PERCENTS

The other major way to express a part–whole relationship (in addition to decimals and fractions) is to use percents. Percent literally means "per one hundred." One can conceive of percents as simply a special type of fraction or decimal that involves the number 100.

75% of the students like chocolate ice cream.

This means that, out of every 100 students, 75 like chocolate ice cream. In fraction form, we write this as $\frac{75}{100}$, which simplifies to $\frac{3}{4}$.

In decimal form, we write this as 0.75 or seventy–five hundredths. Note that the last digit of the percent is in the ***hundredths*** place value.

One common error is to mistake 100% for 100. This is not correct. In fact, 100% means $\frac{100}{100}$, or one hundred hundredths. Therefore, 100% = 1.

Percent problems occur frequently on the GRE. The key to these percent problems frequntly is to make them concrete by picking **real numbers** with which to work.

Percents as Fractions: The Percent Table

A simple but useful way of structuring basic percent problems on the GRE is by relating percents to fractions through a percent table as shown below.

	Numbers	Percentage Fraction
PART		
WHOLE		100

A PART is some PERCENT of a WHOLE.

$$\frac{\text{PART}}{\text{WHOLE}} = \frac{\text{PERCENT}}{100}$$

Example 1: What is 30% of 80?

We are given the whole amount and the percent, and we are looking for the part. First, we fill in the percent table. Then we set up a proportion, cancel, cross–multiply, and solve:

PART	x	30
WHOLE	80	100

$$\frac{x}{80} = \frac{3\cancel{0}}{10\cancel{0}} = \frac{3}{10} \qquad 10x = 240 \qquad x = 24$$

We can also solve this problem using decimal equivalents: $(0.30)(80) = (3)(8) = 24$

Example 2: 75% of what number is 21?

We are given the part and the percent, and we are looking for the whole amount. First, we fill in the percent table. Then we set up a proportion, cancel, cross–multiply, and solve:

PART	21	75
WHOLE	x	100

$$\frac{21}{x} = \frac{75}{100} = \frac{3}{4} \qquad 3x = 84 \qquad x = 28$$

Likewise, we can also solve this problem using decimal equivalents:

$(0.75)x = 21$ then move the decimal → $75x = 2{,}100$ $x = 28$

Example 3: 90 is what percent of 40?

This time we are given the part and the whole amount, and we are looking for the percent. Note that the "part" (90) is BIGGER than the "whole" (40). That is potentially confusing but can happen, so watch the wording of the question *carefully*. Just make sure that you are taking the percent OF the "whole." Here, we are taking a percent OF 40, so 40 is the "whole."

First, we fill in the percent table. Then we set up a proportion again and solve:

PART	90	x
WHOLE	40	100

$\frac{9\cancel{0}}{4\cancel{0}} = \frac{9}{4} = \frac{x}{100}$ $4x = 900$ $x = 225$

90 is 225% of 40. Notice that you wind up with a percent BIGGER than 100%. That is what you should expect when the "part" is bigger than the "whole."

Check Your Skills

1. 84 is 70% of what number?
2. 30 is what percent of 50?

Answers can be found on page 97.

Benchmark Values: 10% and 5%

To find 10% of any number, just move the decimal point to the left one place.

10% of 500 is 50 10% of 34.99 = 3.499 10% of 0.978 is 0.0978

Once you know 10% of a number, it is easy to find 5% of that number: 5% is half of 10%.

10% of 300 is 30 5% of 300 is $30 \div 2 = 15$

These quick ways of calculating 10% and 5% of a number can be useful for more complicated percentages.

What is 35% of 640?

Instead of multiplying 640 by 0.35, begin by finding 10% and 5% of 640.

10% of 640 is 64 5% of 640 is $64 \div 2 = 32$

35% of a number is the same as (3 × 10% of a number) + (5% of a number).

$3 \times 64 + 32 = 192 + 32 = 224$

You can also use the benchmark values to estimate percents. For example:

Karen bought a new television, originally priced at $690. However, she had a coupon that saved her $67. For what percent discount was Karen's coupon?

You know that 10% of 690 would be 69. Therefore, 67 is slightly less than 10% of 690.

