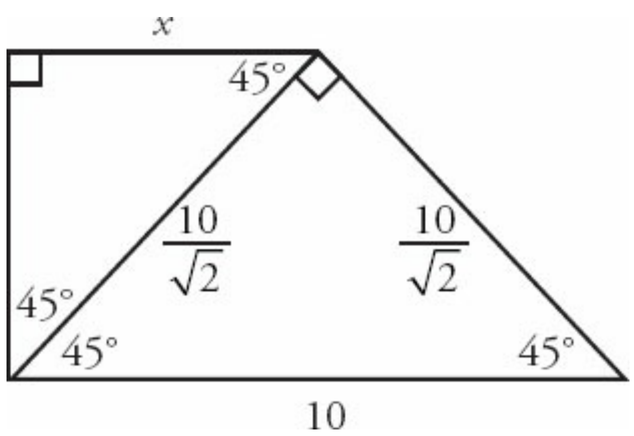
$$x + x + 115 = 180$$
$$2x = 65$$
$$x = 32.5$$

So y is greater than x. Quantity B is greater.

38. **(C).** Redraw the figure and label all angles, applying the rule that the angles in a triangle sum to 180°:



These are two separate 45–45–90 triangles. In a 45–45–90 triangle, the sides are in a $1 : 1 : \sqrt{2}$ ratio. Thus, the length of each leg equals the length of the hypotenuse divided by $\sqrt{2}$. The hypotenuse of the larger triangle is 10, so each leg of that triangle is $\frac{10}{\sqrt{2}}$.



The hypotenuse of the smaller triangle is $\frac{10}{\sqrt{2}}$. Divide by $\sqrt{2}$ again according to the 45–45–90 triangle ratio ($1 : 1 : \sqrt{2}$) to see that $x = \frac{10}{\sqrt{2}\sqrt{2}} = \frac{10}{2} = 5$.

39. **Angle ACB equals 90° and AB = 17 only.** If angle ACB = 90°, then 8 and 15 are the base and height, and you can calculate the area. The first statement is sufficient.

If $AB = 17$, you can plug 8, 15, and 17 into the Pythagorean theorem to see whether you get a true statement. Use 17 as the hypotenuse in the Pythagorean theorem because 17 is the longest side:

$8^2 + 15^2 = 17^2$
$64 + 225 = 289$
$289 = 289$

Since this is true, the triangle is a right triangle with the right angle at *C*. If angle *ACB* = 90°, then 8 and 15 are the base and height, and you can calculate the area. (Since 8–15–17 is a Pythagorean triple, if you had that fact memorized, you could skip the step above.) The second statement is sufficient.

Knowing that *ABC* is a right triangle (the third statement) is *not* sufficient to calculate the area because it's not specified which angle is the right angle. A triangle with sides of 8 and 15 could have hypotenuse 17, but another scenario is possible: perhaps 15 is the hypotenuse. In this case, the third side is shorter than 15, and the area is smaller than in the 8–15–17 scenario.

40. **(A).** The three interior angles of the triangle sum to 180°. Try an example: say each interior angle is 60°. In that case, *a*, *b*, and *c* would each equal 120° (since two angles that make up a straight line sum 180), and Quantity A would equal 360°.

It is possible to prove this result in general by expressing each interior angle in terms of *a*, *b*, and *c*, and then setting their sum equal to 180°:

$(180 - a) + (180 - b) + (180 - c) = 180$
$540 - a - b - c = 180$
$360 = a + b + c$

Quantity A is greater.

41. **(B).** Since both triangles have a 90° angle and an angle *x*°, the third angle of each is the same as well (because the three angles in each triangle sum to 180°). All the corresponding angles are equal, so the triangles are similar, and the ratio of corresponding sides is constant.

The smaller triangle is a 3–4–5 Pythagorean triple (the missing hypotenuse is 5). Set up a proportion that includes two pairs of corresponding sides. The words "4 is to 10 as 5 is to *m*" become this equation:

$\frac{4}{10} = \frac{5}{m}$
$4m = 50$
$m = 12.5$

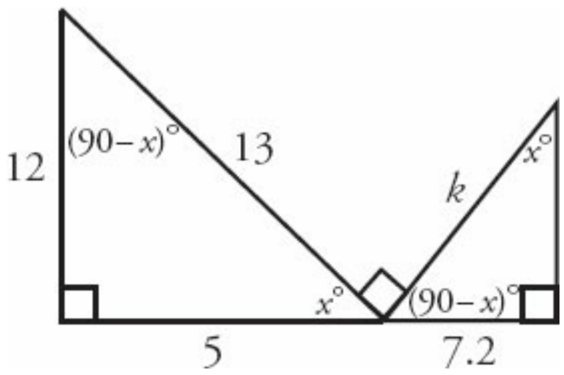
Quantity B is greater.

42. **7.8.** Begin by noting that the triangle on the left is a 5–12–13 Pythagorean triple, so the bottom side is 5. Subtract 12.2 – 5 = 7.2 to get the bottom side of the triangle on the right.

Next, the two unmarked angles that "touch" at the middle must sum to 90°, because they form a straight line together with the right angle of 90° between them, and all three angles must sum to 180°.

Mark the angle on the left x. The angle on the right must then be $90 - x$.

Now the other angles that are still unmarked can be labeled in terms of x. Using the rule that the angles in a triangle sum to 180°, the angle between 12 and 13 must be $90 - x$, while the last angle on the right must be x, as shown:



Since each triangle has angles of 90, x, and $90 - x$, the triangles are similar. This observation is the key to the problem. Now you can make a proportion, carefully tracking which side corresponds to which. The 7.2 corresponds to 12, since each side is across from angle x. Likewise, k corresponds to 13, since each side is the hypotenuse. Write the equation and solve for k:

$$\frac{7.2}{12} = \frac{k}{13}$$

$$\frac{(13)7.2}{12} = k$$

$$7.8 = k$$