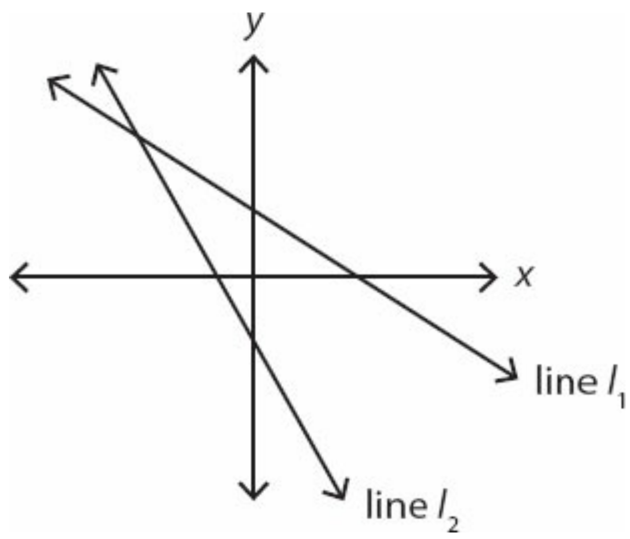
Coordinate Geometry

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

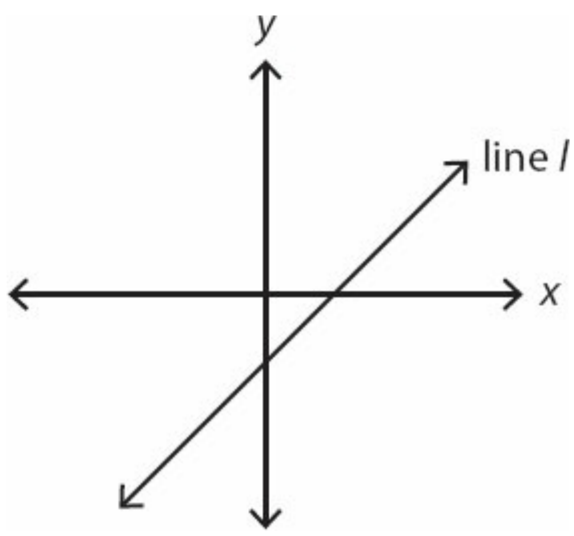
(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box $\boxed{\quad\quad}$, you are to enter your own answer in the box. For questions followed by a fraction-style numeric entry box $\frac{\boxed{\quad\quad}}{\boxed{\quad\quad}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is $\frac{1}{4}$, you may enter $\frac{25}{100}$ or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations, such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

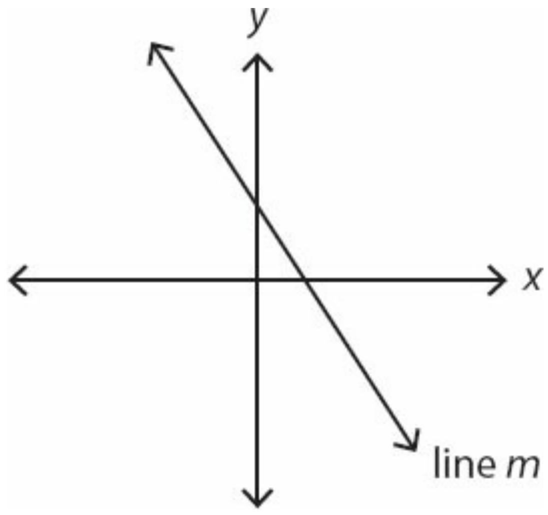


	Quantity A	**Quantity B**
1.	The slope of line l_1	The slope of line l_2



2. If the figure above is drawn to scale, which of the following could be the equation of line l?

(A) $y = 4x + 4$
(B) $y = 4x - 4$
(C) $y = x - 6$
(D) $y = x + \frac{1}{2}$
(E) $y = -x - 3$



3. If the figure above is drawn to scale, which of the following could be the equation of line m?

(A) $6y + 6x = 7$
(B) $3y = -4x - 3$
(C) $5y + 10 = -4x$
(D) $y = 2$
(E) $x = -2$

4. What is the slope of a line that passes through the points (–4, 5) and (1, 2)?

(A) $-\frac{3}{5}$
(B) –1
(C) $-\frac{5}{3}$
(D) $-\frac{7}{3}$
(E) –3

5. Which of the following could be the slope of a line that passes through the point (–2, –3) and crosses the y-axis above the origin?

Indicate all such slopes.

