RATES & WORK

The GMAT's favorite Word Translation type is the RATE problem. Rate problems come in a variety of forms on the GMAT, but all are marked by three primary components: RATE, TIME, & DISTANCE or WORK.

These three elements are related by the equation:

Rate × Time = Distance

or **Rate × Time = Work**

These equations can be abbreviated as $RT = D$ or as $RT = W$. Basic rate problems involve simple manipulations of these equations.

For simple motion problems, use the equation $RT = D$. Simply plug in the values you know and solve for the unknown.

Note that rate-of-travel problems (with a physical distance) and work problems are really the same from the point of view of the math. The main difference is that for work problems, the right side of the equation is not a distance but an ***output*** (e.g., hamburgers cooked). Also, the rate is measured not in units of distance per unit of time (e.g., 10 miles per hour), but in units of ***output*** per unit of time (e.g., 5 hamburgers cooked per minute).

Rate problems on the GMAT come in five main forms:

(1) Basic Motion Problems
(2) Average Rate Problems
(3) Simultaneous Motion Problems
(4) Work Problems
(5) Population Problems

Basic Motion: The RTD Chart

All basic motion problems involve three elements: Rate, Time, and Distance.

Rate is expressed as a ratio of distance and time, with two corresponding units.
Some examples of rates include: 30 miles per hour, 10 meters/second, 15 kilometers/day.

Time is expressed using a unit of time.
Some examples of times include: 6 hours, 23 seconds, 5 months, etc.

Distance is expressed using a unit of distance.
Some examples of distances include: 18 miles, 20 meters, 100 kilometers.

You can make an "RTD chart" to solve a basic motion problem. Read the problem and fill in two of the variables. Then use the $RT = D$ formula to find the missing variable.

> If a car is traveling at 30 miles per hour, how long does it take to travel 75 miles?

An RTD chart is shown to the right. Fill in your RTD chart with the given information. Then solve for the time:

$30t = 75$, or $t = 2.5$ hours

	Rate (mi/hr)	×	*Time* (hr)	=	*Distance* (mi)
Car	30 mi/hr	×		=	75 mi

Matching Units in the RTD Chart

All the units in your RTD chart must match up with one another. The two units in the rate should match up with the unit of time and the unit of distance.

For example:

> It takes an elevator four seconds to go up one floor. How many floors will the elevator rise in two minutes?

The rate is 1 floor/4 seconds, which simplifies to 0.25 floors/second. Note: the rate is NOT 4 seconds per floor! This is an extremely frequent error. **Always express rates as "distance over time,"** not as "time over distance."

Make units match before you substitute any values into $RT = D$ or $RT = W$.

The time is 2 minutes. The distance is unknown.

	R (floors/sec)	×	T (min)	=	W (floors)
Elevator	0.25	×	2	=	?

Watch out! There is a problem with this RTD chart. The rate is expressed in floors per second, but the time is expressed in minutes. This will yield an incorrect answer.

To correct this table, we change the time into seconds. Then all the units will match. To convert minutes to seconds, multiply 2 minutes by 60 seconds per minute, yielding 120 seconds.

	R (floors/sec)	×	T (sec)	=	W (floors)
Elevator	0.25	×	120	=	?

Once the time has been converted from 2 minutes to 120 seconds, the time unit will match the rate unit, and we can solve for the distance using the $RT = D$ equation:

$0.25(120) = d$ $\quad\quad d = 30$ floors

Another example:

> **A train travels 90 kilometers/hr. How many hours does it take the train to travel 450,000 meters? (1 kilometer = 1,000 meters)**

First we divide 450,000 meters by 1,000 to convert this distance to 450 km. By doing so, we match the distance unit (kilometers) with the rate unit (kilometers per hour).

	R (km/hr)	×	T (hr)	=	W (km)
Train	90	×	?	=	450

We can now solve for the time: $90t = 450$. Thus, $t = 5$ hours. Note that this time is the "stopwatch" time: if you started a stopwatch at the start of the trip, what would the stopwatch read at the end of the trip? This is not what a clock on the wall would read, but if you take the *difference* of the start and end clock times (say, 1 pm and 6 pm), you will get the stopwatch time of 5 hours.

The RTD chart may seem like overkill for relatively simple problems such as these. In fact, for such problems, you can simply set up the equation $RT = D$ or $RT = W$ and then substitute. However, the RTD chart comes into its own when we have more complicated scenarios that contain more than one RTD relationship, as we see in the next section.

Multiple RTD Problems

Difficult GMAT rate problems often involve rates, times, and distances for ***more than one trip or traveler***. For instance, you might have more than one person taking a trip, or you might have one person making multiple trips. We expand the RTD chart by adding rows for each trip. Sometimes, we also add a third row, which may indicate a total.

	Rate (miles/hour)	×	*Time* (hour)	=	*Distance* (miles)
Trip 1		×		=	
Trip 2		×		=	
Total					

When you have more than one trip or traveler, make a row in your RTD chart for each traveler or trip.

