POLYGONS

Polygons are a very familiar sight on the GRE. As you saw in the last chapter, many questions about triangles will often involve other polygons, most notably quadrilaterals. Mastery of polygons will ultimately involve understanding the basic properties, such as perimeter and area, and will also involve the ability to distinguish polygons from other shapes in diagrams that include other polygons or circles.

A polygon is defined as a closed shape formed by line segments. The polygons tested on the GRE include the following:

- Three-sided shapes (Triangles)
- Four-sided shapes (Quadrilaterals)
- Other polygons with *n* sides (where *n* is five or more)

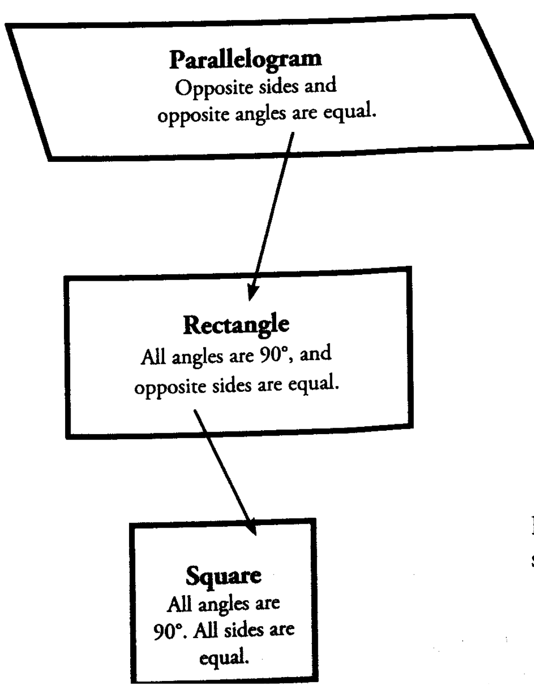
This section will focus on polygons of four or more sides. In particular, the GRE emphasizes quadrilaterals—four-sided polygons—squares, rectangles, parallelograms, and trapezoids.

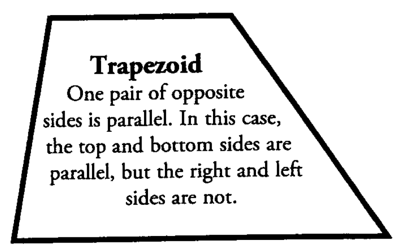
Polygons are two-dimensional shapes—they lie in a plane. The GRE tests your ability to work with different measurements associated with polygons. The measurements you must be adept with are (1) interior angles, (2) perimeter, and (3) area.

The GRE also tests your knowledge of three-dimensional shapes formed from polygons, particularly rectangular solids and cubes. The measurements you must be adept with are (1) surface area and (2) volume.

Quadrilaterals: An Overview

The most common polygon tested on the GRE, aside from the triangle, is the quadrilateral (any four-sided polygon). Almost all GRE polygon problems involve the special types of quadrilaterals shown below.





Rectangles are special types of parallelograms, and squares are special types of rectangles.

Polygons and Interior Angles

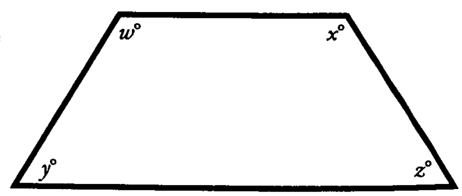
The sum of the interior angles of a given polygon depends on the **number of sides in the polygon.** The following chart displays the relationship between the type of polygon and the sum of its interior angles.

The sum of the interior angles of a polygon follows a specific pattern that depends on n, the number of sides that the polygon has. This sum is always $(n - 2) \times 180$ (where n is the number of sides), because the polygon can be cut into $(n - 2)$ triangles, each of which contains 180°.

Polygon	# of Sides	Sum of Interior Angles
Triangle	3	180°
Quadrilateral	4	360°
Pentagon	5	540°
Hexagon	6	720°

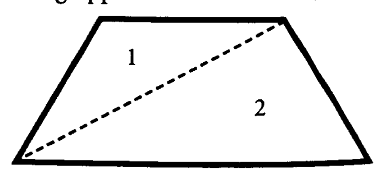
$$\boxed{(n - 2) \times 180 = \textbf{Sum of Interior Angles of a Polygon}}$$

If you forget this formula, you can always say "okay, a triangle has 180°, a rectangle has 360°," and so on. Add 180° for each new side.

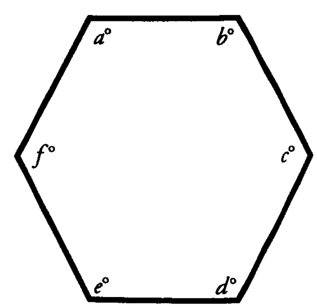


Since this polygon has four sides, the sum of its interior angles is $(4 - 2)180 = 2(180) = 360°$.

Alternatively, note that a quadrilateral can be cut into two triangles by a line connecting opposite corners. Thus, the sum of the angles = $2(180) = 360°$.

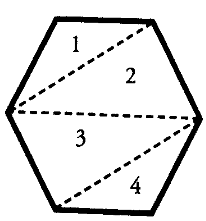


If a polygon has six sides, as in the figure below, the sum of its interior angles would be $(6 - 2)180 = 4(180) = 720°$.



Alternatively, note that a hexagon can be cut into four triangles by three lines connecting corners.