- [ ] $-\frac{2}{3}$
- [ ] $\frac{3}{7}$
- [ ] $\frac{3}{2}$
- [ ] $\frac{5}{3}$
- [ ] $\frac{9}{4}$
- [ ] 4

6. If a line has a slope of –2 and passes through the points (4, 9) and (6, y), what is the value of y?

$\boxed{}$

7. What is the distance between the points (–2, –2) and (4, 6)?

(A) 6
(B) 7
(C) 8
(D) 10
(E) $8\sqrt{2}$

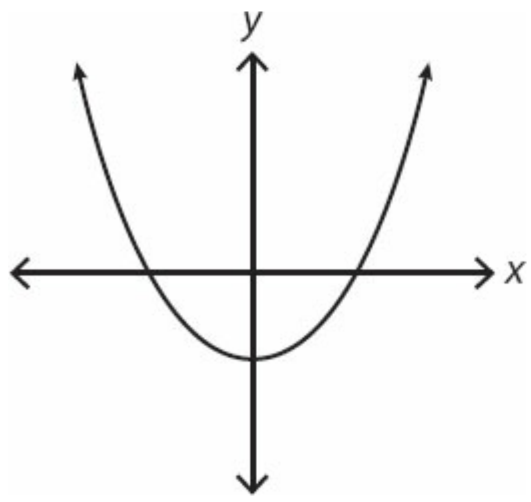
8. Which of the following points in the coordinate plane lies on the line $y = 2x - 8$?

Indicate <u>all</u> such points.

- ☐ (3, –2)
- ☐ (–8, 0)
- ☐ $(\frac{1}{2}, -7)$

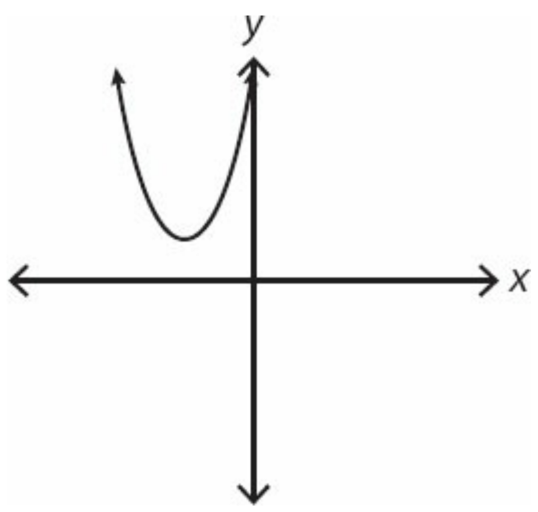
9. Which of the following points in the coordinate plane does <u>not</u> lie on the curve $y = x^2 - 3$?

(A) (3, 6)
(B) (–3, 6)
(C) (0, –3)
(D) (–3, 0)
(E) (0.5, –2.75)



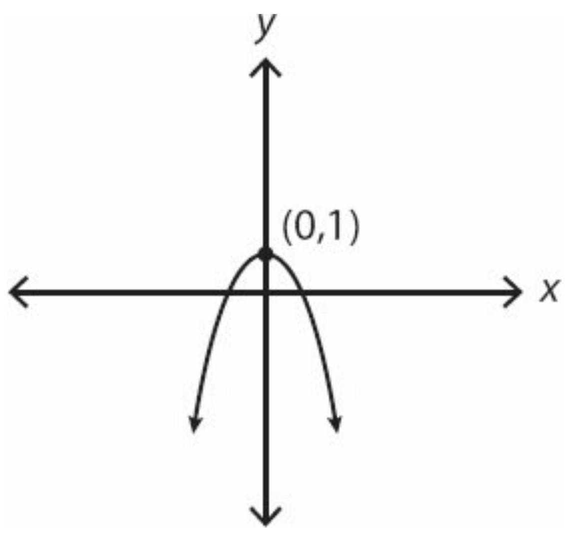
10. Which of the following could be the equation of the figure above?

(A) $y = x - 3$
(B) $y = 2x^2 - x$
(C) $y = x^2 - 3$
(D) $y = x^2 + 3$
(E) $y = x^3 - 3$



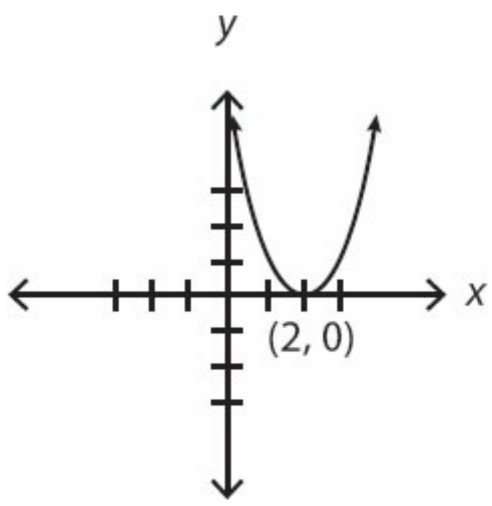
11. Which of the following could be the equation of the parabola in the coordinate plane above?

