Probability, Combinatorics, and Overlapping Sets

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box $\boxed{}$, you are to enter your own answer in the box. For questions followed by a fraction-style numeric entry box $\frac{\boxed{}}{\boxed{}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is $\frac{1}{4}$, you may enter $\frac{25}{100}$ or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations, such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1. A number is randomly chosen from a list of 10 consecutive positive integers. What is the probability that the number selected is greater than the average (arithmetic mean) of all 10 integers?

   (A) $\frac{3}{10}$

   (B) $\frac{2}{5}$

   (C) $\frac{1}{2}$

   (D) $\frac{7}{10}$

   (E) $\frac{4}{5}$

2. A number is randomly chosen from the first 100 positive integers. What is the probability that it is a multiple of 3?

(A) $\frac{32}{100}$

(B) $\frac{33}{100}$

(C) $\frac{1}{3}$

(D) $\frac{34}{100}$

(E) $\frac{2}{3}$

3. A restaurant menu has several options for tacos. There are 3 types of shells, 4 types of meat, 3 types of cheese, and 5 types of salsa. How many distinct tacos can be ordered assuming that any order contains exactly one of each of the above choices?

4. A history exam features five questions. Three of the questions are multiple-choice with four options each. The other two questions are true or false. If Caroline selects one answer for every question, how many different ways can she answer the exam?

5. The probability is $\frac{1}{2}$ that a certain coin will turn up heads on any given toss and the probability is $\frac{1}{6}$ that a number cube with faces numbered 1 to 6 will turn up any particular number. What is the probability of turning up a heads and a 6?

(A) $\frac{1}{36}$

(B) $\frac{1}{12}$

(C) $\frac{1}{6}$

(D) $\frac{1}{4}$

(E) $\frac{2}{3}$

6. An integer is randomly chosen from 2 to 20 inclusive. What is the probability that the number is prime?

Give your answer as a fraction.

7. An Italian restaurant boasts 320 distinct pasta dishes. Each dish contains exactly 1 pasta, 1 meat, and 1 sauce. If there are 8 pastas and 4 meats available, how many sauces are there to choose from?

8. A 10-student class is to choose a president, vice president, and secretary from the group. If no person can occupy more than one post, in how many ways can this be accomplished?

9. BurgerTown offers many options for customizing a burger. There are 3 types of meats and 7 condiments: lettuce, tomatoes, pickles, onions, ketchup, mustard, and special sauce. A burger must include meat, but may include as many or as few condiments as the customer wants. How many different burgers are possible?

(A) 8!
(B) (3)(7!)
(C) (3)(8!)
(D) $(8)(2^7)$
(E) $(3)(2^7)$

10. The probability of rain is $\frac{1}{6}$ for any given day next week. What is the probability that it will rain on both Monday and Tuesday?

(A) $\frac{1}{36}$
(B) $\frac{1}{12}$
(C) $\frac{1}{6}$
(D) $\frac{1}{3}$
(E) $\frac{2}{3}$

11. How many five-digit numbers can be formed using the digits 5, 6, 7, 8, 9, 0 if no digits can be repeated?

(A) 64
(B) 120
(C) 240
(D) 600

(E) 720

12. A bag contains 3 red, 2 blue, and 7 white marbles. If a marble is randomly chosen from the bag, what is the probability that it is <u>not</u> blue?

Give your answer as a fraction.

13. A man has 3 different suits, 4 different shirts, 2 different pairs of socks, and 5 different pairs of shoes. If an outfit consists of exactly 1 suit, 1 shirt, 1 pair of socks, and 1 pair of shoes, how many different outfits can be made with the man's clothing?

A state issues automobile license plates that begin with two letters selected from a 26-letter alphabet, followed by four numerals selected from the digits 0 through 9, inclusive. Repeats are permitted. For example, one possible license plate combination is GF3352.

14.

Quantity A	Quantity B
The number of possible unique license plate combinations	6,000,000

15. A bag contains 6 black chips numbered 1–6 respectively and 6 white chips numbered 1–6 respectively. If Pavel reaches into the bag of 12 chips and removes 2 chips, one after the other, without replacing them, what is the probability that he will pick black chip #3 and then white chip #3?

Give your answer as a fraction.

Tarik has a pile of 6 green chips numbered 1 through 6 respectively and another pile of 6 blue chips numbered 1 through 6 respectively. Tarik will randomly pick 1 chip from the green pile and 1 chip from the blue pile.

	Quantity A	**Quantity B**
16.	The probability that both chips selected by Tarik will display a number less than 4.	$\frac{1}{2}$

17. A bag contains 6 red chips numbered 1 through 6, respectively, and 6 blue chips numbered 1 through 6, respectively. If 2 chips are to be picked sequentially from the bag of 12 chips, without replacement, what is the probability of picking a red chip and then a blue chip with the same number?

Give your answer as a fraction.

$$\frac{\boxed{}}{\boxed{}}$$

In a school of 150 students, 75 study Latin, 110 study Spanish, and 11 study neither.

	Quantity A	**Quantity B**
18.	The number of students who study only Latin	46

19. How many 10-digit numbers can be formed using only the digits 2 and 5?

(A) 2^{10}
(B) (22)(5!)
(C) (5!)(5!)
(D) $\frac{10!}{2}$
(E) 10!

20. A 6-sided cube has faces numbered 1 through 6. If the cube is rolled twice, what is the probability that the sum of the two rolls is 8?

(A) $\frac{1}{9}$

(B) $\frac{1}{8}$

(C) $\frac{5}{36}$

(D) $\frac{1}{6}$

(E) $\frac{7}{36}$

21. A certain coin with heads on one side and tails on the other has a $\frac{1}{2}$ probability of landing on heads. If the coin is flipped 5 times, how many distinct outcomes are possible if the last flip must be heads? Outcomes are distinct if they do not contain exactly the same results in exactly the same order.

In a class of 25 students, each student studies either Spanish, Latin, or French, or two of the three, but no students study all three languages. 9 study Spanish, 7 study Latin, and 5 study exactly two languages.

	Quantity A	**Quantity B**
22.	The number of students who study French	14

23. Pedro has a number cube with 24 faces and an integer between 1 and 24 on each face. Every number is featured exactly once. When he rolls, what is the probability that the number showing is a factor of 24?

Give your answer as a fraction.

24. A baby has x total toys. If 9 of the toys are stuffed animals, 7 of the toys were given to the baby by her grandmother, 5 of the toys are stuffed animals given to the baby by her grandmother, and 6 of the toys are neither stuffed animals nor given to the baby by her grandmother, what is the value of x?