Thus, the sum of the angles = $4(180) = 720°$.



By the way, the corners of polygons are also known as vertices (singular: vertex).

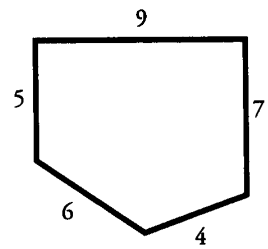
Check Your Skills

1. What is the sum of the interior angles of an octagon (eight-sided polygon)?
2. A regular polygon is a polygon in which every line is of equal length and every interior angle is equal. What is the degree measure of each interior angle in a regular hexagon (six-sided polygon)?

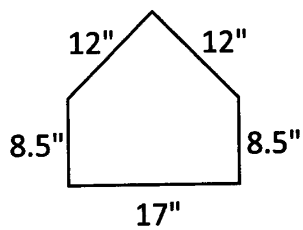
Polygons and Perimeter

The perimeter refers to the distance around a polygon, or the sum of the lengths of all the sides. The amount of fencing needed to surround a yard would be equivalent to the perimeter of that yard (the sum of all the sides).

The perimeter of the pentagon to the right is:
$9 + 7 + 4 + 6 + 5 =$ **31.**



Check Your Skills



3. The figure above represents a standard baseball home plate. What is the perimeter of this figure?

Answers can be found on page 81.

Polygons and Area

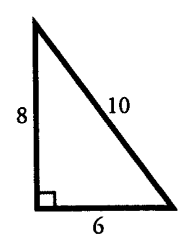
The area of a polygon refers to the space inside the polygon. Area is measured in square units, such as cm^2 (square centimeters), m^2 (square meters), or ft^2 (square feet). For example, the amount of space that a garden occupies is the area of that garden.

On the GRE, there are two polygon area formulas you MUST know:

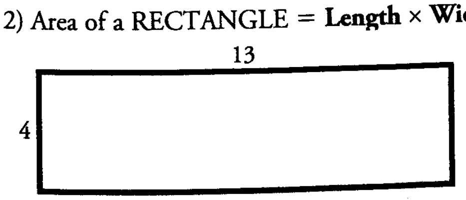
1) Area of a TRIANGLE = $\dfrac{\textbf{Base} \times \textbf{Height}}{\textbf{2}}$

The height ALWAYS refers to a line that is perpendicular (at a 90° angle) to the base.

In this triangle, the base is 6 and the height (perpendicular to the base) is 8.
The area = $(6 \times 8) \div 2 = 48 \div 2 = 24$.



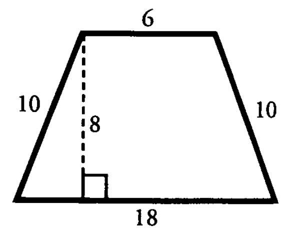
2) Area of a RECTANGLE = **Length** × **Width**



The length of this rectangle is 13, and the width is 4. Therefore, the area = $13 \times 4 = 52$.

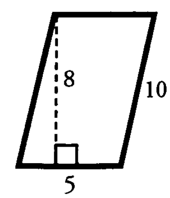
The GRE will occasionally ask you to find the area of a polygon more complex than a simple triangle or rectangle. The following formulas can be used to find the areas of other types of quadrilaterals:

3) Area of a TRAPEZOID = $\frac{(\mathbf{Base_1} + \mathbf{Base_2}) \times \mathbf{Height}}{\mathbf{2}}$



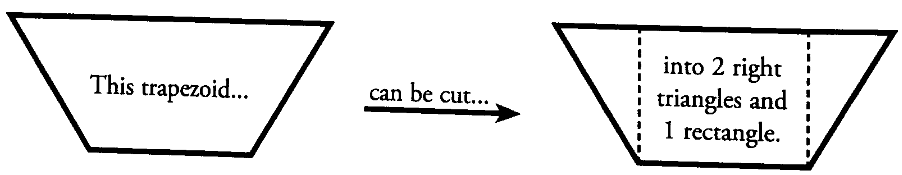
Note that the height refers to a line perpendicular to the two bases, which are parallel. (You often have to draw in the height, as in this case.) In the trapezoid shown, $base_1 = 18$, $base_2 = 6$, and the height = 8. The area = $(18 + 6) \times 8 \div 2 = 96$. Another way to think about this is to take the *average* of the two bases and multiply it by the height.

4) Area of any PARALLELOGRAM = **Base × Height**



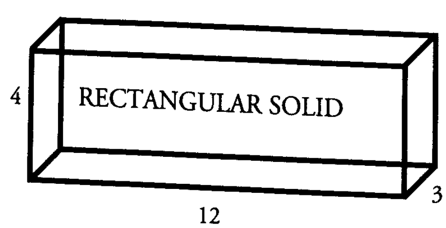
Note that the height refers to the line perpendicular to the base. (As with the trapezoid, you often have to draw in the height.) In the parallelogram shown, the base = 5 and the height = 8. Therefore, the area is $5 \times 8 = 40$.

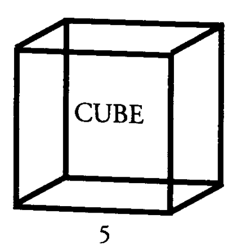
Although these formulas are very useful to memorize for the GRE, you may notice that all of the above shapes can actually be divided into some combination of rectangles and right triangles. Therefore, if you forget the area formula for a particular shape, simply cut the shape into rectangles and right triangles, and then find the areas of these individual pieces. For example:



3 Dimensions: Surface Area

The GRE tests two particular three-dimensional shapes formed from polygons: the rectangular solid and the cube. Note that a cube is just a special type of rectangular solid.





