INEQUALITIES

Earlier we explored how to solve equations. Now let's look at how we can solve *inequalities.*

Inequalities are expressions that use <, >, ≤ or ≥ to describe the relationship between two values.

Examples of inequalities:

$5 > 4$ $\qquad$ $y \le 7$ $\qquad$ $x < 5$ $\qquad$ $2x + 3 \ge 0$

The table below illustrates how the various inequality symbols are translated. Notice that when we translate these inequalities, we read from left to right.

$x < y$	x is less than y	
$x > y$	x is greater than y	
$x \le y$	x is less than or equal to y	x is at most y
$x \ge y$	x is greater than or equal to y	x is at least y

We can also have two inequalities in one statement (sometimes called **compound inequalities**):

$9 < g < 200$	9 is less than g, and g is less than 200
$-3 < y \le 5$	-3 is less than y, and y is less than or equal to 5
$7 \ge x > 2$	7 is greater than or equal to x, and x is greater than 2

To visualize an inequality, it is helpful to represent it on a number line:

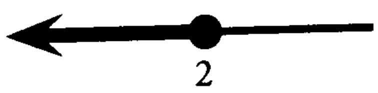
Example 1

$y > 5$

Note: 5 is *not* included in the line (as shown by the circle *around* 5), because it is not a part of the solution—y is greater than 5, but not equal to 5.

Example 2

$b \le 2$



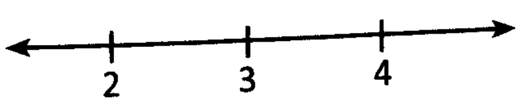
Here, 2 is included in the solution, (as shown by the solid circle at 2) because b can equal 2.

Visually, any number covered by the black arrow will make the inequality true and so is a possible solution to the inequality. Conversely, any number not covered by the black arrow will make the inequality untrue and is not a solution.

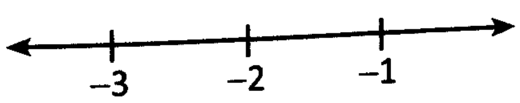
Check Your Skills

Represent the following equations on the number line provided:

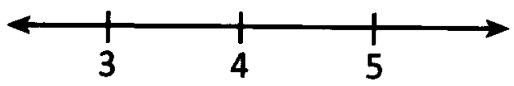
1. $x > 3$



2. $b \ge -2$



3. $y = 4$



Translate the following into inequality statements:

4. z is greater than v.
5. The total amount is greater than \$2,000.

Answers can be found on page 95.

Solving Inequalities

What does it mean to "solve an inequality"?

You may be asking yourself, "I know what it means to solve an equation (such as $x = 2$), but what does it mean to solve an inequality?" Essentially, the principle is the same.

A solution is a number that makes an equation or inequality true. When you plug a solution back into the original equation or inequality, you get a ***true statement.*** This idea works the same for both equations and inequalities.

However, equations have only one, or just a few, values as solutions, but inequalities give a whole *range* of values as solutions—way too many to list individually.

Here's an example to help illustrate:

Equation: $x + 3 = 8$

The solution to $x + 3 = 8$ is $x = 5$.
5 is the **only** number that will make the equation true.

Plug back in to check:
$5 + 3 = 8$. True.

Inequality: $x + 3 < 8$

The solution to $x + 3 < 8$ is $x < 5$. Now, 5 itself is not a solution because $5 + 3 < 8$ is not a true statement. But, 4 is a solution because $4 + 3 < 8$ is true. For that matter, 4.99, 3, 2, 2.87, −5, and −100 are all also solutions. And the list goes on. Whichever of the correct answers you plug in, you need to arrive at something that looks like:

(Any number less than 5) + 3 < 8. True.

Check Your Skills

6. Which of the following numbers are solutions to the inequality $x < 10$?

Indicate *all* that apply.

[A] −3
[B] 2.5
[C] −3/2
[D] 9.999

Answer can be found on page 95.

Cleaning Up Inequalities

As with equations, our objective is to isolate our variable on one side of the inequality. When the variable is by itself, it is easiest to see what the solution (or range of solutions) really is. Although $2x + 6 < 12$ and $x < 3$ provide the same information (the second inequality is a simplified form of the first), we understand the full range of solutions much more easily when we look at the second inequality, which literally tells us that "x is less than 3."

