Averages, Weighted Averages, Median, and Mode

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box [ ], you are to enter your own answer in the box. For questions followed by a fraction-style numeric entry box $\frac{[\ ]}{[\ ]}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is $\frac{1}{4}$, you may enter $\frac{25}{100}$ or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations, such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1. Husain and Dino had an average (arithmetic mean) of \$20 each. Dino then won a cash prize, which increased the average amount of money they had to \$80. If no other changes occurred, how many dollars did Dino win?

$ [ ]

Janani is 6 centimeters taller than Preeti, who is 10 centimeters taller than Rey.

	Quantity A	Quantity B
2.	The average (arithmetic mean)height of the three people	The median height of the three people

The average (arithmetic mean) of x and y is 55.
The average of y and z is 75.

	Quantity A	Quantity B
3.	$z - x$	40

4. What is the average (arithmetic mean) of x, $x - 6$, and $x + 12$?

(A) x
(B) $x + 2$
(C) $x + 9$
(D) $3x + 6$
(E) It cannot be determined from the information given.

$$ab < 0$$

	Quantity A	**Quantity B**
5.	$\dfrac{a+b}{2}$	0

6. If x is negative, what is the median of the list 20, x, 7, 11, 3?

(A) 3
(B) 7
(C) 9
(D) 11
(E) 15.5

7. If the average (arithmetic mean) of n and 11 is equal to $2n$, what is the average of n and $\frac{13}{3}$?

(A) 4
(B) 8
(C) 11
(D) 14
(E) 19

	Quantity A	**Quantity B**
8.	The average (arithmetic mean) of $x - 3$, x, $x + 3$, $x + 4$, and $x + 11$	The median of $x - 3$, x, $x + 3$, $x + 4$, and $x + 11$

9. John bought 5 books with an average (arithmetic mean) price of \$12. If John then buys another book with a price of \$18, what is the average price of all 6 books?

(A) \$12.50
(B) \$13
(C) \$13.50
(D) \$14

(E) $15

10. Every week, Renee is paid \$40 per hour for the first 40 hours she works, and \$80 per hour for each hour she works after the first 40 hours. How many hours would Renee have to work in one week to earn an average (arithmetic mean) of \$60 per hour that week?

(A) 60
(B) 65
(C) 70
(D) 75
(E) 80

At a certain school, all 118 juniors have an average (arithmetic mean) final exam score of 88 and all 100 seniors have an average final exam score of 92.

	Quantity A	Quantity B
11.	The average (arithmetic mean) final exam score for all of the juniors and seniors combined	90

	Quantity A	Quantity B
12.	The average (arithmetic mean) of x, y, and z	The average (arithmetic mean) of $0.5x$, $0.5y$, and $0.5z$

13. Aaron’s first three quiz scores were 75, 84, and 82. If his score on the fourth quiz reduced his average (arithmetic mean) quiz score to 74, what was his score on the fourth quiz?

Four people have an average (arithmetic mean) age of 18, and none of the people are older than 30.

	Quantity A	Quantity B
14.	The range of the four people’s ages	25

Dataset A consists of 5 numbers, which have an average (arithmetic mean) value of 43. Dataset B consists of 5 numbers.

	Quantity A	Quantity B
15.	The value of x if the average of x and the 5 numbers in dataset A is 46.	The average of dataset B if the average of the 10 numbers in datasets A and B combined is 52.

16. The average (arithmetic mean) of 7 numbers in a certain list is 12. The average of the 4 smallest numbers in this list is 8, while the average of the 4 greatest numbers in this list is 20. How much greater is the sum of the 3 greatest numbers in the list than the sum of the 3 smallest numbers in the list?

(A) 4
(B) 14
(C) 28
(D) 48
(E) 52

17. If the average (arithmetic mean) of a, b, c, 5, and 6 is 6, what is the average of a, b, c, and 13?

(A) 8
(B) 8.5
(C) 9
(D) 9.5
(E) It cannot be determined from the information given.

18. A group consists of both men and women. The average (arithmetic mean) height of the women is 66 inches, and the average (arithmetic mean) height of the men is 72 inches. If the average (arithmetic mean) height of all the people in the group is 70 inches, what is the ratio of women to men in the group?

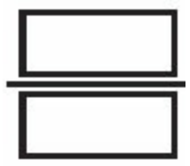
(A) 1:1
(B) 1:2
(C) 2:1
(D) 2:3
(E) 3:2

19. The average (arithmetic mean) of 13 numbers is 70. If the average of 10 of these numbers is 90, what is the average of the other 3 numbers?

