

Difficult Problems from the Math Section

1. The sum of the even numbers between 1 and n is 79×80 , where n is an odd number, then $n = ?$

Sol: First term $a = 2$, common difference $d = 2$ since even number

therefore sum to first n numbers of Arithmetic progression would be

$$\frac{n}{2}(2a + (n-1)d)$$

$$= \frac{n}{2}(2 \times 2 + (n-1) \times 2) = n(n+1) \text{ and this is equal to } 79 \times 80$$

therefore $n = 79$ which is odd...

2. The price of a bushel of corn is currently \$3.20, and the price of a peck of wheat is \$5.80. The price of corn is increasing at a constant rate of $5x$ cents per day while the price of wheat is decreasing at a constant rate of $\sqrt{2}(x) - x$ cents per day. What is the approximate price when a bushel of corn costs the same amount as a peck of wheat?

- (A) \$4.50
- (B) \$5.10
- (C) \$5.30
- (D) \$5.50
- (E) \$5.60

Soln: $320 + 5x = 580 - .41x$; $x = \#$ of days after price is same;

solving for x ; x is approximately 48, thus the required price is $320 + 5 \times 48 = 560$ cents = \$5.6

3. How many randomly assembled people do you need to have a better than 50% prob. that at least 1 of them was born in a leap year?

Soln: Prob. of a randomly selected person to have NOT been born in a leap yr = $3/4$

Take 2 people, probability that none of them was born in a leap = $3/4 \times 3/4 = 9/16$. The probability at least one born in leap = $1 - 9/16 = 7/16 < 0.5$

Take 3 people, probability that none born in leap year = $3/4 \times 3/4 \times 3/4 = 27/64$.

The probability that at least one born = $1 - 27/64 = 37/64 > 0.5$

Thus min 3 people are needed.

4. In a basketball contest, players must make 10 free throws. Assuming a player has 90% chance of making each of his shots, how likely is it that he will make all of his first 10 shots?

Ans: The probability of making all of his first 10 shots is given by

$$(9/10) \times (9/10) \times (9/10) \times (9/10) \times (9/10) \times (9/10) \times (9/10) \times (9/10) \times (9/10) \times (9/10) = (9/10)^{10} = 0.348 \Rightarrow 35\%$$

5. **$AB + CD = AAA$, where AB and CD are two-digit numbers and AAA is a three digit number; A , B , C , and D are distinct positive integers. In the addition problem above, what is the value of C ?**

(A) 1

(B) 3

(C) 7

(D) 9

(E) Cannot be determined

Ans: $AB + CD = AAA$

Since AB and CD are two digit numbers, then AAA must be 111

Therefore $1B + CD = 111$

B can assume any value between 3 and 9

If $B = 3$, then $CD = 111 - 13 = 98$ and $C = 9$

If $B = 9$, then $CD = 111 - 19 = 92$ and $C = 9$

So for all B between 3 & 9, $C = 9$

Therefore the correct answer is D ($C = 9$)

6. **A and B ran a race of 480 m. In the first heat, A gives B a head start of 48 m and beats him by $1/10^{\text{th}}$ of a minute. In the second heat, A gives B a head start of 144 m and is beaten by $1/30^{\text{th}}$ of a minute. What is B's speed in m/s?**

(A) 12

(B) 14

(C) 16

(D) 18

(E) 20

Ans: race 1 :- $t_a = t_b - 6$ (because A beats B by 6 sec)

race 2 :- $T_a = t_b + 2$ (because A loses to B by 2 sec)

By the formula $D = S * T$

we get two equations

$$480/S_a = 432/S_b - 6 \text{ -----1)}$$

$$480/S_a = 336/S_b + 2 \text{ -----2)}$$

Equating these two equations we get $S_b = 12$

t_a, S_a stand for time taken by A and speed of A resp.

7. **A certain quantity of 40% solution is replaced with 25% solution such that the new concentration is 35%. What is the fraction of the solution that was replaced?**

- (A) $\frac{1}{4}$
- (B) $\frac{1}{3}$
- (C) $\frac{1}{2}$
- (D) $\frac{2}{3}$
- (E) $\frac{3}{4}$

Ans: Let X be the fraction of solution that is replaced.

$$\text{Then } X \cdot 25\% + (1-X) \cdot 40\% = 35\%$$

Solving, you get $X = \frac{1}{3}$

8. A person buys a share for \$ 50 and sells it for \$ 52 after a year. What is the total profit made by him from the share?
- (I) A company pays annual dividend
 - (II) The rate of dividend is 25%
 - (A) Statement (I) ALONE is sufficient, but statement (II) alone is not sufficient
 - (B) Statement (II) ALONE is sufficient, but statement (I) is not sufficient
 - (C) BOTH statements TOGETHER are sufficient, but NEITHER statement alone is sufficient
 - (D) Each statement ALONE is sufficient
 - (E) Statements (I) and (II) TOGETHER are NOT sufficient
9. A bag contains 3 red, 4 black and 2 white balls. What is the probability of drawing a red and a white ball in two successive draws, each ball being put back after it is drawn?
- (A) $\frac{2}{27}$
 - (B) $\frac{1}{9}$
 - (C) $\frac{1}{3}$
 - (D) $\frac{4}{27}$
 - (E) $\frac{2}{9}$

Ans: Case I: Red ball first and then white ball

$$P_1 = \frac{3}{9} \cdot \frac{2}{9} = \frac{2}{27}$$

Case 2: White ball first and then red ball

$$P_2 = \frac{2}{9} \cdot \frac{3}{9} = \frac{2}{27}$$

Therefore total probability: $p_1 + p_2 = \frac{4}{27}$

10. What is the least possible distance between a point on the circle $x^2 + y^2 = 1$ and a point on the line $y = \frac{3}{4}x - 3$?

- A) 1.4
- B) $\sqrt{2}$
- C) 1.7
- D) $\sqrt{3}$
- E) 2.0

Ans: The equation of the line will be $3x - 4y - 12 = 0$.
This crosses the x and y axis at (0,-3) and (4,0)

The circle has the origin at the center and has a radius of 1 unit.

So it is closest to the given line when, a perpendicular is drawn to the line, which passes through the origin.

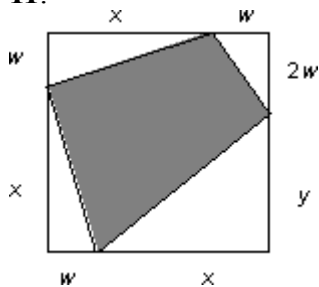
This distance of the line from the origin is $12 / \sqrt{9 + 16}$ which is 2.4
[Length of perpendicular from origin to line $ax + by + c = 0$ is

$$\frac{|c|}{\sqrt{a^2 + b^2}}]$$

The radius is 1 unit.

So the shortest distance is $2.4 - 1 \text{ unit} = 1.4 \text{ units}$

11.



In the square above, $12w = 3x = 4y$. What fractional part of the square is shaded?

- A) $\frac{2}{3}$
- B) $\frac{14}{25}$
- C) $\frac{5}{9}$
- D) $\frac{11}{25}$
- E) $\frac{3}{7}$

Sol: Since $12w=3x=4y$,

$w:x=3:12=1:4$ and $x:y=4:3$

so, $w = 1$

$x = 4$

$y = 3$

the fractional part of the square is shaded:

$$\{(w+x)^2 - [(1/2)wx + (1/2)wx + (1/2)xy + (1/2)w(2w)]\}/(w+x)^2$$

$$= \{(w+x)^2 - [wx + (1/2)xy + w^2]\}/[(w+x)^2]$$

$$=[(5^2) - (4+6+1)]/(5^2)$$

$$= (25 - 11)/25$$

$$= 14/25$$

12. The average of temperatures at noontime from Monday to Friday is 50; the lowest one is 45, what is the possible maximum range of the temperatures?

☒ 20 ☒ 25 ☐ 40 ☐ 45 ☐ 75

Ans: The answer 25 doesn't refer to a temperature, but rather to a range of temperatures.

The average of the 5 temps is: $(a + b + c + d + e) / 5 = 50$

One of these temps is 45: $(a + b + c + d + 45) / 5 = 50$

Solving for the variables: $a + b + c + d = 205$

In order to find the greatest range of temps, we minimize all temps but one. Remember, though, that 45 is the lowest temp possible, so: $45 + 45 + 45 + d = 205$

Solving for the variable: $d = 70$

$$70 - 45 = 25$$

13. If n is an integer from 1 to 96, what is the probability for $n*(n+1)*(n+2)$ being

divisible by 8? ☐ 25% ☒ 50% ☒ 62.5% ☐ 72.5% ☐ 75%

Soln: $E = n*(n+1)*(n+2)$

E is divisible by 8, if n is even.

No of even numbers (between 1 and 96) = 48

E is divisible by 8, also when $n = 8k - 1$ ($k = 1, 2, 3, \dots$)

Such numbers total = 12(7, 15, ...)

Favorable cases = $48 + 12 = 60$.

Total cases = 96

$P = 60/96 = 62.5$

Method 2:

From 1 to 10, there are 5 sets, which are divisible by 8.

$(2*3*4) (4*7*6) (6*7*8)(7*8*9)(8*9*10)$

So till **96**, there will be $12 * 5$ such sets = 60 sets

so probability will be $60/96 = 62.5$

14. Kurt, a painter, has 9 jars of paint:

4 are yellow

2 are red

rest are brown

Kurt will combine 3 jars of paint into a new container to make a new color, which he will name accordingly to the following conditions:

Brun Y if the paint contains 2 jars of brown paint and no yellow

Brun X if the paint contains 3 jars of brown paint

Jaune X if the paint contains at least 2 jars of yellow

Jaune Y if the paint contains exactly 1 jar of yellow

What is the probability that the new color will be Jaune

a) $5/42$

b) $37/42$

c) $1/21$

d) $4/9$

e) $5/9$

Sol: 1. This has at least 2 yellow meaning..

a> there can be all three Y $\Rightarrow 4c3$

OR

b> 2 Y and 1 out of 2 R and 3 B $\Rightarrow 4c2 \times 5c1$

Total 34

2. This has exactly 1 Y and remaining 2 out of 5 $\Rightarrow 4c1 \times 5c2$

Total 40

Total possibilities = $(9!/3!6!) = 84$

Adding the two probabilities: probability = $74/84 = 37/42$

15. TWO couples and a single person are to be seated on 5 chairs such that no couple is seated next to each other. What is the probability of the above??

Soln:

Ways in which the first couple can sit together = $2*4!$ (1 couple is considered one unit)

Ways for second couple = $2*4!$

These cases include an extra case of both couples sitting together

Ways in which both couple are seated together = $2*2*3! = 4!$ (2 couples considered as 2 units- so each couple can be arrange between themselves in 2 ways and the 3 units in $3!$ Ways)

Thus total ways in which at least one couple is seated together = $2*4! + 2*4! - 4! = 3*4!$

Total ways to arrange the 5 ppl = $5!$

Thus, prob of at least one couple seated together = $3*4! / 5! = 3/5$

Thus prob of none seated together = $1 - 3/5 = 2/5$

16. An express train traveled at an average speed of 100 kilometers per hour, stopping for 3 minutes after every 75 kilometers. A local train traveled at an average speed of 50 kilometers, stopping for 1 minute after every 25 kilometers. If the trains began traveling at the same time, how many kilometers did the local train travel in the time it took the express train to travel 600 kilometers?