25. How many integers between 2,000 and 3,999 have a ones digit that is a prime number?

26. A group of 12 people who have never met are in a classroom. How many handshakes are exchanged if each person shakes hands exactly once with each of the other people in the room?

(A) 12
(B) 22
(C) 66
(D) 132
(E) 244

27. A class consists of 12 girls and 20 boys. One quarter of the girls in the class have blue eyes. If a child is selected at random from the class, what is the probability that the child is a girl who does not have blue eyes?

(A) $\frac{3}{32}$
(B) $\frac{9}{32}$
(C) $\frac{3}{8}$
(D) $\frac{23}{32}$
(E) $\frac{29}{32}$

28. A certain coin with heads on one side and tails on the other has a $\frac{1}{2}$ probability of landing on heads. If the coin is flipped three times, what is the probability of flipping 2 tails and 1 head, in any order?

(A) $\frac{1}{8}$

(B) $\frac{1}{3}$

(C) $\frac{3}{8}$

(D) $\frac{5}{8}$

(E) $\frac{2}{3}$

29. A number cube has six faces numbered 1 through 6. If the cube is rolled twice, what is the probability that at least one of the rolls will result in a number greater than 4?

(A) $\frac{2}{9}$

(B) $\frac{1}{3}$

(C) $\frac{4}{9}$

(D) $\frac{5}{9}$

(E) $\frac{2}{3}$

30. 100 tiles are labeled with the integers from 1 to 100 inclusive; no numbers are repeated. If Alma chooses one tile at random, replaces it in the group, and chooses another tile at random, what is the probability that the product of the two integer values on the tiles is odd?

(A) $\frac{1}{8}$

(B) $\frac{1}{4}$

(C) $\frac{1}{3}$

(D) $\frac{1}{2}$

(E) $\frac{3}{4}$

31. If the word “WOW” can be rearranged in exactly 3 ways (WOW, OWW, WWO), how many different arrangements of the letters in “MISSISSIPPI” are possible?

The probability of rain is $\frac{1}{2}$ on any given day next week.

	Quantity A	**Quantity B**
32.	The probability that it rains on at least one of the 7 days next week	$\frac{127}{128}$

33. Two number cubes with six faces numbered with the integers from 1 through 6 are tossed. What is the probability that the sum of the exposed faces on the cubes is a prime number?

Give your answer as a fraction.

34. Jan and 5 other children are in a classroom. The principal of the school will choose two of the children at random. What is the probability that Jan will be chosen?

(A) $\frac{4}{5}$

(B) $\frac{1}{3}$

(C) $\frac{2}{5}$

(D) $\frac{7}{15}$

(E) $\frac{1}{2}$

The probability that Maria will eat breakfast on any given day is 0.5. The probability that Maria will wear a sweater on any given day is 0.3. The two probabilities are independent of each other.

	Quantity A	Quantity B
35.	The probability that Maria eats breakfast or wears a sweater	0.8

The probability of rain in Greg's town on Tuesday is 0.3. The probability that Greg's teacher will give him a pop quiz on Tuesday is 0.2. The events occur independently of each other.

	Quantity A	**Quantity B**
36.	The probability that either or both events occur	The probability that neither event occurs

37. A certain city has a $\frac{1}{3}$ chance of rain occurring on any given day. In any given 3-day period, what is the probability that the city experiences rain?

(A) $\frac{1}{3}$
(B) $\frac{8}{27}$
(C) $\frac{2}{3}$
(D) $\frac{19}{27}$
(E) 1

38. Five students, Adnan, Beth, Chao, Dan, and Edmund are to be arranged in a line. How many such arrangements are possible if Beth is not allowed to stand next to Dan?

(A) 24
(B) 48
(C) 72
(D) 96
(E) 120

39. A polygon has 12 edges. How many different diagonals does it have? (A diagonal is a line drawn from one vertex to any other vertex inside the given shape. This line cannot touch or cross any of the edges of the shape. For example, a triangle has zero diagonals and a rectangle has two.)

(A) 54
(B) 66
(C) 108
(D) 132
(E) 144

	Quantity A	Quantity B
40.	The number of possible 4-person teams that can be selected from 6 people	The number of possible 2-person teams that can be selected from 6 people
41.	The number of ways 1st, 2nd, and 3rd place prizes could be awarded to 3 out of 6 contestants	The number of ways 1st, 2nd, 3rd, 4th, and 5th place prizes could be awarded to 5 contestants

An inventory of coins contains 100 different coins.

	Quantity A	Quantity B
42.	The number of possible collections of 56 coins that can be selected (the order of the coins does not matter)	The number of possible collections of 44 coins that can be selected (the order of the coins does not matter)

An office supply store carries an inventory of 1,345 different products, all of which it categorizes as “business use,” “personal use,” or both. There are 740 products categorized as “business use” only and 520 products categorized as both “business use” and “personal use.”

	Quantity A	Quantity B
43.	The number of products characterized as “personal use”	600

44. Eight women and two men are available to serve on a committee. If three people are picked, what is the probability that the committee includes at least one man?

(A) $\frac{1}{32}$

(B) $\frac{1}{4}$

(C) $\frac{2}{5}$

(D) $\frac{7}{15}$

(E) $\frac{8}{15}$

45. At Lexington High School, each student studies at least one language—Spanish, French, or Latin—and no student studies all three languages. If 100 students study Spanish, 80 study French, 40 study Latin, and 22 study exactly two languages, how many students are there at Lexington High School?

(A) 198
(B) 220
(C) 242
(D) 264
(E) 286

46. Of 60 birds found in a certain location, 20 are songbirds and 23 are migratory. (It is possible for a songbird to be either migratory or not migratory.)

Quantity A	**Quantity B**
The number of the 60 birds that are neither migratory nor songbirds	16

Probability, Combinatorics, and Overlapping Sets Answers

1. **(C).** In a list of 10 consecutive integers, the mean is the average of the 5th and 6th numbers. Therefore, the 6th through 10th integers (five total integers) is greater than the mean. Since probability is determined by the number of desired items divided by the total number of choices, the probability that the number chosen is greater than the average of all 10 integers is $\frac{5}{10} = \frac{1}{2}$.