For each trip, the rate, time, and distance work in the usual manner ($RT = D$), but you have additional relationships among the multiple trips. Below is a list of typical relationships among the multiple trips or travelers.

RATE RELATIONS

Twice / half / n times as fast as

"Train A is traveling at twice the speed of Train B."

	Rate (miles/hour)	×	*Time* (hour)	=	*Distance* (miles)
Train A	$2r$	×		=	
Train B	r	×		=	

(Do not reverse these expressions!)

Slower / faster

"Wendy walks 1 mile per hour more slowly than Maurice."

	Rate (miles/hour)	×	*Time* (hour)	=	*Distance* (miles)
Wendy	$r - 1$	×		=	
Maurice	r	×		=	

Relative rates

"Car A and Car B are driving directly toward each other."

	Rate (miles/hour)	×	*Time* (hour)	=	*Distance* (miles)
Car A	a	×		=	
Car B	b	×		=	
Shrinking Distance Between	$a + b$				

For example, if Car A is going 30 miles per hour and Car B is going 40 miles per hour, then the distance between them is shrinking at a rate of 70 miles per hour. If the cars are driving away from each other, then the distance grows at a rate of (a + b) miles per hour. Either way, the rates add up.

"Car A is chasing Car B and catching up."

	Rate (miles/hour)	×	Time (hour)	=	Distance (miles)
Car A	a	×		=	
Car B	b	×		=	
Shrinking Distance Between	$a - b$				

For example, if Car A is going 55 miles per hour, but Car B is going only 40 miles per hour, then Car A is catching up at 15 miles per hour—that is, the gap shrinks at that rate.

Be sure not to reverse any relationships between rows.

"Car A is chasing Car B and falling behind."

	Rate (miles/hour)	×	Time (hour)	=	Distance (miles)
Car A	a	×		=	
Car B	b	×		=	
Growing Distance Between	$b - a$				

TIME RELATIONS

Slower / faster

"Joey runs a race 30 seconds faster than Tommy."

	Rate (meters/sec)	×	Time (sec)	=	Distance (meters)
Joey		×	$t - 30$	=	
Tommy		×	t	=	

These signs are the *opposites* of the ones for the "slower / faster" rate relations. If Joey runs a race faster than Tommy, then Joey's speed is higher, but his time is lower.

Left … and met / arrived

"Sue left her office at the same time as Tara left hers. They met some time later."

	Rate (miles/hour)	×	Time (hour)	=	Distance (miles)
Sue		×	t	=	
Tara		×	t	=	

Sue and Tara traveled for the same amount time.

"Sue and Tara left at the same time, but Sue arrived home 1 hour before Tara did."

	Rate (miles/hour)	×	Time (hour)	=	Distance (miles)
Sue		×	$t - 1$	=	
Tara		×	t	=	

Sue traveled for 1 hour less than Tara.

"Sue left the office 1 hour after Tara, but they met on the road."

	Rate (miles/hour)	×	Time (hour)	=	Distance (miles)
Sue		×	$t-1$	=	
Tara		×	t	=	

Again, Sue traveled for 1 hour less than Tara.

SAMPLE SITUATIONS

The numbers in the same *row* of an RTD table will *always* multiply across: Rate × Time always equals Distance. However, the specifics of the problem determine which *columns* (R, T, and/or D) will add up into a total row.

The most common Multiple RTD situations are described below. Whenever you encounter a new Multiple RTD problem, try to make an analogy between the new problem and one of the following situations.

Most rate problems fit into one of several typical situations. Use these models as guidelines.

The Kiss (or Crash):

"Car A and Car B start driving toward each other at the same time. Eventually they crash into each other."

	Rate (miles/hour)	×	Time (hour)	=	Distance (miles)
Car A	a	×	t	=	A's distance
Car B	b	×	t	=	B's distance
Total	$a+b$		t		Total distance covered
	ADD		SAME		ADD

(unless one car starts earlier than the other)

The Quarrel: Same math as *The Kiss*.

"Car A and Car B start driving away from each other at the same time…"

The Chase:

"Car A is chasing Car B. How long does it take for Car A to catch up to Car B?"

	Rate (miles/hour)	×	Time (hour)	=	Distance (miles)
Car A	a	×	t	=	A's distance
Car B	b	×	t	=	B's distance
Relative Position	$a-b$		t		Change in the gap between the cars
	SUBTRACT		SAME		SUBTRACT

(unless one car starts earlier than the other)

The Round Trip:

"Jan drives from home to work in the morning, then takes the same route home in the evening."

	Rate (miles/hour)	×	Time (hour)	=	Distance (miles)
Going		×	time going	=	d
Return		×	time returning	=	d
Total			total time		$2d$
	VARIES		ADD		ADD

Often, you can ***make up*** a convenient value for the distance. Pick a Smart Number—a value that is a multiple of all the given rates or times.