The surface area of a three-dimensional shape is the amount of space on the surface of that particular object. For example, the amount of paint that it would take to fully cover a rectangular box could be determined by finding the surface area of that box. As with simple area, surface area is measured in square units such as inches^2 (square inches) or ft^2 (square feet).

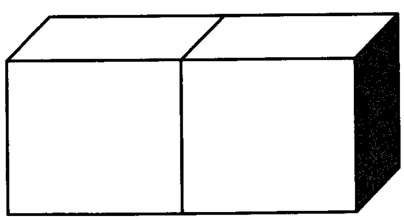
Surface Area = the SUM of the areas of ALL of the faces

Both a rectangular solid and a cube have **six faces.**

To determine the surface area of a rectangular solid, you must find the area of each face. Notice, however, that in a rectangular solid, the front and back faces have the same area, the top and bottom faces have the same area, and the two side faces have the same area. In the solid above, the area of the front face is equal to $12 \times 4 = 48$. Thus, the back face also has an area of 48. The area of the bottom face is equal to $12 \times 3 = 36$. Thus, the top face also has an area of 36. Finally, each side face has an area of $3 \times 4 = 12$. Therefore, the surface area, or the sum of the areas of all six faces equals $48(2) + 36(2) + 12(2) = 192$.

To determine the surface area of a cube, you only need the length of one side. We can see from the cube above that a cube is made of six identical square surfaces. First, find the area of one face: $5 \times 5 = 25$. Then, multiply by six to account for all of the faces: $6 \times 25 = 150$.

Check Your Skills

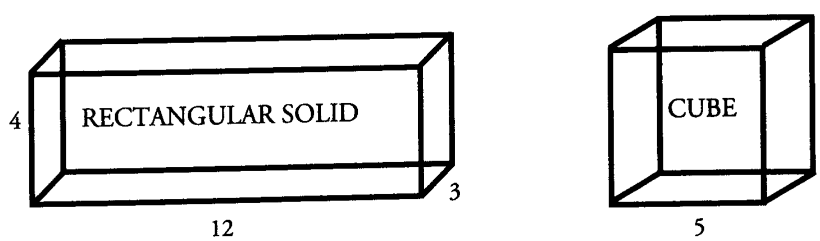


4. The figure to the left shows two wooden cubes joined to form a rectangular solid. If each cube has a surface area of 24, what is the surface area of the resulting rectangular solid?

Answers can be found on page 81.

3 Dimensions: Volume

The volume of a three-dimensional shape is the amount of "stuff" it can hold. For example, the amount of liquid that a rectangular milk carton holds can be determined by finding the volume of the carton. Volume is measured in cubic units such as inches^3 (cubic inches) or ft^3 (cubic feet).



Volume = Length × Width × Height

By looking at the rectangular solid above, we can see that the length is 12, the width is 3, and the height is 4. Therefore, the volume is $12 \times 3 \times 4 = 144$.

In a cube, all three of the dimensions—length, width, and height—are identical. Therefore, knowing the measurement of just one side of the cube is sufficient to find the volume. In the cube above, the volume is $5 \times 5 \times 5 = 125$.

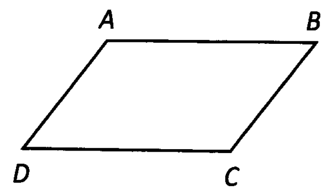
Check Your Skills

5. The volume of a rectangular solid with length 8, width 6, and height 4 is how many times the volume of a rectangular solid with length 4, width 3, and height 2?

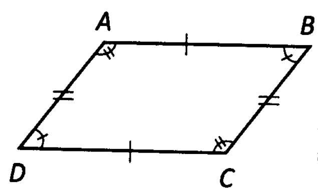
Answers can be found on page 81.

Quadrilaterals

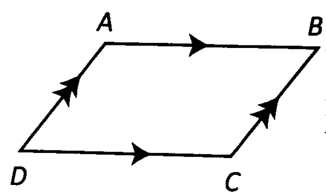
A quadrilateral is any figure with 4 sides. The GRE largely deals with one class of quadrilaterals known as **parallelograms.** A parallelogram is any 4 sided figure in which the opposite sides are parallel and equal and opposite angles are equal. This is an example of a parallelogram.



In this figure, sides *AB* and *CD* are parallel and have equal lengths, sides *AD* and *BC* are parallel and equal length, angles *ADC* and *ABC* are equal and angles *DAB* and *DCB* are equal.



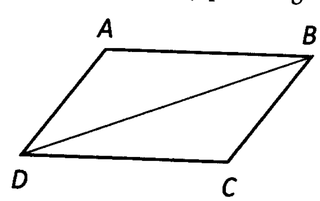
Use hash marks to indicate equal lengths or equal angles.



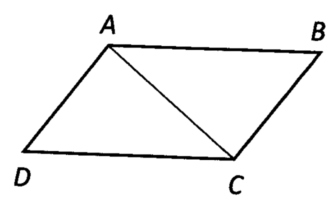
Use arrows to indicate parallel lines.

Any quadrilateral with two sets of opposite and equal sides is a parallelogram, as is any quadrilateral with two sets of opposite and equal angles.