Fortunately, the similarities between equations and inequalities don't end there—the techniques we will be using to clean up inequalities are the same that we used to clean up equations. (We will discuss one important difference shortly.)

Inequality Addition and Subtraction

If we told you that $x = 5$, what would $x + 3$ equal? $x + 3 = (5) + 3$, or $x + 3 = 8$. In other words, if we add the same number to both sides of an equation, the equation is still true.

The same holds true for inequalities. If we add or subtract the same number from both sides of an inequality, the inequality remains true.

Example 1

$$\begin{array}{rl} a - 4 & > 6 \\ +4 & +4 \\ \hline a & > 10 \end{array}$$

Example 2

$$\begin{array}{rl} y + 7 & < 3 \\ -7 & -7 \\ \hline y & < -4 \end{array}$$

We can also add or subtract variables from both sides of an inequality. There is no difference between adding/subtracting numbers and adding/subtracting variables.

$$\begin{array}{rl} 3 - y & > 0 \\ +y & +y \\ \hline 3 & > y \end{array}$$

Check Your Skills

Isolate the variable in the following inequalities.

7. $x - 6 < 13$
8. $y + 11 \geq -13$
9. $x + 7 > 7$

Answers can be found on page 95.

Inequality Multiplication and Division

We can also use multiplication and division to isolate our variables, *as long as we recognize one very important distinction.* If we multiply or divide by a negative number, we must ***switch the direction of the inequality sign.*** If we are multiplying or dividing by a positive number, the direction of the sign stays the same.

Let's look at a couple of examples to illustrate.

Multiplying or dividing by a POSITIVE number—the sign stays the same.

Example 1
$2x > 10$
$2x/2 > 10/2$ Divide each side by 2
$x > 5$

Example 2
$z/3 \leq 2$
$z/3 \times (3) \leq 2 \times (3)$ Multiply each side by 3
$z \leq 6$

In both instances, the sign remains the same because we are multiplying or dividing by a positive number.

Multiplying or Dividing by a NEGATIVE Number—Switch the Sign!

Example 1
$-2x > 10$
$-2x/-2 > 10/-2$ Divide each side by -2
Switch the sign!
$x < -5$

Example 2
$-4b \geq -8$
$-4b/-4 \geq -8/-4$ Divide each side by -4
Flip the inequality sign!
$b \leq 2$

Why do we do this? Take a look at the following example that illustrates why we need to switch the signs when multiplying or dividing by a negative number.

Incorrect if you DON'T switch		**Switch the sign—Correct!**	
$5 < 7$	TRUE	$5 < 7$	TRUE
$(-1) \times 5 < (-1) \times 7$	Multiply both sides by -1	$(-1) \times 5 < (-1) \times 7$	Multiply both sides by -1 AND switch the sign!
$-5 < -7$?!?	NOT TRUE!	$-5 > -7$	STILL TRUE

In each case, we begin with a true inequality statement: $5 < 7$ and then multiply by -1. We see that we have to switch the sign in order for the inequality statement to remain true.

What about multiplying or dividing an inequality by a *variable?* The short answer is… **try not to do it**! The issue is that you don't know the sign of the "hidden number" that the variable represents. If the variable has to be positive (e.g., it counts people or measures a length), then you can go ahead and multiply or divide.

If the variable must be negative, then you are also free to multiply or divide—just remember to flip the sign. However, if you don't know whether the variable is positive or negative, try to work through the problem with the inequality as-is. (If the problem is a Quantitative Comparison, consider whether not knowing the sign of the variable you want to multiply or divide by means that the answer is D!)

Check Your Skills

Isolate the variable in each equation.

10. $x + 3 \geq -2$
11. $-2y < 8$
12. $a + 4 \geq 2a$

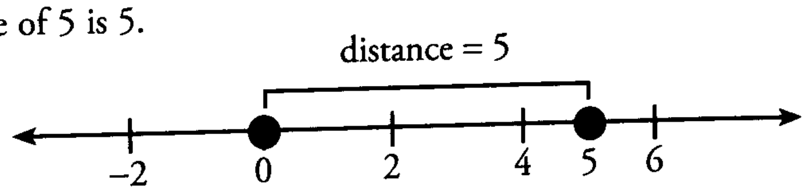
Answers can be found on page 95.