Label the rows in your RTD chart clearly. Notice what adds up and what does not for the type of problem at hand.

Following footsteps:

"Jan drives from home to the store along the same route as Bill."

	Rate (miles/hour)	×	Time (hour)	=	Distance (miles)
Jan		×		=	d
Bill		×		=	d
	VARIES		VARIES		SAME

DISTANCE is the same for each person. Again, pick a Smart Number if necessary.

Second-guessing:

"Jan drove home from work. If she had driven home along the same route 10 miles per hour faster…"

	Rate (miles/hour)	×	Time (hour)	=	Distance (miles)
Actual	r	×		=	d
Hypothetical	$r + 10$	×		=	d
	VARIES		VARIES		SAME

No matter what situation exists in a problem, you will often have a choice as you name variables. When in doubt, use variables to stand for either Rate or Time, rather than Distance. This strategy will leave you with easier and faster calculations (products rather than ratios).

Use the following step-by-step method to solve Multiple RTD problems such as this:

> Stacy and Heather are 20 miles apart and walk towards each other along the same route. Stacy walks at a constant rate that is 1 mile per hour faster than Heather's constant rate of 5 miles/hour. If Heather starts her journey 24 minutes after Stacy, how far from her original destination has Heather walked when the two meet?
>
> (A) 7 miles (B) 8 miles (C) 9 miles (D) 10 miles (E) 12 miles

First, make sure that you understand the physical situation portrayed in the problem. The category is "The Kiss": two people walk toward each other and meet. Notice that Stacy starts walking first. If necessary, you might even draw a picture to clarify the scene.

Go ahead and convert any mismatched units. Because all the rates are given in miles per *hour*, you should convert the time that is given in *minutes*: $24 \text{ min} \times \frac{1 \text{ hr}}{60 \text{ min}} = 0.4 \text{ hr}$.

Now start setting up your RTD chart. Fill in all the numbers that you know or can compute very simply: Heather's speed is 5 miles/hour, and Stacy's speed is 5 + 1 = 6 miles/hour. Next, you should try to introduce only *one variable*. If you introduce more than one variable, you will have to eliminate it later to solve the problem; this elimination can cost you valuable time. Let t stand for Heather's time. Also, we know that Stacy walked for 0.4 hours more than Heather, so Stacy's time is $t + 0.4$.

Be sure that whoever walks for <u>more</u> time has a <u>larger</u> time expression in your RTD chart.

	Rate (mi/h)	×	*Time* (hr)	=	*Distance* (mi)
Stacy	6	×	$\mathbf{t+0.4}$	=	
Heather	5	×	$\mathbf{t}$	=	
Total					20 mi

Finish the table by multiplying across rows (as always) and by adding the one column that *can* be added in this problem (distance). (Because Stacy started walking earlier than Heather, you should <u>not</u> simply add the rates in this scenario. You can only add the rates for the period during which the women are <u>both</u> walking.)

	Rate (mi/h)	×	*Time* (hr)	=	*Distance* (mi)
Stacy	6	×	$t+0.4$	=	$\mathbf{6t+2.4}$ mi
Heather	5	×	t	=	$\mathbf{5t}$ mi
Total					20 mi

The table produces the equation $(6t + 2.4) + 5t = 20$, yielding $t = 1.6$. Heather's distance is therefore $5t$, or 8 miles.

Finally, notice that if you were stuck, you could have eliminated some wrong answer choices by thinking about the physical situation. Heather started later *and* walked more slowly; therefore, she cannot have covered half the 20 miles before Stacy reached her. Thus, answer choices D (10 miles) and E (12 miles) are impossible.

Alternate solution: Relative rates

You can simplify this problem by thinking further about the "Kiss" scenario. First, find the distance Stacy walks in the first 24 minutes (= 0.4 hours) by herself: $d = r \times t =$ (6 mi/h) $\times$ (0.4 h) = 2.4 mi. Therefore, once Heather starts walking, the two women have 20 − 2.4 = 17.6 miles left to travel. Because the two women are now traveling for the *same time* in *opposite directions* (in this case, toward each other), you can just use the concept of relative rate: the distance between them is shrinking at the rate of 6 + 5 = 11 miles per hour.

You can often approach Multiple RTD problems in more than one way. Choose an approach that works for you, but be sure to understand the others.

This idea of relative rates eliminates the need for two separate equations, leading to the simplified table shown at right. Solving the resulting equation gives $t = 1.6$ hours. This is the time during which both women are walking.

R (mi/hr)	$\times$	T (hr)	$=$	D (mi)
11	$\times$	t	$=$	17.6

Now set up another *simple* RTD table for Heather by herself.

R (mi/hr)	$\times$	T (hr)	$=$	D (mi)
5	$\times$	1.6	$=$	D

Heather's distance is therefore $5 \times 1.6 = 8$ miles.

