OVERLAPPING SETS

Translation problems which involve 2 or more given sets of data that partially intersect with each other are termed Overlapping Sets. For example:

> 30 people are in a room. 20 of them play golf. 15 of them play golf and tennis. If everyone plays at least one of the two sports, how many of the people play tennis only?

This problem involves two sets: (1) people who play golf and (2) people who play tennis. The two sets overlap because some of the people who play golf also play tennis. Thus, these 2 sets can actually be divided into 4 categories:

(1) People who only play golf
(2) People who only play tennis
(3) People who play golf and tennis
(4) People who play neither sport

Use a double-set matrix to solve problems that involve overlapping sets.

Solving double-set GMAT problems, such as the example above, involves finding values for these four categories.

The Double-Set Matrix

For GMAT problems involving only *two* categorizations or decisions, the most efficient tool is the *Double-Set Matrix*: a table whose rows correspond to the options for one decision, and whose columns correspond to the options for the other decision. The last row and the last column contain totals, so the bottom right corner contains the total number of everything or everyone in the problem.

Even if you are accustomed to using Venn diagrams for these problems, you should switch to the double-set matrix for problems with only two sets of options. The double-set matrix conveniently displays *all* possible combinations of options, including totals, whereas the Venn diagram only displays a few of them easily.

> Of 30 integers, 15 are in set A, 22 are in set B, and 8 are in both set A and B. How many of the integers are in NEITHER set A nor set B?

This box shows the overlap.

	A	NOT A	TOTAL
B	8		22
NOT B			
TOTAL	15		30

This box shows the total members in SET B.

This box in the lower right corner is the key. This tells you how many distinct members exist in the overall group being considered.

This box shows the total members in SET A.

This box shows the members in NEITHER set.

Once the information given in the problem has been filled in, complete the chart, using the totals to guide you. (Each row and each column sum to a total value.)

	A	NOT A	TOTAL
B	8	**14**	22
NOT B	7	**1**	**8**
TOTAL	15	**15**	30

The question asks for the number of integers in neither set. We look at the chart and find the number of integers that are NOT A and NOT B; we find that the answer is 1.

Make sure that the column labels represent opposite situations. Do the same for the row labels.

When you construct a double-set matrix, be careful! As mentioned above, the rows should correspond to the *mutually exclusive options* for one decision. Likewise, the columns should correspond to the mutually exclusive options for the other. For instance, if a problem deals with students getting either right or wrong answers on problems 1 and 2, the columns should **not** be "problem 1" and "problem 2," and the rows should **not** be "right" and "wrong." Instead, the columns should list options for *one* decision—problem 1 correct, problem 1 incorrect, total—and the rows should list options for the other decision—problem 2 correct, problem 2 incorrect, total.

INCORRECT

	Prob 1	**Prob 2**	**TOTAL**
Right			
Wrong			
TOTAL			

CORRECT

Mutually exclusive

	Prob 1 Right	**Prob 1 Wrong**	**TOTAL**
Prob 2 Right			
Prob 2 Wrong			
TOTAL			

Mutually exclusive

Overlapping Sets and Percents

Many overlapping-sets problems involve *percents* or *fractions*. The double-set matrix is still effective on these problems, especially if you pick a "Smart Number" for the grand total. For problems involving percents, pick a total of 100. For problems involving fractions, pick a common denominator for the total. For example, pick 15 or 30 if the problem mentions categories that are 1/3 and 2/5 of the total.

> 70% of the guests at Company X's annual holiday party are employees of Company X. 10% of the guests are women who are not employees of Company X. If half the guests at the party are men, what percent of the guests are female employees of Company X?

First, fill in 100 for the total number of guests at the party. Then, fill in the other information given in the problem: 70% of the guests are employees, and 10% are women who are not employees. We also know that half the guests are men. (Therefore, we also know that half the guests are women.)

The "smart" number for percents is 100.

	Men	Women	TOTAL
Employee			70
Not Emp.		10	
TOTAL	50	50	100

Next, use subtraction to fill in the rest of the information in the matrix:
100 − 70 = 30 guests who are not employees
30 − 10 = 20 men who are not employees
50 − 10 = 40 female employees

	Men	Women	TOTAL
Employee	**30**	**40**	70
Not Emp.	**20**	10	**30**
TOTAL	50	50	100

40% of the guests at the party are female employees of Company X. Note that the problem does not require us to complete the matrix with the number of male employees, since we have already answered the question asked in the problem. However, completing the matrix is an excellent way to check your computation. The last box you fill in must work both vertically and horizontally.