Check Your Skills

3. What is 10% of 145.028?
4. What is 20% of 73?

Answers can be found on page 97.

Percent Increase and Decrease

Some percent problems involve the concept of percent change. For example:

The price of a cup of coffee increased from 80 cents to 84 cents. By what percent did the price change?

Percent change problems can be solved using our handy percent table, with a small adjustment. The price ***change*** (84 – 80 = 4 cents) is considered the part, while the ***original*** price (80 cents) is considered the whole.

CHANGE	4	x
ORIGINAL	80	100

$$\frac{\text{CHANGE}}{\text{ORIGINAL}} = \frac{\text{PERCENT}}{100}$$

$\frac{\cancel{4}}{\cancel{80}} = \frac{1}{20} = \frac{x}{100}$ $\quad 20x = 100 \quad x = 5$ Thus, the price increased by 5%.

By the way, do not forget to divide by the original! The percent change is NOT 4%, which may be a wrong answer choice.

Alternatively, a question might be phrased as follows:

If the price of a $30 shirt decreased by 20%, what was the final price of the shirt?

The whole is the original price of the shirt. The percent change is 20%. In order to find the answer, we must first find the part, which is the amount of the decrease:

CHANGE	x	20
ORIGINAL	30	100

$\frac{x}{30} = \frac{\cancel{20}}{\cancel{100}} = \frac{1}{5}$ $\quad 5x = 30 \quad x = 6$

Therefore, the price of the shirt decreased by $6. The final price of the shirt was $30 – $6 = $24.

Check Your Skills

5. A GRE score (math + verbal) increased from 1250 to 1600. By what percent did the score increase?
6. 15% of the water in a full 30 gallon drum evaporated. How much water is remaining?

Answers can be found on page 97.

Percent Change vs. Percent of Original

Looking back at the cup of coffee problem, we see that the new price (84 cents) was higher than the original price (80 cents).

We can ask what percent OF the original price is represented by the new price.

$$\frac{\cancel{84}}{\cancel{80}} = \frac{21}{20} = \frac{x}{100} \qquad 20x = 2{,}100 \qquad x = 105$$

Thus, the new price is 105% OF the original price. Remember that the percent CHANGE is 5%. That is, the new price is 5% HIGHER THAN the original price. There is a fundamental relationship between these numbers, resulting from the simple idea that the CHANGE equals the NEW value minus the ORIGINAL value, or equivalently, ORIGINAL + CHANGE = NEW:

If a quantity is increased by x percent, then the new quantity is $(100 + x)$% OF the original. Thus a 15% increase produces a quantity that is 115% OF the original.

We can write this relationship thus: $\text{ORIGINAL} \times \left(1 + \frac{\text{Percent Increase}}{100}\right) = \text{NEW}$

In the case of the cup of coffee, we see that $80 \times \left(1 + \frac{5}{100}\right) = 80(1.05) = 84$.

Likewise, in the shirt problem, we had a 20% decrease in the price of a \$30 shirt, resulting in a new price of \$24.

The new price is some percent OF the old price. Let us calculate that percent.

$$\frac{\cancel{24}}{\cancel{30}} = \frac{4}{5} = \frac{x}{100} \qquad 5x = 400 \qquad x = 80$$

Thus, the new price (20% LESS THAN the original price) is 80% OF the original price.

If a quantity is decreased by x percent, then the new quantity is $(100 - x)$% OF the original. Thus a 15% decrease produces a quantity that is 85% OF the original.

We can write this relationship thus: $\text{ORIGINAL} \times \left(1 - \frac{\text{Percent Decrease}}{100}\right) = \text{NEW}.$

In the case of the shirt, we see that $30 \times \left(1 - \frac{20}{100}\right) = 30(0.80) = 24$.

These formulas are all just another way of saying ORIGIANL ± CHANGE = NEW.

Example 4: What number is 50% greater than 60?

The <u>whole</u> amount is the original value, which is 60. The percent <u>change</u> (i.e., the percent "greater than") is 50%. In order to find the answer, we must first find the <u>part</u>, which is the amount of the increase:

CHANGE	x	50
ORIGINAL	60	100

$$\frac{x}{60} = \frac{\cancel{50}}{\cancel{100}} = \frac{1}{2} \qquad 2x = 60 \qquad x = 30$$

We know that ORIGINAL ± CHANGE = NEW. Therefore, the number that is 50% greater than 60 is 60 + 30 = 90, which is also 150% of 60.

