Circles and Cylinders

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box $\boxed{}$, you are to enter your own answer in the box. For questions followed by a fraction-style numeric entry box $\frac{\boxed{}}{\boxed{}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is $\frac{1}{4}$, you may enter $\frac{25}{100}$ or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations, such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1. A circle has an area of 16π. What is its circumference?

   (A) 4π
   (B) 8π
   (C) 16π
   (D) 32π
   (E) It cannot be determined from the information given.

2. A circle has a circumference of 16. What is its area?

   (A) $\frac{8}{\pi}$
   (B) $\frac{8}{\pi^2}$
   (C) $\frac{64}{\pi}$
   (D) $\frac{64}{\pi^2}$
   (E) 64π

3. A circle has a diameter of 5. What is its area?

(A) $\frac{25\pi}{4}$

(B) $\frac{25\pi}{2}$

(C) $\frac{25\pi^2}{2}$

(D) 10π

(E) 25π

4. A circle's area equals its circumference. What is its radius?

(A) 1
(B) 2
(C) 4
(D) 8
(E) 16

Circle C has a radius r such that $1 < r < 5$.

	Quantity A	Quantity B
5.	The area of circle C	The circumference of circle C

6. A circle has radius 3.5. What is its area?

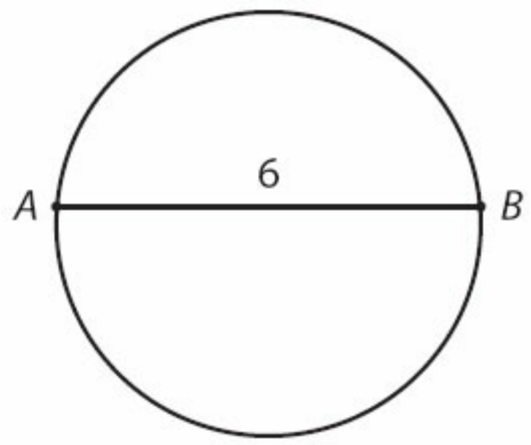
(A) $\frac{7}{2}\pi$

(B) 9.5π

(C) 10.5π

(D) $\frac{49}{4}\pi$

(E) $\frac{49}{2}\pi$



AB is <u>not</u> a diameter of the circle.

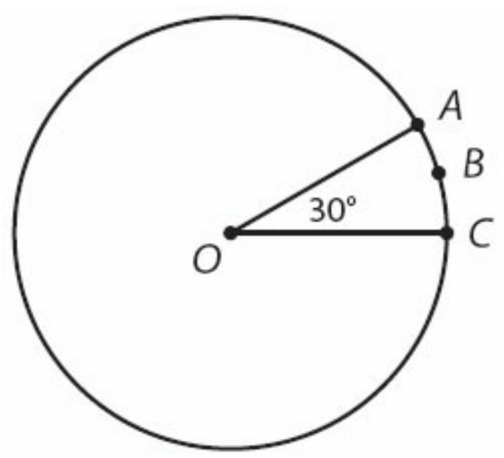
	Quantity A	**Quantity B**
7.	The area of the circle	9π

8. A circle has radius 0.001. What is its area?

   (A) $\pi \times 10^{-2}$
   (B) $\pi \times 10^{-3}$
   (C) $\pi \times 10^{-4}$
   (D) $\pi \times 10^{-6}$
   (E) $\pi \times 10^{-9}$

9. A circle has an area of 4π. If the radius were doubled, the new area of the circle would be how many times the original area?

   (A) 2
   (B) 3
   (C) 4
   (D) 5
   (E) It cannot be determined from the information given.



The radius of the circle with center *O* is 6.

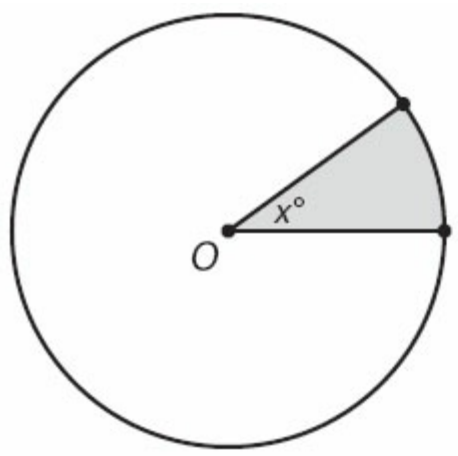
	Quantity A	Quantity B
10.	The length of arc *ABC*	3

11. A sector of a circle has a central angle of 120°. If the circle has a diameter of 12, what is the area of the sector?

