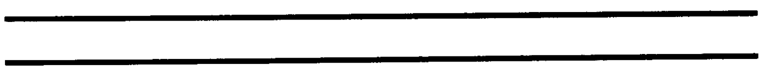
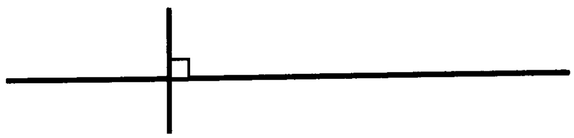
LINES & ANGLES

A straight line is 180°. Think of a line as half of a circle.

Parallel lines are lines that lie in a plane and that never intersect. No matter how far you extend the lines, they never meet. Two parallel lines are shown below:



Perpendicular lines are lines that intersect at a 90° angle. Two perpendicular lines are shown below:



There are two major line-angle relationships that you must know for the GRE:

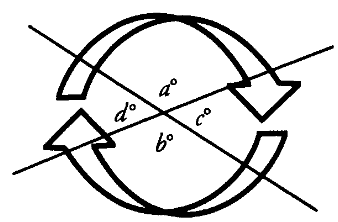
(1) The angles formed by any intersecting lines.
(2) The angles formed by parallel lines cut by a transversal.

Intersecting Lines

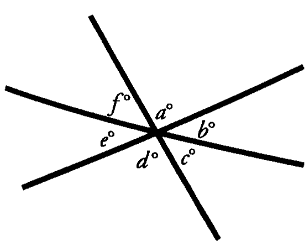
Intersecting lines have three important properties.

First, the interior angles formed by intersecting lines form a circle, so the sum of these angles is 360°. In the diagram shown, $a + b + c + d = 360$.

Second, interior angles that combine to form a line sum to 180°. These are termed **supplementary angles.** Thus, in the diagram shown, $a + d = 180$, because angles a and d form a line together. Other supplementary angles are $b + c = 180$, $a + c = 180$, and $d + b = 180$.



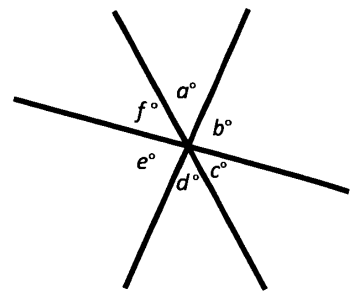
Third, angles found opposite each other where these two lines intersect are equal. These are called **vertical angles.** Thus, in the diagram above, $a = b$, because these angles are opposite one another, and are formed from the same two lines. Additionally, $c = d$ for the same reason.



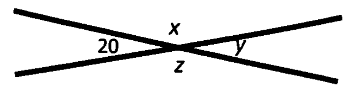
Note that these rules apply to more than two lines that intersect at a point, as shown to the left. In this diagram, $a + b + c + d + e + f = 360$, because these angles combine to form a circle. In addition, $a + b + c = 180$, because these three angles combine to form a line. Finally, $a = d$, $b = e$, and $c = f$, because they are pairs of vertical angles.

Check Your Skills

1. If $b + f = 150$, what is angle d?



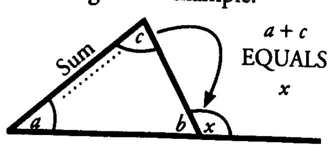
2. What is $x - y$?



Answers can be found on page 119.

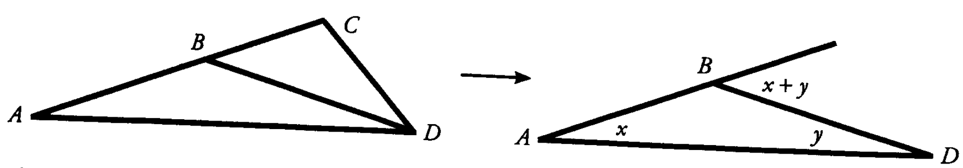
Exterior Angles of a Triangle

An **exterior angle** of a triangle is equal in measure to the sum of the two non-adjacent (opposite) **interior angles** of the triangle. For example:



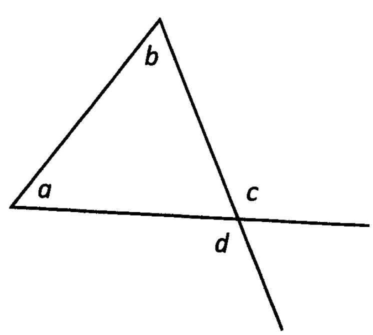
$a + b + c = 180$ (sum of angles in a triangle).
$b + x = 180$ (supplementary angles).
Therefore, $x = a + c$.

