Functions, Formulas, and Sequences

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box $\boxed{}$, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes $\frac{\boxed{}}{\boxed{}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1. If $f(x) = x^2 + 1$, what is $f(2) + f(-2)$?

(A) 0
(B) 1
(C) 4
(D) 5
(E) 10

2. If $f(x) = 2x$ and $g(x) = x^3$, what is $f(g(-3))$?

(A) -6
(B) -27
(C) 54
(D) -54
(E) -216

3. If $h(x) = 2x^3 - 3$ and $h(m) = -19$, what is the value of m?

(A) -3

(B) -2
(C) 2
(D) 6,856
(E) 6,862

4. If $f(x) = x - 3$ and $2[f(g)] = 14$, what is the value of $f(4g)$?

5. If $f(a, b) = a^2b^4$, and $f(m, n) = 5$, what is $f(3m, 2n)$?

6. If $f(x) = x^2 - 1$, what is the value of $f(y) + f(-1)$?

(A) $y^2 - 1$
(B) y^2
(C) $y^2 + 1$
(D) $y^2 - 2y$
(E) $y^2 - 2y - 1$

7. If $g(x) = 3x - 3$, what is the value of $g(3) + g\left(-\frac{1}{3}\right)$? ?

8. If $f(x) = \frac{x}{2} - 1$, what is the value of $f(f(10))$?

9. If $h(x) = 5x^2 + x$, then $h(a + b) =$

(A) $5a^2 + 5b^2$
(B) $5a^3 + 5b^3$
(C) $5a^2 + 5b^2 + a + b$
(D) $5a^3 + 10ab + 5b^3$
(E) $5a^2 + 10ab + 5b^2 + a + b$

10. If $\lceil x = 2x^2 + 2$, what is $\lceil 4$?

(A) $\lceil\lceil -1$
(B) $\lceil -2$
(C) $\lceil\lceil 2$
(D) $\lceil 17$
(E) $\lceil 34$

11. $\boxed{x}$ is defined as the least integer greater than x for all odd values of x, and the greatest integer less than x for all even values of x. What is $\boxed{-2} - \boxed{5}$?

(A) -12
(B) -9
(C) -8
(D) -7
(E) 3

12. $g(x) = x^2 - 4$ and $g(c) = 12$. If $c < 0$, what is $g(c - 2)$?

13. If $h(x) = 2x - 1$ and $g(x) = x^2 - 3$, what is $h(g(5))$?

14. $h(x) = 2x - 1$ and $g(x) = x^2 - 3$. If $g(m) = 61$, what is $h(m)$ if $m > 0$?

15. If $\&x\&$ is defined as one-half the square of x, what is the value of $\frac{\&4\&}{\&6\&}$?

16. If $\sim x = |14x|$, which of the following must be true?

Indicate <u>all</u> such answers.

☐ $\sim 2 = \sim(-2)$
☐ $\sim 3 + \sim 4 = \sim 7$
☐ The minimum possible value of $\sim x$ is zero

17. $\#x$ = the square of the number that is 2 less than x. What is the value of $\#5 - \#(-1)$?

$\boxed{}$

18. If $g(x) = \dfrac{x^2(4x+9)}{(3x-3)(x+2)}$, which answer represents all values of x for which $g(x)$ is undefined?

(A) 0
(B) $-\frac{9}{4}$
(C) -2, 1
(D) -2, 0, 1
(E) $-2, -\frac{9}{4}, 1$

19. If $f(x) = \dfrac{\sqrt{x-2}}{x}$ for all integer values of x, for how many values of x is $f(x)$ undefined?

(A) 0
(B) 1
(C) 2
(D) 3
(E) more than 3

20.

$$f(x) = 2x - 3$$
$$f(m) = -11$$

Quantity A	**Quantity B**
The value of m	Half the value of $f(m)$

21. The price of a phone call consists of a standard connection fee, which does not change, plus a per-minute charge. A 10-minute call costs $2.90 and a 16-minute call costs $4.40. How much does a 13-minute call cost?

(A) $3.55
(B) $3.57
(C) $3.58
(D) $3.65

(E) \$3.77

22. The first three terms in an arithmetic sequence are 30, 33, and 36. What is the 80th term?

$\boxed{}$

23. The sequence S is defined as $S_n = 2(S_{n-1}) - 4$. If $S_1 = 6$, what is S_5?

(A) -20
(B) 16
(C) 20
(D) 24
(E) 36

24. The sequence S is defined as $S_n = 3(S_{n-1}) + 1$. If $S_1 = -2$, what is S_4?

(A) -39
(B) -41
(C) -43
(D) -45
(E) -47

25. If $S_n = S_{n-1} + S_{n-2} - 3$, then what is S_6 when $S_1 = 5$ and $S_2 = 0$?

(A) -6
(B) -5
(C) -3
(D) -1
(E) 1

26. If $S_n = S_{n-1} + S_{n-2} - 1$, then what is S_4 when $S_0 = -10$ and $S_2 = 0$?

(A) -3
(B) 0
(C) 9
(D) 10
(E) 14

27. If $S_n = S_{n-1} + S_{n-2} + S_{n-3} - 5$, what is S_6 when $S_1 = 4$, $S_2 = 0$, and $S_4 = -4$?

(A) -2
(B) -12
(C) -16
(D) -20
(E) -24

28. The sequence P is defined as $P_n = 10(P_{n-1}) - 2$. If $P_1 = 2$, what is P_4?

$\boxed{}$

29. The sequence S is defined as $S_{n-1} = \frac{1}{4}(S_n)$. If S_1 = -4, what is S_4?

(A) -256
(B) -64
(C) -1/16
(D) 1/16
(E) 256

30. In sequence A_n, $A_1 = 45$, and $A_n = A_{n-1} + 2$ for all integers $n > 1$. What is the sum of the first 100 terms in sequence A_n?

(A) 243
(B) 14,400
(C) 14,500
(D) 24,300
(E) 24,545

31. In a certain sequence, the term a_n is given by the formula $a_n = a_{n-1} + 10$. What is the positive difference between the 10th term and the 15th term?

(A) 5
(B) 10
(C) 25
(D) 50
(E) 100

32. In a certain sequence, the term a_n is given by the formula $a_n = 10(a_{n-1})$. How many times greater is a_{10} than a_8?

(A) 1
(B) 3
(C) 10
(D) 30
(E) 100

33. Ashley and Beatrice received the same score on a physical fitness test. The scores for this test, t, are determined by the formula $t = 3ps - 25m$ where s and p are the numbers of sit-ups and push-ups the athlete can do in one minute and m is the number of minutes she takes to run a mile. Ashley did 10 sit-ups and 10 push-ups and ran an 8-minute mile. Beatrice did half as many sit-ups and twice as many push-ups. If both girls received the same overall score, how many minutes did it take Beatrice to run the mile?

(A) 4
(B) 8
(C) 10
(D) 16
(E) 20

34.

The expression $a\{\}b$ is defined as $a\{\}b = (a - b)(a + b)$.

Quantity A	**Quantity B**

The value of (7{}6) {} (11{}11) 26

35. If 5||10 = 5 and 1||(-2) = 1, which of the following could define the expression $a||b$?

(A) $b - a$

(B) $\frac{a^2 - b}{3}$

(C) $-ab/4$

(D) $\frac{b+15}{a}$

(E) $a + b + 4$

36. The maximum height reached by a ball thrown straight up into the air can be determined by the formula $h = -16t^2 + vt + d$, where t is the number of seconds since it was thrown, v is the initial speed of the throw (in feet per second), d is the height (in feet) at which the ball was released, and h is the height of ball t seconds after the throw. Two seconds after a ball is thrown, how high in the air is the ball if it was released at a height of 6 feet and a speed of 80 feet per second?

(A) 96 feet
(B) 100 feet
(C) 102 feet
(D) 134 feet
(E) 230 feet

37. If $a\#b = a^2\sqrt{b} - a$, where $b \geq 0$, what is the value of (-4)#4?

(A) -36
(B) -28
(C) 12
(D) 28
(E) 36

38. The expression $x\$y$ is defined as $\frac{x^2}{y}$, where $y \neq 0$. What is the value of 9\$(6\$2)?

(A) 1/2
(B) 9/4
(C) 9/2
(D) 18
(E) 108

39. Amy deposited \$1,000 into an account that earns 8% annual interest compounded every 6 months. Bob deposited \$1,000 into an account that earns 8% annual interest compounded quarterly. If neither Amy nor Bob makes any additional deposits or withdrawals, in 6 months how much more money will Bob have in his account than Amy?