(A) 4π
(B) 8π
(C) 12π
(D) 18π
(E) 36π

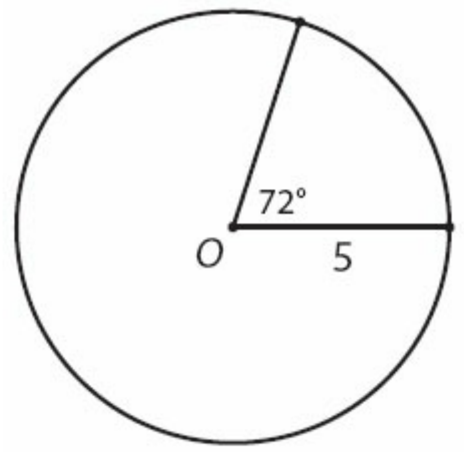
Within a circle with radius 12, a sector has an area of 24π.

	Quantity A	**Quantity B**
12.	The measure of the central angle of the sector, in degrees	90



In the circle with center O, the area of the shaded sector is $\frac{1}{10}$ of the area of the full circle.

	Quantity A	**Quantity B**
13.	$2x$	75



14. If O is the center of the circle, what is the perimeter of the sector with central angle 72°?

(A) $5 + 2\pi$
(B) $10 + 2\pi$
(C) $10 + 4\pi$
(D) $10 + 5\pi$
(E) $20 + 2\pi$

15. A sector of a circle has a radius of 10 and an area of 20π. What is the arc length of the sector?

(A) π
(B) 2π
(C) 4π
(D) 5π
(E) 10π

Sector A and sector B are sectors of two different circles.
Sector A has a radius of 4 and a central angle of 90°.
Sector B has a radius of 6 and a central angle of 45°.

	Quantity A	**Quantity B**
16.	The area of sector A	The area of sector B

17. What is the height of a right circular cylinder with radius 2 and volume 32π?

A right circular cylinder has volume 24π.

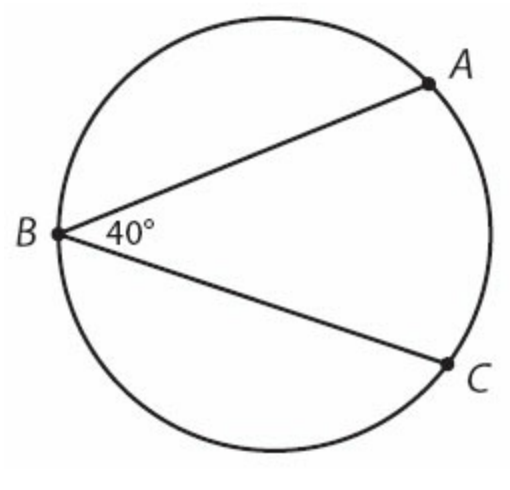
	Quantity A	**Quantity B**
18.	The height of the cylinder	The radius of the cylinder

19. If a half-full 4-inch by 2-inch by 8-inch box of soymilk is poured into a right circular cylindrical glass with radius 2 inches, how many inches high will the soymilk reach? (Assume that the capacity of the glass is greater than the volume of the soymilk.)

(A) 8
(B) 16
(C) $\frac{4}{\pi}$
(D) $\frac{8}{\pi}$
(E) $\frac{16}{\pi}$

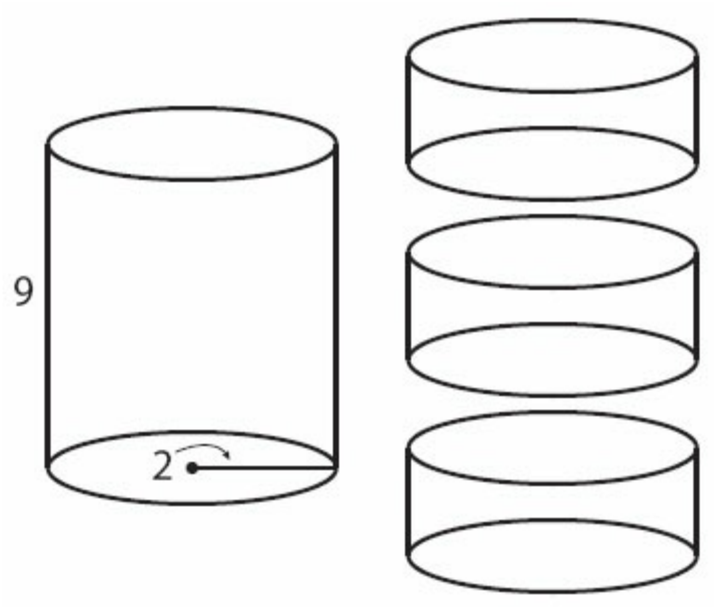
20. If a right circular cylinder's radius is halved and its height doubled, by what percent will the volume increase or decrease?

