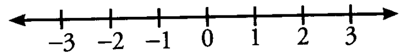
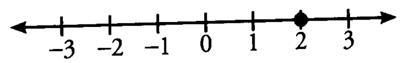
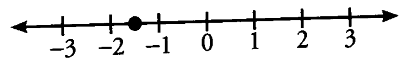
COORDINATE PLANE

Before we discuss the coordinate plane, let's review the number line.

The Number Line



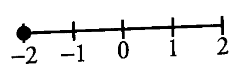
The number line is a ruler or measuring stick that goes as far as we want in both directions. With the number line, we can say where something is positioned with a single number. In other words, we can link a position with a number.

Position	Number	Number Line
"Two units right of zero"	2	
"One and a half units left of zero"	–1.5	

We use both positive and negative numbers, because we want to indicate positions both left and right of zero.

You might be wondering "The position of what?" The answer is, a **point**, which is just a dot. When we are dealing with the number line, a point and a number mean the same thing.

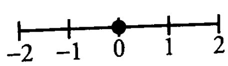
If you show me where the point is on the number line, I can tell you the number.



→ *the point is at –2*

If you tell me the number, I can show you where the point is on the number line.

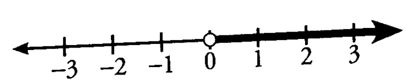
the point is at 0 →



This works even if we only have partial information about our point. If you tell me *something* about where the point is, I can tell you *something* about the number, and vice-versa.

For instance, if I say that the number is positive, then I know that the point lies somewhere to the right of 0 on the number line. Even though I don't know the exact location of the point, I do know a range of potential values.

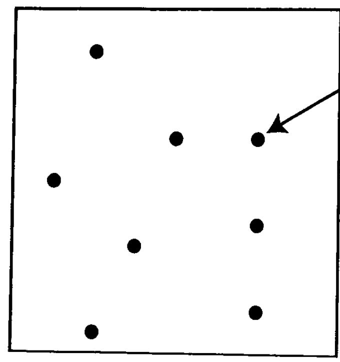
The number is positive.
In other words, the number is greater than (>) 0.



This also works in reverse. If I see a range of potential positions on a number line, I can tell you what that range is for the number.

The number is less than (<) 0.

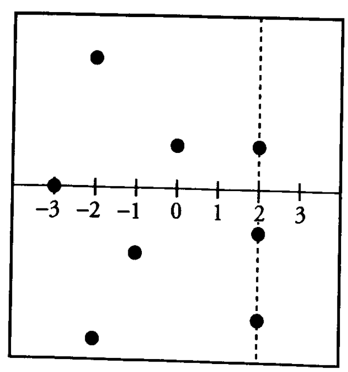
Let's make things more complicated. What if we want to be able to locate a point that's not on a straight line, but on a page?



The point we want

Now one number line won't be enough to tell us where the point is.

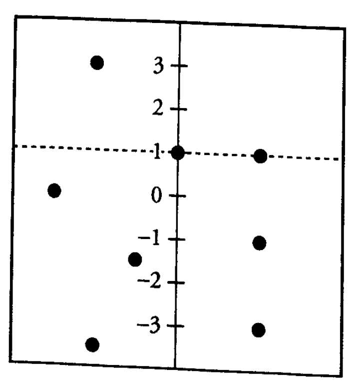
Let's begin by inserting our number line into the picture. This will help us determine how far to the right or left of 0 our point is.



The point is two units to the right of zero.

But all three points that touch the dotted line are two units to the right of zero. We don't have enough information to determine the unique location of our point.

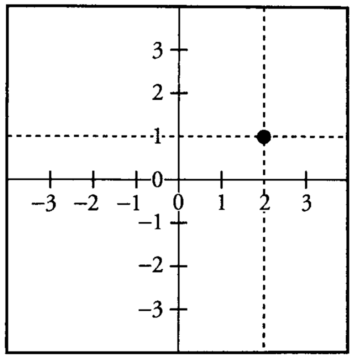
In order to know the location of our point, we also need to know how far up or down the dotted line we need to go. To determine how far up or down we need to go, we're going to need another number line. This number line, however, is going to be vertical. Using this vertical number line, we will be able to measure how far above or below 0 a point is.



The point is one unit above zero.

Notice that this number line by itself also does not provide enough information to determine the unique location of the point.

But, if we combine the information from the two number lines, we can determine both how far left or right *and* how far up or down the point is.



The point is 2 units to the right of 0, because a page has two dimensions.

AND

The point is 1 unit above 0.

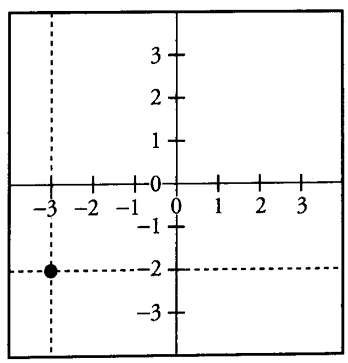
Now we have a unique description of the point's position. There is only one point on the page that is BOTH 2 units to the right of 0 AND 1 unit above 0. So, on a page, we need two numbers to indicate position.

Just as with the number line, information can travel in either direction. If we know the two numbers that give the location, we can place that point on the page.

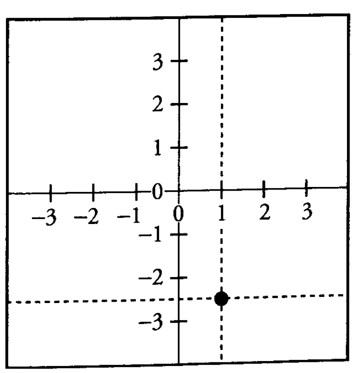
The point is 3 units to the left of 0.

AND

The point is 2 units below 0.



If, on the other hand, we see a point on the page, we can identify its location and determine the two numbers.



The point is 1 unit to the right of 0.

AND

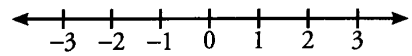
The point is 2.5 units below 0.

Now that we have two pieces of information for each point, we need to keep straight which number is which. In other words, we need to know which number gives the left-right position and which number gives the up-down position.

To represent the difference, we use some technical terms:

The **x-coordinate** is the left right-number.

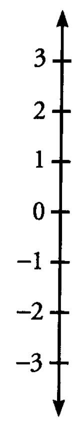
Numbers to the right of 0 are positive.
Numbers to the left of 0 are negative.



This number line is the ***x*-axis.**

The ***y*-coordinate** is the up-down number.

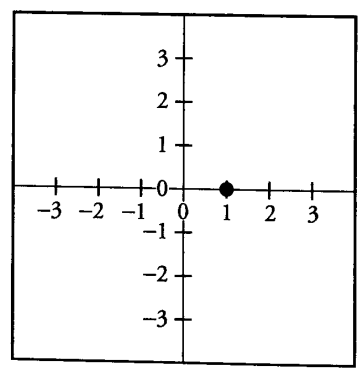
Numbers above 0 are positive.
Numbers below 0 are negative.



This number line is the ***y*-axis.**

Now, when describing the location of a point, we can use the technical terms.

The x-coordinate of the point is 1 and the y-coordinate of the point is 0.

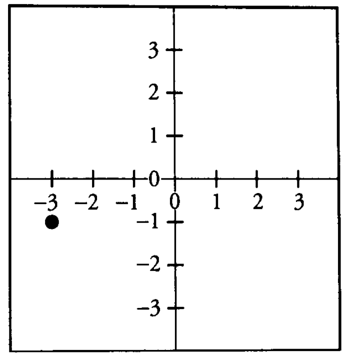


We can condense this and say that, for this point, $x = 1$ and $y = 0$. In fact, we can go even further. We can say that the point is at (1, 0). This shorthand always has the same basic layout. The first number in the parentheses is the x-coordinate, and the second number is the y-coordinate. One easy way to remember this is that x comes before y in the alphabet.

The point is at (−3, −1),

OR

the point has an x-coordinate of −3 and a y-coordinate of −1.



Now we have a fully functioning **coordinate plane:** an x-axis and a y-axis drawn on a page. The coordinate plane allows us to determine the unique position of any point on a **plane** (essentially, a really big and flat sheet of paper).

And in case you were ever curious about what one-dimensional and two-dimensional mean, now you know. A line is one dimensional, because you only need *one* number to identify a point's location. A plane is two-dimensional because you need *two* numbers to identify a point's location.