(A) $y = x^2 + 3$
(B) $y = (x - 3)^2 + 3$
(C) $y = (x + 3)^2 - 3$
(D) $y = (x - 3)^2 - 3$
(E) $y = (x + 3)^2 + 3$



12. Which of the following could be the equation of the parabola in the coordinate plane above?

(A) $y = -x - 1$
(B) $y = x^2 + 1$
(C) $y = -x^2 - 1$
(D) $y = -x^2 + 1$
(E) $y = -(x - 1)^2$

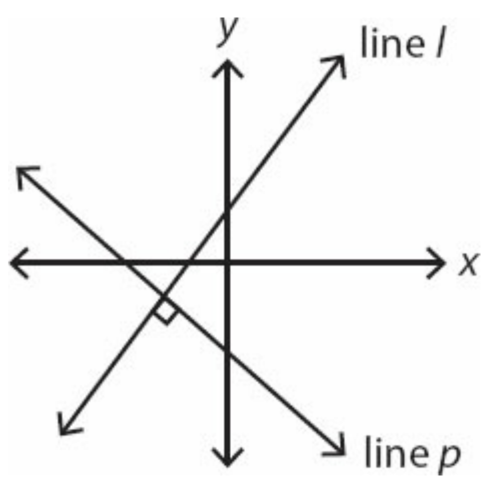


13. If the equation of the parabola in the coordinate plane above is $y = (x - h)^2 + k$ and $(-3, n)$ is a point on the parabola, what is the value of n?

In the coordinate plane, the equation of line p is $3y - 9x = 9$.

	Quantity A	**Quantity B**
14.	The slope of line p	The x-intercept of line p

15. In the xy-coordinate plane, lines j and k intersect at point $(1, 3)$. If the equation of line j is $y = ax + 10$, and the equation of k is $y = bx + a$, where a and b are constants, what is the value of b?

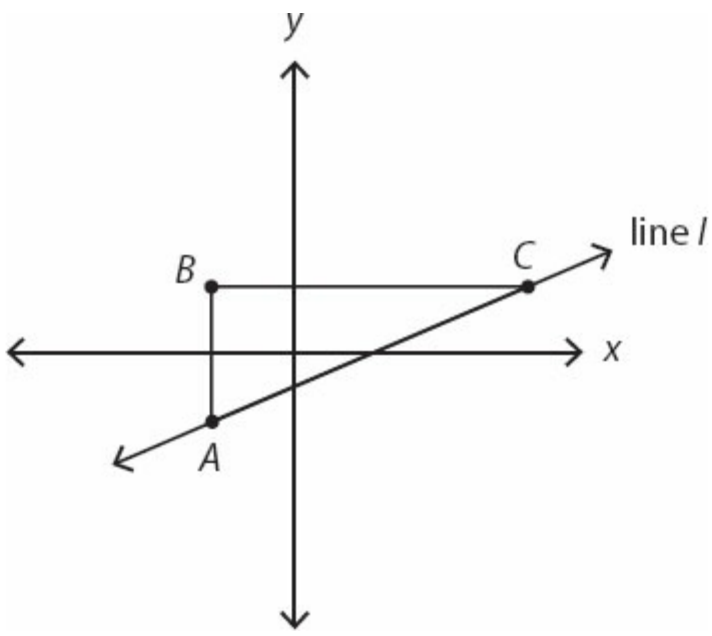


The slope of line l is greater than 1.

	Quantity A	**Quantity B**
16.	The slope of line p	-1

Lines l_1 and l_2 are parallel, and their respective slopes sum to less than 1.

	Quantity A	Quantity B
17.	The slope of a line perpendicular to lines l_1 and l_2	$-\frac{1}{2}$



18. In the coordinate system above, the slope of line l is $\frac{1}{3}$ and the length of line segment BC is 4, how long is line segment AB?

(A) $\frac{3}{4}$
(B) $\frac{4}{3}$
(C) 3
(D) 4
(E) 12

19. What is the area of a triangle with vertices (–2, 4), (2, 4) and (–6, 6) in the coordinate plane?

Lines m and n are perpendicular, neither line is vertical, and line m passes through the origin.

	Quantity A	Quantity B
20.	The product of the slopes of lines m and n	The product of the x-intercepts of lines m and n

In the coordinate plane, points (a, b) and (c, d) are equidistant from the origin.

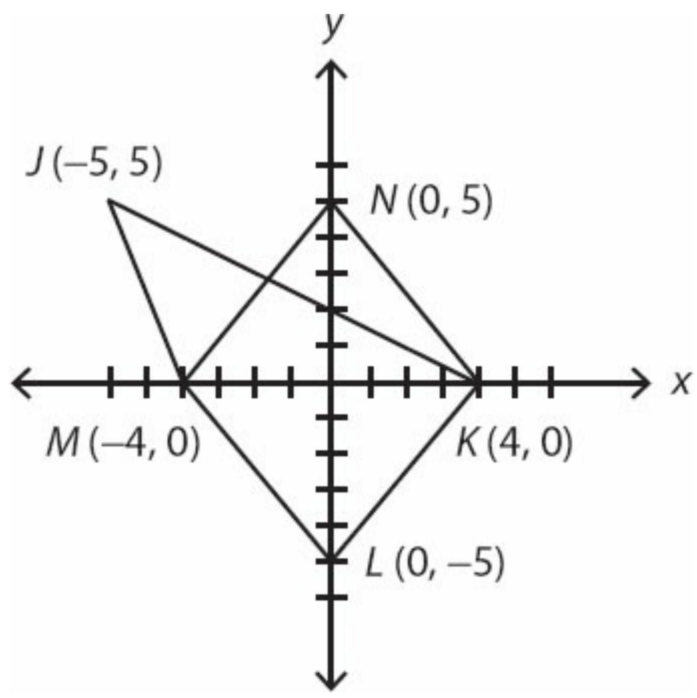
$$|a| > |c|$$

	Quantity A	**Quantity B**
21.	$\lvert b \rvert$	$\lvert d \rvert$

In the coordinate plane, lines j and k are parallel and the product of their slopes is positive.