Another approach to this problem is to create a set of 10 consecutive integers; the easiest such list contains the numbers {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. The mean is one-half the sum of the first element plus the last element, or $\frac{1+10}{2}$ = 5.5. Therefore, there are 5 numbers greater than the mean in the list: 6, 7, 8, 9 and 10. Again, the probability of choosing a number greater than the average of all 10 integers is $\frac{5}{10} = \frac{1}{2}$.

2. **(B).** The first 100 positive integers comprise the set of numbers containing the integers 1 to 100. Of these numbers, the only ones that are divisible by 3 are {3, 6, 9, …, 96, 99}, which adds up to exactly 33 numbers. This can be determined in several ways. One option is to count the multiples of 3, but that's a bit slow. Alternatively, compute $\frac{99}{3}$ = 33 and realize that there are 33 multiples of 3 up to and including 99. The number 100 is not divisible by 3, so the correct answer is $\frac{33}{100}$.

Alternatively, use the "add one before you're done" trick, subtracting the first multiple of 3 from the last multiple of 3, dividing by 3 and then adding 1: $\frac{(99-3)}{3}$ + 1 = 33. Then, since probability is determined by the number of desired options divided by the total number of options, the probability that the number chosen is a multiple of 3 is $\frac{33}{100}$.

3. **180.** This problem tests the fundamental counting principle, which states that the total number of choices is equal to the product of the independent choices. Therefore, the total number of tacos is (3)(4)(3)(5) = 180 tacos.

4. **256.** This question tests the fundamental counting principle, which states that the total number of choices is equal to the product of the independent choices. The five separate test questions means there are five independent choices. For the three multiple-choice questions there are four options each, whereas for the two true/false questions there are two options each. Multiplying the independent choices yields (4)(4)(4)(2)(2) = 256 different ways to answer the exam.

5. **(B).** The probability of independent events A *and* B occurring is equal to the product of the probability of event A and the probability of event B. In this case, the probability of the coin turning up heads is $\frac{1}{2}$ and the probability of rolling a 6 is $\frac{1}{6}$. Therefore, the probability of heads *and* a 6 is equal to $\left(\frac{1}{2}\right)\left(\frac{1}{6}\right) = \frac{1}{12}$. Alternatively, list all the possible outcomes: H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6. There are 12 total outcomes and only 1 with heads and a 6. Therefore, the desired outcome divided by the total number of outcomes is equal to $\frac{1}{12}$.

6. $\frac{8}{19}$ **(or any equivalent fraction).** Among the integers 2 through 20, inclusive, there are 8 primes: 2, 3, 5, 7, 11, 13, 17, and 19. From 2 to 20, inclusive, there are exactly 20 – 2 + 1 = 19 integers; remember to "add one before you're done" to include both endpoints. Alternatively, there are 20 integers from 1 to 20, inclusive, so there must be 19 integers from 2 to 20, inclusive. Since probability is defined as the number of desired items divided by the total number of choices, the probability that the number chosen is prime is $\frac{8}{19}$.

7. **10.** This problem tests the fundamental counting principle, which states that the total number of choices is equal to the product of the independent choices. Let the number of sauces be represented by the variable S. The total number of possible pasta dishes can be represented by each separate choice multiplied together: $(8)(4)(S)$, or $32S$. The problem also indicates that the total number of pasta dishes must be equal to 320. Therefore, $32S = 320$, so $S = 10$.

8. **720.** One possible approach is to ask, "How many choices do I have for each of the class positions?" Begin by considering the president of the class. Since no one has been chosen yet, there are 10 students from whom to choose. Then, for the vice president there are 9 options because now one student has already been chosen as president. Similarly, there are 8 choices for the secretary. Using the fundamental counting principle, the total number of possible selections is (10)(9)(8) = 720.

Alternatively, use factorials. In this case, order matters because people are selected for specific positions. This problem is synonymous to asking, "How many different ways can you line up 3 students as first, second, and third from a class of 10?" The number of ways to arrange the entire class in line is 10!. However, the problem is only concerned with the first 3 students in line, so exclude rearrangements of the last 7. The way in which these "non-chosen" 7 students can be ordered is 7!. Thus, the total number of arrangements for 3 students from a class of 10 is $\frac{10!}{7!} = (10)(9)(8) =$ 720 choices.

9. **(E).** This problem tests the fundamental counting principle, which states that the total number of choices is equal to the product of the independent choices. The key to this problem is realizing how many choices there are for each option. For the meat, there are 3 choices. For each of the condiments there are exactly 2 choices: yes or no. The only real choice regarding each condiment is whether to include it at all. As there are 7 condiments, the total number of choices is $(3)(2)(2)(2)(2)(2)(2)(2) = (3)(2^7)$.

Note: the condiment options cannot be counted as 8! (0 through 7 = 8 options) because, in this case, the order in which the options are chosen does not matter; a burger with lettuce and pickles is the same as a burger with pickles and lettuce.

10. **(A).** For probability questions, always begin by separating out the probabilities of each individual event. Then, if all the events happen (an "*and* question"), multiply the probabilities

together. If only one of the multiple events happens (an “*or* question”), add the probabilities together.

In this case, there are two events: rain on Monday and rain on Tuesday. The question asks for the probability that it will rain on Monday *and* on Tuesday, so multiply the individual probabilities together:

$$\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

11. **(D).** This problem relies on the fundamental counting principle, which says that the total number of ways for something to happen is the product of the number of options for each individual choice. The problem asks how many five-digit numbers can be created from the digits 5, 6, 7, 8, 9, and 0. For the first digit, there are only five options (5, 6, 7, 8, and 9) because a five-digit number must start with a non-zero integer. For the second digit, there are 5 choices again, because now zero can be used but one of the other numbers has already been used, and numbers cannot be repeated. For the third number, there are 4 choices, for the fourth number there are 3 choices, and for the fifth number there are 2 choices. Thus, the total number of choices is (5)(5)(4)(3)(2) = 600.

Alternatively, use the same logic and realize there are 5 choices for the first digit. (Separate out the first step because you have to remove the zero from consideration.) The remaining five digits all have an equal chance of being chosen, so choose four out of the remaining five digits to complete the number. The number of ways in which this second step can be accomplished is $\frac{(5!)}{(1!)}$ = (5)(4)(3)(2).

Thus, the total number of choices is again equal to (5)(5)(4)(3)(2) = 600.

12. $\frac{5}{6}$ **(or any equivalent fraction).** In the bag of marbles, there are 3 red marbles and 7 white marbles, for a total of 10 marbles that are not blue. There are a total of 3 + 7 + 2 = 12 marbles in the bag. Since probability is defined as the number of desired items divided by the total number of choices, the probability that the marble chosen is *not* blue is $\frac{10}{12} = \frac{5}{6}$.