This property is frequently tested on the GRE! In particular, look for exterior angles within more complicated diagrams. You might even redraw the diagram with certain lines removed to isolate the triangle and exterior angle you need.



Check Your Skills

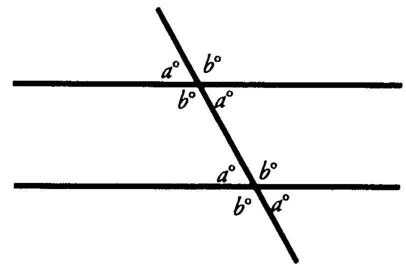
3. If $c + d = 200$, what is $a + b$?



Answers can be found on page 119.

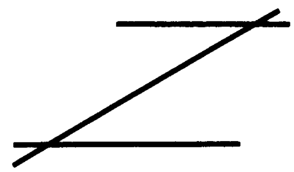
Parallel Lines Cut By a Transversal

The GRE makes frequent use of diagrams that include parallel lines cut by a **transversal.**



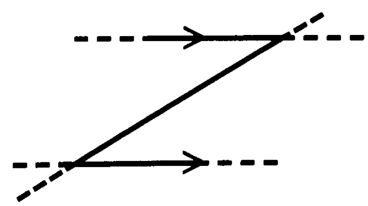
Notice that there are 8 angles formed by this construction, but there are only TWO different angle measures (a and b). All the **acute** angles (less than 90°) in this diagram are equal. Likewise, all the **obtuse** angles (more than 90° but less than 180°) are equal. Any acute angle is supplementary to any obtuse angle. Thus, $a + b = 180°$.

When you see a third line intersecting two lines that you know to be parallel, fill in all the a (acute) and b (obtuse) angles, just as in the diagram above.



Sometimes the GRE disguises the parallel lines and the transversal so that they are not readily apparent, as in the diagram pictured to the right.

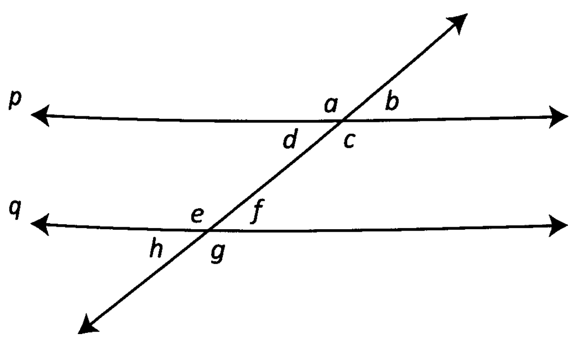
In these disguised cases, it is a good idea to extend the lines so that you can easily see the parallel lines and the transversal. Just remember always to be on the lookout for parallel lines. When you see them, extend lines and label the acute and obtuse angles.



You might also mark the parallel lines with arrows.

Check Your Skills

Refer to the following diagram for questions 4–5.



(lines p and q are parallel)

4. If angle $g = 120$, what is a?
5. If angle $g = 120$, what is $a + b + c$?

Answers can be found on page 119.

Check Your Skills Answers

1. **30:** Because they are vertical angles, angle a is equal to angle d.

Because they add to form a straight line, $a + b + f = 180$.

Substitute d for a to yield, $(d) + b + f = 180$. Substitute 150 for $b + f$ to yield $d + (150) = 180$. So $d = 180 - 150 = 30$.

2. **140:** Because x and 20 are supplementary, $x = 180 - 20 = 160$. Because y and 20 are vertical, $y = 20$. So $x - y = 160 - 20 = 140$.