- a. 300
- b. 305
- c. 307.5
- d. 1200
- e. 1236

Sol: the answer is C: 307.5 km

Express Train:

600 km --> 6 hours (since 100 km/h) + stops * 3 min.

stops = $600 / 75 = 8$, but as it is an integer number, the last stop in km 600 is not a real stop, so it would be 7 stops

so, time = 6 hours + 7 * 3 min. = 6 hours 21 min

Local Train:

in 6 hours, it will make 300 km (since its speed is 50 km/hour)

in 300 km it will have $300 / 25$ stops = 12, 12 stops 1 min each = 12 min

we have 6 hours 12 min, but we need to calculate how many km can it make in 6 hours 21 min, so $21 - 12 = 9$ min

if 60 min it can make 50 km, 9 min it can make 7.5 km

so, the distance is $300 + 7.5 = 307.5$ km

17. Matt starts a new job, with the goal of doubling his old average commission of \$400. He takes a 10% commission, making commissions of \$100.00, \$200.00,

\$250.00, \$700.00, and \$1,000 on his first 5 sales. If Matt made two sales on the last day of the week, how much would Matt have had to sell in order to meet his goal?

Sol: The two sales on Matt's last day of the week must total \$33,500.

$$(100 + 200 + 250 + 700 + 1000 + x) / 7 = 800$$

$$x = 3350$$

since x is Matt's 10% commission, the sale is \$33,500.

18. On how many ways can the letters of the word "COMPUTER" be arranged?

- 1) Without any restrictions.**
- 2) M must always occur at the third place.**
- 3) All the vowels are together.**
- 4) All the vowels are never together.**
- 5) Vowels occupy the even positions.**

Sol:

$$1) 8! = 40320$$

$$2) 7*6*1*5*4*3*2*1=5,040$$

3) Considering the 3 vowels as 1 letter, there are five other letters which are consonants C, M, P, T, R

CMPTR (AUE) = 6 letters which can be arranged in 6p6 or 6! Ways

and A, U, E themselves can be arranged in another 3! Ways for a total of 6!*3! Ways

$$4) \text{ Total combinations - all vowels always together} \\ = \text{what u found in 1) - what u found in 3)} \\ = 8! - 6! * 3!$$

$$5) \text{ I think it should be } 4 * 720$$

there are 4 even positions to be filled by three even numbers.

in 5*3*4*2*3*1*2*1 It is assumed that Last even place is NOT filled by a vowel. There can be total 4 ways to do that.

$$\text{Hence } 4 * 720$$

19. In the infinite sequence A , $A_n = x^{n-1} + x^n + x^{n+1} + x^{n+2} + x^{n+3}$, where x is a positive integer constant. For what value of n is the ratio of A_n to $x(1 + x(1 + x(1 + x(1 + x))))$ equal to x^5 ?

- (A) 8**
- (B) 7**
- (C) 6**
- (D) 5**
- (E) 4**

Sol: The method I followed was to reduce the Q to $(x^6/x) * (Y/Y)$
the eqn $An = (x^n - 1)(1 + x + x^2 + x^3 + \dots)$ -----(1)

and the eqn $x(1+x(1+x(1+x\dots)))$
which I call Z can be reduced to $x(1+x+x^2+x^3\dots)$ -----(2)

from (1) and (2) we get $An / Z = x^{(n-1)} / x$
therefore for getting answer $x^5 (n-1) = 6$
therefore $n=7$

Ans: B

20. In how many ways can one choose 6 cards from a normal deck of cards so as to have all suits present?

- a. $(13^4) \times 48 \times 47$
- b. $(13^4) \times 27 \times 47$
- c. $48C6$
- d. 13^4
- e. $(13^4) \times 48C6$

Sol: 52 cards in a deck -13 cards per suit
First card - let us say from suit hearts = $13C1 = 13$
Second card - let us say from suit diamonds = $13C1 = 13$
Third card - let us say from suit spade = $13C1 = 13$
Fourth card - let us say from suit clubs = $13C1 = 13$
Remaining cards in the deck = $52 - 4 = 48$
Fifth card - any card in the deck = $48C1$
Sixth card - any card in the deck = $47C1$

Total number of ways = $13 * 13 * 13 * 13 * 48 * 47 = 13^4 * 48 * 47 \rightarrow$ choice A

21. Each of the integers from 0 to 9, inclusive, is written on a separate slip of blank paper and the ten slips are dropped into a hat. If the slips are then drawn one at a time without replacement, how many must be drawn to ensure that the numbers on two of the slips drawn will have a sum of 10?

- 3
- 4
- 5
- 6
- 7 *

Sol:
ok consider this

0 + 1 + 2 + 3 + 4 + 5 + 6..Stop --> Don't go further. Why? Here's why.

At the worst the order given above is how u could pick the out the slips. Until u add the slip with no. 6 on it, no two slips before that add up to 10 (which is what the Q wants)

the best u can approach is a sum of 9 (slip no. 5 + slip no. 4)

but as soon as u add slip 6. Voila u get your first sum of 10 from two slips and that is indeed the answer = 7 draws

22. Two missiles are launched simultaneously. Missile 1 launches at a speed of x miles per hour, increasing its speed by a factor of \sqrt{x} every 10 minutes (so that after 10 minutes its speed is $x\sqrt{x}$, after 20 minutes its speed is $x\sqrt{x}\sqrt{x}$, and so forth. Missile 2 launches at a speed of y miles per hour, doubling its speed every 10 minutes. After 1 hour, is the speed of Missile 1 greater than that of Missile 2?

- 1) $x = \sqrt{y}$
- 2) $x > 8$

(A) Statement (1) ALONE is sufficient to answer the question, but statement (2) alone is not.

(B) Statement (2) ALONE is sufficient to answer the question, but statement (1) alone is not.

(C) Statements (1) and (2) TAKEN TOGETHER are sufficient to answer the question, but NEITHER statement ALONE is sufficient.

(D) EACH statement ALONE is sufficient to answer the question.

(E) Statements (1) and (2) TAKEN TOGETHER are NOT sufficient to answer the question.

Sol:

Since Missile 1's rate increases by a factor of \sqrt{x} every 10 minutes, Missile 1 will be traveling at a speed of x^4 miles per hour after 60 minutes:

minutes 0-10 10-20 20-30 30-40 40-50 50-60 60+ speed $x \times \sqrt{x} \times^2 \times^2 \sqrt{x} \times^3 \times^3 \sqrt{x} \times^4$

And since Missile 2's rate doubles every 10 minutes, Missile 2 will be traveling at a speed of $2^6 y$ after 60 minutes:

minutes 0-10 10-20 20-30 30-40 40-50 50-60 60+ speed $y \ 2y \ 2^2 y \ 2^3 y \ 2^4 y \ 2^5 y \ 2^6 y$

The question then becomes: Is $x^4 > 2^6 y$?

Statement (1) tells us that $x = \sqrt{y}$. Squaring both sides yields $x^2 = y$. We can substitute

for y : Is $x^4 > 2^6 x^2$? If we divide both sides by x^2 , we get: Is $x^2 > 2^6$? We can further simplify by taking the square root of both sides: Is $x > 2^3$? We still cannot answer this, so statement (1) alone is NOT sufficient to answer the question.

Statement (2) tells us that $x > 8$, which tells us nothing about the relationship between x and y . Statement (2) alone is NOT sufficient to answer the question.

Taking the statements together, we know from statement (1) that the question can be rephrased: Is $x > 2^3$? From statement (2) we know certainly that $x > 8$, which is another way of expressing $x > 2^3$. So using the information from both statements, we can answer definitively that after 1 hour, Missile 1 is traveling faster than Missile 2.

The correct answer is C: Statements (1) and (2) taken together are sufficient to answer the question, but neither statement alone is sufficient.

23. If $\frac{(13!)^{16} - (13!)^8}{(13!)^8 + (13!)^4} = a$, what is the unit's digit of $(13!)^4$?

- (A) 0
- (B) 1
- (C) 3
- (D) 5
- (E) 9

Sol:

$$\begin{aligned} \frac{(13!)^{16} - (13!)^8}{(13!)^8 + (13!)^4} &= a \\ \frac{\cancel{(13!)^8 + (13!)^4} \left[(13!)^8 - (13!)^4 \right]}{\cancel{(13!)^8 + (13!)^4}} &= a \\ (13!)^8 - (13!)^4 &= a \\ (13!)^4 \left[(13!)^4 - 1 \right] &= a \\ (13!)^4 - 1 &= \frac{a}{(13!)^4} \end{aligned}$$

The unit's digit of the left side of the equation is equal to the unit's digit of the right side of the equation (which is what the question asks about). Thus, if we can determine the unit's digit of the expression on the left side of the equation, we can answer the question.

Since $(13!) = 13 \times 12 \times 11 \times 10 \dots \times 1$, we know that $13!$ Contains a factor of 10, so its

unit's digit must be 0. Similarly, the unit's digit of $(13!)^4$ will also have a unit's digit of 0. If we subtract 1 from this, we will be left with a number ending in 9.

Therefore, the unit's digit of $\frac{a}{(13!)^4}$ is 9. The correct answer is E.

24. The dimensions of a rectangular floor are 16 feet by 20 feet. When a rectangular rug is placed on the floor, a strip of floor 3 feet wide is exposed on all sides. What are the dimensions of the rug, in feet?

(A) 10 by 14 (B) 10 by 17 (C) 13 by 14 (E) (D) 13 by 17 (E) 14 by 16

Soln: The rug is placed in the middle of the room. The rug leaves 3m on either side of it both lengthwise and breadth wise. Now, the dimensions of the rug would be the dimensions of the room - the space that it does not occupy. With three 3 on either side, 6 m is not occupied by the rug in both dimensions.

So, rug size = $(16-6) \times (20-6) = 10 \times 14$

25. How many different subsets of the set {10,14,17,24} are there that contain an odd number of elements?

(a) 3 (b) 6 (c) 8 (d) 10 (e) 12

Soln: 8 is the answer. The different subsets are

10,
14,
17,
24,
10, 14, 17

14 17 24

17 24 10

24 10 14

26. Seven men and seven women have to sit around a circular table so that no 2 women are together. In how many different ways can this be done?
a.24 b.6 c.4 d.12 e.3

Soln: I suggest to first arranging men. This can be done in $6!$ Ways. Now to satisfy above condition for women, they should sit in spaces between each man. This can be done in $7!$

Ways (because there will be seven spaces between each man on round table)
Total ways = $6! * 7!$

27. Find the fourth consecutive even number:

(I) the sum of the last two numbers is 30

(II) the sum of the first two numbers is 22

the ans is D

please explain why I alone is sufficient?

Soln: I guess, there are 4 consecutive even numbers to start with..