The x-intercept of line j is greater than the x-intercept of line k.

	Quantity A	**Quantity B**
22.	The y-intercept of line j	The y-intercept of line k



	Quantity A	**Quantity B**
23.	The area of parallelogram *KLMN*	The area of quadrilateral *JKLM*

24. Which of the following could be the equation of a line parallel to the line $3x + 2y = 8$?

(A) $y = \frac{2}{3}x + 7$

(B) $y = -\frac{2}{3}x + 7$

(C) $y = \frac{3}{2}x + 7$

(D) $y = -\frac{3}{2}x + 7$

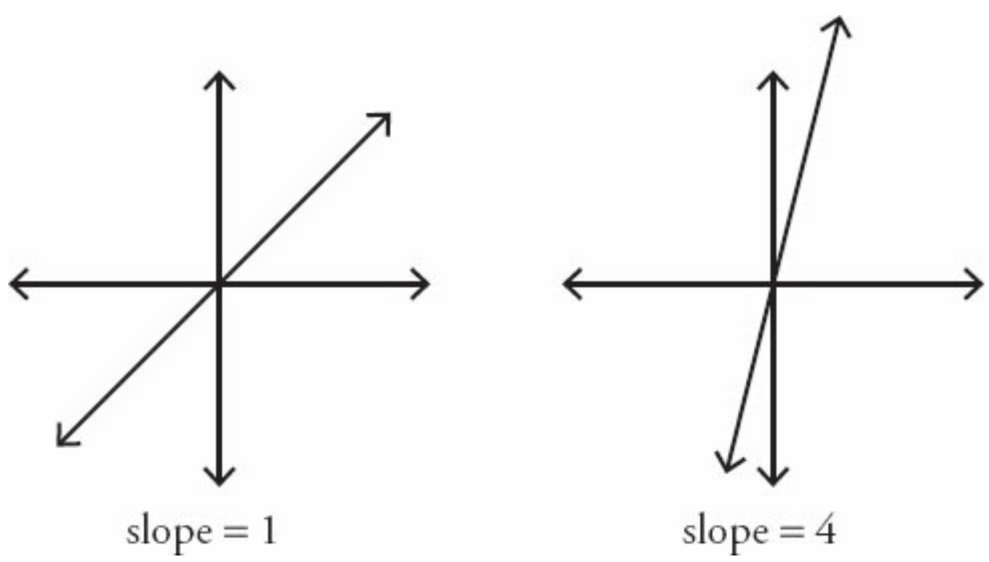
(E) $y = \frac{3}{2}x - 7$

Coordinate Geometry Answers

1. **(A).** Both slopes are negative (pointing down when reading from left to right), and line l_2 is steeper than line l_1. Thus, the slope of l_2 has a greater *absolute value*. But since the values are both negative, the slope of l_1 is a greater number. For instance, the slope of l_1 could be –1 and the slope of l_2 could be –2. Whatever the actual numbers are, the slope of l_1 is closer to 0 and therefore greater.

2. **(C).** Since there are no numbers on the graph, the exact equation of the line cannot be determined, but the line has a positive slope (it points upward when reading from left to right) and a negative y-intercept (it crosses the y-axis below the origin). All of the answers are already in slope-intercept form ($y = mx + b$, where m = slope and b = y-intercept). Choices (A), (B), (C), and (D) have positive slope. Of those, only choices (B) and (C) have a negative y-intercept.

Now, is the slope closer to positive 4 or positive 1? A slope of 1 makes 45° angles when it cuts through the x and y axes, and this figure looks very much like it represents a slope of 1. A slope of 4 would look much steeper than this picture. Note that xy-planes are drawn to scale on the GRE, and units on the x-axis and on the y-axis are the same, unless otherwise noted.



The correct answer is (C). Note that the GRE would only give questions in which the answers are far enough apart that you can determine the intended answer.

3. **(A).** Since there are no numbers on the graph, the exact equation of the line cannot be determined, but the line has a negative slope (it points down when reading from left to right) and a positive y-intercept (it crosses the y-axis above the origin).

Change the answers to slope-intercept form ($y = mx + b$, where m = slope and b = y-intercept). First note that (D) and (E) cannot be the answers—choice (D) represents a horizontal line crossing through (0, 2), and choice (E) represents a vertical line passing through (–2, 0).

Choice (A):

$$\begin{aligned} 6y + 6x &= 7 \\ 6y &= -6x + 7 \\ y &= -x + \frac{7}{6} \end{aligned}$$

This line, choice (A), has a slope of –1 and y-intercept of $\frac{7}{6}$.