13. **120.** This problem utilizes the fundamental counting principle, which states that the total number of choices is equal to the product of the independent choices. Since the man must choose one suit, one shirt, one pair of socks, and one pair of shoes, the total number of outfits is the number of suits times the number of shirts times the number of socks times the number of shoes: (3)(4)(2)(5) = 120.

14. **(A).** This is a combinatorics problem. The license plates have 2 letters followed by 4 numbers, so make 6 "slots" and determine how many possibilities there are for each slot. There are 26 letters in the alphabet and 10 digits to pick from, so:

$$\underline{26}\quad \underline{26}\quad \underline{10}\quad \underline{10}\quad \underline{10}\quad \underline{10}$$

Multiply 26 × 26 on the calculator to get 676. Add four zeros for the four 10's to get 6,760,000. Quantity A is greater.

15. $\mathbf{\frac{1}{132}}$ **(or any equivalent fraction).** The probability of picking black chip #3 is $\frac{1}{12}$. Once Pavel has removed the first chip, only 11 chips remain, so the probability of picking white chip #3 is

$\frac{1}{11}$. Multiply $\frac{1}{12}\times\frac{1}{11}=\frac{1}{132}$.

16. **(B).** In this problem, Tarik is *not* picking 1 chip out of all 12. Rather, he is picking 1 chip out of 6 green ones, and then picking another chip out of 6 blue ones. There are 3 green chips with numbers less than 4, so Tarik has a $\frac{3}{6}$ chance of selecting a green chip showing a number less than 4. Likewise, Tarik has a $\frac{3}{6}$ chance of selecting a blue chip showing a number less than 4. Therefore, Quantity A is equal to $\frac{3}{6}\times\frac{3}{6}=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$. Quantity B is greater.

17. $\frac{\mathbf{1}}{\mathbf{22}}$ **(or any equivalent fraction).** The trap answer in this problem is $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$. This is *not* the answer to the question being asked—rather, this is the answer to the question, "What is the probability of picking a red chip and then a blue chip that both have #3?" (or any other specific number). This is a more specific question than the one actually asked. In the question, asked, there are six possible ways to fulfill the requirements of the problem, not one, because the problem does not specify whether the number should be 1, 2, 3, 4, 5, or 6.

Thus, *any* of the 6 red chips is acceptable for the first pick. However, on the second pick, only the blue chip with the same number as the red one that was just picked is acceptable (the chip must "match" the first one picked). Thus:

$$\frac{6}{12} \times \frac{1}{11} = \frac{1}{2} \times \frac{1}{11} = \frac{1}{22}$$

18. **(B).** Use the overlapping sets formula for two groups: Total = Group 1 + Group 2 – Both + Neither. (Adding the two groups—in this case Latin and Spanish—double-counts the students who study both languages, so the formula subtracts the "both" students.) Set up your equation:

$150 = 75 + 110 - B + 11$
$150 = 196 - B$
$46 = B$

Careful! This is not the value of Quantity A. Since 46 students study both Latin and Spanish, subtract 46 from the total who study Latin to find those who study only Latin:

$75 - 46 = 29$

Thus, Quantity A is 29. Therefore Quantity B is greater.

19. **(A).** This problem relies on the fundamental counting principle, which says that the total number of ways for something to happen is the product of the number of options for each individual choice. For any digit of the 10-digit number there are exactly two options, a 2 or a 5. Thus, since there are two choices for each digit and it is a 10-digit number, there are $(2)(2)(2)(2)(2)(2)(2)(2)(2)(2) = 2^{10}$ total choices.

20. **(C).** The probability of any event equals the number of ways to get the desired outcome divided by the total number of ways for the event to happen. Starting with the denominator, use the fundamental counting principle to compute the total number of ways to roll a cube twice. There are 6 possibilities (1, 2, 3, 4, 5, or 6) for the first roll and 6 for the second, giving a total of $(6)(6) = 36$ possibilities for the two rolls. For the numerator, determine the number of possible combinations that will sum to 8. For example, rolling a 2 the first time and a 6 the second time. The full set of options is (2, 6), (3, 5), (4, 4), (5, 3), and (6, 2). Thus, there are 5 possible combinations that sum to 8, yielding

a probability of $\frac{5}{36}$.

21. **16.** This problem utilizes the fundamental counting principle, which states that the total number of choices is equal to the product of the independent choices. For the first flip, there are 2 options: heads or tails. Similarly, for the second flip, there 2 options; for the third, there are 2 options; for the fourth, there are 2 options; and for the fifth there is only one option because the problem restricts this final flip to heads. Therefore, the total number of outcomes

is (2)(2)(2)(2)(1) = 16. A good rephrasing of this question is, "How many different outcomes are there if the coin is flipped 4 times?" The fifth flip, having been restricted to heads, is irrelevant. Therefore, the total number of ways to flip the coin five times with heads for the fifth flip is equal to the total number of ways to flip the coin four times; either way, the answer is 16.

22. **(C).** The problem specifies that no one studies all three languages. In addition, a total of 5 people study two languages. Thus, 5 people have been double-counted. Since the total number of people who have been double-counted (5) and triple-counted (0) is known, use the standard overlapping sets formula:

Total = Spanish + French + Latin – (Two of the three) – 2(All three)

25 = 9 + French + 7 – 5 – 2(0)
25 = 11 + French
14 = French

The two quantities are equal.

23. $\mathbf{\frac{1}{3}}$ **(or any equivalent fraction).** Probability equals the number of desired outcomes divided by the total number of possible outcomes. Among the integers 1 through 24, there are four factor pairs of 24: (1, 24), (2, 12), (3, 8), and (4, 6), for a total of 8 factors. The total number of possible outcomes when rolling the cube once is 24. The probability that the number chosen is a factor of 24 is $\frac{8}{24} = \frac{1}{3}$.

24. **17.** Use the overlapping sets formula for two groups: Total = Group 1 + Group 2 – Both + Neither. Here, the groups are "stuffed animal" and "given by the baby's grandmother." The problem indicates that the "both" category is equal to 5 and that the "neither" number is 6. The total is x.

