Number Properties

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box $\boxed{}$, you are to enter your own answer in the box. For questions followed by a fraction-style numeric entry box $\frac{\boxed{}}{\boxed{}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is $\frac{1}{4}$, you may enter $\frac{25}{100}$ or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations, such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

On a number line, the distance from A to B is 4 and the distance from B to C is 5.

1.

Quantity A	Quantity B
The distance from A to C	9

a, b, c, and d are consecutive integers such that $a < b < c < d$.

2.

Quantity A	Quantity B
The average (arithmetic mean) of a, b, c, and d	The average (arithmetic mean)of b and c

3. w, x, y, and z are consecutive odd integers such that $w < x < y < z$. Which of the following statements must be true?

Indicate <u>all</u> such statements.

- ☐ $wxyz$ is odd
- ☐ $w + x + y + z$ is odd
- ☐ $w + z = x + y$

	Quantity A	Quantity B
4.	The sum of all the odd integers from 1 to 100, inclusive	The sum of all the even integers from 1 to 100, inclusive

$$x > 0 > y$$

5.

Quantity A	Quantity B
$x - y$	$(x + y)^2$

$$a < b < c < d < 0$$

6.

Quantity A	Quantity B
$a - d$	bc

7. If set S consists of all positive integers that are multiples of both 2 and 7, how many numbers in set S are between 140 and 240, inclusive?

$$ab > 0$$
$$bc < 0$$

8.

Quantity A	Quantity B
ac	0

$$abc < 0$$
$$b^2c > 0$$

9.

Quantity A	Quantity B
ab	0

a, b, and c are integers such that $a < b < c$.

10.

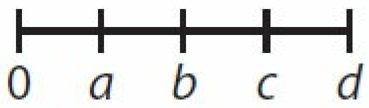
Quantity A	Quantity B
$\dfrac{a+b+c}{3}$	b

11. If $x^2 = y^2$, which of the following must be true?

- ☐ $x = y$
- ☐ $x^2 - y^2 = 0$
- ☐ $|x| - |y| = 0$

12. If $0 < a < \frac{1}{b} < 1$, then which of the following must be true?

(A) $a^2 > a > b > b^2$
(B) $b > a > a^2 > b^2$
(C) $b^2 > a > a^2 > b$
(D) $b^2 > a^2 > b > a$
(E) $b^2 > b > a > a^2$



	Quantity A	**Quantity B**
13.	$a \times c$	$b \times d$

	Quantity A	**Quantity B**
14.	The number of distinct factors of 32	The number of distinct factors of 20

15. If $y^2 = 4$ and $x^2y = 18$, which of the following values could equal $x + y$?

Indicate two such values.

- ☐ –5
- ☐ –1
- ☐ 1
- ☐ 5
- ☐ 6

	Quantity A	**Quantity B**
16.	The remainder when 10^{11} is divided by 2	The remainder when 3^{13} is divided by 3

q is odd.

	Quantity A	**Quantity B**
17.	$(-1)^q$	$(-1)^{q+1}$

n is a positive integer.

	Quantity A	**Quantity B**
18.	$(-1)^{4n} \times (-1)^{202}$	$(3)^3 \times (-5)^5$

19. If x is a positive integer, which one of the following could be the remainder when 73^x is divided by 10?

Indicate <u>all</u> such remainders.

- ☐ 0
- ☐ 1
- ☐ 2
- ☐ 3
- ☐ 4
- ☐ 5
- ☐ 6
- ☐ 7
- ☐ 8
- ☐ 9

20. If x, y, and z are integers, $y + z = 13$, and $xz = 9$, which of the following must be true?

(A) x is even
(B) $x = 3$
(C) y is odd
(D) $y > 3$
(E) $z < x$

	Quantity A	**Quantity B**
21.	The least prime number greater than 13	The greatest prime number less than 16

22. The average (arithmetic mean) of 11 integers is 35. What is the sum of all the integers?

☐

23. What is the sum of all the integers from 1 to 80, inclusive?

(A) 3,200
(B) 3,210
(C) 3,230
(D) 3,240
(E) 3,450

24. If p is the sum of all the integers from 1 to 150, inclusive, and q is the sum of all the integers from 1 to 148, inclusive, what is the value of $p - q$?