The algebraic manipulations are actually very similar in both solutions, but the second approach is more intuitive, and the intermediate calculations make sense. By reformulating problems, you can often increase your understanding and your confidence, even if you do not save that much algebraic work.

Average Rate: Don't Just Add and Divide

Consider the following problem:

> If Lucy walks to work at a rate of 4 miles per hour, but she walks home by the same route at a rate of 6 miles per hour, what is Lucy's average walking rate for the round trip?

It is very tempting to find an average rate as you would find any other average: add and divide. Thus, you might say that Lucy's average rate is 5 miles per hour (4 + 6 = 10 and 10 ÷ 2 = 5). However, this is INCORRECT!

If an object moves the **same distance** twice, but at **different rates**, then ***the average rate will NEVER be the average of the two rates given for the two legs of the journey***. In fact, because the object spends more time traveling at the slower rate, *the average rate will be closer to the slower of the two rates than to the faster.*

In order to find the average rate, you must first find the TOTAL combined time for the trips and the TOTAL combined distance for the trips.

First, we need a value for the distance. Since all we need to know to determine the average rate is the *total time* and *total distance*, we can actually pick any number for the distance. The portion of the total distance represented by each part of the trip ("Going" and "Return") will dictate the time.

Pick a Smart Number for the distance. Since 12 is a multiple of the two rates in the problem, 4 and 6, 12 is an ideal choice.

Set up a Multiple RTD Chart:

	Rate (mi/hr)	×	Time (hr)	=	Distance (mi)
Going	4 mi/hr	×		=	12 mi
Return	6 mi/hr	×		=	12 mi
Total	?	×		=	24 mi

The times can be found using the *RTD* equation. For the GOING trip, $4t = 12$, so $t = 3$ hrs. For the RETURN trip, $6t = 12$, so $t = 2$ hrs. Thus, the total time is 5 hrs.

	Rate (mi/hr)	×	Time (hr)	=	Distance (mi)
Going	4 mi/hr	×	**3 hrs**	=	12 mi
Return	6 mi/hr	×	**2 hrs**	=	12 mi
Total	?	×	**5 hrs**	=	24 mi

The average rate is NOT the simple average of the two rates in the problem!

Now that we have the total Time and the total Distance, we can find the Average Rate using the RTD formula:

$RT = D$

$r(5) = 24$

$r = 4.8$ miles per hour

Again, 4.8 miles per hour is *not* the simple average of 4 miles per hour and 6 miles per hour. In fact, it is the weighted average of the two rates, with the *times* as the weights.

You can test different numbers for the distance (try 24 or 36) to prove that you will get the same answer, regardless of the number you choose for the distance.

Basic Work Problems

Work problems are just another type of rate problem. Just like all other rate problems, work problems involve three elements: rate, time, and "distance."

WORK: In work problems, distance is replaced by work, which refers to the number of jobs completed or the number of items produced.

TIME: This is the time spent working.

RATE: In motion problems, the rate is a ratio of distance to time, or the amount of distance traveled in one time unit. In work problems, the rate is a ratio of work to time, or the amount of work completed in one time unit.

Figuring Work Rates

Work rates usually include one major twist not seen in distance problems: you often have to *calculate* the work rate.

In distance problems, if the rate (speed) is known, it will normally be *given* to you as a ready-to-use number. In work problems, though, you will usually have to *figure out* the rate from some given information about how many jobs the agent can complete in a given amount of time:

$$\text{Work rate} = \frac{\text{Given \# of jobs}}{\text{Given amount of time}}, \text{ or } \frac{1}{\text{Time to complete 1 job}}$$

Work problems are just like distance problems, except that the distance traveled is now the work performed.

For instance, if Oscar can perform one hand surgery in 1.5 hours, his work rate is

$$\frac{1 \text{ operation}}{1.5 \text{ hours}} = \frac{2}{3} \text{ operation per hour}$$

Remember the rate is NOT 1.5 hours per hand surgery! **Always express work rates as jobs per unit time, not as time per job.** Also, you need to distinguish this type of general information—which is meant to specify the work rate—from the data given about the actual work performed, or the time required to perform that specific work.

For example:

> If a copier can make 3 copies every 2 seconds, how long will it take to make 40 copies?

Here, the work is 40 copies, because this is the number of items that will be produced. The time is unknown. The rate is 3 copies/2 seconds, or 1.5 copies per second. Notice the use of the verb "can" with the general rate.

> If it takes Anne 5 hours to paint one fence, and she has been working for 7 hours, how many fences has she painted?

Here the time is 7 hours, because that is the time which Anne spent working. The work done is unknown. Anne's general working rate is 1 fence per 5 hours, or 1/5 fence per hour. Be careful: her rate is not 5 hours per fence, but rather 0.2 fence per hour. Again, always express rates as work per time unit, not time per work unit. Also, notice that the "5 hours" is part of the general rate, whereas the "7 hours" is the actual time for this specific situation. Distinguish the general description of the work rate from the specific description of the episode or task. Here is a useful test: you should be able to add the phrase "in general" to the rate information. For example, we can easily imagine the following:

> If, in general, a copier can make 3 copies every 2 seconds...
>
> If, in general, it takes Anne 5 hours to paint one fence...

