Algebra

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:
(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box $\boxed{}$, you are to enter your own answer in the box. For questions followed by a fraction-style numeric entry box $\dfrac{\boxed{}}{\boxed{}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is $\frac{1}{4}$, you may enter $\frac{25}{100}$ or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations, such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1. If $4(-3x - 8) = 8(-x + 9)$, what is the value of x^2?

$\boxed{}$

2. If $2x(4 - 6) = -2x + 12$, what is the value of x?

$\boxed{}$

3. If $x \neq 0$ and $\dfrac{3(6-x)}{2x} = -6$, what is the value of x?

$\boxed{}$

4. If $x \neq 2$ and $\dfrac{8-2(-4+10x)}{2-x} = 17$, what is the value of x?

$\boxed{}$

-5 is 7 more than $-z$.

	Quantity A	**Quantity B**
5.	z	-12

6. If $(x + 3)^2 = 225$, which of the following could be the value of $x - 1$?

(A) 13
(B) 12
(C) –12
(D) –16
(E) –19

$$x = 2$$

	Quantity A	**Quantity B**
7.	$x^2 - 4x + 3$	1

$$p = 300c^2 - c$$
$$c = 100$$

	Quantity A	**Quantity B**
8.	p	$29{,}000c$

$$-(x)^3 = 64$$

	Quantity A	**Quantity B**
9.	x^4	x^5

10. If $3t^3 - 7 = 74$, what is the value of $t^2 - t$?

(A) –3
(B) 3
(C) 6
(D) 9
(E) 18

11. If $y = 4x + 10$ and $y = 7x - 5$, what is the value of y?

$\boxed{}$

12. If $x - y = 4$ and $2x + y = 5$, what is the value of x?

$\boxed{}$

13. $4x + y + 3z = 34$
$4x + 3z = 21$

What is the value of y?

$\boxed{}$

	Quantity A	**Quantity B**
14.	$(x + 2)\ (x - 3)$	$x^2 - x - 6$

$$xy > 0$$

	Quantity A	**Quantity B**
15.	$(2x - y)(x + 4y)$	$2x^2 + 8xy - 4y^2$

$$x^2 - 2x = 0$$

	Quantity A	**Quantity B**
16.	x	2

	Quantity A	**Quantity B**
17.	$d(d^2 - 2d + 1)$	$d(d^2 - 2d) + 1$

	Quantity A	**Quantity B**
18.	$xy^2z(x^2z + yz^2 - xy^2)$	$x^3y^2z^2 + xy^3z^3 - x^2y^4z$

$a = 2b = 4c$ and a, b, and c are integers.

	Quantity A	**Quantity B**
19.	$a + b$	$a + c$

$k = 2m = 4n$ and k, m, and n are non-negative integers.

	Quantity A	**Quantity B**
20.	km	kn

For the positive integers *a, b, c,* and *d, a* is half of *b*, which is one-third of *c*. The value of *d* is three times the value of *c*.

	Quantity A	**Quantity B**
21.	$\dfrac{a+b}{c}$	$\dfrac{a+b+c}{d}$

22. If $x^2 - y^2 = 0$ and $xy \neq 0$, which of the following must be true?

Indicate <u>all</u> such statements.

☐ $x = y$
☐ $|x| = |y|$
☐ $\dfrac{x^2}{y^2} = 1$

$$3x + 6y = 27$$
$$x + 2y + z = 11$$

	Quantity A	**Quantity B**
23.	$z + 5$	$x + 2y - 2$

24. If $(x - y) = \sqrt{12}$ and $(x + y) = \sqrt{3}$, what is the value of $x^2 - y^2$?

(A) 3
(B) 6
(C) 9
(D) 36
(E) It cannot be determined from the information given.