We could also solve this problem using the formula: $\text{ORIGINAL} \times \left(1 + \frac{\text{Percent Increase}}{100}\right) = \text{NEW}$

$$60\left(1+\frac{50}{100}\right) = 60(1.5) = 90$$

Example 5: What number is 150% greater than 60?

The whole amount is the original value, which is 60. The percent change (i.e., the percent "greater than") is 150%. In order to find the answer, we must first find the part, which is the amount of the increase:

CHANGE	x	150
ORIGINAL	60	100

$$\frac{x}{60} = \frac{\cancel{150}}{\cancel{100}} = \frac{3}{2} \qquad 2x = 180 \qquad x = 90$$

Now, x is the CHANGE, NOT the new value! **It is easy to forget to add back the original amount when the percent change is more than 100%.** Thus, the number that is 150% greater than 60 is 60 + 90 = 150, which is also 250% of 60.

We could also solve this problem using the formula: $\text{ORIGINAL} \times \left(1 + \frac{\text{Percent Increase}}{100}\right) = \text{NEW}$

$$60\left(1+\frac{150}{100}\right) = 60(2.5) = 150$$

Check Your Skills

7. A plant originally cost \$35. The price is increased by 20%. What is the new price?
8. 70 is 250% greater than what number?

Answers can be found on page 97–98.

Successive Percents

One of the GRE's favorite tricks involves successive percents.

If a ticket increased in price by 20%, and then increased again by 5%, by what percent did the ticket price increase in total?

Although it may seem counterintuitive, the answer is NOT 25%.

To understand why, consider a concrete example. Let us say that the ticket initially cost \$100. After increasing by 20%, the ticket price went up to \$120 (\$20 is 20% of \$100).

Here is where it gets tricky. The ticket price goes up again by 5%. However, it increases by 5% of the **NEW PRICE** of \$120 (not 5% of the ***original*** \$100 price). 5% of \$120 is 0.05(120) = \$6. Therefore, the final price of the ticket is \$120 + \$6 = \$126, not \$125.

You can now see that two successive percent increases, the first of 20% and the second of 5%, DO NOT result in a combined 25% increase. In fact, they result in a combined 26% increase (because the ticket price increased from \$100 to \$126).

Successive percents CANNOT simply be added together! This holds for successive increases, successive decreases, and for combinations of increases and decreases. If a ticket goes up in price by 30% and then goes down by 10%, the price has NOT in fact gone up a net of 20%. Likewise, if an index increases by 15% and then falls by 15%, it does NOT return to its original value! (Try it—you will see that the index is actually <u>down</u> 2.25% overall!)

A great way to solve successive percent problems is to choose real numbers and see what happens. The preceding example used the real value of \$100 for the initial price of the ticket, making it easy to see exactly what happened to the ticket price with each increase. **Usually, 100 will be the easiest real number to choose for percent problems.** We will explore this in greater detail in the next section.

You could also solve by converting to decimals. Increasing a price by 20% is the same as multiplying the price by 1.20.

Increasing the new price by 5% is the same as multiplying that new price by 1.05.

Thus, you can also write the relationship this way:

ORIGINAL × (1.20) × (1.05) = FINAL PRICE

When you multiply 1.20 by 1.05, you get 1.26, indicating that the price increased by 26% overall.

This approach works well for problems that involve many successive steps (e.g., compound interest, which we will address later). However, in the end, it is still usually best to pick \$100 for the original price and solve using concrete numbers.

Check Your Skills

9. If your stock portfolio increased by 25% and then decreased by 20%, what percent of the original value would your new stock portfolio have?

Answer can be found on page 98.

Smart Numbers: Pick 100

Sometimes, percent problems on the GRE include unspecified numerical amounts; often these unspecified amounts are described by variables.

> A shirt that initially cost *d* dollars was on sale for 20% off. If *s* represents the sale price of the shirt, *d* is what percentage of *s*?