Total = Group 1 + Group 2 – Both + Neither

$x = 9 + 7 - 5 + 6$
$x = 17$

25. **800.** This is a combinatorics problem. Make four "slots" (since the numbers are all four-digit numbers), and determine how many possibilities there are for each slot:

$\underline{\quad\quad}\ \ \underline{\quad\quad}\ \ \underline{\quad\quad}\ \ \underline{\quad\quad}$

Since the number must begin with 2 or 3, there are two possibilities for the first slot. Because the ones digit must be prime and there are only four prime one-digit numbers (2, 3, 5, and 7), there are four possibilities for the last slot:

$\underline{\ \ 2\ \ }\ \ \underline{\quad\quad}\ \ \underline{\quad\quad}\ \ \underline{\ \ 4\ \ }$

The other slots have no restrictions, so put 10 in them, since there are ten digits from 0–9:

$$\underline{\ 2\ }\quad \underline{\ 10\ }\quad \underline{\ 10\ }\quad \underline{\ 4\ }$$

Multiply to get 800.

Alternatively, figure out the pattern and add up the number of qualifying four-digit integers. In the first ten numbers, 2000–2009, there are exactly four numbers that have a prime units digit: 2002, 2003, 2005, and 2007. The pattern then repeats in the next group of ten numbers, 2010–2020, and so on. In any group of ten numbers, then, four qualify. Between 2,000 and 3,999 there are 3,999 – 2,000 + 1 = 2,000 numbers, or $\frac{2,000}{10}$ = 200 groups of ten numbers, so there are a total of 400 × 2 = 800 numbers that have a prime units digit.

26. **(C).** Multiple approaches are possible here. One way is to imagine the scenario and count up the number of handshakes. How many hands does everyone need to shake? There are 11 other people in the room, so the first person needs to shake hands 11 times. Now, move to the second person: how many hands must he shake? He has already shaken one hand, leaving him 10 others with whom to shake hands. The third person will need to shake hands with 9 others, and so on. Therefore, there are a total of 11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 handshakes. The fastest way to find the sum of a group of consecutive numbers is to take the average of the first and last terms and multiply it by the number of terms. The average is $\frac{(11+1)}{2}$ = 6 and there are 11 – 1 + 1 = 11 terms (find the difference between the terms and "add one before you're done"). The sum is 6 × 11 = 66.

Alternatively, rephrase the question as "How many different ways can any 2 people be chosen from a group of 12?" (This works because the problem ultimately asks you to "choose" each distinct pair of 2 people one time.) The key here is to realize that handshakes are independent of order, that is, it doesn't matter if A shakes hands with B or if B shakes hands with A; it's the same outcome. Thus, it only matters how many pairs you can make. Any time a question presents a group of order-independent items selected from a larger set, apply the formula $\frac{\text{total!}}{\text{in!out!}}$ to arrive at the total number of combinations. Thus: $\frac{12!}{2!10!} = \frac{12 \times 11}{2}$ = 66.

27. **(B).** The probability of any outcome is equal to the number of desired outcomes divided by the total number of outcomes. There are 12 girls and 20 boys in the classroom. If one-quarter of the girls have blue eyes, then there are $(12)\left(\frac{1}{4}\right)$ = 3 girls with blue eyes. Therefore, there are 12 – 3 = 9 girls who do *not* have blue eyes. The total number of ways in which a child could be chosen is the total number of children in the class, namely 12 + 20 = 32. Therefore, the probability of choosing a girl who does not have blue eyes equals the number of girls without blue eyes divided by the total

number of children, which is $\frac{9}{32}$.

28. **(C).** There are only 2 possible outcomes for each flip and only 3 flips total. The most straightforward approach is to list all of the possible outcomes: {HHH, HHT, HTH, HTT, TTT, TTH THH, THT}. Of these 8 possibilities, 3 of the outcomes have one head and two tails, so the probability of this event is $\frac{3}{8}$.

Alternatively, count the total number of ways of getting 1 head without listing all the possibilities. If the coin is flipped 3 times and you want only 1 head, then there are 3 possible positions for the single head: on the first flip alone, on the second flip alone, or on the third flip alone. Since there are 2 possible outcomes for each flip, heads or tails, there are (2)(2)(2) = 8 total outcomes. Again, the probability is $\frac{3}{8}$.

Finally, another alternative is to compute the probability directly. The probability of flipping heads is $\frac{1}{2}$ and the probability of flipping tails is also $\frac{1}{2}$. The probability of getting heads in the first position alone, or HTT, is $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)=\frac{1}{8}$, multiplied because the coin was flipped heads *and* tails *and* tails. This represents the probability of heads in position 1, but heads could also be in position 2 alone or in position 3 alone. Since there are 3 possible positions for the heads, multiply by 3 to get the total probability $(3)\left(\frac{1}{8}\right)=\frac{3}{8}$.

29. **(D).** Because this problem is asking for an "at least" solution, use the 1 – x shortcut. The probability that at least one roll results in a number greater than 4 is equal to 1 minus the probability that both of the rolls result in numbers 4 or lower. For one roll, there are 6 possible outcomes (1 through 6) and 4 ways in which the outcome can be 4 or lower, so the probability is $\frac{4}{6}=\frac{2}{3}$. Thus, the probability that both rolls result in numbers 4 or lower is $\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)=\frac{4}{9}$. This is the result that you do *not* want; subtract this from 1 to get the probability that you do want. The probability that at least one of the rolls results in a number greater than 4 is 1 – $\left(\frac{4}{9}\right)=\frac{5}{9}$.

Alternatively, write out the possibilities. The total number of possibilities for two rolls is (6)(6) = 36. Here are the ways in which at least one number greater than 4 can be rolled:

51, 52, 53, 54, 55, 56
61, 62, 63, 64, 65, 66
15, 25, 35, 45 (note: 55 and 65 have already been counted above)
16, 26, 36, 46 (note: 56 and 66 have already been counted above)

There are 20 elements (be careful not to double-count any options). The probability of at least one roll resulting in a number greater than 4 is $\frac{20}{36}=\frac{5}{9}$.

30. **(B).** Use both probability and number properties concepts in order to answer this question. First, in order for two integers to produce an odd integer, the two starting integers must be odd. An odd times an odd equals an odd. An even times an odd, by contrast, produces an even, as does an even times an even.

Within the set of tiles, there are 50 even numbers (2, 4, 6, …, 100) and 50 odd numbers (1, 3, 5, …, 99). One randomly-chosen tile will have a $\frac{50}{100} = \frac{1}{2}$ probability of being even, and a $\frac{1}{2}$ probability of being odd. The probability of choosing an odd tile first is $\left(\frac{1}{2}\right)$ and the probability of choosing an odd tile second is also $\left(\frac{1}{2}\right)$, so the probability of "first odd *and* second odd" is $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4}$.