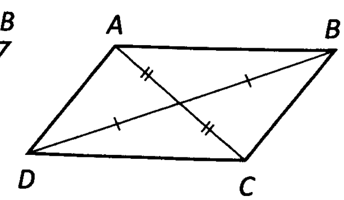
An additional property of any parallelogram is that the diagonal will divide the parallelogram into 2 equal triangles.



Triangle *ABD* = Triangle *BCD*

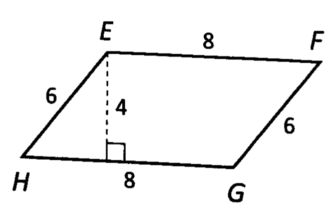


Triangle *ADC* = Triangle *ABC*



The diagonals also cut each other in half (bisect each other)

For any parallelogram, the perimeter is the sum of the lengths of all the sides and the area is equal to (base) × (height). With parallelograms, as with triangles, it is important to remember that the base and the height MUST be perpendicular to one another.



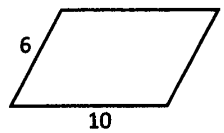
In this parallelogram, what is the perimeter, and what is the area? The perimeter is the sum of the sides, so it's 6 + 8 + 6 + 8 = 28. Alternatively, you can use one of the properties of parallelograms to calculate the perimeter in a different way.

We know that parallelograms have two sets of equal sides. In this parallelogram, two of the sides have a length of 6, and two of the sides have a length of 8. So the perimeter equals $2 \times 6 + 2 \times 8$. We can factor out a 2, and say that perimeter $= 2 \times (6 + 8) = 28$.

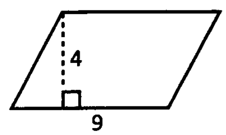
To calculate the area, we need a base and a height. It might be tempting to say that the area is $6 \times 8 = 48$. But the two sides of this parallelogram are not perpendicular to each other. The dotted line drawn into the figure, however, is perpendicular to side *HG*. The area of parallelogram *EFGH* is $8 \times 4 = 32$.

Check Your Skills

6. What is the perimeter of the parallelogram?



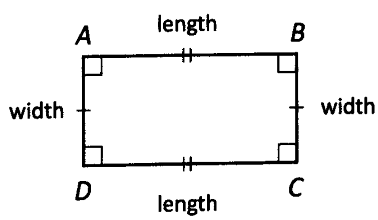
7. What is the area of the parallelogram?



Answers can be found on page 81.

Rectangles

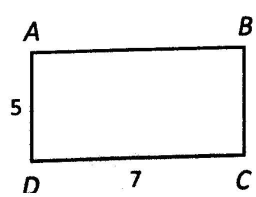
Rectangles are a specific type of parallelogram. Rectangles have all the same properties as parallelograms, with one additional property—all four internal angles of a rectangle are right angles. Additionally, with rectangles, we refer to one pair of sides as the length and one pair of sides as the width.



The formula for the perimeter of a rectangle is the same as for the perimeter of a parallelogram—either sum the lengths of the four sides or add the length and the width and multiply by 2.

The formula for the area of a rectangle is also the same as for the area of a parallelogram, but for any rectangle, the length and width are by definition perpendicular to each other, so you don't need a separate height. For this reason, the area of a rectangle is commonly expressed as (length) × (width).

Let's practice. For the following rectangle, find the perimeter and the area.



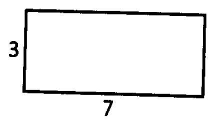
Let's start with the perimeter. Again, we can either fill in the missing sides and add them all up, or recognize that we have two sides with a length of 5 and two sides with a length of 7. Therefore, perimeter = $2 \times (5 + 7)$, which equals 24. Alternatively, $5 + 5 + 7 + 7$ also equals 24.

Now to find the area. The formula for area is (length) $\times$ (width). For the purposes of finding the area, it is irrelevant which side is the length and which side is the width. If we make AD the length and DC the width, then the area = $(5) \times (7) = 35$. If, instead, we make DC the length and AD the width, then we have area = $(7) \times (5) = 35$. The only thing that matters is that we choose two sides that are perpendicular to each other.

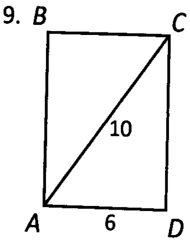
Check Your Skills

Find the area and perimeter of each rectangle.

8.



9.

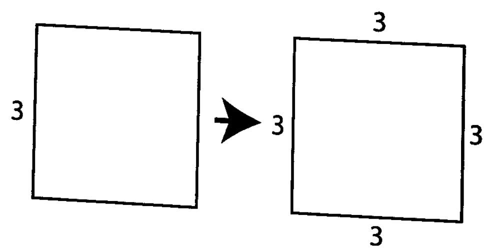


Answers can be found on pages 81–82.

Squares

One particular type of rectangle warrants mention—the square. A square is a rectangle in which the lengths of all four sides are equal. Everything that is true of rectangles is true of squares as well. What this means is that knowing only one side of a square is enough to determine the perimeter and area of a square.

For instance, if we have a square, and we know that the length of one of its sides is 3, we know that all 4 sides have a length of 3.



The perimeter of the square is $3 + 3 + 3 + 3$, which equals 12. Alternatively, once you know the length of one side of a square, you can multiply that length by 4 to find the perimeter. $3 \times 4 = 12$.