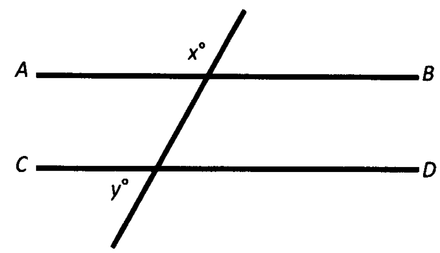
3. **100:** Since c and d are vertical angles, they are equal. Since they sum to 200, each must be 100. $a + b = c$, because c is an exterior angle of the triangle shown, and a and b are the two non-adjacent interior angles. $a + b = c = 100$.

4. **120:** In a system of parallel lines cut by a transversal, opposite exterior angles (like a and g) are equal. $g = a = 120$.

5. **300:** From question 4, we know that $a = 120$. Since $a = 120$, its supplementary angle $d = 180 - 120 = 60$. Since $a + b + c + d = 360$, and $d = 60$, $a + b + c = 300$.

Problem Set (Note: Figures are not drawn to scale.)

Problems 1–4 refer to the diagram on the right, where line AB is parallel to line CD.



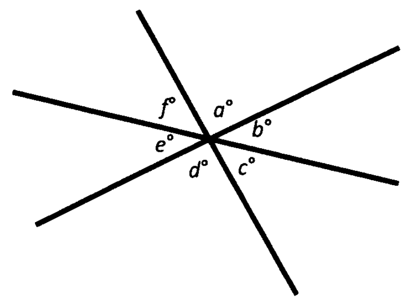
1. If $x - y = 10$, what is x?

2. If the ratio of x to y is $3 : 2$, what is y?

3. If $x + (x + y) = 320$, what is x?

4. If $\dfrac{x}{x - y} = 2$, what is x?

Problems 5–8 refer to the diagram on the right.



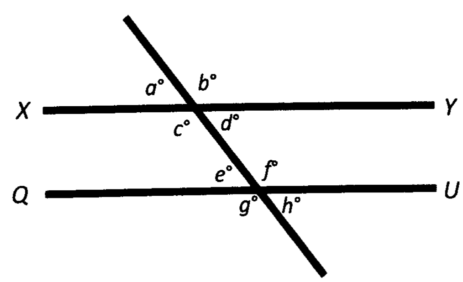
5. If a is 95, what is $b + d - e$?

6. If $c + f = 70$, and $d = 80$, what is b?

7. If a and b are **complementary angles** (they sum to 90°), name three other pairs of complementary angles.

8. If e is 45, what is the sum of all the other angles?

Problems 9–12 refer to the diagram on the right, where line XY is parallel to line QU.



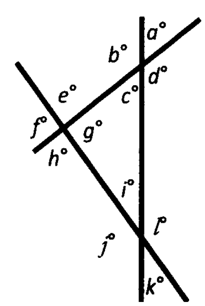
9. If $a + e = 150$, find f.

10. If $a = y$, $g = 3y + 20$, and $f = 2x$, find x.

11. If $g = 11y$, $a = 4x - y$, and $d = 5y + 2x - 20$, find h.

12. If $b = 4x$, $e = x + 2y$, and $d = 3y + 8$, find h.

Problems 13–15 refer to the diagram to the right.

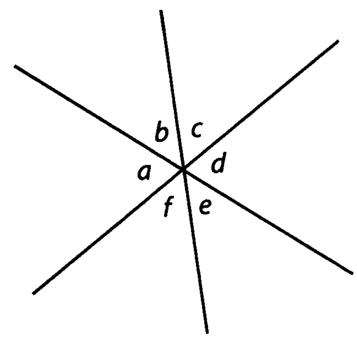


13. If $c + g = 140$, find k.

14. If $g = 90$, what is $a + k$?

15. If $f + k = 150$, find b.

16.



