Data Interpretation

For questions in the Quantitative Comparison format (“Quantity A” and “Quantity B” given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box $\boxed{}$, you are to enter your own answer in the box. For questions followed by a fraction-style numeric entry box $\frac{\boxed{}}{\boxed{}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is $\frac{1}{4}$, you may enter $\frac{25}{100}$ or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations, such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

Problem Set A

9th Grade Students at Millbrook High School

	Boys	Girls
Enrolled in Spanish	12	13
Not Enrolled in Spanish	19	16

1. Approximately what percent of the 9th grade girls at Millbrook High School are enrolled in Spanish?

   (A) 21%
   (B) 37%
   (C) 45%
   (D) 50%
   (E) 57%

2. What fraction of the students in 9th grade at Millbrook High School are boys who are enrolled in Spanish?

(A) $\frac{1}{5}$

(B) $\frac{19}{60}$

(C) $\frac{5}{12}$

(D) $\frac{12}{31}$

(E) $\frac{12}{25}$

3. What is the ratio of 9th grade girls not enrolled in Spanish to all 9th grade students at Millbrook Middle School?

(A) 1 : 16
(B) 13 : 60
(C) 4 : 15
(D) 19 : 60
(E) 16 : 29

4. If *x* percent more 9th grade students at Millbrook High School are not enrolled in Spanish than are enrolled in Spanish, what is the value of x*?*

(A) 20
(B) 25
(C) 30
(D) 40
(E) 50

5. If two of the 9th grade boys at Millbrook High school who are not enrolled in Spanish decided to enroll in Spanish, and then eight additional girls and seven additional boys attended the 9th grade at Millbrook Middle School and also enrolled in Spanish, what percent of 9th grade students at Millbrook would then be enrolled in Spanish?

(A) 52%
(B) 53%
(C) 54%

(D) 55%
(E) 56%

Problem Set B

Number of Hours Worked per Week per Employee at Marshville Toy Company

# of Employees	Hours Worked Per Week
4	15
9	25
15	35
27	40
5	50

6. What is the median number of hours worked per week per employee at Marshville Toy Company?

   (A) 25
   (B) 30
   (C) 35
   (D) 37.5
   (E) 40

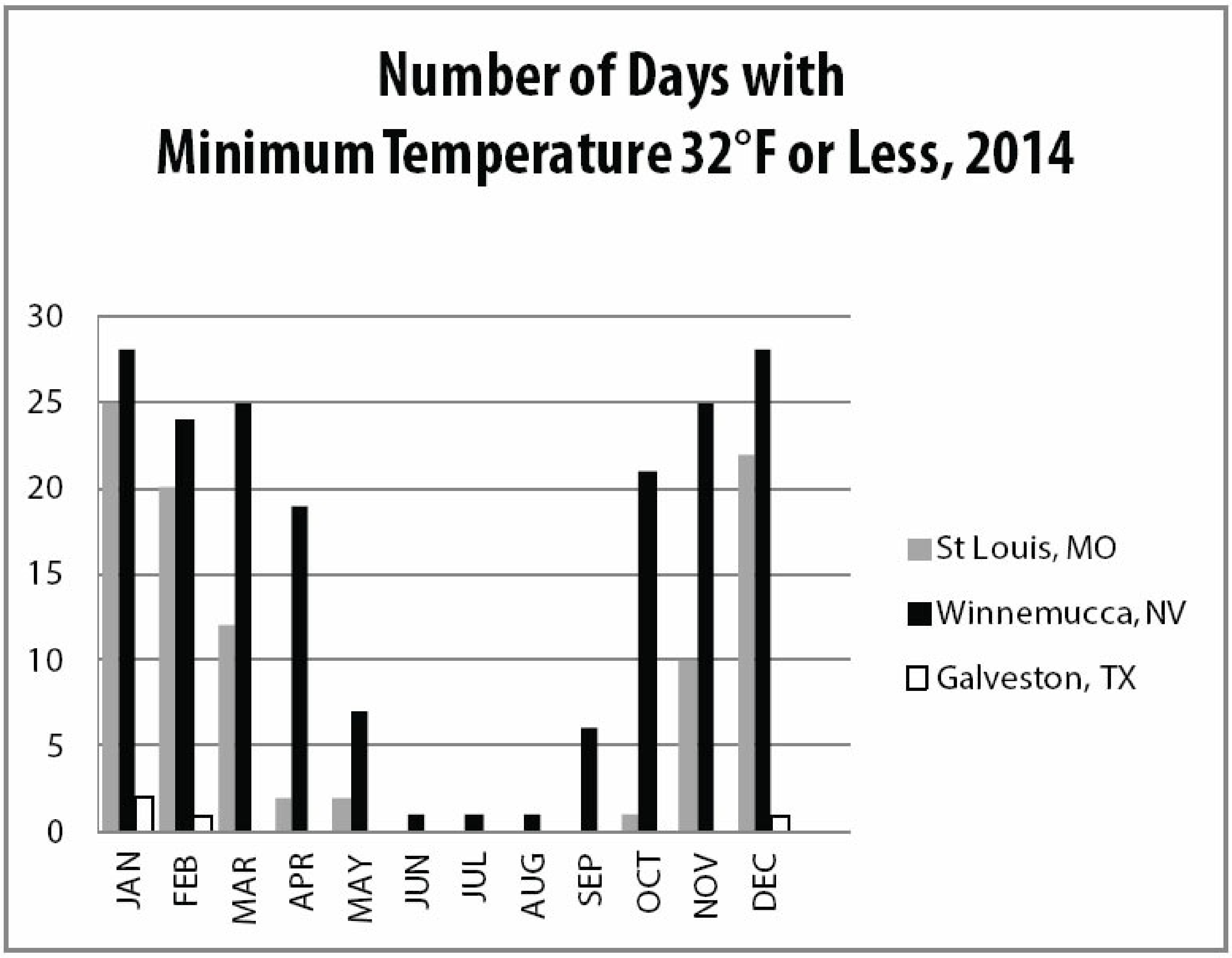
7. What is the average (arithmetic mean) number of hours worked per week per employee at Marshville Toy Company?

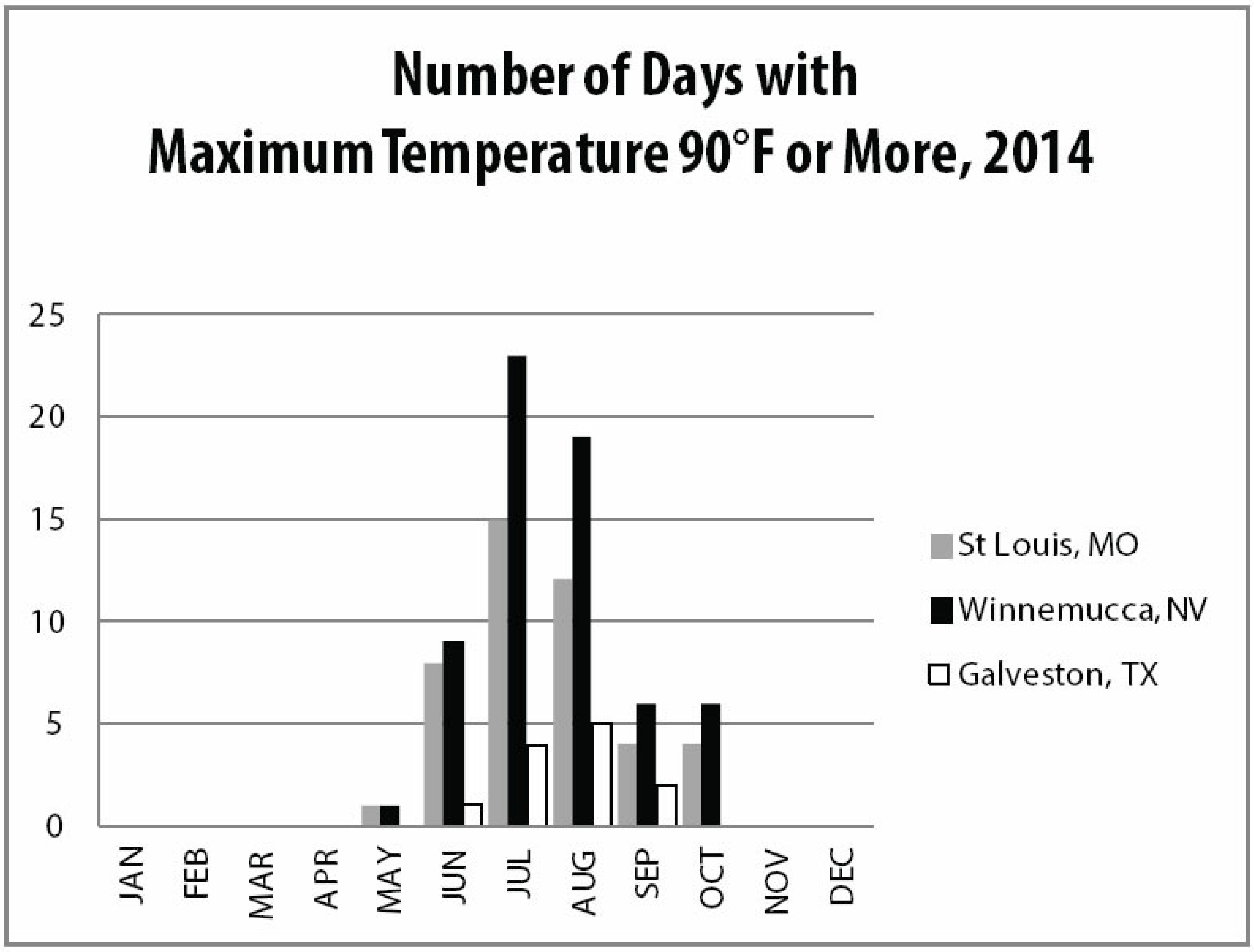
   (A) 32
   (B) 33
   (C) 35
   (D) $35\frac{2}{3}$
   (E) $36\frac{1}{3}$

8. What is the positive difference between the mode and the range of the number of hours worked per week per employee at Marshville Toy Company?

   (A) 0
   (B) 4
   (C) 5
   (D) 8
   (Ed) 26

Problem Set C




Number of Days with
Maximum Temperature 90°F or More, 2014
25
20
15
10
5
0
JAN
FEB
MAR
APR
MAY
JUN
JUL
AUG
SEP
OCT
NOV
DEC
St Louis, MO
Winnemucca, NV
Galveston, TX

9. In how many months of the year were there more than 20 days with temperatures 32°F or less in Winnemucca?

(A) 2
(B) 3
(C) 4
(D) 6
(E) 7

10. On how many days in the entire year did the temperature in Galveston rise to at least 90°F or fall at least as low as 32°F?