$$a \neq b$$

	Quantity A	**Quantity B**
25.	$\dfrac{a-b}{b-a}$	1

$$a = \frac{b}{2}$$
$$c = 3b$$

	Quantity A	**Quantity B**
26.	a	c

27. If $xy \neq 0$ and $x \neq -y$, $\dfrac{x^{36} - y^{36}}{\left(x^{18} + y^{18}\right)\left(x^{9} + y^{9}\right)}$

(A) 1
(B) $x^2 - y^2$
(C) $x^9 - y^9$
(D) $x^{18} - y^{18}$
(E) $\dfrac{1}{x^9 - y^9}$

28. If $x \neq -y$, what is the value of $\dfrac{x^2 + 2xy + y^2}{2(x+y)^2}$?

(A) 1
(B) $\dfrac{1}{2}$
(C) $\dfrac{1}{x+y}$
(D) xy
(E) $2xy$

$$x > y$$
$$xy \neq 0$$

	Quantity A	**Quantity B**
29.	$\dfrac{x^2}{y + \frac{1}{y}}$	$\dfrac{y^2}{x + \frac{1}{x}}$

30. If $x + y = -3$ and $x^2 + y^2 = 12$, what is the value of $2xy$?

31. If $x - y = \frac{1}{2}$ and $x^2 - y^2 = 3$, what is the value of $x + y$?

$\boxed{}$

32. If $x^2 - 2xy = 84$ and $x - y = -10$, what is the value of $|y|$?

$\boxed{}$

33. Which of the following is equal to $(x - 2)^2 + (x - 1)^2 + x^2 + (x + 1)^2 + (x + 2)^2$?

(A) $5x^2$
(B) $5x^2 + 10$
(C) $x^2 + 10$
(D) $5x^2 + 6x + 10$
(E) $5x^2 - 6x + 10$

34. If $a = (x + y)^2$ and $b = x^2 + y^2$ and $xy > 0$, which of the following must be true?

Indicate <u>all</u> such statements.

☐ $a = b$
☐ $a > b$
☐ a is positive

35. a is directly proportional to b. If $a = 8$ when $b = 2$, what is a when $b = 4$?

(A) 10
(B) 16
(C) 32
(D) 64
(E) 128

Algebra Answers

1. **676.** Distribute, group like terms, and solve for x:

$$\begin{aligned} 4(-3x - 8) &= 8(-x + 9) \\ -12x - 32 &= -8x + 72 \\ -32 &= 4x + 72 \\ -104 &= 4x \\ -26 &= x \end{aligned}$$

Then, multiply 26 by 26 in the calculator (or –26 by –26, although the negatives will cancel each other out) to get x^2, which is 676.

2. –**6.**

$$\begin{aligned} 2x(4 - 6) &= -2x + 12 \\ 2x(-2) &= -2x + 12 \\ -4x &= -2x + 12 \\ -2x &= 12 \\ x &= -6 \end{aligned}$$

3. –**2.** $\dfrac{3(6-x)}{2x} = -6$

Multiply both sides by $2x$, distribute the left side, combine like terms, and solve:

$$\begin{aligned} 3(6 - x) &= -6(2x) \\ 18 - 3x &= -12x \\ 18 &= -9x \\ -2 &= x \end{aligned}$$

4. –**6.** $\dfrac{8-2(-4+10x)}{2-x} = 17$

Multiply both sides by the expression $2 - x$, distribute both sides, combine like terms, and solve:

$$
\begin{aligned}
8 - 2(-4 + 10x) &= 17(2 - x) \\
8 + 8 - 20x &= 34 - 17x \\
16 - 20x &= 34 - 17x \\
16 &= 34 + 3x \\
-18 &= 3x \\
-6 &= x
\end{aligned}
$$

5. **(A).** Translate the question stem into an equation and solve for z:

$$
\begin{aligned}
-5 &= -z + 7 \\
-12 &= -z \\
12 &= z
\end{aligned}
$$

Because $z = 12 > -12$, Quantity A is greater.

6. **(E).** Begin by square-rooting both sides of the equation, but remember that 225 could be the square of either 15 or –15. (The calculator will not remind you of this! It's your job to keep this in mind). So:

$$\begin{aligned} x + 3 &= 15 \\ x &= 12 \\ \text{so, } x - 1 &= 11 \end{aligned}$$

OR

$$\begin{aligned} x + 3 &= -15 \\ x &= -18 \\ \text{so, } x - 1 &= -19 \end{aligned}$$

Only –19 appears in the choices.