(A) 50% decrease
(B) 0%
(C) 25% increase
(D) 50% increase
(E) 100% increase



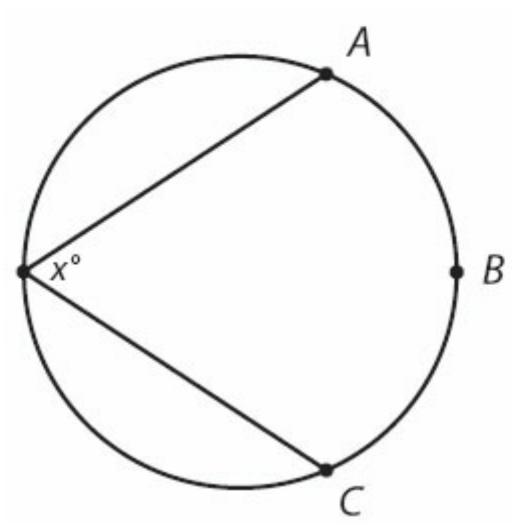
21. If the diameter of the circle is 36, what is the length of arc *ABC*?

(A) 8
(B) 8π
(C) 28π
(D) 32π
(E) 56π



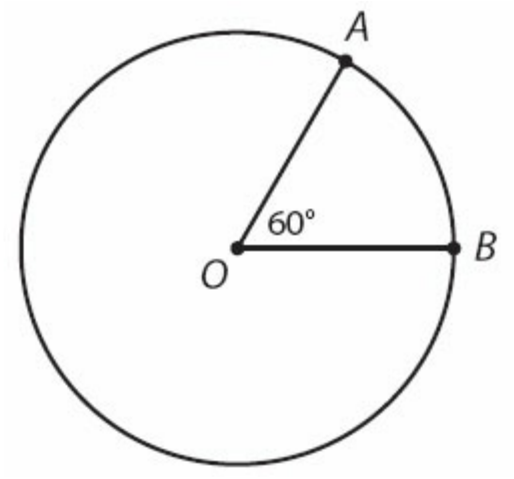
22. If a solid right circular cylinder with height 9 and radius 2 is cut as shown into three new cylinders, each of equal and uniform height, how much new surface area is created?

(A) 4π
(B) 12π
(C) 16π
(D) 24π
(E) 36π



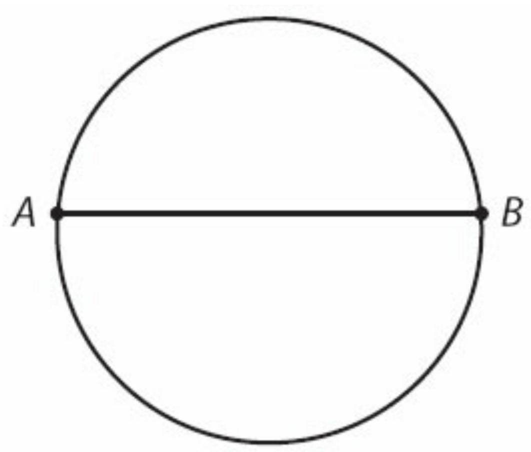
$x > 60°$

	Quantity A	**Quantity B**
23.	The ratio of the length of arc ABC to the circumference of the circle	$\frac{1}{3}$



Point O is the center of the circle above.

	Quantity A	Quantity B
24.	The ratio of the length of minor arc AB to major arc AB	$\frac{1}{6}$



The circle above has area 25.