Since the insertion of "in general" makes sense, we know that these parts of the problem contain the general rate information.

Basic work problems are solved like basic rate problems, using an RTW chart or the RTW equation. Simply replace the distance with the work. They can also be solved with a simple proportion. Here are both methods for Anne's work problem:

RTW CHART

R (fence/hr)	×	T (hr)	=	W (fences)
1/5 fence/hr	×	7 hours	=	x

$$RT = W$$

$$\frac{1}{5}(7) = \frac{7}{5}$$

PROPORTION

$$\frac{5 \text{ hours}}{1 \text{ fence}} = \frac{7 \text{ hours}}{x \text{ fences}}$$

$$5x = 7$$

$$x = \frac{7}{5}$$

When two or more workers work together on a job, their rates add, not their times. The resulting time will be lower than any individual worker's.

Anne has painted 7/5 of a fence, or 1.4 fences. Note that you can set up the proportion either as "hours/fence" or as "fences/hour." You must simply be consistent on both sides of the equation. However, any rate in an $RT = W$ relationship must be in "fences/hour." (Verify for yourself that the answer to the copier problem above is 80/3 seconds or 26 2/3 seconds.)

Working Together: Add the Rates

The GMAT often presents problems in which several workers working together to complete a job. The trick to these "working together" problems is to determine the combined rate of all the workers working together. This combined working rate is equal to the sum of all the individual working rates. For example, if Machine A can make 5 boxes in an hour, and Machine B can make 12 boxes in an hour, then working together, Machines A and B can make 17 boxes in an hour.

Note that work problems, which are mathematically equivalent to distance problems, feature much less variety than distance problems.

Almost every work problem with multiple people or machines doing work has those people or machines working together. Thus, we can almost always follow this rule: **If two or more agents are performing simultaneous work, add the work rates.**

You can think of "two people working together" as "two people working alongside each other." If Lucas can assemble 1 toy in an hour, and Serena can assemble 2 toys in an hour, then working together, Lucas and Serena can assemble 3 toys in an hour. In other words, Lucas's rate (1 toy per hour) plus Serena's rate (2 toys per hour) equals their joint rate (3 toys per hour).

The only exception to this rule comes in the rare case when one agent's work *undoes* the other agent's work; in that case, you would subtract the rates. For example, one pump might put water *into* a tank, while another pump draws water *out* of that same tank. Again, such problems are very rare.

Use a Population Chart to track the "exponential growth" of populations that double or triple in size over constant intervals of time.

If work problems involve *time* relations, or relations such as "second-guessing," then the calculations are exactly the same as for the corresponding distance problems (with total work substituted for distance).

> Larry can wash a car in 1 hour, Moe can wash a car in 2 hours, and Curly can wash a car in 4 hours. How long will it take them to wash a car together?

First, find their individual rates, or the amount of work they can do in one hour: Larry's rate is 1 (or 1 car/1 hour), Moe's rate is 1 car/2 hours, and Curly's rate is 1 car/4 hours. To find their combined rate, sum their individual rates (***not*** their times):

$$1+\frac{1}{2}+\frac{1}{4}=\frac{7}{4}\text{ cars/hr.}$$

Then, create an RTW chart:

R (cars/hr)	×	*T* (hr)	=	*W* (cars)
7/4	×	*t*	=	1

Using the formula $RT=W$, solve for the time:

$$RT=W$$

$$\frac{7}{4}t=1 \qquad t=\frac{4}{7}\text{ hours, or approximately 34 minutes.}$$

Population Problems

The final type of rate problem on the GMAT is the population problem. In such problems, some population typically increases by a common factor every time period.These can be solved with a Population Chart. Consider the following example:

> The population of a certain type of bacterium triples every 10 minutes. If the population of a colony 20 minutes ago was 100, in approximately how many minutes from now will the bacteria population reach 24,000?

You can solve simple population problems, such as this one, by using a Population Chart. Make a table with a few rows, labeling one of the middle rows as "NOW." Work forward, backward, or both (as necessary in the problem), obeying any conditions given in the problem statement about the rate of growth or decay. In this case, simply triple each population number as you move down a row. Notice that while the population increases by a constant factor, it does not increase by a constant amount each time period.

Time Elapsed	Population
20 minutes ago	100
10 minutes ago	300
NOW	900
in 10 minutes	2,700
in 20 minutes	8,100
in 30 minutes	24,300

For this problem, the Population Chart at right shows that the bacterial population will reach 24,000 about 30 minutes from now.

In some cases, you might pick a Smart Number for a starting point in your Population Chart. If you do so, pick a number that makes the computations as simple as possible.