7. **(B).** To evaluate the expression in Quantity A, replace x with 2.

$$\begin{aligned} x^2 - 4x + 3 &= \\ (2)^2 - 4(2) + 3 &= \\ 4 - 8 + 3 &= -1 < 1 \end{aligned}$$

Therefore, Quantity B is greater.

8. **(A).** To find the value of p, first replace c with 100 to find the value for Quantity A:

$$\begin{aligned} p &= 300c^2 - c \\ p &= 300(100)^2 - 100 \\ p &= 300(10{,}000) - 100 \\ p &= 3{,}000{,}000 - 100 = 2{,}999{,}900 \end{aligned}$$

Since $c = 100$, the value for Quantity B is $29{,}000(100) = 2{,}900{,}000$. Quantity A is greater.

9. **(A).** First, solve for x:

$$\begin{aligned} -(x)^3 &= 64 \\ (x)^3 &= -64 \end{aligned}$$

The GRE calculator will not do a cube root. As a result, cube roots on the GRE tend to be quite small and easy to puzzle out. What number times itself three times equals –64? The answer is $x = -4$.

Since x is negative, Quantity A is positive (a negative number times itself four times is positive) and Quantity B is negative (a negative number times itself five times is negative). No further calculations are needed to conclude that Quantity A is greater. Notice that solving for the value of x here was not strictly necessary. Knowing that the cube root of a negative number is negative gives you all the information you need to solve.

10. **(C).** First, solve for t:

$$\begin{aligned} 3t^3 - 7 &= 74 \\ 3t^3 &= 81 \\ t^3 &= 27 \\ t &= 3 \end{aligned}$$

Now, plug $t = 3$ into $t^2 - t$:

$$(3)^2 - 3 = 9 - 3 = 6$$

11. **30.** Since each equation is already solved for y, set the right side of each equation equal to the other.

$$\begin{aligned} 4x + 10 &= 7x - 5 \\ 10 &= 3x - 5 \\ 15 &= 3x \\ 5 &= x \end{aligned}$$

Substitute 5 for x in the first equation and solve for y.

$$\begin{aligned} y &= 4(5) + 10 \\ y &= 30 \end{aligned}$$

$x = 5$ and $y = 30$. Be sure to answer for y, not x.

12. **3.** Notice that the first equation has the term $-y$ while the second equation has the term $+y$. While it is possible to use the substitution method, summing the equations together will make $-y$ and y cancel, so this is the easiest way to solve for x.

$$\begin{array}{rl} x - y &= 4 \\ 2x + y &= 5 \\ \hline 3x &= 9 \\ x &= 3 \end{array}$$

13. **13.** This question contains only two equations, but three variables. To isolate y, both x and z must be eliminated. Notice that the coefficients of x and z are the same in both equations. Subtract the

second equation from the first to eliminate x and z.

$$\begin{array}{rcl} 4x + y + 3z & = & 34 \\ -(4x \qquad + 3z) & = & 21) \\ \hline y & = & 13 \end{array}$$

14. **(C).** FOIL the terms in Quantity A:

$$(x + 2)(x - 3) = x^2 - 3x + 2x - 6 = x^2 - x - 6$$

The two quantities are equal.

15. **(B).** FOIL the terms in Quantity A:

$$(2x - y)(x + 4y) = 2x^2 + 8xy - xy - 4y^2 = 2x^2 + 7xy - 4y^2$$

Since $2x^2$ and $-4y^2$ appear in both quantities, eliminate them. Quantity A is now equal to $7xy$ and Quantity B is now equal to $8xy$. Because $xy > 0$, Quantity B is greater. (Don't assume! If xy were zero, the two quantities would have been equal. If xy were negative, Quantity A would have been greater.)

16. **(D).** Factor $x^2 - 2x = 0$:

$$x^2 - 2x = 0$$
$$x(x - 2) = 0$$
$$x = 0 \text{ OR } (x - 2) = 0$$

$x = 0$ or 2.