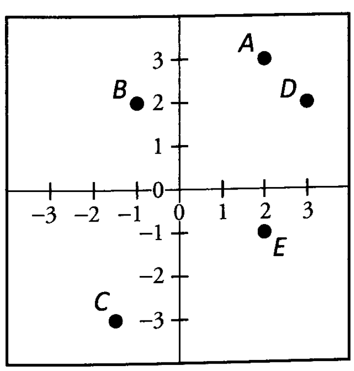
Check Your Skills

1. Draw a coordinate plane and plot the following points:

   1. (3, 1)   2. (−2, 3.5)   3. (0, −4.5)   4. (1, 0)

2. Which point on the coordinate plane below is indicated by the following coordinates?

   1. (2, −1)   2. (−1.5, −3)   3. (−1, 2)   4. (3, 2)

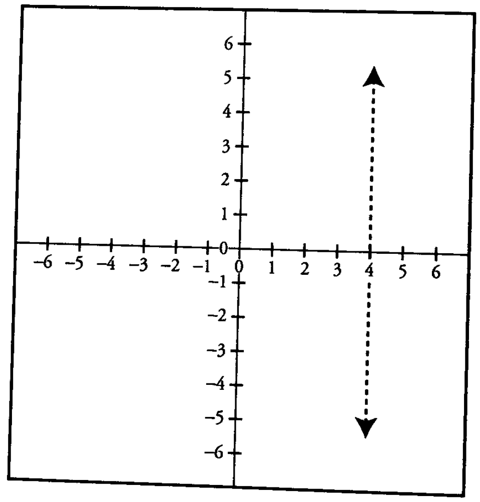


Answers can be found on page 151.

Knowing Just One Coordinate

As we've just seen, we need to know both the x-coordinate and the y-coordinate to plot a point exactly on the coordinate plane. If we only know one coordinate, we can't tell precisely where the point is, but we can narrow down the possibilities.

Consider this situation. Let's say that this is all we know: the point is 4 units to the right of 0.

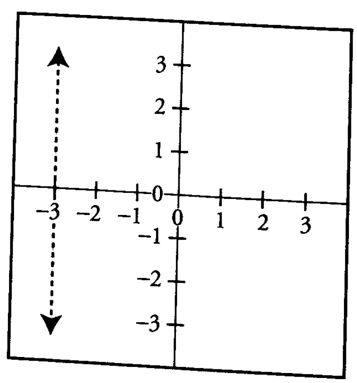


As we saw earlier, any point along the vertical dotted line is 4 units to the right of 0. In other words, every point on the dotted line has an x-coordinate of 4. We could shorten that and say $x = 4$. We don't know anything about the y-coordinate, which could be any number. All the points along the dotted line have different y-coordinates but the same x-coordinate, which equals 4.

So, if we know that $x = 4$, then our point can be anywhere along a vertical line that crosses the x-axis at (4, 0). Let's try with another example.

If we know that $x = -3$...

Then we know

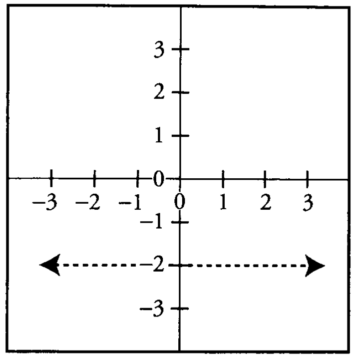


Every point on the dotted line has an x-coordinate of -3.

Points on the dotted line include (–3, 1), (–3, –7), (–3, 100) and so on. In general, if we know the x-coordinate of a point and not the y-coordinate, then all we can say about the point is that it lies on a vertical line.

The x-coordinate still indicates left-right position. If we fix that position but not the up-down position, then the point can only move up and down—forming a vertical line.

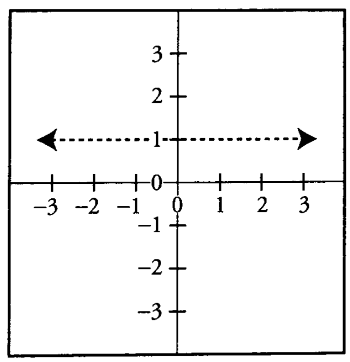
Now imagine that all we know is the y-coordinate of a number. Let's say we know that $y = -2$. How could we represent this on the coordinate plane? In other words, what are all the points for which $y = -2$?



Every point 2 units below 0 fits this condition. These points form a horizontal line. We don't know anything about the x-coordinate, which could be any number. All the points along the horizontal dotted line have different x-coordinates but the same y-coordinate, which equals −2. For instance, (−3, −2), (−2, −2), (50, −2) are all on the line.

Let's try another example. If we know that $y = 1$...

Then we know



Every point on the dotted line has an y-coordinate of 1.

If we know the y-coordinate but not the x-coordinate, then we know the point lies somewhere on a horizontal line.

Check Your Skills

Draw a coordinate plane and plot the following lines.

3. $x = 6$

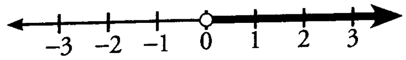
4. $y = -2$

5. $x = 0$

Answers can be found on pages 151–152.

Knowing Ranges

Now let's provide even less information. Instead of knowing the actual x-coordinate, let's see what happens if all we know is a range of possible values for x. What do we do if all we know is that $x > 0$? To answer that, let's return to the number line for a moment. As we saw earlier, if $x > 0$, then the target is anywhere to the right of 0.



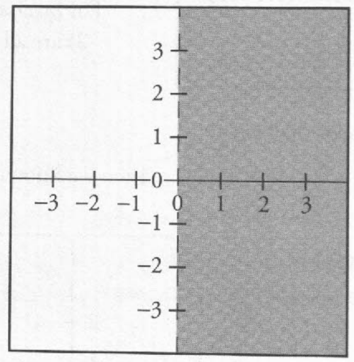
$x > 0$

Now let's look at the coordinate plane. All we know is that x is greater than 0. And we don't know *anything* about y, which could be any number.

How do we show all the possible points? We can shade in part of the coordinate plane: the part to the right of 0.

If we know that $x > 0$...

Then we know:

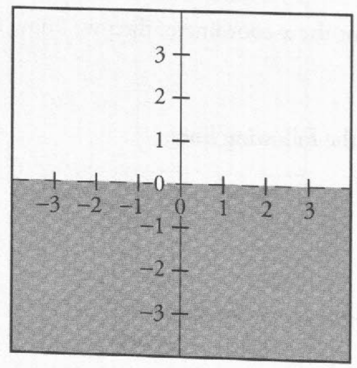


Every point in the shaded region has an x-coordinate greater than 0.

Now let's say that all we know is $y < 0$. Then we can shade in the bottom half of the coordinate plane—where the y-coordinate is less than 0. The x-coordinate can be anything. Notice that the dashed line indicates that x cannot be zero. It must be below the dashed line.

If we know that $y < 0$...

Then we know:

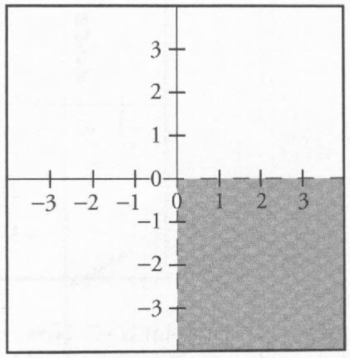


Every point in the shaded region has a y-coordinate less than 0.

Finally, if we know information about both x and y, then we can narrow down the shaded region.

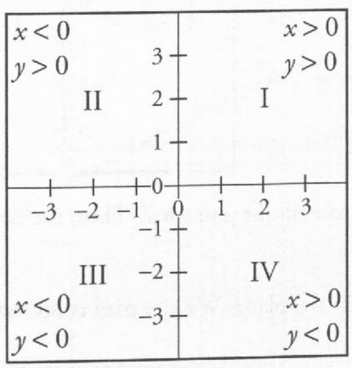
If we know that $x > 0$ AND $y < 0$...

Then we know:



The only place where x is greater than 0 AND y is less than 0 is the bottom right quarter of the plane. So we know that the point lies somewhere in the bottom right quarter of the coordinate plane.