(A) –130
(B) $\frac{10}{3}$
(C) 30
(D) 90
(E) 290

20. Town A has 6,000 citizens and an average (arithmetic mean) of 2 radios per citizen. Town B has 10,000 citizens and an average of 4 radios per citizen. What is the average number of radios per citizen in both towns combined?

Give your answer as a fraction.



The average (arithmetic mean) weight of 4 people is 85 kilograms. Two of the people weigh 75 and 90 kilograms, respectively.

	Quantity A	**Quantity B**
21.	The average (arithmetic mean) weight of the other two people, in kilograms	85 kilograms

22. Fiber X cereal is 55% fiber. Fiber Max cereal is 70% fiber. Sheldon combines an amount of the two cereals in a single bowl of mixed cereal that is 65% fiber. If the bowl contains a total of 12 ounces of cereal, how much of the cereal, in ounces, is Fiber X?

(A) 3
(B) 4
(C) 6
(D) 8
(E) 9

23. The average (arithmetic mean) population in town X was recorded as 22,455 during the years 2000–2010, inclusive. However, an error was later uncovered: the figure for 2009 was erroneously recorded as 22,478, but should have been correctly recorded as 22,500. What was the average population in town X during the years 2000–2010, inclusive, once the error was corrected?

While driving from city A to city B, a car got 22 miles per gallon and while returning on the same road, the car got 30 miles per gallon.

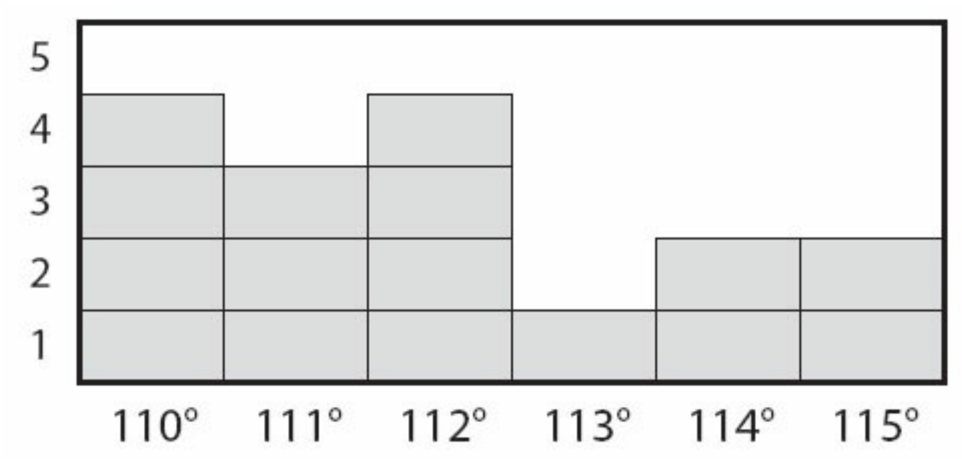
	Quantity A	**Quantity B**
24.	The car's average gas mileage for the entire trip, in miles per gallon	26

$S_n = 3n + 3$

Sequence S is defined for each integer n such that $0 < n < 10{,}000$.

	Quantity A	**Quantity B**
25.	The median of sequence S	The mean of sequence S

26. The bar graph below displays the number of temperature readings at each value from a sample, measured in degrees Fahrenheit. What was the average (arithmetic mean) temperature reading?



degrees Fahrenheit

27. Score results on a college mathematics proficiency exam

	Freshmen	Sophomores
Number of students taking the exam	120	80
Average (arithmetic mean) score on the exam	78 points	84 points

What was the average (arithmetic mean) score for all the freshmen and sophomores taking the exam? Give your answer rounded to the nearest 0.1 points.

[ ] points

28. Set A: 1, 3, 5, 7, 9
Set B: 6, 8, 10, 12, 14

For the sets of numbers above, which of the following statements are true?

Indicate all such statements.

- ☐ The mean of set B is greater than the mean of set A.
- ☐ The median of set B is greater than the median of set A.
- ☐ The standard deviation of set B is greater than the standard deviation of set A.
- ☐ The range of set B is greater than the range of set A.

29. Three people have $32, $72, and $98, respectively. If they pool their money then redistribute it among themselves, what is the maximum possible value for the median amount of money?

(A) $72
(B) $85
(C) $98
(D) $101
(E) $202

30.

Weekly Revenue Per Product Category at Office Supply Store *X*

Product Category	Weekly Revenue
Pens	\$164
Pencils	\$111
Legal pads	\$199
Erasers	\$38
Average (arithmetic mean) of categories above	\$128

According to the chart above, the average (arithmetic mean) revenue per week per product category is \$128. However, there is an error in the chart; the revenue for Pens is actually \$176, not \$164. What is the new, correct average revenue per week per product category?