25. If m is the product of all the integers from 2 to 11, inclusive, and n is the product of all the integers from 4 to 11, inclusive, what is the value of $\frac{n}{m}$?

Give your answer as a fraction.

26. If $\sqrt{x}$ is an integer and $xy^2 = 36$, how many values are possible for the integer y?

(A) Two
(B) Three
(C) Four
(D) Six
(E) Eight

a, b, and c are positive even integers such that $8 > a > b > c$.

	Quantity A	**Quantity B**
27.	The range of a, b, and c	The average (arithmetic mean) of a, b, and c

28. If x is a non-zero integer and $0 < y < 1$, which of the following must be greater than 1?

(A) x
(B) $\frac{x}{y}$
(C) xy^2
(D) x^2y
(E) $\frac{x^2}{y}$

a, b, and c are consecutive integers such that $a < b < c < 4$.

	Quantity A	Quantity B
29.	The range of a, b, and c	The average of a, b, and c

$\sqrt{xy}$ is a prime number, xy is even, and $x > 4y > 0$.

	Quantity A	Quantity B
30.	y	1

x is even, $\sqrt{x}$ is a prime number, and $x + y = 11$.

	Quantity A	Quantity B
31.	x	y

The product of positive integers f, g, and h is even and the product of integers f and g is odd.

	Quantity A	Quantity B
32.	The remainder when f is divided by 2	The remainder when h is divided by 2

33. If x is odd, all EXCEPT which one of the following must be odd?

(A) $x^2 + 4x + 6$

(B) $x^3 + 5x + 3$

(C) $x^4 + 6x + 7$

(D) $x^5 + 7x + 1$

(E) $x^6 + 8x + 4$

$x^2 > 25$ and $x + y < 0$

	Quantity A	Quantity B
34.	x	y

The positive integer a is divisible by 2 and $0 < ab < 1$.

	Quantity A	Quantity B
35.	b	$\frac{1}{2}$

p and w are single-digit prime numbers such that $p + w < 6$. p^2 is odd.

	Quantity A	Quantity B
36.	w	3

$x^2 > y^2$ and $x > -|y|$

	Quantity A	Quantity B
37.	x	y

The sum of four consecutive integers is –2.

	Quantity A	Quantity B
38.	The smallest of the four integers	–2

39. If g is an integer and x is a prime number, which of the following must be an integer?

Indicate all such expressions.

☐ $\dfrac{g^2x + 5gx}{x}$

☐ $g^2 - x^2\left(\dfrac{1}{3}\right)$

☐ $6\left(\dfrac{g}{2}\right) - 100\left(\dfrac{g}{2}\right)^2$

Number Properties Answers

1. **(D).** Whenever a question looks this straightforward ($4 + 5 = 9$, so the quantities initially appear equal), be suspicious. Draw the number line described. If the points A, B, and C are in alphabetical order from left to right, then the distance from A to C will be 9. However, alphabetical order is not required. If the points are in the order C, A, and B from left to right, then the distance from A to C is $5 - 4 = 1$. Therefore, the relationship cannot be determined.

2. **(C).** When integers are consecutive (or just evenly spaced), the average equals the median. Since the median of this list is the average of the two middle numbers, Quantity A and Quantity B both equal the average of b and c. Alternatively, try this with real numbers. If the set is 2, 3, 4, 5, both quantities equal 3.5. No matter what consecutive integers are tested, the two quantities are equal.

3. **$wxyz$ is odd and $w + z = x + y$ only.** This question tests the properties of odd numbers as well as of consecutives.

The first choice is TRUE, as multiplying only odd integers together (and no evens) always yields an odd answer.