(A) 11
(B) 16
(C) 28
(D) 42
(E) 59

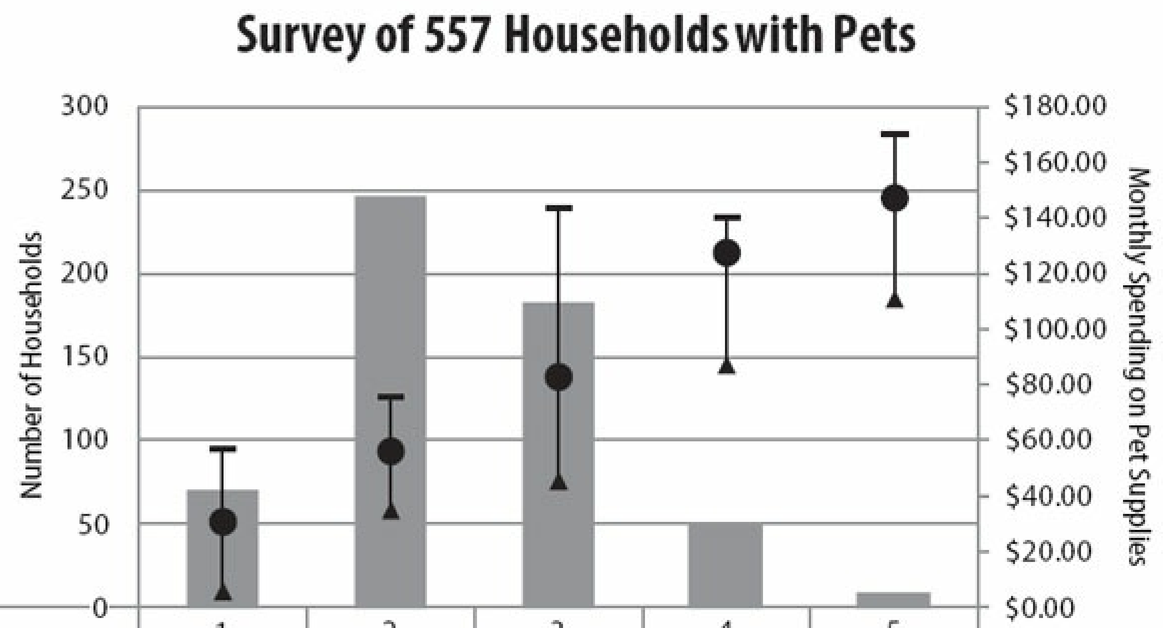
11. Approximately what percent of the days with maximum temperature of 90°F or more in St. Louis occurred in July?

(A) 6%
(B) 15%
(C) 17%
(D) 34%
(E) 44%

12. The number of freezing January days in Winnemucca was approximately what percent more than the number of freezing January days in St. Louis? (A "freezing" day is one in which the minimum temperature is 32°F or less.)

(A) 3%
(B) 6%
(C) 12%
(D) 24%
(E) 28%

Problem Set D



Number of Pets	1	2	3	4	5
■ Number of Households with This Number of Pets	70	246	183	49	9
‒ Maximum Monthly Spending on Pet Supplies	\$57.32	\$75.98	\$143.57	\$140.48	\$170.23
▲ Minimum Monthly Spending on Pet Supplies	\$6.34	\$35.47	\$45.84	\$87.46	\$111.20
● Average Monthly Spending on Pet Supplies	\$31.25	\$56.42	\$83.11	\$127.74	\$147.38

13. Approximately what percent of the surveyed households have more than three pets?

(A) 10%
(B) 20%
(C) 30%
(D) 40%
(E) 50%

14. Which of the following is the median number of pets owned by the households in the survey?

(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

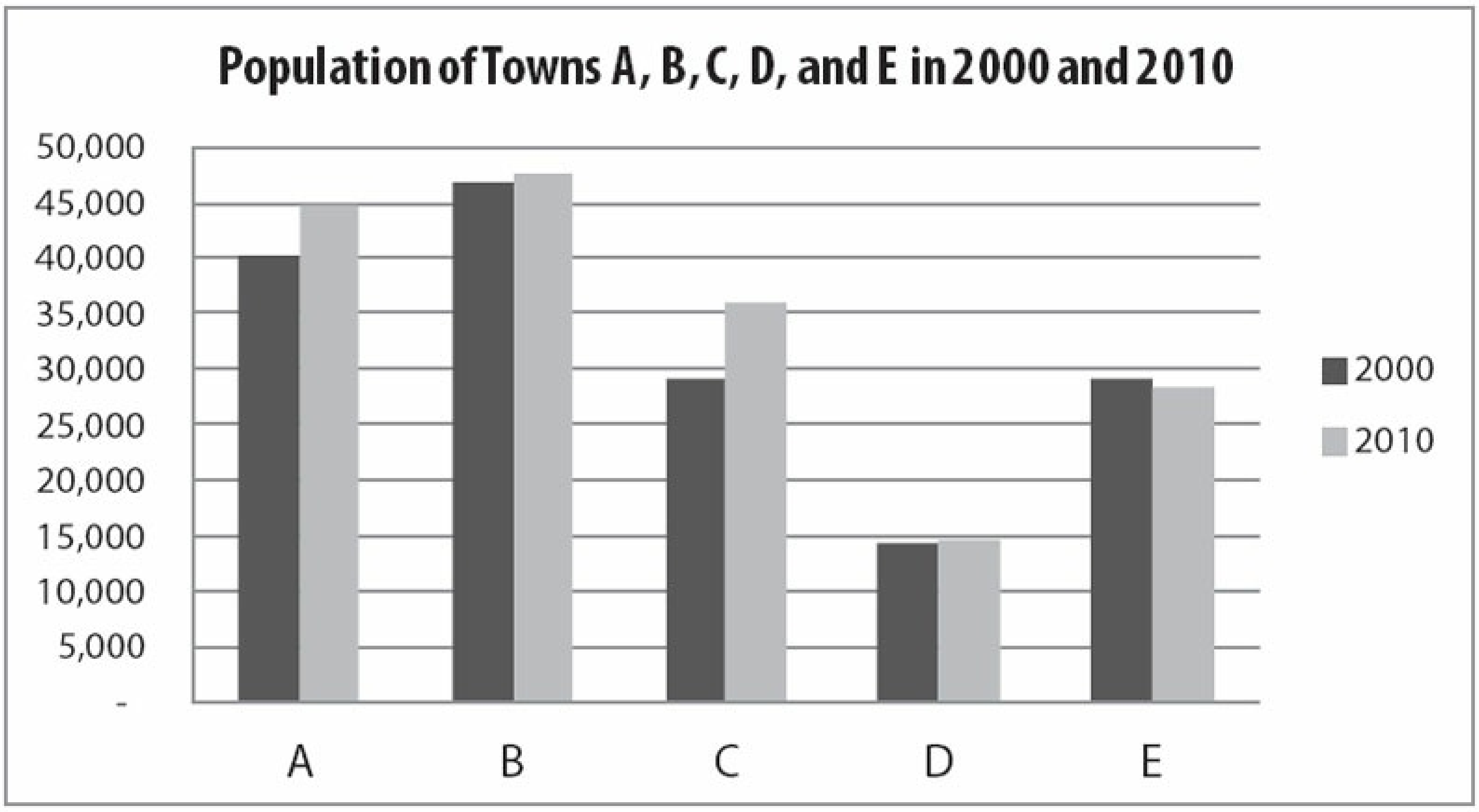
15. What is the range of monthly spending on pet supplies for the household group with the largest such range?

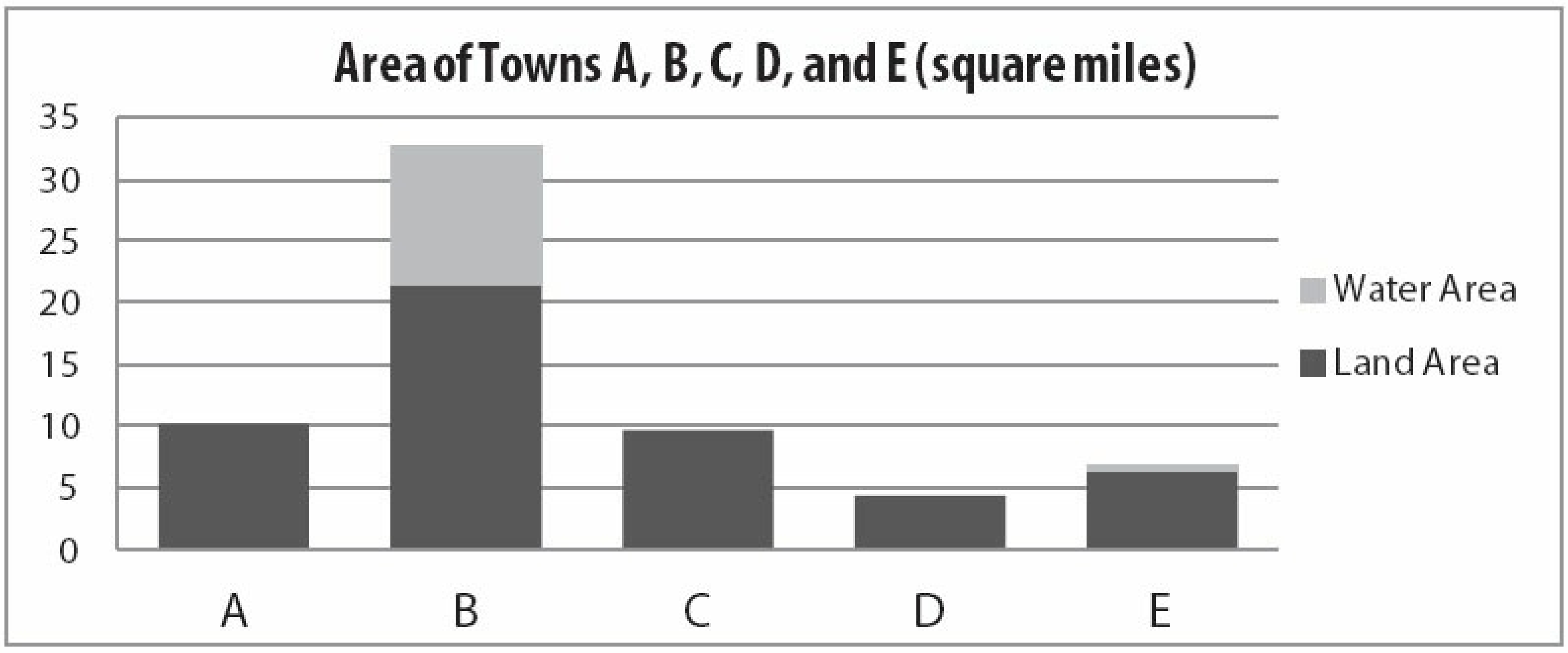
(A) $69.03
(B) $97.73
(C) $116.13
(D) $138.98
(E) $170.23