Choice (B):

$$3y = -4x - 3$$

$$y = -\frac{4}{3}x - 1y$$

This line, choice (B), has a slope of $-\frac{4}{3}$ and y-intercept of –1.

Choice (C):

$$5y + 10 = -4x$$

$$5y = -4x - 10$$

$$y = -(-\frac{4}{5})x - 2$$

This line, choice (C), has a slope of $-\frac{4}{5}$ and y-intercept of –2.

The only choice with a negative slope and a positive y-intercept is choice (A).

4. **(A).** The slope formula is $m = \frac{y_2 - y_1}{x_2 - x_1}$. It doesn't matter which point is first; just be consistent. Using (–4, 5) as x_1 and y_1 and (1, 2) as x_2 and y_2:

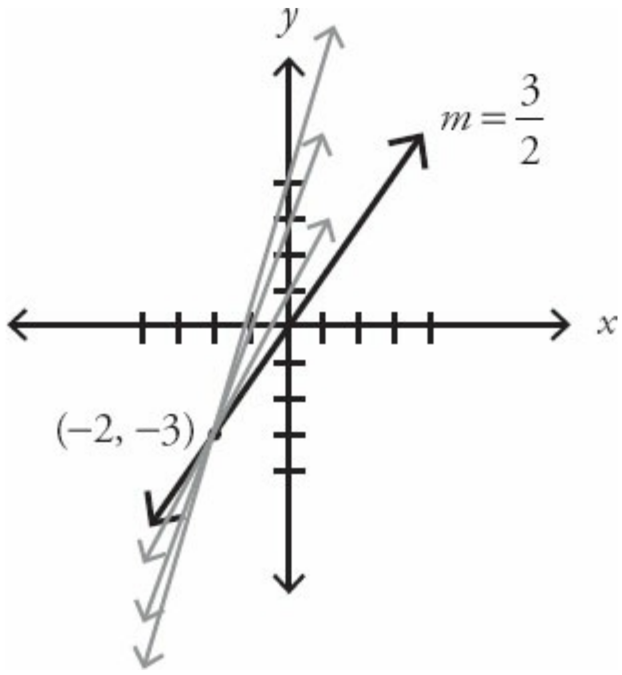
$$m = \frac{2-5}{1-(-4)} = -\frac{3}{5}$$

5. **$\frac{5}{3}$, $\frac{9}{4}$, and 4 only.** The line must hit a point on the y-axis above (0, 0). That means the line could include (0, 0.1), (0, 25), or even (0, 0.00000001). Since the y-intercept could get very, very close to (0, 0), use the point (0, 0) to calculate the slope—and then reason that since the line can't *actually* go through (0, 0), the slope will actually have to be steeper than that.

The slope formula is $m = \frac{y_2 - y_1}{x_2 - x_1}$. Using (0, 0) as x_1 and y_1 and (–2, –3) as x_2 and y_2 (you can make either pair of points x_1 and y_1, so make whatever choice is most convenient):

$$m = \frac{-3-0}{-2-0} = \frac{-3}{-2} = \frac{3}{2}$$

Since the slope is positive and the line referenced in the problem needs to hit the x-axis above (0, 0), the slope of that line will have to be greater than $\frac{3}{2}$, as in the gray lines below:



Select all answers with a slope greater than $\frac{3}{2}$. Thus, only $\frac{5}{3}$, $\frac{9}{4}$, and 4 are correct.

6. **5.** The slope formula is $m = \frac{y_2 - y_1}{x_2 - x_1}$. Using (4, 9) as x_1 and y_1 and (6, y) as x_2 and y_2, and plugging in –2 for the slope:

$$-2 = \frac{y-9}{6-4}$$

$$-2 = \frac{y-9}{2}$$

$$-4 = y - 9$$

$$5 = y$$

7. **(D).** Use the distance formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$:

$$d = \sqrt{(4-(-2))^2 + (6-(-2))^2}$$

$$d = \sqrt{(6)^2 + (8)^2}$$

$$d = \sqrt{36+64}$$

$d = \sqrt{100}$

$d = 10$

Alternatively, recognize the Pythagorean triple 6–8–10.

8. **(3, –2) and ($\frac{1}{2}$, –7) only.** For the point (3, –2) to lie on the line $y = 2x - 8$, y needs to equal –2 when 3 is plugged in for x:

$y = 2(3) - 8$
$y = 6 - 8 = -2$

Since y does equal –2 when x equals 3, the point (3, –2) does lie on the line. However, when –8 is plugged in for x, y does not equal 0, so (–8, 0) is not a point on the line. When $\frac{1}{2}$ is plugged in for x, y equals –7, so point $\left(\frac{1}{2}, -7\right)$, lies on the line.

9. **(D).** The problem asks for the point that does <u>not</u> lie on the curve. $y = x^2 - 3$ is the equation of a parabola, but you don't need to know that fact in order to answer this question. For each choice, plug in the coordinates for x and y. For instance, try choice (A):

$$6 = (3)^2 - 3$$
$$6 = 6$$

Since this is a true statement, choice (A) lies on the curve. The only choice that yields a false statement when plugged in is choice (D), the correct answer.