As in other problems involving Smart Numbers, you can only assign a number to the total if it is **undetermined** to start with. If the problem contains only fractions and/or percents, but no actual *numbers* of items or people, then go ahead and pick a total of 100 (for percent problems) or a common denominator (for fraction problems). But if actual quantities appear anywhere in the problem, then all the totals are already determined. In that case, you cannot assign numbers, but must solve for them instead.

Overlapping Sets and Algebraic Representation

When solving overlapping sets problems, you must pay close attention to the wording of the problem. For example, consider the problem below:

> Santa estimates that 10% of the children in the world have been good this year but do not celebrate Christmas, and that 50% of the children who celebrate Christmas have been good this year. If 40% of the children in the world have been good, what percentage of children in the world are not good and do not celebrate Christmas?

Read the problem very carefully to determine whether you need to use algebra to represent unknowns.

It is tempting to fill in the number 50 to represent the percent of good children who celebrate Christmas. However, this approach is incorrect.

WRONG

	Good	Not Good	TOTAL
X-mas	50		
No X-mas	10		
TOTAL			100

CORRECT

	Good	Not Good	TOTAL
X-mas	$0.5x$		x
No X-mas	10		
TOTAL	40		100

Notice that we are told that 50% of the children *who celebrate Christmas* have been good. This is different from being told that 50% of the children in the world have been good. In this problem, this information we have is a fraction of an unknown number. We do not yet know how many children celebrate Christmas. Therefore, we cannot yet write a number for the good children who celebrate Christmas. Instead, we represent the unknown total number of children who celebrate Christmas with the variable x. Thus, we can represent the number of good children who celebrate Christmas with the expression $0.5x$.

From the relationships in the table, we can set up an equation to solve for x.

$$0.5x + 10 = 40$$
$$x = 60$$

With this information, we can fill in the rest of the table.

	Good	Not Good	TOTAL
X-mas	**$0.5x = 30$**	**30**	**$x = 60$**
No X-mas	10	**30**	**40**
TOTAL	40	**60**	100

30% of the children are not good and do not celebrate Christmas.

2 Sets, 3 Choices: Still Double-Set Matrix

Very rarely, you might need to consider more than 2 options for one or both of the dimensions of your chart. As long as each set of distinct options is complete and has no overlaps, you can simply extend the chart.

For instance, if respondents can answer "Yes," "No," or "Maybe" to a survey question, and we care about the gender of the respondents, then we might set up the following matrix:

	Yes	No	Maybe	Total
Female				
Male				
Total				

The set of 3 answer choices is complete (there are no other options). Also, the choices do not overlap (no respondent can give more than 1 response). So, this extended chart is fine.

You rarely need to do real computation, but setting up an extended chart such as this can be helpful on certain Data Sufficiency problems, so that you can see what information is or is not sufficient to answer the given question.

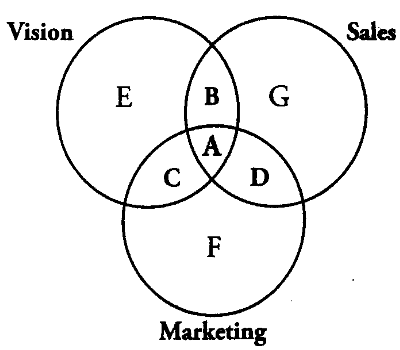
Use a Venn Diagram to solve problems with 3 overlapping sets.

3-Set Problems: Venn Diagrams

Problems that involve 3 overlapping sets can be solved by using a Venn Diagram. The three overlapping sets are usually 3 teams or clubs, and each person is either <u>on</u> or <u>not</u> <u>on</u> any given team or club. That is, there are only 2 choices for any club: member or not.

> Workers are grouped by their areas of expertise and are placed on at least one team. 20 workers are on the Marketing team, 30 are on the Sales team, and 40 are on the Vision team. 5 workers are on both the Marketing and Sales teams, 6 workers are on both the Sales and Vision teams, 9 workers are on both the Marketing and Vision teams, and 4 workers are on all three teams. How many workers are there in total?

In order to solve this problem, use a Venn Diagram. A Venn Diagram should be used ONLY for problems that involve three sets. Stick to the double-set matrix for two-set problems.



Begin your Venn Diagram by drawing three overlapping circles and labeling each one.