Stmnt 1) Sum of 3rd even + Sum of 4th even = 30 $\Rightarrow 14+16 = 30 \Rightarrow$ 4th even num = 16..

Sufficient

Stmnt 2) Sum of 1st even + Sum of 2nd even = 22 \Rightarrow 10 and 12 are the 2 numbers to begin with, then 3rd num = 14 and 4th num = 16.. Sufficient

Ans – D

28. If the sum of five consecutive positive integers is A, then the sum of the next five consecutive integers in terms of A is:

a) A+1

b) A+5

c) A+25

d) 2A

e) 5A

Soln: If you divide the sum obtained by adding any 5consecutive numbers by '5', and then you will get the Center number of the sequence itself.

i.e. $1 + 5 = 15/5 = 3$. 1, 2, 3, 4, 5

so, sixth consecutive number will be '3' more than the 'Middle term'

i.e. $3+3=6$, similarly $3+4=7$

Hence going by this. Asked sum would be

$$[(A/5) + 3] + [(A/5) + 4] + [(A/5) + 5] + [(A/5) + 6] + [(A/5) + 7] = A + 25$$

29. If P represents the product of the first 15 positive integers, then P is not a multiple of:

a) 99 b) 84 c) 72 d) 65 e) 57

Solution: If P represents the product of the first 15 integers, P would consist of the prime numbers that are below 15.

2,3,5,7,11,13

Any value that has a prime higher than 13 would not be a value of P.

$57 = 3, 19$

19 is a prime greater than 13, so the answer is E.

30. 5 girls and 3 boys are arranged randomly in a row. Find the probability that:

A) there is one boy on each end.

B) There is one girl on each end.

Solution: For the first scenario:

A) there is one boy on each end.

The first seat can be filled in 3C_1 (3 boys 1 seat) ways = 3

the last seat can be filled in 2C_1 (2 boys 1 seat) ways = 2

the six seats in the middle can be filled in $6!$ (1 boy and 5 girls) ways

Total possible outcome = $8!$

Probability = $({}^3C_1 * {}^2C_1 * 6!) / 8! = 3/28$

For the second scenario:

A) there is one girl on each end.

The first seat can be filled in 5C_1 (5 girls 1 seat) ways = 5

the last seat can be filled in 4C_1 (4 girls 1 seat) ways = 4

the six seats in the middle can be filled in $6!$ (3 boys and 3 girls) ways

Total possible outcome = $8!$

Probability = $({}^5C_1 * {}^4C_1 * 6!) / 8! = 5/14$

31. If Bob and Jen are two of 5 participants in a race, how many different ways can the race finish where Jen always finishes in front of Bob?

Solution: approach: first fix Jen, and then fix Bob. Then fix the remaining three.

Case 1: When Jen is in the first place.

=> Bob can be in any of the other four places. => 4.

The remaining 3 can arrange themselves in the remaining 3 places in $3!$ Ways.

Hence total ways = $4 * 3!$

Case 2: When Jen is in the second place.

\Rightarrow Bob can be in any of three places $\Rightarrow 3$.

The remaining can arrange themselves in 3 places in $3!$ Ways.

Continuing the approach.

Answer = $4*3! + 3*3! + 2*3! + 1*3! = 10*3! = 60$ ways.

32. A set of numbers has the property that for any number t in the set, $t + 2$ is in the set. If -1 is in the set, which of the following must also be in the set?

I. -3 II. 1 III. 5

A. I only

B. II only

C. I and II only

D. II and III only

E. I, II, and III

Soln: Series property: $t \Rightarrow t+2$. (Note: for any given number N , ONLY $N + 2$ is compulsory. $N - 2$ is not a necessity as N could be the first term...this can be used as a trap.)

Given: -1 belongs to the series. $\Rightarrow 1 \Rightarrow 3 \Rightarrow 5$. DOES NOT imply -3 .

Hence, II and III (D).

33. A number is selected at random from first 30 natural numbers. What is the probability that the number is a multiple of either 3 or 13?

(A) $17/30$

(B) $2/5$

(C) $7/15$

(D) $4/15$

(E) $11/30$

Solution: Total no from 1 to 30 = 30

total no from 1 to 30 which r multiple of 3 = 10 (eg(3,6,9,12,15,18,21,24,27,30))

total no from 1 to 30 which r multiple of 13 = 2 (eg 13,26)

$P(a \text{ or } b) = p(a) + p(b)$

$p(a) = 10/30$

$p(b) = 2/30$

$p(a) + p(b) = 10/30 + 2/30 = 2/5$

34. Two numbers are less than a third number by 30% and 37 % respectively. How much percent is the second number less than the first?

a) 10 % b) 7 % c) 4 % d) 3 %

Solution: .7 & .63

diff in % = $(.7 - .63)/.7 * 100 = .07/.7 * 100 = 10\%$

35. If $y \neq 3$ and $2x/y$ is a prime integer greater than 2, which of the following must be true?

I. $x = y$

II. $y = 1$

III. x and y are prime integers.

(A) None

(B) I only

(C) II only

(D) III only

(E) I and II

37. Someone passed a certain bridge, which needs fee. There are 2 ways for him to choice. A : \$13/month+\$0.2/time , B: \$0.75/time . He passes the bridge twice a day. How many days at least he passes the bridge in a month, it is economic by A way?

A) 11 B) 12 C) 13 D) 14 E) 15

Soln: Let x be the no days where both are equal cost

$$13 + 0.2 \cdot 2x = 0.75 \cdot 2x$$

$$13 + .4x = 1.5x$$

$$13 = 1.1x$$

$$x = 11.81$$

by plugging in for 12 days

For A

$$13 + (12 \cdot 2) \cdot .2 = 13 + 4.8 = 17.8$$

For B

$$24 \cdot .75 = 18$$

therefore answer is B.

38. Two measure standards R and S. 24 and 30 measured with R are 42 and 60 when they are measured with S, respectively. If 100 is acquired with S, what would its value be measured with R?

39. Every student of a certain school must take one and only one elective course. In last year, $1/2$ of the students took biology as an elective, $1/3$ of the students took chemistry as an elective, and all of the other students took physics. In this year, $1/3$ of the students who took biology and $1/4$ of the students who took chemistry left school, other students did not leave, and no fresh student come in. What fraction of

all students took biology and took chemistry?

A. 7/9 B .6/7 C.5/7 D.4/9 E.2/5

Soln: If total =1

Last year

$$B=1/2$$

$$C=1/3$$

$$P=1/6$$

This year

$$B= 1/2 * 2/3 = 1/3$$

$$C= 1/3 * 3/4= 1/4$$

$$\text{Tot } 1/3+1/4=7/12$$

student left this year

$$B = 1/2 * 1/3 = 1/6$$

$$C= 1/3 * 1/4 = 1/12$$

$$\text{tot}= 1/6 + 1/12 = 1/4$$

So the school has total student this year

= Last year student no - total no of student left this year

$$= 1 - 1/4$$

$$= 3/4$$

$$\text{Answer} = 7/12 / 3/4$$

$$= 7/9$$

40. If $X > 0.9$, which of the following equals to X?

A. $0.81^{1/2}$ B. $0.9^{1/2}$ C. 0.9^2 D. $1-0.01^{1/2}$

41. There are 8 students. 4 of them are men and 4 of them are women. If 4 students are selected from the 8 students. What is the probability that the number of men is equal to that of women?

A.18/35 B.16/35 C.14/35 D.13/35 E.12/35

Soln: there has to be equal no of men & women so out of 4 people selected there has to be 2M & 2W.

Total ways of selecting 4 out of 8 is $8C4$

total ways of selecting 2 men out of 4 is $4C2$

total ways of selecting 2 women out of 4 is $4C2$

so probability is $(4C2 * 4C2) / 8C4 = 18/35$

42. The area of an equilateral triangle is 9.what is the area of it circumcircle.

A.10PI B.12PI C.14PI D.16PI E.18PI

$$\text{Soln: area} = \frac{\sqrt{3}}{4} * (\text{side})^2 = 9$$

$$\text{so, } (\text{side})^2 = \frac{9*4}{\sqrt{3}}$$

$$\text{Height} = H = \frac{\sqrt{3}}{2} * (\text{side}), \text{ so } H^2 = \frac{3}{4} * \text{Side}^2 = \frac{3}{4} * \frac{9*4}{\sqrt{3}}$$

$$\text{Radius of CircumCircle} = R = \frac{2}{3} \text{ of (Height of the Equilateral Triangle)}$$

$$\text{so, area of Circumcircle} = \pi * R^2$$

$$\Rightarrow \pi * \frac{4}{9} * H^2$$

$$\Rightarrow \pi * \frac{4}{9} * \frac{3}{4} * \frac{9*4}{\sqrt{3}}$$

$$\Rightarrow \pi * 4 * \sqrt{3}$$

43. A group of people participate in some curriculums, 20 of them practice Yoga, 10 study cooking, 12 study weaving, 3 of them study cooking only, 4 of them study both the cooking and yoga, 2 of them participate all curriculums. How many people study both cooking and weaving?

A.1 B.2 C.3 D.4 E.5

Soln: We know there are 10 people who do cooking as an activity.

3 -> people who do only cooking

4 -> do cooking and Yoga

2 -> do all of the activities

x -> number of people doing cooking and weaving

When you sum all this up, we should have 10. So $3+4+2+x=10 \rightarrow x=10-9=1$

44. There 3 kinds of books in the library fiction, non-fiction and biology. Ratio of fiction to non-fiction is 3 to 2; ratio of non-fiction to biology is 4 to 3, and the total of the books is more than 1000?which one of following can be the total of the book?

A 1001 B. 1009 C.1008 D.1007 E.1006

Soln: fiction : non-fiction = 3 : 2 = 6 : 4

non-fiction : biology = 4 : 3

fiction : non-fiction : biology = 6 : 4 : 3

$$6x + 4x + 3x = 1000$$

$$x = 76 \frac{12}{13}$$

if we add 1, 13 will divide evenly ($1001/13 = 77$)
 $1000 + 1 = 1001$

45. In a consumer survey, 85% of those surveyed liked at least one of three products: 1, 2, and 3. 50% of those asked liked product 1, 30% liked product 2, and 20% liked product 3. If 5% of the people in the survey liked all three of the products, what percentage of the survey participants liked more than one of the three products?

A) 5

B) 10

C) 15

D) 20

E) 25

Soln: $n(1 \cup 2 \cup 3) = n(1) + n(2) + n(3) - n(1 \cap 2) - n(2 \cap 3) - n(1 \cap 3) + n(1 \cap 2 \cap 3)$
 $85 = 50 + 30 + 20 - [n(1 \cap 2) - n(2 \cap 3) - n(1 \cap 3)] + 5$
 $[n(1 \cap 2) - n(2 \cap 3) - n(1 \cap 3)] = 20$

46. For a certain company, operating costs and commissions totaled \$550 million in 1990, representing an increase of 10 percent from the previous year. The sum of operating costs and commissions for both years was

(A) \$1,000 million

(B) \$1,050 million

(C) \$1,100 million

(D) \$1,150 million

(E) \$1,155 million

Solution: $1998 = \$500 \text{ M}$
 $1999 = \$550 \text{ M}$

Sum = \$ 1050 M

Ans: B

47. Fox jeans regularly sell for \$15 a pair and Pony jeans regularly sell for \$18 a pair. During a sale these regular unit prices are discounted at different rates so that a total of \$9 is saved by purchasing 5 pairs of jeans: 3 pairs of Fox jeans and 2 pairs of Pony jeans. If the sum of the two discounts rates is 22 percent, what is the discount rate on Pony jeans?