The four quarters of the coordinate plane are called **quadrants**. Each quadrant corresponds to a different combination of signs of x and y. The quadrants are always numbered as shown below, starting on the top right and going counter-clockwise.



Check Your Skills

6. Which quadrant do the following points lie in?

1. (1, −2) 2. (−4.6, 7) 3. (−1, −2.5) 4. (3, 3)

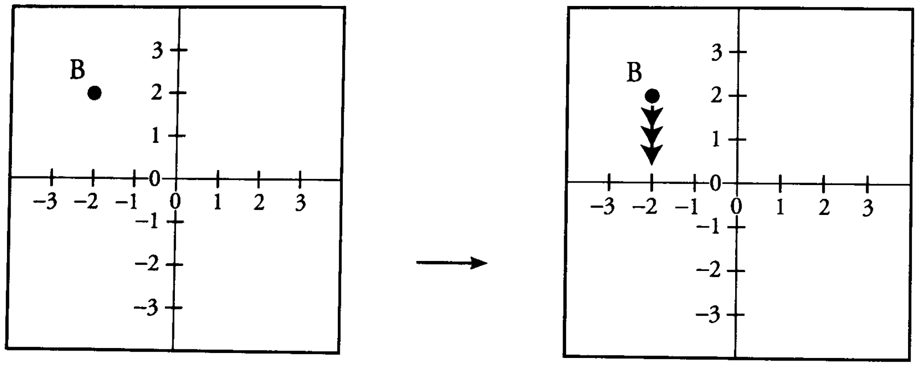
7. Which quadrant or quadrants are indicated by the following?

1. $x < 0, y > 0$ 2. $x < 0, y < 0$ 3. $y > 0$ 4. $x < 0$

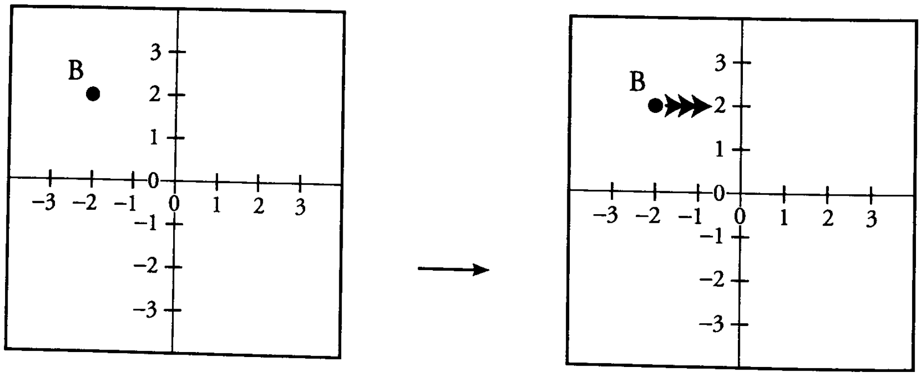
Answers can be found on page 152.

Reading a Graph

If we see a point on a coordinate plane, we can read off its coordinates as follows. To find an x-coordinate, drop an imaginary line down to the x-axis (if the point is above the x-axis) or draw a line up to the x-axis (if the point is below the x-axis) and read off the number.



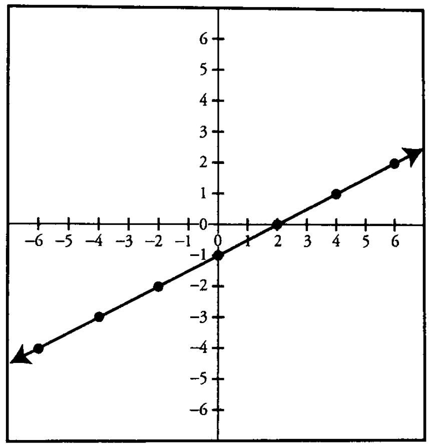
The line hit the *x*-axis at −2, which means the *x*-coordinate of our point is −2. Now, to find the *y*-coordinate, we employ a similar technique, only now we draw a horizontal line instead of a vertical line.



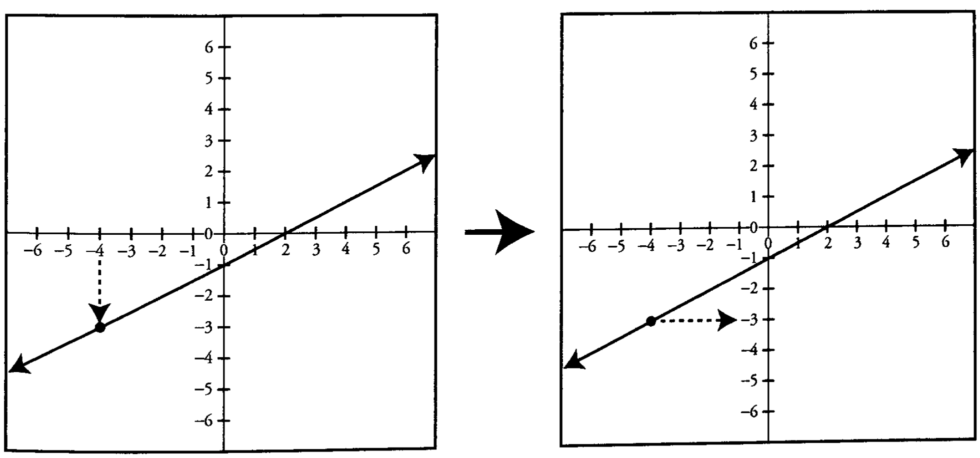
Our line touched the *y*-axis at 2, which means the *y*-coordinate of our point is 2. Thus, the coordinates of point B are (−2, 2).

Now suppose that we know the target is on a slanted line in the plane. We can read coordinates off of this slanted line. Try this problem on your own first.

> On the line shown, what is the *y*-coordinate of the point that has an *x*-coordinate of −4?



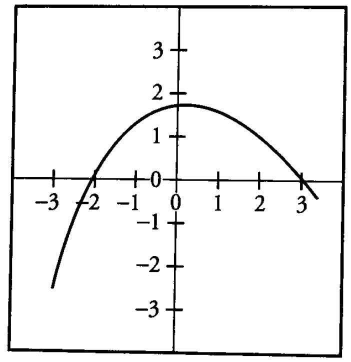
To answer this question, think about reading the coordinates of a point. We went from the point to the axes. Here, we will go from the axis that we know (here, the x-axis) to the line that contains the point, and then to the y-axis (the axis we don't know).



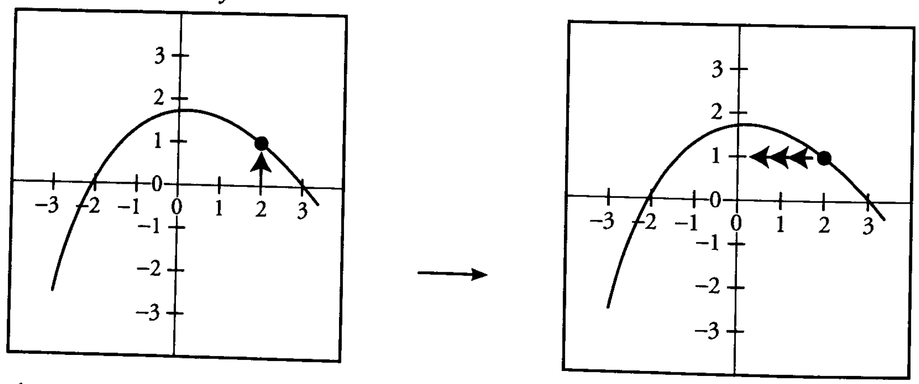
So the point on the line that has an x-coordinate of -4 has a y-coordinate of -3.

This method of locating points applies equally well to any shape or curve you may encounter on a coordinate plane. Try this next problem.

On the curve shown, what is the value of y when $x = 2$?



Once again, we know the *x*-coordinate, so we draw a line from the *x*-axis (where we know the coordinate) to the curve, and then draw a line to the *y*-axis.