Notice that there are 7 different sections in a Venn Diagram. There is one innermost section (A) where all 3 circles overlap. This contains individuals who are on all 3 teams.

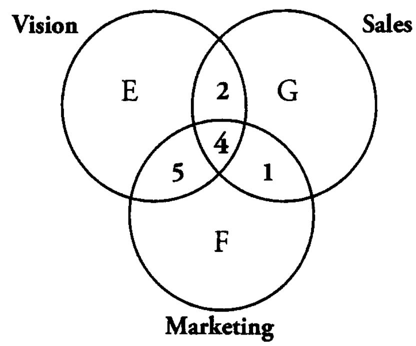
There are three sections **(B, C, and D)** where 2 circles overlap. These contain individuals who are on 2 teams. There are three non-overlapping sections **(E, F, and G)** that contain individuals who are on only 1 team.

Venn Diagrams are easy to work with, if you remember one simple rule: **Work from the Inside Out.**

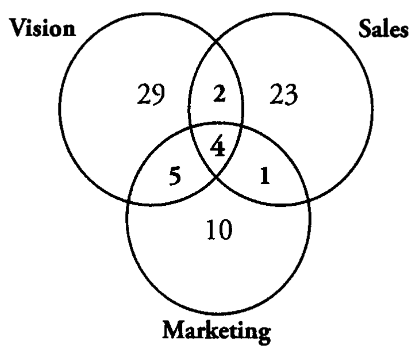
That is, it is easiest to begin by filling in a number in the innermost section **(A)**. Then, fill in numbers in the middle sections **(B, C, and D)**. Fill in the outermost sections **(E, F, and G)** last.

When you use a Venn Diagram, work from the INSIDE OUT.

First: Workers on all 3 teams: Fill in the innermost circle. This is given in the problem as 4.



Second: Workers on 2 teams: Here we must remember to subtract those workers who are on all 3 teams. For example, the problem says that there are 5 workers on the Marketing and Sales teams. However, this includes the 4 workers who are on all three teams. Therefore, in order to determine the number of workers who are on the Marketing and Sales teams exclusively, we must subtract the 4 workers who are on all three teams. We are left with 5 – 4 = 1. The number of workers on the Marketing and Vision teams exclusively is 9 – 4 = 5. The number of workers on the Sales and Vision teams exclusively is 6 – 4 = 2.



Third: Workers on 1 team only: Here we must remember to subtract those workers who are on 2 teams and those workers who are on 3 teams. For example, the problem says that there are 20 workers on the Marketing team. But this includes the 1 worker who is on the Marketing and Sales teams, the 5 workers who are on the Marketing and Vision teams, and the 4 workers who are on all three teams. We must subtract all of these workers to find that there are 20 – 1 – 5 – 4 = 10 people who are on the Marketing team exclusively. There are 30 – 1 – 2 – 4 = 23 people on the Sales team exclusively. There are 40 – 2 – 5 – 4 = 29 people on the Vision team exclusively.

In order to determine the total, just add all 7 numbers together = 74 total workers.

Problem Set

1. X and Y are sets of integers. X | Y denotes the set of integers that belong to set X or set Y, but not both. If X consists of 10 integers, Y consists of 18 integers, and 5 of the integers are in both X and Y, then X | Y consists of how many integers?

2. All of the members of Gym 1 live in Building A or Building B. There are 350 members of Gym 1. 200 people live in Building A across the street. 400 people live in Building B. 100 people from Building A are members of Gym 1. How many people live in Building B that do not belong to Gym 1?

3. Of 28 people in a park, 12 are children and the rest are adults. 8 people have to leave at 3pm; the rest do not. If after 3pm, there are 6 children still in the park, how many adults are still in the park?

4. Of 30 snakes at the reptile house, 10 have stripes, 21 are poisonous, and 5 have no stripes and are not poisonous. How many of the snakes have stripes AND are poisonous?

5. There are 30 stocks. 8 are volatile; the rest are blue-chip. 14 are tech; the rest are non-tech. If there are 3 volatile tech stocks, how many blue-chip non-tech stocks are there?

6. Students are in clubs as follows: Science–20, Drama–30, and Band–12. No student is in all three clubs, but 8 are in both Science and Drama, 6 are in both Science and Band, and 4 are in Drama and Band. How many different students are in at least one of the three clubs?