Absolute Value—Distance on the Number Line

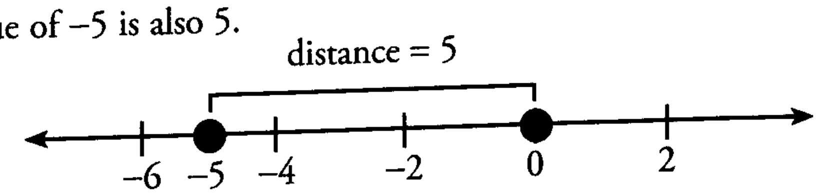
The GRE adds another level of difficulty to equations and inequalities in the form of *absolute value.*

The "absolute value" of a number describes how far that number is away from 0. It is the distance between that number and 0 on a number line. The symbol for absolute value is |number|. For instance, we would write the absolute value of –5 as |–5|.

Example 1: The absolute value of 5 is 5.



Example 2: The absolute value of –5 is also 5.



When you face an expression like |4 – 7|, treat the absolute value symbol like parentheses. Solve the arithmetic problem inside first, and then find the absolute value of the answer. In this case, 4 – 7 = –3, and –3 is 3 units from zero, so |4 – 7| = |–3| = 3.

Check Your Skills

Mark the following expressions as TRUE or FALSE.

13. $|3| = 3$
14. $|-3| = -3$
15. $|3| = -3$
16. $|-3| = 3$
17. $|3 - 6| = 3$
18. $|6 - 3| = -3$

Answers can be found on page 95.

Solving Absolute Value Equations

On the GRE, some absolute value equations place a variable inside the absolute value signs.

Example: $|y| = 3$

What's the trap here? The trap is that there are two numbers, 3 and −3, that are 3 units away from 0. That means both of these numbers could be possible values for y. So how do we figure that out? Here, we can't. All we can say is that y could be either the positive value or the negative value; y is *either* 3 or −3.

When there is a variable inside an absolute value, you should look for the variable to have two possible values. Although you will not always be able to determine which of the two is the correct value, it is important to be able to find both values. Next we will go through a step-by-step process for finding all solutions to an equation that contains a variable inside an absolute value.

$\lvert y\rvert = 3$	Step 1: Isolate the absolute value expression on one side of the equation. In this case, the absolute value expression is already isolated.
$+(y) = 3$ or $-(y) = 3$	Step 2: Take what's inside the absolute value sign and set up two equations. The first sets the positive value equal to the other side of the equation, and the second sets the negative value equal to the other side.
$y = 3$ or $-y = 3$ $y = 3$ or $y = -3$	Step 3: Solve both equations. Note: We have two possible values for y.

Sometimes people take a shortcut and go right to "y equals plus or minus 3." This shortcut works as long as the absolute value expression is by itself on one side of the equation.

Here's a slightly more difficult problem, using the same technique:

Example: $6 \times |2x + 4| = 30$

To solve this, you can use the same approach.

$6 \times \lvert 2x + 4\rvert = 30$ $\lvert 2x + 4\rvert = 5$	Step 1: Isolate the absolute value expression on one side of the equation or inequality.
$(2x + 4) = 5$ or $-(2x + 4) = 5$ $2x + 4 = 5$ or $-2x - 4 = 5$	Step 2: Set up two equations—the positive and the negative values are set equal to the other side.
$2x = 1$ or $-2x = 9$	Step 3: Solve both equations/inequalities.
$x = 1/2$ or $x = -9/2$	Note: We have two possible values for x.

Check Your Skills

Solve the following equations with absolute values in them:

19. $|a| = 6$
20. $|x + 2| = 5$
21. $|3y - 4| = 17$
22. $4|x + 1/2| = 18$

Answers can be found on page 96.

Putting Them Together: Inequalities and Absolute Values

Some problems on the GRE include both inequalities and absolute values. We can solve these problems by combining what we have learned about solving inequalities with what we have learned about solving absolute values.

Example 1: $|x| \geq 4$

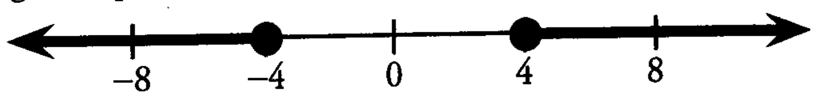
Even though we're now dealing with an inequality, and not an equals sign, the basic process is the same. The absolute value is already isolated on one side, so now we need to set up our two equations or, in this case, inequalities. The first inequality replaces the absolute value with the *positive* of what's inside, and the second replaces the absolute value with the *negative* of what's inside.