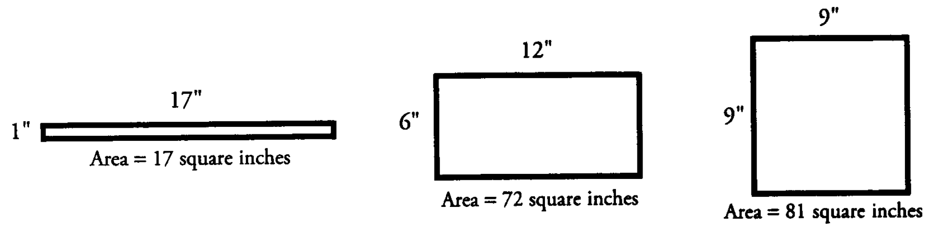
To find the area, we use the same formula as for a rectangle—Area = (length) $\times$ (width). But, because the shape is a square, we know that the length and the width are equal. Therefore, we can say that the area of a square is Area = $(\text{side})^2$. In this case, Area = $(3)^2 = 9$.

Maximum Area of Polygons

In some problems, the GRE may require you to determine the maximum or minimum area of a given figure. Following a few simple shortcuts can help you solve certain problems quickly.

Maximum Area of a Quadrilateral

Perhaps the best-known maximum–area problem is one which asks you to maximize the area of a *quadrilateral* (usually a rectangle) with a *fixed perimeter.* If a quadrilateral has a fixed perimeter, say, 36 inches, it can take a variety of shapes:



Of these figures, the one with the largest area is the square. This is a general rule: **Of all quadrilaterals with a given perimeter, the SQUARE has the largest area.** This is true even in cases involving non-integer lengths. For instance, of all quadrilaterals with a perimeter of 25 feet, the one with the largest area is a square with 25/4 = 6.25 feet per side.

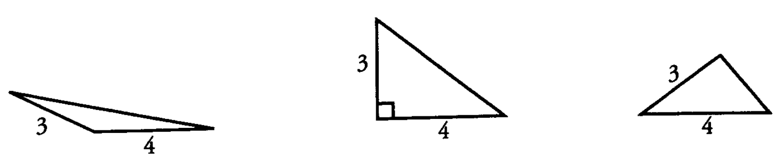
This principle can also be turned around to yield the following corollary: **Of all quadrilaterals with a given area, the SQUARE has the minimum perimeter.**

Both of these principles can be generalized for polygons with *n* sides: **a regular polygon with all sides equal and all angles equal will maximize area for a given perimeter and minimize perimeter for a given area.**

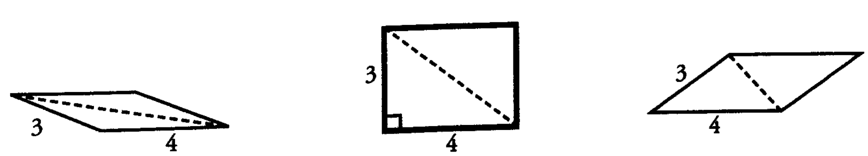
Maximum Area of a Parallelogram or Triangle

Another common optimization problem involves maximizing the area of a *triangle or parallelogram with given side lengths.*

For instance, there are many triangles with two sides 3 and 4 units long. Imagine that the two sides of length 3 and 4 are on a hinge. The third side can have various lengths:



There are many corresponding parallelograms with two sides 3 and 4 units long:



The area of a triangle is given by $A = \frac{1}{2}bh$, and the area of a parallelogram is given by $A = bh$. Because both of these formulas involve the perpendicular height h, the maximum area of each figure is achieved when the 3-unit side is perpendicular to the 4-unit side, so that the height is 3 units. All the other figures have lesser heights. (Note that in this case, the triangle of maximum area is the famous 3–4–5 right triangle.) If the sides are not perpendicular, then the figure is squished, so to speak.

The general rule is this: **if you are given two sides of a triangle or parallelogram, you can maximize the area by placing those two sides PERPENDICULAR to each other.**

Check Your Skills Answers

1. **1,080:** One way to calculate the sum of the interior angles of a polygon is by applying the formula $(n-2)180$ = Sum of the interior angles, where n is the number of sides. Substituting 8 for n yields:

Sum of the interior angles = (8 — 2)180
= (6)180
= 1,080

2. **120:** Since each interior angle is the same, we can determine the angle of any one by dividing the sum of the interior angles by 6 (the number of interior angles). Use the formula $(n-2)180$ = Sum of the interior angles, where n is the number of sides. Substituting 6 for n yields: Sum = (4)180 = 720. Divide 720 by 6 to get 120.

3. **58:** The sum of the five sides is 58". It is simplest to arrange them as $12 + 12 + 17 + (8\frac{1}{2} + 8\frac{1}{2}) = 12 + 12 + 17 + 17 = 58$.

4. **40:** Since the surface area of a cube is 6 times the area of one face, each square face of each cube must have an area of 4. One face of each cube is lost when the two cubes are joined, so the total surface area of the figure will be the sum of the surface areas of both cubes minus the surface areas of the covered faces.

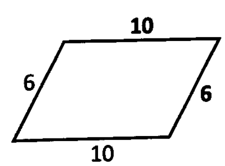
Each cube has surface area of 24, so the total surface area is 48. Subtract the surface area of each covered face (4). $48 - 2(4) = 40$.

5. **8:** The volume of a rectangular solid is the product of its three dimensions, length, width, and height.

