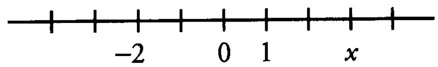
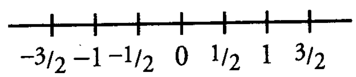
NUMBER LINES

Number lines can appear in a variety of different forms on the GRE. They also provide varying amounts of information.

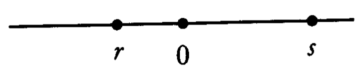
The most structured version of a number line will contain evenly spaced tick marks. These provide the most detail about the position of points on a number line and about the distance between points.



These number lines will almost always contain numbers, and will often contain variables as well. Also note that the distance between tick marks can be an integer amount (like in the number line pictured above) or a fractional amount (like in the number line pictured below).

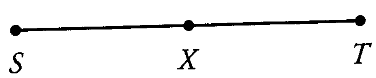


Not all number lines will provide this level of detail. Many number lines will only display a handful of points that are not evenly spaced.



These number lines are likely to contain fewer actual numbers, and will always contain at least one variable. On these number lines, it is more likely that you won't have specific information about the distance between two points.

Additionally, questions that talk about line segments or points that all lie on a line can be thought of as number lines. For example, a question might state that point X is the midpoint of line segment ST. This is the picture you would draw.



These number lines will rarely contain any real numbers. Often, the only points on the line will be designated by variables. Questions that require this type of number line may or may not provide information about the specific distance between points, although they may provide proportional information. For instance, in the number line above, although we don't know the length of line segment ST, we do know that ST is twice as long as segments SX and XT (because X is the midpoint of ST).

Relative Position & Relative Distance

Questions that involve number lines overwhelmingly ask for information about the *position* of a point or points or the *distance* between two points.

Position

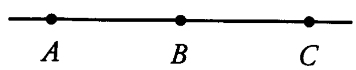
On any number line you will see, numbers get bigger as they move from left to right.

B is greater than A — B is more positive than A (if both positive)

A is less than B — A is more negative than B (if both negative)

The above statements are true regardless of where zero is shown on the number line. A and B could both be positive or both be negative, or A could be negative and B could be positive.

Number lines on the GRE follow rules similar to the rules for geometric shapes. If there is more than one point on a number line, you KNOW the *Relative Position* of each point.

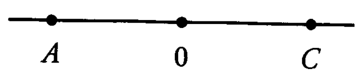


$A < B < C$

While you do know the relative position of each point, you DO NOT KNOW the *Relative Distance* between points (unless that information is specifically provided).

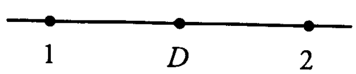
On the number line above, B could be closer to A than to C, closer to C than to A, or equidistant between A and C. Without more information, there is no way to know.

The rules are similar if a number line contains both numbers and variables.



$A < 0$, A is negative

$C > 0$, C is positive

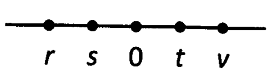


$1 < D < 2$

D looks like it is halfway between 1 and 2, but that does not mean that it is 1.5. D could be 1.5, but it could also be 1.000001, or 1.99999 or, in fact, any number between 1 and 2.

Check Your Skills

Refer to the following number line for questions 1–3.

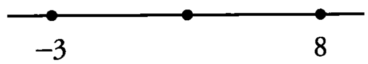


Which of the following MUST be true?

1. $v > s + t$
2. $v + s > t + r$
3. $rs > v$

Distance

If you know the specific location of two points on a number line, the distance between them is the absolute value of their difference.

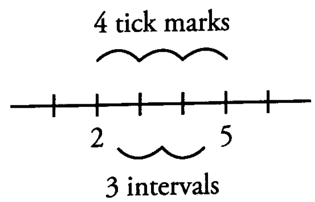


$$\text{Distance} = |8 - (-3)| = |(-3) - 8| = 11$$

If a number line contains tick marks and specifically tells you they are evenly spaced, it may be necessary to calculate the distance between tick marks.

On an evenly spaced number line, tick marks represent specific values, and the intervals between tick marks represent the distance between tick marks.

For any specific range, there will always be 1 more tick mark than interval.



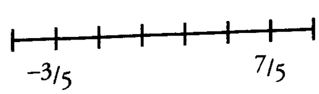
On the number line above, there are 4 ticks marks between 2 and 5 (inclusive). There is one fewer interval than tick marks. There are only 3 intervals between 2 and 5. Now calculate the length of the intervals on this number line. To calculate the distance between any two tick marks (which is the same as the length of the intervals), subtract the lower bound from the upper bound and divide the difference by the number of intervals.