$+ (x) \geq 4$ or $- (x) \geq 4$

Now that we have our two equations, we isolate the variable in each equation.

$+ (x) \geq 4$	$- (x) \geq 4$	
$x \geq 4$	$-x \geq 4$	Divide by -1
	$x \leq -4$	Remember to flip the sign when dividing by a negative

So the two solutions to the original equation are $x \geq 4$ and $x \leq -4$. Let's represent that on a number line.



As before, any number that is covered by the black arrow will make the inequality true. Because of the absolute value, there are now two arrows instead of one, but nothing else has changed. Any number to the left of −4 will make the inequality true, as will any number to the right of 4.

Looking back at the inequality $|x| \geq 4$, we can now interpret it in terms of distance. $|x| \geq 4$ means "x is at least 4 units away from zero, in either direction." The black arrows indicate all numbers for which that statement is true.

Example 2: $|x + 3| < 5$

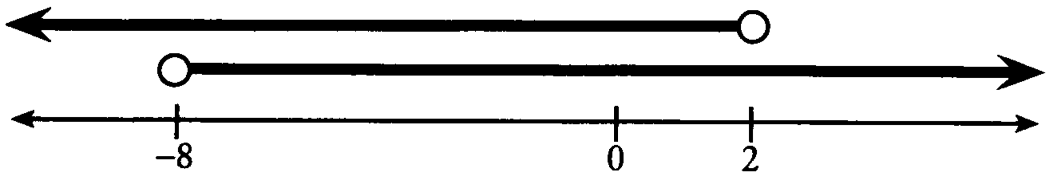
Once again, the absolute value is already isolated on one side, so now we need to set up our two equations. The first inequality replaces the absolute value with the *positive* of what's inside, and the second replaces the absolute value with the *negative* of what's inside.

$+ (x + 3) < 5$ and $- (x + 3) < 5$

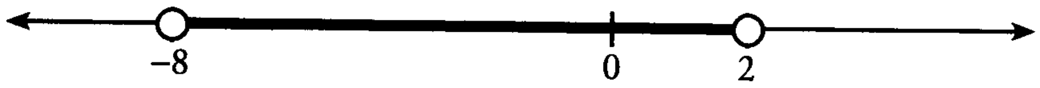
Next we isolate the variable in each equation.

$x + 3 < 5$	$-x - 3 < 5$
$x < 2$	$-x < 8$
	$x > -8$

So our two equations are $x < 2$ and $x > -8$. But now something curious happens if we plot those two equations on our number line.



It seems like every number should be a solution to the equation. But if you start testing numbers, that isn't the case. Test out the number 5 for example. Is $|5 + 3| < 5$? No, it isn't. As it turns out, the only numbers that make the original inequality true are those that are true for **both** inequalities. Really, our number line should look like this:



In our first example, it was the case that x could be greater than or equal to 4 OR less than or equal to –4. For this example, however, it seems to make more sense to say that x is greater than –8 AND less than 2.

The inequality we just graphed means "$(x + 3)$ is less than 5 units away from from zero, in either direction." The shaded segment indicates all numbers x for which this is true. As the inequalities become more complicated, don't worry about interpreting their meaning—simply solve them algebraically!

To summarize, when representing inequalities on the number line, absolute value expressions that are *greater than some quantity* will show up *as two ranges in opposite directions* (or "double arrows"); however, absolute value expressions that are *less than some quantity* will show up *as a single range* (or "line segment").

Check Your Skills

23. $|x + 1| > 2$
24. $|-x - 4| \geq 8$
25. $|x - 7| < 9$

Answers can be found on page 96.

Manipulating Compound Inequalities

Sometimes a problem with compound inequalities will require you to manipulate the inequalities in order to solve the problem. You can perform operations on a compound inequality as long as you remember to perform those operations on **every term** in the inequality, not just the outside terms. For example:

$x + 3 < y < x + 5 \xrightarrow{\times} x < y < x + 2$ **WRONG**: you must subtract 3 from EVERY term in the inequality

$x + 3 < y < x + 5 \xrightarrow{\checkmark} x < y - 3 < x + 2$ CORRECT

$\frac{c}{2} \leq b - 3 \leq \frac{d}{2} \xrightarrow{\times} \leq c \leq b - 3 \leq d$ **WRONG**: you must multiply by 2 in EVERY term in the inequality

$\frac{c}{2} \leq b - 3 \leq \frac{d}{2} \xrightarrow{\checkmark} \leq c \leq 2b - 6 \leq d$ CORRECT

If $1 > 1 - ab > 0$, which of the following must be true?

