DIVISIBILITY

In This Chapter:

- Divisibility rules
- How to find the factors of a number
- The connection between factors and divisibility
- How to answer questions on the GRE related to divisibility

There is a category of problems on the GRE that tests what could broadly be referred to as "Number Properties." These questions are focused on a very important subset of numbers known as integers. Before we explore divisibility any further, it will be necessary to understand exactly what integers are and how they function.

Integers are whole numbers. That means that they are numbers that do not have any decimals or fractions attached. Some people think of them as counting numbers, i.e. 1, 2, 3… etc. Integers can be positive, and they can also be negative. −1, −2, −3… etc. are all integers as well. And there's one more important number that qualifies as an integer: 0.

So numbers such as 7, 15,003, −346, and 0 are all integers. Numbers such as 1.3, 3/4, and π are not integers.

Now that we know what integers are, let's see what we know about them when dealing with the four basic operations: addition, subtraction, multiplication and division.

integer + integer = always an integer	ex. $4 + 11 = 15$
integer − integer = always an integer	ex. $-5 - 32 = -37$
integer × integer = always an integer	ex. $14 \times 3 = 42$

None of these properties of integers turn out to be very interesting. But what happens when we *divide* an integer by another integer? Well, $18 \div 3 = 6$, which is an integer, but $12 \div 8 = 1.5$, which is not an integer.

If an integer divides another integer and the result, or quotient, is an integer, we say the first number is divisible by the second. So 18 is divisible by 3 because $18 \div 3 =$ an integer. On the other hand, we would say that 12 is NOT divisible by 8, because $12 \div 8$ is not an integer.

Divisibility Rules

The Divisibility Rules are important shortcuts to determine whether an integer is divisible by 2, 3, 4, 5, 6, 8, 9, and 10. You can always use your calculator to test divisibility, but these shortcuts will save you time.

An integer is divisible by:

2 if the integer is EVEN.
12 is divisible by 2, but 13 is not. Integers that are divisible by 2 are called "even" and integers that are not are called "odd." You can tell whether a number is even by checking to see whether the units (ones) digit is 0, 2, 4, 6, or 8. Thus, 1,234,567 is odd, because 7 is odd, whereas 2,345,678 is even, because 8 is even.

3 if the SUM of the integer's DIGITS is divisible by 3.
72 is divisible by 3 because the sum of its digits is 9, which is divisible by 3. By contrast, 83 is not divisible by 3, because the sum of its digits is 11, which is not divisible by 3.

4 if the integer is divisible by 2 TWICE, or if the LAST TWO digits are divisible by 4.
28 is divisible by 4 because you can divide it by 2 twice and get an integer result ($28 \div 2 = 14$, and $14 \div 2 = 7$). For larger numbers, check only the last two digits. For example, 23,456 is divisible by 4 because 56 is divisible by 4, but 25,678 is not divisible by 4 because 78 is not divisible by 4.

5 if the integer ends in 0 or 5.
75 and 80 are divisible by 5, but 77 and 83 are not.

6 if the integer is divisible by BOTH 2 and 3.
48 is divisible by 6 since it is divisible by 2 (it ends with an 8, which is even) AND by 3 ($4 + 8 = 12$, which is divisible by 3).

8 if the integer is divisible by 2 THREE TIMES, or if the LAST THREE digits are divisible by 8.
32 is divisible by 8 since you can divide it by 2 three times and get an integer result ($32 \div 2 = 16$, $16 \div 2 = 8$, and $8 \div 2 = 4$). For larger numbers, check only the last 3 digits. For example, 23,456 is divisible by 8 because 456 is divisible by 8, whereas 23,556 is not divisible by 8 because 556 is not divisible by 8.

9 if the SUM of the integer's DIGITS is divisible by 9.
4,185 is divisible by 9 since the sum of its digits is 18, which is divisible by 9. By contrast, 3,459 is not divisible by 9, because the sum of its digits is 21, which is not divisible by 9.

10 if the integer ends in 0.
670 is divisible by 10, but 675 is not.