(A) \$130
(B) \$131
(C) \$132
(D) \$164
(E) \$176

31. Set M consists of 20 evenly spaced integers, 10 numbers of which are positive and 10 of which are negative.

Quantity A	**Quantity B**
The average (arithmetic mean) of all the numbers in set M	0

32. The average (arithmetic mean) of $3x$, x, and y is equal to $2x$.

Quantity A	**Quantity B**
$2x$	y

33. The average (arithmetic mean) age of the buildings on a certain city block is greater than 40 years old. If four of the buildings were built two years ago and none of the buildings are more than 80 years old, which of the following could be the number of buildings on the block?

Indicate <u>all</u> such numbers.

- ☐ 4
- ☐ 6
- ☐ 8
- ☐ 11
- ☐ 40

34. Four students contributed to a charity drive, and the average (arithmetic mean) amount contributed by each student was $20. If no student gave more than $25, what is the minimum amount that any student could have contributed?

$ ☐

The average (arithmetic mean) of seven distinct integers is 12, and the least of these integers is –15.

	Quantity A	**Quantity A**
35.	The maximum possible value of the greatest of these integers	84

36. The average (arithmetic mean) of fifteen consecutive integers is 88. What is the greatest of these integers?

☐

Three numbers have a range of 2 and a median of 4.4.

	Quantity A	**Quantity A**
37.	The greatest of the numbers	5.4

Averages, Weighted Averages, Median, and Mode Answers

1. **$120.** If the two people had an average of $20 each, they held a sum of 2($20) = $40. After Dino won a cash prize, the new sum is 40 + p and the new average is 80. Plug into the average formula:

$$\begin{aligned} \text{Average} &= \frac{\text{Sum}}{\text{Number of Terms}} \\ 80 &= \frac{40+p}{2} \\ 160 &= 40+p \\ 120 &= p \end{aligned}$$

Dino won $120.

2. **(B).** Pick numbers that agree with the given height constraints. Rey is the shortest person, and if Rey is 100 cm tall, Preeti is 110 cm tall, and Janani is 116 cm tall. The average height is $\frac{100+110+116}{3}$ = 108.67 cm (rounded to nearest 0.01 cm). The median height is the middle height, which is 110 cm. Quantity B is greater.

Alternatively, note that Preeti's height is the median. Preeti's height is closer to Janani's than to Rey's. Since the average of Janani and Rey's heights would be midway between those heights, and Preeti's height is greater than that middle, the median is greater than the average.

3. **(C).** Since the average of x and y is 55, $\frac{x+y}{2} = 55$, so $x + y = 110$.

Since the average of y and z is 75, $\frac{y+z}{2} = 75$, so $y + z = 150$.

Stack the two equations and subtract to cancel the y's and get $z - x$ directly:

$$\begin{array}{rcl} z+y &=& 150 \\ -(x+y &=& 110) \\ \hline z-x &=& 40 \end{array}$$

The two quantities are equal.

4. **(B).** The average formula can be applied to algebraic expressions, just as to arithmetic ones:

$$\text{Average} = \frac{\text{Sum}}{\text{Number of Terms}}$$

$$\text{Average} = \frac{(x)+(x-6)+(x+12)}{3}$$

$$\text{Average} = \frac{3x+6}{3} = x+2$$

5. **(D).** The best way to solve Quantitative Comparisons problems with variables is to plug in multiple values for the variables, trying to prove (D).

If $ab < 0$, then one of the variables is positive and the other negative.

Try $a = 2$ and $b = -3$

$\frac{2+-3}{2} = -\frac{1}{2}$ and Quantity B is greater. Therefore, the answer cannot be (A) or (C).

Try $a = 3$ and $b = -2$:

$\frac{3+-2}{2} = \frac{1}{2}$ and now Quantity A is greater. Therefore, the answer cannot be (B) or (C). If you plug in two different sets of numbers and get two different results for which quantity is greater, the answer must be (D).

6. **(B).** The easiest way to start thinking about a question like this is to plug in a value and see what happens. If $x = -1$, the list looks like this when ordered from least to greatest:

$-1, 3, 7, 11, 20$

The median is 7. Because any negative x used will be the least term in the list, the order of the list won't change, so the median will always be 7.