Indicate *all* that apply.

[A] $\frac{a}{b} > 0$

[B] $\frac{a}{b} < 1$

[C] $ab < 1$

We can manipulate the original compound inequality as follows, making sure to perform each manipulation on every term:

$1 > 1 - ab > 0$	
$0 > \quad -ab > -1$	Subtract 1 from all three terms
$0 < \quad ab < 1$	Multiply all three terms by -1 and flip the inequality signs

Therefore we know that $0 < ab < 1$. This tells us that ab is positive, so $\frac{a}{b}$ must be positive (a and b have the same sign). Therefore, A must be true. However, we do not know whether $\frac{a}{b} < 1$, so B is not necessarily true. But we do know that ab must be less than 1, so C must be true. Therefore, the correct answers are **A and C**.

Check Your Skills

26. Find the range of values for x if $-7 < 3 - 2x < 9$.

Answer can be found on page 97.

Using Extreme Values

One effective technique for solving GRE inequality problems is to focus on the EXTREME VALUES of a given inequality. This is particularly helpful when solving the following types of inequality problems:

(1) Problems with multiple inequalities where the question involves the potential range of values for variables in the problem
(2) Problems involving both equations and inequalities

INEQUALITIES WITH RANGES

Whenever a question asks about the possible range of values for a problem, consider using extreme values:

> Given that $0 \leq x \leq 3$, and $y < 8$, which of the following could NOT be the value of xy?
>
> (A) 0 (B) 8 (C) 12 (D) 16 (E) 24

To solve this problem, consider the EXTREME VALUES of each variable.

Extreme Values for x	Extreme Values for y
The lowest value for x is **0**.	The lowest value for y is negative infinity.
The highest value for x is **3**.	The highest value for y is **less than 8**.

(Since y cannot be 8, we term this upper limit "less than 8" or "LT8" for shorthand.)

What is the lowest value for xy? Plug in the lowest values for both x and y. In this problem, y has no lower limit, so there is no lower limit to xy.

What is the highest value for xy? Plug in the highest values for both x and y. In this problem, the highest value for x is **3**, and the highest value for y is **LT8**.

Multiplying these two extremes together yields: $3 \times \text{LT}8 = \text{LT}24$. Notice that we can multiply LT8 by another number (as long as that other number is positive) just as though it were 8. We just have to remember to include the "LT" tag on the result.

Because the upper extreme for xy is less than 24, xy CANNOT be 24, and the answer is (E).

Notice that we would run into trouble if x did not have to be non-negative. Consider this slight variation:

> Given that $-1 \leq x \leq 3$, and $y < 8$, what is the possible range of values for xy?

Because x could be negative and because y could be a large negative number, there is no longer an upper extreme on xy. For example, if $x = -1$ and $y = -1{,}000$, then $xy = 1{,}000$. Obviously, much larger results are possible for xy if both x and y are negative. Therefore, xy can equal any number.

Check Your Skills

27. If $-4 < a < 4$ and $-2 < b < -1$, which of the following could NOT be the value of ab?

(A) -3
(B) 0
(C) 4
(D) 6
(E) 9

Answer can be found on page 97.

Optimization Problems

Related to extreme values are problems involving optimization: specifically, minimization or maximization problems. In these problems, you need to **focus on the largest and smallest possible values for each of the variables**, as some combination of them will usually lead to the largest or smallest possible result.

If $-7 \leq a \leq 6$ and $-7 \leq b \leq 8$, what is the maximum possible value for ab?

Once again, we are looking for a maximum possible value, this time for ab. We need to test the extreme values for a and for b to determine which combinations of extreme values will maximize ab:

Extreme Values for a	Extreme Values for b
The lowest value for a is -7.	The lowest value for b is -7.
The highest value for a is 6.	The highest value for b is 8.