(A) \$40
(B) \$8
(C) \$4
(D) \$0.40
(E) \$0.04

40. The half-life of an isotope is the amount of time required for 50% of a sample of the isotope to undergo radioactive decay. The half-life of the carbon-14 isotope is 5,730 years. How many years must pass until a sample that starts out with 16,000 carbon-14 isotopes decays into a sample with only 500 carbon-14 isotopes?

(A) 180 years
(B) 1,146 years
(C) 5,730 years
(D) 28,650 years
(E) 183,360 years

41. $f(x) = \dfrac{2-x}{5}$ and $g(x) = 3x - 2$. If $f(g(x)) = 1$, what is the value of x?

(A) -5/3
(B) -1/3
(C) 2/3
(D) 1
(E) 5/3

42. $f(x) = 2x - 2$ and $g(x) = \dfrac{x}{2} + 2$. What is the value of $2f(g(2))$?

(A) 0
(B) 4
(C) 6
(D) 8
(E) 12

43. In a particular competition, a skater's score, s, is calculated by the formula $s = \dfrac{10(3t-5f)}{m}$ where t is the number of successful triple axels performed, f is the number of times the skater fell, and m is the length, in minutes, of the performance. If Seiko received a score of 6 for a 5 minute performance in which she had twice as many successful triple axels as she had falls, how many times did she fall?

(A) 2
(B) 3
(C) 4
(D) 5
(E) 6

44. An investor doubles his money every 8 years. At 33 years old, he had 13 million dollars ($13m). How much money will he have when he retires at 65 years old?

(A) $26m
(B) $104m
(C) $130m
(D) $208m
(E) $260m

45. $a \sim b$ is defined as $a - \dfrac{5-b}{a}$. What is the value of $1 \sim ((-1) \sim 1)$?

(A) -9
(B) -1
(C) 1
(D) 2
(E) 5

46. An archer's score is calculated by the formula $\frac{50b - 10a}{10 + s}$ where b is the number of bull's-eyes hit, a is the total number of arrows shot, and s is the time in seconds it took the archer to shoot. By how many points would an archer who took 10 seconds to shoot 10 arrows and hit all bull's-eyes beat an archer who shot twice as many arrows and hit half as many bull's-eyes in 15 seconds?

(A) 2
(B) 7
(C) 10
(D) 18
(E) 20

47. Each term of a certain sequence is calculated by adding a particular constant to the previous term. The second term of this sequence is 27 and the fiftjh term is 84. What is the 1st term of this sequence?

(A) 20
(B) 15
(C) 13
(D) 12
(E) 8

48. If $a\#b = \frac{1}{2a - 3b}$ and $a@b = 3a - 2b$, what is the value of 1@2 - 3#4 ?

(A) -7/6
(B) -1
(C) -5/6
(D) 2/3
(E) 7/6

49. In a certain sequence, the term a_n is given by the formula $a_n = a_{n-1} + 5$ where $a_1 = 1$. What is the sum of the first 75 terms of this sequence?

(A) 10,150
(B) 11,375
(C) 12,500
(D) 13,950
(E) 15,375

50. In a certain sequence, the term a_n is given by the formula $a_n = 2 \times a_{n-1}$ where $a_1 = 1$. What is the positive difference between the sum of the first 10 terms of the sequence and the sum of the 11th and 12th terms of the same sequence?

(A) 1
(B) 1,024
(C) 1,025

(D) 2,048
(E) 2,049

51.

An operation @ is defined by the equation $a@b = (a - 1)(b - 2)$.
$x@5 = 3@x$

Quantity A	**Quantity B**
The value of x	1

52. The wait time in hours, w, for a certain popular restaurant can be estimated by the formula $w = \frac{n+ks}{10}$ where k is a constant, n is the number of parties waiting ahead of you, and s is the size of your party. If a family of 4 has a wait time of 30 minutes when 2 other parties are ahead of it, how long would a family of 6 expect to wait if there are 8 parties ahead of it?

(A) 45.5 minutes
(B) 1 hour 15 minutes
(C) 1 hour 25 minutes
(D) 1 hour 45 minutes
(E) 2 hours

53. A sequence is defined as $a_n = 5(a_{n-1}) - 3$ where $a_4 = 32$. What is the first term of the sequence, a_1?

(A) 1
(B) 7
(C) 16
(D) 128
(E) 157

54.

A certain sequence is defined by the formula $a_n = a_{n-1} - 7$.
$a_7 = 7$

Quantity A	**Quantity B**
The value of a_1	-35

55. Monthly rent for units in a certain apartment building is determined by the formula $k\left(\frac{5r^2 + 10t}{f+5}\right)$ where k is a constant, r and t are the number of bedrooms and bathrooms in the unit, respectively, and f is the floor number of the unit. A 2-bedroom, 2-bathroom unit on the first floor is going for \$800/month. How much is the monthly rent on a 3-bedroom unit with 1 bathroom on the 3rd floor?

(A) \$825
(B) \$875
(C) \$900

(D) \$925
(E) \$1,000

56.

Quantity A	Quantity B
The sum of all the multiples of 3 between 250 and 350	9,990

57. Town A has a population of 160,000 and is growing at a rate of 20% annually. Town B has a population of 80,000 and is growing at a rate of 50% annually.

Quantity A	Quantity B
The number of years until Town B's population is larger than that of Town A	3

58. If $f(x) = x^2$, what is $f(m + n) + f(m - n)$?

(A) $m^2 + n^2$
(B) $m^2 - n^2$
(C) $2m^2 + 2n^2$
(D) $2m^2 - 2n^2$
(E) m^2n^2

59. S_n is a sequence such that $S_n = (-1)^n$, where $n \geq 1$. What is the sum of the first 20 terms in S_n?

60. If //x// means "the least integer greater than or equal to x," what is //-2.5// + //3.6//?

61. If $f(x, y) = x^2y$ and $f(a, b) = 6$, what is $f(2a, 4b)$?

62.

$f(x) = m$ where m is the number of distinct prime factors of x.

Quantity A	Quantity B

$f(30)$ $f(64)$

63. In a particular sequence, S_n is equal to the units digit of 3^n where n is a positive integer. If $S_1 = 3$, how many of the first 75 terms of the sequence are equal to 9?

64. The sequence $a_1, a_2, a_3, \ldots, a_n$ is such that $a_n = 9 + a_{n-1}$ for all $n > 1$. If $a_1 = 11$, what is the value of a_{35}?

65. In sequence Q, the first number is 3, and each subsequent number in the sequence is determined by doubling the previous number and then adding 2. How many times does the digit 8 appear in the units digit of the first 10 terms of the sequence?

66. If $g(2m) = 2g(m)$ and $g(3) = 5.5$, what is $g(6)$?

67. For which of the following functions $f(x)$ is $f(a + b) = f(a) + f(b)$?

(A) $f(x) = x^2$
(B) $f(x) = 5x$
(C) $f(x) = 2x + 1$
(D) $f(x) = \sqrt{x}$
(E) $f(x) = x - 2$

68.

Sam invests a principal of $10,000, which earns annually compounded interest over a period of years.

Quantity A	**Quantity B**
The final value of the investment after 2 years at 8% interest, compounded annually	The final value of the investment after 4 years at 4% interest, compounded annually

69. The number of years it would take for the value of an investment to double, at 26% interest compounded annually, is approximately

(A) 2
(B) 3
(C) 4
(D) 5
(E) 6

70. The interest rate, compounded annually, that would bring a principal of \$1,200 to a final value of \$1,650 in 2 years is closest to

(A) 17%
(B) 18%
(C) 19%
(D) 20%
(E) 21%

71. An investment is made at 12.5% annual simple interest. The number of years it will take for the cumulative value of the interest to equal the original investment is

(A) 4
(B) 5
(C) 6
(D) 7
(E) 8

72. If $f(2a) = 2f(a)$ and $f(6) = 11$, what is $f(24)$?

(A) 22
(B) 24
(C) 44
(D) 66
(E) 88

73. $\frac{1}{2}f(x) = f\left(\frac{1}{2}x\right)$, which is true for all values of $f(x)$?