Thus, Quantity A could be less than or equal to Quantity B. The relationship cannot be determined from the information given.

(Note that you *cannot* simply divide both sides of the original equation by x. It is illegal to divide by a variable unless it is certain that the variable does not equal zero.)

17. **(D).** In Quantity A, multiply d by every term in the parentheses:

$$d(d^2 - 2d + 1) =$$
$$(d \times d^2) - (d \times 2d) + (d \times 1) =$$
$$d^3 - 2d^2 + d$$

In Quantity B, multiply d by the two terms in the parentheses:

$$d(d^2 - 2d) + 1 =$$
$$(d \times d^2) - (d \times 2d) + 1 =$$
$$d^3 - 2d^2 + 1$$

Because $d^3 - 2d^2$ is common to both quantities, it can be ignored. The comparison is really between d and 1. Without more information about d, the relationship cannot be determined from the information given.

18. **(C).** In Quantity A, the term xy^2z on the outside of the parentheses must be multiplied by each of the three terms inside the parentheses. Then simplify the expression as much as possible.

Taking one term at a time, the first is $xy^2z \times x^2z = x^3y^2z^2$, because there are three factors of x, two factors of y, and two factors of z. Similarly, the second term is $xy^2z \times yz^2 = xy^3z^3$ and the third is $xy^2z \times (-xy^2) = -x^2y^4z$. Adding these three terms together gives the distributed form of Quantity A: $x^3y^2z^2 +$

$xy^3z^3 - x^2y^4z$.

This is identical to Quantity B, so the two quantities are equal.

19. **(D).** Since a is common to both quantities, it can be ignored. The comparison is really between b and c. Because $2b = 4c$, it is true that $b = 2c$, so the comparison is really between $2c$ and c. Watch out for negatives. If the variables are positive, Quantity A is greater, but if the variables are negative, Quantity B is greater.

20. **(D).** If the variables are positive, Quantity A is greater. However, all three variables could equal zero, in which case the two quantities are equal. Watch out for the word "non-negative," which means "positive or zero."

21. **(C).** The following relationships are given: $a = \frac{b}{2}$, $b = \frac{c}{3}$, and $d = 3c$. Pick one variable and put everything in terms of that variable. For instance, variable a:

$b = 2a$
$c = 3b = 3(2a) = 6a$
$d = 3c = 3(6a) = 18a$

Substitute into the quantities and simplify.

Quantity A: $\frac{a+b}{c} = \frac{a+2a}{6a} = \frac{3a}{6a} = \frac{1}{2}$

Quantity B: $\frac{a+b+c}{d} = \frac{a+2a+6a}{18a} = \frac{9a}{18a} = \frac{1}{2}$

The two quantities are equal.

22. **$|x| = |y|$ and $\frac{x^2}{y^2} = 1$.** Since $x^2 - y^2 = 0$, add y^2 to both sides to get $x^2 = y^2$. It might look as though $x = y$, but this is not necessarily the case. For example, x could be 2 and y could be -2. Algebraically, taking the square root of both sides of $x^2 = y^2$ does *not* yield $x = y$, but rather $|x| = |y|$. Thus, the 1st statement is not necessarily true and the 2nd statement is true. The 3rd statement is also true and can be generated algebraically:

$$\begin{aligned} x^2 - y^2 &= 0 \\ x^2 &= y^2 \\ \frac{x^2}{y^2} &= 1 \end{aligned}$$

23. **(C).** This question may at first look difficult, as there are three variables and only two equations.

However, notice that the top equation can be divided by 3, yielding $x + 2y = 9$. This can be plugged into the second equation:

$$\begin{aligned} (x + 2y) + z &= 11 \\ (9) + z &= 11 \\ z &= 2 \end{aligned}$$

Quantity A is thus $2 + 5 = 7$. For Quantity B, remember that $x + 2y = 9$. Thus, Quantity B is $9 – 2 = 7$.

The two quantities are equal.