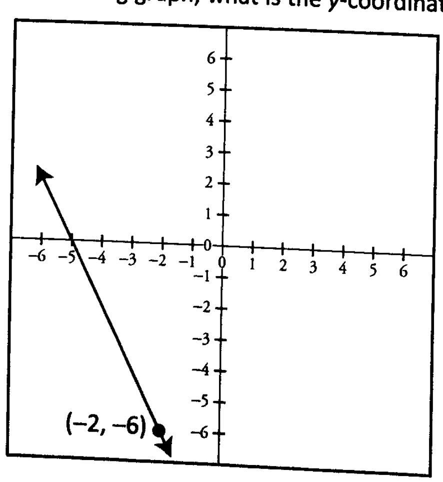


On the curve shown, the point that has an *x*-coordinate of 2 has a *y*-coordinate of 1.

Note that the GRE will mathematically define each line or curve, so you will never be forced to guess visually where a point falls. In fact, if more specific information is <u>not</u> given for a coordinate problem on the GRE, you <u>cannot</u> infer the location of a point based solely on visual cues. This discussion is **only** meant as an exercise to convey how to use any graphical representation.

Check Your Skills

8. On the following graph, what is the *y*-coordinate of the point on the line that has an *x*-coordinate of −3?



The answer can be f

Plotting a Relationship

The most frequent use of the coordinate plane is to display a relationship between x and y. Often, this relationship is expressed this way: if you tell me x, I can tell you y.

As an equation, this sort of relationship looks like this:

y = some expression involving x — Another way of saying this is we have y "in terms of" x

Examples: $y = 2x + 1$ — If you plug a number in for x in any of these equations, you can calculate a value for y.

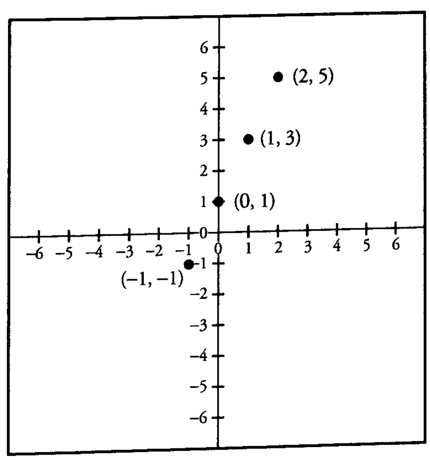
$y = x^2 - 3x + 2$

$$y = \frac{x}{x+2}$$

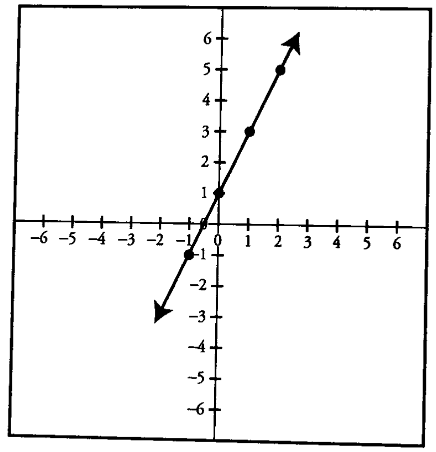
Let's take $y = 2x + 1$. We can generate a set of y's by plugging in various values of x. Start by making a table.

x	$y = 2x + 1$
-1	$y = 2(-1) + 1 = -1$
0	$y = 2(0) + 1 = 1$
1	$y = 2(1) + 1 = 3$
2	$y = 2(2) + 1 = 5$

Now that we have some values, let's see what we can do with them. We can say that when x equals 0, y equals 1. These two values form a pair. We express this connection by plotting the point (0, 1) on the coordinate plane. Similarly, we can plot all the other points that represent an x-y pair from our table:

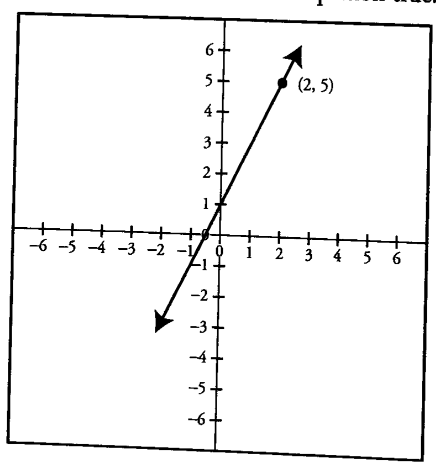


You might notice that these points seem to lie on a straight line. You're right—they do. In fact, any point that we can generate using the relationship $y = 2x + 1$ will also lie on the line.



This line is the graphical representation of $y = 2x + 1$

So now we can talk about equations in visual terms. In fact, that's what lines and curves on the coordinate plane are—they represent all the *x*-*y* pairs that make an equation true. Take a look at the following example:



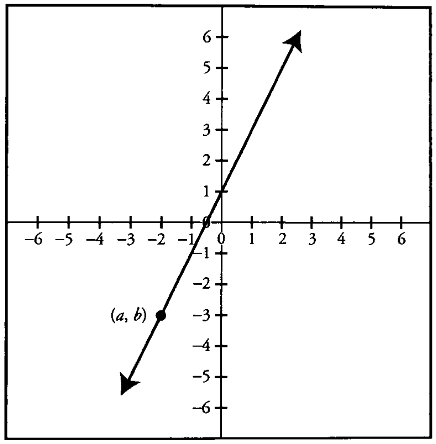
$\longleftrightarrow$

$y = 2x + 1$

$5 = 2(2) + 1$

The point (2, 5) lies on the line $y = 2x + 1$ $\longleftrightarrow$ If we plug in 2 for x in $y = 2x + 1$, we get 5 for y

We can even speak more generally, using variables.



$\longleftrightarrow$

$y = 2x + 1$

$b = 2(a) + 1$

The point (a, b) lies on the line $y = 2x + 1$ $\longleftrightarrow$ If we plug in a for x in $y = 2x + 1$, we get b for y

Check Your Skills

9. True or False? The point (9, 21) is on the line $y = 2x + 1$

10. True or False? The point (4, 14) is on the curve $y = x^2 - 2$

Answers can be found on page 153.

Lines in the Plane

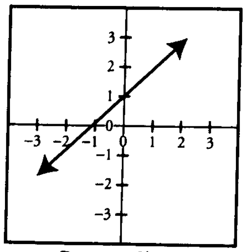
The relationship $y = 2x + 1$ formed a line in the coordinate plane, as we saw. We can actually generalize this relationship. *Any* relationship of the following form represents a line:

$y = mx + b$ — m and b represent numbers (positive or negative)

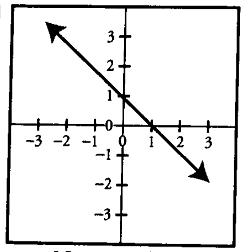
For instance, in the equation $y = 2x + 1$, we can see that $m = 2$ and $b = 1$.

Lines		Not Lines
$y = 3x - 2$	$m = 3,\ b = -2$	$y = x^2$
$y = -x + 4$	$m = -1,\ b = 4$	$y = \frac{1}{x}$
These are called linear equations.		These equations are not linear.

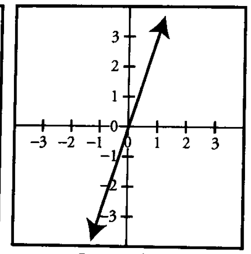
The numbers m and b have special meanings when we are dealing with linear equations. m = **slope.** This tells us how steep the line is and whether the line is rising or falling.



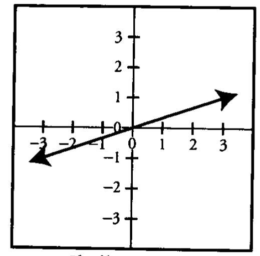
Positive Slope
$m > 0$



Negative Slope
$m < 0$

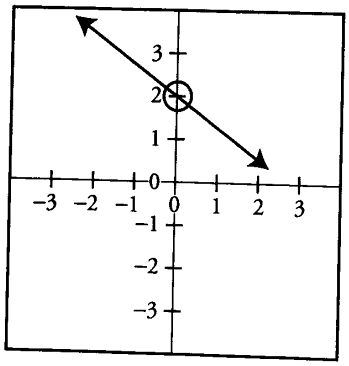


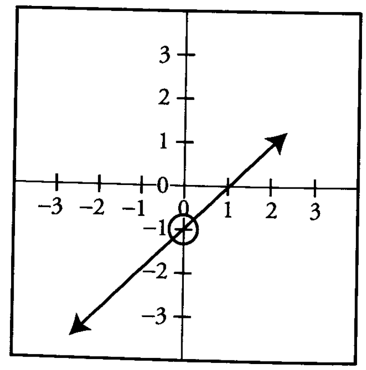
Steep Slope
$m > 1$



Shallow Slope
$0 < m < 1$

***b* = *y*-intercept.** This tells you where the line crosses the *y*-axis. Any line or curve crosses the *y*-axis when $x = 0$. To find the *y*-intercept, plug in 0 for x into the equation.