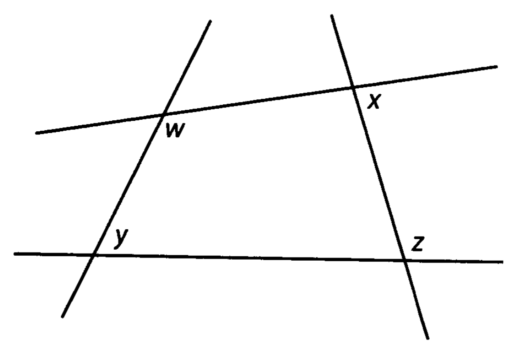
Quantity A

$a + f + b$

Quantity B

$c + d + e$

17.



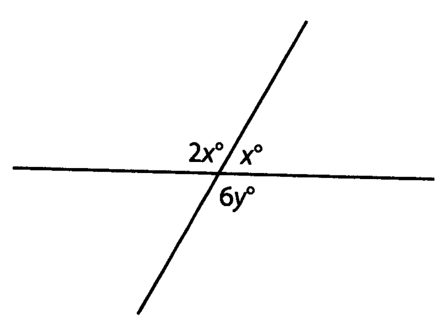
Quantity A

$w + y$

Quantity B

$x + z$

18.



Quantity A

y

Quantity B

10

1. **95:** We know that $x + y = 180$, since any acute angle formed by a transversal that cuts across two parallel lines is supplementary to any obtuse angle. Use the information given to set up a system of two equations with two variables:

$$\begin{aligned} x + y &= 180 \\ \underline{x - y} &= \ \ 10 \\ 2x &= 190 \\ x &= \ \ 95 \end{aligned}$$

2. **72:** Set up a ratio, using the unknown multiplier, a.

$$\frac{x}{y} = \frac{3a}{2a}$$

$180 = x + y = 3a + 2a = 5a$

$180 = 5a$

$a = 36$

$y = 2a = 2(36) = 72$

3. **140:** Use the fact that $x + y = 180$ to set up a system of two equations with two variables:

$$x + y = 180 \quad \rightarrow \quad \begin{array}{rrcl} & -x - y & = & -180 \\ + & 2x + y & = & 320 \\ \hline & x & = & 140 \end{array}$$

4. **120:** Use the fact that $x + y = 180$ to set up a system of two equations with two variables:

$$\frac{x}{x - y} = 2 \quad \rightarrow \quad \begin{array}{rrcl} & x - 2y & = & 0 \\ - & x + y & = & 180 \\ \hline & -3y & = & -180 \\ & y & = & 60 \end{array} \quad \rightarrow \quad \text{Therefore, } x = 120.$$

5. **95:** Because a and d are vertical angles, they have the same measure: $a = d = 95°$. Likewise, since b and e are vertical angles, they have the same measure: $b = e$. Therefore, $b + d - e = d = 95°$.

6. **65:** Because c and f are vertical angles, they have the same measure: $c + f = 70$, so $c = f = 35$. Notice that b, c, and d form a straight line: $b + c + d = 180$. Substitute the known values of c and d into this equation:

$$\begin{aligned} b + 35 + 80 &= 180 \\ b + 115 &= 180 \\ b &= 65 \end{aligned}$$

7. ***b* and *d*, *a* and *e*, & *d* and *e*:** If a is complementary to b, then d (which is equal to a, since they are vertical angles), is also complementary to b. Likewise, if a is complementary to b, then a is also complementary to e (which is equal to b, since they are vertical angles). Finally, d and e must be complementary, since $d = a$ and $e = b$. You do not need to know the term
"complementary," but you should be able to work with the concept (two angles adding up to 90°).

8. **315°:** If $e = 45$, then the sum of all the other angles is $360° — 45° = 315°$.

9. **105:** We are told that $a + e = 150$. Since they are both acute angles formed by a transversal cutting across two parallel lines, they are also congruent. Therefore, $a = e = 75$. Any acute angle in this diagram is supplementary to any obtuse angle, so $75 + f = 180$, and $f = 105$.

10. **70:** We know that angles a and g are supplementary; their measures sum to 180. Therefore:

$$y + 3y + 20 = 180$$
$$4y = 160$$
$$y = 40$$

Angle f is congruent to angle g, so its measure is also $3y + 20$. The measure of angle $f = g = 3(40) + 20 = 140$. If $f = 2x$, then $140 = 2x \rightarrow x = 70$.

