ODDS & EVENS

Even numbers are integers that are divisible by 2. Odd numbers are integers that are not divisible by 2. All integers are either even or odd.

Evens: 0, 2, 4, 6, 8, 10, 12... Odds: 1, 3, 5, 7, 9, 11...

Consecutive integers alternate between even and odd:

9, 10, 11, 12, 13...
O, E, O, E, O...

Negative integers are also either even or odd:

Evens: $-2, -4, -6, -8, -10, -12\ldots$ Odds: $-1, -3, -5, -7, -9, -11\ldots$

Arithmetic Rules of Odds & Evens

The GRE tests your knowledge of how odd and even numbers combine through addition, subtraction, multiplication, and division. Rules for adding, subtracting, multiplying and dividing odd and even numbers can be derived by simply picking numbers and testing them out. While this is certainly a valid strategy, it also pays to memorize the following rules for operating with odds and evens, as they are extremely useful for certain GRE math questions.

Addition and Subtraction:
Add or subtract 2 odds or 2 evens, and the result is EVEN. $7 + 11 = 18$ and $14 - 6 = 8$
Add or subtract an odd with an even, and the result is ODD. $7 + 8 = 15$

Multiplication:
When you multiply integers, if ANY of the integers are even, the result is EVEN. $3 \times \mathbf{8} \times 9 \times 13 = 2{,}808$

Likewise, if NONE of the integers are even, then the result is ODD. $3 \times 5 \times 7 = 105$

If you multiply together several even integers, the result will be divisible by higher and higher powers of 2. This result should make sense from our discussion of prime factors. Each even number will contribute at least one 2 to the factors of the product.

For example, if there are TWO even integers in a set of integers being multipled together, the result will be divisible by 4. $\mathbf{2} \times 5 \times \mathbf{6} = 60$ (divisible by 4)

If there are THREE even integers in a set of integers being multipled together, the result will be divisible by 8.
$\mathbf{2} \times 5 \times \mathbf{6} \times \mathbf{10} = 600$ (divisible by 8)

To summarize so far:

Odd ± Even = ODD
Odd ± Odd = EVEN
Even ± Even = EVEN

Odd × Odd = ODD
Even × Even = EVEN (and divisible by 4)
Odd × Even = EVEN

Division:

There are no guaranteed outcomes in division, because the division of two integers may not yield an integer result. There are several potential outcomes, depending upon the value of the dividend and divisor.

Divisibility of Odds & Evens

	Even?	Odd?	Non-Integer?
Even ÷ Even	✓ Example: $12 \div 2 = 6$	✓ Example: $12 \div 4 = 3$	✓ Example: $12 \div 8 = 1.5$
Even ÷ Odd	✓ Example: $12 \div 3 = 4$	✗	✓ Example: $12 \div 5 = 2.4$
Odd ÷ Even	✗	✗	✓ Example: $9 \div 6 = 1.5$
Odd ÷ Odd	✗	✓ Example: $15 \div 5 = 3$	✓ Example: $15 \div 25 = 0.6$

An odd number divided by any other integer CANNOT produce an even integer. Also, an odd number divided by an even number CANNOT produce an integer, because the odd number will never be divisible by the factor of 2 concealed within the even number.

Check Your Skills

For 1–3, say whether the expression will be odd or even.

1. $1{,}007{,}425 \times 305{,}313 + 2$
2. $5 \times 778 \times 3 \times 4 + 1$
3. The sum of four consecutive integers.
4. Will the product of two odd integers divided by a multiple of two be an integer?

The Sum of Two Primes

Notice that all prime numbers are odd, except the number 2. (All larger even numbers are divisible by 2, so they cannot be prime.) Thus, the sum of any two primes will be even ("Add two odds..."), unless one of those primes is the number 2. So, if you see a sum of two primes that is odd, one of those primes must be the number 2. Conversely, if you know that 2 CANNOT be one of the primes in the sum, then the sum of the two primes must be even.

> If a and b are both prime numbers greater than 10, which of the following *must* be true?
> Indicate *all* that apply.
>
> [A] b is an even number.
> [B] The difference between a and b equals 117.
> [C] The sum of a and b is even.

Since a and b are both prime numbers greater than 10, they must both be odd. Therefore ab must be an odd number, so Choice A cannot be true. Similarly, if a and b are both odd, then $a - b$ cannot equal 117 (an odd number). This difference must be even. Therefore, Choice B cannot be true. Finally, since a and b are both odd, $a + b$ must be even, so Choice C will always be true.

Check Your Skills

5. The difference between the factors of prime number x is one. The difference between the factors of prime number y is two. Is xy even?

Answers can be found on page 63.

Testing Odd & Even Cases

Sometimes multiple variables can be odd or even, and you need to determine the implications of each possible scenario. In that case, set up a table listing all the possible odd/even combinations of the variables, and determine what effect that would have on the question.