By recognizing linear equations and identifying m and b, we can plot a line more quickly than by plotting several points on the line.

Check Your Skills

What are the slope and *y*-intercept of the following lines?

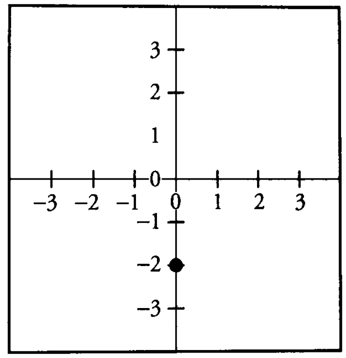
11. $y = 3x + 4$

12. $2y = 5x - 12$

Answers can be found on page 153.

Now the question becomes, how do we use m and b to sketch a line? Let's plot the line $y = \frac{1}{2}x - 2$.

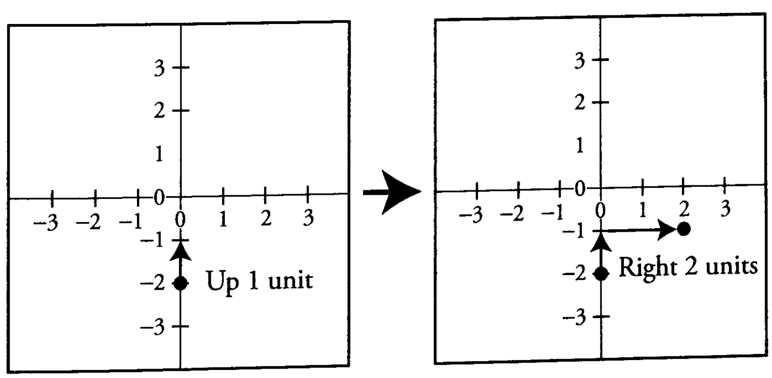
The easiest way to begin graphing a line is to begin with the *y*-intercept. We know that the line crosses the *y*-axis at $y = -2$, so let's begin by plotting that point on our coordinate plane.



Now we need to figure out how to use slope in order to finish drawing our line. Every slope, whether an integer or a fraction, should be thought of as a fraction. In this equation, our m is $\frac{1}{2}$. Let's look at the parts of the fraction and see what they can tell us about our slope.

$$\frac{1}{2} \rightarrow \frac{\text{Numerator}}{\text{Denominator}} \rightarrow \frac{\text{Rise}}{\text{Run}} \rightarrow \frac{\text{Change in } y}{\text{Change in } x}$$

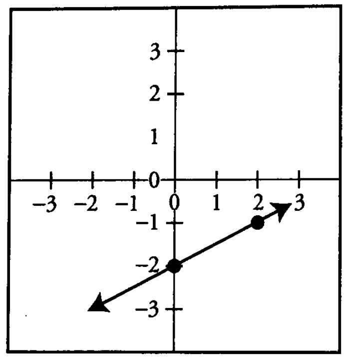
The numerator of our fraction tells us how many units we want to move in the y direction—in other words, how far up or down we want to move. The denominator tells us how many units we want to move in the x direction—in other words, how far left or right we want to move. For this particular equation, the slope is $\frac{1}{2}$, which means we want to move up 1 unit and right 2 units.



After we went up 1 unit and right 2 units, we ended up at the point (2, −1). What that means is that the point (2, −1) is also a solution to the equation $y = \frac{1}{2}x - 2$. In fact, we can plug in the x value and solve for y to check that we did this correctly.

$$y = \frac{1}{2}x - 2 \rightarrow y = \frac{1}{2}(2) - 2 \rightarrow y = 1 - 2 \rightarrow y = -1$$

What this means is that we can use the slope to generate points and draw our line. If we go up another 1 unit and right another 2 units, we will end up with another point that appears on the line. Although we could keep doing this indefinitely, in reality, with only 2 points we can figure out what our line looks like. Now all we need to do is draw the line that connects the 2 points we have, and we're done.

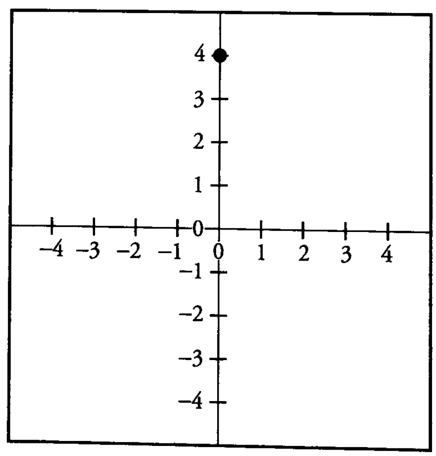


$$y = \frac{1}{2}x - 2$$

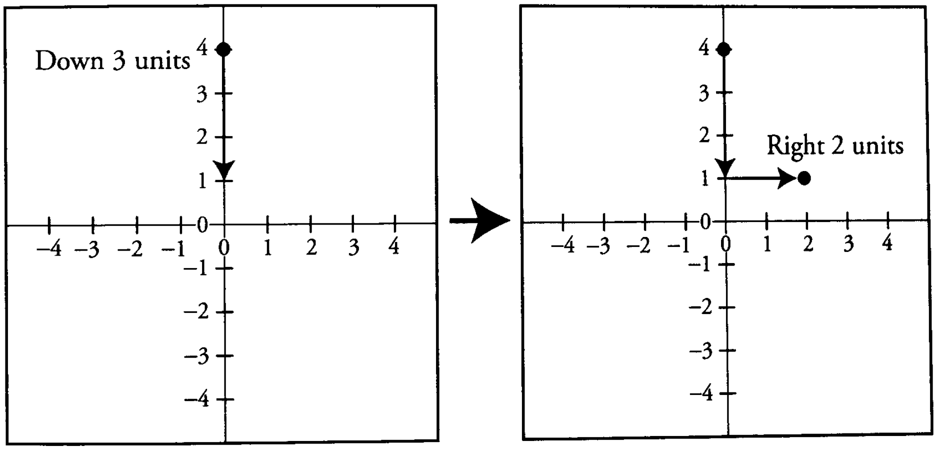
That means that this line is the graphical representation of $y = \frac{1}{2}x - 2$.

Let's try another one. Graph the equation $y = \left(-\frac{3}{2}\right)x + 4$.

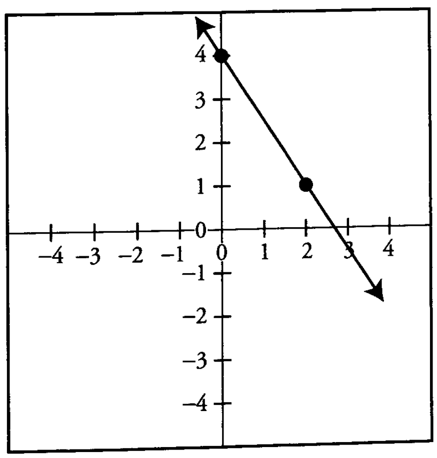
Once again, the best way to start is to plot the y-intercept. In this equation, $b = 4$, so we know the line crosses the y-axis at the point (0, 4):



Now we can use the slope to find a second point. This time, the slope is $-\frac{3}{2}$, which is a negative slope. While positive slopes go up and to the right, negative slopes go down and to the right. Now, to find the next point, we need to go *down* 3 units and right 2 units.



That means that (2, 1) is another point on the line. Now that we have 2 points, we can draw our line.



$$y = \left(-\frac{3}{2}\right)x + 4$$

Check Your Skills

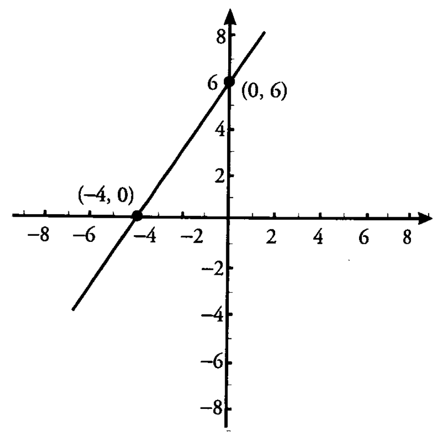
13. Draw a coordinate plane and graph the line $y = 2x - 4$. Identify the slope and the y-intercept.