(A) $f(x) = 2x + 2$
(B) $f(x) = 13x$
(C) $f(x) = x^2$
(D) $f(x) = x - 10$
(E) $f(x) = \sqrt{x-4}$

Functions, Formulas, and Sequences Answers

1. **(E).** The notation "$f(x)$" and "$f(2)$" indicates that you should plug 2 in for x in the given equation:

$$f(2) = (2)^2 + 1$$
$$f(2) = 5$$

Likewise, plug -2 in for x:

$$f(-2) = (-2)^2 + 1$$
$$f(-2) = 5$$

Now add: 5 + 5 = 10.

2. **(D).** Start with the innermost function, $g(-3)$. The notation indicates that you should plug -3 in for x in the given equation:

$$g(-3) = (-3)^3$$
$$g(-3) = -27$$

Now the problem reads "what is $f(-27)$?" Plug -27 in for x in the $f(x)$ function:

$$f(-27) = 2(-27) = -54$$

3. **(B).** Be careful with the notation here. The problem indicates that $h(m) = -19$, *not* that $h(-19)$ = something else. Do not plug -19 in for x; rather, plug m in for x and set the answer equal to -19:

$$2m^3 - 3 = -19$$
$$2m^3 = -16$$
$$m^3 = -8$$
$$m = -2$$

4. **37.** Be careful with the notation here. $2[f(g)] = 14$ represents some number (denoted by variable g) plugged into the function $f(g)$, and then multiplied by 2, to yield the answer 14. If $2[f(g)] = 14$, then divide both sides by 2 to get $f(g) = 7$.

The main function is $f(x) = x - 3$. The notation $f(g)$ indicates that you should plug g in for all instances of x: $f(g) = g - 3$. You also determined that $f(g) = 7$, so set the two right-hand halves of the equations equal to each other: $g - 3 = 7$. The value of g is therefore 10.

The question asks for the value of $f(4g)$. Since $g = 10$, $4g = 40$, and $f(40)$:

$$f(40) = 40 - 3 = 37$$

5. **720.** Plug m and n into the function in place of a and b. If $f(m, n) = 5$, then:

$$m^2n^4 = 5$$

This cannot be further simplified, so continue to the second part of the problem: plug $3m$ and $2n$ into the function for a and b:

$$f(3m, 2n) = (3m)^2(2n)^4 = 9m^2\ 16n^4 = 144m^2n^4$$

Since $m^2n^4 = 5$, $144m^2n^4 = 144(5) = 720$.

6. **(A).** The question is asking you to plug y into the function, then plug -1 into the function, then add the two answers together.

$$f(y) = y^2 - 1$$
$$f(-1) = (-1)^2 - 1$$
$$f(-1) = 0$$

Thus, $f(y) + f(-1) = y^2 - 1 + 0 = y^2 - 1$, or choice (A).

7. **2.** The question is asking you to plug 3 into the function, then plug $-\frac{1}{3}$ into the function, and then add the two answers together.

$$g(3) = 3(3) - 3$$
$$g(3) = 6$$
$$g\left(-\frac{1}{3}\right) = 3\left(-\frac{1}{3}\right) - 3$$
$$g\left(-\frac{1}{3}\right) = -1 - 3$$
$$g\left(-\frac{1}{3}\right) = -4$$

Thus, $g(3) + g\left(-\frac{1}{3}\right) = 6 + (-4) = 2.$.

8. **1.** When dealing with "nested" functions, tackle the innermost function first.

$$f(10) = \frac{10}{2} - 1 = 4$$

$$f(4) = \frac{4}{2} - 1 = 1$$

Thus, $f(f(10)) = 1$.

9. **(E).** The notation $h(a + b)$ indicates that you need to replace each x with the expression $(a + b)$

$h(a + b) = 5(a + b)^2 + (a + b)$
$h(a + b) = 5(a^2 + 2ab + b^2) + a + b$
$h(a + b) = 5a^2 + 10ab + 5b^2 + a + b$

This is equivalent to answer choice (E).

10. **(A).** The question uses a made-up symbol in place of the traditional notation $f(x)$. The question "If $\ulcorner x = 2x^2 + 2$, what is $\ulcorner 4$?" is asking you to plug 4 into the function.

$$\ulcorner 4 = 2(4)^2 + 2$$
$$\ulcorner 4 = 34$$

Do not fall for trap answer choice (E). The correct answer is 34, which does not appear in the choices in that form.

Trap choice (E) is $\ulcorner 34$, which equals $2(34)^2 + 2$; this is much larger than 34.

You need to solve each answer choice until you find one that equals 34. Choice (A), $\ulcorner\ulcorner -1$, uses the function symbol twice, which requires you to plug -1 into the function, then plug your answer back into the function again:

$$\ulcorner -1 = 2(-1)^2 + 2 = 4$$

$\ulcorner 4 = 2(4)^2 + 2 = 34$ (Note: you do not need to complete this math if you notice that $\ulcorner 4$ must have the same value as the original $\ulcorner 4$ in the question stem.)

Thus, $\ulcorner\ulcorner -1 = 34$ and choice (A) is correct. It is not necessary to try the other answer choices.

11. **(B).** This problem uses a made-up symbol which is then defined verbally, rather than with a formula. $\boxed{x}$ has two different definitions:

If x is odd, $\boxed{x}$ equals the least integer greater than x (for example, if $x = 3$, then the "least integer greater than 3" is equal to 4)

If x is even, $\boxed{x}$ equals the greatest integer less than x (for example, if $x = 6$, the "greatest integer less than x" is equal to 5.

Since -2 is even, $\boxed{-2}$ = the greatest integer less than -2, or -3.

Since 5 is odd, $\boxed{5}$ = the least integer greater than 5, or 6.

Thus, $\boxed{-2} - \boxed{5} = -3 - 6 = -9$.

12. **32.** For the function $g(x) = x^2 - 4$, plugging c in for x gives the answer 12. Thus:

$c^2 - 4 = 12$
$c^2 = 16$
$c = 4$ or -4

The problem indicates that $c < 0$, so c must be -4.

The problem then asks for $g(c - 2)$. Since $c = -4$, $c - 2 = -6$. Plug -6 into the function:

$g(-6) = (-6)^2 - 4$
$g(-6) = 36 - 4 = 32$

13. **43.** The problem introduces two functions and asks for $h(g(5))$. When dealing with "nested" functions, begin with the innermost function.

$g(5) = 5^2 - 3 = 22$
$h(22) = 2(22) - 1 = 43$

Thus, $h(g(5)) = 43$.

14. **15.** The problem introduces two functions as well as the fact that $g(m) = 61$. First, solve for m:

$m^2 - 3 = 61$
$m^2 = 64$
$m = 8$ or -8

The problem indicates that $m > 0$, so m must equal 8. The problem asks for $h(m)$. Since $m = 8$, find $h(8)$:

$h(8) = 2(8) - 1$
$h(8) = 15$

15. $\frac{4}{9}$ **(or any equivalent fraction).** This function defines a made-up symbol rather than using traditional function notation such as $f(x)$. Since &x& is defined as "one-half the square of x":

$$\&x\& = \frac{1}{2}x^2$$

The problem asks for &4& divided by &6&:

$$\&4\& = \frac{1}{2}(4)^2 = 8$$
$$\&6\& = \frac{1}{2}(6)^2 = 18$$

Therefore, $\frac{\&4\&}{\&6\&} = \frac{8}{18} = \frac{4}{9}$.

16. **I, II, and III.** This function defines a made-up symbol: ~x is equivalent to $|14x|$. The question asks which statements must be true, so test each one.

$\sim 2 = \sim(-2)$
$|14(2)| = |14(-2)|$
$|28| = |-28|$
$28 = 28$

This statement must be TRUE.

Similarly test the second statement:

$\sim 3 + \sim 4 = \sim 7$
$|14(3)| + |14(4)| = |14(7)|$
$42 + 56 = 98$
$98 = 98$

This statement must be TRUE.