This is an easy problem that might look confusing. To solve percent problems such as this one, simply pick 100 for the unspecified amount (just as we did when solving successive percents).

If the shirt initially cost $100, then $d = 100$. If the shirt was on sale for 20% off, then the new price of the shirt is $80. Thus, $s = 80$.

The question asks: d is what percentage of s, or 100 is what percentage of 80? Using a percent table, we fill in 80 as the whole amount and 100 as the part. We are looking for the percent, so we set up a proportion, cross-multiply, and solve:

PART	100	x
WHOLE	80	100

$$\frac{100}{80} = \frac{x}{100} \qquad 80x = 10{,}000 \qquad x = 125$$

Therefore, d is 125% of s.

The important point here is that, like successive percent problems and other percent problems that include unspecified amounts, this example is most easily solved by plugging in a real value. For percent problems, the easiest value to plug in is generally 100. **The fastest way to success with GRE percent problems with unspecified amounts is to pick 100 as a value.** (Note that, as we saw in the fractions chapter, if any amounts are specified, we cannot pick numbers—we must solve the problem algebraically.)

Check Your Skills

10. If your stock portfolio decreased by 25% and then increased by 20%, what percent of the original value would your new stock portfolio have?

Answer can be found on page 98.

Interest Formulas: Simple and Compound

Certain GRE percent problems require a working knowledge of basic interest formulas. The compound interest formula may look complicated, but it just expresses the idea of "successive percents" for a number of periods.

Especially for compound interest questions, be prepared to use the GRE onscreen calculator to help with the math involved!

	Formula	**Example**
SIMPLE INTEREST	Principal × Rate × Time	\$5,000 invested for 6 months at an annual rate of 7% will earn \$175 in simple interest. Principal = \$5,000, Rate = 7% or 0.07, Time = 6 months or 0.5 years. ***Prt* = \$5,000(0.07)(0.5) = \$175**
COMPOUND INTEREST	$P\left(1+\frac{r}{n}\right)^{nt}$, where P = principal, r = rate (decimal) n = **number of times per year** t = **number of years**	\$5,000 invested for 1 year at a rate of 8% compounded quarterly will earn approximately \$412: $\$5{,}000\left(1+\frac{0.08}{4}\right)^{4(1)} = \$5{,}412$ (or \$412 of interest)

Check Your Skills

11. Assume an auto loan in the amount of $12,000 is made. The loan carries an interest charge of 14%. What is the amount of interest owed in the first three years of the loan, assuming there is no compounding?
12. For the same loan, what is the loan balance after 3 years assuming no payments on the loan, and annual compounding?
13. For the same loan, what is the loan balance after 3 years assuming no payments, and quarterly compounding?

Answers can be found on page 98.

Check Your Skills Answer Key

1. **120:**

PART	84	70
WHOLE	x	100

$$\frac{84}{x} = \frac{\cancel{70}}{\cancel{100}} = \frac{7}{10} \qquad 7x = 840 \qquad x = 120$$

2. **60:**

PART	30	x
WHOLE	50	100

$$\frac{x}{100} = \frac{\cancel{30}}{\cancel{50}} = \frac{3}{5} \qquad 5x = 300 \qquad x = 60$$

3. **14.5028:** Move the decimal to the left one place. 145.028 → 14.5028

4. **14.6:** To find 20% of 73, first find 10% of 73. Move the decimal to the left one place. 73 → 7.3. 20% is twice 10%:

$7.3 \times 2 = 14.6$

5. **28%:** First find the change: 1600 – 1250 = 350.

$$\frac{\text{CHANGE}}{\text{ORIGINAL}} = \frac{\cancel{350}}{\cancel{1250}} = \frac{7}{25} = \frac{7 \times 4}{25 \times 4} = \frac{28}{100} = 28\%$$

6. **25.5:**

CHANGE	x	15
ORIGINAL	30	100

$$\frac{x}{30} = \frac{\cancel{15}}{\cancel{100}} = \frac{3}{20} \qquad 20x = 90 \qquad x = 4.5$$

However, the question asks how much water is <u>remaining</u>. 4.5 gallons have evaporated, so 30 – 4.5 = 25.5 gallons remain.