24. **(B).** The factored form of the Difference of Squares (one of the "special products" you need to memorize for the exam) is comprised of the terms given in this problem:

$$x^2 - y^2 = (x + y)(x - y)$$

Substitute the values $\sqrt{12}$ and $\sqrt{3}$ in place of $(x - y)$ and $(x + y)$, respectively:

$$x^2 - y^2 = \sqrt{12} \times \sqrt{3}$$

Combine 12 and 3 under the same root sign and solve:

$$\begin{aligned} x^2 - y^2 &= \sqrt{12} \times \sqrt{3} \\ x^2 - y^2 &= \sqrt{36} \\ x^2 - y^2 &= 6 \end{aligned}$$

25. **(B).** Plug in any two unequal values for a and b, and Quantity A will always be equal to -1. This is because a negative sign can be factored out of the top or bottom of the fraction to show that the top and bottom are the same, except for their signs:

$$\frac{a-b}{b-a} = \frac{a-b}{-(a-b)} = -1$$

26. **(D).** To compare a and c, put c in terms of a. Multiply the first equation by 2 to find that $b = 2a$. Substitute into the second equation: $c = 3b = 3(2a) = 6a$. If all three variables are positive, then $6a > a$. If all three variables are negative, then $a > 6a$. Finally, all three variables could equal zero, making the two quantities equal.

27. **(C).** The Difference of Squares (one of the "special products" you need to memorize for the exam) is $x^2 - y^2 = (x + y)(x - y)$. This pattern works for any perfect square minus another perfect square. Thus, $x^{36} - y^{36}$ will factor according to this pattern. Note that $\sqrt{x^{36}} = \left(x^{36}\right)^{1/2} = x^{36/2} = x^{18}$, or $x^{36} = (x^{18})^2$. First, factor $x^{36} - y^{36}$ in the numerator, then cancel $x^{18} + y^{18}$ with the $x^{18} + y^{18}$ on the bottom:

$$\frac{x^{36} - y^{36}}{\left(x^{18} + y^{18}\right)\left(x^{9} + y^{9}\right)} = \frac{\cancel{\left(x^{18} + y^{18}\right)}\left(x^{18} - y^{18}\right)}{\cancel{\left(x^{18} + y^{18}\right)}\left(x^{9} + y^{9}\right)} = \frac{\left(x^{18} - y^{18}\right)}{\left(x^{9} + y^{9}\right)}$$

The $x^{18} - y^{18}$ in the numerator will also factor according to this pattern. Then cancel $x^9 + y^9$ with the

$x^9 + y^9$ on the bottom:

$$\frac{(x^{18} - y^{18})}{(x^9 + y^9)} = \frac{\cancel{(x^9 + y^9)}(x^9 - y^9)}{\cancel{(x^9 + y^9)}} = x^9 - y^9$$

28. **(B).** First, recognize that $x^2 + 2xy + y^2 = (x + y)^2$. This is one of the "special products" you need to memorize for the exam. Factor the top, then cancel:

$$\frac{x^2+2xy+y^2}{2(x+y)^2}=\frac{\cancel{(x+y)^2}}{2\cancel{(x+y)^2}}=\frac{1}{2}$$

29. (**D**). It is possible to simplify first and then plug in examples, or to just plug in examples without simplifying. For instance, if $x = 2$ and $y = 1$:

Quantity A: $$\frac{x^2}{y+\frac{1}{y}}=\frac{2^2}{1+\frac{1}{1}}=\frac{4}{2}=2$$

Quantity B: $$\frac{y^2}{x+\frac{1}{x}}=\frac{1^2}{2+\frac{1}{2}}=\frac{1}{\frac{5}{2}}=\frac{2}{5}$$

In this case, Quantity A is greater. Next, try negatives. If $x = -1$ and $y = -2$ (remember, x must be greater than y):

Quantity A: $$\frac{x^2}{y+\frac{1}{y}}=\frac{(-1)^2}{-2+\frac{1}{-2}}=\frac{1}{\frac{5}{-2}}=\frac{-2}{5}$$