7. **(A).** This question can be solved with the average formula:

$$\text{Average} = \frac{\text{Sum}}{\text{Number of terms}}$$

$$2n = \frac{n+11}{2}$$

$$4n = n + 11$$

$$3n = 11$$

$$n = \frac{11}{3}$$

Since $n = \frac{11}{3}$, the average of n and $\frac{13}{3}$ is:

$$\frac{\frac{11}{3}+\frac{13}{3}}{2}=\frac{\frac{24}{3}}{2}=\frac{8}{2}=4$$

Alternatively, just notice that the midpoint between $\frac{11}{3}$ and $\frac{13}{3}$ is $\frac{12}{3}$, just as 12 is the midpoint between 11 and 13. The average is $\frac{12}{3}$ = 4.

8. **(C).** To find the median of the numbers, notice that they are already in order from least to greatest: $x - 3, x, x + 3, x + 4, x + 11$.

The median is the middle, or third, term: $x + 3$.

Now find the average of the numbers:

$$\frac{(x-3)+(x)+(x+3)+(x+4)+(x+11)}{5} = \frac{5x+15}{5} = x+3$$

The median and the mean are both $(x + 3)$, therefore, the two quantities are equal.

9. **(B).** First, calculate the cost of the first 5 books.

Sum = (Average cost)(Number of books) = (\$12)(5) = \$60

Total cost of all 6 books = \$60 + \$18 = \$78 Total number of books = 6

Average = \$78/6 = \$13 per book.

10. **(E).** Let h = number of hours Renee would have to work. The average rate Renee gets paid is equal to the total wages earned divided by the total number of hours worked. Rene earns \$40 per hour for the first 40 hours, so she makes $40 \times 40 =$ \$1,600 in the first 40 hours. She also earns \$80 for every hour after 40 hours, for additional pay of \$80(h – 40). Therefore, her total pay can be calculated as:

$$\frac{1{,}600 + 80(h-40)}{h} = 60$$

$$1{,}600 + 80h - 3{,}200 = 60h$$

Now isolate h:

$$\begin{aligned} 80h - 1{,}600 &= 60h \\ -1{,}600 &= -20h \\ 80 &= h \end{aligned}$$

You could also notice that 60 is exactly halfway between 40 and 80. Therefore, Renee needs to work an equal number of hours at \$40 per hour and \$80 per hour. If she works 40 hours at \$40 per hour, she also needs to work 40 hours at \$80 per hour, yielding 80 hours of total work.

11. **(B).** This is a weighted average problem. Because the number of juniors is greater than the number of seniors, the overall average will be closer to the juniors' average than the seniors' average. Since 90 is halfway between 88 and 92, and the weighted average is closer to 88, Quantity B is greater.

It is not necessary to do the math because this is a Quantitative Comparison question with a very convenient number as Quantity B. However, you can actually calculate the overall average by summing up all 218 scores and dividing by the number of people: $\frac{118(88)+100(92)}{118+100}$ = 89.83…

12. **(D).** The average of *x, y*, and *z* is $\frac{x+y+z}{3}$. Calculated similarly, the average of 0.5*x*, 0.5*y*, and 0.5*z* is exactly half that. If the sum of the variables is positive, Quantity A is greater. However, if the sum of the variables is negative, Quantity B is greater. If the sum of the variables is zero, the two quantities are equal.

13. **55.** To find Aaron's fourth quiz score, set up an equation:

$$\begin{aligned}\frac{75+84+82+x}{4} &= 74\\ 241+x &= 296\\ x &= 55\end{aligned}$$

14. **(D).** If 4 people have an average age of 18, then the sum of their ages is 4 × 18 = 72. Since the question is about range, try to minimize and maximize the range. Minimizing the range is easy—if everyone were exactly 18, the average age would be 18 and the range would be 0. So clearly, the range can be smaller than 25.

To maximize the range, make the oldest person the maximum age of 30, and see whether the youngest person could be just 1 year old while still obeying the other rules of the problem: the sum of the ages is 72 and, of course, no one can be a negative age.

One such set: 1, 20, 21, 30

This is just one example that would work. In this case, the range is 30 – 1 = 29, which is greater than 25.

15. **(C).** If the average of the 5 numbers in dataset A is 43, the sum of dataset A is (5)(43) = 215.

For Quantity A, use the average formula and sum all 6 numbers and divide by 6:

$$\begin{aligned}\frac{\text{Sum of the 5 numbers in dataset A}+x}{6} &= 46\\ \frac{215+x}{6} &= 46\end{aligned}$$

$$\begin{aligned} 215 + x &= 276 \\ x &= 61 \end{aligned}$$

For Quantity B, use the average formula again:

$$\frac{\text{Sum of dataset A} + \text{Sum of dataset B}}{10} = 52$$

$$215 + \text{Sum of dataset B} = 520$$

$$\text{Sum of dataset B} = 305$$

The average of the 5 numbers in dataset B is $\frac{305}{5} = 61$.