7. **42:** Recall that $\text{ORIGIANL} \times \left(1 + \frac{\text{Percent Increase}}{100}\right) = \text{NEW}$.

$$35 \times \left(1 + \frac{20}{100}\right) = 35(1.2) = 4.2$$

8. **20:** Recall that $\text{ORIGIANAL} \times \left(1 + \frac{\text{Percent Increase}}{100}\right) = \text{NEW}$. Designate the original value x.

$$x \times \left(1+\frac{250}{100}\right)=70$$

$$3.5x = 70$$

$$x = 20$$

9. **100%:** Pick 100 for the original value of the portfolio. A 25% increase is:

$$100\left(1+\frac{25}{100}\right)=100(1.25)=125.$$

A 20% decrease is:

$$125\left(1-\frac{20}{100}\right)=125(0.8)=100.$$

The final value is 100. Because the starting value was also 100, the portfolio is 100% of its original value.

10. **90%:** Pick 100 for the original value of the portfolio. A 25% decrease is:

$$100\left(1-\frac{25}{100}\right)=100(0.75)=75.$$

A 20% increase is:

$$75\left(1+\frac{20}{100}\right)=75(1.2)=90.$$

The final value is 90 and the original value was 100. $\frac{90}{100}=90\%$ of the original value.

11. **\$5,040:** $P \times r \times t = \$12{,}000 \times 14\% \times 3 = \$5{,}040$.

12. **\$17,778.53:** $P\left(1+\frac{r}{n}\right)^{nt}$, where P – \$12,000, $r = 14\%$, $n = 1$ (annual compounding), and $t = 3$ years.

$\$12{,}000\left(1+\frac{14\%}{1}\right)^{1\times3} = \$12{,}000\times(1.14)^3 = \$17{,}778.53$ (rounded to the nearest penny). This represents \$17,778.53 – \$12,000 = \$5,778.53 in interest.

13. **\$18,132.82:** $P\left(1+\frac{r}{n}\right)^{nt}$, where $P = \$12{,}000$, $r = 14\%$, $n = 4$ (quarterly compounding), and $t = 3$ years.

$\$12{,}000\left(1+\frac{14\%}{4}\right)^{4\times3} = \$12{,}000\times(1.035)^{12} = \$18{,}132.82$ (rounded to the nearest penny). This represents \$18,132.82 – \$12,000 = \$6,132.82 in interest.

Problem Set

Solve the following problems. Use a percent table to organize percent problems, and pick 100 when dealing with unspecified amounts.

1. x% of y is 10. y% of 120 is 48. What is x?

2. A stereo was marked down by 30% and sold for $84. What was the pre-sale price of the stereo?

3. From 1980 to 1990, the population of Mitannia increased by 6%. From 1991 to 2000, it decreased by 3%. What was the overall percentage change in the population of Mitannia from 1980 to 2000?

4. If y is decreased by 20% and then increased by 60%, what is the new number, expressed in terms of y?

5. A 7% car loan, which is compounded annually, has an interest payment of $210 after the first year. What is the principal on the loan?

6. A bowl was half full of water. 4 cups of water were then added to the bowl, filling the bowl to 70% of its capacity. How many cups of water are now in the bowl?

7. A large tub is filled with 920 liters of alcohol and 1,800 liters of water. 40% of the water evaporates. What percent of the remaining liquid is water?

8. x is 40% of y. 50% of y is 40. 16 is what percent of x?

9. 800, increased by 50% and then decreased by 30%, yields what number?

10. If 1,600 is increased by 20%, and then reduced by y%, yielding 1,536, what is y?

11. Lori deposits $100 in a savings account at 2% interest, compounded annually. After 3 years, what is the balance on the account? (Assume Lori makes no withdrawals or deposits.)

12.

Steve uses a certain copy machine that reduces an image by 13%.

<u>Quantity A</u>	<u>Quantity B</u>
The percent of the original if Steve reduces the image by another 13%	75%

13.

y is 50% of *x*% of *x*.

Quantity A	Quantity B
y	x

14.

Quantity A	Quantity B
10% of 643.38	20% of 321.69

1. **25:** We can use two percent tables to solve this problem. Begin with the fact that y% of 120 is 48:

PART	48	y
WHOLE	120	100

$$4{,}800 = 120y$$
$$y = 40$$

Then, set up a percent table for the fact that x% of 40 is 10.