However, when adding, the rule is “an odd number of odds makes an odd.” Summing an even number of odds produces an even, so the second choice is FALSE.

The third choice is TRUE. Since w, x, y, and z are consecutive odd integers, all can be defined in terms of w:

$w = w$
$x = w + 2$
$y = w + 4$
$z = w + 6$

Thus, $w + z = w + (w + 6) = 2w + 6$, and $x + y = (w + 2) + (w + 4) = 2w + 6$

Therefore, $w + z = x + y$. Alternatively, try real numbers, such as 1, 3, 5, and 7. It is true that $1 + 7 = 3 + 5$. This would hold true for any set of four consecutive, ordered odd numbers tested.

4. **(B).** No math is required to solve this problem. Note that the numbers from 1 to 100 include 50 even integers and 50 odd integers. The first few odds are 1, 3, 5, etc. The first few evens are 2, 4, 6, etc. Every even is 1 greater than its counterpart (2 is 1 greater than 1, 4 is 1 greater than 3, 6 is 1 greater than 5, etc.) Not only is Quantity B greater, it’s greater by precisely 50.

5. **(D).** From the constraint, x is positive and y is negative. So Quantity A is definitely positive: $x - y$ = positive – negative = positive. Quantity B is the square of a number, which cannot be negative. Quantity B could be zero, if, for example, $x = 2$ and $y = -2$: $(x + y)^2 = (2 + -2)^2 = (0)^2$. In this case, Quantity A is greater. But if $x = 100$ and $y = -1$, Quantity A is $100 - (-1) = 101$ and Quantity B is $(100 + -1)^2 = 99^2$, which is much greater (close to 10,000). The relationship cannot be determined from the information given.

6. **(B).** This problem can be approached either conceptually or by picking values. For the former, anytime a greater number is subtracted from a smaller one, the result will be negative. Thus, $a - d < 0$. Conversely, since the product of two negatives is positive, $bc > 0$. Because any positive value is greater than all negative values, Quantity B must be greater. Alternatively, picking simple values for the variables would also lead to the same result.

7. **8.** A positive integer that is a multiple of both 2 and 7 is a multiple of 14. Since 140 is a multiple of 14, start listing there and count the terms in the range: 140, 154, 168, 182, 196, 210, 224, 238.

Alternatively, note that 140 is the 10th multiple of 14, and $240/14 \approx 17.143$ (use the calculator). Therefore, the 10th through the 17th multiples of 14, inclusive, are in this range. The number of terms is $17 - 10 + 1 = 8$ ("add one before you are done" for an inclusive list).

8. **(B).** If $ab > 0$, then a and b have the same sign. If $bc < 0$, then b and c have opposite signs. Therefore, a and c must have opposite signs. Therefore, ac is negative, so Quantity B is greater.

If you find the logic difficult (a and b are same sign, b and c are opposite signs, therefore a and c are opposite signs), you could make a quick chart of the possibilities using plus and minus signs:

a	b	c	
+	+	–	⟵ First possibility, a and c have different signs.
–	–	+	⟵ Second possibility, a and c have different signs.

9. **(B).** If abc is negative, then either exactly 1 of or all 3 of the values a, b, and c are negative:

– – – ⟵ First possibility, all are negative.

– + + ⟵ Second possibility, 1 negative and 2 positives (order can vary).

If b^2c is positive, then c must be positive, since b^2 cannot be negative. If c is positive, eliminate the first possibility since all 3 variables cannot be negative. Thus, only one of a, b, and c are negative, but the one negative cannot be c. Either a or b is negative, and the other is positive. It doesn't matter which one of a or b is negative—that's enough to know that ab is negative and Quantity B is greater.