> If a, b, and c are integers and $ab + c$ is odd, which of the following *must* be true?
> Indicate *all* that apply.
>
> [A] $a + c$ is odd
> [B] $b + c$ is odd
> [C] abc is even

Here, a, b and c could all possibly be odd or even. Some combinations of Odds & Evens for a, b and c will lead to an odd result. Other combinations will lead to an even result. We need to test each possible combination to see what the result will be for each. Set up a table, as shown below, and fill in the possibilities.

Scenario	***a***	***b***	***c***	***ab* + *c***
1	ODD	ODD	ODD	$O \times O + O = E$
2	**ODD**	**ODD**	**EVEN**	$\mathbf{O \times O + E = O}$
3	**ODD**	**EVEN**	**ODD**	$\mathbf{O \times E + O = O}$
4	ODD	EVEN	EVEN	$O \times E + E = E$
5	**EVEN**	**ODD**	**ODD**	$\mathbf{E \times O + O = O}$
6	EVEN	ODD	EVEN	$E \times O + E = E$
7	**EVEN**	**EVEN**	**ODD**	$\mathbf{E \times E + O = O}$
8	EVEN	EVEN	EVEN	$E \times E + E = E$

Scenarios 2, 3, 5 and 7 yield an odd result, and so we must focus only on those scenarios. We can conclude that Choice A need not be true (Scenario 3 yields $a + c$ = EVEN), Choice B need not be true (Scenario 5 yields $b + c$ = EVEN), and Choice C must be true (all 4 working scenarios yield abc = EVEN). Therefore, the only correct answer is Choice C.

Check Your Skills

6. If $\frac{x}{y}$ is even, which of the following could be true?

   Indicate *all* that apply.

   [A] xy is odd
   [B] xy is even
   [C] $x + y$ is odd

7. If xyz is even, $x + z$ is odd, and $y + z$ is odd, z is:

   Choose just *one* answer.

   (A) Even
   (B) Odd
   (C) Indeterminable (could be even or odd, or a fraction)

Check Your Skills Answers

1. **Odd:** We have an odd multiplied by an odd, which always results in an odd. Then we add an even to the odd, which also results in an odd.

2. **Odd:** At least one of the numbers multiplied together is even, meaning the product will be even. When we add an odd to that even, we get an odd.

3. **Even:** Because integers go back and forth between evens and odds, the sum of any four consecutive integers can be expressed as Even + Even + Odd + Odd. Taking these one by one, we start with Even + Even = Even. Then we add an Odd to that Even, resulting in an Odd. Finally, we add another Odd to that Odd, resulting in an Even.

4. **No:** The product of two odd integers is always odd. Any multiple of two is even, and as the chart showed, an odd divided by an even cannot be an integer.

5. **Yes:** Prime numbers only have two factors: one and themselves. So if the difference between the factors of a prime number is one, its factors must be one and two. This means $x = 2$. By the same logic, y must be equal to 3 ($3 - 1 = 2$). The product of 2 and 3 is 6, so xy is even.

6. **B and C:** If x/y is even, then either x and y are both even, or x is even and y is odd. Let's make a chart:

Scenario	x	y	x/y	xy	$x+y$
1	**E**	**E**	**Even or Odd or Non-int.**	**Even**	**Even**
2	**E**	**O**	**Even or Non-int.**	**Even**	**Odd**
3	O	E	Non-int.	Even	Odd
4	O	O	Odd or Non-int.	Odd	Even

The question stem stipulates that x/y is even. This is only possible in the first two scenarios. In both of those situations, xy is even. This means that Choice A is untrue, but Choice B is true. While $x + y$ can be either even or odd, that means that it *could* be odd, so Choice C also works.

7. **C:** Indeterminable. Once again, let's make a chart.

As we can see, there are two rows where xyz is even, $x + z$ is odd, and $y + z$ is odd. z is even in one case and odd in another.

Scenario	x	y	z	xyz	$x + z$	$y + z$
1	E	E	E	E	E	E
2	**E**	**E**	**O**	**E**	**O**	**O**
3	E	O	E	E	E	O
4	E	O	O	E	O	E
5	O	E	E	E	O	E
6	O	E	O	E	E	O
7	**O**	**O**	**E**	**E**	**O**	**O**
8	O	O	O	O	E	E

Problem Set

For questions #1–15, answer each question ODD, EVEN, or CANNOT BE DETERMINED. Try to explain each answer using the rules you learned in this section. All variables in problems #1–15 are assumed to be integers unless otherwise indicated.