Finally, the third statement is also true. Since ~x is equal to a statement inside an absolute value, this value can never be negative. If $x = 0$, then the value of $|14x|$ is also 0. The minimum possible value for ~x is 0.

17. **0.** This function defines a made-up symbol, rather than using traditional notation such as $f(x)$. First, translate the function:

$\#x = (x - 2)^2$
$\#5 = (5 - 2)^2 = 9$ and $\#(-1) = (-1 - 2)^2 = 9$.

$\#5 - \#(-1) = 9 - 9 = 0$.

18. **(C).** The term "undefined" refers to the circumstance when the solution is not a real number — for example, when a value would ultimately cause you to divide by 0, that situation is considered "undefined." There aren't many circumstances that result in an undefined answer. Essentially, you can't take the square root of a negative, and you can't divide by 0. There are no square roots in this problem, but it's possible that 0 could end up on the denominator of the fraction. Set each of the terms in the denominator equal to 0:

$3x - 3 = 0$
$3x = 3$
$x = 1$

$x + 2 = 0$
$x = -2$

Thus, if $x = 1$ or $x = -2$, then you'd have to divide by 0, making $g(x)$ undefined. All other values are acceptable.

19. **(E).** The term "undefined" refers to the circumstance when the solution is not a real number — for example, when a value would ultimately cause you to divide by 0, that situation is considered "undefined." There aren't many circumstances that result in an undefined answer. Essentially, you can't take the square root of a negative, and you can't divide by 0.

Since you can't divide by 0 and the bottom of the fraction is simply x, then x cannot be 0. So far, there is 1 "prohibited" value for x.

Since the top of the fraction is a square root and you can't take the square root of a negative, you can conclude that the quantity inside the absolute value, $x - 2$, must be positive or zero:

$x - 2 \geq 0$
$x \geq 2$

Therefore, there are an infinite number of values that x cannot be (1, 0, -1, -2, -3, -4, -5…).

20. **(A).** The problem gives a function, $f(x) = 2x - 3$, and then indicates that, when m is plugged in to the function, the answer is -11. Therefore:

$2m - 3 = -11$
$2m = -8$
$m = -4$

Quantity A is equal to -4. The problem indicates that $f(m) = -11$, so Quantity B is equal to $-\frac{11}{2} = -5.5$. Quantity A is larger.

21. **(D).** Since "the price of a phone call consists of a standard connection fee, which does not change, plus a per-minute charge," you can write a formula, using variables for the unknown information. Let c equal the connection fee and r equal the per-minute rate:

$2.90 = c + r(10)$
$4.40 = c + r(16)$

Now, either substitute and solve, or stack and combine the equation. Note that there is one c in each equation, so subtracting is likely going to be fastest:

$$\begin{array}{l} \;\;4.40 = c + 16r \\ \underline{-\,(2.90 = c + 10r)} \\ \;\;1.50 = 6r \end{array}$$

$r = 0.25$

The calls cost 25 cents per minute. Note that most people will next plug r back into either equation to find c, but c isn't necessary to solve!

A 10-minute call costs \$2.90. That \$2.90 already includes the basic connection fee (which never changes) as well as the per-minute fee for 10 minutes. The problem asks how much a 13-minute call costs. Add the cost for another 3 minutes (\$0.75) to the cost for a 10-minute call (\$2.90): 2.90 + 0.75 = \$3.65.

In fact, if you notice earlier that both the 10-minute and 16-minute calls include the same connection fee (which never changes), you can use a shortcut to solve. The extra 6 minutes for the 16-minute call cost a total of \$4.40 - \$2.90 = \$1.50. From there, you can calculate the cost per minute ($1.5 \div 6 = 0.25$) or you can notice that 13 minutes is halfway between 10 minutes and 16 minutes, so the cost for a 13-minute call must also be halfway between the cost for a 10-minute call and the cost for a 16-minute call. Add half of \$1.50, or \$0.75, to \$2.90 to get \$3.65.

22. **267.** While the sequence is clear (30, 33, 36, 39, 42, etc.), you don't have time to count to the 80th term. Instead, find a pattern. Each new term in the list adds 3 to the previous term, so determine how many times you need to add 3. (By the way, the term "arithmetic sequence" means a sequence in which the same number is added or subtracted for each new term.)

Start with the first term, 30. To get from the first term to the second term, you start with 30 and add 3 *once*. To get from the first term to the third term, you start with 30 and add 3 *twice*. In other words, for the third term, you add one fewer instance of 3: twice rather than three times. To write this mathematically, say: $30 + 3(n-1)$, where n is the number of the term. (Note: you don't need to write that, as long as you understand the pattern.)

To get to the 80th term, then, start with 30 and add 3 exactly 79 times:

$30 + (79 \times 3) = 267$

23. **(E).** The sequence $S_n = 2(S_{n-1}) - 4$ can be read as "to get any term in sequence S, double the previous term and subtract 4."

The problem gives S_1 (the first term) and asks for S_5 (the fifth term):

6				
S_1	S_2	S_3	S_4	S_5

To get any term, double the previous term and subtract 4. To get S_2, double S_1 (which is 6) and subtract 4: $S_2 = 2(6) - 4 = 8$. Continue doubling each term and subtracting 4 to get the subsequent term:

6	8	12	20	36
S_1	S_2	S_3	S_4	S_5

24. **(B).** The sequence $S_n = 3(S_{n-1}) + 1$ can be read as "to get any term in sequence *S*, triple the previous term and add 1."

The problem gives S_1 (the first term) and asks for S_4 (the fourth term):

−2			
S_1	S_2	S_3	S_4

To get any term, triple the previous term and add 1. To get S_2, triple S_1 (which is -2) and add 1. Thus, $S_2 = 3(-2) + 1 = -5$. Continue tripling each term and adding 1 to get the subsequent term:

−2	−5	−14	−41
S_1	S_2	S_3	S_4

25. **(A).** The sequence $S_n = S_{n-1} + S_{n-2} - 3$ can be read as "to get any term in sequence *S*, add the two previous terms and subtract 3."

The problem gives the first two terms and asks for the sixth term:

5	0				
S_1	S_2	S_3	S_4	S_5	S_6

To get any term, add the two previous terms and subtract 3. So the third term will equal 5 + 0 - 3 = 2. The fourth term will equal 0 + 2 - 3 = -1. The fifth term will equal 2 + (-1) - 3 = -2. The sixth term will equal -1 + (-2) - 3 = -6.

5	0	2	−1	−2	−6
S_1	S_2	S_3	S_4	S_5	S_6

26. **(C).** The sequence $S_n = S_{n-1} + S_{n-2} - 1$ can be read as "to get any term in sequence *S*, add the two previous terms and subtract 1."

The problem gives the zero-th term and the *second* term and asks for the fourth term:

−10		0		
S_0	S_1	S_2	S_3	S_4

Within the sequence S_0 to S_2, the problem gives two values but not the third (S_1). What version of the formula would include those three terms?

$S_2 = S_1 + S_0 - 1$
$0 = S_1 + (-10) - 1$
$0 = S_1 - 11$
$11 = S_1$

–10	11	0		
S_0	S_1	S_2	S_3	S_4

To get each subsequent term, add the two previous terms and subtract 1. $S_3 = 0 + 11 - 1 = 10$. $S_4 = 10 + 0 - 1 = 9$.

–10	11	0	10	9
S_0	S_1	S_2	S_3	S_4

27. **(E).** The sequence $S_n = S_{n-1} + S_{n-2} + S_{n-3} - 5$ can be read as "to get any term in sequence *S*, add the three previous terms and subtract 5."

The problem gives the first, second, and *fourth* terms and asks for the sixth term:

4	0		–4		
S_1	S_2	S_3	S_4	S_5	S_6

Within the sequence S_1 to S_4, the problem gives three values but not the fourth (S_3). What version of the formula would include those four terms?

$S_4 = S_3 + S_2 + S_1 - 5$
$-4 = S_3 + 4 + 0 - 5$
$-4 = S_3 - 1$
$-3 = S_3$

Fill in the newly-calculated value. To find each subsequent value, continue to add the three previous terms and subtract 5. $S_5 = -4 + (-3) + 0 - 5 = -12$. $S_6 = -12 + (-4) + (-3) - 5 = -24$.