Alternatively, recognize that there are only four options for odd/even pairs if two tiles are chosen: OO, OE, EO, EE. The only one of these combinations that yields an odd product is OO. Since all of these combinations are equally likely, and since OO is exactly one out of the four possibilities, the probability of choosing OO is $\frac{1}{4}$.

31. **34,650.** This is a combinatorics problem, and the WOW example is intended to make it clear that any W is considered identical to any other W—switching one W with another would *not* result in a different combination, just as switching one S with another in MISSISSIPPI would not result in a different combination.

Therefore, solve this problem using the classic combinatorics formula for accounting for subgroups among which order does not matter:

$$\frac{\text{Total number of items!}}{\text{First group! Second group! Etc.}}$$

Because MISSISSIPPI has 11 letters, including 1 M, 4 S's, 4 I's, and 2 P's:

$$\frac{11!}{1!4!4!2!}$$

Now expand the factorials and cancel; use the calculator for the last step of the calculation:

$$\frac{11\times10\times9\times8\times7\times6\times5\times\cancel{4!}}{\cancel{1!}\cancel{4!}(4\times3\times2\times1)(2\times1)}=\frac{11\times10\times9\times\cancel{8}\times7\times\cancel{6}\times5}{(\cancel{4}\times\cancel{3}\times\cancel{2}\times\cancel{1})(\cancel{2}\times\cancel{1})}=11\times10\times9\times7\times5=34{,}650$$

32. **(C).** Since Quantity A is an "at least" problem, use the $1 - x$ shortcut. Rather than calculate the probability of rain on exactly 1 day next week, and then the probability of rain on exactly 2 days next week, and so on (after which you would still have to add all of the probabilities together!), instead calculate the probability of no rain at all on any day, and then subtract that number from 1. That will give the combined probabilities for any scenarios that include rain on at least 1 day.

Probability of NO rain for any of the 7 days = $\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{128}$

Subtract this probability from 1:

$$1-\frac{1}{128}=\frac{128}{128}-\frac{1}{128}=\frac{127}{128}$$

Quantities A and B are equal.

33. $\frac{5}{12}$**.** First think about the prime numbers less than 12, the maximum sum of the numbers on the cube. These primes are 2, 3, 5, 7, 11.

The probability of rolling 2, 3, 5, 7, or 11 is equal to the number of ways to roll any of these sums divided by the total number of possible rolls. The total number of possible cube rolls is $6 \times 6 = 36$.

Make a list:

Sum of 2 can happen 1 way: 1 + 1.
Sum of 3 can happen 2 ways: 1 + 2 or 2 + 1.
Sum of 5 can happen 4 ways: 1 + 4, 2 + 3, 3 + 2, 4 + 1.
Sum of 7 can happen 6 ways: 1 + 6, 2 + 5, 3 + 4, 4 + 3, 5 + 2, 6 + 1.

Sum of 11 can happen 2 ways: 5 + 6, 6 + 5.

That's a total of 1 + 2 + 4 + 6 + 2 = 15 ways to roll a prime sum.

Thus, the probability is $\frac{15}{36} = \frac{5}{12}$.

34. **(B).** The probability of any event equals the number of ways to get the desired outcome divided by the total number of outcomes.

Start with the denominator, which is the total number of ways that the principal can choose two children from the classroom. Use the fundamental counting principle. There are 6 possible options for the first choice and 5 for the second, giving (6)(5) = 30 possibilities. However, this double-counts some cases; for example, choosing Jan and then Robert is the same as choosing Robert and then Jan. Divide the total number of pairs by 2: $\frac{(6)(5)}{2}$ = 1. Alternatively, use the formula for a set in which the order doesn't matter: $\frac{\text{total!}}{\text{in! out!}}$. In this case: $\frac{6!}{2!4!} = \frac{6 \times 5 \times 4!}{(2)(4!)} = \frac{6 \times 5}{2} = 15$.

Now compute the numerator, which is the number of pairs that include Jan. Since the pair only includes two children and one is already decided (Jan), there are exactly 5 options for the other child. Thus, there are 5 total pairs that include Jan: Jan with each of the other students.

The probability of choosing a pair with Jan is $\frac{5}{15} = \frac{1}{3}$.

Alternatively, you can calculate the probability of not choosing Jan and use the $1 - x$ shortcut. The probability of not choosing Jan as the first student is $\frac{5}{6}$, and the probability of choosing again and not choosing Jan is $\frac{4}{5}$ so $1 - \left(\frac{5}{6} \times \frac{4}{5}\right) = 1 - \frac{20}{30} = \frac{10}{30} = \frac{1}{3}$.

As a final alternative, list all the pairs of students and count how many of them include Jan. Label the students in the class as J, 1, 2, 3, 4, and 5, where J is Jan. Then all the pairs can be listed as (J1), (J2), (J3), (J4), (J5), (12), (13), (14), (15), (23), (24), (25), (34), (35), and (45). (Be careful not to include repeats.) There are 15 total elements in this list and 5 that include Jan, yielding a probability of $\frac{5}{15} = \frac{1}{3}$.

35. **(B).** The problem indicates that the events occur independently of each other. Therefore, in calculating Quantity A, the first step is to calculate the "or" situation, but don't stop there. Adding 0.5 + 0.3 = 0.8 double counts the occurrences when both events occur. To compensate, subtract out the

probability of both events occurring in order to get rid of the “double counted” occurrences.

Notice that this is a Quantitative Comparison. Because the 0.8 figure includes at least one “both” occurrence, the real figure for Quantity A must be less than 0.8. Therefore, Quantity B must be greater.

To do the actual math, find the probability of both events occurring (breakfast *and* sweater): (0.5)(0.3) = 0.15. Subtract the “and” occurrences from the total “or” probability: 0.8 – 0.15 = 0.65.

Thus, Quantity B is greater.

36. **(B).** The problem indicates that the events occur independently of each other. Therefore, in calculating Quantity A, do not just add both events, even though it is an "or" situation. Adding 0.3 + 0.2 = 0.5 is incorrect because the probability that both events occur is counted twice. (Only add probabilities in an "or" situation when the probabilities are mutually exclusive.)

While Quantity A's value should include the probability that both events occur, make sure to count this probability only once, not twice. Since the probability that both events occur is 0.3(0.2) = 0.06, subtract this value from the "or" probability.