For the point (–3, 0) to lie on the curve $y = x^2 - 3$, y needs to equal 0 when –3 is plugged in for x:

$$y = (-3)^2 - 3$$
$$y = 9 - 3 = 6$$

y does not equal 0 when x equals –3, so the point does not lie on the curve.

10. **(C).** The graph is of a parabola, so its equation must be in the general form of $y = ax^2 + bx + c$. That eliminates choices (A) and (E). Of the remaining answer choices, only answer choice (C) gives a negative y value when $x = 0$ is plugged in. Also, it should be noted that when a parabola lacks a bx term, that is $b = 0$, it will be centered around the y-axis, just as this graph is.

11. **(E).** The standard equation of a parabola in vertex form is $y = a(x - h)^2 + k$, where the vertex is (h, k). Here is the vertex of the parabola described by each answer choice:

(A)	(0, 3)	On the axis
(B)	(3, 3)	Incorrect quadrant
(C)	(–3, –3)	Incorrect quadrant
(D)	(3, –3)	Incorrect quadrant
(E)	(–3, 3)	Correct

Only choice (E) places the vertex in the correct quadrant.

12. **(D).** The standard equation of a parabola in vertex form is $y = a(x - h)^2 + k$, where the vertex is (h, k). Eliminate choice (A), as it is not the equation of a parabola. Here is the vertex of the parabola described by each remaining answer choice:

(B) $(0, 1)$ Correct

(C) $(0, -1)$ Incorrect

(D) $(0, 1)$ Correct

(E) $(1, 0)$ Incorrect

Both (B) and (D) have the correct vertex. However, choice (B) describes a parabola pointing upward from that vertex, because the x^2 term is positive. The negative in front of choice (D) indicates a parabola pointing downward from that vertex.

13. **25.** The equation of the given parabola is $y = (x - h)^2 + k$. The standard equation of a parabola in vertex form is $y = a(x - h)^2 + k$, where the vertex is (h, k). (Since the equation of this particular parabola does not have constant a, a must be equal to 1.)

Using $y = (x - h)^2 + k$ and the vertex (2, 0) shown in the graph:

$$y = (x - 2)^2 + 0$$
$$y = (x - 2)^2$$

Since (–3, n) is a point on the parabola, plug in –3 and n for x and y, respectively:

$$n = (-3 - 2)^2$$
$$n = (-5)^2$$
$$n = 25$$

14. **(A).** In slope intercept form ($y = mx + b$, where m is the slope and b is the y-intercept):

$$\begin{aligned} 3y - 9x &= 9 \\ 3y &= 9x + 9 \\ y &= 3x + 3 \end{aligned}$$

The slope is 3. The y-intercept is also 3, but the problem asks for the x-intercept. To get an x-intercept, substitute 0 for y:

$$0 = 3x + 3$$
$$-3 = 3x$$
$$-1 = x$$

Thus, the slope is 3 and the x-intercept is –1. Quantity A is greater.

15. **10.** If lines k and m intersect at the point (1, 3), then 1 can be plugged in for x and 4 plugged in for y in either line equation.

For line j:

$$y = ax + 10$$
$$3 = a(1) + 10$$
$$-7 = a$$

For line k, plug in not only $x = 1$ and $y = 3$, but also the fact that $a = -7$:

$$y = bx + a$$
$$3 = (b)(1) + (-7)$$
$$10 = b$$

16. **(A).** If the slope of line l is greater than 1 and line p is perpendicular (because of the right angle symbol on the figure), then line p has a negative slope between –1 and 0, because perpendicular lines have negative reciprocal slopes—that is, the product of the two slopes is –1.

Try a few examples to better illustrate this: line l could have a slope of 2, in which case line p would have a slope of $-\frac{1}{2}$. Line l could have a slope of $\frac{3}{2}$, in which case line p would have a slope of $-\frac{2}{3}$. Or line l could have a slope of 100, in which case line p would have a slope of $-\frac{1}{100}$.

All of these values ($-\frac{1}{2}$, $-\frac{2}{3}$, and $-\frac{1}{100}$) are greater than –1. This will work with any example you try. Since line l has a slope greater than 1, line p has a slope with an absolute value less than 1. Because that value will also be negative, it will always be the case that $-1 <$ the slope of line $p < 0$.

17. **(D).** Since lines l_1 and l_2 are parallel, they have the same slope. Call that slope m. Since the slopes sum to less than 1:

$$m + m < 1$$
$$2m < 1$$
$$m < \frac{1}{2}$$

Thus, lines l_1 and l_2 each have the same slope that is less than $\frac{1}{2}$. A line perpendicular to those lines would have a negative reciprocal slope. However, there isn't much more you can do here. Lines l_1 and l_2 could have slopes of $\frac{1}{4}$ (in which case a perpendicular line would have slope = –4) or slopes of –100 (in which case a perpendicular line would have slope = $\frac{1}{100}$). Thus, the slope of the perpendicular line could be less than or greater than $-\frac{1}{2}$.