4	0	–3	–4	–12	–24
S_1	S_2	S_3	S_4	S_5	S_6

28. **1,778.** The sequence $P_n = 10(P_{n-1}) - 2$ can be read as "to get any term in sequence *P*, multiply the previous term by 10 and subtract 2."

The problem gives the first term and asks for the fourth:

$$\underset{P_1}{\underline{\;2\;}} \quad \underset{P_2}{\underline{\quad}} \quad \underset{P_3}{\underline{\quad}} \quad \underset{P_4}{\underline{\quad}}$$

To get P_2, multiply 2 × 10, then subtract 2 to get 18. Continue this procedure to find each subsequent term ("to get any term in sequence P, multiply the previous term by 10 and subtract 2.") $P_3 = 10(18) - 2 = 178$. $P_4 = 10(178) - 2 = 1{,}778$.

$$\underset{P_1}{\underline{\;2\;}} \quad \underset{P_2}{\underline{\;18\;}} \quad \underset{P_3}{\underline{\;178\;}} \quad \underset{P_4}{\underline{\;1{,}778\;}}$$

29. **(A).** The sequence $S_{n-1} = \frac{1}{4}(S_n)$ can be read as "to get any term in sequence S, multiply the term *after* that term by $\frac{1}{4}$." Since this formula is "backwards" (usually, you define later terms with regard to previous terms), you may wish to solve the formula for for S_n:

$$S_{n-1} = \frac{1}{4}(S_n)$$
$$4S_{n-1} = S_n$$
$$S_n = 4S_{n-1}$$

This can be read as "to get any term in sequence S, multiply the previous term by 4."

The problem gives the first term and asks for the fourth:

$$\underset{S_1}{\underline{\;-4\;}} \quad \underset{S_2}{\underline{\quad}} \quad \underset{S_3}{\underline{\quad}} \quad \underset{S_4}{\underline{\quad}}$$

To get S_2, multiply the previous term by 4: (4)(-4) = -16. Continue this procedure to find each subsequent term. $S_3 = (4)(-16) = -64$. $S_4 = (4)(-64) = -256$.

$$\underset{S_1}{\underline{\;-4\;}} \quad \underset{S_2}{\underline{\;-16\;}} \quad \underset{S_3}{\underline{\;-64\;}} \quad \underset{S_4}{\underline{\;-256\;}}$$

30. **(B).** The first term of the sequence is 45, and each subsequent term is determined by adding 2. The problem asks for the sum of the first 100 terms, which cannot be calculated directly in the given time frame; instead, find the pattern. The first few terms of the sequence are 45, 47, 49, 51, ….

What's the pattern? To get to the 2nd term, start with 45 and add 2 once. To get to the 3rd term, start with 45 and add 2 twice. To get to the 100th term, then, start with 45 and add 2 ninety-nine times: 45 + (2)(99) = 243.

Next, find the sum of all odd integers from 45 to 243, inclusive. To sum up any evenly-spaced set, multiply the average by the number of elements in the set. To get the average, average the first and last terms. Since $\frac{45+243}{2} = 144$, the average is 144.

To find the total number of elements in the set, subtract 243 - 45 = 198, then divide by 2 (count only the odd numbers, not the even ones). 198/2 = 99 terms. Now, add 1 (to count both endpoints in a consecutive set, first subtract and then "add 1 before you're done"). The list has 100 terms.

Multiply the average and the number of terms:

$144 \times 100 = 14{,}400$

31. **(D).** This is an arithmetic sequence where the difference between successive terms is always +10. The difference between, for example, a_{10} and a_{11}, is exactly 10, regardless of the actual values of the two terms. The difference between a_{10} and a_{12} is $10 + 10 = 20$ or $10 \times 2 = 20$, because there are two "steps," or terms, to get from a_{10} to a_{12}. Starting from a_{10}, there is a sequence of 5 terms to get to a_{15}. Therefore, the difference between a_{10} and a_{15} is $10 \times 5 = 50$.

32. **(E).** This is a geometric sequence, in which every term is 10 times the term before. The problem does not provide the actual value of any terms in the sequence, but the sequence could be something like 10, 100, 1,000…, or 35, 350, 3,500…. Thus, any term is 100 times as large as the term that comes two before it.

Alternatively, try an algebraic approach. From the formula, $a_9 = 10a_8$ and $a_{10} = 10a_9$. Substitute for a_9 in the second equation to give: $a_{10} = 10(10a_8) = 100a_8$.

33. **(B).** First, calculate Ashley's score: $t_a = 3 \times 10 \times 10 - 25 \times 8 = 300 - 200 = 100$. If Ashley and Beatrice scored the same score and Beatrice did half as many sit-ups (5) and twice as many push-ups (20): $t_b = 3 \times 5 \times 20 - 25m = 100$. Solve for m: $300 - 25m = 100$ and $25m = 200$ so $m = 8$.

Alternatively, you could use logic. Since p and s are multiplied together in the score formula, if you multiply s by $\frac{1}{2}$ and p by 2, the two girls will have the same overall value for $3ps$. Beatrice will need the same mile time, 8 minutes, in order to achieve the same overall score as Ashley.

34. **(A).** This problem defines a function for the made-up symbol {}. To calculate the value of Quantity A, follow the rules of PEMDAS. First calculate the expressions within parentheses:

$7\{\}6 = (7 - 6)(7 + 6) = 1 \times 13 = 13$
$11\{\}11 = (11 - 11)(11 + 11) = 0$

Then substitute these values back into the original expression:

$(13)\{\}(0) = (13 - 0)(13 + 0) = 13 \times 13 = 169$. Quantity A is larger.

35. **(B).** This question provides two examples of the input and output into a made-up function, and asks which answer choice could be that function. In other words, the function in which answer choice gives the answer 5 when you evaluate 5||10, and also gives the answer 1 when you evaluate 1||(-2)?

Only one answer can work for both examples, so first test $a = 5$ and $b = 10$ to determine whether the function returns 5. If it doesn't, cross off that choice. If it does, test $a = 1$ and $b = -2$. When you find an answer that works for both, you can stop.

(A) Does 10 - 5 = 5? YES. Does (-2) -1 = 1? No. Cross off answer (A).

(B) Does $\frac{5^2-10}{3}=5$? YES. Does $\frac{1^2-(-2)}{3}=1$? YES. This is the correct answer.

It is not necessary to test the remaining answers; the work is shown below for completeness.

(C) Does $\frac{-5\times 10}{4}=5$ NO. Cross off answer (C).

(D) Does $\frac{10+15}{5}=5$? YES. Does $\frac{-2+15}{1}=1$? No. Cross off answer (D).

(E) Does 5 + 10 + 4 = 5? NO. Cross off answer (E).
The answer is (B).

36. **(C).** The question presents a formula with a number of variables and also provides values for all but one of those variables (h, the height of the ball). Solve for h by plugging in the values given for the other variables: $t = 2$ seconds, $v = 80$ feet/second, $d = 6$ feet.

$h = -16(2)^2 + (80)(2) + 6 = 102$ feet

37. **(E).** This problem defines a function for the made-up symbol #. In this problem $a = (-4)$ and $b = 4$. Plug the values into the function: $(-4)^2\sqrt{4}-(-4) = 16 \times 2 + 4 = 36$. Do not forget to keep the parentheses around the -4! Also note that you take only the positive root of 4 (2) because the problem has been presented in the form of a real number underneath the square root sign.

38. **(B).** This problem defines a function for the made-up symbol \$. Order of operation rules (PEMDAS) stay the same even when the problem uses made-up symbols. First, calculate the value of the expression in parentheses, 6\$2. Plug $x = 6$ and $y = 2$ into the function: $6^2/2 = 36/2 = 18$. Replace 6\$2 with 18 in the original expression to give 9\$18. Again, plug $x = 9$ and $y = 18$ into the function: 9\$18 = $9^2/18 = 81/18 = 9/2$.