$8 \times 6 \times 4 = 192$ and $4 \times 3 \times 2 = 24$

$\frac{192}{24} = 8$, so the volume of the larger cube is 8 times the volume of the smaller cube.

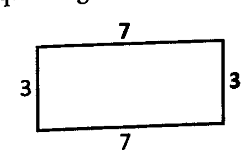
6. **32:** In parallelograms, opposite sides have equal lengths, so we know that two of the sides of the parallelogram have a length of 6 and two sides have a length of 10.



So the perimeter is 6 + 10 + 6 + 10, which equals 32.

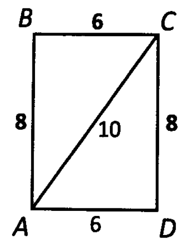
7. **36:** Area of a parallelogram is $b \times h$. In this parallelogram, the base is 9 and the height is 4, so the area is $(9) \times (4)$, which equals 36. The area of the parallelogram is 36.

8. **20, 21:** In rectangles, opposite sides have equal lengths, so our rectangle looks like this:



So the perimeter is $3 + 7 + 3 + 7$, which equals 20. The area of a rectangle is $b \times h$, so the area is $(7) \times (3)$, which equals 21. So the perimeter is 20, and the area is 21.

9. **28, 48:** To find the area and perimeter of the rectangle, we need to know the length of either side *AB* or side *CD*. The diagonal of the rectangle creates a right triangle, so we can use the Pythagorean Theorem to find the length of side *CD*. Alternatively, we can recognize that triangle *ACD* is a 6–8–10 triangle, and thus the length of side *CD* is 8. Either way, our rectangle now looks like this:



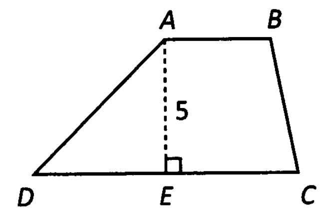
So the perimeter of the rectangle is $6 + 8 + 6 + 8$, which equals 28. The area is $(6) \times (8)$, which equals 48.

Problem Set (Note: Figures are not drawn to scale.)

1. Frank the Fencemaker needs to fence in a rectangular yard. He fences in the entire yard, except for one 40-foot side of the yard. The yard has an area of 280 square feet. How many feet of fence does Frank use?

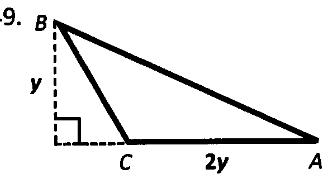
2. A pentagon has three sides with length x, and two sides with the length $3x$. If x is $\frac{2}{3}$ of an inch, what is the perimeter of the pentagon?

3. *ABCD* is a quadrilateral, with *AB* parallel to *CD* (see figure). *E* is a point between *C* and *D* such that *AE* represents the height of *ABCD*, and *E* is the midpoint of *CD*. If *AB* is 4 inches long, *AE* is 5 inches long, and the area of triangle *AED* is 12.5 square inches, what is the area of *ABCD*?



4. A rectangular tank needs to be coated with insulation. The tank has dimensions of 4 feet, 5 feet, and 2.5 feet. Each square foot of insulation costs $20. How much will it cost to cover the surface of the tank with insulation?

5. Triangle *ABC* (see figure) has a base of $2y$, a height of y, and an area of 49. What is y?



6. 40 percent of Andrea's living room floor is covered by a carpet that is 4 feet by
9 feet. What is the area of her living room floor?

7. If the perimeter of a rectangular flower bed is 30 feet, and its area is 44 square feet, what is the length of each of its shorter sides?

8. There is a rectangular parking lot with a length of $2x$ and a width of x. What is the ratio of the perimeter of the parking lot to the area of the parking lot, in terms of x?

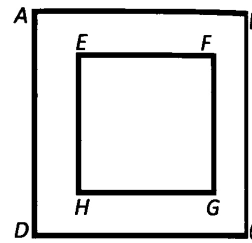
9. A rectangular solid has a square base, with each side of the base measuring 4 meters. If the volume of the solid is 112 cubic meters, what is the surface area of the solid?

10. A swimming pool has a length of 30 meters, a width of 10 meters, and an average depth of 2 meters. If a hose can fill the pool at a rate of 0.5 cubic meters per minute, how many hours will it take the hose to fill the pool?

11. A Rubix cube has an edge 5 inches long. What is the ratio of the cube's surface area to its volume?

12. If the length of an edge of Cube A is one third the length of an edge of Cube B, what is the ratio of the volume of Cube A to the volume of Cube B?

13. *ABCD* is a square picture frame (see figure). *EFGH* is a square inscribed within *ABCD* as a space for a picture. The area of *EFGH* (for the picture) is equal to the area of the picture frame (the area of *ABCD* minus the area of *EFGH*). If $AB = 6$, what is the length of *EF*?



14. What is the maximum possible area of a quadrilateral with a perimeter of 80 centimeters?

15. What is the minimum possible perimeter of a quadrilateral with an area of 1,600 square feet?

16. What is the maximum possible area of a parallelogram with one side of length 2 meters and a perimeter of
24 meters?

17. What is the maximum possible area of a triangle with a side of length 7 units and another side of length
8 units?

18. The lengths of the two shorter legs of a right triangle add up to 40 units. What is the maximum possible area of the triangle?