Problem Set

Solve the following problems, using the strategies you have learned in this section. Use RTD or RTW charts as appropriate to organize information.

1. A cat travels at 60 inches/second. How long will it take this cat to travel 300 feet? (12 inches = 1 foot)

2. Water is being poured into a tank at the rate of approximately 4 cubic feet per hour. If the tank is 6 feet long, 4 feet wide, and 8 feet deep, how many hours will it take to fill up the tank?

3. The population of grasshoppers doubles in a particular field every year. Approximately how many years will it take the population to grow from 2,000 grasshoppers to 1,000,000 or more?

4. Two hoses are pouring water into an empty pool. Hose 1 alone would fill up the pool in 6 hours. Hose 2 alone would fill up the pool in 4 hours. How long would it take for both hoses to fill up two-thirds of the pool?

5. One hour after Adrienne started walking the 60 miles from X to Y, James started walking from X to Y as well. Adrienne walks 3 miles per hour, and James walks 1 mile per hour faster than Adrienne. How far from X will James be when he catches up to Adrienne?

   (A) 8 miles (B) 9 miles (C) 10 miles (D) 11 miles (E) 12 miles

6. Machine A produces widgets at a uniform rate of 160 every 40 minutes, and Machine B produces widgets at a uniform rate of 100 every 20 minutes. If the two machines run simultaneously, how long will it take them to produce 207 widgets in total?

7. An empty bucket being filled with paint at a constant rate takes 6 minutes to be filled to 7/10 of its capacity. How much more time will it take to fill the bucket to full capacity?

8. Three workers can fill a tank in 4, 5, or 6 minutes, respectively. How many tanks can be filled by all three workers working together in 2 minutes?

9. 4 years from now, the population of a colony of bees will reach 1.6×10^8. If the population of the colony doubles every 2 years, what was the population 4 years ago?

10. The Technotronic can produce 5 bad songs per hour. Wanting to produce bad songs more quickly, the record label also buys a Wonder Wheel, which works as fast as the Technotronic. Working together, how many bad songs can the two produce in 72 minutes?

11. A car travels from Town A to Town B at an average speed of 40 miles per hour, and returns immediately along the same route at an average speed of 50 miles per hour. What is the average speed in miles per hour for the round-trip?

12. Jack is putting together gift boxes at a rate of 3 per hour in the first hour. Then Jill comes over and yells, "Work faster!" Jack, now nervous, works at the rate of only 2 gift boxes per hour for the next 2 hours. Then Alexandra comes to Jack and whispers, "The steadiest hand is capable of the divine." Jack, calmer, then puts together 5 gift boxes in the fourth hour. What is the average rate at which Jack puts together gift boxes over the entire period?

13. Andrew drove from A to B at 60 miles per hour. Then he realized that he forgot something at A, and drove back at 80 miles per hour. He then zipped back to B at 90 mph. What was his approximate average speed in miles per hour for the entire night?

14. A bullet train leaves Kyoto for Tokyo traveling 240 miles per hour at 12 noon. Ten minutes later, a train leaves Tokyo for Kyoto traveling 160 miles per hour. If Tokyo and Kyoto are 300 miles apart, at what time will the trains pass each other?

    (A)12:40 pm (B) 12:49 pm (C) 12:55 pm (D) 1:00 pm (E) 1:05 pm

15. Nicky and Cristina are running a 1,000 meter race. Since Cristina is faster than Nicky, she gives him a 12 second head start. If Cristina runs at a pace of 5 meters per second and Nicky runs at a pace of only 3 meters per second, how many seconds will Nicky have run before Cristina catches up to him?

    (A) 15 seconds (B) 18 seconds (C) 25 seconds (D) 30 seconds (E) 45 seconds

1. **1 minute:** This is a simple application of the $RT = D$ formula, involving one unit conversion. First convert the rate, 60 inches/second, into 5 feet/second (given that 12 inches = 1 foot). Substitute this value for R. Substitute the distance, 300 feet, for D. Then solve:

$$(5 \text{ ft/s})(t) = 300 \text{ ft}$$

$$t = \frac{300 \text{ ft}}{5 \text{ ft/s}} = 60 \text{ seconds} = 1 \text{ minute}$$

R (ft/sec)	×	*T* (sec)	=	*D* (ft)
5	×	*t*	=	300

2. **48 hours:** The capacity of the tank is 6 × 4 × 8, or 192 cubic feet. Use the $RT = W$ equation, substituting the rate, 4 ft^3/hour, for R, and the capacity, 192 cubic feet, for W.

R (ft^3/hr)	×	*T* (hr)	=	*W* (ft^3)
4	×	*t*	=	192

$$(4 \text{ cubic feet/hr})(t) = 192 \text{ cubic feet}$$

$$t = \frac{192 \text{ cubic feet}}{4 \text{ cubic feet/hr}} = 48 \text{ hours}$$

3. **9 years:** Organize the information given in a population chart. Notice that since the population is increasing exponentially, it does not take very long for the population to top 1,000,000.