7. 40% of all high school students hate roller coasters; the rest love them. 20% of those students who love roller coasters own chinchillas. What percentage of students love roller coasters but do not own a chinchilla?

8. There are 26 students who have read a total of 56 books among them. The only books they have read, though, are Aye, Bee, Cod, and Dee. If 10 students have only read Aye, and 8 students have read only Cod and Dee, what is the smallest number of books any of the remaining students could have read?

9. Scout candies come in red, white, or blue. They can also be hard or soft. There are 50 candies: 20 red, 20 white, and 10 blue. There are 25 hard and 25 soft. If there are 5 soft blue candies and 12 soft red candies, how many hard white candies are there?

10. Of 60 children, 30 are happy, 10 are sad, and 20 are neither happy nor sad. There are 20 boys and 40 girls. If there are 6 happy boys and 4 sad girls, how many boys are neither happy nor sad?

11. 10% of all aliens are capable of intelligent thought and have more than 3 arms, and 75% of aliens with 3 arms or less are capable of intelligent thought. If 40% of all aliens are capable of intelligent thought, what percent of aliens have more than 3 arms?

12. There are three country clubs in town: Abacus, Bradley, and Claymore. Abacus has 300 members, Bradley 400, and Claymore has 450. 30 people belong to both Abacus and Bradley, 40 to both Abacus and Claymore, and 50 to both Bradley and Claymore. 20 people are members of all three clubs. How many people belong to at least 1 country club in town?

13. There are 58 vehicles in a parking lot. 24 are trucks, 30 are cars, and the rest are some other vehicle. 20 of the vehicles are red, 16 are blue, and the rest are some other color. If there are 12 red trucks in the parking lot, 5 blue trucks, and 4 red cars, what is the largest possible number of blue cars in the parking lot?

14. The 38 movies in the video store fall into the following three categories: 10 action, 20 drama, and 18 comedy. However, some movies are classified under more than one category: 5 are both action and drama, 3 are both action and comedy, and 4 are both drama and comedy. How many action–drama–comedies are there?

15. There are 6 stores in town that had a total of 20 visitors on a particular day. However, only 10 people went shopping that day; some people visited more than one store. If 6 people visited exactly two stores each, and everyone visited at least one store, what is the largest number of stores anyone could have visited?

1. **18:** Use a Double-Set Matrix to solve this problem. First, fill in the numbers given in the problem: There are 10 integers in set X and 18 integers in set Y. There are 5 integers that are in both sets. Then, use subtraction to figure out that there are 5 integers that are in set X and not in set Y, and 13 integers that are in set Y and not in set X. This is all the information you need to solve this problem: X | Y = 5 + 13 = 18.

	Set X	NOT Set X	TOTAL
Set Y	5	13	18
NOT Set Y	**5**		
TOTAL	10		

2. **150:** Use a Double-Set Matrix to solve this problem. First, fill in the numbers given in the problem: 350 people who belong to the gym, 200 people who live in Building A, 400 people who live in Building B, and 100 gym members from Building A. Then, use subtraction to figure out that there are 250 people from Building B who belong to the gym and 150 people from Building B who do not belong to the gym.

	Building A	Building B	TOTAL
Gym	100	**250**	350
NOT Gym		**150**	
TOTAL	200	400	

3. **14:** Use a Double-Set Matrix to solve this problem. First, fill in the numbers given in the problem: 28 total people in the park, 12 children and the rest (16) adults; 8 leave at 3 pm and the rest (20) stay. Then, we are told that there are 6 children left in the park after 3pm. Since we know there are a total of 20 people in the park after 3pm, the remaining 14 people must be adults.

	Children	Adults	TOTAL
Leave at 3			8
Stay	6	**14**	**20**
TOTAL	12	**16**	28

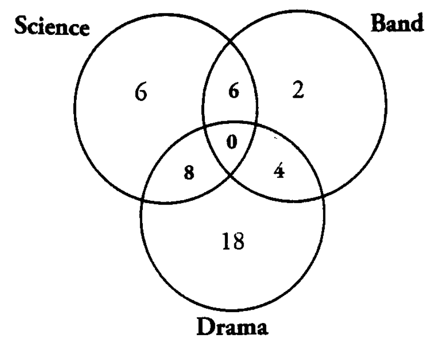
4. **6:** Use a Double-Set Matrix to solve this problem. First, fill in the numbers given in the problem: 30 snakes, 10 with stripes (and therefore 20 without), 21 that are poisonous (and therefore 9 that are not), and 5 that are neither striped nor poisonous. Use subtraction to fill in the rest of the chart. 6 snakes have stripes and are poisonous.