In the number line above, the lower bound is 2, the upper bound is 5, and there are 3 intervals between 2 and 5. Use these numbers to calculate the distance between tick marks on the number line.

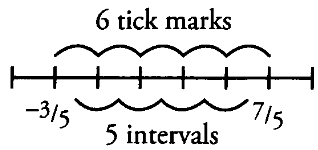
$$\frac{\text{upper} - \text{lower}}{\#\text{ of intervals}} = \frac{5-2}{3} = 1$$

That means that each tick mark in the number line above is 1 unit away from each of the two tick marks to which it is adjacent.

Not every number line will have interval lengths with integer values. Note that this method is equally effective if the intervals are fractional amounts. What is the distance between adjacent tick marks on the following number line?

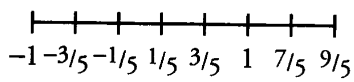


Now the range contains 6 tick marks and 5 intervals.

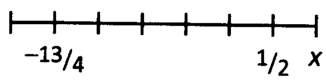


$$\frac{\text{upper} - \text{lower}}{\#\,\text{of intervals}} = \frac{\frac{7}{5} - \left(-\frac{3}{5}\right)}{3} = \frac{\frac{10}{5}}{5} = \frac{10}{25} = \frac{2}{5}$$

The distance between tick marks is $\frac{2}{5}$.



Check Your Skills



4. On the number line above, what is the value of point x?

The answer can be found on page 133.

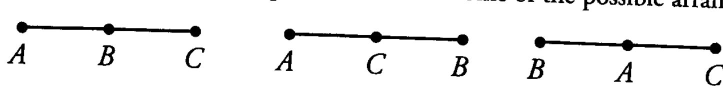
Line Segments

Some questions on the GRE will describe either several points that all lie on a line or line segments that also lie on the same line. In order to answer these questions correctly, you will need to use the information in the question to construct a number line. Ultimately, position and distance will be of prime importance.

Position

In order to correctly draw number lines, you need to remember one thing. If a question mentions a line segment, there are two possible versions of that segment. Suppose a question tells you that the length of line segment *BD* is 4. These are the two possible versions of *BD*:

We can take this even further. Suppose there are three points on a line: *A*, *B*, and *C*. Without more information, we don't know the order of the three points. Below are some of the possible arrangements:

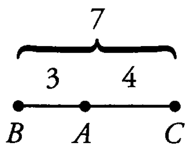


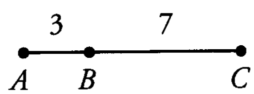
When questions provide incomplete information about the relative position of points, make sure that you account for the lack of information by drawing multiple number lines.

Distance

Distance on this type of number line can potentially be made more difficult by a lack of complete information about the positions of points on the line.

Suppose that A, B, and C all lie on a number line. Further suppose that $\overline{AB} = 3$ and $\overline{BC} = 7$. Because $\overline{AB}$ is shorter than $\overline{BC}$, there are two possible positions for point A: in between B and C, or on one side of B, with C on the other side.

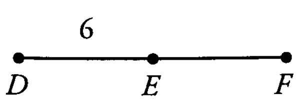


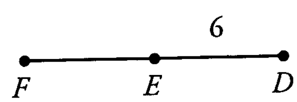


Constructing number lines can be made more difficult by many pieces of information in the question. To construct number lines efficiently and accurately, while remembering to keep track of different possible scenarios, always start with the most restrictive pieces of information first.

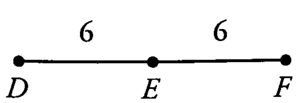
> On a line, E is the midpoint of $\overline{DF}$, and $\overline{DE}$ has a length of 6. Point G does not lie on the line and $\overline{EG} = 4$. What is the range of possible values of $\overline{FG}$?

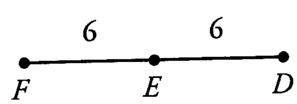
The best way to start this problem is to draw $\overline{DF}$, with E in the middle. There are two possible versions. Also note that $\overline{DE} = 6$.



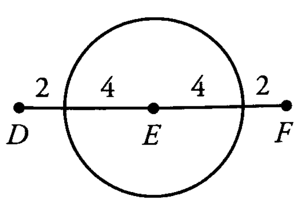


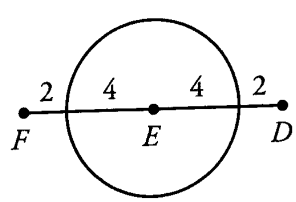
Also, because E is the midpoint of $\overline{DF}$, we know that $\overline{EF}$ also has a length of 6.