Now let us consider the different extreme value scenarios for a, b, and ab:

a		***b***		***ab***
Min	-7	Min	-7	$(-7) \times (-7) = \mathbf{49}$
Min	-7	Max	8	$(-7) \times 8 = -56$
Max	6	Min	-7	$6 \times (-7) = -42$
Max	6	Max	8	$6 \times 8 = 48$

This time, ab is maximized when we take the NEGATIVE extreme values for both a and b, resulting in $ab = 49$. Notice that we could have focused right away on the first and fourth scenarios, because they are the only scenarios which produce positive products.

If $-4 \leq m \leq 7$ and $-3 < n < 10$, what is the maximum possible integer value for $m - n$?

Again, we are looking for a maximum possible value, this time for $m - n$. We need to test the extreme values for m and for n to determine which combinations of extreme values will maximize $m - n$:

Extreme Values for m	Extreme Values for n
The lowest value for m is -4.	The lowest value for n is greater than -3.
The highest value for m is 7.	The highest value for n is less than 10.

Now let us consider the different extreme value scenarios for m, n, and $m - n$:

m		n		$m - n$
Min	-4	Min	GT(-3)	(-4) – GT(-3) = LT(-1)
Min	-4	Max	LT10	(-4) – LT10 = GT(-14)
Max	7	Min	**GT(-3)**	**7 – GT(-3) = LT10**
Max	7	Max	LT10	7 – LT10 = GT(-3)

$m - n$ is maximized when we take the POSITIVE extreme for m and the NEGATIVE extreme for n, resulting in $m - n$ = LESS THAN 10. The largest integer less than 10 is 9, so the correct answer is $\boldsymbol{m - n = 9}$. Let's look at another, slightly different, problem.

If $x \geq 4 + (z + 1)^2$, what is the minimum possible value for x?

The key to this type of problem—where we need to maximize or minimize when one of the variables has an even exponent—is to recognize that the squared term will be minimized when it is set equal to zero. Therefore, we need to set $(z + 1)^2$ equal to 0:

$$(z + 1)^2 = 0$$
$$z + 1 = 0$$
$$z = -1$$

The minimum possible value for x occurs when $z = -1$, and $x \geq 4 + (-1 + 1)^2 = 4 + 0 = 4$. Therefore $\boldsymbol{x \geq 4}$, so 4 is the minimum possible value for x.

Check Your Skills

28. If $-1 \leq a \leq 4$ and $-6 \leq b \leq -2$, what is the minimum value for $b - a$?
29. If $(x + 2)^2 \leq 2 - y$, what is the maximum possible value for y?

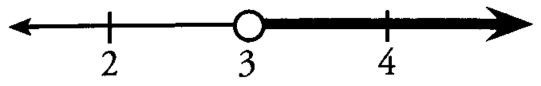
Answers can be found on page 97.

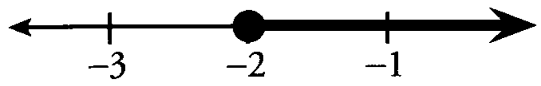
Summary of Inequality Techniques

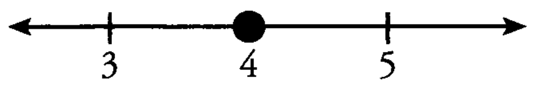
We have covered many topics in inequalities. Here is a quick recap of "DOs and DON'Ts" when working with inequalities:

DOs	DON'Ts
• DO think about inequalities as ranges on a number line. • DO treat inequalities like equations when adding or subtracting terms, or when multiplying/dividing by a positive number on both sides of the inequality. • DO use extreme values to solve inequality range problems, problems containing both inequalities and equations, and many optimization problems. • DO set terms with even exponents equal to zero when trying to solve minimization problems.	• DON'T forget to flip the inequality sign if you multiply or divide both sides of an inequality by a negative number. • DON'T multiply or divide an inequality by a variable unless you know the sign of the variable. • DON'T forget to perform operations on every expression when manipulating a compound inequality.

Check Your Skills Answer Key:

1. 

2. 

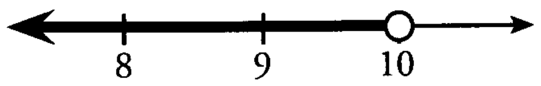
3. 

4. $\boldsymbol{z > v}$

5. Let a = amount.

   $\boldsymbol{a > \$2,000}$

6. **A, B, C, D:** All of these numbers are to the left of 10 on the number line.