The answer can be found on page 153.

The Intercepts of a Line

A point where a line intersects a coordinate axis is called an **intercept.** There are two types of intercepts: the x-intercept, where the line intersects the x-axis, and the y-intercept, where the line intersects the y-axis.

The x-intercept is expressed using the ordered pair $(x, 0)$, where x is the point where the line intersects the x-axis. **The x-intercept is the point on the line at which $y = 0$.** In this diagram, the x-intercept is -4, as expressed by the ordered pair $(-4, 0)$.



The y-intercept is expressed using the ordered pair $(0, y)$, where y is the point where the line intersects the y-axis. **The y-intercept is the point on the line at which $x = 0$.** In this diagram, the y-intercept is 6, as expressed by the ordered pair $(0, 6)$.

To find x-intercepts, plug in 0 for y. To find y-intercepts, plug in 0 for x.

Check Your Skills

14. What are the x- and y-intercepts of the equation $x - 2y = 8$?

Answers can be found on page 153–154.

The Intersection of Two Lines

Recall that a line in the coordinate plane is defined by a linear equation relating x and y. That is, if a point (x, y) lies on the line, then those values of x and y satisfy the equation. For instance, the point $(3, 2)$ lies on the line defined by the equation $y = 4x - 10$, since the equation is true when we plug in $x = 3$ and $y = 2$:

$$y = 4x - 10$$
$$2 = 4(3) - 10 = 12 - 10$$
$$2 = 2 \quad \text{TRUE}$$

On the other hand, the point $(7, 5)$ does not lie on that line, because the equation is false when we plug in $x = 7$ and $y = 5$:

$$y = 4x - 10$$
$$5 = 4(7) - 10 = 28 - 10 = 18? \quad \text{FALSE}$$

So, what does it mean when two lines intersect in the coordinate plane? It means that at the point of intersection, BOTH equations representing the lines are true. That is, the pair of numbers (x, y) that represents the point of intersection solves BOTH equations. Finding this point of intersection is equivalent to solving a system of two linear equations. You can find the intersection by using algebra more easily than by graphing the two lines.

At what point does the line represented by $y = 4x - 10$ intersect the line represented by $2x + 3y = 26$?

Since $y = 4x - 10$, replace y in the second equation with $4x - 10$ and solve for x:

$$
\begin{aligned}
2x + 3(4x - 10) &= 26 \\
2x + 12x - 30 &= 26 \\
14x &= 56 \\
x &= 4
\end{aligned}
$$

Now solve for y. You can use either equation, but the first one is more convenient:

$$
\begin{aligned}
y &= 4x - 10 \\
y &= 4(4) - 10 \\
y &= 16 - 10 = 6
\end{aligned}
$$

Thus, the point of intersection of the two lines is (4, 6).

If two lines in a plane do not intersect, then the lines are parallel. If this is the case, there is NO pair of numbers (x, y) that satisfies both equations at the same time.

Two linear equations can represent two lines that intersect at a single point, or they can represent parallel lines that never intersect. There is one other possibility: the two equations might represent the same line. In this case, infinitely many points (x, y) along the line satisfy the two equations (which must actually be the same equation in disguise).

The Distance Between 2 Points

The distance between any two points in the coordinate plane can be calculated by using the Pythagorean Theorem. For example:

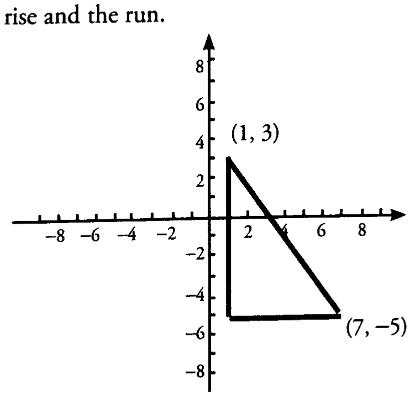
What is the distance between the points (1, 3) and (7, –5)?

(1) Draw a right triangle connecting the points.

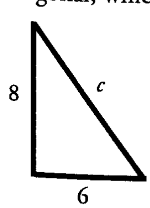
(2) Find the lengths of the two legs of the triangle by calculating the rise and the run.

The y-coordinate changes from 3 to –5, a difference of 8 (the vertical leg).

The x-coordinate changes from 1 to 7, a difference of 6 (the horizontal leg).



(3) Use the Pythagorean Theorem to calculate the length of the diagonal, which is the distance between the points.



$$
\begin{aligned}
6^2 + 8^2 &= c^2 \\
36 + 64 &= c^2 \\
100 &= c^2 \\
c &= 10
\end{aligned}
$$

The distance between the two points is 10 units.

Alternatively, to find the hypotenuse, we might have recognized this triangle as a variation of a 3–4–5 triangle (specifically, a 6–8–10 triangle).

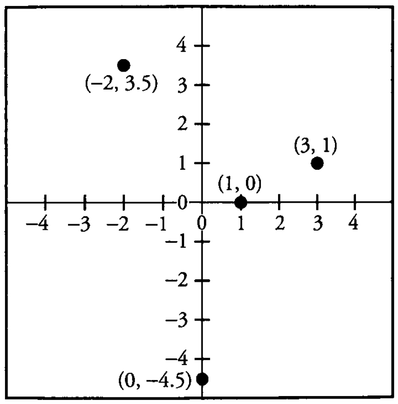
Check Your Skills

15. What is the distance between (−2, −4) and (3, 8)?

Answers can be found on page 154.

Check Your Skills Answers

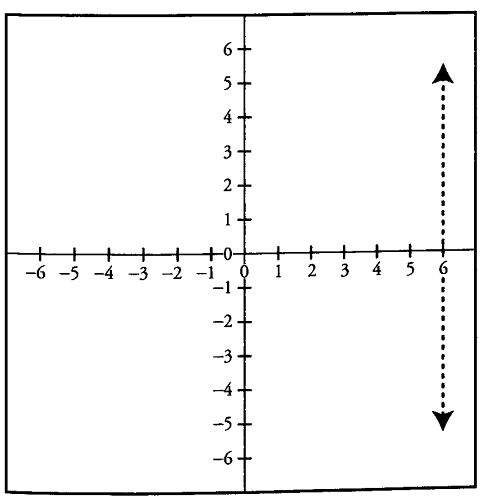
1.



2.

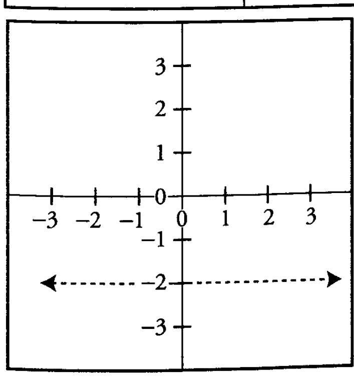
1. (2, −1): **E**
2. (−1.5, −3): **C**
3. (−1, 2): **B**
4. (3, 2): **D**

3.



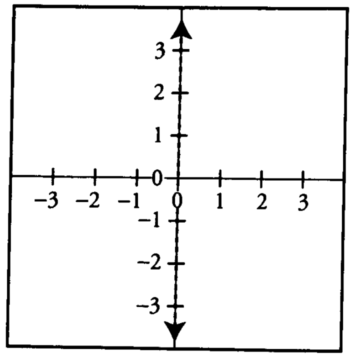
$x = 6$

4.



$y = -2$

5.



$x = 0$ is the y-axis.

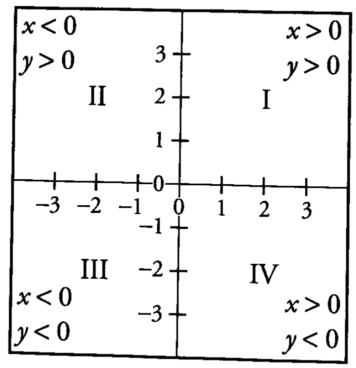
6.

1. (1, −2) is in Quadrant IV

2. (−4.6, 7) is in Quadrant II

3. (−1, −2.5) is in Quadrant III

4. (3, 3) is in Quadrant I



7.