10. **(D).** Note that $\frac{a+b+c}{3}$ is just another way to express "the average of a, b, and c." The average of a, b, and c would equal b if the numbers were evenly spaced (such as 1, 2, 3 or 5, 7, 9), but that is not specified. For instance, the integers could be 1, 2, 57 and still satisfy the $a < b < c$ constraint. In that case, the average is 20, which is greater than $b = 2$. The relationship cannot be determined from the information given.

11. **$x^2 - y^2 = 0$ and $|x| - |y| = 0$ only.** When you take the square root of $x^2 = y^2$, the result is not $x = y$. Actually, it is $|x| = |y|$. After all, if $x^2 = y^2$, the variables could represent 2 and –2, 5 and 5, –1 and –1, etc. The information about the signs of x and y is lost when the numbers are squared; thus, taking the

square root results in absolute values, which allow both sign possibilities for x and y. Thus, the first choice is not necessarily true.

From $x^2 = y^2$, subtract y^2 from both sides to yield the second choice, providing algebraic proof that it must be true.

To prove the third choice, take the square root of both sides of $x^2 = y^2$ to get $|x| = |y|$, then subtract $|y|$ from both sides.

12. **(E).** The goal in this question is to order a, a^2, b, and b^2 by magnitude. Based on the original inequality $0 < a < \frac{1}{b} < 1$, several things are true. First, a and $\frac{1}{b}$ are positive, and thus b itself is positive. If $\frac{1}{b} < 1$ and b is positive, multiply both sides of the inequality by b to get $1 < b$, and then again to get $b < b^2$. (Multiplying by a positive value both times meant there was no need to flip the inequality sign.) Two of the expressions in the answer choices have been ordered: $b < b^2$. Eliminate choices that contradict this fact: choices (A) and (B) are wrong.

Second, note that $a < 1$ in the given inequality. Since a is a positive number less than 1, $a^2 < a$. Show this either by multiplying both sides of $a < 1$ by a (again, no need to flip the inequality sign when multiplying by a positive value) or by number properties (squaring positive fractions less than 1 always yields a smaller fraction). Eliminate choices that contradict the fact that $a^2 < a$: choices (A) and (D) are wrong.

Now, what is the relationship between the terms with a and the terms with b? From the first paragraph above, $1 < b$. From the second paragraph (and the given inequality), $a < 1$. Put these together: $a < 1 < b$, or just $a < b$. Eliminate the choices that contradict this fact: choices (A) and (C) are wrong.

At this point, choice (A) has been eliminated for three reasons, and (B), (C), and (D) for one reason each. The only choice remaining is (E), so it must be right by elimination.

(E) can be proven right by putting together the three separate inequalities ($b < b^2$) and ($a^2 < a$) and ($a < b$) into a single inequality: $a^2 < a < b < b^2$. This is equivalent to choice (E): $b^2 > b > a > a^2$.

13. **(B).** The exact values of a, b, c, and d are unknown, as is whether they are evenly spaced (do not assume that they are, just because the figure looks that way). However, it is known that all of the variables are positive such that $0 < a < b < c < d$.

Because $a < b$ and $c < d$ and all the variables are positive, $a \times c < b \times d$. In words, the product of the two smaller numbers is less than the product of the two greater numbers. Quantity B is greater.

You could also try this with real numbers. You could try $a = 1$, $b = 2$, $c = 3$, and $d = 4$, or you could mix up the spacing, as in $a = 0.5$, $b = 7$, $c = 11$, $d = 45$. For any scenario that matches the conditions of the problem, Quantity B is greater.

14. **(C).** This question asks for the greater number of distinct factors, not prime factors. The approach to determine the number of distinct factors is to create a chart that systematically lists all the combinations of two integers that equal the number in question. When you find the same pair in reverse order, the chart is done.

Quantity A (32) **Quantity B (20)**

1×32	1×20
2×16	2×10
4×8	4×5

Each has 6 distinct factors and the two quantities are equal.