The GRE can also test these divisibility rules in reverse. For example, if you are told that a number has a ones digit equal to 0, you can infer that that number is divisible by 10. Similarly, if you are told that the sum of the digits of x is equal to 21, you can infer that x is divisible by 3 but NOT by 9.

Note also that there is no rule listed for divisibility by 7. The simplest way to check for divisibility by 7, or by any other number not found in this list, is to use the calculator.

Check Your Skills

1. Is 123,456,789 divisible by 2?
2. Is 732 divisible by 3?
3. Is 989 divisible by 9?
4. Is 4,578 divisible by 4?
5. Is 4,578 divisible by 6?
6. Is 603,864 divisible by 8?

Answers can be found on page 45.

Factors

Let's continue to explore the question of divisibility by asking the question, what numbers is 6 divisible by? Questions related to divisibility are only interested in positive integers, so we really only have 6 possible numbers: 1, 2, 3, 4, 5, and 6. So let's see which numbers 6 is divisible by.

$6 \div 1 = 6$ Any number divided by 1 equals itself, so an integer divided by 1 will be an integer.

$6 \div 2 = 3$
$6 \div 3 = 2$ > Note that these form a pair

$6 \div 4 = 1.5$
$6 \div 5 = 1.2$ > Not integers, so 6 is NOT divisible by 4 or by 5.

$6 \div 6 = 1$ Any number divided by itself equals 1, so an integer is always divisible by itself.

So 6 is divisible by 1, 2, 3 and 6. That means that 1, 2, 3 and 6 are **factors** of 6. There are a variety of ways you might see this relationship expressed on the GRE.

2 is a factor of 6	6 is a multiple of 2
2 is a divisor of 6	6 is divisible by 2
2 divides 6	2 goes into 6

Sometimes it will be necessary to find the factors of a number in order to answer a question. An easy way to find all the factors of a small number is to use factor pairs. Factor pairs for any integer are the pairs of factors that, when multiplied together, yield that integer.

Here's a step-by-step way to find all the factors of the number 60 using a **factor pairs table:**

(1) Make a table with 2 columns labeled "Small" and "Large."

(2) Start with 1 in the small column and 60 in the large column. (The first set of factor pairs will always be 1 and the number itself.)

(3) The next number after 1 is 2. If 2 is a factor of 60, then write "2" underneath the "1" in your table. It is, so divide 60 by 2 to find the factor pair: $60 \div 2 = 30$. Write "30" in the large column.

(4) The next number after 2 is 3. Repeat this process until the numbers in the small and the large columns run into each other. In this case, we find that 6 and 10 are a factor pair. But 7, 8 and 9 are not factors of 60, and the next number after 9 is 10, which appears in the large column, so we can stop.

Small	Large
1	60
2	30
3	20
4	15
5	12
6	10

The advantage of using this method, as opposed to thinking of factors and listing them out, is that this is an organized, methodical approach that makes it easier to find every factor of a number quickly. Let's practice. (This is also a good opportunity to practice your long division.)

Check Your Skills

7. Find all the factors of 90.
8. Find all the factors of 72.
9. Find all the factors of 105.
10. Find all the factors of 120.

Answers can be found on pages 45–46.

Prime Numbers

Let's backtrack a little bit and try finding the factors of another small number: 7. Our only possibilities are the positive integers less than or equal to 7, so let's check every possibility.

$7 \div 1 = 7$	Every number is divisible by 1—no surprise there!
$7 \div 2 = 3.5$	
$7 \div 3 = 2.33\ldots$	
$7 \div 4 = 1.75$	7 is not divisible by *any* integer besides 1 and itself
$7 \div 5 = 1.4$	
$7 \div 6 = 1.16\ldots$	
$7 \div 7 = 1$	Every number is divisible by itself—boring!

So 7 only has two factors—1 and itself. Numbers that only have 2 factors are known as **prime numbers.** As we will see, prime numbers play a very important role in answering questions about divisibility. Because they're so important, it's critical that we learn to identify what numbers are prime and what numbers aren't.