	Stripes	No Stripes	TOTAL
Poisonous	**6**		21
NOT Poison	**4**	5	**9**
TOTAL	10	**20**	30

5. **11:** Use a Double-Set Matrix to solve this problem. First, fill in the numbers given in the problem. There are 30 stocks. 8 are volatile; the rest are blue-chip. 14 are tech; the rest are non-tech. We also know that there are 3 volatile tech stocks. Therefore, by subtraction, there are 5 volatile non-tech stocks, and there are 11 blue-chip non-tech stocks.

	Volatile	Blue-Chip	TOTAL
Tech	3		14
Non-Tech	**5**	**11**	**16**
TOTAL	8	**22**	30

6. **44:** There are three overlapping sets here. Therefore, use a Venn diagram to solve the problem. First, fill in the numbers given in the problem, working from the inside out: no students in all three clubs, 8 in Science and Drama, 6 in Science and Band, and 4 in Drama and Band. Then, use the totals for each club to determine how many students are in only one club. For example, we know that there are 30 students in the Drama club. So far, we have placed 12 students in the circle that represents the Drama club (8 who are in Science and Drama, and 4 who are in Band and Drama). 30 − 12 = 18, the number of students who are in only the Drama Club. Use this process to determine the number of students in just the Science and Band clubs as well. To find the number of students in at least one of the clubs, sum all the numbers in the diagram:
6 +18 + 2 + 6 + 8 + 4 = 44.



7. **48:** Since all the numbers in this problem are given in percentages, assign a grand total of 100 students. We know that 40% of all high school students hate roller coasters, so we fill in 40 for this total and 60 for the number of students who love roller coasters. We also know that 20% **of those students who love roller coasters** own chinchillas. It does not say that 20% of all students own chinchillas. Since 60% of students love roller coasters, 20% of 60% own chinchillas. Therefore, we fill in 12 for the students who both love roller coasters and own chinchillas. The other 48 roller coaster lovers do not own chinchillas.

	Love R.C.	Do not	TOTAL
Chinchilla	**12**		
No Chinch.	**48**		
TOTAL	**60**	40	100

8. **2:** According to the problem, 10 students have read only 1 book: Aye, and 8 students have read 2 books: Cod and Dee. This accounts for 18 students, who have read a total of 26 books among them. Therefore, there are 8 students left to whom we can assign books, and there are 30 books left to assign. We can assume that one of these 8 students will have read the smallest possible number if the other 7 have read the maximum number: all 4 books. If 7 students have read 4 books each, this accounts for another 28 books, leaving only 2 for the eighth student to have read. Note that it is impossible for the eighth student to have read only one book. If we assign one of the students to have read only 1 book, this leaves 29 books for 7 students. This is slightly more than 4 books per students. However, we know that there are only four books available. It is therefore impossible for one student to have read more than four books.

Students	**Books Read**
10	10
8	16
7	28
1	2
26	56

9. **12:** Use a Double-Set Matrix to solve this problem, with the "color" set divided into 3 categories instead of only 2. First, fill in the numbers given in the problem: 20 red, 20 white, and 10 blue, 25 hard and 25 soft. We also know there are 5 soft blue candies and 12 soft red candies. Therefore, by subtraction, there are 8 soft white candies, and there are 12 hard white candies.

	Red	White	Blue	TOTAL
Hard		**12**		25
Soft	12	**8**	5	25
TOTAL	20	20	10	50

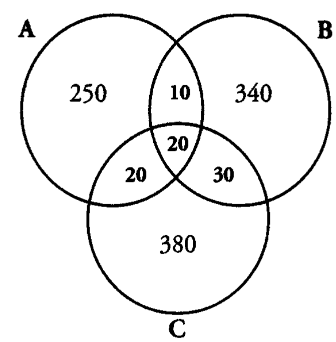
10. **8:** Use a Double-Set Matrix to solve this problem, with the "mood" set divided into 3 categories instead of only 2. First, fill in the numbers given in the problem: of 60 children, 30 are happy, 10 are sad, and 20 are neither happy nor sad; 20 are boys and 40 are girls. We also know there are 6 happy boys and 4 sad girls. Therefore, by subtraction, there are 6 sad boys and there are 8 boys who are neither happy nor sad.