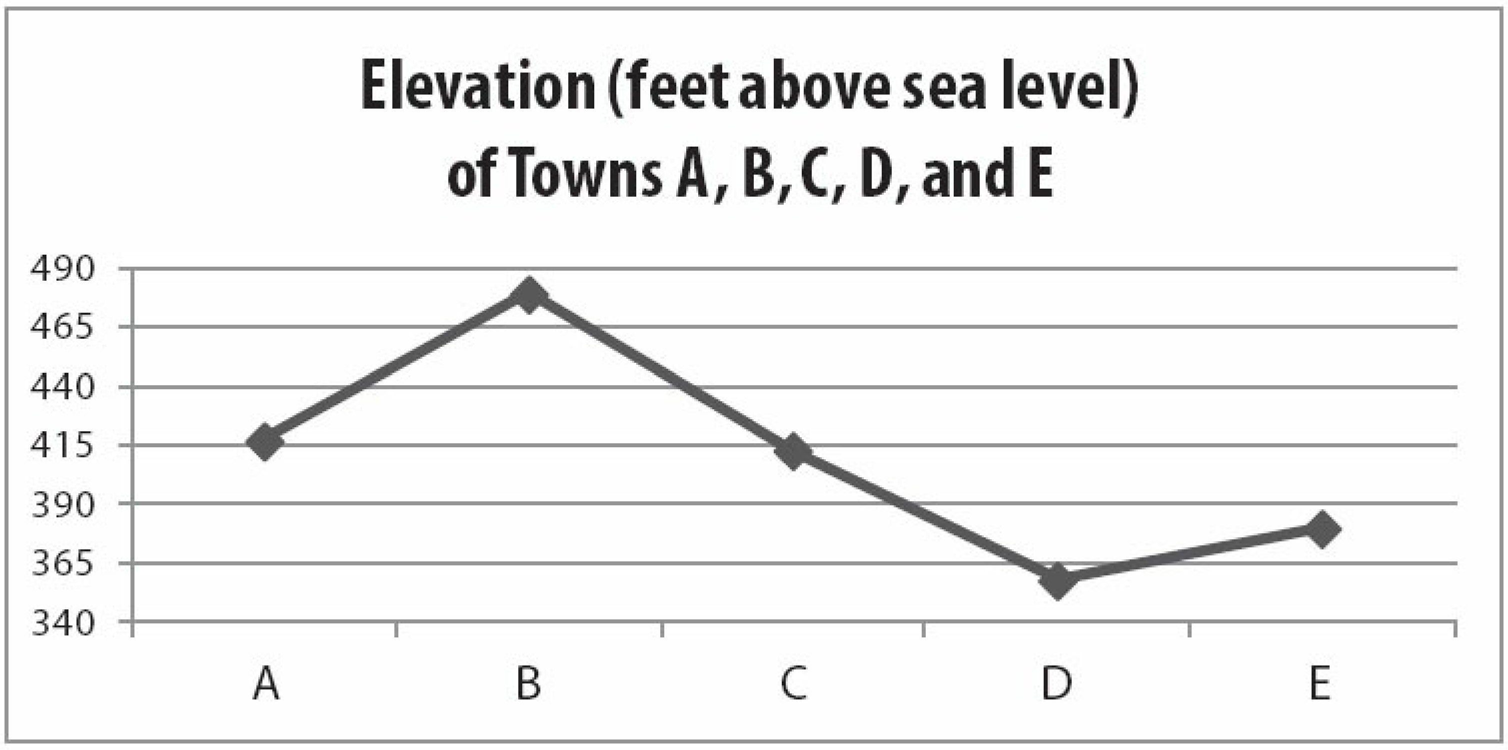
16. The household group with which number of pets had the greatest average (arithmetic mean) monthly spending per pet?

(A) 1 pet
(B) 2 pets
(C) 3 pets
(D) 4 pets
(E) 5 pets

Problem Set E






Elevation (feet above sea level)
of Towns A, B, C, D, and E
490
465
440
415
390
365
340
A
B
C
D
E

17. In what town did the population increase by the greatest percent between 2000 and 2010?

(A) Town A
(B) Town B
(C) Town C
(D) Town D
(E) Town E

18. The water area of town B is most nearly equal the to the sum of the land areas of which two towns?

Indicate <u>two</u> such towns.

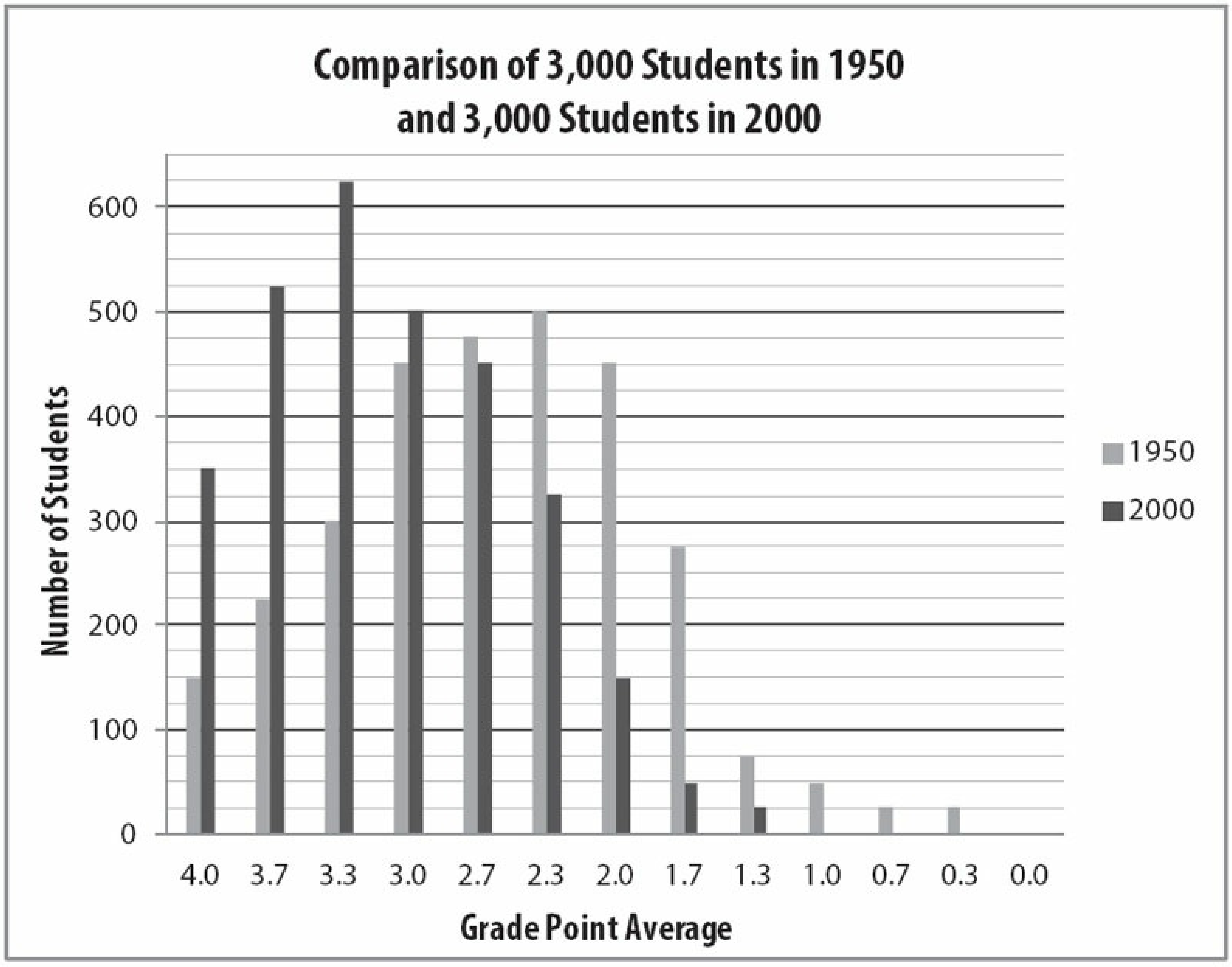
☐ Town A
☐ Town B
☐ Town C
☐ Town D
☐ Town E

19. Which two towns have approximately the same elevation in feet above sea level?

Indicate <u>two</u> such towns.

☐ Town A
☐ Town B
☐ Town C
☐ Town D
☐ Town E

Problem Set F



20. What was the mode for grade point average of the 3,000 students in 2000?

    (A) 3.7
    (B) 3.3
    (C) 3.0
    (D) 2.7
    (E) 2.3

21. What was the median grade point average of the 3,000 students in 1950?

    (A) 3.7
    (B) 3.3
    (C) 3.0

(D) 2.7
(E) 2.3

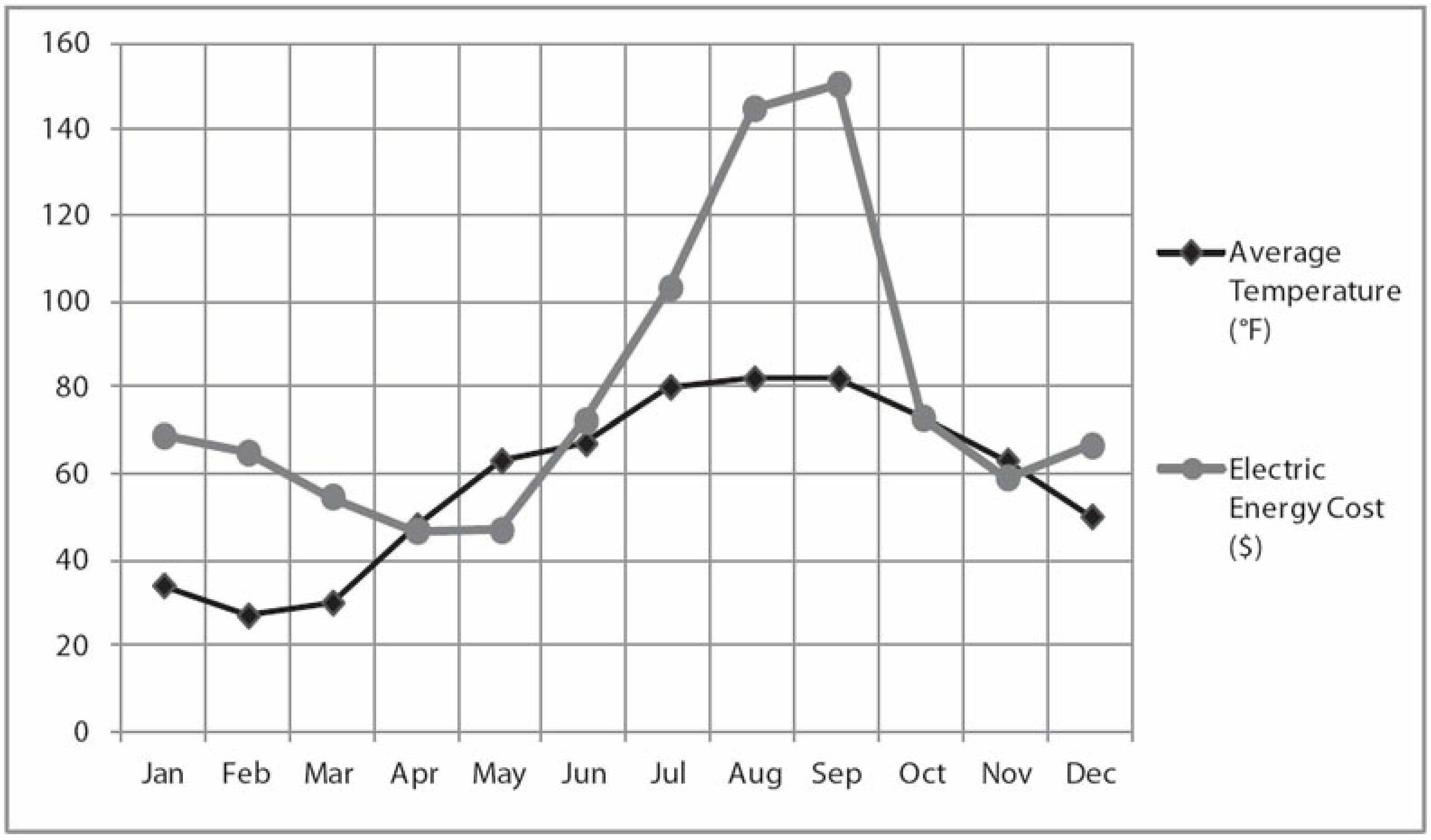
22. Approximately what percent of the students in 2000 earned at least a 3.0 grade point average?