The prime numbers that appear most frequently on the test are prime numbers less than 20. They are 2, 3, 5, 7, 11, 13, 17 and 19. Two things to note about this list: 1 is not prime, and out of *all* the prime numbers, 2 is the *only* even prime number.

2 is prime because it has only two factors—1 and itself. The reason that it's the only even prime number is that *every* even number is divisible by 2, and thus has another factor besides 1 and itself. For instance, we can immediately tell that 12,408 isn't prime, because we know that it has at least one factor besides 1 and itself: 2.

So every positive integer can be placed into one of two categories—prime or not prime.

Primes	Non-Primes
2, 3, 5, 7, 11, etc.	4, 6, 8, 9, 10, etc.
exactly two factors: 1 and itself	*more than* 2 factors
ex. $7 = 1 \times 7$	ex. $6 = 1 \times 6$
only factor pair	*and* $6 = 2 \times 3$
	more than 2 factors *and*
	more than 1 factor pair

Check Your Skills

11. List all the prime numbers between 20 and 50.

The answer can be found on page 46.

Prime Factorization

Let's take another look at 60. When we found the factor pairs of 60, we discovered that it had 12 factors and 6 factor pairs.

$60 = 1 \times 60$ Always the first factor pair—boring!

and 2×30

and 3×20

and 4×15 5 other factor pairs—interesting! Let's look at these in a little more detail.

and 5×12

and 6×10

From here on, we will be referring to boring and interesting factor pairs. These are not technical terms, but the boring factor pair is the factor pair that involves 1 and the number itself. All other pairs are interesting pairs. Keep reading to see why!

Let's look at one of these factor pairs—4×15. One way to think about this pair is that 60 ***breaks down*** into 4 and 15. One way to express this relationship visually is to use a **factor tree.**

```
   60
  /  \
 4    15
```

Now, the question arises—can we go further? Sure! Neither 4 nor 15 is prime, which means they both have factor pairs that we might find *interesting*. 4 breaks down into 2×2, and 15 breaks down into 3×5:

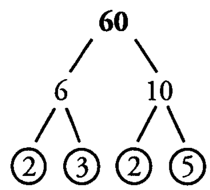
```
     60
    /  \
   4    15
  / \   / \
 2   2 3   5
```

Can we break it down any further? Not with *interesting* factor pairs. We could say that $2 = 2 \times 1$, for instance, but that doesn't provide us any new information. The reason we can't go any further is that 2, 2, 3 and 5 are all *prime numbers*. Prime numbers only have one boring factor pair. So when we find a prime factor, we know that that branch of our factor tree has reached its end. We can go one step further and circle every prime number as we go, reminding us that we can't break down that branch any further. The factor tree for 60 would look like this:

```
     60
    /  \
   4    15
  / \   / \
 2   2 3   5
```

So after we broke down 60 into 4 and 15, and broke 4 and 15 down, we ended up with $60 = 2 \times 2 \times 3 \times 5$.

What if we start with a different factor pair of 60? Let's create a factor tree for 60 in which the first breakdown we make is 6×10.



According to this factor tree $60 = 2 \times 3 \times 2 \times 5$. Notice that, even though they're in a different order, this is the same group of prime numbers we had before. In fact, *any* way we break down 60, we will end up with the same prime factors: two 2's, one 3 and one 5. Another way to say this is that $2 \times 2 \times 3 \times 5$ is the **prime factorization** of 60.

One way to think about prime factors is that they are the DNA of a number. Every number has a unique prime factorization. 60 is the only number that can be written as $2 \times 2 \times 3 \times 5$. Breaking down numbers into their prime factors is the key to answering many divisibility problems.

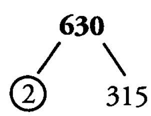
As we proceed through the chapter, we'll discuss what prime factors can tell us about a number and some different types of questions the GRE may ask. But because the prime factorization of a number is so important, first we need a fast, reliable way to find the prime factorization of *any* number.

A factor tree is the best way to find the prime factorization of a number. A number like 60 should be relatively straightforward to break down into primes, but what if you need the prime factorization of 630?