(A) 9%

(B) 10%

(C) 11%

(D) 12%

(E) 15%

Soln: Ans : B

Total discount is 22% = \$9

by back solving, in this case ,the discount percent for pony jeans should be less than 11% (22/2)because the price of this product is more.

take choice B

10% of 18= 1.8

total discount on 2 pony jeans= \$3.6

22%-10%=12%

12% of 15 = \$1.8

total discount on 3 fox jeans = \$5.4

3fox jeans discount +2 pony jeans discount =\$9

5.4 +3.6=9

so answer is B.

48. There are 2 kinds of staff members in a certain company, PART TIME AND FULL TIME. 25 percent of the total members are PART TIME members others are FULL TIME members. The work time of part time members is $\frac{3}{5}$ of the full time members. Wage per hour is same. What is the ratio of total wage of part time members to total wage of all members.

A. $\frac{1}{4}$ B. $\frac{1}{5}$ C $\frac{1}{6}$ D $\frac{1}{7}$ E $\frac{1}{8}$

Soln: What is the ratio of total wage of part time members to **total wage of all members**.

You have calculated ratio of Part time-to-Full time.

P $\times \frac{4}{3}$ $\frac{3y}{5}$ $\frac{3xy}{4 \times 5}$

A $\frac{3x}{4}$ Y + $\frac{x}{4}$ $\frac{3y}{5}$ $\frac{18xy}{20}$

Ratio= p/a= $\frac{3xy}{18xy} = \frac{1}{6}$

49. If 75% of a class answered the 1st question on a certain test correctly, 55% answered the 2nd question on the test correctly and 20% answered neither of the questions correctly, what percent answered both correctly?

10%

20%

30%

50%

65%

Soln: This problem can be easily solved by Venn Diagrams

Lets think the total class consists of 100 students

so 75 students answered question 1
and 55 students answered question 2

Now 20 students not answered any question correctly

Therefore out of total 100 students only 80 students answered either question 1 or question 2 or both the questions...

So $75+55=130$ which implies $130-80=50\%$ are the students who answered both correctly and are counted in both the groups...that's why the number was 50 more..

Let me know if someone has problems understanding...

50. A set of data consists of the following 5 numbers: 0,2,4,6, and 8. Which two numbers, if added to create a set of 7 numbers, will result in a new standard deviation that is close to the standard deviation for the original 5 numbers?

- A). -1 and 9
- B). 4 and 4
- C). 3 and 5
- D). 2 and 6
- E). 0 and 8

Soln: $SD = \sqrt{\text{Sum}(X-x)^2/N}$

Since N is changing from 5 to 7. Value of $\text{Sum}(X-x)^2$ should change from 40(current) to 48. So that SD remains same.

so due to new numbers it adds 8. Choice D only fits here.

51. How many integers from 0 to 50, inclusive, have a remainder of 1 when divided by 3?

A.14 B.15. C.16 D.17 E.18

Soln: if we arrange this in AP, we get
 $4+7+10+\dots+49$

so $4+(n-1)3=49$: $n=16$
C is my pick

52. If $k=m(m+4)(m+5)$ k and m are positive integers. Which of the following could divide k evenly?

I.3 II.4 III.6

Soln: The idea is to find what are the common factors that we get in the answer.

$m = 1$, $k = 30$ which is divisible by 1,2,3,5,6,10, 15 and 30

$m = 2$, $k = 84$ which is divisible by 1,2,3,4,6,

As can be seen, the common factors are 1,2,3,6

So answer is 3 and 6

53. If the perimeter of square region S and the perimeter of circular region C are equal, then the ratio of the area of S to the area of C is closest to

(A) $\frac{2}{3}$

(B) $\frac{3}{4}$

(C) $\frac{4}{3}$

(D) $\frac{3}{2}$

(E) 2

Soln: and the answer would be B...here is the explanation...

Let the side of the square be s ..then the perimeter of the square is $4s$

Let the radius of the circle be r ..then the perimeter of the circle is $2\pi r$

it is given that both these quantities are equal..therefore

$$4s = 2\pi r$$

which is then $s/r = \pi/2$

Now the ratio of area of square to area of circle would be

$$\frac{s^2}{\pi r^2}$$

$$(\frac{1}{\pi}) * (\frac{s}{r})^2$$

$$= (\frac{1}{\pi}) * (\frac{\pi}{2})^2 \text{ from the above equality relation}$$

$$\pi = \frac{22}{7} \text{ or } 3.14$$

the value of the above expression is approximate $= 0.78$ which is near to answer B

54. Two people walked the same distance, one person's speed is between 25 and 45, and if he used 4 hours, the speed of another people is between 45 and 60, and if he used 2 hours, how long is the distance?

A.116 B.118 C.124 D.136 E.140

Soln: First person-- speed is between 25mph and 45mph

so for 4 hrs he can travel 100 miles if he goes at 25mph speed

and for 4 hrs he can travel 130 miles if he goes at 45 mph speed

similarly

Second person-- speed is between 45mph and 60mph
so for 2 hrs he can travel 90 miles if he goes at 45mph speed
and for 2 hrs he can travel 120 miles if he goes at 60 mph speed

So for the first person---distance traveled is greater than 100 and Less than 130

and for the second person---distance traveled is greater than 90 and less than 120

so seeing these conditions we can eliminate C, D, and E answers...

but I didn't understand how to select between 116 and 118 as both these values are satisfying the conditions...

55. How many number of 3 digit numbers can be formed with the digits 0,1,2,3,4,5 if no digit is repeated in any number? How many of these are even and how many odd?

Soln: Odd: fix last as odd, 3 ways $\underline{\quad}\underline{\quad}\underline{3}$
now, left are 5, but again leaving 0, 4 for 1st digit & again 4 for 2nd digit: $\underline{4}\underline{\quad}\underline{4}\underline{\quad}\underline{3}$
=48 Odd.

$100-48=52$ Even

56. How many 3-digit numerals begin with a digit that represents a prime and end with a digit that represents a prime number?

A) 16 B) 80 c) 160 D) 180 E) 240

Soln: The first digit can be 2, 3, 5, or 7 (4 choices)
The second digit can be 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9 (10 choices)
The third digit can be 2, 3, 5, or 7 (4 choices)

$4 * 4 * 10 = 160$

57. There are three kinds of business A, B and C in a company. 25 percent of the total revenue is from business A; t percent of the total revenue is from B, the others are from C. If B is \$150,000 and C is the difference of total revenue and 225,000, what is t?

A.50 B.70 C.80 D.90 E.100

Soln: Let the total revenue be X.

So $X = A + B + C$

which is $X = \frac{1}{4}X + 150 + (X - 225)$

$4X = X + 600 + 4X - 900$

Solving for X you get $X = 300$

And 150 is 50% of 300 so the answer is 50 % (A)

58. A business school club, Friends of Foam, is throwing a party at a local bar. Of the business school students at the bar, 40% are first year students and 60% are second year students. Of the first year students, 40% are drinking beer, 40% are drinking mixed drinks, and 20% are drinking both. Of the second year students, 30% are drinking beer, 30% are drinking mixed drinks, and 20% are drinking both. A business school student is chosen at random. If the student is drinking beer, what is the probability that he or she is also drinking mixed drinks?

- A. $\frac{2}{5}$
- B. $\frac{4}{7}$
- C. $\frac{10}{17}$
- D. $\frac{7}{24}$
- E. $\frac{7}{10}$

Soln: The probability of an event A occurring is the number of outcomes that result in A divided by the total number of possible outcomes.

The total number of possible outcomes is the total percent of students drinking beer.

40% of the students are first year students. 40% of those students are drinking beer. Thus, the first years drinking beer make up $(40\% * 40\%)$ or 16% of the total number of students.

60% of the students are second year students. 30% of those students are drinking beer. Thus, the second years drinking beer make up $(60\% * 30\%)$ or 18% of the total number of students.

$(16\% + 18\%)$ or 34% of the group is drinking beer.

The outcomes that result in A is the total percent of students drinking beer and mixed drinks.

40% of the students are first year students. 20% of those students are drinking both beer and mixed drinks. Thus, the first years drinking both beer and mixed drinks make up $(40\% * 20\%)$ or 8% of the total number of students.

60% of the students are second year students. 20% of those students are drinking both beer and mixed drinks. Thus, the second years drinking both beer and mixed drinks make up $(60\% * 20\%)$ or 12% of the total number of students.

$(8\% + 12\%)$ or 20% of the group is drinking both beer and mixed drinks.

If a student is chosen at random is drinking beer, the probability that they are also drinking mixed drinks is $(20/34)$ or $10/17$.

59. A merchant sells an item at a 20% discount, but still makes a gross profit of 20 percent of the cost. What percent of the cost would the gross profit on the item have been if it had been sold without the discount?

A) 20% B) 40% C) 50% D) 60% E) 75%

Soln: Lets suppose original price is 100.

And if it sold at 20% discount then the price would be 80

but this 80 is 120% of the actual original price...so 66.67 is the actual price of the item

now if it sold for 100 when it actually cost 66.67 then the gross profit would be 49.99%
i.e. approx 50%

60. If the first digit cannot be a 0 or a 5, how many five-digit odd numbers are there?

- A. 42,500**
- B. 37,500**
- C. 45,000**
- D. 40,000**
- E. 50,000**

Soln: This problem can be solved with the Multiplication Principle. The Multiplication Principle tells us that the number of ways independent events can occur together can be determined by multiplying together the number of possible outcomes for each event.

There are 8 possibilities for the first digit (1, 2, 3, 4, 6, 7, 8, 9).

There are 10 possibilities for the second digit (0, 1, 2, 3, 4, 5, 6, 7, 8, 9)

There are 10 possibilities for the third digit (0, 1, 2, 3, 4, 5, 6, 7, 8, 9)

There are 10 possibilities for the fourth digit (0, 1, 2, 3, 4, 5, 6, 7, 8, 9)

There are 5 possibilities for the fifth digit (1, 3, 5, 7, 9)

Using the Multiplication Principle:

$$\begin{aligned} &= 8 * 10 * 10 * 10 * 5 \\ &= 40,000 \end{aligned}$$

61. A bar is creating a new signature drink. There are five possible alcoholic ingredients in the drink: rum, vodka, gin, peach schnapps, or whiskey. There are five possible non-alcoholic ingredients: cranberry juice, orange juice, pineapple juice, lime juice, or lemon juice. If the bar uses two alcoholic ingredients and two non-alcoholic ingredients, how many different drinks are possible?