11. **70:** We are given the measure of one acute angle (a) and one obtuse angle (g). Since any acute angle in this diagram is supplementary to any obtuse angle, $11y + 4x - y = 180$, or $4x + 10y = 180$. Since angle d is congruent to angle a, we know that $5y + 2x - 20 = 4x - y$, or $2x - 6y = -20$. We can set up a system of two equations with two variables:

$$2x - 6y = -20 \quad \rightarrow \quad \begin{array}{r} -4x + 12y = 40 \\ 4x + 10y = 180 \\ \hline 22y = 220 \\ y = 10;\ x = 20 \end{array}$$

Since h is one of the acute angles, h has the same measure as a: $4x - y = 4(20) - 10 = 70$.

12. **68:** Because b and d are supplementary, $4x + 3y + 8 = 180$, or $4x + 3y = 172$. Since d and e are congruent, $3y + 8 = x + 2y$, or $x - y = 8$. We can set up a system of two equations with two variables:

$$x - y = 8 \quad \rightarrow \quad \begin{array}{rl} 4x + 3y & = 172 \\ 3x - 3y & = 24 \\ \hline 7x & = 196 \\ x & = 28;\ y = 20 \end{array}$$

Since h is congruent to d, $h = 3y + 8$, or $3(20) + 8 = 68$.

13. **40:** If $c + g = 140$, then $i = 40$, because there are 180° in a triangle. Since k is vertical to i, k is also $= 40$. Alternately, if $c + g = 140$, then $j = 140$, since j is an exterior angle of the triangle and is therefore equal to the sum of the two remote
interior angles. Since k is supplementary to j, $k = 180 - 140 = 40$.

14. **90:** If $g = 90$, then the other two angles in the triangle, c and i, sum to 90. Since a and k are vertical angles to c and i, they sum to 90 as well.

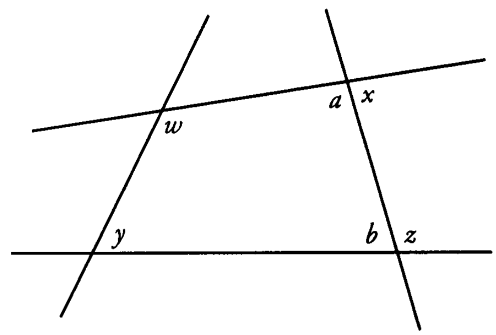
15. **150:** Angles f and k are vertical to angles g and i. These two angles, then, must also sum to 150. Angle b, an exterior angle of the triangle, must be equal to the sum of the two remote interior angles g and i. Therefore, $b = 150$.

16. **C:** You can substitute each of the values in Quantity A for a corresponding value in Quantity B. $a = d$, $c = f$, and $b = e$, in each case because the equal angles are vertical angles. Rewrite Quantity A.

<u>**Quantity A**</u>	<u>**Quantity B**</u>
$a + f + b = (d) + (c) + (e)$	$c + d + e$

Therefore **the two quantities are equal.**

17. **C:** To see why the sums in the two quantities are equal, label the remaining two interior angles of the quadrilateral a and b.



Quantity A	**Quantity B**
$w + y$	$x + z$

There are several relationships we can describe based on the diagram. For instance, we know the sum of the four internal angles of the quadrilateral is 360.

$w + y + a + b = 360$

We also have two pairs of supplementary angles.

$a + x = 180$
$b + z = 180$

Add the two equations together:

$a + b + x + z = 360$

$w + y + a + b$ sum to 360, as do $a + b + x + z$. Therefore the two sums equal each other.

$w + y + a + b = a + b + x + z$ Subtract $a + b$ from both sides
$w + y = x + z$

The **two quantities are equal**.

18. **A:** First solve for x. The two angles x and $2x$ are supplementary.

$x + 2x = 180$
$3x = 180$
$x = 60$

Next note that $2x = 6y$, because $2x$ and $6y$ are vertical angles. Plug in 60 for x and solve for y.

$2(60) = 6y$
$120 = 6y$
$20 = y$

Quantity A	**Quantity B**
$y =$ **20**	10

Therefore **Quantity A is larger**.