	Quantity A	Quantity B
25.	The length of chord AB	10

Circles and Cylinders Answers

1. **(B).** Since the area formula for a circle is $A = \pi r^2$:

$$16\pi = \pi r^2$$
$$16 = r^2$$
$$4 = r$$

Since the circumference formula is $C = 2\pi r$ and $r = 4$:

$$C = 2\pi(4)$$
$$C = 8\pi$$

2. **(C).** Since the circumference formula is $C = 2\pi r$:

$$16 = 2\pi r$$

Note that the circumference is just 16, not 16π, so the radius is going to look a bit unusual. Divide both sides by 2π to solve for r:

$$\frac{16}{2\pi} = r$$
$$\frac{8}{\pi} = r$$

Now, plug the radius $\frac{8}{\pi}$ into the area formula for a circle:

$$A = \pi\left(\frac{8}{\pi}\right)^2$$
$$A = \pi \times \frac{64}{\pi^2}$$
$$A = \frac{64}{\pi}$$

3. **(A).** If a circle's diameter is 5, its radius is $\frac{5}{2}$. Plug this into the area formula:

$$A = \pi \left(\frac{5}{2}\right)^2$$

$$A = \pi \times \frac{25}{4}$$

$$A = \frac{25\pi}{4}$$

4. **(B).** To find the radius that would make the area and the circumference of a circle equal, set the area and circumference formulas equal to one another:

$$\pi r^2 = 2\pi r$$

Since both sides have both r and π, divide both sides by πr:

$$r = 2$$

5. **(D).** Picking numbers is the easiest way to prove (D). If the radius is 3, the area is 9π and the circumference is 6π, so Quantity A is greater. If the radius is 4, the area is 16π and the circumference is 8π, so once again Quantity A is greater. But if the radius is 2, both the area and the circumference equal 4π. Therefore, Quantity A is not always greater. Note also that r is not required to be an integer. If you try a radius close to the minimum, such as 1.1, Quantity B would be greater.

6. **(D).** The area formula for a circle is $A = \pi r^2$, so plug the radius in. Since the decimal will be unwieldy, it is easier to plug the fractional version of 3.5 $\left(\text{i.e., } \frac{7}{2}\right)$ into the formula:

$$A = \pi \left(\frac{7}{2}\right)^2$$
$$A = \frac{49\pi}{4}$$

7. **(A).** Since a diameter is the longest straight line you can draw from one point on a circle to another (that is, a diameter is the longest chord in a circle), the actual diameter must be *greater* than 6.

If the diameter were exactly 6, the radius would be 3, and the area would be:

$$A = \pi(3)^2$$
$$A = 9\pi$$

However, since the diameter must actually be greater than 6, the area must be greater than 9π. Do *not* make the mistake of picking (D) for Quantitative Comparison geometry questions in which you cannot "solve." There is often still a way to determine which quantity is greater.

8. **(D).** The formula for the area of a circle is $A = \pi r^2$, so plug radius 0.001 into the formula. However, since the answers are in exponential form, it would be easier to first convert 0.001 to 1×10^{-3}, or just 10^{-3}, and use that in the formula:

$$A = \pi(10^{-3})^2$$
$$A = \pi(10^{-6})$$

9. **(C).** To begin, find the original radius of the circle: Area = $\pi r^2 = 4\pi$, so $r = 2$. Once doubled, the new radius is 4. A circle with a radius of 4 has an area of 16π. The new area of 16π is 4 times the old area of 4π.

10. **(A).** If the sector has a central angle of 30°, then it is $\frac{1}{12}$ of the circle, because $\frac{30}{360} = \frac{1}{12}$. To find the arc length of the sector, first find the circumference of the entire circle. The radius of the circle is 6, so the circumference is $2\pi(6) = 12\pi$. That means that the arc length of the sector is $\frac{1}{12}(12\pi) = \pi$. Since π is about 3.14, Quantity A is greater.

11. **(C).** The sector is $\frac{1}{3}$ of the circle, because $\frac{120}{360} = \frac{1}{3}$. To find the area of the sector, first find the area of the whole circle. The diameter of the circle is 12, so the radius of the circle is 6, and the area is $\pi(6)^2 = 36\pi$. That means the area of the sector is $\frac{1}{3}(36\pi) = 12\pi$.

12. **(B).** First find the area of the whole circle. The radius is 12, which means the area is $\pi(12)^2 = 144\pi$. Since the sector has an area of 24π and $\frac{24\pi}{144\pi} = \frac{1}{6}$, the sector is $\frac{1}{6}$ of the entire circle. That means that the central angle is $\frac{1}{6}$ of 360, or 60°. Quantity B is greater.

13. **(B).** If the area of the sector is $\frac{1}{10}$ of the area of the full circle, then the central angle is $\frac{1}{10}$ of the degree measure of the full circle, or $\frac{1}{10}$ of $360° = 36° = x°$. Thus, Quantity A = 2(36) = 72, so Quantity B is greater.