7. $x - 6 < 13$

   $\boldsymbol{x < 19}$

8. $y + 11 \geq -13$

   $\boldsymbol{y \geq -24}$

9. $x + 7 > 7$

   $\boldsymbol{x > 0}$

10. $x + 3 \geq -2$

    $\boldsymbol{x \geq -5}$

11. $-2y < 8$

    $\boldsymbol{y > -4}$

12. $a + 4 \geq 2a$

    $\boldsymbol{4 \geq a}$

13. **True**

14. **False**— *(Note that absolute value is always positive!)*

15. **False**

16. **True**

17. **True** ($|3 - 6| = |-3| = 3$)

18. **False**

19. $|a| = 6$

$\boldsymbol{a = 6}$ or $\boldsymbol{a = -6}$

20. $\boldsymbol{x = 3}$ **or** $\boldsymbol{-7}$**:** $|x + 2| = 5$

$+ \quad (x + 2) = 5$ or $-(x + 2) = 5$

$x + 2 = 5$ or $-x - 2 = 5$

$x = 3$ or $-x = 7$

$x = -7$

21. $\boldsymbol{y = 7}$ **or** $\boldsymbol{-13/3}$**:** $|3y - 4| = 17$

$+ \quad (3y - 4) = 17$ or $-(3y - 4) = 17$

$3y - 4 = 17$ or $-3y + 4 = 17$

$3y = 21$ or $-3y = 13$

$y = 7$ or $y = -13/3$

22. $\boldsymbol{x = 4}$ **or** $\boldsymbol{-5}$**:** or $4|x + 1/2| = 18$

$+ \quad (x + 1/2) = 4\ 1/2$ or $-(x + 1/2) = 4\ 1/2$

$x + 1/2 = 4\ 1/2$ or $-x - 1/2 = 4\ 1/2$

$x = 4$ or $-x = 5$

$x = -5$

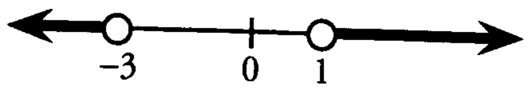
23. $|x + 1| > 2$

$+ \quad (x + 1) > 2$ or $-(x + 1) > 2$

$x + 1 > 2$ or $-x - 1 > 2$

$x > 1$ or $-x > 3$

$x < -3$



$\boldsymbol{x < -3}$ OR $\boldsymbol{x > 1}$

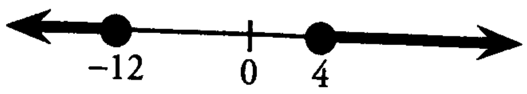
24. $|-x - 4| \geq 8$

$+ \quad (-x - 4) \geq 8$ or $-(-x - 4) \geq 8$

$-x - 4 \geq 8$ or $x + 4 \geq 8$

$-x \geq 12$ or $x \geq 4$

$x \leq -12$



$\boldsymbol{x \leq -12}$ OR $\boldsymbol{x \geq 4}$

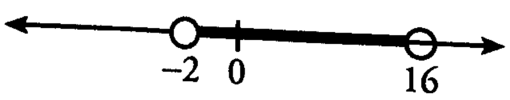
25. $|x - 7| < 9$

$+ \quad (x - 7) < 9$ or $-(x - 7) < 9$

$x - 7 < 9$ or $-x + 7 < 9$

$x < 16$ or $-x < 2$

$x > -2$



$x > -2$ AND $x < 16$, $\boldsymbol{-2 < x < 16}$

26. **$-3 < x < 5$:** $-7 < 3 - 2x < 9$

$-10 < -2x < 6$	Subtract 3 from all three terms.
$5 > x > -3$	Divide all three terms by -2, and flip the inequality signs.
or **$-3 < x < 5$**	

27. **E:** Extreme Values for a are greater than -4, or GT(-4), and less than 4, or LT(4).
Extreme values for b are greater than -2, or GT(-2), and less than -1, or LT(-1).

a can be positive or negative, while b can only be negative, so ab can be positive and negative.

The most negative ab can be is GT(-2) × GT(4) = GT(-8).
The most positive ab can be is GT(-4) × GT(-2) = LT($+8$).