Time Elapsed	Population
NOW	2,000
1 year	4,000
2 years	8,000
3 years	16,000
4 years	32,000
5 years	64,000
6 years	128,000
7 years	256,000
8 years	512,000
9 years	1,024,000

4. **$1\frac{3}{5}$ hours :** If Hose 1 can fill the pool in 6 hours, its rate is 1/6 "pool per hour," or the fraction of the job it can do in one hour. Likewise, if Hose 2 can fill the pool in 4 hours, its rate is 1/4 pool per hour. Therefore, the combined rate is 5/12 pool per hour (1/4 + 1/6 = 5/12).

$$RT = W$$

$$(5/12)t = 2/3$$

$$t = \left(\frac{2}{3}\right)\left(\frac{12}{5}\right) = \frac{8}{5} = 1\frac{3}{5} \text{ hours}$$

R (pool/hr)	×	*T* (hr)	=	*W* (pool)
5/12	×	*t*	=	2/3

5. **(E) 12 miles:** Organize this information in an RTD chart as follows.

	R (mi/hr)	$\times$	T (hr)	$=$	D (mi)
Adrienne	3	$\times$	$t+1$	$=$	d
James	4	$\times$	t	$=$	d
Total	—				$2d$

Set up algebraic equations to relate the information in the chart, using the $RT = D$ equation.

ADRIENNE: $3(t + 1) = d$
JAMES: $4t = d$

Substitute $4t$ for d in the first equation:

$$3(t+1) = 4t$$
$$3t + 3 = 4t$$
$$t = 3$$

Therefore, $d = 4(3) = 12$ miles.

Alternatively, you can model this problem as a "Chase." Adrienne has a 3-mile headstart on James (since Adrienne started walking 1 hour before James, and Adrienne's speed is 3 miles per hour). Since James is walking 1 mile per hour faster than Adrienne, it will take 3 hours for him to catch up to Adrienne. Therefore, he will have walked (4 miles per hour)(3 hours) = 12 miles by the time he catches up to Adrienne.

6. **23 minutes:** Machine A produces 160 widgets every 40 minutes; therefore, it produces 4 widgets every minute. Machine B produces 100 widgets every 20 minutes, or 5 widgets a minute. Together the two machines will produce 4 + 5 = 9 widgets per minute. Substitute this rate into the $RT = W$ equation, using the target work of 207 widgets for W:

R (wid/min)	$\times$	T (min)	$=$	W (wid)
9	$\times$	t	$=$	207

(9 widgets/min) t = 207 widgets
$t = 207 \div 9 = 23$ minutes

7. $2\frac{4}{7}$ **minutes:** Use the $RT = W$ equation to solve for the rate, with $t = 6$ minutes and $w = 7/10$.

R (bkt/min)	$\times$	T (min)	$=$	W (bucket)
r	$\times$	6	$=$	7/10

r(6 minutes) = 7/10

$r = 7/10 \div 6 = \frac{7}{60}$ buckets per minute.

Then, substitute this rate into the equation again, using 3/10 for w (the remaining work to be done).

R (bkt/min)	$\times$	T (min)	$=$	W (bucket)
7/60	$\times$	t	$=$	3/10

$$\left(\frac{7}{60}\right)t = \frac{3}{10}$$

$$t = \frac{3}{10} \div \frac{7}{60} = \frac{18}{7} = 2\frac{4}{7} \text{ minutes}$$

8. $\mathbf{1\frac{7}{30}}$ **tanks:** Since this is a "working together" problem, add the individual rates.

Remember that the rates need to be in tanks per minute, so we need to take the reciprocal of the given "minutes per tank":

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=\frac{1}{x}$$

$$\frac{1}{4}+\frac{1}{5}+\frac{1}{6}=\frac{1}{x}$$

	R (tank/min)	×	*T* (min)	=	*W* (tanks)
Worker 1	1/4	×	4	=	1
Worker 2	1/5	×	5	=	1
Worker 3	1/6	×	6	=	1
Total	1/4 + 1/5 + 1/6				1

Remember to find a common denominator:

$$\frac{15}{60}+\frac{12}{60}+\frac{10}{60}=\frac{37}{60}$$

The 3 workers have a combined rate of 37/60 tanks per minute. Use the $RT = W$ equation to find the total work that can be done in 2 minutes:

$$\left(\frac{37}{60}\right)(2\text{ minutes})=\frac{37}{30}=1\frac{7}{30}\text{ tanks.}$$

9. $\mathbf{1 \times 10^7}$**:** Organize the information given in a population chart.

Time Elapsed	Population
4 years ago	0.1×10^8
2 years ago	0.2×10^8
NOW	0.4×10^8
in 2 years	0.8×10^8
in 4 years	1.6×10^8

Then, convert:

$0.1 \times 10^8 = 10{,}000{,}000 = 1 \times 10^7$ bees.