Quantity A: Add the two probabilities (rain *or* pop quiz) and subtract *both* scenarios (rain *and* pop quiz):

$$0.3 + 0.2 - (0.3)(0.2) = 0.44$$

Quantity B: Multiply the probability that rain does *not* occur (0.7) and the probability that the pop quiz does *not* occur (0.8):

$$0.7(0.8) = 0.56$$

Alternatively, note that the two quantities, collectively, include every possibility and are mutually exclusive of one another (Quantity A includes "rain and no quiz," "quiz and no rain," and "both rain and quiz," and Quantity B includes "no rain and no quiz"). Therefore, the values of Quantities A and B must sum to 1. Calculating the value of either Quantity A or Quantity B would automatically indicate the value for the other quantity.

If you do this, calculate Quantity B first (because it's the easier of the two quantities to calculate) and then subtract Quantity B from 1 in order to get Quantity A's value. That is, 1 – 0.56 = 0.44.

37. **(D).** In essence, the question is asking, "What is the probability that one or more days are rainy days?" since any single rainy day would mean the city experiences rain. In this case, employ the $1 - x$ shortcut, where the probability of rain on one or more days is equal to 1 minus the probability of no rain on any day. Since the probability of rain is $\frac{1}{3}$ on any given day, the probability of no rain on any given day is $1 - \frac{1}{3} = \frac{2}{3}$. Therefore, the probability of no rain on three consecutive days is $\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right) = \frac{8}{27}$. Finally, subtract from 1 to find the probability that it rains on one or more days: P(1 or more days) = 1 – P(no rain) = $1 - \frac{8}{27} = \frac{19}{27}$.

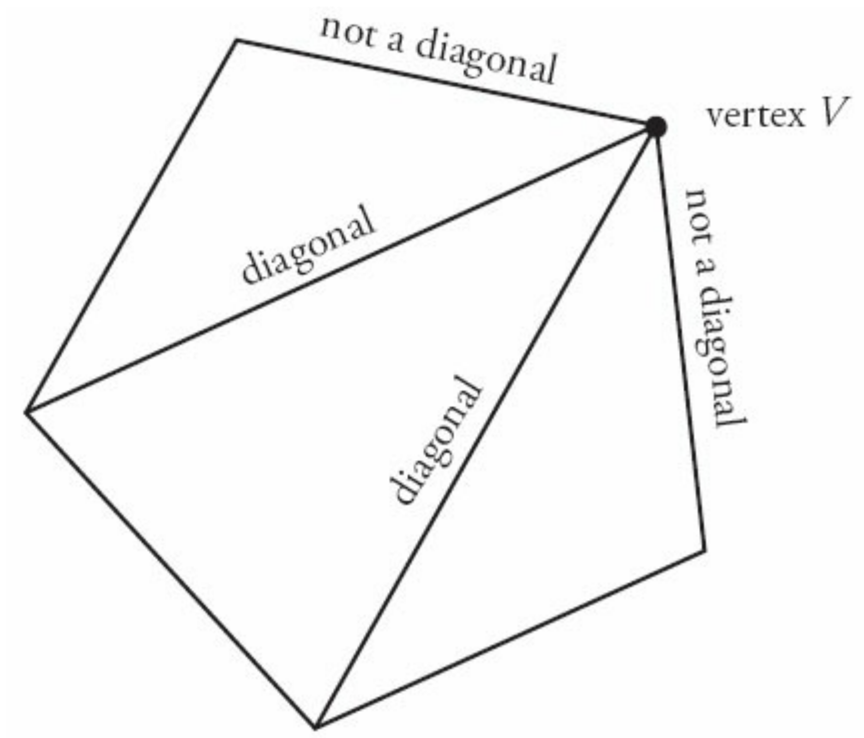
38. **(C).** The number of ways in which the students can be arranged with Beth and Dan separated is equal to the total number of ways in which the students can be arranged minus the number of ways they can be arranged with Beth and Dan together. The total number of ways to arrange 5 students in a line is 5! = 120. To compute the number of ways to arrange the 5 students such that Beth and Dan are

together, group Beth and Dan as “one” person, since they must be lined up together. Then the problem becomes one of lining up 4 students, which gives 4! possibilities. However, remember that there are actually two options for the Beth and Dan arrangement: Beth first and then Dan or Dan first and then Beth. Therefore, there are (4!)(2) = (4)(3)(2)(1)(2) = 48 total ways in which the students can be lined up with Dan and Beth together. Finally, there are 120 – 48 = 72 arrangements in which Beth is separated from Dan.

Alternatively, compute the number of ways to arrange the students directly by considering individual cases. In this case, investigate how many ways there are to arrange the students if Beth occupies each spot in line and sum them to find the total. If Beth is standing in the first spot in line, then there are 3 options for the second spot (since Dan cannot occupy this position), 3 options for the next spot, 2 options for the next spot, and finally 1 option for the last spot. This yields (3)(3)(2)(1) = 18 total possibilities if Beth is first. If Beth is second, then there are 3 options for the first person (Dan cannot be this person), 2 options for the third person (Dan cannot be this person either), 2 options for the fourth person, and 1 option for the fifth. This yields (3)(2)(2)(1) = 12 possibilities. In fact, if Beth is third or fourth in line, the situation is the same as when Beth is second. Thus, there are 12 possible arrangements whether Beth is 2nd, 3rd, or 4th in line, yielding 36 total arrangements for these 3 cases. Using similar logic, the situation in which Beth is last in line is exactly equal to the situation in which she is first in line. Thus, there are (18)(2) = 36 possibilities in which Beth is first or last. In total, this yields 36 + 36 = 72 possible outcomes when considering all of the possible placements for Beth.

39. **(A).** A diagonal of a polygon is an internal line segment connecting any two unique vertices; this line segment does not lie along an edge of the given shape. Consider a polygon with 12 vertices. Construct a diagonal by choosing any two vertices and connecting them with a line. Remember that this is order independent; the line is the same regardless of which is the starting vertex. Therefore, this is analogous to choosing any 2 elements from a set of 12, and can be written as $\frac{12!}{10!2!} = \frac{12 \times 11 \times 10!}{10!(2)(1)} = \frac{12 \times 11}{2}$ = 6 × 11 = 66. However, this method includes the vertices connected to their adjacent vertices, which form edges instead of diagonals. In order to account for this, subtract the number of edges on the polygon from the above number: 66 – 12 = 54.