39. **(D).** Both Amy and Bob start with \$1,000 and earn 8% interest annually; the difference is in how often this interest is compounded. Amy's interest is compounded twice a year at 4% each time (8% annual interest compounded 2 times a year means that she gets half the interest, or 4%, every six months). Bob's interest is compounded four times a year at 2% (8% divided by 4 times per year) each time. After 6 months, Amy has \$1,000 × 1.04 = \$1,040.00 (1 interest

payment at 4%) and Bob has $\$1,000 \times (1.02)^2 = \$1,040.40$ (2 interest payments at 2%). The difference is \$1,040.40 - \$1,040.00 = \$0.40.

For Bob's interest, you can also calculate the two separate payments. After three months, Bob will have \$1,000 × 1.02 = \$1,020.00. After six months, Bob will have \$1,020 × 1.02 = \$1,040.40.

40. **(D).** After each half-life, the sample is left with half of the isotopes it started with in the previous period. After one half-life, the sample goes from 16,000 isotopes to 8,000. After two half-lives, it goes from 8,000 to 4,000. Continue this pattern to determine the total number of half-lives that have passed: 4,000 becomes 2,000 after 3 half-lives, 2,000 becomes 1,000 after 4 half-lives, 1,000 becomes 500 after 5 half-lives. The sample will have 500 isotopes after 5 half-lives. Thus, multiply 5 times the half-life, or 5 × 5730 = 28,650 years.

Note that the answer choices are very spread apart. Once you have determined that 5 half-lives have passes, you can estimate: 5 × 5000 = 25,000 years; answer (D) is the only possible answer.

41. **(B).** The question is asking you to substitute the expression for $g(x)$ into the function for $f(x)$, and set the answer equal to 1. Since $g(x) = 3x - 2$, substitute the expression $3x - 2$ in for x in the expression for $f(x)$:

$$f(g(x)) = \frac{2-g(x)}{5} = \frac{2-(3x-2)}{5} = \frac{4-3x}{5}$$

Since $f(g(x)) = 1$, solve the equation $\frac{4-3x}{5} = 1$: :

$$\begin{aligned} 4 - 3x &= 5 \\ -3x &= 1 \\ x &= -\frac{1}{3} \end{aligned}$$

42. **(D).** Start with the innermost portion of $2f(g(2))$:

$g(2) = 2/2 + 2 = 3$

Since $g(2) = 3$, now the expression is $2f(3)$. First evaluate $f(3)$: $2(3) - 2 = 4$.

Therefore $2f(3) = 2 \times 4 = 8$.

43. **(B).** If Seiko had twice as many triple axels as falls then $t = 2f$. Substitute $2f$ in for t, along with the other given information:

$$6 = \frac{10(3(2f) - 5f)}{5}$$

$$6 = \frac{10(6f - 5f)}{5}$$

$$6 = \frac{10f}{5}$$

$$30 = 10f$$

$$f = 3$$

Note that you want to substitute $2f$ in for t, and not the reverse, because the problem asks for the value of f.

44. **(D).** The investor's amount of money doubles every 8 years. Calculate the amount of money for each 8-year period:

Age	$ (millions)
33	13
33 + 8 = 41	13 × 2 = 26
41 + 8 = 49	26 × 2 = 52
49 + 8 = 57	52 × 2 = 104
57 + 8 = 65	104 × 2 = 208

At age 65, the investor will have $208 million.

45. **(B).** This problem defines a function for the made-up symbol ~. Follow PEMDAS order and start with the inner most parentheses, (-1)~1. In this case a = -1 and b = 1: $-1 - \left(\frac{5-1}{-1}\right) = -1 - \left(\frac{4}{-1}\right) = -1 + 4 = 3$. (Note: it's important to insert the parentheses! Replace (-1)~1 with 3 and evaluate the next function: 1~3. Now a = 1 and b = 3:

$$1 - \left(\frac{5-3}{1}\right) = 1 - \left(\frac{2}{1}\right) = 1 - 2 = -1$$

46. **(D).** Calculate each of the archer's scores by plugging in the appropriate values for b, a, and s. For the first archer, $b = a = s = 10$ and the score is $\frac{(50 \times 10) - (10 \times 10)}{10 + 10} = \frac{400}{20} = 20$. For the second archer, b = half of 10 = 5, a = twice as many as 10 = 20, and s = 15. The score for the second archer is $\frac{(50 \times 5) - (10 \times 20)}{10 + 15} = \frac{50}{25} = 2$.

The difference in scores is 20 - 2 = 18.

47. **(E).** Let k equal the constant added to a term to get the next term. If the second term = 27, then the third term = 27 + k, the 4th term = 27 + 2k, and the 5th term = 27 + 3k. The 5th term equals 84, so create an equation:

$27 + 3k = 84$
$3k = 57$
$k = 19$

To find the first term, subtract k from the second term. The first term = 27 - 19 = 8.

48. **(C).** This problem defines functions for the made-up symbols # and @. Substitute a = 1 and b = 2 into the function for $a@b$: 3(1) - 2(2) = -1. Substitute a = 3 and b = 4 into the function for $a\#b$: $\frac{1}{2(3)-3(4)} = \frac{1}{-6} = -\frac{1}{6}$. Subtract: $(-1) - \left(-\frac{1}{6}\right) = -1 + \frac{1}{6} = -\frac{5}{6}$.

49. **(D).** This is an arithmetic sequence: each new number is created by adding 5 to the previous number in the sequence. Calculate the first few terms of the sequence: 1, 6, 11, 16, 21, and so on. Arithmetic sequences can be written in this form: $a_n = a_1 + k(n - 1)$, where k is the added constant and n is the number of the desired term. In this case, the function is: $a_n = 1 + 5(n - 1)$. The 75th term of this sequence is $a_{75} = 1 + 5(74) = 371$.

To find the sum of an arithmetic sequence, multiply the average value of the terms by the number of terms. The average of any evenly-spaced set is equal to the midpoint between the first and last terms. The average of the 1st and 75th terms is $\frac{1+371}{2} = 186$. There are 75 terms. Therefore, the sum of the first 75 terms = 186 × 75 = 13,950.

50. **(E).** This is a geometric sequence: each new number is created by multiplying the previous number by 2. Calculate the first few terms of the series to find the pattern: 1, 2, 4, 8, 16, and so on. Geometric sequences can be written in this form: $a_n = r^{n-1}$, where r is the multiplied constant and n is the number of the desired term. In this case, the function is $a_n = 2^{n-1}$.

The question asks for the difference between the sum of the first 10 terms and the sum of the 11th and 12th terms. While there is a clever pattern at play, it is hard to spot. If you don't see the pattern, one way to solve is to use the calculator to add the first ten terms: 1 + 2 + 4 + 8 + 16 + 32 + 64 + 128 + 256 + 512 = 1,023.

The 11th + 12th terms = 1,024 + 2,048 = 3,072

Subtract to get 2,049.

Alternatively, you may see the pattern in the first few terms (1, 2, 4, 8, 16…): Every term is equal to 1 more than the sum of the ones before it. For example, 1 + 2 = 3 and the next term is 4. 1 + 2 + 4 = 7 and the next term is 8. Thus, the sum of the first 10 terms of the sequence is 1 less than the 11th term. The 11th term = 2^{10} = 1,024, so the sum of the first 10 terms = 1,023. and the difference between the 10th and 11th terms equals 1. Add the value of the 12th term or 1 + 2,048 = 2,049.

51. **(B).** This problem defines a function for the made-up symbol @. Use the definition of the new symbol to rewrite the equation $x@5 = 3@x$ without the @ operator.

For $x@5$, $a = x$ and $b = 5$: $x@5 = (x - 1)(5 - 2) = 3x - 3$.
For $3@x$, $a = 3$ and $b = x$: $3@x = (3 - 1)(x - 2) = 2x - 4$.

Equating these two expressions gives us:

$3x - 3 = 2x - 4$
$x = -1$

Quantity B is larger.