(A) 25%
(B) 50%
(C) 67%
(D) 80%
(E) 97.5%

23. Approximately what percent of the students in 1950 earned a grade point average less than 3.0?

(A) 33%
(B) 37.5%
(C) 50%
(D) 62.5%
(E) 75%

Problem Set G



24. According to the chart, which two-month period had the greatest increase in electric energy cost?
    - (A) Between January and February
    - (B) Between May and June
    - (C) Between June and July
    - (D) Between July and August
    - (E) Between November and December

25. According to the chart, in which two-month period did electric energy cost increase the least?
    - (A) Between January and February
    - (B) Between April and May
    - (C) Between May and June
    - (D) Between June and July
    - (E) Between November and December

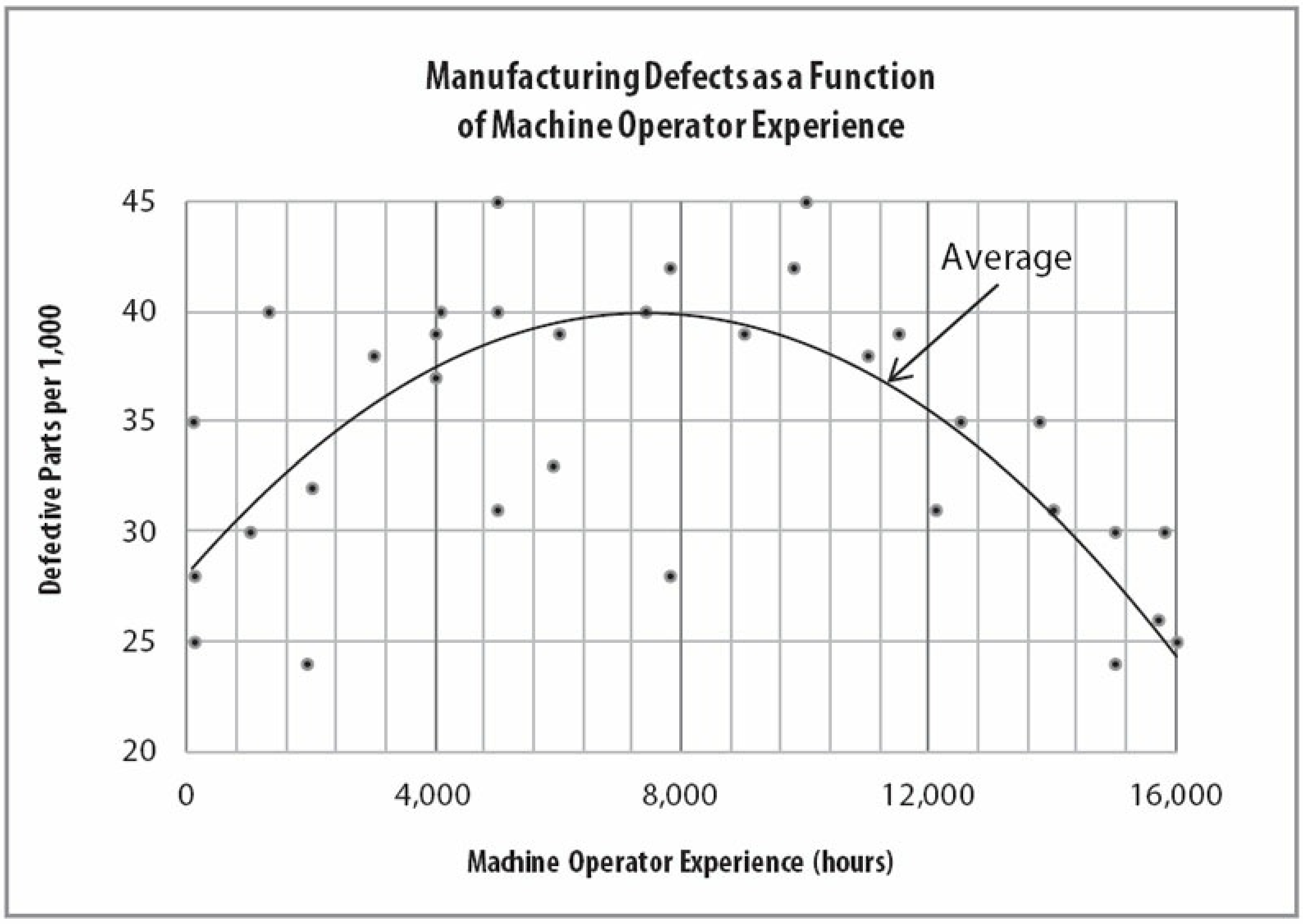
26. Approximately what was the average (arithmetic mean) electric energy cost per month for the first half of the year?

(A) $45
(B) $50
(C) $60
(D) $70
(E) $75

27. In which month was the electric energy cost per Fahrenheit degree (°F) of average temperature the least?

(A) April
(B) May
(C) October
(D) November
(E) December

Problem Set H



28. On average, the machine operators that produce the fewest defective parts per 1,000 have how many hours of experience?

    (A) 40
    (B) 4,000
    (C) 8,000
    (D) 12,000
    (E) 16,000

29. On average, the machine operators with approximately how many hours of experience have the same defective part rate as those with 12,000 hours of experience?

    (A) 2,000
    (B) 2,700
    (C) 4,400

(D) 8,400

(E) 12,800

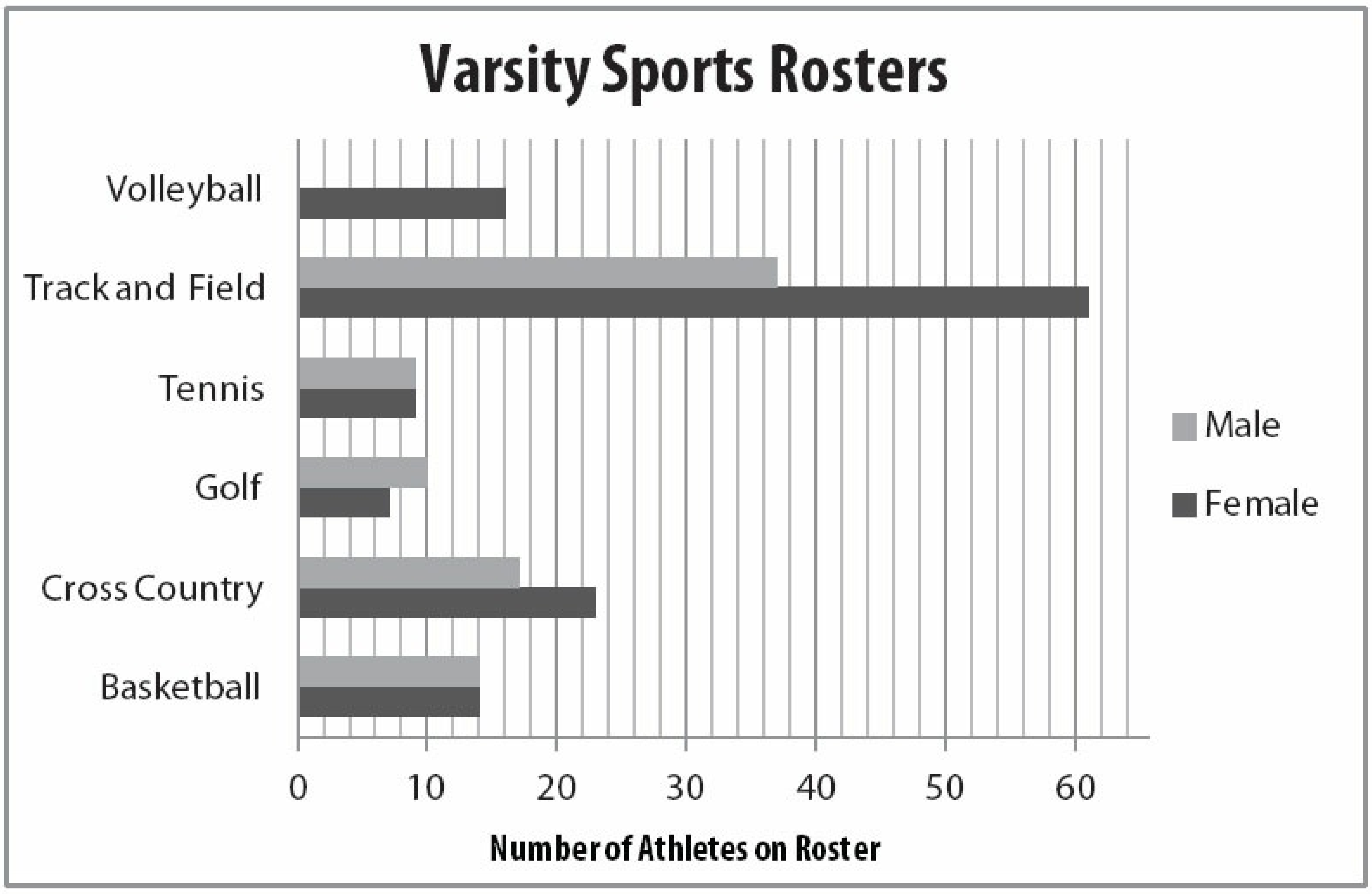
30. On average, approximately how many hours of experience do machine operators who produce the most defective parts per 1,000 have?

(A) 40
(B) 4,000
(C) 8,000
(D) 12,000
(E) 16,000

31. Of the individual machine operators who recorded a defective part rate of 4.2%, approximately how many hours of experience did the least experienced operator have?

(A) 2,300
(B) 5,000
(C) 7,700
(D) 9,800
(E) 15,100

Problem Set I



32. What is the ratio of male athletes to female athletes on the track and field roster?

(A) $\frac{37}{61}$

(B) $\frac{9}{14}$

(C) $\frac{17}{23}$

(D) $\frac{14}{9}$

(E) $\frac{61}{37}$

33. All athletes are on only one varsity sports roster EXCEPT those who are on both the Track and Field team and the Cross Country team. If there are 76 male athletes in total on the varsity sports

rosters, how many male athletes are on both the Track and Field team and the Cross Country team?