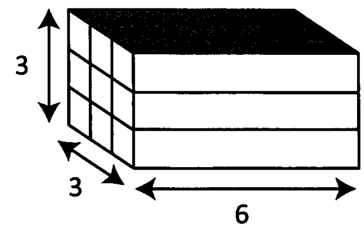
19.

Quantity A	**Quantity B**
The surface area in square inches of a cube with edges of length 6	The volume in cubic inches of a cube with edges of length 6

20.

Quantity A	**Quantity B**
The total volume of 3 cubes with edges of length 2	The total volume of 2 cubes with edges of length 3

21.



The large rectangular solid above
is formed by binding together nine
identical rectangular rods, as shown.

Quantity A

The combined surface area of four
of the individual, identical
rectangular rods.

Quantity B

The surface area of the large
rectangular solid

1. **54 feet:** We know that one side of the yard is 40 feet long; let us call this the length. We also know that the area of the yard is 280 square feet. In order to determine the perimeter, we must know the width of the yard.

$$A = l \times w$$
$$280 = 40w$$
$$w = 280 \div 40 = 7 \text{ feet}$$

Frank fences in the two 7-foot sides and one of the 40-foot sides. $40 + 2(7) = 54$.

2. **6 inches:** The perimeter of a pentagon is the sum of its five sides: $x + x + x + 3x + 3x = 9x$. If x is $\frac{2}{3}$ of an inch, the perimeter is $9\frac{2}{3}$, or 6 inches.

3. **35 in²:** If E is the midpoint of CD, then $CE = DE = x$. We can determine the length of x by using what we know about the area of triangle AED.

$$A = \frac{b \times h}{2} \quad 12.5 = \frac{5x}{2}$$
$$25 = 5x$$
$$x = 5$$

Therefore, the length of CD is $2x$, or 10.

To find the area of the trapezoid, use the formula: $A = \frac{b_1 + b_2}{2} \times h$

$$= \frac{4+10}{2} \times 5$$
$$= 35 \text{ in}^2$$

4. **$1,700:** To find the surface area of a rectangular solid, sum the individual areas of all six faces:

Top and Bottom:	$5 \times 4 = 20$	$\rightarrow$	$2 \times 20 = 40$
Side 1:	$5 \times 2.5 = 12.5$	$\rightarrow$	$2 \times 12.5 = 25$
Side 2:	$4 \times 2.5 = 10$	$\rightarrow$	$2 \times 10 = 20$

$$40 + 25 + 20 = 85 \text{ ft}^2$$

Covering the entire tank will cost $85 \times \$20 = \$1,700$.

5. **7:** The area of a triangle is equal to half the base times the height. Therefore, we can write the following relationship:

$$\frac{2y(y)}{2} = 49$$
$$y^2 = 49$$
$$y = 7$$

6. **90 ft²:** The area of the carpet is equal to $l \times w$, or 36 ft². Set up a percent table or a proportion to find the area of the whole living room floor:

$$\frac{40}{100} = \frac{36}{x}$$ Cross-multiply to solve.

$$40x = 3600$$
$$x = 90 \text{ ft}^2$$

7. **4:** Set up equations to represent the area and perimeter of the flower bed:

$$A = l \times w \qquad\qquad P = 2(l + w)$$

Then, substitute the known values for the variables A and P:

$$44 = l \times w \qquad\qquad 30 = 2(l + w)$$

Solve the two equations with the substitution method:

$$l = \frac{44}{w}$$

$$30 = 2\left(\frac{44}{w} + w\right)$$

$$30 = \frac{88}{w} + 2w$$ Multiply the entire equation by $\frac{w}{2}$.

$$15w = 44 + w^2$$

$$w^2 - 15w + 44 = 0$$

$$(w - 11)(w - 4) = 0$$

$$w = \{4,\ 11\}$$

Solving the quadratic equation yields two solutions: 4 and 11. Since we are looking only for the length of the shorter side, the answer is 4.

Alternatively, you can arrive at the correct solution by picking numbers. What length and width add up to 15 (half of the perimeter) and multiply to produce 44 (the area)? Some experimentation will demonstrate that the longer side must be 11 and the shorter side must be 4.

8. $\frac{3}{x}$**:** If the length of the parking lot is $2x$ and the width is x, we can set up a fraction to represent the ratio of the perimeter to the area as follows:

$$\frac{\text{perimeter}}{\text{area}} = \frac{2(2x + x)}{(2x)(x)} = \frac{6x}{2x^2} = \frac{3}{x}$$

9. **144 m²:** The volume of a rectangular solid equals (length) × (width) × (height). If we know that the length and width are both 4 meters long, we can substitute values into the formulas as shown:

$$112 = 4 \times 4 \times h$$
$$h = 7$$

To find the surface area of a rectangular solid, sum the individual areas of all six faces:

Top and Bottom:	$4 \times 4 = 16$	$\rightarrow$	$2 \times 16 = 32$
Sides:	$4 \times 7 = 28$	$\rightarrow$	$4 \times 28 = 112$

$$32 + 112 = 144 \text{ m}^2$$

10. **20 hours:** The volume of the pool is (length) $\times$ (width) $\times$ (height), or $30 \times 10 \times 2 = 600$ cubic meters. Use a standard work equation, $RT = W$, where W represents the total work of 600 m^3.

$0.5t = 600$
$t = 1{,}200$ minutes   Convert this time to hours by dividing by 60: $1{,}200 \div 60 = 20$ hours.