15. **–1 and 5 only.** From the first equation it seems that y could equal either 2 or –2, but if $x^2y = 18$, then y must equal only 2 (otherwise, x^2y would be negative). Still, the squared x indicates that x can equal 3 or –3. So the possibilities for $x + y$ are:

$3 + 2 = 5$
$(-3) + 2 = -1$

16. **(C).** It is not necessary to calculate 10^{11} or 3^{13}. Because 10 is an even number, so is 10^{11}, and 0 is the remainder when any even is divided by 2. Similarly, 3^{13} is a multiple of 3 (it has 3 among its prime factors), and 0 is the remainder when any multiple of 3 is divided by 3. Therefore, the quantities are equal.

17. **(B).** The negative base –1 to any odd power is –1, and the negative base –1 to any even power is 1. Since q is odd, Quantity A = –1 and Quantity B = 1.

18. **(A).** Before doing any calculations on a problem with negative bases raised to integer exponents, check to see whether one quantity is positive and one quantity is negative, in which case no further calculation is necessary. Note that a negative base to an even exponent is positive, while a negative base to an odd exponent is negative.

Since n is an integer, $4n$ is even. Thus, in Quantity A, $(-1)^{4n}$ and $(-1)^{202}$ are both positive, so Quantity A is positive. In Quantity B, $(3)^3$ is positive but $(-5)^5$ is negative, and thus Quantity B is negative. Since a positive is by definition greater than a negative, Quantity A is greater.

19. **1, 3, 7, and 9 only.** As with multiplication, when an integer is raised to a power, the units digit is determined solely by the product of the units digits. Those products will form a repeating pattern. Here, $3^1 = \underline{3}$, $3^2 = \underline{9}$, $3^3 = 2\underline{7}$, $3^4 = 8\underline{1}$, and $3^5 = 24\underline{3}$. Here the pattern returns to its original value of 3 and any larger power of 3 will follow this same pattern: 3,9, 7, and then 1. Thus, the units digit of 73^x must be 1, 3, 7, or 9. When dividing by 10, the remainder is the units digits, so those same values are the complete list of possible remainders.

20. **(D).** If $xz = 9$ and x and z must both be integers, then they are 1 and 9 (or –1 and –9) or 3 and 3 (or –3 and –3). Therefore, they are both odd. More generally, the product of two integers will only be odd if the component integers themselves are both odd. Because z is odd, and $y + z$ equals 13 (an odd), y must be even.

(A): x is NOT even. Eliminate.
(B): x could be 3 but doesn't have to be. Eliminate.
(C): y is NOT odd. Eliminate.
(E): z does not have to be less than x (for instance, they could both be 3). Eliminate.

At this point, only (D) remains, so it must be the answer. To prove it, consider the constraint that limits the value of y: $y + z = 13$. Since z could be –1, 1, –3, 3, –9, or 9, the maximum possible value for z is 9, so y must be at least 4. All values that are at least 4 are also greater than 3, so (D) must be true.

21. **(A).** This question draws upon knowledge of the smaller prime numbers. It might be helpful to list out the first few prime numbers on your paper: 2, 3, 5, 7, 11, 13, 17, 19…

The smallest prime number greater than 13 is 17, and the greatest prime number less than 16 is 13. Therefore, Quantity A is greater.

22. **385.** To find the sum of a set of numbers, given the average and number of terms, use the average formula. Average = $\frac{\text{Sum}}{\text{Number of Terms}}$, so Sum = Average × Number of Terms = 35 × 11 = 385.

23. **(D).** To find the sum of a set of evenly spaced numbers, multiply the median (which is also the average) by the number of terms in the set. The median of the numbers from 1 to 80 inclusive is 40.5 (the first 40 numbers are 1 through 40, and the second 40 numbers are 41 through 80, so the middle is 40.5). You can also use the formula $\frac{\text{First} + \text{Last}}{2}$ to calculate the median of an evenly spaced set: $\frac{1+80}{2}$ = 40.5. Multiply 40.5 times 80 to get the answer: 3,240.