1. If n is odd, p is even, and q is odd, what is $n + p + q$?

2. If r is a prime number greater than 2, and s is odd, what is rs?

3. If t is odd, what is t^4?

4. If u is even and w is odd, what is $u + uw$?

5. If $x \div y$ yields an odd integer, what is x?

6. If $a + b$ is even, what is ab?

7. If c, d, and e are consecutive integers, what is cde?

8. If f and g are prime numbers, what is $f + g$?

9. If h is even, j is odd, and k is odd, what is $k(h + j)$?

10. If m is odd, what is $m^2 + m$?

11. If n, p, q, and r are consecutive integers, what is their sum?

12. If $t = s - 3$, what is $s + t$?

13. If u is odd and w is even, what is $(uw)^2 + u$?

14. If xy is even and z is even, what is $x + z$?

15. If a, b, and c are consecutive integers, what is $a + b + c$?

16.

202 divided by some prime number x yields an odd number. 411 multiplied by some prime number y yields an even number.

Quantity A	**Quantity B**
x	y

17.

Quantity A	Quantity B
The tenths digit of the product of two even integers divided by 4	The tenths digit of the product of an even and an odd integer divided by 4

18.

x is a non-negative even integer

Quantity A	Quantity B
x	1

1. **EVEN:** O + E = O. O + O = E. If in doubt, try plugging in actual numbers: $7 + 2 + 3 = 12$ (even).

2. **ODD:** O × O = O. If in doubt, try plugging in actual numbers: $3 \times 5 = 15$ (odd).

3. **ODD:** O × O × O × O = O. If in doubt, try plugging in actual numbers: $3 \times 3 \times 3 \times 3 = 81$ (odd).

4. **EVEN:** uw is even. Therefore, E + E = E.

5. **CANNOT BE DETERMINED:** There are no guaranteed outcomes in division.

6. **CANNOT BE DETERMINED:** If $a + b$ is even, a and b are either both odd or both even. If they are both odd, ab is odd. If they are both even, ab is even.

7. **EVEN:** At least one of the consecutive integers, c, d, and e, must be even. Therefore, the product cde must be even.

8. **CANNOT BE DETERMINED:** If either f or g is 2, then $f + g$ will be odd. If f and g are odd primes, or if f and g are both 2, then $f + g$ will be even.

9. **ODD:** $h + j$ must be odd (E + O = O). Therefore, $k(h + j)$ must be odd (O × O = O).

10. **EVEN:** m^2 must be odd (O × O = O). $m^2 + m$, therefore, must be even (O + O = E).

11. **EVEN:** If n, p, q, and r are consecutive integers, two of them must be odd and two of them must be even. You can pair them up to add them: O + O = E, and E + E = E. Adding the pairs, you will see that the sum must be even: E + E = E.

12. **ODD:** If s is even, then t must be odd. If s is odd, then t must be even. Either way, the sum must be odd: E + O = O, or O + E = O.

13. **ODD:** $(uw)^2$ must be even. Therefore, E + O = O.

14. **CANNOT BE DETERMINED:** If xy is even, then either x or y (or both x and y) must be even. Given that z is even, $x + z$ could be O + E or E + E. Therefore, we cannot determine whether $x + z$ is odd or even.

15. **CANNOT BE DETERMINED:** If a, b, and c are consecutive, then there could be either one or two even integers in the set. $a + b + c$ could be O + E + O or E + O + E. In the first case, the sum is even; in the second, the sum is odd.

16. **C:** An even divided by an odd can never yield an odd quotient. This means the prime number x must be even (because otherwise you'd have 202/odd, which wouldn't yield an odd quotient). The only even prime number is 2, so $x = 2$. Similarly, an odd times an odd will always be odd, so y must be even. The only prime even number is 2, so $y = 2$.

Quantity A	Quantity B
$\frac{202}{x} = \text{odd} \rightarrow \frac{202}{2} = 101$	$411 \times y = \text{even} \rightarrow 411 \times 2 = 822$
$x = 2$	$y = 2$

17. **D:** This question could be solved either by trying out numbers or making a chart. For Quantity A, the product of two even integers will always divide evenly by 4 because each even number has a 2 in its prime tree. For instance, $2 \times 2 = 4$, $2 \times 4 = 8$, $2 \times 6 = 12$. All of these numbers are divisible by 4. The tenths digit will always have a zero in it (ie. 4.*0*, 8.*0*, 12.*0*).

The tenths digit of the product of an even and an odd integer *could* be divisible by 4. For example, $4 \times 5 = 20$, and $\frac{20}{4} = 5$, the tenths digit of which is 0. However, it could also *not* be divisible by 4. For example, $2 \times 5 = 10$, and $\frac{10}{4} = 2.5$, the tenths digit of which is 5. Because the two quantities could be equal or different, the answer must be Choice D. **We do not have enough information** to determine which quantity is greater.

18. **D:** Always be careful when dealing with evens and odds. While 0 is neither positive nor negative, it *is* even. Thus, the first possible value of x here is 0, not 2. Thus x could be either less than or greater than 1. **We do not have enough information** to determine which quantity is greater.