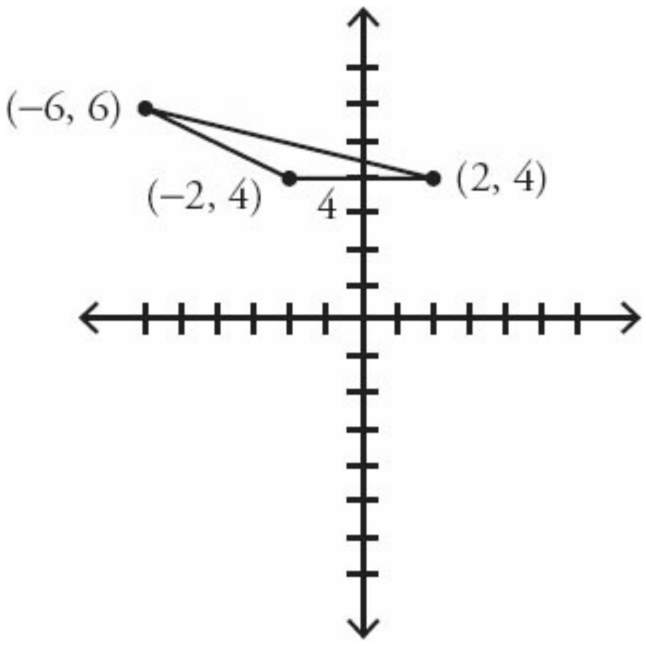
18. **(B).** The slope of line l is $\frac{1}{3}$. Since slope $= \frac{\text{rise}}{\text{run}}$ (or "change in y" divided by "change in x"), for every 1 unit the line moves up, it will move 3 units to the right.

Since the *actual* move to the right is equal to 4, create a proportion:

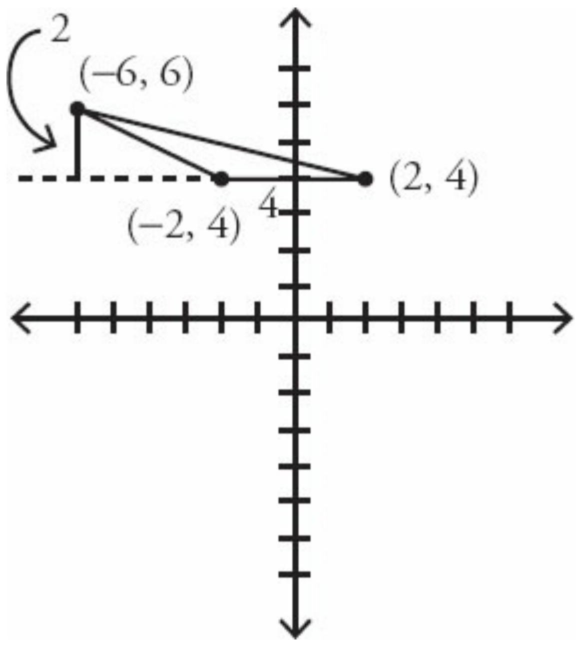
$$\frac{1}{3} = \frac{AB}{4}$$

Cross-multiply to get $3AB = 4$ or $AB = \frac{4}{3}$.

19. **4.** Make a quick sketch of the three points, joining them to make a triangle. Since (–2, 4) and (2, 4) make a horizontal line, use this line as the base. Since these two points share a y-coordinate, the distance between them is the distance between their x-coordinates: 2 – (–2) = 4, as shown below:



The height of a triangle is always perpendicular to the base. Drop a height vertically from (–6, 6). Subtract the y-coordinates to get the distance: 6 – 4 = 2.

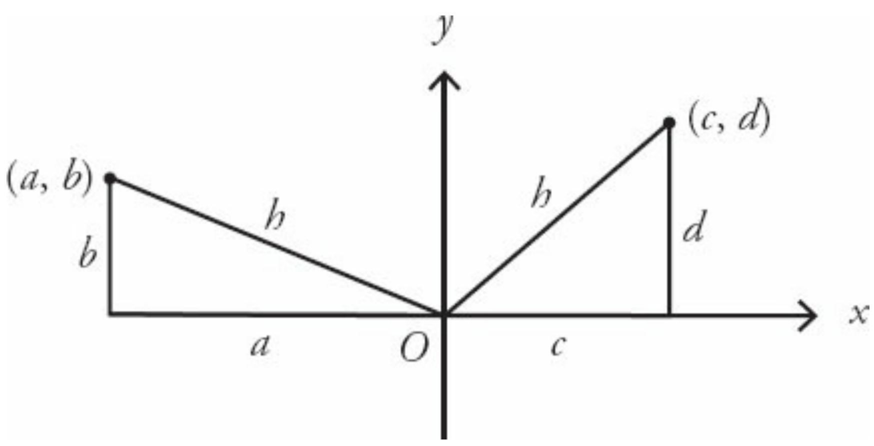


The formula for area of a triangle is $\frac{bh}{2}$. Thus, the area is $\frac{(4)(2)}{2}$, or 4.