	Happy	**Sad**	**Neither**	**TOTAL**
Boys	6	**6**	**8**	20
Girls		4		40
TOTAL	30	10	20	60

11. **60%:** Since all the numbers in this problem are given in percentages, assign a grand total of 100 aliens. We know that 10% of all aliens are capable of intelligent thought and have more than 3 arms. We also know that 75% **of aliens with 3 arms or less** are capable of intelligent thought. It does not say that 75% of all aliens are capable of intelligent thought. Therefore, assign the variable x to represent the percentage of aliens with three arms or less. Then, the percentage of aliens with three arms or less who are capable of intelligent thought can be represented by $0.75x$. Since we know that 40% of all aliens are capable of intelligent thought, we can write the equation $10 + 0.75x = 40$, or $0.75x = 30$. Solve for x: $x = 40$. Therefore, 40% of the aliens have three arms or less, and 60% of aliens have more than three arms.

	Thought	**No Thought**	**TOTAL**
> 3 arms	10		$100 - x$
≤ 3 arms	$0.75x$		x
TOTAL	40		100

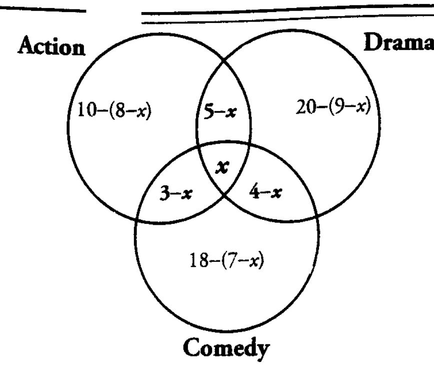
12. **1050:** There are three overlapping sets here; therefore, use a Venn diagram to solve the problem. First, fill in the numbers given in the problem, working from the inside out. We know that 20 people are in all three clubs. If 30 people are in both A & B, then 10 are in A & B, but not C. If 40 people are in both A & C, then 20 are in A & C, but not B. If 50 people are in both B & C, then 30 are in B & C, but not A. Then, use the totals for each club to determine how many students are in only one club. For example, we know that Abacus has 300 members. So far, we have placed 50 people in the circle that represents Abacus (10 who are in A and B, 20 who are in A and C, and 20 who are in all three clubs). $300 - 50 = 250$, the number of people who are in only the Abacus club. Use this process to determine the number of students in just the Bradley and Claymore clubs as well. To find the number of people in at least one of the clubs, sum all the numbers in the diagram:
$250 + 340 + 380 + 10 + 20 + 30 + 20 = 1050$.



13. **11:** Use a Double-Set Matrix to solve this problem. First, fill in the numbers given in the problem: 58 vehicles are in a parking lot. 24 are trucks, 30 are cars, and the rest some other vehicle. 20 of the vehicles are red, 16 are blue, and the rest are some other color. We also know there are 12 red trucks, 5 blue trucks, and 4 red cars. The critical total in this problem is that there are 16 blue vehicles. Since 5 of them are blue trucks, and (by filling in the matrix we see that) there are 0 "other" blue vehicles, there must be 11 blue cars in the lot.

	Trucks	**Cars**	**Other**	**TOTAL**
Red	12	4	4	20
Blue	5		**0**	16
Other	7			**22**
TOTAL	24	30	4	58

14. **2:** There are three overlapping sets here; therefore, use a Venn diagram to solve the problem. First, fill in the numbers given in the problem, working from the inside out. Assign the variable x to represent the number of action–drama–comedies. Then, create variable expressions, using the totals given in the problem, to represent the number of movies in each of the other categories. We know that there is a total of 38 movies; therefore, we can write the following equation to represent the total number of movies in the store:



$$\begin{array}{r} 10-8+x \\ 20-9+x \\ 18-7+x \\ 5-x \\ 4-x \\ 3-x \\ +\quad x \\ \hline 36+x=38 \\ x=2 \end{array}$$

If you are unsure of the algebraic solution, you can also guess a number for x and fill in the rest of the diagram until the total number of movies reaches 38.

15. **5:** If 6 people visited exactly 2 stores each, this accounts for 12 of the visitors counted in the total. This leaves 4 people to account for the remaining 8 visitors. In order to assign the maximum number of stores to any one person, assign the minimum to the first three of the remaining people: 1 store. This leaves 5 stores for the fourth person to visit.

People	Store Visits
6	12
3	3
1	5
10	20