Now we need to deal with point G. Although we do not know the precise position of G, we know it is a fixed distance from E. The set of all points that are equidistant from a fixed point is actually a circle—in other words, to represent the possible positions of G, draw a circle around point E with a radius of 4.





As it turns out, both number lines behave the exact same way, so there is no need to continue to look at both.

On this diagram, you can see that G would be closest to F when it is on the line between E and F. That point is 2 away from F. Similarly, G is farthest away from F when it is on the line between D and E. That point is 10 away from F.

If G could be on the line, the range of possible values of $\overline{FG}$ would be $2 \leq \overline{FG} \leq 10$. Because it can't be on the line, the range is instead $2 < \overline{FG} < 10$.

Check Your Skills

5. X, Y, and Z all lie on a number line. $\overline{XY}$ has a length of 5 and $\overline{YZ}$ has a length of 7. If point U is the midpoint of $\overline{XZ}$, and $\overline{UZ} > 2$, what is the length of $\overline{UZ}$?

The answer can be found on page 133.

Check Your Skills Answer Key:

1. **MUST BE TRUE.** v is already greater than t, and adding a negative number to t will only make it smaller.

2. **MUST BE TRUE.** One way to prove this statement is always true is to add inequalities. We know that $v > t$, and that $s > r$.

$$\begin{array}{r} v > t \\ + \; s > r \\ \hline v + s > t + r \end{array}$$

3. **NOT ALWAYS TRUE.** v could be any positive number, and r and s could be any negative number.

If $r = -3$, $s = -2$ and $v = 4$, then $rs > v$.

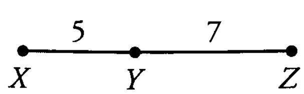
If, however, $r = -2$, $s = -1/2$ and $v = 3$, $v > rs$.

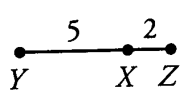
4. $\frac{5}{4}$**:** To find x, we need to figure out how far apart tick marks are. We can use the two given points $\left(-\frac{13}{4}, \frac{1}{2}\right)$ to do so. There are five intervals between the two points.

$$\frac{\frac{1}{2} - \left(-\frac{13}{4}\right)}{5} = \frac{\frac{15}{4}}{5} = \frac{15}{20} = \frac{3}{4}$$

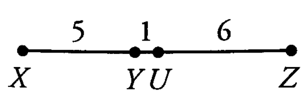
If the distance between tick marks is $\frac{3}{4}$, then x is $\frac{1}{2} + \frac{3}{4} = \frac{5}{4}$.

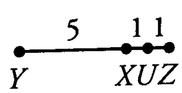
5. **6:** Start with the points X, Y, and Z. There are two possible arrangements:





Now place U on each number line.

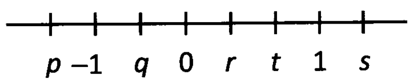




On one number line, $\overline{UZ} = 1$, but the question stated that $\overline{UZ} > 2$, so $\overline{UZ}$ must equal 6.

Problem Set

For questions 1–6, refer to the number line below. Decide whether each statement MUST be true, COULD be true, or will NEVER be true.



1. $s + q > 0$

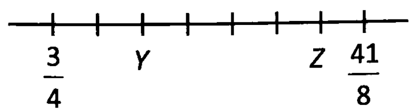
2. $pq > t$

3. $p^2 > s^4$

4. $s - p > r - q$

5. $t - q = 2$

6. $rs > 1$

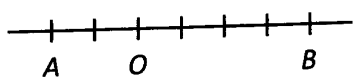


7. If the tick marks on the number line above are evenly spaced, what is the distance between Y and Z?

8. A, B, and C all lie on a line. D is the midpoint of AB and E is the midpoint of BC. $AB = 4$ and $BC = 10$. Which of the following could be the length of AE?

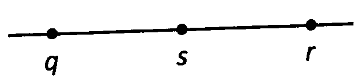
   A) 1 B) 2 C) 3 D) 4 E) 5

9.



Quantity A	Quantity B
$(A)(B)$	-1

10.



s is the midpoint of $\overline{qr}$

$r = -2q$

Quantity A	Quantity B
s	0

11.

A, B, C, and *D* all lie on a number line. *C* is the midpoint of $\overline{AB}$ and *D* is the midpoint of $\overline{AC}$.