52. **(B).** Start by solving for the constant, k. A family of 4 ($s = 4$) has a wait time of 30 minutes ($w = 0.5$ hours — don't forget that w is in hours!) when 2 parties are ahead of it ($n = 2$). Plug these values into the formula: $0.5 = \frac{2+4k}{10}$. Solve for k:

$5 = 2 + 4k$
$3 = 4k$
$k = \frac{3}{4}$

To solve for the wait time of the family of 6 with 8 parties ahead of it, plug these values into the formula along with $k = \frac{3}{4}$: $w = \frac{8+\left(\frac{3}{4}\right)6}{10} = 1.25$ hours. The answer choices are shown in hours and minutes. 0.25 hours is equal to 0.25×60 minutes = 15 minutes. The answer is 1 hour 15 minutes.

53. **(A).** The given sequence can be read as, "To get any term in a, multiply the previous term by 5 and then subtract 3." The problem indicates that $a_4 = 32$ and asks for the value of a_1.

$$\underset{A_1}{\underline{\qquad}} \quad \underset{A_2}{\underline{\qquad}} \quad \underset{A_3}{\underline{\qquad}} \quad \underset{A_4}{\underline{\quad 32 \quad}}$$

Start with a_4 and find a_3:

$32 = 5a_3 - 3$
$35 = 5a_3$
$7 = a_3$

Use a_3 to find a_2:

$7 = 5a_2 - 3$

$$10 = 5a_2$$
$$2 = a_2$$

Use a_2 to find a_1:

$$2 = 5a_1 - 3$$
$$5 = 5a_1$$
$$1 = a_1$$

54. **(A).** The sequence $a_n = a_{n-1} - 7$ can be read as "to get any term in sequence a, subtract 7 from the previous term." The problem provides the 7th term; plug the term into the function in order to determine the pattern. Note that Quantity A asks for the value of a_1, so try to find the 6th term:

$$7 = a_6 - 7$$
$$a_6 = 14$$

In other words, each previous term will be 7 larger than the subsequent term. Therefore, $a_7 = 7$, $a_6 = 14$, $a_5 = 21$, and so on. The term a_1, then, is larger than the starting point, 7, and must also be larger than the negative value in Quantity B. Quantity A is larger. Note that the value in Quantity B is the result of incorrectly *subtracting* 7 six times, rather than adding it.

55. **(A).** First, solve for the constant k using the price information of the 2-bedroom, 2-bath unit ($m = 800$, $r = t = 2$ and $f = 1$):

$$800 = k\left(\frac{5(2)^2 + 10(2)}{1+5}\right)$$

$$800 = k\left(\frac{20+20}{6}\right)$$

$$800 = k\left(\frac{20}{3}\right)$$

$$800\left(\frac{3}{20}\right) = k$$

$$40(3) = k$$

$$120 = k$$

Next, solve for the rent on the 3-bedroom, 1-bath unit on the 3rd floor ($r = 3$, $t = 1$, and $f = 3$):

$$m = 120\left(\frac{5(3)^2 + 10(1)}{3+5}\right)$$
$$m = 120\left(\frac{45+10}{8}\right)$$
$$m = 120\left(\frac{55}{8}\right)$$
$$m = 15(55)$$
$$m = 825$$

56. **(B).** First, find the smallest multiple of 3 in this range: 250 is not a multiple of 3 (2 + 5 + 0 = 7, which is not a multiple of 3). The smallest multiple of 3 in this range is 252 (2 + 5 + 2 = 9, which is a multiple of 3). Next, find the largest multiple of 3 in this range. 350 is not a multiple of 3 (3 + 5 + 0 = 8); the largest multiple of 3 in this range is 348.

The sum of an evenly-spaced set of numbers equals the average value multiplied by the number of terms. The average value is the midpoint between 252 and 348: (252 + 348) ÷ 2 = 300. To find the number of terms, first subtract 348 - 252 = 96. This figure represents all numbers between 348 and 252, inclusive. To count only the multiples of 3, divide 96 by the 3: 96/3 = 32. Finally, "add 1 before you're done" because you do want to count both end points of the range: 32 + 1 = 33.

The sum is 300 × 33 = 9,900. Since 9,900 is smaller than 9,990, Quantity B is larger.

57. **(A).** Set up a table and calculate the population of each town after every year; use the calculator to calculate Town A's population. If you feel comfortable multiplying by 1.5 yourself, you do not need to use the calculator for Town B. Instead, add 50% each time (e.g., from 80,000, add 50% or 40,000 to get 120,000).

	Town A	**Town B**
Now	160,000	80,000
Year 1	160,000(1.2) = 192,000	80,000 + 40,000 = 120,000
Year 2	192,000(1.2) = 230,400	120,000 + 60,000 = 180,000
Year 3	230,400(1.2) = 276,480	180,000 + 90,000 = 270,000

Note that, after three years, Town A still has more people than Town B. It will take longer than 3 years, then, for Town B to surpass Town A, so Quantity A is larger.

58. **(C).** The problem provides the function $f(x) = x^2$ and asks you to evaluate $f(m + n) + f(m - n)$. Plug into this function twice — first, to insert $m + n$ in place of x, and then to insert $m - n$ in place of x.

$f(m + n) = (m + n)^2 = m^2 + 2mn + n^2$
$f(m - n) = (m - n)^2 = m^2 - 2mn + n^2$

Now add the two:

$(m^2 + 2mn + n^2) + (m^2 - 2mn + n^2) = 2m^2 + 2n^2$

59. **0.** Adding 20 individual terms would take quite a long time. Look for a pattern. The first several terms in $S_n = (-1)^n$, where $n \geq 1$:

$S_1 = (-1)^1 = -1$
$S_2 = (-1)^2 = 1$
$S_3 = (-1)^3 = -1$
$S_4 = (-1)^4 = 1$

The terms alternate -1, 1, -1, 1, and so on. If the terms are added, every pair of -1 and 1 will add to zero; in other words, for an even number of terms, the sum will be zero. 20 is an even number, so the first 20 terms add to zero.

60. **2.** The "least integer greater than or equal to" language is tricky because of the words "least" and "greater" in the same sentence. First, is the problem asking for a number that is larger or smaller than the starting number? "Greater than or equal to" indicates that the resulting number will be larger than the starting number. Rephrase this as "what is the next largest integer?"

The next largest integer starting from -2.5 is -2. (Remember that, for negative numbers, larger means "closer to 0.")

The next largest integer starting from 3.6 is 4.

$-2 + 4 = 2$

61. **96.** The problem provides the function $f(x, y) = x^2y$ and also the fact that when a and b are plugged in for x and y, the answer is 6. In other words:

$f(x, y) = x^2y$
$f(a, b) = a^2b = 6$

The problem asks for the value of $f(2a, 4b)$. First, plug $2a$ in for x and $4b$ in for y:

$f(2a, 4b) = (2a)^2(4b)$
$f(2a, 4b) = 4a^2(4b)$
$f(2a, 4b) = 16a^2b$

The problem already provides the value for the variables: $a^2b = 6$. Therefore, $16a^2b = 16(6) = 96$.

62. **(A).** The problem indicates that $f(x) = m$ where m is the number of distinct (or different) prime factors of x. For example, if $x = 6$, 6 has two distinct prime factors: 2 and 3. Therefore, the corresponding answer (m value) would be 2.

For Quantity A $f(30)$: 30 has 3 distinct prime factors (2, 3, and 5), so $f(30) = 3$.

For Quantity B $f(64)$: 64 is made of the prime factors 2, 2, 2, 2, 2, and 2). This is only one distinct prime factor, so $f(64) = 1$. Quantity A is larger.

63. **19.** It's too much work to write out the first 75 terms of a sequence, so there must be some kind of pattern. Figure it out (the units digit is bolded):

$3^1 = \mathbf{3}$
$3^2 = \mathbf{9}$
$3^3 = 2\mathbf{7}$
$3^4 = 8\mathbf{1}$
$3^5 = 24\mathbf{3}$
$3^6 = 72\mathbf{9}$

The pattern is 3, 9, 7, 1 and repeats every 4 terms. (Note: you can memorize this pattern or recreate it when you need it. For the numbers 0 to 9, there are no more than 4 units digits in the pattern; if you remember this, you only need to test the first 4 terms.)

The 2nd term in the pattern of 4 terms is equal to 9; for each set of 4 terms, then, there will be one number with a units digit of 9. Since 75 divided by 4 = 18 remainder 3, the entire pattern will repeat 18 times, for a total of 18 terms with a units digit of 9, followed by 3 extra terms. The three extra terms (3, 9, 7) include one extra 9. Therefore, a units digit of 9 appears 18 + 1 = 19 times in this sequence.