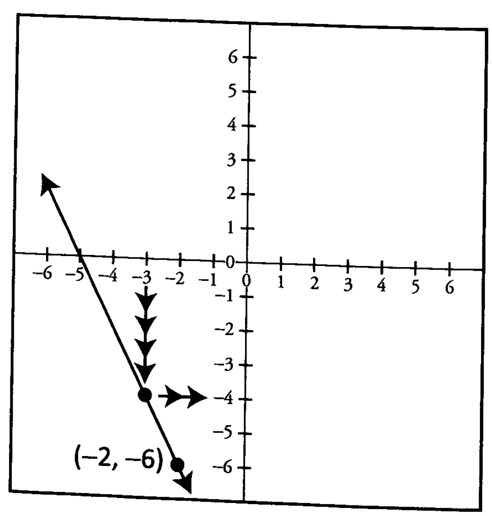
1. $x < 0, y > 0$ indicates Quadrant II

2. $x < 0, y < 0$ indicates Quadrant III

3. $y > 0$ indicates Quadrants I and II

4. $x < 0$ indicates Quadrants II and III

8. The point on the line with $x = -3$ has a y-coordinate of -4.



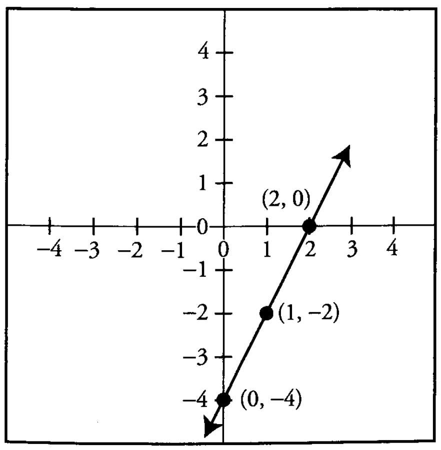
9. **False.** The relationship is $y = 2x + 1$, and the point we are testing is (9, 21). So we plug in 9 for x and see if we get 21 for y. $y = 2(9) + 1 = 19$. The point (9, 21) does not lie on the line.

10. **True.** The relationship is $y = x^2 - 2$, and the point we are testing is (4, 14). So we plug in 4 for x and see if we get 14 for y. $y = (4)^2 - 2 = 14$. The point (4, 14) lies on the curve.

11. **Slope is 3, *y*-intercept is 4.** The equation $y = 3x + 4$ is already in $y = mx + b$ form, so we can directly find the slope and y-intercept. The slope is 3, and the y-intercept is 4.

12. **Slope is 2.5, *y*-intercept is –6.** To find the slope and y-intercept of a line, we need the equation to be in $y = mx + b$ form. We need to divide our original equation by 2 to make that happen. So $2y = 5x - 12$ becomes $y = 2.5x - 6$. So the slope is 2.5 (or 5/2) and the y-intercept is -6.

13.

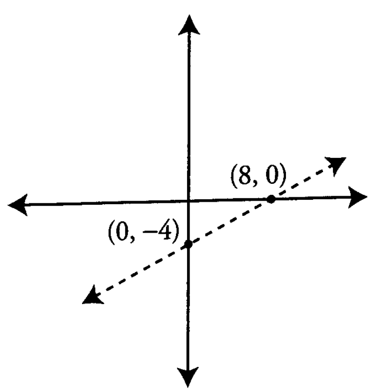


$y = 2x - 4$

slope = 2

y-intercept = –4

14. ***x*-intercept is 8, *y*-intercept is –4:**



We've illustrated the line on the coordinate plane above, but you can also answer this question using algebra.

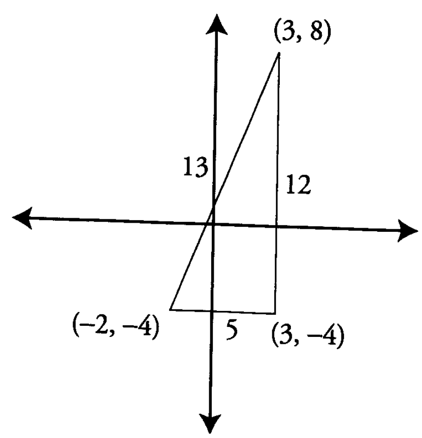
To determine the x-intercept, set y equal to 0, then solve for x:

$$x - 2y = 8$$
$$y = 0$$
$$x - 0 = 8$$
$$x = 8$$

To determine the y-intercept, set x equal to 0, then solve for y:

$$x - 2y = 8$$
$$x = 0$$
$$0 - 2y = 8$$
$$-2y = 8$$
$$y = -4$$

15. **13:** Answer: 13



The illustration above shows the two points. We have constructed a right triangle by finding a point directly below (3, 8) and directly to the right of (−2, −4). This right triangle has legs of 5 (the change from −2 to 3) and 12 (the change from −4 to 8). We can plug those values into the Pythagorean Theorem and solve for the hypotenuse:

$$A^2 + B^2 = C^2$$
$$5^2 + 12^2 = C^2$$
$$25 + 144 = C^2$$
$$C^2 = 169$$
$$C = \sqrt{169} = 13$$

Alternatively, we could recognize the common Pythagorean triplet 5, 12, 13.

Problem Set

1. A line has the equation $y = 3x + 7$. At which point will this line intersect the y-axis?

2. A line has the equation $x = \frac{y}{80} - 20$. At which point will this line intersect the x-axis?

3 A line has the equation $x = -2y + z$. If (3, 2) is a point on the line, what is z?

4. A line is represented by the equation $y = zx + 18$. If this line intersects the x-axis at (–3, 0), what is z?

5. A line has a slope of 1/6 and intersects the x-axis at (–24, 0). Where does this line intersect the y-axis?

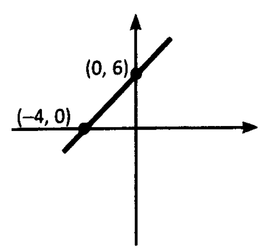
6. Which quadrants, if any, do not contain any points on the line represented by $x - y = 18$?

7. Which quadrants, if any, do not contain any points on the line represented by $x = 10y$?

8. Which quadrants contain points on the line $y = \frac{x}{1{,}000} + 1{,}000{,}000$?

9. Which quadrants contain points on the line represented by $x + 18 = 2y$?

10. What is the equation of the line shown to the right?



11. What is the intersection point of the lines defined by the equations $2x + y = 7$ and $3x - 2y = 21$?

12.

Quantity A	Quantity B
The y-intercept o\f the line $y = \frac{3}{2}x - 2$	The x-intercept of the line $y = \frac{3}{2}x - 2$

13.

Quantity A	Quantity B
The slope of the line $2x + 5y = 10$	The slope of the line $5x + 2y = 10$

14.

Quantity A	Quantity B
The distance between points (0, 9) and (–2, 0)	The distance between points (3, 9) and (10, 3)

1. **(0, 7):** A line intersects the y-axis at the y-intercept. Since this equation is written in slope-intercept form, the y-intercept is easy to identify: 7. Thus, the line intersects the y-axis at the point (0, 7).

2. **(−20, 0):** A line intersects the x-axis at the x-intercept, or when the y-coordinate is equal to zero. Substitute zero for y and solve for x:

$$x = 0 - 20$$
$$x = -20$$

3. **7:** Substitute the coordinates (3, 2) for x and y and solve for z.

$$3 = -2(2) + z$$
$$3 = -4 + z$$
$$z = 7$$

4. **6:** Substitute the coordinates (−3, 0) for x and y and solve for z.

$$0 = z(-3) + 18$$
$$3z = 18$$
$$z = 6$$

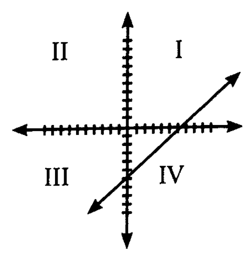
5. **(0, 4):** Use the information given to find the equation of the line:

$$y = \frac{1}{6}x + b$$
$$0 = \frac{1}{6}(-24) + b$$
$$0 = -4 + b$$
$$b = 4$$

The variable b represents the y-intercept. Therefore, the line intersects the y-axis at (0, 4).