11. $\mathbf{\frac{6}{5}}$: To find the surface area of a cube, find the area of 1 face, and multiply that by 6: $6(5^2) = 150$.

To find the volume of a cube, cube its edge length: $5^3 = 125$.

The ratio of the cube's surface area to its volume, therefore, is $\frac{150}{125}$, or $\frac{6}{5}$.

12. **1 to 27:** First, assign the variable x to the length of one side of Cube A. Then the length of one side of Cube B is $3x$. The volume of Cube A is x^3. The volume of Cube B is $(3x)^3$, or $27x^3$.

Therefore, the ratio of the volume of Cube A to Cube B is $\frac{x^3}{27x^3}$, or 1 to 27. You can also pick a number for the length of a side of Cube A and solve accordingly.

13. $\mathbf{3\sqrt{2}}$: The area of the frame and the area of the picture sum to the total area of the image, which is 6^2, or 36. Therefore, the area of the frame and the picture are each equal to half of 36, or 18. Since *EFGH* is a square, the length of *EF* is $\sqrt{18}$, or $3\sqrt{2}$.

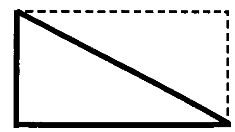
14. **400 cm²:** The quadrilateral with maximum area for a given perimeter is a square, which has four equal sides. Therefore, the square that has a perimeter of 80 centimeters has sides of length 20 centimeters each. Since the area of a square is the side length squared, the area = (20 cm)(20 cm) = 400 cm^2.

15. **160 ft:** The quadrilateral with minimum perimeter for a given area is a square. Since the area of a square is the side length squared, we can solve the equation $x^2 = 1{,}600 \text{ ft}^2$ for the side length x, yielding $x = 40$ ft. The perimeter, which is four times the side length, is (4)(40 ft) = 160 ft.

16. **20 m²:** If one side of the parallelogram is 2 meters long, then the opposite side must also be 2 meters long. We can solve for the unknown sides, which are equal in length, by writing an equation for the perimeter: $24 = 2(2) + 2x$, with x as the unknown side. Solving, we get $x = 10$ meters. The parallelogram with these dimensions and maximum area is a *rectangle* with 2-meter and 10-meter sides. Thus the maximum possible area of the figure is (2 m)(10 m) = 20 m^2.

17. **28 square units:** A triangle with two given sides has maximum area if these two sides are placed at right angles to each other. For this triangle, one of the given sides can be considered the base, and the other side can be considered the height (because they meet at a right angle). Thus we plug these sides into the formula $A = \frac{1}{2}bh$: $A = \frac{1}{2}(7)(8) = 28$.

18. **200 square units:** You can think of a right triangle as half of a rectangle. Constructing this right triangle with legs adding to 40 is equivalent to constructing the rectangle with a perimeter of 80. Since the area of the triangle is half that of the rectangle, you can use the previously mentioned technique for maximizing the area of a rectangle: of all rectangles with a given perimeter, the ***square*** has the greatest area. The desired rectangle is thus a 20 by 20 square, and the right triangle has area $\frac{1}{2}(20)(20) = 200$ units.



19. **C:** The surface area of a cube is 6 times e^2, where e *is* the length of each edge (that is, the surface area is the number of faces times the area of each face). Apply this formula to Quantity A.

<u>**Quantity A**</u>	<u>**Quantity B**</u>
The surface area in square inches of a cube with edges of length 6 = **6 × (6 × 6)**	The volume in cubic inches of a cube with edges of length 6.

The volume of a cube is e^3, where e is the length of each edge. Apply this formula to Quantity B.

<u>**Quantity A**</u>	<u>**Quantity B**</u>
6 × (6 × 6)	The volume in cubic inches of a cube with edges of length 6 = **6** × **6** × **6**

It is not generally the case that the volume of a cube in cubic units is equal to the surface area of the cube in square inches; they are only equal when the edge of the cube is of length 6. In this case, **the two quantities are equal.**

20. **B:** The volume of a cube is e^3, where e is the length of each edge. Apply this formula to each quantity.

<u>**Quantity A**</u>	<u>**Quantity B**</u>
The total volume of 3 cubes with edges of length 2 = $3 \times (2^3) =$ **24**	The total volume of 2 cubes with edges of length 3 = $2 \times (3^3) =$ **54**

Quantity B is larger.

21. **A:** A rectangular solid has three pairs of opposing equal faces, each pair representing two of the dimensions of the solid (length × width; length × height; height × width). The total surface area of a rectangular solid is the sum of the surface areas of those three pairs of opposing sides.

According to the diagram, the dimensions of each rod must be $1 \times 1 \times 6$. So each of the rods described in Quantity A has a surface area of:

$$2(1 \times 1) + 2(1 \times 6) + 2(1 \times 6), \quad \text{or} \quad 2[(1 \times 1) + (1 \times 6) + (1 \times 6)]$$

That is, each rod has a total surface area of 26, and the four rods together have a surface area of $4 \times 26 = 104$.

<u>**Quantity A**</u>	<u>**Quantity B**</u>
The combined surface area of four of the identical rectangular rods = **104**	The surface area of the large rectangular solid.

The large rectangular solid has a total surface area of:

$$2(3 \times 3) + 2(3 \times 6) + 2(3 \times 6), \text{ or } 90.$$

<u>**Quantity A**</u>	<u>**Quantity B**</u>
104	The surface area of the large rectangular solid = **90**

Quantity A is larger.