20. **(B).** The slopes of perpendicular lines are the negative inverse of each other, so their product is –1. For example, perpendicular lines could have slopes of 2 and $-\frac{1}{2}$, or $-\frac{5}{7}$ and $\frac{7}{5}$. In all of these cases, Quantity A is –1. (The only exception is when one of the lines has an undefined slope because it's vertical, but that case has been specifically excluded.) If line m passes through the origin, its x-

intercept is 0, so regardless of the x-intercept of line n, Quantity B is 0. Quantity B is greater.

21. **(B).** A point's distance from the origin can be calculated by constructing a right triangle in which the legs are the vertical and horizontal distances. Sketch a diagram in which you place (a, b) and (c, d) anywhere in the coordinate plane that you wish; then construct two right triangles using (0, 0) as a vertex.



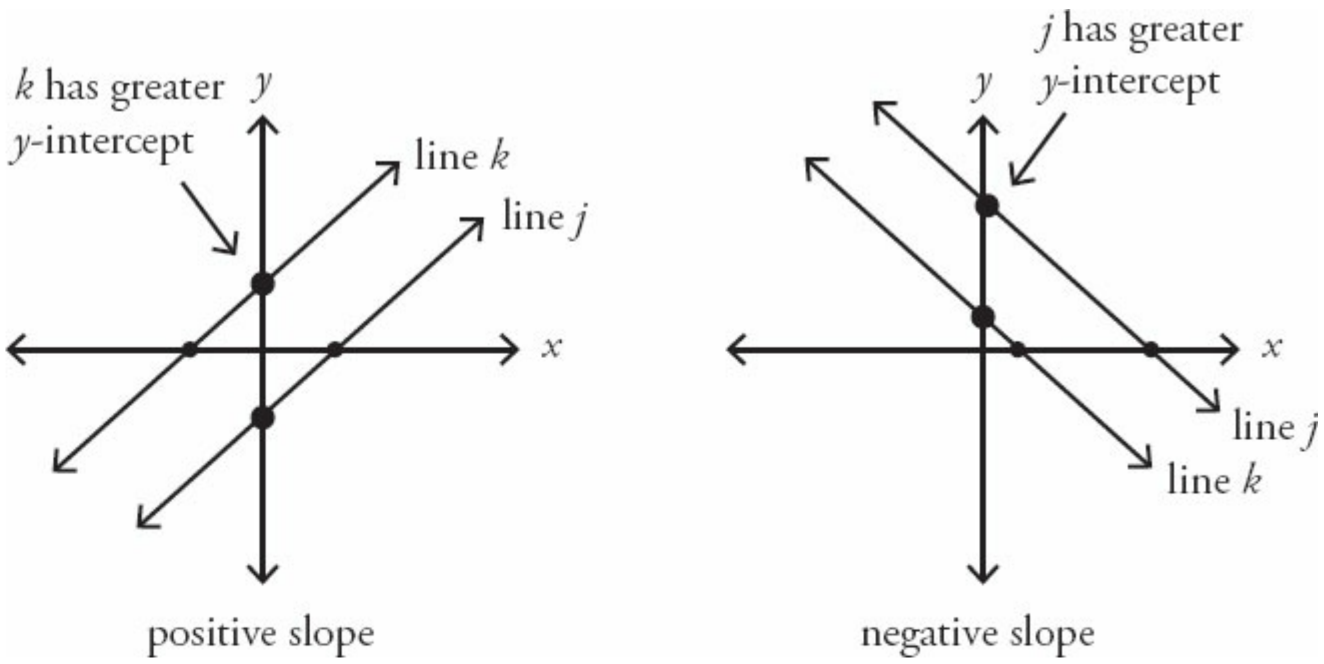
Both hypotenuses are labeled h, since the points are equidistant from the origin. Set up two Pythagorean equations:

$$a^2 + b^2 = h^2$$
$$c^2 + d^2 = h^2$$

So $a^2 + b^2 = c^2 + d^2$.

Since $|a| > |c|$, it is true that $a^2 > c^2$. (Try it with test numbers.) To make the equation $a^2 + b^2 = c^2 + d^2$ true, you must have $b^2 < d^2$. This means that $|b| < |d|$, and Quantity B is greater.

22. **(D).** Parallel lines have the same slope. Since the product of the two slopes is positive, either both slopes are positive or both slopes are negative. Here are two examples in which line j has a greater x-intercept, as specified by the problem:



If the slopes are positive, k will have the greater y-intercept, but if the slopes are negative, j will have the greater y-intercept. The relationship cannot be determined from the information given.

23. **(C).** Both figures share triangle MLK, so there is no need to calculate anything for this part of the figure. Parallelogram $KLMN$ and quadrilateral $JKLM$ each have a "top" (the part above the x-axis)

that is a triangle with base MK (= 8) and height 5. If two triangles have the same base and equal heights, their areas are equal. No calculation is needed to pick (C).

24. **(D).** Rearrange the equation to get it into $y = mx + b$ format, where m is the slope:

$$3x + 2y = 8$$
$$2y = -3x + 8$$
$$y = -\frac{3}{2}x + 4$$

The slope is $-\frac{3}{2}$. Parallel lines have the same slope, so only choice (D) is parallel.