14. **(B).** To find the perimeter of a sector, first find the radius of the circle and the arc length of the sector. Begin by determining what fraction of the circle the sector is. The central angle of the sector is 72°, so the sector is $\frac{72}{360} = \frac{1}{5}$ of the circle. The radius is 5, so the circumference of the circle is $2\pi(5) = 10\pi$. The arc length of the sector is $\frac{1}{5}$ of the circumference: $\frac{1}{5}(10\pi) = 2\pi$. The perimeter of the sector is this 2π plus the two radii that make up the straight parts of the sector: $10 + 2\pi$.

15. **(C).** Compare the given area of the sector to the calculated area of the whole circle. The radius of the circle is 10, so the area of the whole circle is $\pi(10)^2 = 100\pi$. The area of the sector is 20π, or $\frac{20\pi}{100\pi} = \frac{1}{5}$ of the circle. The radius is 10, so the circumference of the whole circle is $2\pi(10) = 20\pi$.

Since the sector is $\frac{1}{5}$ of the circle, the arc length is $\left(\frac{1}{5}\right)(20\pi) = 4\pi$.

16. **(B).** Sector A is $\frac{90}{360} = \frac{1}{4}$ of the circle with radius 4. The area of this circle is $\pi(4)^2 = 16\pi$, so the area of sector A is $\frac{1}{4}$ of 16π, or 4π.

Sector B is $\frac{45}{360} = \frac{1}{8}$ of the circle with radius 6. The area of this circle is $\pi(6)^2 = 36\pi$, so the area of sector B is $\frac{1}{8}$ of 36π, or 4.5π.

Since 4.5π is greater than 4π, Quantity B is greater.

17. **8.** Use the formula for the volume of a right circular cylinder, $V = \pi r^2 h$:

$$\begin{aligned} 32\pi &= \pi(2)^2 h \\ 32 &= 4h \\ 8 &= h \end{aligned}$$

18. **(D).** Plugging into the formula for volume of a right circular cylinder, $V = 24\pi = \pi r^2 h$. However, there are many combinations of r and h that would make the volume 24π. For instance, $r = 1$ and $h = 24$, or $r = 4$ and $h = 1.5$. Keep in mind that the radius and height don't even have to be integers, so there truly are an infinite number of possibilities, some for which h is greater and some for which r is greater.

19. **(D).** A box is a rectangular solid whose volume formula is V = length × width × height. Thus, the volume of the box is 4 inches × 2 inches × 8 inches = 64 inches^3. Since the box is half full, there are 32 inches^3 of soymilk. This volume will not change when the soymilk is poured from the box into the cylinder. The formula for the volume of a cylinder is $V = \pi r^2 h$, so:

$$32 = \pi(2)^2 h, \text{ where } r \text{ and } h \text{ are in units of inches.}$$

$$\frac{32}{4\pi} = h$$

$$\frac{8}{\pi} = h$$

The height is $\frac{8}{\pi}$ inches. Note that the height is "weird" (divided by π) because the volume of the cylinder did *not* have a π.

20. **(A).** According to the formula for the volume of a right circular cylinder, the original volume is $V = \pi r^2 h$. To halve the radius, replace r with $\frac{r}{2}$. To double the height, replace h with $2h$. The only caveat: be sure to use parentheses!

$$V = \pi\left(\frac{r}{2}\right)^2 (2h) = \frac{2\pi r^2 h}{2^2} = \frac{\pi r^2 h}{2}$$

Thus, the volume, which was once $\pi r^2 h$, is now $\frac{\pi r^2 h}{2}$. In other words, it has been cut in half, or reduced by 50%.

Alternatively, plug in numbers. If the cylinder originally had radius 2 and height 1, the volume would be $V = \pi(2)^2(1) = 4\pi$. If the radius were halved to become 1 and the height were doubled to become 2, the volume would be $V = \pi(1)^2(2) = 2\pi$. Again, the volume is cut in half, or reduced by 50%.

21. **(C).** Note that a *minor* arc is the "short way around" the circle from one point to another, and a *major* arc is the "long way around." Arc *ABC* is thus the same as major arc *AC*.