- A. 100**
- B. 25**
- C. 50**
- D. 75**
- E. 3600**

Soln: The first step in this problem is to calculate the number of ways of selecting two alcoholic and two non-alcoholic ingredients. Since order of arrangement does not matter, this is a combination problem.

The number of combinations of n objects taken r at a time is

$$C(n, r) = \frac{n!}{r!(n-r)!}$$

The number of combinations of alcoholic ingredients is

$$\begin{aligned} C(5, 2) &= \frac{5!}{2!(3!)} \\ C(5, 2) &= \frac{120}{(2)(6)} \\ C(5, 2) &= 10 \end{aligned}$$

The number of combinations of non-alcoholic ingredients is

$$\begin{aligned} C(5, 2) &= \frac{5!}{2!(3!)} \\ C(5, 2) &= \frac{120}{(2)(6)} \\ C(5, 2) &= 10 \end{aligned}$$

The number of ways these ingredients can be combined into a drink can be determined by the Multiplication Principle. The Multiplication Principle tells us that the number of ways independent events can occur together can be determined by multiplying together the number of possible outcomes for each event.

The number of possible drinks is

$$\begin{aligned} &= 10 * 10 \\ &= 100 \end{aligned}$$

62. The sum of the even numbers between 1 and n is $79 \cdot 80$, where n is an odd number. $N = ?$

Soln: The sum of numbers between 1 and n is $= \frac{n(n+1)}{2}$

$$1+2+3+\dots+n = \frac{n(n+1)}{2} \text{ \{formula\}}$$

we are looking for the sum of the even numbers between 1 and n, which means:

$$\begin{aligned}
 &2+4+6+\dots+(n-1) \text{ n is ODD} \\
 &=1*2+2*2+2*3+\dots+2*((n-1)/2) \\
 &=2*(1+2+3+\dots+*((n-1)/2)) \\
 &\text{from the formula we obtain :} \\
 &=2*(((n-1)/2)*((n-1)/2+1))/2 \\
 &=((n-1)/2)*((n+1)/2)=79*80 \\
 &\Rightarrow (n-1)*(n+1)=158*160 \\
 &\Rightarrow n=159
 \end{aligned}$$

63. A committee of 6 is chosen from 8 men and 5 women so as to contain at least 2 men and 3 women. How many different committees could be formed if two of the men refuse to serve together?

- (A) 3510
- (B) 2620
- (C) 1404
- (D) 700
- (E) 635

$$\begin{aligned}
 \text{Soln: } 4W \ 2M &= 5C_4 \cdot (8C_2 - 1) = 5 \cdot (27) = 135 \\
 3W \ 3M &= 5C_3 \cdot (8C_3 - 6) = 10 \cdot 50 = 500 \\
 \text{total } &635
 \end{aligned}$$

Max. number of possibilities considering we can choose any man $8C_2 * 5C_4 + 8C_3 * 5C_3 = 700$.

consider it this way.... from my previous reply max possible ways considering we can chose any man = 700

now we know that 2 man could not be together... now think opposite... how many ways are possible to have these two man always chosen together...

since they are always chosen together...

For chosing 2 men and 4 women for the committee there is only 1 way of chosing 2 men for the committee since we know only two specific have to be chosen and there are $5C_4$ ways of choosing women

$$1 * 5C_4 = 5$$

For choosing 3 men and 3 women for the committee there are exactly $6C_1$ ways choosing 3 men for the committee since we know two specific have to be chosen so from the remaining 6 men we have to chose 1 and there are $5C_4$ ways of choosing women

$$6C1 * 5C3 = 60$$

So total number of unfavorable cases = $5 + 60 = 65$

Now since we want to exclude these 65 cases... final answer is $700 - 65 = 635$

64. In how many ways can the letters of the word 'MISSISSIPPI' be arranged?

- a) 1260
- b) 12000
- c) 12600
- d) 14800
- e) 26800

Soln: Total # of alphabets = 10
so ways to arrange them = $10!$

Then there will be duplicates because 1st S is no different than 2nd S.
we have 4 Is
3 S
and 2 Ps

Hence # of arrangements = $10! / 4! * 3! * 2!$

65. Goldenrod and No Hope are in a horse race with 6 contestants. How many different arrangements of finishes are there if No Hope always finishes before Goldenrod and if all of the horses finish the race?

- (A) 720
- (B) 360
- (C) 120
- (D) 24
- (E) 21

Soln: two horses A and B, in a race of 6 horses... A has to finish before B

if A finishes 1... B could be in any of other 5 positions in 5 ways and other horses finish in $4!$ Ways, so total ways $5 * 4!$

if A finishes 2... B could be in any of the last 4 positions in 4 ways. But the other positions could be filled in $4!$ ways, so the total ways $4 * 4!$

if A finishes 3rd... B could be in any of last 3 positions in 3 ways, but the other positions could be filled in $4!$ ways, so total ways $3 * 4!$

if A finishes 4th... B could be in any of last 2 positions in 2 ways, but the other positions could be filled in $4!$ ways, so total ways... $2 * 4!$

if A finishes 5th .. B has to be 6th and the top 4 positions could be filled in 4! ways..

A cannot finish 6th, since he has to be ahead of B

therefore total number of ways

$$5*4! + 4*4! + 3*4! + 2*4! + 4! = 120 + 96 + 72 + 48 + 24 = 360$$

66. On how many ways can the letters of the word "COMPUTER" be arranged?

1. M must always occur at the third place

2. Vowels occupy the even positions.

Soln: For 1.

$$7*6*1*5*4*3*2*1=5,040$$

For 2.) I think It should be $4 * 720$

there are 4 even positions to be filled by three even numbers.

in $5*3*4*2*3*1*2*1$ It is assumed that Last even place is NOT filled by a vowel. There can be total 4 ways to do that.

Hence $4 * 720$

67. A shipment of 10 TV sets includes 3 that are defective. In how many ways can a hotel purchase 4 of these sets and receive at least two of the defective sets?

Soln: There are 10 TV sets; we have to choose 4 at a time. So we can do that by $10C4$ ways. We have 7 good TV's and 3 defective.

Now we have to choose 4 TV sets with at least 2 defective. We can do that by

2 defective 2 good

3 defective 1 good

That stands to $3C2*7C2 + 3C3*7C1$ (shows the count)

If they had asked probability for the same question then

$$3C2*7C2 + 3C3*7C1 / 10C4.$$

68. A group of 8 friends want to play doubles tennis. How many different ways can the group be divided into 4 teams of 2 people?

A. 420

B. 2520

C. 168

D. 90

E. 105

Soln: 1st team could be any of 2 guys... there would be 4 teams (a team of A&B is same as a team of B&A)... possible ways $8C2 / 4$.

2nd team could be any of remaining 6 guys. There would be 3 teams (a team of A&B is same as a team of B&A)... possible ways $6C2 / 3$

3rd team could be any of remaining 2 guys... there would be 2 teams (a team of A&B is same as a team of B&A). Possible ways $4C2 / 2$

4th team could be any of remaining 2 guys... there would be 1 such teams... possible ways $2C2 / 1$

total number of ways...

$$8C2 * 6C2 * 4C2 * 2C2$$

$$4 * 3 * 2 * 1$$

=

$$8 * 7 * 6 * 5 * 4 * 3 * 2 * 1$$

$$4 * 3 * 2 * 1 * 2 * 2 * 2 * 2$$

$$= 105 \text{ (ANSWER)...}$$

Another method: say you have 8 people ABCDEFGH

now u can pair A with 7 others in 7 ways.

Remaining now 6 players.

Pick one and u can pair him with the remaining 5 in 5 ways.

Now you have 4 players.

Pick one and u can pair him with the remaining in 3 ways.

Now you have 2 players left. You can pair them in 1 way

so total ways is $7 * 5 * 3 * 1 = 105$ ways i.e. E

69. In how many ways can 5 people sit around a circular table if one should not have the same neighbors in any two arrangements?

Soln: The ways of arranging 5 people in a circle = $(5-1)! = 4!$

For a person seated with 2 neighbors, the number of ways of that happening is 2:

AXB or BXA, where X is the person in question.

So, for each person, we have two such arrangements in $4!$. Since we don't want to repeat arrangement, we divide $4!/2$ to get 12

70. There are 4 copies of 5 different books. In how many ways can they be arranged on a shelf?

A) $20!/4!$

B) $20!/5(4!)$

C) $20!/(4!)^5$

D) $20!$

E) $5!$

Soln: 4 copies each of 5 types.

Total = 20 books.

Total ways to arrange = $20!$

Taking out repeat combos = $20!/(4! * 4! * 4! * 4! * 4!)$ – each book will have 4 copies that are duplicate. So we have to divide $20!$ by the repeated copies.

71. In how many ways can 5 rings be worn on the four fingers of the right hand?

Soln: 5 rings, 4 fingers

1st ring can be worn on any of the 4 fingers \Rightarrow 4 possibilities

2nd ring can be worn on any of the 4 fingers \Rightarrow 4 possibilities

3rd ring can be worn on any of the 4 fingers \Rightarrow 4 possibilities

4th ring can be worn on any of the 4 fingers \Rightarrow 4 possibilities

5th ring can be worn on any of the 4 fingers \Rightarrow 4 possibilities

Total possibilities = $4*4*4*4*4 = 4^5$.

72. If both 5^2 and 3^3 are factors of $n \times (2^5) \times (6^2) \times (7^3)$, what is the smallest possible positive value of n ?

Soln: Write down $n \times (2^5) \times (6^2) \times (7^3)$ as

$= n \times (2^5) \times (3^2) \times (2^2) \times (7^3)$,

$= n \times (2^7) \times (3^2) \times (7^3)$

now at a minimum 5^2 and a 3 is missing from this to make it completely divisible by $5^2 \times 3^3$

Hence answer = $5^2 \times 3 = 75$

73. Obtain the sum of all positive integers up to 1000, which are divisible by 5 and not divisible by 2.

(1) 10050 (2) 5050

(3) 5000 (4) 50000

Soln: Consider 5 15 25 ... 995

$$l = a + (n-1)*d$$

$$l = 995 = \text{last term}$$

$$a = 5 = \text{first term}$$

$$d = 10 = \text{difference}$$

$$995 = 5 + (n-1)*10$$

$$\text{thus } n = 100 = \# \text{ of terms}$$

consider 5 10 15 20.... 995

$$995 = 5 + (n-1)*5$$

$$\Rightarrow n = 199$$

Another approach...

Just add up $995 + 985 + 975 + 965 + 955 + 945 = 5820$, so it has to be greater than 5050, and the only possible choices left are 1) & 4)

Also, series is 5 15 25.... 985 995

$$\# \text{ of terms} = 100$$

$$\text{sum} = (100/2)*(2*5 + (100-1)*10) = 50*1000 = 50000$$

74. If the probability of rain on any given day in city x is 50% what is the probability it with rain on exactly 3 days in a five day period?