Alternatively, choose a random vertex of the 12-sided shape. There are 12 – 1 = 11 lines that can be drawn to other vertices since no line can be drawn from the vertex to itself. However, the lines from this vertex to the two adjacent vertices will lie along the edges of the polygon and therefore cannot be included as diagonals (see the figure of a pentagon below for an example):



Thus, there are $12 - 1 - 2 = 9$ diagonals for any given vertex. Since there are 12 vertices, it is tempting to think that the total number of diagonals is equal to $(12)(9) = 108$. However, this scheme counts each diagonal twice, using each side of the diagonal once as the starting point. Therefore, there are half this many different diagonals: $\frac{108}{2} = 54$.

40. **(C).** This is a classic combinatorics problem in which *order doesn't matter*—that is, picking Javier and Sonya is the same as picking Sonya and Javier. A person is either on the team or not. Use the standard "order doesn't matter" formula:

$$\frac{\text{total!}}{\text{in!out!}}$$

For Quantity A:

$$\frac{6!}{2!4!} = \frac{6 \times 5 \times 4!}{(2)(1)4!} = \frac{6 \times 5}{2} = 15$$

For Quantity B:

$$\frac{6!}{4!2!} = \frac{6 \times 5 \times 4!}{4!(2)(1)} = \frac{6 \times 5}{2} = 15$$

The two quantities are equal. Note that it is not actually necessary to reduce each quantity. The factorials are the same in each, so the resulting quantities must be equal.

This will always work—when order doesn't matter, the number of ways to pick 4 and leave out 2 is the same as the number of ways to pick 2 and leave out 4. Either way, it's one group of 4 and one group of 2. What actually happens to those groups (getting picked, not getting picked, getting a prize, losing a contest, etc.) is irrelevant to the ultimate solution.

41. **(C).** In this problem, order matters; if Mari comes in 1st place and Rohit comes in 2nd, there is a different outcome than when Rohit places 1st and Mari places 2nd. Use the fundamental counting principle to solve. To determine Quantity A, make three slots (one for each prize). Six people are available to win 1st, and then five people could win 2nd, and four people could win 3rd:

$$\underline{\;6\;} \quad \underline{\;5\;} \quad \underline{\;4\;}$$

Multiply: (6)(5)(4) = 120.

For Quantity B, make 5 slots, one for each prize. Five people can win 1st prize, then 4 people can win 2nd prize, and so on:

$$\underline{\;5\;} \quad \underline{\;4\;} \quad \underline{\;3\;} \quad \underline{\;2\;} \quad \underline{\;1\;}$$

Multiply (5)(4)(3)(2)(1) = 120. The two quantities are equal.

42. **(C).** This is a classic combinatorics problem in which *order doesn't matter*—in fact, the problem states that explicitly. Use the standard "order doesn't matter" formula:

$$\frac{\text{total!}}{\text{in! out!}}$$

For Quantity A:

$$\frac{100!}{56!44!}$$

Because the numbers are so large, there must be a way to solve the problem without actually simplifying (even with a calculator, this is unreasonable under GRE time limits). Try Quantity B and compare:

$$\frac{100!}{44!56!}$$

The quantities are equal. Note that this will always work—when order doesn't matter, the number of ways to pick 56 and leave out 44 is the same as the number of ways to pick 44 and leave out 56. Either way, it's one group of 56 and one group of 44. What actually happens to those groups (being part of a collection, being left out of the collection, etc.) is irrelevant to the ultimate solution.

43. **(A).** Use the overlapping sets formula for two groups: Total = Group 1 + Group 2 – Both + Neither. But first, add 740 ("business use" *only*) + 520 ("business use" and "personal use") to get 1,260, the total number of products categorized as "business use."

Also note that the problem indicates that *all* of the products fall into one or both of the two categories, so "neither" in this formula is equal to zero:

$$\begin{aligned} \text{Total} &= \text{Business} + \text{Personal} - \text{Both} + \text{Neither} \\ 1{,}345 &= 1{,}260 + P - 520 + 0 \\ 1{,}345 &= 740 + P \\ 605 &= P \end{aligned}$$

Quantity A is greater. Note that the question asked for the number of products characterized as "personal use" (which includes products in the "both" group). If the problem had asked for the number of products characterized as "personal use" *only*, you would have had to subtract the "both" group to get 605 – 520 = 85. In this problem, however, Quantity A equals 605.

44. **(E).** Because this is an "at least" question, use the $1 - x$ shortcut:

(The probability of picking at least one man) + (The probability of picking no men) = 1

The probability of picking no men is an *and* setup: woman *and* woman *and* woman.

For the first choice, there are 8 women out of 10 people: $\frac{8}{10} = \frac{4}{5}$.

For the second choice, there are $\frac{7}{9}$ (because one woman has already been chosen).

For the third choice, there are $\frac{6}{8} = \frac{3}{4}$.

Multiply the three probabilities together to find the probability that the committee will be comprised of woman *and* woman *and* woman:

$$\frac{4}{5}\times\frac{7}{9}\times\frac{3}{4}=\frac{1}{5}\times\frac{7}{3}\times\frac{1}{1}=\frac{7}{15}$$

To determine the probability of picking at least one man, subtract this result from 1:

$$1-\frac{7}{15}=\frac{8}{15}$$

45. **(A).** This overlapping sets question can be solved with the following equation:

Total # of people = Group 1 + Group 2 + Group 3 – (# of people in two groups) – (2)(# of people in all three groups) + (# of people in no groups)

The problem indicates that everyone studies at least one language, so the number of people in no groups is zero. The problem also indicates that nobody studies all three languages, so that value is also zero:

Total # of students = 100 + 80 + 40 – 22 – (2)(0) + 0 = 198.

46. **(A).** It is not possible to solve for a single value for Quantity A, but it is possible to tell that Quantity A will always be greater than 16. Since 20 birds are songbirds and 23 are migratory, the total of these groups is 43, which is less than 60. It is possible for the overlap (the number of migratory songbirds) to be as little as 0, which would result in 20 songbirds, 23 non-songbird migratory birds, and 60 – 20 – 23 = 17 birds that are neither songbirds nor migratory.

It is also possible that there could be as many as 20 birds that overlap the two categories. (Find this figure by taking the number of birds in the smaller group; in this case, there are 20 songbirds). In the case that there are 20 migratory songbirds, there would also be 3 migratory birds that are not songbirds, in which case there would be 60 – 20 – 3 = 37 birds that are neither migratory nor songbirds.

Thus, the number of birds that are neither migratory nor songbirds is at least 17 and at most 37. No matter where in the range that number may be, it is greater than Quantity B, which is only 16.