Quantity A	**Quantity B**
The ratio of $\overline{AD}$ to $\overline{CB}$	The ratio of $\overline{AC}$ to $\overline{AB}$

IN ACTION ANSWER KEY

1. **MUST be true:** Although we don't have specific values for either s or q, we know that s is greater than 1, and we know that q is between 0 and -1. Even if s was as small as it could be (≈ 1.00001) and q was as negative as it could be (≈ -0.99999), the sum would still be positive.

2. **COULD be true:** t must be positive, and the product pq will also be positive. We know that t must be between 0 and 1, but the product pq could be either less than 1 or greater than 1, depending on the numbers chosen. If $t = 0.9$, $q = -0.1$ and $p = -2$, then $t > pq$. However, if $t = 0.5$, $q = -0.9$ and $p = -5$, $pq > t$.

3. **COULD be true:** Both p^2 and s^4 will be positive, but depending on the numbers chosen for p and s, either value could be larger. If $p = -2$ and $s = 3$, $s^4 > p^2$. If $p = -8$ and $s = 2$, $s^4 < p^2$.

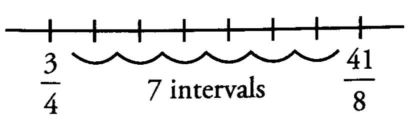
4. **MUST be true:** s is greater than 1 and p is less than -1. The smallest that the difference can be is greater than 2.

r must be between 0 and 1 and q must be between 0 and -1. The greatest the difference can be is less than 2. $s - p$ will always be greater than $r - q$.

5. **NEVER be true:** Even if t is as large as it can be and q is as small as it can be, the difference will still have to be less than 2. If $t = 0.999999$ and $q = -0.999999$, $t - q = 1.999998$.

6. **COULD be true:** If $r = 0.1$ and $s = 2$, $rs < 1$. If $r = 0.5$ and $s = 3$, $rs > 1$.

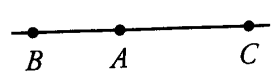
7. **2.5:** To figure out the distance between Y and Z, we first need to figure out the distance between tick marks. We can use the two points on the number line $\left(\frac{3}{4} \text{ and } \frac{41}{8}\right)$ to find the distance. There are 7 intervals between the two points.



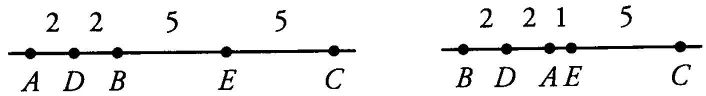
$$\frac{41/8 - 3/4}{7} = \frac{41/8 - 6/8}{7} = \frac{35/8}{7} = \frac{5}{8}$$

We actually do not need to know the positions of Y and Z to find the distance between them. We know that there are 4 intervals between Y and Z, so the distance is $4 \times \frac{5}{8} = \frac{20}{8} = 2.5$.

8. **A:** The trick to this problem is recognizing that there is more than one possible arrangement for the points on the number line. Because $\overline{BC}$ is longer than $\overline{AB}$, A could be either be in between B and C or on one side of B with C on the other side of B.



Using the information about the midpoints (D and E) and the lengths of the line segments, we can fill in all the information for our two number lines.

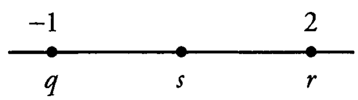


We can see that $\overline{AE}$ has two possible lengths: 1 and 9. 1 is the only option that is an answer choice.

9. **D:** With only one actual number displayed on the number line, we have no way of knowing the distance between tick marks. If the tick marks are a small fractional distance away from each other, then AB will be greater than -1. For instance, if the distance between tick marks is $\frac{1}{8}$, then A is $-\frac{1}{4}$, B is $\frac{1}{2}$ and AB is $-\frac{1}{8}$, which is greater than -1. If the distance between tick marks is 1, then A is -2, B is 4, and AB is -8, which is less than -1.

Therefore **we do not have enough information** to answer the question.

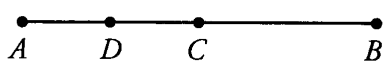
10. **A:** The easiest approach is to pick numbers. q must be a negative number and r must be positive. If $q = -1$, then $r = 2$.



If s is the midpoint of q and r, then s must be 0.5. Therefore $s > 0$.

For any numbers we pick, s will be positive. Therefore **Quantity A is larger.**

11. **C:**



Visualizing the number line above, the ratio of AD to CB is $\frac{1}{2}$. Similarly, the ratio of AC to AB is $\frac{1}{2}$. Therefore **the two quantities are equal.**