24. **299.** p is a large number, but it consists entirely of $q + 149 + 150$. Thus, $p - q$ is what's left of p once the common terms are subtracted: 149 + 150 = 299.

25. $\mathbf{\frac{1}{6}}$**.** There is a trick to this problem—all of the integers in the product n will be canceled out by the same integers appearing in the product m:

$$\frac{n}{m} = \frac{\cancel{4\times5\times6\times7\times8\times9\times10\times11}}{2\times3\times\cancel{4\times5\times6\times7\times8\times9\times10\times11}} = \frac{1}{2\times3} = \frac{1}{6}$$

26. **(E).** If $\sqrt{x}$ is an integer, then x must be a perfect square. If x is a perfect square and $xy^2 = 36$, then x could actually equal *any* of the perfect square factors of 36, which are 1, 4, 9, or 36. (Only consider positive factors, because in order to have a valid square root, x must be positive.) Thus, y^2 could equal 36, 9, 4, or 1, respectively.

If y^2 is positive, y itself could be positive or negative. Thus, $y = \pm6, \pm3, \pm2$, or ±1, for a total of 8 possible values.

27. **(C).** Integers *a, b*, and *c* must be 6, 4, and 2, respectively, as they are positive even integers less than 8 and ordered according to the given inequality. The range of *a, b*, and *c* is 6 – 2 = 4. The average of *a, b*, and *c* is $\frac{6+4+2}{3} = \frac{12}{3}$ = 4. The two quantities are equal.

28. **(E).** Find the choices that do not have to be greater than 1. It is possible that x could be negative, which eliminates (A), (B), and (C). For choice (D), if $x^2 = 1$, that times the positive fraction y would be less than 1. In choice (E), x^2 must be positive and at least 1, so dividing by the positive fraction y increases the value.

29. **(D)**. If the variables were also constrained to be positive, they would have to be 1, 2, and 3, making the quantities both equal to 2. However, the variables could be negative, for example, $a = -10$, $b = -9$, $c = -8$. The range of *a, b*, and *c* will always be 2 because the integers are consecutive, but the average can vary depending on the specific values. There is not enough information to determine the relationship.

30. **(B).** If $\sqrt{xy}$ is a prime number, $\sqrt{xy}$ could be 2, 3, 5, 7, 11, 13, etc. Square these possibilities to get a list of possibilities for *xy:* 4, 9, 25, 49, 121, 169, etc. However, *xy* is even, so *xy* must equal 4.

Finally, $x > 4y > 0$, which implies that both x and y are positive. Solve $xy = 4$ for x, then substitute to eliminate the variable x and solve for y:

If $xy = 4$, then $x = \frac{4}{y}$.

If $x > 4y$, then $\frac{4}{y} > 4y$.

Because y is positive, you can multiply both sides of the inequality by y and you don't have to flip the sign of the inequality: $4 > 4y^2$

Finally, divide both sides of the inequality by 4: $1 > y^2$

Thus, y is a positive fraction less than 1 (it was already given that $y > 0$). Quantity B is greater.

31. **(B).** If $\sqrt{x}$ is a prime number, $x = \left(\sqrt{x}\right)\left(\sqrt{x}\right)$ is the square of a prime number. Squaring a number does not change whether it is odd or even (the square of an odd number is odd and the square of an even number is even). Since x is even, it must be the square of the only even prime number. Thus, $\sqrt{x} = 2$ and $x = 4$. Since $x + y = 11$, $y = 7$ and Quantity B is greater.

32. **(A).** If fg is odd and both f and g are positive integers, both f and g are odd. The remainder when odd f is divided by 2 is 1. Since fgh is even and f and g are odd, integer h must be even. Thus, when h is divided by 2, the remainder is 0. Quantity A is greater.