Alternatively, note that each dataset of 5 numbers has the same "weight" in the average of all 10 numbers. The average of dataset A is 43, which is 52 – 43 = 9 below the average of all 10 numbers. The average of dataset B must be 9 above the average of all 10 numbers: 52 + 9 = 61.

16. **(D).** Using the average formula, Average = $\frac{\text{Sum}}{\text{Number of terms}}$, build three separate equations:

All 7 numbers:

$$12 = \frac{\text{Sum of all 7 numbers}}{7}$$

Sum of all 7 numbers = 84

The 4 smallest numbers:

$$8 = \frac{\text{Sum of the 4 smallest numbers}}{4}$$

Sum of the 4 smallest numbers = 32

The 4 greatest numbers:

$$20 = \frac{\text{Sum of the 4 greatest numbers}}{4}$$

Sum of the 4 greatest numbers = 80

There are only 7 numbers, yet information is given about the 4 smallest and the 4 greatest, which is a total of 8 numbers! The middle number has been counted twice—it is included in both the 4 greatest and the 4 smallest.

The sum of all 7 numbers is 84, but the sum of the 4 greatest and 4 smallest is 80 + 32 = 112. The difference can only be attributed to the double counting of the middle number in the set of 7: 112 – 84 = 28.

The middle number is 28, so subtract it from the sum of the 4 smallest numbers to get the sum of the 3 smallest numbers: 32 – 28 = 4.

Now subtract the middle number from the sum of the 4 greatest numbers to get the sum of the 3 greatest numbers: 80 – 28 = 52.

The difference between the sum of the 3 greatest numbers and the sum of the 3 smallest numbers is 52 – 4 = 48.

17. **(A).** Since Average = $\frac{\text{Sum}}{\text{Number of terms}}$:

$$6 = \frac{a+b+c+5+6}{5}$$

$$30 = a+b+c+11$$

$$19 = a + b + c$$

It is not necessary, or possible, to determine the values of a, b, and c individually. The second average includes all three variables, so the values will be summed again anyway.

$$\text{Average} = \frac{a + b + c + 13}{4}$$

$$\text{Average} = \frac{19 + 13}{4}$$

$$\text{Average} = \frac{32}{4} = 8$$

18. **(B).** Use the weighted average formula:

$$\text{Avgerage height of all} = \frac{(\text{Total height of all women}) + (\text{Total height of all men})}{(\text{Number of women}) + (\text{Number of men})}$$

From the average formula, Sum = Average × Number of terms. If w is the number of women and m is the number of men, the total heights of women and men respectively are:

Total height of all women = $66w$
Total height of all men = $72m$

Plug into the average formula, recalling that the average height for the entire group is 70 inches:

$$\text{Avgerage height of all} = \frac{66w + 72m}{w + m} = 70$$

Cross-multiply and simplify:

$$\begin{aligned} 66w + 72m &= 70(w + m) \\ 66w + 72m &= 70w + 70m \\ 72m &= 4w + 70m \\ 2m &= 4w \\ m &= 2w \end{aligned}$$

At this point, it might be tough to determine whether the answer is (B) or (C). This is an ideal time to plug in numbers. For instance, if $w = 3$, then $m = 6$. Now, the ratio of women to men is 3:6 or 1:2, answer choice (B).

Alternatively, continue with the algebra, solving for the $\frac{w}{m}$ ratio:

$$m = 2w$$

$$\frac{m}{2m} = \frac{2w}{2m}$$

$$\frac{1}{2} = \frac{w}{m}$$

19. **(B).** Remember the Average formula, Average = $\frac{\text{Sum}}{\text{Number of terms}}$, can also be rewritten as:

Sum = Average × Number of Terms.

The average of 13 numbers is 70, so:

Sum of all 13 terms = 70 × 13 = 910

The average of 10 of these numbers is 90, so:

Sum of 10 of these numbers = 90 × 10 = 900

Subtract to find the sum of "the other 3 numbers": 910 – 900 = 10.

Average of the other 3 numbers = $\frac{\text{Sum}}{\text{Number of terms}} = \frac{10}{3}$.

20. $\frac{13}{4}$ **(or any equivalent fraction).** To find this weighted average, first find the sum of all the radios in towns A and B, and then divide by the total number of people in both towns:

Average = $\frac{6{,}000(2)+10{,}000(4)}{16{,}000}$

Cancel three zeros from each term:

Average = $\frac{6(2)+10(4)}{16}$

Average = $\frac{52}{16}$

This reduces to $\frac{13}{4}$, though reduction is not required.

21. **(A).** The formula for averages is: Average = $\frac{\text{Sum}}{\text{Number of terms}}$.