PART	10	x
WHOLE	40	100

$$1{,}000 = 40x$$
$$x = 25$$

We can also set up equations with decimal equivalents to solve:
$(0.01y)(120) = 48$, so $1.2\,y = 48$ or $y = 40$. Therefore, since we know that $(0.01x)(y) = 10$, we have:

$(0.01x)(40) = 10$ $\qquad 40x = 1{,}000$ $\qquad x = 25$

2. **$120:** We can use a percent table to solve this problem. Remember that the stereo was marked down 30% from the original, so we have to solve for the original price.

CHANGE	x	30
ORIGINAL	$84 + x	100

$$\frac{x}{84+x} = \frac{30}{100} \qquad 100x = 30(84+x) \qquad 100x = 30(84) + 30x$$
$$70x = 30(84) \qquad x = 36$$

Therefore, the original price was (84 + 36) = $120.

We could also solve this problem using the formula: $\text{ORIGIANAL} \times \left(1 - \frac{\text{Percent Decrease}}{100}\right) = \text{NEW}$

$$x\left(1-\frac{30}{100}\right) = 84 \qquad 0.7x = 84 \qquad x = 120$$

3. **2.82% increase:** For percent problems, the Smart Number is 100. Therefore, assume that the population of Mitannia in 1980 was 100. Then, apply the successive percents procedure to find the overall percent change:

From 1980–1990, there was a 6% increase: $100(1 + 0.06) = 100(1.06) = 106$
From 1991–2000, there was a 3% decrease: $106(1 - 0.03) = 106(0.97) = 102.82$
Overall, the population increased from 100 to 102.82, representing a 2.82% increase.

4. **1.28*y*:** For percent problems, the Smart Number is 100. Therefore, assign y a value of 100. Then, apply the successive percents procedure to find the overall percentage change:

(1) y is decreased by 20%: $100(1 - 0.20) = 100(0.8) = 80$
(2) Then, it is increased by 60%: $80(1 + 0.60) = 80(1.6) = 128$
Overall, there was a 28% increase. If the original value of y is 100, the new value is $1.28y$.

5. **$3,000:** We can use a percent table to solve this problem, which helps us find the decimal equivalent equation.

PART	210	7
WHOLE	x	100

$21{,}000 = 7x$

$x = 3{,}000$

6. **14:** For some problems we cannot use Smart Numbers, since the total amount can be calculated. This is one of those problems. Instead, use a percent table:

PART	$0.5x + 4$	70
WHOLE	x	100

$\frac{0.5x+4}{x} = \frac{70}{100} = \frac{7}{10}$

$5x + 40 = 7x$

$40 = 2x$

$x = 20$

The capacity of the bowl is 20 cups. There are 14 cups in the bowl {70% of 20, or 0.5(20) + 4}.

PART	4	20
WHOLE	x	100

Alternately, the 4 cups added to the bowl represent 20% of the total capacity. Use a percent table to solve for x, the whole. Since $x = 20$, there are 14 (50% of 20 + 4) cups in the bowl.

7. **54%:** For this liquid mixture problem, set up a table with two columns: one for the original mixture and one for the mixture after the water evaporates from the tub.

	Original	After Evaporation
Alcohol	920	920
Water	1,800	0.60(1,800) = 1,080
TOTAL	2,720	2,000

The remaining liquid in the tub is $\frac{1{,}080}{2{,}000}$, or 54%, water.

We could also solve for the new amount of water using the formula:

$$\text{ORIGIANAL} \times \left(1 - \frac{\text{Percent Decrease}}{100}\right) = \text{NEW}$$

$1{,}800\left(1 - \frac{40}{100}\right) = (1{,}800)(0.6) = 1{,}080$ units of water. Water is $\frac{1{,}080}{920 + 1{,}080} = \frac{1{,}080}{2{,}000} = 54\%$ of the total.

8. **50%:** Use two percent tables to solve this problem. Begin with the fact that 50% of y is 40:

PART	40	50
WHOLE	y	100

$4{,}000 = 50y$

$y = 80$

Then, set up a percent table for the fact that x is 40% of y.