(A) 11
(B) 17
(C) 37
(D) 54
(E) 76

34. On which varsity sports rosters do male athletes outnumber female athletes?

Indicate all such rosters.

- ☐ Volleyball
- ☐ Track and Field
- ☐ Tennis
- ☐ Golf
- ☐ Cross Country
- ☐ Basketball

35. What is the ratio of female tennis players to male basketball players on the varsity sports rosters?

(A) $\frac{5}{12}$

(B) $\frac{9}{14}$

(C) $\frac{7}{8}$

(D) $\frac{14}{9}$

(E) $\frac{12}{5}$

Problem Set J

	Change in Total Revenue (2011–2012)	Percent Change in Number of Distinct Customers (2011–2012)	Percent Change in Total Costs (2011–2012)
Store W	–$400,000	+2%	+15%
Store X	+$520,000	+14%	+4%
Store Y	–$365,000	+5%	+12%
Store Z	+$125,000	–7%	–20%

36. For which store was the revenue per distinct customer greatest in 2012?

(A) Store W
(B) Store X
(C) Store Y
(D) Store Z
(E) It cannot be determined from the information given.

37. Between 2011 and 2012, total costs per distinct customer increased by the greatest percent at which store?

(A) Store W
(B) Store X
(C) Store Y
(D) Store Z
(D) It cannot be determined from the information given.

38. At which of the following stores could the profit in 2012 have been less than that same store's profit in 2011?

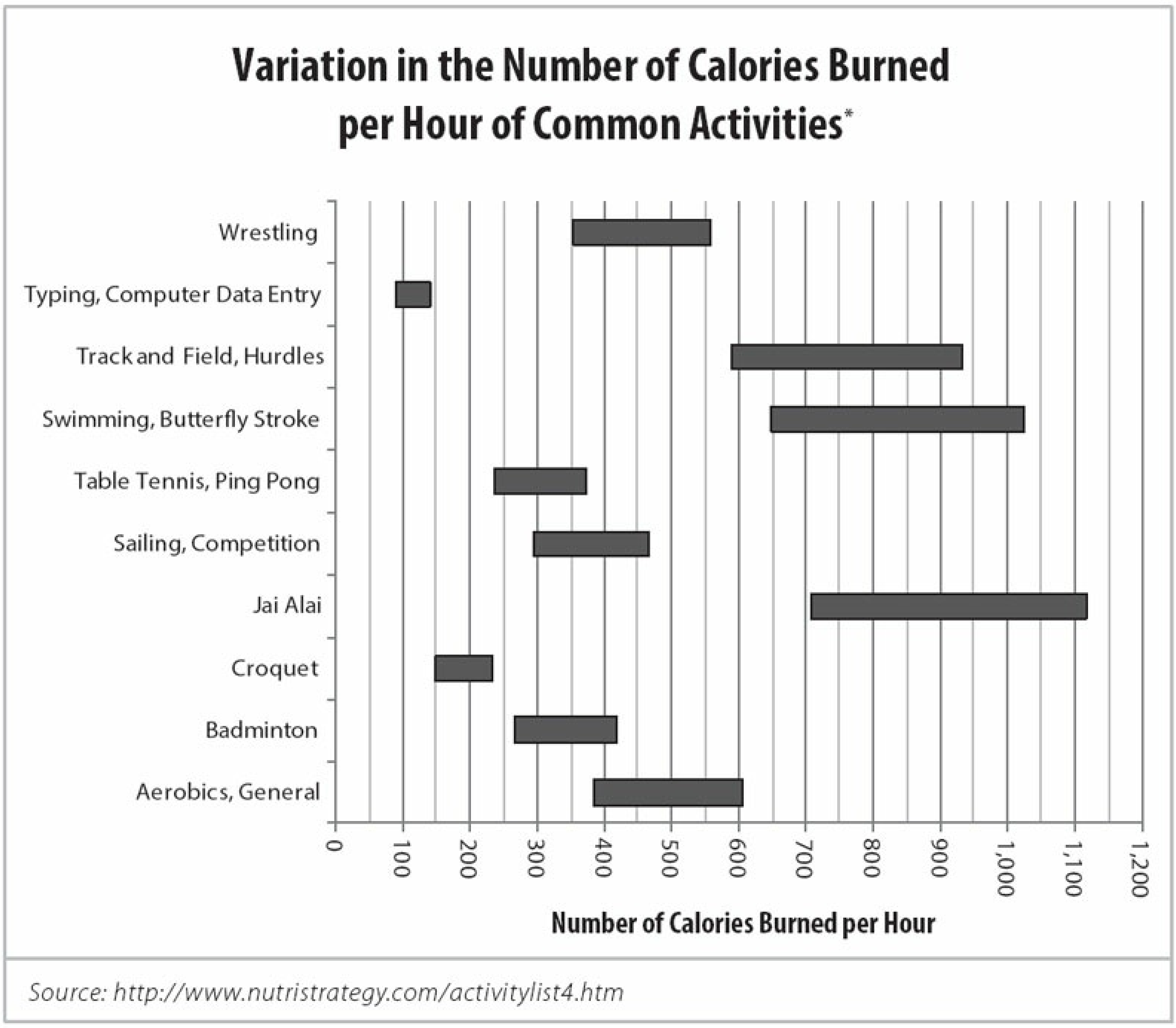
Indicate <u>all</u> such stores.

☐ Store W
☐ Store X
☐ Store Y
☐ Store Z
☐ None of the above

39. Which of the following statements must be true?

(A) Of the four stores, store X had the greatest percent increase in revenue from 2011 to 2012.
(B) Per customer revenue increased at store Z from 2011 to 2012.
(C) Of the four stores, store W had the greatest increase in total costs from 2011 to 2012.
(D) Of the four stores, store Y had the highest percentage of repeat customers.
(E) In 2012, store W and store Z combined had fewer distinct customers than did store X.

Problem Set K



* Based on body weight of exercise subject. The lower limit represents the calories burned by a person weighing 130 pounds, while the upper limit represents the calories burned by a person weighing 205 pounds.

40. Which of the following statements could be true?

Indicate <u>all</u> such statements.

- ☐ A person weighing between 130 and 205 pounds performs one of the above activities for 10 hours yet burns fewer calories than another person in the same weight range performing another activity for 1 hour.
- ☐ A 175-pound person playing jai alai for 1 hour burns fewer calories than a 180-pound person swimming the butterfly stroke for 1 hour.
- ☐ If all the people in question weigh between 130 and 205 pounds, the average calories burned by one person playing table tennis for 1 hour is more than the total calories burned by two people typing for 3 hours.

41. Which combination of activities burns the fewest calories total?

(A) A 130-pound person playing badminton for 1 hour and a 205-pound person playing table tennis for 1 hour

(B) A 130-pound person wrestling for 1 hour and a 205-pound person running track and field, hurdles for 1 hour

(C) A 130-pound person typing for 1 hour and a 205-pound person swimming the butterfly stroke for 1 hour

(D) A 130-pound person sailing in a competition for 1 hour and a 205-pound person doing aerobics for 1 hour

(E) A 130-pound person typing for 1 hour and a 205-pound person playing croquet for 1 hour

Problem Set L

Population and GDP for 50 African Countries

		Population					
		More Than 50 Million	20–50 Million	10–20 Million	2–10 Million	Less Than 2 Million	Total
Gross Domestic Product	More Than $100 Billion	3	2	0	0	0	5
	$20–100 Billion	1	7	1	1	0	10
	$10–20 Billion	1	3	3	3	3	13
	Less Than $10 Billion	0	0	7	8	7	22
	Total	5	12	11	12	10	50

42. Among the 50 African countries represented in the chart above, how many countries have a population between 10 million and 50 million people and a GDP between $10 billion and $20 billion?

(A) 6
(B) 7
(C) 13
(D) 16
(E) 23

43. Among the 50 African countries represented in the chart above, what percent of the countries have a population of less than 20 million people and a GDP of less than $20 billion?

(A) 38%
(B) 44%
(C) 62%
(D) 68%
(E) 90%

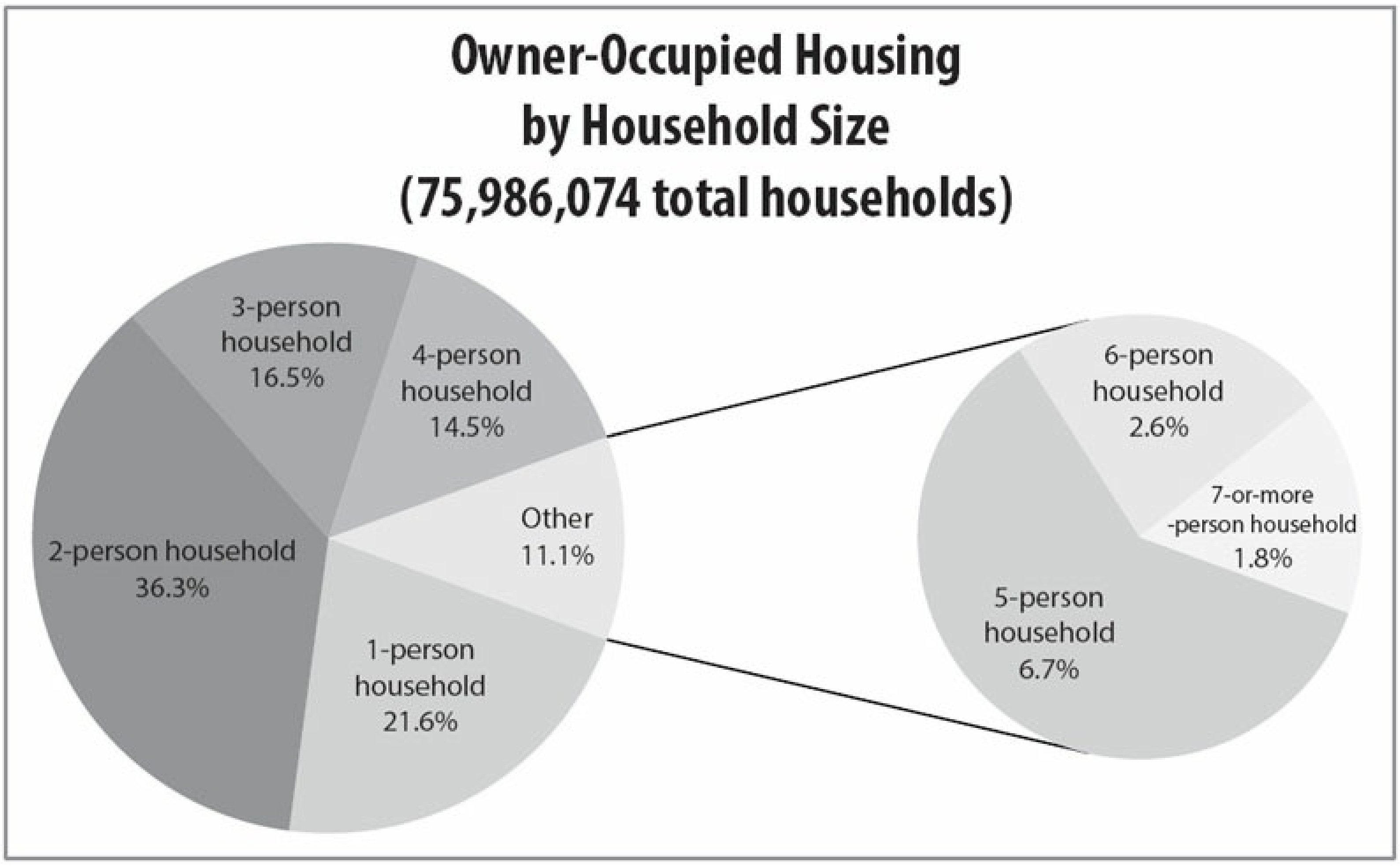
44. Approximately what percent of the African countries in the chart above that have a GDP between \$10 billion and \$20 billion also have a population between 10 million and 20 million?