The sum of the weights of the people in this group is:

Sum = Average × Number of terms = 85 × 4 = 340

If two of the people weigh 75 and 90 kilograms, subtract them from the total to see what the other two people weigh, combined:

340 – 75 – 90 = 175 kg

If the total weight of the other two people is 175, their average weight is:

$\frac{175}{2}$ = 87.5 kg.

Quantity A is greater.

22. **(B).** Use the weighted average formula to get the ratio of Fiber X to Fiber Max:

$\frac{0.55x + 0.70m}{x + m}$ = 0.65, where x is the amount of Fiber X and m is the amount of Fiber Max.

This is not that different from the regular average formula—on the top, there is the total amount of fiber (55% of Fiber X and 70% of Fiber Max), which is divided by the total amount of cereal ($x + m$) to get the average. Simplify by multiplying both sides by ($x + m$):

$0.55x + 0.70m = 0.65(x + m)$
$0.55x + 0.70m = 0.65x + 0.65m$

To simplify, multiply both sides of the equation by 100 to eliminate all the decimals:

$55x + 70m = 65x + 65m$
$55x + 5m = 65x$
$5m = 10x$

$\frac{m}{x} = \frac{10}{5}$ or $\frac{2}{1}$

Since m and x are in a 2 to 1 ratio, 2/3 of the total is m and 1/3 of the total is x. Since the total is 12 ounces, Fiber X accounts for $\frac{1}{3}$ (12) = 4 ounces of the mixed cereal.

One shortcut to this procedure is to note that the weighted average (65%) is 10% away from Fiber X's percent and 5% away from Fiber Max's percent. Since 10 is twice as much as 5, the ratio of the two cereals is 2 to 1. However, it is a 2 to 1 ratio of Fiber Max to Fiber X, not the reverse! Whichever number is closer to the weighted average (in this case, 70% is closer to 65%) gets the larger of the ratio parts. Since the ratio is 2 to 1 (Fiber Max to Fiber X), again, 1/3 of the cereal is Fiber X and $\frac{1}{3}(12) = 4$.

23. **22,457.** There is a simple shortcut for a change to an average. The figure for 2009 was recorded as 22,478, but actually should have been recorded as 22,500, meaning 22 people in that year were not counted. Thus, the sum should have been 22 higher when the average was originally calculated.

2000–2010, inclusive, is 11 years (subtract low from high and then add 1 to count an inclusive list of consecutive numbers). When taking an average, divide the sum by the number of things being averaged (in this case, 11). So the shortcut is to take the change to the sum and "spread it out" over all of the values being averaged by dividing the change by the number of things being averaged.

Divide 22 by 11 to get 2. The average should have been 2 greater. Thus, the correct average for the 11-year period is 22,457.

Alternatively, use the traditional method: 22,455 × 11 years = 247,005, the sum of all 11 years' recorded populations. Add the 22 uncounted people, making the corrected sum 247,027. Divide by 11 to get the corrected average: 22,457. (Note that while the traditional method is faster to explain, the shortcut is faster to actually execute!)

24. **(B).** One trap is to mistakenly pick (C), thinking that the car got a simple average of 22 and 30 miles per gallon. However, the trip from *A* to *B* required *more* gallons of gas, therefore, the average will be "weighted" more to the side of 22 (same number of miles, but *more* gallons), and wind up less than 26 mpg. Quantity (B) is therefore greater.

To show this explicitly, use a smart number of miles from city A to B, for example, a multiple of 22 and 30, such as 660 miles. Set up a chart using the formula

(Miles per gallon) × (Gallons) = (Miles)

	Miles per gallon	×	Gallons	=	Miles
A to *B*	22	×	30	=	660
B to *A*	30	×	22	=	660
Total	x	×	52	=	1,320

The average miles per gallon for the whole trip is the total number of miles (1,320) divided by the total number of gallons (52):

$\frac{1,320}{52}$ = 25.38… , which is less than 26.

Quantity B is greater.

25. **(C).** Sequence S is an evenly spaced set, which can be seen by plugging in a few n values:

$S_1 = 3(1) + 3 = 6$
$S_2 = 3(2) + 3 = 9$
$S_3 = 3(3) + 3 = 12\ldots$

Terms increase by 3 every time n increases by 1; this meets the definition of an evenly spaced set. For *any* evenly spaced set, the median equals the mean.

26. **112.** This is a weighted average problem. *Do not* simply average 110, 111, 112, 113, 114, and 115. Instead, take into account how many times each number appears. The chart is really another way of writing:

110, 110, 110, 110
111, 111, 111
112, 112, 112, 112
113
114, 114
115, 115

In other words, the average temperature reading is really an average of 16 numbers. The easiest way to do this is:

$$\frac{4(110) + 3(111) + 4(112) + 1(113) + 2(114) + 2(115)}{16}$$

Use the calculator—the correct answer is 112.