PART	x	40
WHOLE	80	100

$3{,}200 = 100x$

$x = 32$

Finally, 16 is 50% of 32. We could alternatively set up equations with decimal equivalents to solve: $x = (0.4)y$ We also know that $(0.5)y = 40$, so $y = 80$ and $x = (0.4)(80) = 32$. Therefore, 16 is half, or 50%, of x.

9. **840:** Apply the successive percents procedure to find the overall percentage change:

(1) 800 is increased by 50%: $800 \times 1.5 = 1{,}200$

(2) Then, the result is decreased by 30%: $1{,}200 \times 0.7 = 840$

10. **20:** Apply the percents in succession with two percent tables.

PART	x	120
WHOLE	1,600	100

$$192{,}000 = 100x$$
$$x = 1{,}920$$

Then, fill in the "change" for the part (1,920 – 1,536 = 384) and the original for the whole (1,920).

PART	384	y
WHOLE	1,920	100

$$1{,}920y = 38{,}400$$
$$y = 20$$

Alternatively we could solve for the new number using formulas. Because this is a successive percents problem, we need to "chain" the formula: once to reflect the initial increase in the number, then twice to reflect the subsequent decrease:

$$\text{ORIGINAL} \times \left(1 + \frac{\text{Percent Increase}}{100}\right) \times \left(1 - \frac{\text{Percent Decrease}}{100}\right) = \text{NEW}$$

$$1{,}600 \times \left(1 + \frac{20}{100}\right) \times \left(1 - \frac{y}{100}\right) = 1{,}536 \qquad 1{,}920 \times \left(1 - \frac{y}{100}\right) = 1{,}536 \qquad 1{,}920 - \frac{1{,}920y}{100} = 1{,}536$$

$$1{,}920 - 1{,}536 = 19.2y \qquad 384 = 19.2y \qquad 20 = y$$

11. **$106.12:** Interest compounded annually is just a series of successive percents:

(1) 100.00 is increased by 2%: $100(1.02) = 102$

(2) 102.00 is increased by 2%: $102(1.02) = 104.04$

(3) 104.04 is increased by 2%: $104.04(1.02) \cong 106.12$

12. **A:** In dealing with percents problems, we should choose 100. In this case, the original size of the image is 100. The question tells us that Steve reduces the image by 13%.

$$100 - 0.13(100) = 100 - 13 = 87$$

So our image is at 87 percent of its original size. Quantity A tells us that we have to reduce the image size by another 13%.

If the image size is reduced by 13%, then 87% of the image remains. Multiply 87 (the current size of the image) by 0.87 (87% expressed as a decimal).

$$87 \times 0.87 = 75.69$$

Quantity A	Quantity B
The percent of the original if Steve reduces the image by another 13% = **75.69%**	75%

Therefore **Quantity A is larger.**

13. **D:** First translate the statement in the question stem into an equation.

$$y = 50\% \times \frac{x}{100} \times x \longrightarrow y = 0.5 \times \frac{x}{100} \times x = \frac{x^2}{200} \longrightarrow 200y = x^2$$

Now try to pick some easy numbers. If $y = 1$, then $x = \sqrt{200}$, which is definitely bigger than 1.

Quantity A	Quantity B
$\boldsymbol{y = 1}$	$\boldsymbol{x = \sqrt{200}}$

However, if $y = 200$, then x must also equal 200.

Quantity A	Quantity B
$\boldsymbol{y = 200}$	$\boldsymbol{x = 200}$

y can be less than x, but y can also be <u>equal</u> to x. We could also choose values for which y is greater than x. Therefore **we do not have enough information** to answer the question.

14. **C:** To calculate 10% of 643.38, move the decimal to the left one place. 643.38 → 64.338

Quantity A	Quantity B
10% of 643.38 = **64.338**	20% of 321.69

To calculate 20% of 321.69, don't multiply by 0.2. Instead, find 10% first by moving the decimal to the left one place. 321.69 → 32.169

Now multiply by 2: 32.169 × 2 = 64.338

Quantity A	Quantity B
64.338	20% of 321.69 = **64.338**

Therefore **the two quantities are equal.**