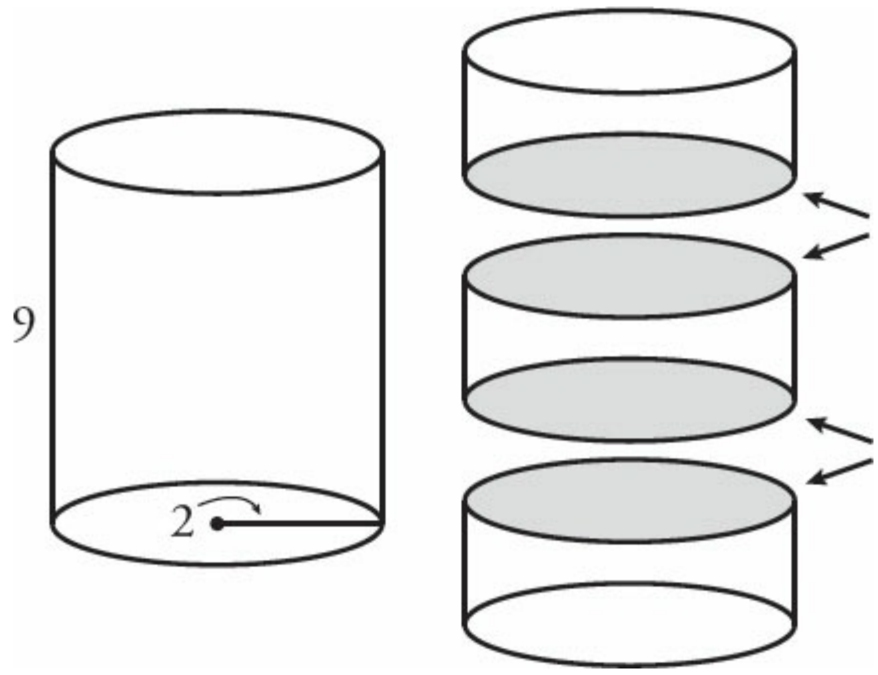
For a given arc, an inscribed angle is always half the central angle, which would be 80° in this case. The minor arc *AC* is thus $\frac{80}{360} = \frac{2}{9}$ of the circle. Since the circumference is 36π:

$$\text{minor arc } AC = \frac{2}{9}(36\pi) = 8\pi$$

Arc *ABC*, or major arc *AC*, is the entire circumference minus the minor arc:

$36\pi - 8\pi = 28\pi$

22. **(C).** One method is to find the surface area of the large cylinder, then the surface areas of the three new cylinders, then subtract the surface area of the large cylinder from the combined surface areas of the three new cylinders. However, there is a much faster way. When the large cylinder is cut into three smaller ones, only a few *new* surfaces are created—the bottom base of the top cylinder, the top and bottom bases of the middle cylinder, and the top surface of the bottom cylinder.



Thus, these four circular bases represent the new surface area created. Since the radius of each base is 2, use the area formula for a circle, $A = \pi r^2$:

$$A = \pi(2)^2$$
$$A = 4\pi$$

Since there are 4 such bases, multiply by 4 to get 16π.

23. **(A).** If $x°$ were equal to 60°, arc *ABC* would have a central angle of 120°. (Inscribed angles, with the vertex at the far side of the circle, are always half the central angle.) A 120° arc is $\frac{120}{360} = \frac{1}{3}$ of the circumference of the circle. Since x is actually greater than 60°, the arc is actually greater than $\frac{1}{3}$ of the circumference. Thus, the ratio of the arc length to the circumference is greater than $\frac{1}{3}$.

24. **(A).** Since the angle that determines the arc is equal to 60 and $\frac{60}{360} = \frac{1}{6}$, minor arc *AB* is $\frac{1}{6}$ of the circumference of the circle. (There are always 360° in a circle. Minor arc *AB* is the "short way around" from *A* to *B*, while major arc *AB* is the "long way around.")

Since minor arc *AB* is $\frac{1}{6}$ of the circumference, major arc *AB* must be the other $\frac{5}{6}$. Therefore, the ratio of the minor arc to the major arc is 1 to 5 (*not* 1 to 6). You could calculate this as $\frac{\frac{1}{6}}{\frac{5}{6}} = \frac{1}{6} \times \frac{6}{5} = \frac{1}{5}$, or you could just reason the ratio of 1 of *anything* (such as sixths) to 5 of the same thing (again, sixths) is a 1 to 5 ratio.

The trap answer here is (C). This is a common mistake: $\frac{1}{6}$ of the total is not the same as a 1 to 6 ratio of two parts.

25. **(B).** The equation for the area of a circle is $A = \pi r^2$. Note that the given area is just 25, *not* 25π! So:

$$\begin{aligned} \pi r^2 &= 25 \\ r^2 &= \frac{25}{\pi} \approx 8 \\ r &= \text{a bit less than 3.} \end{aligned}$$

So the diameter of the circle is a bit less than 6. The diameter is the chord with maximum length, so wherever *AB* is on this circle, it's significantly shorter than 10.