6. **II:** First, rewrite the line in slope-intercept form:

$$y = x - 18$$



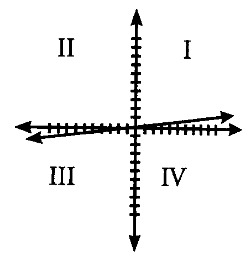
Find the intercepts by setting x to zero and y to zero:

$$y = 0 - 18 \qquad 0 = x - 18$$
$$y = -18 \qquad x = 18$$

Plot the points: (0, −18), and (18, 0). From the sketch, we can see that the line does not pass through quadrant II.

7. **II and IV:** First, rewrite the line in slope-intercept form:

$$y = \frac{x}{10}$$



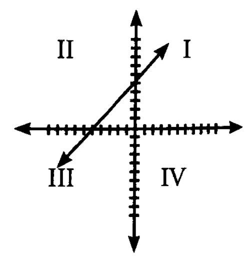
Notice from the equation that the y-intercept of the line is (0,0). This means that the line crosses the y-intercept at the origin, so the x- and y-intercepts are the same. To find another point on the line, substitute any convenient number for x; in this case, 10 would be a convenient, or "smart," number.

$$y = \frac{10}{10} = 1 \qquad \text{The point (10, 1) is on the line.}$$

Plot the points: (0, 0) and (10, 1). From the sketch, we can see that the line does not pass through quadrants II and IV.

8. **I, II, and III:** The line is already written in slope-intercept form:

$$y = \frac{x}{1{,}000} + 1{,}000{,}000$$



Find the intercepts by setting x to zero and y to zero:

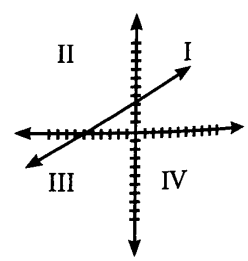
$$0 = \frac{x}{1{,}000} + 1{,}000{,}000 \qquad y = \frac{0}{1{,}000} + 1{,}000{,}000$$

$$x = -1{,}000{,}000{,}000 \qquad y = 1{,}000{,}000$$

Plot the points: (−1,000,000,000, 0) and (0, 1,000,000). From the sketch, we can see that the line passes through quadrants I, II, and III.

9. **I, II, and III:** First, rewrite the line in slope-intercept form:

$$y = \frac{x}{2} + 9$$



Find the intercepts by setting x to zero and y to zero:

$$0 = \frac{x}{2} + 9 \qquad y = \frac{0}{2} + 9$$

$$x = -18 \qquad y = 9$$

Plot the points: (−18, 0) and (0, 9). From the sketch, we can see that the line passes through quadrants I, II, and III.

10. $\boldsymbol{y = \frac{3}{2}x + 6}$**:** First, calculate the slope of the line:

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{6-0}{0-(-4)} = \frac{6}{4} = \frac{3}{2}$$

We can see from the graph that the line crosses the y-axis at (0, 6). The equation of the line is:

$$y = \frac{3}{2}x + 6$$

11. **(5, –3):** To find the coordinates of the point of intersection, solve the system of 2 linear equations. You could turn both equations into slope-intercept form and set them equal to each other, but it is easier is to multiply the first equation by 2 and then add the second equation:

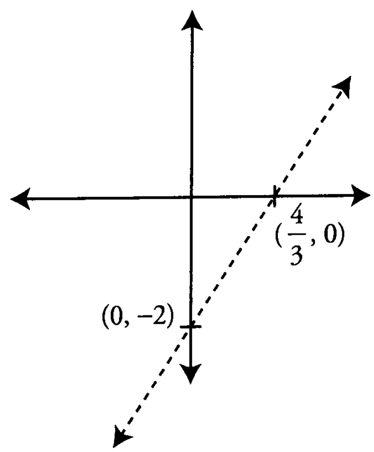
$2x + y = 7$	(first equation)	$7x = 35$	(sum of previous two eqations)
$4x + 2y = 14$	(multiply by 2)	$x = 5$	
$3x - 2y = 21$	(second equation)		

Now plug $x = 5$ into either equation:

$2x + y = 7$	(first equation)	$10 + y = 7$
$2(5) + y = 7$		$y = -3$

Thus, the point (5, –3) is the point of intersection. There is no need to graph the two lines to find the point of intersection.

12. **B:**



We've illustrated the line on the coordinate plane above. Because the equation is already in slope intercept form ($y = mx + b$), you can read the y-intercept directly from the b position, and use the slope to determine the x-intercept. A slope of 3/2 means that the line corresponding to this equation will rise 3 for every 2 that it runs. You don't need to determine the exact x-intercept to see that it is positive, and so greater than –2.

Alternatively, you could set each variable equal to 0, and determine the intercepts.

To determine the y-intercept, set x equal to 0, then solve for y:

$$y = \frac{3}{2}x - 2$$

$$y = \frac{3}{2}(0) - 2$$

$$y = 0 - 2 = -2$$

To determine the x-intercept, set y equal to 0, then solve for x:

$$y = \frac{3}{2}x - 2$$

$$(0) = \frac{3}{2}x - 2$$

$$2 = \frac{3}{2}x$$

$$\frac{4}{3} = x$$

Quantity A	**Quantity B**
The y-intercept of the line	The x-intercept of the line
-2	$\frac{4}{3}$

Therefore **Quantity B is greater.**

13. **A:** The best method would be to put each equation into slope-intercept form ($y = mx + b$), and see which has the greater value for m, which represents the slope. Start with the equation in **Quantity A:**

$$2x + 5y = 10$$
$$5y = -2x + 10$$
$$y = -\frac{2}{5}x + 2$$

Quantity A	**Quantity B**
The slope of the line $2x + 5y = 10$ is $-\frac{2}{5}$	The slope of the line $5x + 2y = 10$

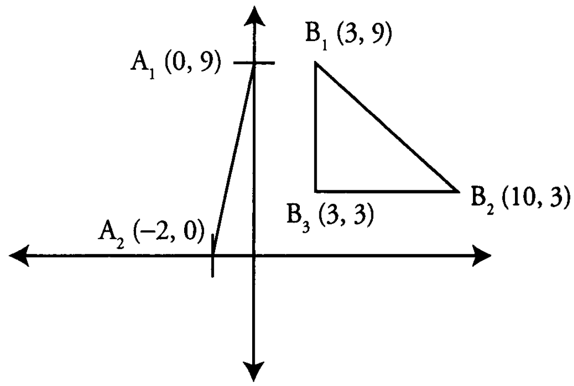
Now find the slope of the equation in **Quantity B:**

$$5x + 2y = 10$$
$$2y = -5x + 10$$
$$y = -\frac{5}{2}x + 5$$

Quantity A	**Quantity B**
−2/5	The slope of the line $5x + 2y = 10$ is $-\frac{5}{2}$

Be careful. Remember that $-\frac{2}{5} > -\frac{5}{2}$. Therefore **Quantity A is larger.**

14. **C:**



The illustration above shows the two points from **Quantity A,** here labeled A_1 and A_2, and the two points from **Quantity B,** here labeled B_1 and B_2. We have constructed a right triangle from the A values by finding a point (0, 0) directly below A_1 and directly to the right of A_2. This right triangle has legs of 2 (the change from −2 to 0) and 9 (the change from 0 to 9). We can plug those values into the Pythagorean Theorem and solve for the hypotenuse:

$A^2 + B^2 = C^2$
$(2)^2 + (9)^2 = C^2$
$4 + 81 = C^2$
$C^2 = 85$
$C = \sqrt{85}$

Quantity A	Quantity B
The distance between points (0, 9) and (−2, 0) = $\sqrt{85}$	The distance between points (3, 9) and (10, 3)

We have constructed a right triangle from the B values by finding a point (3, 3) directly below B_1 and directly to the left of B_2. This right triangle has legs of 7 (the change from 3 to 10) and 6 (the change from 3 to 9). We can plug those values into the Pythagorean Theorem and solve for the hypotenuse:

$A^2 + B^2 = C^2$
$(7)^2 + (6)^2 = C^2$
$49 + 36 = C^2$
$C^2 = 85$
$C = \sqrt{85}$

Quantity A	Quantity B
$\sqrt{85}$	The distance between points (3, 9) and (10, 3) = $\sqrt{85}$

Therefore **the two quantities are equal.**