10. **12 songs:** Since this is a "working together" problem, add the individual rates: $5 + 5 = 10$ songs per hour.

The two machines together can produce 10 bad songs in 1 hour. Convert the given time into hours:

$$(72\text{ minutes})\left(\frac{1\text{ hour}}{60\text{ minutes}}\right)=\frac{72}{60}=1.2\text{ hours}$$

Then, use the $RT = W$ equation to find the total work done:

R (songs/hr)	×	*T* (hr)	=	*W* (songs)
10	×	1.2	=	*w*

$(10)(1.2\text{ hours}) = w$

$w = 12$ bad songs

11. $\mathbf{44\frac{4}{9}}$ **miles per hour:** Use a Multiple RTD chart to solve this problem. Start by selecting a Smart Number for *d*: 200 miles. (This is a common multiple of the 2 rates in the problem.) Then, work backwards to find the time for each trip and the total time:

	R (mi/hr)	×	*T* (hr)	=	*D* (mi)
A to B	40	×	t_1	=	200
B to A	50	×	t_2	=	200
Total	—		t		400

$t_1 = \frac{200}{40} = 5$ hrs $\quad t_2 = \frac{200}{50} = 4$ hrs $\quad t = 4 + 5 = 9$ hours

The average speed $= \frac{\text{total distance}}{\text{total time}} = \frac{400}{9} = 44\frac{4}{9}$ miles per hour.

Do NOT simply average 40 miles per hour and 50 miles per hour to get 45 miles per hour. The fact that the right answer is very close to this wrong result makes this error especially pernicious: avoid it at all costs!

12. **3 boxes per hour:** The average rate is equal to the total work done divided by the time in which the work was done. Remember that you cannot simply average the rates. You must find the total work and total time. The total time is 4 hours. To find the total work, add up the boxes Jack put together in each hour: $3 + 2 + 2 + 5 = 12$. Therefore, the average rate is $\frac{12}{4}$, or 3 boxes per hour. The completed chart looks like this:

	R (box/hr)	×	*T* (hr)	=	*W* (box)
Phase 1	3	×	1	=	3
Phase 2	2	×	2	=	4
Phase 3	5	×	1	=	5
Total	3 = 12/4		4 Sum		12 Sum

13. **Approximately 74.5 mph:** Use a Multiple RTD chart to solve this problem. Start by selecting a Smart Number for *d*: 720 miles. (This is a common multiple of the 3 rates in the problem.) Then, work backwards to find the time for each trip and the total time:

$t_A = \frac{720}{60} = 12$ hrs

$t_B = \frac{720}{80} = 9$ hrs

$t_C = \frac{720}{90} = 8$ hrs

$t = 12 + 9 + 8 = 29$ hours

	R (mi/hr)	×	*T* (hr)	=	*D* (mi)
A to B	60	×	t_A	=	720
B to A	80	×	t_B	=	720
A to B	90	×	t_C	=	720
Total	—		t		2,160

The average speed = $\frac{\text{total distance}}{\text{total time}} =$

$\frac{2{,}160}{29} \approx 74.5$ miles per hour .

	R (mi/hr)	×	T (hr)	=	D (mi)
Train K to T	240	×	$t + 1/6$	=	$240(t + 1/6)$
Train T to K	160	×	t	=	$160t$
Total	—		—		300

14. **(B) 12:49 P.M.:** This is a "Kiss" problem in which the trains are moving TOWARDS each other.

Solve this problem by filling in the RTD chart. Note that the train going from Kyoto to Tokyo leaves first, so its time is longer by 10 minutes, which is 1/6 hour.

Next, write the expressions for the distance that each train travels, in terms of t. Now, sum those distances and set that total equal to 300 miles.

$$240\left(t+\frac{1}{6}\right)+160t=300$$
$$240t+40+160t=300$$
$$400t=260$$
$$20t=13$$
$$t=\frac{13}{20}\text{hour}=\frac{39}{60}\text{hour}=39\,\text{minutes}$$

The first train leaves at 12 noon. The second train leaves at 12:10 P.M. Thirty-nine minutes after the second train has left, at 12:49 P.M., the trains pass each other.

15. **(D) 30 seconds:** This is a "Chase" problem in which the people are moving in the SAME DIRECTION.

Fill in the RTD chart. Note that Nicky starts 12 seconds before Cristina, so Nicky's time is $t + 12$.

	R (m/s)	×	T (second)	=	D (meter)
Cristina	5	×	t	=	$5t$
Nicky	3	×	$t + 12$	=	$3(t + 12)$

Write expressions for the total distance, and then set these two distances equal to each other.

CRISTINA: $5t$ = distance NICKY: $3(t + 12)$ = distance

COMBINE:
$$5t = 3(t + 12)$$
$$5t = 3t + 36$$
$$2t = 36$$
$$t = 18$$

Therefore, Nicky will have run for 18 + 12 = 30 seconds before Cristina catches up to him.