8/125

2/25

5/16

8/25

3/4

Soln: Use binomial theorem to solve the problem....

$$p = 1/2$$

$$q = 1/2$$

of favorable cases = 3 = r

of unfavorable cases = 5-3 = 2

total cases = 5 = n

$P(\text{probability of } r \text{ out of } n) = nCr \cdot p^r \cdot q^{(n-r)}$

75. The Full House Casino is running a new promotion. Each person visiting the casino has the opportunity to play the Trip Aces game. In Trip Aces, a player is randomly dealt three cards, without replacement, from a deck of 8 cards. If a player receives 3 aces, they will receive a free trip to one of 10 vacation destinations. If the deck of 8 cards contains 3 aces, what is the probability that a player will win a trip?

A. 1/336

B. 1/120

C. 1/56

D. 1/720

E. 1/1440

The probability of an event A occurring is the number of outcomes that result in A divided by the total number of possible outcomes.

There is only one result that results in a win: receiving three aces.

Since the order of arrangement does not matter, the number of possible ways to receive 3 cards is a combination problem.

The number of combinations of n objects taken r at a time is

$$C(n,r) = n!/(r!(n-r)!)$$

$$C(8,3) = 8!/(3!(8-3)!)$$

$$C(8,3) = 8!/(3!(5!))$$

$$C(8,3) = 40320/(6(120))$$

$$C(8,3) = 40320/720$$

$$C(8,3) = 56$$

The number of possible outcomes is 56.

Thus, the probability of being dealt 3 aces is 1/56.

76. The Full House Casino is running a new promotion. Each person visiting the casino has the opportunity to play the Trip Aces game. In Trip Aces, a player is randomly dealt three cards, without replacement, from a deck of 8 cards. If a player receives 3 aces, they will receive a free trip to one of 10 vacation destinations. If the deck of 8 cards contains 3 aces, what is the probability that a player will win a trip?

A. 1/336

B. 1/120

C. 1/56

- D. $1/720$
E. $1/1440$

Soln: Since each draw doesn't replace the cards:

Prob. of getting an ace in the first draw = $3/8$

getting in the second, after first draw is ace = $2/7$

getting in the third after the first two draws are aces = $1/6$

thus total probability for these mutually independent events = $3/8 * 2/7 * 1/6 = 1/56$

77. Find the probability that a 4 person committee chosen at random from a group consisting of 6 men, 7 women, and 5 children contains

- A) exactly 1 woman
B) at least 1 woman
C) at most 1 woman

Soln:

A.) ${}^7C_1 * {}^{11}C_3 / {}^{18}C_4$

B) $1 - ({}^{11}C_4 / {}^{18}C_4)$

C) $({}^{11}C_4 / {}^{18}C_4) + ({}^7C_1 * {}^{11}C_3 / {}^{18}C_4)$

78. A rental car service facility has 10 foreign cars and 15 domestic cars waiting to be serviced on a particular Saturday morning. Because there are so few mechanics, only 6 can be serviced.

(a) If the 6 cars are chosen at random, what is the probability that 3 of the cars selected are domestic and the other 3 are foreign?

(b) If the 6 cars are chosen at random, what is the probability that at most one domestic car is selected?

Soln:

A) ${}^{10}C_3 * {}^{15}C_3 / {}^{25}C_6$

B) Probability of no domestic car + Probability of 1 domestic car =
 ${}^{10}C_6 / {}^{25}C_6 + {}^{15}C_1 * {}^{10}C_5 / {}^{25}C_6$

79. How many positive integers less than 5,000 are evenly divisible by neither 15 nor 21?

- A. 4,514
B. 4,475
C. 4,521
D. 4,428
E. 4,349

Soln: We first determine the number of integers less than 5,000 that are evenly divisible by 15. This can be found by dividing 4,999 by 15:

$= 4,999 / 15$

$= 333 \text{ integers}$

Now we will determine the number of integers evenly divisible by 21:

$$\begin{aligned} &= 4,999/21 \\ &= 238 \text{ integers} \end{aligned}$$

some numbers will be evenly divisible by BOTH 15 and 21. The least common multiple of 15 and 21 is 105. This means that every number that is evenly divisible by 105 will be divisible by BOTH 15 and 21. Now we will determine the number of integers evenly divisible by 105:

$$\begin{aligned} &= 4,999/105 \\ &= 47 \text{ integers} \end{aligned}$$

Therefore the positive integers less than 5000 that are not evenly divisible by 15 or 21 are $4999 - (333 + 238 - 47) = 4475$

80) Find the least positive integer with four different prime factors, each greater than 2.

Soln: $3 \cdot 5 \cdot 7 \cdot 11 = 1155$

81) From the even numbers between 1 and 9, two different even numbers are to be chosen at random. What is the probability that their sum will be 8?

Soln: Initially you have 4 even numbers (2,4,6,8)
you can get the sum of 8 in two ways $\Rightarrow 2 + 6$ or $6 + 2$
so the first time you pick a number you can pick either 2 or 8 - a total of 2 choices out of 8 $\Rightarrow 1/2$
after you have picked your first number and since you have already picked 1 number you are left with only 2 options \Rightarrow either (4,6,8) or (2,4,8) and you have to pick either 6 from the first set or 2 from the second and the probability of this is $1/3$. Since these two events have to happen together we multiply them. $\frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$

82) 5 is placed to the right of two – digit number, forming a new three – digit number. The new number is 392 more than the original two-digit number. What was the original two-digit number?

Soln: If the original number has x as the tens digit and y as the ones digit (x and y are integers less than 10) then we can set up the equation:

$$100x + 10y + 5 = 10x + y + 392$$

$$90x + 9y = 387$$

$$9(10x + y) = 387$$

$$10x + y = 43 \Rightarrow x = 4, y = 3$$

the original number is 43, the new number is 435

83) If one number is chosen at random from the first 1000 positive integers, what is the probability that the number chosen is multiple of both 2 and 8?

Soln: Any multiple of 8 is also a multiple of 2 so we need to find the multiples of 8 from 0 to 1000

the first one is 8 and the last one is 1000

$$\implies ((1000-8)/8) + 1 = 125$$

$$\implies p(\text{picking a multiple of 2 \& 8}) = 125/1000 = 1/8$$

84) A serial set consists of N bulbs. The serial set lights up only if all the N bulbs are in working condition. Even if one of the bulbs fails then the entire set fails. The probability of a bulb failing is x. What is the probability of the serial set failing?

Soln: Probability of x to fail. Probability of a bulb not failing = $1-x$

probability that none of the N bulbs fail, hence **serial set** not failing = $(1-x)^N$

probability of **serial set** failing = $1-(1-x)^N$

85) Brad flips a two-sided coin 8 times. What is the probability that he gets tails on at least 7 of the 8 flips?

1/32

1/16

1/8

7/8

none of the above

Soln: Number of ways 7 tails can turn up = 8C_7

the probability of those is $1/2$ each

Since the question asks for at least 7, we need to find the prob of all 8 tails - the number of ways is ${}^8C_8 = 1$

Add the two probabilities

$${}^8C_7 * (1/2)^8 = 8/2^8 \text{ -- for getting 7}$$

$$\text{Prob of getting 8 tails} = 1/2^8$$

$$\text{Total prob} = 8/2^8 + 1/2^8 = 9/2^8$$

Ans is E.

86. A photographer will arrange 6 people of 6 different heights for photograph by placing them in two rows of three so that each person in the first row is standing in front of someone in the second row. The heights of the people within each row must increase from left to right, and each person in the second row must be taller than the person standing in front of him or her. How many such arrangements of the 6 people are possible?

A. 5

B. 6

C. 9

D. 24

E. 36

Soln: 5 ways

123 124 125 134 135
456 356 346 256 246

87. As a part of a game, four people each much secretly chose an integer between 1 and 4 inclusive. What is the approximate likelihood that all four people will chose different numbers?

Soln: The probability that the first person will pick unique number is 1 (obviously) then the probability for the second is $\frac{3}{4}$ since one number is already picked by the first, then similarly the probabilities for the 3rd and 4th are $\frac{1}{2}$ and $\frac{1}{4}$ respectively. Their product $\frac{3}{4} * \frac{1}{2} * \frac{1}{4} = \frac{3}{32}$

88. Which of the sets of numbers can be used as the lengths of the sides of a triangle?

I. [5,7,12]

II. [2,4,10]

III. [5,7,9]

A. I only

B. III only

C. I and II only

D. I and III only

E. II and III only

Soln: For any side of a triangle. Its length must be greater than the difference between the other two sides, but less than the sum of the other two sides.

Answer is B

89. A clothing manufacturer has determined that she can sell 100 suits a week at a selling price of 200\$ each. For each rise of 4\$ in the selling price she will sell 2 less suits a week. If she sells the suits for x\$ each, how many dollars a week will she receive from sales of the suits?

Soln: Let y be the number of \$4 increases she makes, and let S be the number of suits she sells. Then

$$X = 200 + 4y \implies y = \frac{x}{4} - 50$$

$$S = 100 - 2y \implies S = 100 - 2[\frac{x}{4} - 50] = 100 - \frac{x}{2} + 100 = 200 - \frac{x}{2}$$

so the answer is that the number of suits she'll sell is $200 - \frac{x}{2}$

90. A certain portfolio consisted of 5 stocks, priced at \$20, \$35, \$40, \$45 and \$70, respectively. On a given day, the price of one stock increased by 15%, while the price of another decreased by 35% and the prices of the remaining three remained constant. If the average price of a stock in the portfolio rose by approximately 2%, which of the following could be the prices of the shares that remained constant?

A) 20, 35, 70

B) 20, 45, 70

C) 20, 35, 40

D) 35, 40, 70

E) 35, 40, 45

Soln: Add the 5 prices together:

$$20 + 35 + 40 + 45 + 70 = 210$$

$$2\% \text{ of that is } 210 \times .02 = 4.20$$

Let x be the stock that rises and y be the stock that falls.

$$.15x - .35y = 4.20 \implies x = (7/3)y + 27$$

This tells us that the difference between x and y has to be at least 27. Since the answer choices list the ones that DONT change, we need to look for an answer choice in which the numbers NOT listed have a difference of at least 27.

Thus the answer is (E)

91. if $-2 \leq x \leq 2$ and $3 \leq y \leq 8$, which of the following represents the range of all possible values of $y-x$?

(A) $5 \leq y-x \leq 6$

(B) $1 \leq y-x \leq 5$

(C) $1 \leq y-x \leq 6$

(D) $1 \leq y-x \leq 10$

(E) $1 \leq y-x \leq 10$

Soln: you can easily solve this by **subtracting** the two inequalities. To do this they need to be in the **opposite** direction; when you **subtract** them preserve the sign of the inequality **from** which you are **subtracting**.

$$3 < y < 8$$

multiply the second one by (-1) to reverse the sign

$$2 > x > -2$$

Subtract them to get

$$3 - 2 < y - x < 8 - (-2)$$

$$1 < y - x < 10$$

92. Of a group of 260 people who purchased stocks, 61 purchased A, 88 purchased B, 56 purchased C, 75 purchased D, 60 purchased E. what is the greatest possible number of the people who purchased both B and D?