27. **80.4.** In order to determine the average score for *all* the freshman and sophomores combined, compute the total points for everyone, and divide by the total number of students.

Because Average = $\frac{\text{Sum}}{\text{Number of terms}}$, it can also be written as the Sum = Average × Number of terms. Use the formula to compute the total number of points for all the freshmen and all the sophomores as individual groups:

Freshman total points = 78 points × 120 = 9,360
Sophomore total points = 84 points × 80 = 6,720

Combined, the freshmen and sophomores scored 9,360 + 6,720 = 16,080 points.

The total number of students is 120 + 80 = 200.

Now, apply the average formula, Average = $\frac{\text{Sum}}{\text{Number of terms}}$, to the combined group:

$$\text{Average} = \frac{16,080}{200} = 80.4$$

28. **1st and 2nd only.** In both sets, the numbers are evenly spaced. Moreover, both sets are evenly spaced by the same amount (adjacent terms increase by 2) and have the same number of terms (5 numbers in each set). The difference is that each term in set *B* is 5 greater than the corresponding term in set *A* (i.e., 6 – 1 = 5, 8 – 3 = 5, etc.).

In evenly spaced sets, the mean = median. Also, if an evenly spaced set has an odd number of numbers, the mean and median both equal the middle number. (When such a set has an even number of numbers, the mean and median both equal the average of the two middle numbers).

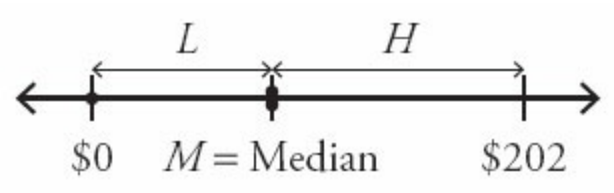
So, set *A* has mean and median of 5 and set B has mean/median of 10. The first and second statements are true.

Since sets *A* and *B* are equally spaced and have the same number of elements, their standard deviations are equal (that is, set *A* is exactly as spread out from its own mean as set *B* is from its own mean), so the third statement is false.

Since 9 – 1 = 8 and 14 – 6 = 8, the ranges are equal and the fourth statement is false.

29. **(D).** The pool of money is \$32 + \$72 + \$98 = \$202. After the redistribution, each person will have an amount between \$0 and \$202, inclusive. Call the amounts *L, M*, and *H* (low, median, high). To maximize *M*, minimize *L* and *H*.

The minimum value for *H* is *M*. The "highest" of the three values can be equal to the median (if *H* were lower than *M*, the term order and therefore which number is the median would change, but if *H* = *M*, *M* can still be the median). Draw it out



Minimum L = \$0
Minimum $H = M$

Maximum M = Total pool of money – Minimum L – Minimum H
M = \$202 – \$0 – M
$2M$ = \$202
M = \$101

The correct answer is (D).

30. **(B).** The chart provides the average and the number of product categories. If the incorrectly calculated average was \$128 for the 4 categories, then the sum was 4 × 128 = \$512. Since the revenue for pens was actually \$176, not \$164, the sum should have been \$12 higher. Thus, the correct sum is \$524. Divide by 4 to get \$131, the answer.

Alternatively, notice that the $128 average given in the question stem actually does a lot of work for you. If $164 jumps up to $176, that's an increase of $12. Distributed over the four categories, it will bring the overall average up by $3, from $128 to $131.

31. **(D).** "Evenly spaced" means ascending by some regular increment (each number greater than the next by some value). If 10 of the integers are positive and 10 are negative, then none of the numbers in the set are 0. Therefore, the 10th number must be less than 0 and the 11th greater than 0. To better understand this, try listing values for set M, starting in the middle. If the middle numbers are

…–1, +1…

then the spacing between the numbers is 2 and set M would look like:

–19, –17, –15, –13, –11, –9, –7, –6, –5, –4, –3, –2, –1, 1, 3, 5, 7, 9, 11, 13, 15, 17, 19

The sum and the average of all the numbers in this set are both 0, so (A) and (B) cannot be the answer. However, is the average always 0? If the middle two numbers were offset a little:

…–1, 4…

then the spacing between the numbers is 5 and set M would look like:

–46, –41, –36, –31, –26, –21, –16, –11, –6, –1, 4, 9, 14, 19, 24, 29, 34, 39, 44, 49

The first and the last terms sum to –46 + 49 = 3. The second term and penultimate term sum to –41 + 44 = 3, and so on. Each term can be paired with another for a sum of 3. There are 10 such pairs, so the total sum is 3 × 10 = 30 and the average is 30/20 = 1.5.