Quantity B: $$\frac{y^2}{x+\frac{1}{x}}=\frac{(-2)^2}{(-1)+\frac{1}{-1}}=\frac{4}{-2}=-2$$

Quantity A is still greater. However, before assuming that Quantity A is *always* greater, make sure you have tried every category of possibilities for x and y. What if x is positive and y is negative? For instance, $x = 2$ and $y = -2$:

Quantity A: $$\frac{x^2}{y+\frac{1}{y}}=\frac{2^2}{-2+\frac{1}{-2}}=\frac{4}{-\frac{5}{2}}=4\times-\frac{2}{5}=-\frac{8}{5}$$

Quantity B: $$\frac{y^2}{x+\frac{1}{x}} = \frac{(-2)^2}{(2)+\frac{1}{2}} = \frac{4}{\frac{5}{2}} = 4 \times \frac{2}{5} = \frac{8}{5}$$

In this case, Quantity B is greater. The relationship cannot be determined from the information given.

30. –**3.** One of the “special products” you need to memorize for the GRE is $x^2 + 2xy + y^2 = (x + y)^2$. Write this pattern on your paper, plug in the given values, and simplify, solving for 2xy:

$$\begin{aligned} x^2 + 2xy + y^2 &= (x + y)^2 \\ (x^2 + y^2) + 2xy &= (x + y)^2 \\ (12) + 2xy &= (-3)^2 \\ 12 + 2xy &= 9 \\ 2xy &= -3 \end{aligned}$$

31. **6.** The Difference of Squares (one of the "special products" you need to memorize for the exam) is $x^2 - y^2 = (x + y)(x - y)$. Write this pattern on your paper and plug in the given values, solving for $x + y$:

$$\begin{aligned} x^2 - y^2 &= (x + y)(x - y) \\ 3 &= (x + y)(1/2) \\ 6 &= x + y \end{aligned}$$

32. **4.** One of the "special products" you need to memorize for the exam is $x^2 - 2xy + y^2 = (x - y)^2$. Write this pattern on your paper and plug in the given values:

$$\begin{aligned} x^2 - 2xy + y^2 &= (x - y)^2 \\ 84 + y^2 &= (-10)^2 \\ 84 + y^2 &= 100 \\ y^2 &= 16 \\ y &= 4 \text{ or } -4, \text{ so } |y| = 4. \end{aligned}$$

33. **(B)**. First, multiply out (remember FOIL = First, Outer, Inner, Last) each of the terms in parentheses:

$$(x^2 - 2x - 2x + 4) + (x^2 - 1x - 1x + 1) + (x^2) + (x^2 + 1x + 1x + 1) + (x^2 + 2x + 2x + 4)$$

Note that some of the terms will cancel each other out (e.g., $-x$ and x, $-2x$ and $2x$):

$$(x^2 + 4) + (x^2 + 1) + (x^2) + (x^2 + 1) + (x^2 + 4)$$

Finally, combine:

$$5x^2 + 10$$

34. ***a* > *b* and *a* is positive.** Distribute for a: $a = (x + y)^2 = x^2 + 2xy + y^2$. Since $b = x^2 + y^2$, a and b are the same except for the "extra" $2xy$ in a. Since xy is positive, a is greater than b. The 1st statement is false and the 2nd statement is true.

Each term in the sum for a is positive: xy is given as positive, and x^2 and y^2 are definitely positive, as they are squared and not equal to zero. Therefore, $a = x^2 + 2xy + y^2$ is positive. The 3rd statement is true.

35. **(B).** To answer this question, it is important to understand what is meant by the phrase "directly

proportional." It means that $a = kb$, where k is a constant. In alternative form: $\frac{a}{b} = k$, where k is a constant.

So, because they both equal the constant, $\frac{a_{\text{old}}}{b_{\text{old}}} = \frac{a_{\text{new}}}{b_{\text{new}}}$. Plugging in values: $\frac{8}{2} = \frac{a_{\text{new}}}{4}$. Cross-multiply and solve:

$$32 = 2a_{\text{new}}$$
$$a_{\text{new}} = 16$$