28. **-10:**

a	*b*	*b* − *a*
-1	-6	$-6 - (-1) = -5$
-1	-2	$-2 - (-1) = -1$
4	-6	$-6 - 4 = -10$
4	-2	$-2 - 4 = -6$

29. **2:** $(x + 2)^2 \leq 2 - y$

$y + (x + 2)^2 \leq 2$	Add y to both sides
$y \leq 2 - (x + 2)^2$	Subtract $(x + 2)^2$ from both sides

y is maximized when $(x + 2)^2$ is minimized. The smallest possible value for $(x + 2)^2$ is 0, when $x = -2$. When $(x + 2)^2 = 0$, $y = 2$.

Problem Set

1. If $4x - 12 \geq x + 9$, which of the following must be true?
(A) $x > 6$ (B) $x < 7$ (C) $x > 7$ (D) $x > 8$ (E) $x < 8$

2. Which of the following is equivalent to $-3x + 7 \leq 2x + 32$?
(A) $x \geq -5$ (B) $x \geq 5$ (C) $x \leq 5$ (D) $x \leq -5$

3. If $G^2 < G$, which of the following could be G?
(A) 1 (B) $\frac{23}{7}$ (C) $\frac{7}{23}$ (D) -4 (E) -2

4. If $|A| > 19$, which of the following could not be equal to A?
(A) 26 (B) 22 (C) 18 (D) -20 (E) -24

5. If $B^3A < 0$ and $A > 0$, which of the following must be negative?
(A) AB (B) B^2A (C) B^4 (D) $\frac{A}{B^2}$ (E) $-\frac{B}{A}$

6.

$$|2x - 5| \leq 7$$

Quantity A	Quantity B
x	3

7.

$$1 \leq x \leq 5 \text{ and } 1 \geq y \geq -2$$

Quantity A	Quantity B
xy	-10

8.

$$x = 4$$

Quantity A	Quantity B
$|2 - x|$	2

1. **A:** $4x - 12 \geq x + 9$

$3x \geq 21$

$x \geq 7$

If $x \geq 7$, then $x > 6$.

2. **A:** $-3x + 7 \leq 2x + 32$

$-5x \leq 25$

$x \geq -5$

When you divide by a negative number, you must reverse the direction of the inequality symbol.

3. **C:** If $G^2 < G$, then G must be positive (since G^2 will never be negative), and G must be less than 1, because otherwise, $G^2 > G$. Thus, $0 < G < 1$. We can eliminate Choices D and E, since they violate the condition that G be positive. Then test Choice A: 1 is not less than 1, so we can eliminate A. Choice B is larger than 1, so only Choice C satisfies the inequality.

4. **C:** If $|A| > 19$, then $A > 19$ OR $A < -19$. The only answer choice that does not satisfy either of these inequalities is Choice C, 18.

5. **A:** If A is positive, B^3 must be negative. Therefore, B must be negative. If A is positive and B is negative, the product AB must be negative.

6. **D:** To evaluate the absolute value, set up two equations and isolate x.

$+(2x - 5) \leq 7$	AND	$-(2x - 5) \leq 7$
$2x - 5 \leq 7$		$-2x + 5 \leq 7$
$2x \leq 12$		$-2x \leq 2$
$x \leq 6$		$x \geq -1$

Combine the information from the two equations.

$$|2x - 5| \leq 7$$

Quantity A	Quantity B
$-1 \leq x \leq 6$	3

There are possible values of x greater than AND less than 3. **We do not have enough information.**

7. **D:** To find the minimum and maximum values of xy, test the boundaries of x and y.

x	y	xy
Min 1	Min −2	$(1) \times (-2) = -2$
Min 1	Max 1	$(1) \times (1) = 1$
Max 5	Min −2	$(5) \times (-2) = -10$
Max 5	Max 1	$(5) \times (1) = 5$

Combine the information from the chart to show the range of xy.

$$1 \leq x \leq 5 \text{ and } 1 \geq y \geq -2$$

Quantity A	Quantity B
$-10 \leq xy \leq 5$	−10

Quantity A can be either greater than or equal to -10. **We do not have enough information.**

8. **C:** Plug in 4 for x in Quantity A.

$$x = 4$$

Quantity A	Quantity B
$\lvert 2 - x \rvert =$ $\lvert 2 - (4) \rvert = \lvert -2 \rvert = 2$	2

The two quantities are equal.