The relationship cannot be determined from the information given.

32. **(C).** Write "the average of $3x$, x, and y is equal to $2x$" as an equation and solve:

$$\frac{3x + x + y}{3} = 2x$$
$$4x + y = 6x$$
$$y = 2x$$

The two quantities are equal.

33. **8, 11, and 40 only.** Because Average = $\frac{\text{Sum}}{\text{Number of terms}}$, this question about averages depends both on x, the total number of buildings on the block, and on the sum of the building ages. The 4 buildings that are 2 years old have a total age of 4(2), and the (x – 4) other buildings have a total age of (x – 4)(no more than 80). Set up an equation to find the average age:

$$\text{Average age} = \frac{4(2) + (x - 4)(\text{no more than } 80)}{x}$$

Having many 80-year-old buildings on the block would raise the average much closer to 80. (For instance, if there were a million 80-year-old buildings and four 2-year-old buildings, the average would be very close to 80 years old.) So, there is some minimum number of older buildings that could raise the average above 40.

Ignore the “greater than” 40 years old constraint on the average building age for a moment. What is the minimum x needs to be to make the average age exactly 40 when the age of the other buildings is maximized at 80?

$$
\begin{aligned}
40x &= 8 + (x - 4)(80) \\
40x &= 8 + 80x - 320 \\
-40x &= -320 \\
x &= \frac{312}{40} = 7.8
\end{aligned}
$$

Because there can’t be a partial building, and the age of the buildings can’t be greater than 80, x must be at least 8 to bring the average age up over 40. (More buildings would be required to bring the average above 40 if those older buildings were only between 50 and 70 years old, for example.)

Alternatively, test the answer choices. Try the first choice, 4 buildings. Since 4 of the buildings on the block are only 2 years old, this choice can’t work—the average age of the buildings would be 2.

Try the second choice. With 6 total buildings, there would be four 2-year-old buildings, plus two others. To maximize the average age, maximize the ages of the two other buildings by making them both 80 years old:

$$\frac{4(2)+2(80)}{6} = 28$$

Since the average is less than 40 years old, this choice is not correct.

Try the third choice. With 8 total buildings, there would be the four 2-year-old buildings, plus four others. To maximize the average age, maximize the ages of the four other buildings by making them each 80 years old:

$$\frac{4(2)+4(80)}{8} = 41$$

Since the average age is greater than 40 years old, this choice is correct. Since the other, greater choices allow the possibility of even more 80-year-old buildings, increasing the average age further, those choices are also correct.

34. **\$5.** The average of four values is \$20. Thus, the sum of the four values is \$80. To determine the minimum contribution one student could have given, maximize the contributions of the other three students. If the three other students each gave the maximum of \$25, the fourth student would only have to give \$5 to make the sum equal to \$80.

35. **(A).** If the average of 7 integers is 12, then their sum must be $7 \times 12 = 84$. To maximize the largest of the numbers, minimize the others.

The smallest number is –15. The integers are distinct (that is, different from each other), so the minimum values for the smallest 6 integers are –15, –14, –13, –12, –11, and –10. To find the maximum value for the 7th integer, sum –15, –14, –13, –12, –11, –10, and x, while setting that sum equal to 84:

$$\begin{aligned} -15 + (-14) + (-13) + (-12) + (-11) + (-10) + x &= 84 \\ -75 + x &= 84 \\ x &= 159 \end{aligned}$$

Quantity A is greater.

36. **95.** In any evenly spaced set, the average equals the median. Thus, 88 is the middle number in the set. Since the set has 15 elements, the 8th element is the middle one:

Lowest seven integers:	81	82	83	84	85	86	87
Middle integer:	88						
Greatest seven integers:	89	90	91	92	93	94	95

The largest integer in the list is 95. Confidence in this process allows you to skip the counting process. Instead reason that to go from 8th integer to the 15th integer, simply add 7: 88 + 7 = 95.

37. **(D).** If the set has an odd number of terms, then the median is the middle number, so the middle number is 4.4. The set has a range of 2. The other two numbers could be 2 apart and also equally distributed around 4.4:

Example 1: 3.4, 4.4, 5.4

Here, the two quantities are equal.

Or, the two other numbers could be 2 apart but both a bit higher, or both a bit lower.

Example 2: 4.3, 4.4, 6.3
Example 3: 2.5, 4.4, 4.5

Thus, Quantity A could be equal to, less than, or greater than Quantity B. The relationship cannot be determined from the information given.