(A) 6%
(B) 23%
(C) 26%
(D) 30%
(E) 51%

45. According to the chart above, which of the following is greatest?

(A) The number of countries with more than \$10 billion of GDP and a population of less than 20 million
(B) The number of countries with less than \$20 billion of GDP and a population of more than 10 million
(C) The number of countries with more than \$20 billion of GDP
(D) The number of countries with less than \$100 billion of GDP and a population of less than 10 million
(E) The number of countries with less than \$100 billion of GDP and a population between 10 million and 50 million

Problem Set M



46. What percent of owner-occupied housing units are households with fewer than four people?

(A) 11.1%
(B) 14.5%
(C) 25.6%
(D) 74.4%
(E) 88.9%

47. Among the owner-occupied housing units represented in the chart above, approximately how many households are 5-person households?

(A) 1 million
(B) 2 million
(C) 3 million
(D) 4 million
(E) 5 million

48. Based on the total number of people living in all such households, which of the following is a correct ordering, from least to greatest, of 1-person households, 3-person households, and 5-person households?

(A) 1-person households, 3-person households, 5-person households
(B) 1-person households, 5-person households, 3-person households
(C) 3-person households, 1-person households, 5-person households
(D) 3-person households, 5-person households, 1-person households
(E) 5-person households, 3-person households, 1-person households

49. Which combination of household sizes accounts for more than 50% of all owner-occupied housing units?

(A) 2- and 3-person
(B) 3- and 4-person
(C) 4- and 5-person
(D) 5- and 6-person
(E) 6- and 7-person

Data Interpretation Answers

Problem Set A: The title of the chart indicates that the total population is the total number of 9th graders at Millbrook Middle School.

When given a chart that depends on addition (boys + girls = total students, and also those enrolled in Spanish + those not enrolled in Spanish = total students), it can be helpful to sketch a quick version of the chart and add a total column. For example:

	Boys	Girls	TOTAL
Enrolled in Spanish	12	13	
Not Enrolled in Spanish	19	16	
TOTAL			

Now add down and across:

	Boys	Girls	TOTAL
Enrolled in Spanish	12	13	25
Not Enrolled in Spanish	19	16	35
TOTAL	31	29	60

1. **(C).** There are 29 total girls and 13 are enrolled in Spanish. The fraction of girls enrolled in Spanish is $\frac{13}{29}$. Convert to a percent: $\left(\frac{13}{29}\times 100\right)\%$ = 44.827…%, or about 45%.

2. **(A).** There are 60 total students and 12 boys enrolled in Spanish. The answer is $\frac{12}{60}$, which reduces to $\frac{1}{5}$. (Read carefully! "What fraction of the students … are boys who are enrolled in Spanish?" is *not* the same as "What fraction of the boys are enrolled in Spanish?")

3. **(C).** There are 16 girls not enrolled in Spanish and 60 total students. The ratio is $\frac{16}{60}$, which reduces to $\frac{4}{15}$ or 4 : 15.

4. **(D).** There are 35 students not enrolled in Spanish and 25 who are. The question can be rephrased

as, “35 is what percent greater than 25?” Using the percent change formula:

$$\text{Percent Change} = \left(\frac{\textit{Difference}}{\textit{Original}} \times 100\right)\%$$

$$\text{Percent Change} = \left(\frac{10}{25} \times 100\right)\% = 40\%$$

Thus, x is 40.

5. **(E)**. Sketch a new chart to reflect the changes. Switch 2 of the boys from "not enrolled" to "enrolled." Then, add 8 new girls and 7 new boys to the "enrolled" groups:

	Boys	Girls	TOTAL
Enrolled in Spanish	12 + 2 + 7 = 21	13 + 8 = 21	42
Not Enrolled in Spanish	19 – 2 = 17	16	33
TOTAL	38	37	75

Update the Total rows and columns as well. Both "Boys" and "Girls," as well as "Enrolled in Spanish" and "Not Enrolled in Spanish," now sum to 75.

What percent of 9th grade students at Millbrook would then be taking Spanish? Since 42 out of 75 students would be enrolled in Spanish, calculate $\frac{42}{75}$ (with the calculator) and multiply by 100 to convert it to a percent. The answer is 56%.

Problem Set B: This chart shows the frequency with which certain values—numbers of hours worked per week—appear. That is, the chart is a short way of showing a list of values that would begin: 15, 15, 15, 15, 25, 25, 25, 25, 25, 25, 25, 25, 25 … (then the number 35 fifteen times, then the number 40 twenty-seven times, then the number 50 five times). Add the numbers in the left-hand column to determine that the total workforce is 60 people.

6. **(E).** The median of any list is the middle number, if there is an odd number of terms, or the average of the two middle numbers, if there is an even number of terms. However, here the median number of hours worked per week is not the middle number in the list 15, 25, 35, 40, 50 because this does not account for the frequency with which each of those numbers occurs. The actual list includes the value 15 four times (once for each of the 4 employees who works 15 hours per week), the value 25 nine times (once for each of the 9 employees who works 25 hours per week), and so on.

There are 60 employees. Thus, the middle of this list is the average of the 30th and 31st values. Since 4 + 9 + 15 = 28, the 30th and 31st values fall into the next highest group—the group of 27 people who work 40 hours per week. The median number of hours worked per week per employee is 40.

7. **(D).** The average number of hours worked per week is not the average of 15, 25, 35, 40, and 50. The calculation must account for how many people work each number of hours:

$$\frac{4(15)+9(25)+15(35)+27(40)+5(50)}{60} = 35.\overline{6} \text{ hours}$$

The answer is $35.\overline{6}$ or $35\frac{2}{3}$.

8. **(C).** The mode is the number that appears in the list with the greatest frequency. Since 27 people worked 40 hours a week and every other group has fewer than 27 people, the mode is 40. The range is the highest value in the list minus the lowest value in the list: 50 – 15 = 35. The positive difference between 40 and 35 is 40 – 35 = 5.

Problem Set C: The two charts show how often daily temperature extremes occurred in each month of the year for three cities. For the sake of simplicity, you can think of the top chart as "cold" and the bottom chart as "hot."

Note that there is no information about exactly how hot or how cold the days tallied are: a day with a minimum temperature of 27°F counts as a "cold" day, just as a day with minimum temperature of –10°F would. Therefore, it is likely that questions will just reference one or both of the two temperature categories broadly (≥ 90 and ≤ 32).

9. **(D).** From the "cold" chart, the black bar referring to Winnemucca rises above 20 in Jan, Feb, Mar, Oct, Nov, and Dec, for a total of 6 months.

10. **(B).** This question asks about number of days with both temperature extremes in Galveston. Galveston had 1 "hot" day in June, 4 in July, 5 in August, and 2 in September, for a total of 12. It had 2 "cold" days in January and 1 each in February and December, for a total of 4. The total number of days with either extreme temperature is 16 days.

11. **(D).** From the grey bars on the "hot" day chart, St. Louis had a total of 1 + 8 + 15 + 12 + 4 + 4 = 44 days when the temperature reached at least 90°F, and 15 of those were in July. These July days account for $\left(\frac{15}{44}\times 100\right)\% \approx 34\%$ of all the hot days in St. Louis (approximately).

12. **(C).** In January, Winnemucca had 28 freezing days, while St. Louis had 25. So the question is asking, "28 is what percent more than 25?" Use the percent change formula:

$$\left(\frac{\textit{Difference}}{\textit{Original}}\times 100\right)\% = \left(\frac{3}{25}\times 100\right)\% = 12\%.$$

Problem Set D: This table tallies the number of households, according to number of pets in the household, and each column captures information about these households. For example, the left most column with numbers indicates that there are 70 households that have one pet, and these households spend an average of $31.25 per month on pet supplies. In that group, the households that spent the least spent $6.34 on pet supplies, while the households that spent the most spent $57.32. Notice that the bars and the max/min/average range lines duplicate the information in the table. For exact calculations, rely on the chart numbers. For broader questions, such as "which is greater," the more visual representation of the data can often provide a quick answer.

13. **(A).** There are 557 households, of which 49 have four pets and 9 have five pets. Thus, a total of 58 households have more than three pets. To express this as a percent, divide this number by the total number of households:

$$\frac{58}{557} \approx 10\%$$

14. **(B).** Since there are 557 households, the median household would be midway between the 1st and 557th households on the list if the households are ranked by how many pets they own. The midpoint between the 1st and 557th households is the $\frac{(1+557)}{2}$ = 279th household. Check: There are 278 households below this one, and 278 households above (because 279 + 278 = 557). Ranked by number of pets, households 1 through 70 (the first 70 households) have one pet, which means households 71 through 316 (in other words, the next 246 households) have two pets. Since the 279th household falls in this interval, the median household owns two pets.

15. **(B).** The group with the largest range of monthly spending is the group of households that own three pets. This can be seen by looking at the length of the vertical line between the maximum spending bar and the minimum spending triangle. Within this group, the maximum amount spent is \$143.57 and the minimum is \$45.84, so the range is \$143.57 – \$45.84 = \$97.73.

16. **(D).** The group with one pet spent an average of \$31.25 per pet, as indicated in the chart. The group with two pets spent an average of \$56.42 on two pets, which is \$28.21 per pet $\left(\frac{\$56.42}{2}\right)$. The 3rd group spent an average of \$83.11 on three pets, or $\frac{\$83.11}{3}$ = \$27.70 per pet (approximately). The 4th group spent an average of \$127.74 on four pets, or $\frac{\$127.74}{4}$ = \$31.94 per pet (approximately). The 5th group spent an average of \$147.38 on five pets, or $\frac{\$147.38}{5}$ = \$29.47 per pet (approximately). The highest average is among the group that has four pets.

Problem Set E: The three charts give information about the same five towns A, B, C, D, and E: population (2000 and 2010), area (water and land, given in square miles), and elevation (in feet above sea level).