A:40 B:50 C:60 D:75 E:80

Soln: ANSWER is D (75)

since they have asked us to find out the greatest possible number buying both B as well as D, the answer has to be the smallest no between the two which is 75...as all the guys purchasing D can also buy B and only 75 out of 88 purchasing B can simultaneously purchase D as well....

93. There are 30 people and 3 clubs M, S, and Z in a company. 10 people joined M, 12 people joined S and 5 people joined Z. If the members of M did not join any other club, at most, how many people of the company did not join any club?

A: 4 B: 5 C: 6 D: 7 E: 8

Soln: total no of people = 30

no joining M = 10

no joining S = 12

no joining Z = 5

question asked - **AT MOST** how many people did not join any group?

solution: now since none of the members of M joined any other group, the no of people left = $30 - 10$ (for M) = 20

since the question says at most how many did not join any group, lets assume the all people who join Z also join S. so no of people joining group S and Z are 12 (note that there will be 5 people in group S who have also joined Z)

therefore no of people not joining any group = $20 - 12 = 8$

Hence e

94. Find the numbers of ways in which 4 boys and 4 girls can be seated alternatively.

1) in a row

2) in a row and there is a boy named John and a girl named Susan amongst the group who cannot be put in adjacent seats

3) around a table

A:

1) $4! * 4! * 2$

2) $4! * 4! * 2$ - number of ways with John and Susan sitting together

= $(4! * 4! * 2) - (7 * 3! * 3! * 2)$

The way that JS arrangements are found is by bracketing J and S and considering it to be a single entity. So a possible arrangement is (B=boy, G=girl)

(JS) B G B G B G number of arrangements is $7 \times 3! \times 3! = 252$

(SJ) G B G B G B number of arrangements is $7 \times 3! \times 3! = 252$

3) Fix one boy and arrange the other 3 boys in $3!$ ways. Arrange the 4 girls in $4!$ ways in the gaps between the boys.

Total arrangements = $3! \times 4!$

= 6×24

= 144

95. From a group of 3 boys and 3 girls, 4 children are to be randomly selected. What is the probability that equal numbers of boys and girls will be selected?

A. $1/10$

B. $4/9$

C. $1/2$

D. $3/5$

E. $2/3$

Soln: Total number of ways of selecting 4 children = ${}^6C_4 = 15$

with equal boys and girls. \Rightarrow 2 boys and 2 girls. $\Rightarrow {}^3C_2 \times {}^3C_2 = 9$.

Hence $p = 9/15 = 3/5$

96. What is the remainder when $9^1 + 9^2 + 9^3 + \dots + 9^9$ is divided by 6?

(1) 0

(2) 3

(3) 4

(4) 2

(5) None of these

Soln: Remainder of $9^{\text{odd}}/6$ is 3
remainder of $9^{\text{even}}/6$ is 0

$$9^1 + 9^2 + 9^3 + \dots + 9^9 = 9(1 + 9 + 9^2 + \dots + 9^8)$$

$1 + 9 + 9^2 + \dots + 9^8$ is odd.

Thus we obtain 3 as a remainder when we divide $9(1 + 9 + 9^2 + \dots + 9^8)$ by 6.

Another way: We get the value as $9 \cdot \text{odd} / 6 = 3 \cdot \text{odd} / 2$. Since $3 \cdot \text{odd} = \text{odd}$; $\text{odd} / 2 = \text{XXXX}.5$

so something divided by 6, gives XXXX.5, hence remainder is $6 \cdot 0.5 = 3$

97. Find the value of $1.1! + 2.2! + 3.3! + \dots + n.n!$

- (1) $n! + 1$
- (2) $(n+1)!$
- (3) $(n+1)! - 1$
- (4) $(n+1)! + 1$
- (5) None of these

Soln: $1.1! + 2.2! + 3.3! + \dots + n.n!$
 $= 1.1! + (3-1)2! + (4-1)3! + \dots + ((n+1)-1)n!$
 $= 1.1! + 3! - 2! + 4! - 3! + \dots + (n+1)! - n!$

So it is $(n+1)! - 1$ (Answer choice 4)

98. The numbers x and y are three-digit positive integers, and $x + y$ is a four-digit integer. The tens digit of x equals 7 and the tens digit of y equals 5. If $x < y$, which of the following must be true?

- I. The units digit of $x + y$ is greater than the units digit of either x or y .
- II. The tens digit of $x + y$ equals 2.
- III. The hundreds digit of y is at least 5.

- A. II only
- B. III only
- C. I and II
- D. I and III
- E. II and III

Soln: $x = abc$
 $y = def$

$x = a7c$
 $y = b5f$

$x > y$ and $x + y = wxyz$.

I. The units digit of $x + y$ is greater than the units digit of either x or y .

It can carryover one digit. False

II. The tens digit of $x + y$ equals 2.

It can be 2 or 3. False

III. The hundreds digit of y is at least 5.

$$a+b+1 \geq 10$$

$a > b$ so a at least 5. True.

Ans: b

99. Among 5 children there are 2 siblings. In how many ways can the children be seated in a row so that the siblings do not sit together?

(A) 38

(B) 46

(C) 72

(D) 86

(E) 102

Soln: The total number of ways 5 of them can sit is 120

when the siblings sit together they can be counted as one entity

therefore the number of ways that they sit together is $4! = 24$, but since

the two siblings can sit in two different ways e.g. AB and BA we multiply 24 by 2 to get the total number of ways in which the 5 children can sit together with the siblings sitting together - 48

In other words $4P4 * 2P2$

the rest is obvious $120 - 48 = 72$

100. There are 70 students in Math or English or German. Exactly 40 are in Math, 30 in German, 35 in English and 15 in all three courses. How many students are enrolled in exactly two of the courses? Math, English and German.

$$\text{Soln: } \text{MuEuG} = M + E + G - \text{MnE} - \text{MnG} - \text{EnG} - 2(\text{MnEnG})$$

$$\text{MnE} + \text{MnG} + \text{EnG} = M + E + G - 2(\text{MnEnG}) - \text{MuEuG}$$

$$\text{MnE} + \text{MnG} + \text{EnG} = 40 + 30 + 35 - 2(15) - 70 = 105 - 30 - 70 = 5$$

Whenever an intersection occurs between 2 sets, (MnEnG) is counted twice, therefore you deduct one of it. If the intersection occurs between 3 sets, it is counted thrice; therefore you deduct two of it. And so forth.

If there are four sets, then the formula is $A + B + C + D - (\text{two}) - (\text{three}) * 2 - (\text{four}) * 3 = \text{total}$

101. John can complete a given task in 20 days. Jane will take only 12 days to complete the same task. John and Jane set out to complete the task by beginning to work together. However, Jane was indisposed 4 days before the work got over. In how many days did the work get over from the time John and Jane started to work

on it together?

A- 6 B- 10 C- 8 D- 7.5 E- 3.5

Soln: Together they do $1/20 + 1/12 = 4/30$ of the task.

Jane and John started the work together, but only John finished the work because Jane gets sick. So let x be the number of days they worked together.

$$x \cdot \frac{3}{40} + 4 \cdot \frac{1}{20} = 1$$

$$x \cdot \frac{4}{30} = \frac{4}{5} \text{ and therefore } x = 6$$

So in total they worked 6 days on it together and John worked 4 days on it. So total days spent = 10, but if the question is asking how many time did they spend working on the project together, then the answer is 6.

102. Jane gave Karen a 5 m head start in a 100 race and Jane was beaten by 0.25m. In how many meters more would Jane have overtaken Karen?

Soln: Jane gave Karen a 5 m head start means Karen was 5 m ahead of Jane. So after the lead, Karen ran 95m and Jane ran 99.75 m when the race ended.

Let speed of Karen and Jane be K and J respectively and lets say after X minutes Jane overtakes Karen.

$$\text{1st condition: } 95/K = 99.75/J$$

$$\text{2nd condition } JX - KX = 0.25$$

Solving for JX , we get $JX = 21/4$.

Hence Jane needs to run 5.25m more (or total of 105m) to overtake Karen.

103. Out of 20 surveyed students 8 study math and 7 study both math and physics. If 10 students do not study either of these subjects, how many students study physics but not math?

(A) 1

(B) 2

(C) 4

(D) 5

(E) 6

Soln: total = gr1 + gr2 - both + neither

$$20 = 8 + P - 7 + 10$$

$$P = 9, 9 \text{ students study both P and M, } 9 - 7 = 2 \text{ study only P}$$

104. Machine A can produce 50 components a day while machine B only 40. The monthly maintenance cost for

machine A is \$1500 while that for machine B is \$550. If each component generates an income of \$10 what is the least number of days per month that the plant has to work to justify the usage of machine A instead of machine B?

- (A) 6
- (B) 7
- (C) 9
- (D) 10
- (E) 11

Soln: Let x be the number of days that need to be worked

$$500x - 1500 > 400x - 550$$

$$100x > 950$$

$$x > 9.5 \text{ (D)}$$

105. Four cups of milk are to be poured into a 2-cup bottle and a 4-cup bottle. If each bottle is to be filled to the same fraction of its capacity, how many cups of milk should be poured into the 4-cup bottle?

- A. $\frac{2}{3}$
- B. $\frac{7}{3}$
- C. $\frac{5}{2}$
- D. $\frac{8}{3}$
- E. 3

Soln: $x + y = 4$

$$x = 4 - y$$

$$\frac{x}{2} = \frac{y}{4}$$

$$4x = 2y$$

$$4(4-y) = 2y$$

$$16 = 6y$$

$$y = \frac{8}{3} \text{ (D)}$$

106. If 10 persons meet at a reunion and each person shakes hands exactly once with each of the others, what is the total number of handshakes?

- (A) 10!
- (B) $10 \cdot 10$
- (C) $10 \cdot 9$
- (D) 45
- (E) 36

Soln: There are $10C_2$ ways to pick 2 different people out of 10 people.

$${}^{10}C_2 = \frac{10!}{2!8!} = 45 \text{ (D)}$$

107. If operation \$ is defined as

\$X = X + 2 if X is even

\$X = X - 1 if X is odd,

what is \$(...\$(\$(\$15)))...) 99 times?

(A) 120

(B) 180

(C) 210

(D) 225

(E) 250

Soln: \$15 gives 14

After that it is an AP with $d=2$, $a=14$ and $n=99$

108. A and B alternately toss a coin. The first one to turn up a head wins. if no more than five tosses each are allowed for a single game.

1- Find the probability that the person who tosses first will win the game?

2- What are the odds against A's losing if she goes first?

Soln: look at the conditions; it says that the first person who tosses a head wins.

Let's say A tosses first.

what is the probability that he wins

$$H + TTH + TTTTH + TTTTTTH + TTTTTTTH$$

i.e. either the first toss is head,
or the first time A tosses the coin he gets a tail and B also gets a tail , n in the second throw A gets a head.....