33. **(C).** For even and odd questions, you can either think it out logically or plug in a number. Since one choice requires raising the number to the 6th power, pick something small! Plug in $x = 1$:

(A) $x^2 + 4x + 6 = 1 + 4 + 6 = 11$
(B) $x^3 + 5x + 3 = 1 + 5 + 3 = 9$
(C) $x^4 + 6x + 7 = 1 + 6 + 7 = 14$
(D) $x^5 + 7x + 1 = 1 + 7 + 1 = 9$
(E) $x^6 + 8x + 4 = 1 + 8 + 4 = 13$

For the logic approach, remember that an odd number raised to an integer power is always odd, an odd number multiplied by an odd number is always odd, and an odd number multiplied by an even number is always even:

(A) $x^2 + 4x + 6$ = odd + even + even = odd
(B) $x^3 + 5x + 3$ = odd + odd + odd = odd
(C) $x^4 + 6x + 7$ = odd + even + odd = even
(D) $x^5 + 7x + 1$ = odd + odd + odd = odd
(E) $x^6 + 8x + 4$ = odd + even + even = odd

34. **(D).** If $x^2 > 25$, then $x > 5$ OR $x < -5$. For instance, x could be 6 or –6.

If $x = 6$:

$$6 + y < 0$$
$$y < -6$$

x is greater than y.

If $x = -6$:

$$-6 + y < 0$$
$$y < 6$$

y could be less than x (e.g., $y = -7$) or greater than x (e.g., $y = 4$). Therefore, you do not have enough information.

35. **(B).** If the positive integer a is divisible by 2, it is a positive even integer. Thus, the minimum value for a is 2. Therefore, since $ab < 1$, b must be less than $\frac{1}{2}$.

36. **(B).** If the sum of two primes is less than 6, either the numbers are 2 and 3 (the two smallest unique primes), or both numbers are 2 (just because the variables are different letters doesn't mean that p cannot equal w). Both numbers cannot equal 3, though, or $p + w$ would be too great. If p^2 is odd, p is odd, and therefore $p = 3$, so w can only be 2.

37. **(A).** If $x^2 > y^2$, x must have a greater absolute value than y. For instance:

	x	y
Example 1	3	2
Example 2	–3	2
Example 3	3	–2
Example 4	–3	–2

If $x > -|y|$ must also be true, which of the examples continue to be valid?

	x	y	$x > -|y|$?	
Example 1	3	2	$3 > -|2|$	TRUE
Example 2	–3	2	$-3 > -|2|$	FALSE
Example 3	3	–2	$3 > -|-2|$	TRUE
Example 4	–3	–2	$-3 > -|-2|$	FALSE

Only Example 1 and Example 3 remain.

	x	y
Example 1	3	2
Example 3	3	–2

Thus, either x and y are both positive and x has a greater absolute value (Quantity A is greater) or x is positive and y is negative (Quantity A is greater). In either case, Quantity A is greater.

38. **(C).** Write an equation: $x + (x + 1) + (x + 2) + (x + 3) = -2$. Now solve:

$$4x + 6 = -2$$
$$4x = -8$$
$$x = -2$$

Thus, the integers are –2, –1, 0, and 1. The smallest of the four integers equals –2, so the quantities are equal.

39. $\boldsymbol{\frac{g^2x+5gx}{x}}$ **and** $\boldsymbol{6\left(\frac{g}{2}\right)-100\left(\frac{g}{2}\right)^2}$ **=only.** In the first choice, x can be factored out and canceled:

$$\frac{g^2+5gx}{x}=\frac{x\left(g^2+5g\right)}{x}=g^2+5g$$

Since g is an integer, so too is $g^2 + 5g$.

In the second choice, g^2 is certainly an integer, but $x^2\left(\frac{1}{3}\right)$ is only an integer if $x = 3$ (since 3 is the only prime number divisible by 3), so the second choice is not necesarily an integer.

When the third choice is simplified, $6\left(\frac{g}{2}\right)-100\left(\frac{g}{2}\right)^2 = 3g - \frac{100g^2}{4} = 3g - 25g^2$ results; since g is an integer, $3g - 25g^2$ is also an integer.