64. **317.** Each term in the sequence is 9 greater than the previous term. To make this obvious, you may want to write a few terms of the sequence: 11, 20, 29, 38, etc.

a_{35} comes 34 terms after a_1 in the sequence. In other words, a_{35} is $34 \times 9 = 306$ greater than a_1.

Thus, $a_{35} = 11 + 306 = 317$.

65. **9.** After the first term in the sequence, every term has a units digit of 8:

$Q_1 = 3$
$Q_2 = 2(3) + 2 = 8$
$Q_3 = 2(8) + 2 = 18$
$Q_4 = 2(18) + 2 = 38$
$Q_5 = 2(38) + 2 = 78$
…

So 8 will be the units digit nine out of the first ten times.

66. **11.** This question concerns some function for which the full formula is not given. The problem indicates that $g(2m) = 2g(m)$. In other words, this function is such that plugging in $2m$ is the same as plugging in m and then multiplying by 2. Plug $g(3)$ into the equation for $g(m)$:

$g(6) = 2g(3)$
$g(6) = 2(5.5)$
$g(6) = 11$

67. **(B).** The question asks which of the functions in the answer choices is such that performing the function on $a + b$ yields the same answer as performing the function to a and b individually and then adding those answers together.

The correct answer should be such that $f(a + b) = f(a) + f(b)$ is true for any values of a and b. Test some numbers, for example $a = 2$ and $b = 3$.

	$f(a + b) = f(5)$	$f(a) = f(2)$	$f(b) = f(3)$	Does $f(a + b) = f(a) + f(b)$?
(A)	$f(5) = 5^2 = 25$	$f(2) = 2^2 = 4$	$f(3) = 3^2 = 9$	No
(B)	$f(5) = 5(5) = 25$	$f(2) = 5(2) = 10$	$f(3) = 5(3) = 15$	Yes
(C)	$f(5) = 2(5) + 1 = 11$	$f(2) = 2(2) + 1 = 5$	$f(3) = 2(3) + 1 = 7$	No
(D)	$f(5) = \sqrt{5}$	$f(2) = \sqrt{2}$	$f(3) = \sqrt{3}$	No
(E)	$f(5) = 5 - 2 = 3$	$f(2) = 2 - 2 = 0$	$f(3) = 3 - 2 = 1$	No

Alternatively, use logic — for what kinds of operations are performing the operation on two numbers and then adding them together the same as adding the original numbers together and then performing the operation? Multiplication or division would work, but squaring, square-rooting, adding, or subtracting would not. The correct function can contain ONLY multiplication and/or division.

68. **(B).** You can solve this problem by applying the compound interest formula:

$$V = P\left(1 + \frac{r}{100}\right)^t$$

Since the principal P is the same in both cases, you can leave it out and just compare the rest.

Quantity A: $\left(1 + \frac{r}{100}\right)^t = \left(1 + \frac{8}{100}\right)^2 = (1.08)^2 = 1.08 \times 1.08 = 1.1664$

Quantity B: $\left(1 + \frac{r}{100}\right)^t = \left(1 + \frac{4}{100}\right)^4 = (1.04)^4 = 1.04 \times 1.04 \times 1.04 \times 1.04 \approx 1.1699$

Quantity B is larger.

Alternatively, you can use logic. Notice that the *simple* interest in each case would be the same: 2 years of 8% *simple* interest (of an unchanging principal) is equal to 4 years of 4% *simple* interest of the same principal. Now go back to the compounded world. If the simple interest scenarios are the same, then it will always be true that the compounded scenario with *more frequent* compounding will give you a larger principal in the end, because you're earning "interest on the interest” more often.

2 periods of 8% compounded interest	<	4 periods of 4% compounded interest	<	8 periods of 2% compounded interest	<	16 periods of 1% compounded interest

The differences are small but real.

69. **(B).** Start with $1, and multiply by $\left(1+\frac{26}{100}\right)$ = 1.26 for each year that passes. In order for the amount to double, it would have to reach $2.

End of Year 1: $1 × 1.26 = $1.26
End of Year 2: $1.26 × 1.26 = $1.5876
End of Year 3: $1.5876 × 1.26 = $2.000376 ≈ $2.00

It takes 3 years for the investment to double in value. If you mistakenly thought in terms of *simple* interest, you might think it would take about 4 years (since 26% is just a tiny bit more than 25% = 1/4). In the compounded case, you’re earning “interest on the interest,” though, so the investment grows more quickly.

70. **(A).** You can try the answer choices to find the answer. Start with either choice (B) or choice (D). An 18% interest rate corresponds to multiplying by 1.18. Compounding over two years means multiplying by 1.18 twice:

$1,200 × 1.18 × 1.18 = $1,670.88.

This is close to $1,650 but 17% could be closer. Try answer choice (A):

$1,200 × 1.17 × 1.17 = $1,642.68

Since this result is in fact closer to $1,650, the interest rate must be closer to 17% than to 18%.

Alternatively, use the compound interest formula and solve for the missing interest rate:

$$V = P\left(1+\frac{r}{100}\right)^t$$

$$1{,}650 = 1{,}200\left(1+\frac{r}{100}\right)^2$$

$$\frac{1{,}650}{1{,}200} = \left(1+\frac{r}{100}\right)^2$$

$$\sqrt{1.375} = 1+\frac{r}{100}$$

$$1.172 = 1+\frac{r}{100}$$

$$(0.172)100 = r$$

$$r \approx 17$$

71. **(E).** "Simple" interest means that the interest is calculated based on the initial amount every time; the interest earned is not included in future calculations. Each year, the investment pays 12.5%, or 1/8, of the original investment as simple interest. As a result, it will take exactly 8 years for the cumulative interest to add up to the original investment.

Be careful not to apply the compound interest formula here. If the 12.5% interest is in fact compounded annually, it will take only about 6 years for the investment to double in value.

72. **(C).** This question concerns some function for which the full formula is not provided. The problem indicates that $f(2a) = 2f(a)$. In other words, this function is such that plugging in $2a$ is the same as plugging in a and then multiplying by 2. Plug $f(6) = 11$ into the equation $f(2a) = 2f(a)$:

$f(2(6)) = 2(11)$
$f(12) = 22$

Use the same process a second time. If $a = 12$ and $f(12) = 22$:

$f(2(12)) = 2(22)$
$f(24) = 44$

Alternatively, use logic. When you plug in $2a$, you'll get the same answer as when you plug in a and then multiply by 2. Plugging in 24 is the same as plugging in 6 a total of 4 times, and will give you an answer 4 times as big as plugging in 6. Since plugging in 6 yields 11, plugging in 24 yields 44.

73. **(B).** The question is asking "For which function is performing the function on x and THEN multiplying by 1/2 the equivalent of performing the function on 1/2 of x?"

The fastest method is to use logic: since the order of operations says that order does not matter with multiplication and division but DOES matter between multiplication and addition/subtraction, or multiplication and exponents, you need a function that has only multiplication and/or division. Only answer choice (B) qualifies.

Alternatively, try each choice.

	$\frac{1}{2}f(x)$	$f\left(\frac{1}{2}x\right)$	equal?
(A)	$f\left(\frac{1}{2}x\right)$	$\frac{1}{2}(2x+2)=x+1$	No
(B)	$\frac{1}{2}(13x)=\frac{13x}{2}$	$13\left(\frac{1}{2}x\right)=\frac{13x}{2}$	Yes
(C)	$\frac{1}{2}x^2$	$\left(\frac{1}{2}x\right)^2=\frac{1}{4}x^2$	No
(D)	$\frac{1}{2}(x-10)=\frac{x}{2}-5$	$\frac{1}{2}x-10=\frac{x}{2}-10$	No
(E)	$\frac{1}{2}\sqrt{x-4}$	$\sqrt{\frac{1}{2}x-4}$	No

If you aren't sure whether the two terms in choice (E) are equal, try plugging in a real number for x. If $x = 8$, then the left-hand value becomes 1 and the right-hand value becomes the square root of 0. The two values are *not* the same.