17. **(C).** The difference in population between 2000 and 2010 is the difference in the heights of the dark- and light-gray bars in the population bar chart. Population decreased in town E, and barely changed in towns B and D. Thus towns *A* and *C* remain. The difference is about 5,000 for town A (45,000 – 40,000), but more than 5,000 for town C (more than 35,000—less than 30,000). The question asks for the town with greatest percent increase in population, so use the percent change formula: Percent Change = $\left(\frac{\text{Difference}}{\text{Original}}\times 100\right)\%$. Not only is the population increase greatest for town C, the population of town C is smaller than for town A. The percent increase in population for town A was $\left(\frac{5,000}{40,000}\times 100\right)\% = 12.5\%$, but the percent increase in population for town C was about $\left(\frac{7,000}{29,000}\times 100\right)\% \approx 24\%$.

18. **Town D and Town E only.** In the bar chart for area, dark gray represents the land area and light gray (stacked on top) represents the water area. Thus, to find water area, subtract the height of the dark-gray land area bar from the total height of the stacked bars.

Water area of town B is about 32– 21 = 11, although the vertical scale is admittedly not that precise. Water area for town B is a little more than 10, as the top of the dark gray bar is slightly *closer* to the horizontal line for 20 than the top of the light gray bar is to the horizontal line for 30. Similarly, water area for town B must be less than 12.5, as the top of the light gray bar is halfway between 30 and 35, but the top of the dark gray bar is clearly higher than 20.

Towns C, D, and E all have land area less than 10 square miles (i.e. all are below horizontal grid line for 10). Adding the land area of town C (a bit less than 10) to that of town D (about 4), the result is too high. The sum of land area in towns D and E is about 4 + about 6 (certainly less than 7.5), for a result closest to 11.

19. **Town A and Town C only.** In the elevation chart, the towns are on the x-axis and the elevation (in feet above sea level) is on the y-axis. Two towns have the same elevation if marked at the same y value, that is, if their data points are on the same horizontal line. Towns A and C are both close to the horizontal line for 415.

Problem Set F: The dark gray bars indicate the number of students with various grade point averages in 2000, and the light gray bars indicate number of students in the same categories in 1950. The title states that the surveys consist of 3,000 students.

Note the general contrast between students in the two years. Connecting the top of each light gray bar with a smooth line, the result would be a sort-of bell curve that peaks at grade point average of 2.3. Similarly, the dark gray bars form a similar bell curve, but its peak is at grade point average of 3.3, so the grades in general are clustered at the higher end of the scale in 2000.

20. **(B).** The mode of a list of numbers is the number that occurs most frequently in the list. In the bar graph for grade point average, dark gray bars represent the students in 2000, and the mode of that dataset is indicated by the tallest dark gray bar. This is at grade point average of 3.3. There were 625 students with a grade point average of 3.3 in the year 2000.

21. **(D).** The median is the middle value of an ordered list of numbers. For the 3,000 students in 1950, the median grade point average is the average of the 1,500th highest grade point average and the 1,501st highest grade point average. The students in 1950 are represented by the light gray bars. From the chart, you know the following:

150 students had a 4.0 grade point average.
225 students had a 3.7 grade point average. (Total with this GPA and higher = 150 + 225 = 375)
300 students had a 3.3 grade point average. (Total with this GPA and higher = 375 + 300 = 675)
450 students had a 3.0 grade point average. (Total with this GPA and higher = 675 + 450 = 1,125)
475 students had a 2.7 grade point average. (Total with this GPA and higher = 1,125 + 475 = 1,600)

The 1,500th and 1,501st students fall between the 1,125th and 1,600th students. Thus, the 1,500th and 1,501st highest grade point averages are both 2.7.

22. **(C).** The students in 2000 are represented by the dark gray bars:

350 students had a 4.0 grade point average.
525 students had a 3.7 grade point average.
625 students had a 3.3 grade point average.
500 students had a 3.0 grade point average.

There were 350 + 525 + 625 + 500 = 2,000 students who earned at least a 3.0 grade point average in the year 2000, out of a total of 3,000 students. This is $\frac{2}{3}$ of the students, or about 67% of the students.

23. **(D).** The students in 1950 are represented by the light gray bars:

150 students had a 4.0 grade point average.

225 students had a 3.7 grade point average.
300 students had a 3.3 grade point average.
450 students had a 3.0 grade point average.

In 1950, 150 + 225 + 300 + 450 = 1,125 students had a grade point average of 3.0 or higher. Thus, 3,000 – 1,125 = 1,875 students earned a grade point average *less than* 3.0. As a percent of the class, this is equal to $\left(\frac{1,875}{3,000}\times 100\right)\%$ = 62.5%.

Problem Set G: The vertical number scale on the left side of the graph applies to both datasets, but for Average Temperature the units are °F and for Electric Energy Cost the units are dollars ($). For example, in January the average temperature was between 30°F and 40°F and the electric energy cost was about $70. Be careful to read data from the correct set.

24. **(D).** Electric energy cost is represented by the light gray line and circular data points. A cost increase from one month to the next would mean a positive slope for the line segment between the two circular data points. The greater the slope of the light gray line segment, the greater the cost increase between those two months. There was an increase each month between May and September, and again between November and December. But the steepest positive slope is between July and August.

The cost increase from July to August was approximately $145 – $103 = $42. For comparison, the cost increase from June to July was only about $103 – $70 = $33. The correct answer is between July and August.

25. **(B).** Electric energy cost is represented by the light gray line and circular data points. A cost increase from one month to the next would mean a positive slope for the line segment between the two data points, and a cost decrease would mean a negative slope. The steeper the slope of the line segment, the greater the cost change between two consecutive months. A cost change of $0 would mean the line segment has a slope of 0 (i.e., it is horizontal).

To find the two consecutive months with the smallest electric energy cost change, look for the light gray line segment that is most horizontal. The line segment between April and May is nearly horizontal. The correct answer is between April and May.

26. **(C).** There are two ways to approximate average electric energy cost per month in the first half of the year.

One way is to use the electric energy costs on the chart and compute the average for the first six months, using the light gray circular data points:

$$\text{Approximate average cost} = \frac{\$70+\$65+\$55+\$47+\$47+\$70}{6} = \frac{\$354}{6} = \$59$$

Answer choice (C) $60 is closest.

The other method is more visual. Consider choice (A), $45, and imagine a horizontal line at $45. All six cost data points for the first half of the year are above this horizontal line, so the average must be more than $45. Similarly, imagine a horizontal line at $75 for choice (E). All six cost data points for the first half of the year are below this horizontal line, so the average must be less than $75. When a horizontal line at $60 is considered, the six cost data points "balance": three are above the line and three are below, by approximately the same amount.

27. **(B).** To minimize $\frac{\text{Electric Energy Cost } (\$)}{\text{Average Temperature } (^\circ\text{F})}$, minimize cost (light gray circular data points) while maximizing average temperature (black diamond data points). Only in April, May, October, and November is the black data point equal to or greater than the gray data point (i.e., the $\frac{\text{Electric Energy Cost } (\$)}{\text{Average Temperature } (^\circ\text{F})}$ ratio is equal to or less than 1). In April, October, and November, this ratio is close to 1. In May, the difference between the cost and the average temperature is greatest, so the electric energy cost per °F of average temperature is least. The correct answer is May.

Problem Set H: The chart shows defective parts per 1,000 as a function of machine operator experience. The dots indicate individual machine operators, and there is quite a bit of variance by individual. The line labeled "Average" shows the average performance of the group as a whole. A trend emerges: inexperienced machine operators and very experienced machine operators make fewer mistakes than those with medium level of experience. Also, certain individual machine operators produce defective parts at a lower rate than others with similar levels of experience.

28. **(E).** Because the question specifies "on average," refer to the curve marked "Average" rather than the individual data points. At the lowest point on this average curve, operators with 16,000 hours of experience produce slightly fewer than 25 defective parts per 1,000. Another low point is for operators with minimal experience, but even they produce between 25 and 30 defective parts per 1,000. In contrast, the defective part rate is maximized at the top of the curve: operators with 8,000 hours of experience produce about 40 defective parts per 1,000.

29. **(B).** Because the question specifies "on average," refer to the curve marked "Average" rather than the individual data points. Machine operators with 12,000 hours of experience produce an average of about 36 defective parts per 1,000.

The other group of machine operators that produces about 36 defective parts per 1,000 has a little less than 3,200 hours of experience. (Note that there are 5 grid lines for every 4,000 hours, so each vertical grid line is 800 hours apart. The grid mark to the left of the 4,000 mark represents 4,000 – 800 = 3,200 hours.) Choice (B) is close to and less than 3,200.

Alternatively, check the average defective part rate for machine operators with the hours of experience listed in the choices:

(A) 2,000 hours (around 33 or 34 defective parts per 1,000)
(B) 2,700 hours (a bit over 35 defective parts per 1,000) CORRECT.
(C) 4,400 hours (around 38 defective parts per 1,000)
(D) 8,400 hours (a bit less than 40 defective parts per 1,000)
(E) 12,800 (around 34 defective parts per 1,000)

30. **(C).** Because the question specifies "on average," refer to the curve marked "Average" rather than the individual data points. The defective part rate is maximized at the top of the curve: operators with

8,000 hours of experience produce about 40 defective parts per 1,000.

31. **(C).** Because the question refers to "individual machine operators," refer to the individual data points rather than the curve marked "Average."

A defective part rate of 4.2% equates to $\frac{4.2}{100} \times 1{,}000 = 42$ defective parts per 1,000. The chart has only two data points at approximately 42 defective parts per 1,000. The less experienced of these two machine operators had just under 8,000 hours of experience.

Problem Set I: Note that there are five vertical grid lines for every 10 athletes, so each vertical grid line accounts for 2 people.

32. **(A).** On the Track and Field roster, there are between 36 and 38 men (therefore 37) represented by the light gray bar. On the Track and Field roster, there are between 60 and 62 women (therefore 61) represented by the dark gray bar. In fraction form, the "ratio of men to women" is $\frac{\text{men}}{\text{women}}$. The correct answer is $\frac{37}{61}$.

33. **(A).** Male athletes are represented by the light gray bars for each sport. Sum the male athletes on each of the separate varsity sports rosters:

Males on Volleyball roster: 0
Males on Track and Field roster: between 36 and 38 (therefore 37)
Males on Tennis roster: between 8 and 10 (therefore 9)
Males on Golf roster: 10
Males on Cross Country roster: between 16 and 18 (therefore 17)
Males on Basketball roster: 14

There are 0 + 37 + 9 + 10 + 17 + 14 = 87 male names on all of the rosters combined, but there are only 76 male athletes total. Since tennis, golf, and basketball players are all on only one roster, there must be 87 – 76 = 11 male athletes who are counted twice, on both the Track and Field team and the Cross Country team. The correct answer is 11.

34. **Golf only.** Male athletes are represented by the light gray bars, female athletes by the dark gray bars. A sport in which male athletes outnumber female athletes will have a shorter dark gray bar than light gray bar.

This is only the case for Golf; there are 10 male athletes and 7 female athletes. Volleyball only has female athletes, so they outnumber the zero male athletes on the roster. In Tennis and Basketball, there are equal numbers of men and women. Female athletes outnumber male athletes on the Cross Country and Track and Field rosters.