This continues for a max till 5 throws, because the game is for 5 throws only.

$$\text{So, 1. } \frac{1}{2} + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^5 + \left(\frac{1}{2}\right)^7 + \left(\frac{1}{2}\right)^9$$

$$2. \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^6 + \left(\frac{1}{2}\right)^8 + \left(\frac{1}{2}\right)^{10}$$

109. How many integers less than 1000 have no factors (other than 1) in common with 1000?

(1) 400

(2) 410

- (3) 411
 (4) 412
 (5) None of the above

Soln: 1000 - multiples of 2 and/or 5

multiples of 2 = 500 (all even #)
 multiples of 5 = $(995 - 5)/10 + 1$ [Using AP formula]
 = 100

Answer = $1000 - (500 + 100)$
 = 400

You cannot calculate for all multiples of 5 because you have already removed all even integers (including 10, 20, and 30). The difference in the AP series should be 10 instead of 5 because you're looking for the integers that have 5 as a unit's digit. Therefore we divide by 10 and not 5.

110. Two different numbers when divided by the same divisor left remainders of 11 and 21 respectively. When the numbers' sum was divided by the same divisor, the remainder was 4. What was the divisor?

36, 28, 12, 9 or none

Soln: Let the divisor be a.

$$x = a*n + 11 \text{ ---- (1)}$$

$$y = a*m + 21 \text{ ----- (2)}$$

$$\text{also given, } (x+y) = a*p + 4 \text{ ----- (3)}$$

$$\text{adding the first 2 equations. } (x+y) = a*(n+m) + 32 \text{ ----- (4)}$$

equate 3 and 4.

$$a*p + 4 = a*(n+m) + 32$$

or

$$a*p + 4 = [a*(n+m) + 28] + 4$$

cancel 4 on both sides.

u will end up with.

$$a*p = a*(n+m) + 28.$$

which implies that 28 should be divisible by a. or in short $a = 28$ works.

Another method:

I think the easiest (not necessarily the shortest), way to solve this is to use given answer choices. Since the remainders are given as 11 and 21, therefore the divisor has to be greater than 21 which leaves with two choices 28 and 36. Try 28 first; let the two numbers be $28+11=39$ and $28+21=49$. Summing them up and dividing by 28 gives $(49+39=88)$, $88/28$ remainder is 4, satisfies the given conditions. Check for 36 with same approach, does not work, answer is 28

111. There are 8 members; among them are Kelly and Ben. A committee of 4 is to be chosen out of the 8. What is the probability that Ben is chosen to be in the committee and Kelly is not?

Soln: let's assume Ben has already been chosen. Then I have to choose 3 more people from the remaining, excluding Kelly, that is, three from six people, that's 6C_3 . so the total is $(1 + {}^6C_3)/8C_4$ which is $2/7$

112. How many 5-digit positive integers exist where no two consecutive digits are the same?

- A.) $9 \cdot 9 \cdot 8 \cdot 7 \cdot 6$
- B.) $9 \cdot 9 \cdot 8 \cdot 8 \cdot 8$
- C.) 9^5
- D.) $9 \cdot 8^4$
- E.) $10 \cdot 9^4$

Soln: C is correct.

The first place has 9 possibilities, since 0 is not to be counted. All others have 9 each, since you cannot have the digit, which is same as the preceding one.

Hence 9^5

113. How many five digit numbers can be formed using the digits 0, 1, 2, 3, 4 and 5 which are divisible by 3, without repeating the digits?

- (A) 15
- (B) 96
- (C) 216
- (D) 120
- (E) 180

Soln: The sum of digits of a multiple of 3 should be div by 3.

for a 5 digit number to be div by 3, the sum of digits (given the digits here) can be only 12 or 15.

For a sum of 12, the digits that can be used : 0,1,2,4,5

for a sum of 15: 1,2,3,4,5

Number of numbers from the first set = $4 \cdot 4!$ (0 cannot be the first digit in the numbers)

for the second set : $5!$

total = $5! + 4 \cdot 4! = 4!(5+4) = 24 \cdot 9 = 216$

Since 0 cannot be the first digit of a number, for the first position, you have 4 choices (all digits except zero). No such constraints exist for the rest of the positions; hence the next choices are 4,3,2,1 - all multiplying up to give a $4!$. Had there been no 0 involved, the choices would've been $5!$ Instead of $4 \cdot 4!$

114. A group of 8 friends want to play doubles tennis. How many different ways can the group be divided into 4 teams of 2 people?

- A. 420
- B. 2520
- C. 168
- D. 90
- E. 105

Soln: out of 8 people one team can be formed in $8C2$ ways.

$$8C2 * 6C2 * 4C2 * 2C2 = 2520.$$

The answer is 105. Divide 2520 by $4!$ to remove the multiples (for example: (A,B) is same as (B,A))

115. My name is AJEET. But my son accidentally types the name by interchanging a pair of letters in my name. What is the probability that despite this interchange, the name remains unchanged?

- a) [5%](#)
- b) [10%](#)
- c) [20%](#)
- d) [25%](#)

Soln: there are actually 20 ways to interchange the letters, namely, the first letter could be one of 5, and the other letter could be one of 4 left. So total pairs by product rule = 20.

Now, there are two cases when it wouldn't change the name. First, keeping them all the same. Second, interchanging the two EEs together. Thus 2 options would leave the name intact.

$$\text{Prob} = 2/20 = 0.1, \text{ or } 10\%.$$

116. A certain right triangle has sides of length x , y , and z , where $x < y < z$. If the area of this triangular region is 1, which of the following indicates all of the possible values of y ?

- A. $y > \text{ROOT}2$**
- B. $\text{ROOT}3/2 < y < \text{ROOT}2$**
- C. $\text{ROOT}2/3 < y < \text{ROOT}3/2$**
- D. $\text{ROOT}3/4 < y < \text{ROOT}2/3$**
- E. $y < \text{ROOT}3/4$**

Soln: right triangle with sides $x < y < z$ and area of 1 $\Rightarrow z = \text{hypotenuse}$ and $xy/2 = 1$
i.e $xy = 2$

If x were equal to y , we would have had $xy = y^2 = 2$. And $y = \text{root}2$

But, $x < y$ and so $y > \text{root}2$.

117. A certain deck of cards contains 2 blue cards, 2 red cards, 2 yellow cards, and 2 green cards. If two cards are randomly drawn from the deck, what is the probability that they will both are not blue?

- A. 15/28**

B. $\frac{1}{4}$

C. $\frac{9}{16}$

D. $\frac{1}{32}$

E. $\frac{1}{16}$

Soln: Chance of drawing a blue on the first draw = $\frac{2}{8}$, so chance of not drawing a blue on the first draw is $\frac{6}{8}$

similarly chance of not drawing blue on second draw = $\frac{5}{7}$

Therefore probability of not drawing blue in 2 draws = $\frac{6}{8} \times \frac{5}{7} = \frac{15}{28}$

118. How many integers between 100 and 150, inclusive can be evenly divided by neither 3 nor 5?

Soln: Number of integers that divide 3:

the range is 100-150

Relevant to this case, we take 102 - 150 (since 102 is the first to div 3)

$$102 = 34 \times 3$$

$$150 = 50 \times 3, \text{ so we have } 50 - 34 + 1 = 17 \text{ multiples of } 3$$

For multiples of 5,

$$100 = 5 \times 20$$

$$150 = 5 \times 30$$

$$30 - 20 + 1 = 11$$

Now we have a total of 27 integers, but we double counted the ones that divide BOTH 3 AND 5, ie 15.

105 is the first to divide 15.

$$105 = 15 \times 7$$

$$150 = 15 \times 10$$

$$10 - 7 + 1 = 4 \text{ integers}$$

So our total is $17 + 11 - 4 = 24$ integers that can be divided by either 3 or 5 or both.

$51 \text{ integers} - 24 \text{ integers} = 27$ that cannot be evenly divided.

119. Seed mixture X is 40 percent ryegrass and 60 percent bluegrass by weight; seed mixture Y is 25 percent ryegrass and 75 percent fescue. If a mixture of X and Y contains 30 percent ryegrass, what percent of the weight of this mixture is X ?

(A) 10%

(B) $33 \frac{1}{3}\%$

(C) 40%

(D) 50%

(E) 66 2/3%

Soln: $(x+y)30/100 = x*40/100 + y*25/100$
 $30x + 30y = 40x + 25y$
 $y = 2x$ or $y/x = 2/1$ or $y:x = 2:1$ hence $x = 33 \frac{1}{3}\%$

120. Sequence A and B. $a_1=1$, $b_1=k$. $a_n=b(n-1)-a(n-1)$ $b_n=b(n-1)+a(n-1)$. What is a_4 =?

Soln: $a_2 = k-1$; $b_2 = k+1$

$a_3 = (k+1)-(k-1) = 2$; $b_3 = (k+1)+(k-1) = 2k$

$a_4 = 2k - 2 = 2(k-1) = 2(b_2 - a_2)$

121. A person put 1000 dollars in a bank at a compound interest 6 years ago. What percentage of the initial sum is the interest if after the first three years the accrued interest amounted to 19% of the initial sum?
A) 38% B) 42% C) 19% D) 40%

Soln: assume, interest = r
so after 3 years total money = $1000*(1+r)^3 = 1000*1.19$
 $(1+r)^3 = 1.19$
so after 6 years total money = $1000*(1+r)^6 = 1000*1.19^2 = 1000*1.42$
so percentage of interest is 42%

122. A, B and C run around a circular track of length 750m at speeds of 3 m/sec, 6 m/sec and 18 m/sec respectively. If all three start from the same point, simultaneously and run in the same direction, when will they meet for the first time after they start the race?

- A. 750 seconds**
- B. 50 seconds**
- C. 250 seconds**
- D. 375 seconds**
- E. 75 seconds**

Soln: When two people are running in the same direction the relative speed is a difference in speeds of the two people.

In this case $A=3$ $B=6$ $C=18$

So relative speed of B wrt A is $6-3 = 3\text{m/s}$
Relative speed of A wrt to C is $18-3 = 15\text{m/s}$

Therefore relative distances will be:

B wrt A is $750/3 = 250$
C wrt to A $750/15 = 50$

So they have to bridge this distance of 250 and 50 between them which is the LCM of 250 and 50 which is 250.

Another Method: Simply put, Runner A's **time** take to run one lap is 250
Runner B's **time** is 125s
and Runner C's **time** is 41.67s

We can notice that $A=2B$
and that $B=3C$

So when 250 s elapse, they will be at their starting point. A will have completed one lap, B 2 laps, and C 6 laps

123. X percents of the rooms are suits, Y percent of the rooms are painted light blue. Which of the following best represents the least percentage of the light blue painted suits?

- 1) $X-Y$
- 2) $Y-X + 100$
- 3) $100X-Y$
- 4) $X+Y-100$
- e) $100-XY$

Soln: Equation from set theory:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

where,

A = % of rooms which are suites

B = % of rooms painted blue

$A \cap B$ means the intersection of the two sets

Now in this case, what we need to find is $n(A \cap B)$, therefore

$$\begin{aligned} n(A \cap B) &= n(A) + n(B) - n(A \cup B) \\ &= X + Y - n(A \cup B) \end{aligned}$$

Now this would be least when $n(A \cup B)$ is maximum, which would happen if these two kinds of rooms are only two kinds available, making $n(A \cup B) = 100$

Therefore the answer should be $X+Y-100$