35. **(B).** There are between 8 and 10 female tennis players (therefore 9) represented by the dark gray bar. There are 14 male basketball players represented by the light gray bar. In fraction form, the "ratio

of female tennis players to male basketball players” is $\frac{\text{female tennis players}}{\text{male basketball players}}$. Thus, the answer is $\frac{9}{14}$.

Problem Set J: This chart compares four stores, providing information about change from 2011 to 2012 in three metrics: total revenue, number of distinct customers, and total costs. It is essential to note that change in total revenue is given in terms of dollars, whereas changes in number of distinct customers and in total costs are given only in percentages. In general, percents provide less information than absolute numbers, as the total (i.e., percent of what?) is needed for context.

36. **(E).** It may be tempting to select store Z, as revenue increased from 2011 to 2012 while the number of distinct customers decreased, but be careful when mixing absolute numbers and percents. Without knowing the revenue in 2012 (only the change from the previous year is known) or the number of customers (only the percent change from the previous year is known) for any of the stores, it cannot be determined from the information given.

37. **(A).** Because the question concerns costs per customer, given in percent change terms in the chart, and the question asks about percent change for this ratio, a comparison can be made among the stores.

The percent change formula in general is $\left(\frac{\text{Difference}}{\text{Original}}\times 100\right)\%$. Thus, the percent change in total costs per distinct customer at a particular store is:

$$\left(\frac{\frac{\text{cost}_{2012}}{\text{customer}_{2012}}-\frac{\text{cost}_{2011}}{\text{customer}_{2011}}}{\frac{\text{cost}_{2011}}{\text{customer}_{2011}}}\times 100\right)\%$$

This looks like a mess, but remember that both cost_{2012} and customer_{2012} can be written in terms of cost_{2011} and customer_{2011}, respectively, based on the percent changes given in the table. Then cost_{2011} and customer_{2011} are in each term of the fraction and can be canceled. For example, for store W, the percent change in total costs per distinct customer is:

$$\left(\frac{\frac{1.15\times\text{cost}_{2011}}{1.02\times\text{customer}_{2011}}-\frac{\text{cost}_{2011}}{\text{customer}_{2011}}}{\frac{\text{cost}_{2011}}{\text{customer}_{2011}}}\times 100\right)\%=\left(\frac{\frac{1.15}{1.02}-1}{1}\times 100\right)\%=\left(\left(\frac{1.15}{1.02}-1\right)\times 100\right)\%$$

In other words, the magnitude of percent change in total costs per distinct customer depends only on the ratio of (1 + Percent change in total costs) to (1 + Percent change in number of distinct customers). Perform this comparison for all of the stores:

(A) Store W: $\frac{1.15}{1.02}=1.12745$ *Greatest.*

(B) Store X: $\dfrac{1.04}{1.14} = 0.91228$

(C) Store Y: $\dfrac{1.12}{1.05} = 1.06667$

(D) Store Z: $\dfrac{0.80}{0.93} = 0.86022$

38. **Store W, Store X, and Store Y only.** Profit = Revenue – Cost.

Store W: Revenue decreased by $400,000, and costs increased by 15%. Both changes negatively affect profit. CORRECT.

Store X: Revenue increased by $520,000, but costs also increased by 4%. Profit in 2012 could be greater than, less than, or equal to profit in 2011, depending on the store's cost structure. Try sample numbers to show that profit could have decreased. If in 2011, revenue was $20,000,000 and costs were $15,000,000, the profit was $5,000,000. In 2012, revenue would be $20,520,000 and costs $15,600,000, making profit $4,920,000, less than in the previous year. CORRECT.

Store Y: Revenue decreased by $365,000, and costs increased by 12%. Both changes negatively affect profit. CORRECT.

Store Z: Revenue increased by $125,000 and costs decreased by 20%. Both changes positively affect profit.

39. **(B)**. Consider each statement individually:

(A) While store X had the greatest increase in revenue, in dollars, it is impossible to calculate percent change in revenue $\left(\frac{\text{Difference}}{\text{Original}} \times 100\right)\%$ for any of the stores without information about the actual dollar amount of their revenue in either year. Not necessarily true.

(B) Per customer revenue is $\frac{\text{Revenue}}{\text{Number of customers}}$. Store Z experienced an increase in revenue and a decrease in number of distinct customers, both of which increase per customer revenue. TRUE.

(C) While store W had the greatest percent increase in total costs, it is impossible to say whether this was the greatest increase in dollars without knowing the actual dollar amount of total costs for each of the stores that experienced a cost increase. Not necessarily true.

(D) The chart says nothing about repeat customers, only "distinct" customers. Not necessarily true.

(E) The chart says nothing about absolute numbers of distinct customers at any of the stores, only percent change from 2011 to 2012. Not necessarily true.

Problem Set K: Much of the detail in this chart is given in the title and other text. According to the title and the * note below, the chart shows the range of calories burned per hour by people in the 130- to 205-pound weight range in the course of performing various activities.

40. **First and second statements only.** Consider each statement individually.

The first statement could be true. A 130-pound person typing for 10 hours would burn less than 1,000 calories, which is less than the number of calories burned by a 205-pound person doing one of several activities on the chart for 1 hour (certainly jai alai and swimming the butterfly stroke burn more than 1,000 calories).

The second statement could be true. In general, the range of calories burned per hour is greater for jai alai than for swimming the butterfly stroke. The people in question are about the same weight, but it cannot be assumed that the number of calories burned is a function of weight in this range or that the relationship is linear. All that matters is that the calorie burning ranges for the two activities overlap, and both people fall in the weight range, so it could be true that a 175-pound person playing jai alai for 1 hour burns fewer calories than a 180-pound person swimming the butterfly stroke for 1 hour.

The third statement must be false. The average calories burned by one person playing table tennis for 1 hour is a maximum of about 375. Two people typing for 3 hours burn as many calories total as one person typing for $2 \times 3 = 6$ hours, which is a minimum of about 550. "At most 375" cannot be greater than "at least 550."

41. **(E).** For each combination of activities, look at the minimum value on the chart for the 130-pound person and the maximum value on the chart for the 205-pound person:

(A) Badminton (minimum) + Table tennis (maximum) = 275 + 375 = 650
(B) Wrestling (minimum) + Track and field, hurdles (maximum) = 350 + 925 = 1,275
(C) Typing (minimum) + Swimming, butterfly stroke (maximum) = under 100 + 1025 = under 1,125
(D) Sailing, competition (minimum) + Aerobics (maximum) = 300 + 600 = 900
(E) Typing (minimum) + Croquet (maximum) = under 100 + over 225 = about 325 CORRECT.

Alternatively, note that typing and croquet are the two activities that burn the fewest calories per hour overall. A 130 pound person and a 205-pound person each doing 1 hour of an activity shown on the chart would only burn fewer calories total if both people were typing.

Problem Set L: The table categorizes 50 African countries according to GDP (rows) and population (columns). Notice that each row sums to a subtotal number of countries in that GDP range, and each column sums to a subtotal number of countries in that population range. Both the subtotal row and subtotal column sum to 50, the grand total. Moreover, notice that both population and GDP are shown in descending order: high population/high GDP countries are in the upper left corner of the table, while low population/low GDP countries are in the lower right corner of the table.

42. **(A).** GDP between \$10 billion and \$20 billion is a single row in the table. Population between 10–50 million people includes two columns in the table. Look at the intersections between this row and two columns. There are three countries with populations of 10–20 million and GDPs of \$10 billion to \$20 billion. There are also three countries with populations of 20–50 million and GDPs of \$10 billion to \$20 billion GDP, for a total of six countries.

43. **(C).** Adding the entries that are in both the bottom two rows (less than \$20 billion GDP) and the last three columns (population less than 20 million), the number of countries is 3 + 3 + 3 + 7 + 8 + 7 = 31. Out of 50 countries, 31 fit this description, so the percent is $\left(\frac{31}{50}\times 100\right)\%$, or 62%.

44. **(B).** There are 13 countries with GDPs between \$10–\$20 billion, and of these, 3 have

populations between 10–20 million. Thus, the percent is $\left(\frac{3}{13}\times 100\right)$, or approximately 23%.

45. **(D).** For each choice, carefully find the row(s)/column(s) that fit the description, and sum all table entries that apply.

(A) More than \$10 billion GDP (the top three rows) and a population of less than 20 million (the three columns on right, before the subtotal column): 0 + 0 + 0 + 1 + 1 + 0 + 3 + 3 + 3 = 11
(B) Less than \$20 billion GDP (the bottom two rows above the subtotal row) and a population of more than 10 million (the three columns on left): 1 + 3 + 3 + 0 + 0 + 7 = 14
(C) More than \$20 billion GDP (the entire top two rows): 5 + 10 = 15
(D) Less than \$100 billion GDP (the bottom three rows above the subtotal row) and a population of less than 10 million (the two columns on the right before the subtotal column): 1 + 0 + 3 + 3 + 8 + 7 = 22
(E) Less than \$100 billion GDP (the bottom three rows above the subtotal row) and a population between 10–50 million (the second and third columns): 7 + 1 + 3 + 3 + 0 + 7 = 21 *Greatest*.

Choice (D), 22, is the greatest.

Problem Set M: This pie chart represents about 76 million owner-occupied housing units, categorized by household population. The smaller pie chart on the right further subdivides the households with at least 5 people. These categories could have been shown as small slivers in the pie chart on the left in the "Other" slice (notice that 6.7% + 2.6% + 1.8% = 11.1%).

46. **(D).** Sum the households with one, two, or three people (i.e., "fewer than four people"). Together these account for 21.6% + 36.3% + 16.5% = 74.4% of the total.

47. **(E).** According to the chart, 6.7% of the 75,986,074 households are 5-person households. Multiply 0.067 by 76 (keep "million" in mind). The result is about 5, so the answer is 5 million households.

48. **(B).** Approximate the total number of households as 76 million (close enough to 75,986,074).

One-person households are 21.6% of the total, or approximately 16.4 million. Since each such household has only one person, this represents about 16.4 million people.

Three-person households are 16.5% of the total, or approximately 12.5 million. Since each of these households has three people, that is about 37.5 million people.

Five-person households are 6.7% of the total, or approximately 5.1 million. Since each of these households has five people, that is about 25.5 million people.

Since 16.4 million < 25.5 million < 37.5 million, the correct ranking is 1-person households, 5-person households, 3-person households.

49. **(A).** The 2- and 3-person households account for 36.3% + 16.5% = 52.8% of all households, so this is the correct answer. Quickly rule out the other choices as a check:

> (B) The 3- and 4-person households account for 16.5% + 14.5% = 31.0% of all households.
> (C) The 4- and 5-person households account for 14.5% + 6.7% = 21.2% of all households.
> (D) The 5- and 6-person households account for 6.7% + 2.6% = 9.3% of all households.
> (E) The 6- and 7-person households account for at most 2.6% + 1.8% = at most 4.4% of all households (remember that some of the 1.8% could consist of households with more than 7 people).

All of the choices other than (A) are less than 50%.