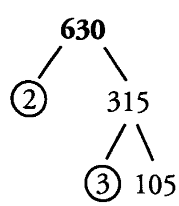
For large numbers, it's often best to start with the smallest prime factors and work your way toward larger primes. This is why it's good to know your divisibility rules!

Take a second to try on your own, and then we'll go through it together.

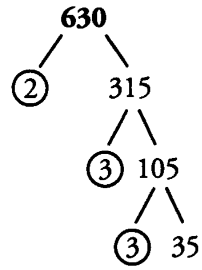
Start by finding the smallest prime number that 630 is divisible by. The smallest prime number is 2. 630 is even, so we know it must be divisible by 2. 630 divided by 2 is 315, so our first breakdown of 630 is into 2 and 315.



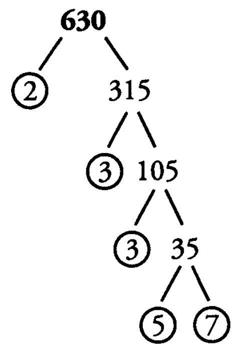
Now we still need to factor 315. It's not even, so we know it's not divisible by 2. Is it divisible by 3? If the digits of 315 add up to a multiple of 3, it is. $3 + 1 + 5 = 9$, which is a multiple of 3, so 315 is divisible by 3. 315 divided by 3 is 105, so our factor tree now looks like this:



If 315 was not divisible by 2, then 105 won't be either (we will discuss why later), but 105 might still be divisible by 3. $1 + 0 + 5 = 6$, so 105 is divisible by 3. $105 \div 3 = 35$, so our tree now looks like this:



35 is not divisible by 3 (3 + 5 = 8, which is not a multiple of 3), so the next number to try is 5. 35 ends in a 5, so we know it is divisible by 5. 35 ÷ 5 = 7, so our tree now looks like this:



Every number on the tree has now been broken down as far as it can go. So the prime factorization of 630 is 2 × 3 × 3 × 5 × 7.

Alternatively, you could have split 630 into 63 and 10, since it's easy to see that 630 is divisible by 10. Then you would proceed from there. Either way will get you to the same set of prime factors.

Now it's time to get a little practice doing prime factorizations.

Check Your Skills

12. Find the prime factorization of 90.
13. Find the prime factorization of 72.
14. Find the prime factorization of 105.
15. Find the prime factorization of 120.

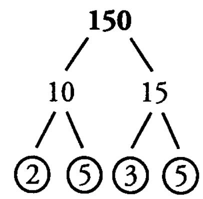
Answers can be found on pages 46–47.

The Factor Foundation Rule

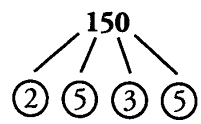
This discussion begins with the **factor foundation rule.** The factor foundation rule states that if a is divisible by b, and b is divisible by c, then a is divisible by c as well. In other words, if we know that 12 is divisible by 6, and 6 is divisible by 3, then 12 is divisible by 3 as well.

This rule also works in reverse to a certain extent. If d is divisible by two different primes, e and f, d is also divisible by $e \times f$. In other words, if 20 is divisible by 2 and by 5, then 20 is also divisible by 2 × 5 (10).

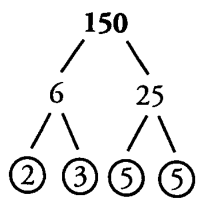
Another way to think of this rule is that divisibility travels up and down the factor tree. Let's walk through the factor tree of 150. We'll break it down, and then we'll build it back up.



150 is divisible by 10 and by 15, so 150 is also divisible by *everything* that 10 and 15 are divisible by. 10 is divisible by 2 and 5, so 150 is also divisible by 2 and 5. 15 is divisible by 3 and 5, so 150 is also divisible by 3 and 5. Taken all together, we know that the prime factorization of 150 is $2 \times 3 \times 5 \times 5$. We could represent that information like this:

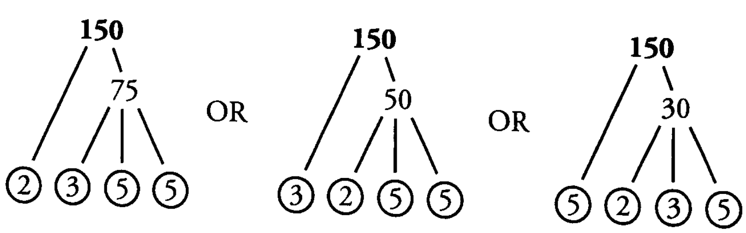


Think of prime factors as building blocks. In the case of 150, we have one 2, one 3 and two 5's at our disposal to build other factors of 150. In our first example, we went down the tree —from 150 down to 10 and 15, and then down again to 2, 5, 3 and 5. But we can also build upwards, starting with our four building blocks. For instance, $2 \times 3 = 6$, and $5 \times 5 = 25$, so our tree could also look like this:



(Even though 5 and 5 are not different primes, 5 appears twice on 150's tree. So we are allowed to multiply those two 5's together to produce another factor of 150, namely 25.)

The tree above isn't even the only other possibility. These are all trees that we could build using different combinations of our prime factors.



We began with four prime factors of 150: 2, 3, 5 and 5. But we were able to build different factors by multiplying 2, 3 or even all 4 of those primes together in different combinations. As it turns out, *all* of the factors of a number (except for 1) can be built with different combinations of its prime factors.

The Factor/Prime Factorization Connection

Let's take one more look at the number 60 and its factors. Specifically, let's look at the prime factorizations of all the factors of 60.

	Small	Large	
1	1	60	$2 \times 2 \times 3 \times 5$
2	2	30	$2 \times 3 \times 5$
3	3	20	$2 \times 2 \times 5$
2×2	4	15	3×5
5	5	12	$2 \times 2 \times 3$
2×3	6	10	2×5

All the factors of 60 are just different combinations of the prime numbers that make up the prime factorization of 60. To say this another way, every factor of a number can be expressed as the product of a combination of its prime factors. Take a look back at your work for Check Your Skills questions 7–10 and 12–15. Break down all the factor pairs from the first section into their prime factors. This relationship between factors and prime factors is true of every number.

Now that you know why prime factors are so important, it's time for the next step. An important skill on the GRE is to take the given information in a question and go further with it. For example, if a question tells you that a number n is even, what else do you know about it? Every even number is a multiple of 2, so n is a multiple of 2. These kinds of inferences often provide crucial information necessary to correctly solving problems.

So far, we've been finding factors and prime factors of numbers—but the GRE will sometimes ask divisibility questions about *variables*. In the next section, we'll take our discussion of divisibility to the next level and bring variables into the picture. But first, we'll recap what we've learned so far and what tools we'll need going forward.

1. If a is divisible by b, and b is divisible by c, then a is divisible by c as well. (ex. 100 is divisible by 20, and 20 is divisible by 4, so 100 is divisible by 4 as well.)

2. If d has e and f as prime factors, d is also divisible by $e \times f$. (ex. 90 is divisible by 5 and by 3, so 90 is also divisible by $5 \times 3 = 15$.) You can let e and f be the same prime, as long as there are at least 2 copies of that prime in d's factor tree.

3. Every factor of a number (except 1) is either prime or the product of a different combination of that number's prime factors. For example, $30 = 2 \times 3 \times 5$. Its factors are 1, 2, 3, 5, 6 (2×3), 10 (2×5), 15 (3×5) and 30 $(2 \times 3 \times 5)$. (ex. 98 has two 7's in its factors, and so is divisible by 49.)

4. To find *all* the factors of a number in an easy, methodical way, set up a factor pairs table.

5. To find *all* the prime factors of a number, use a factor tree. With larger numbers, start with the smallest primes and work your way up to larger primes.

Check Your Skills

16. The prime factorization of a number is 3×5. What is the number and what are all its factors?
17. The prime factorization of a number is $2 \times 5 \times 7$. What is the number and what are all its factors?
18. The prime factorization of a number is $2 \times 3 \times 13$. What is the number and what are all its factors?

Answers can be found on pages 47–48.

Unknown Numbers and Divisibility

Let's say that you are told some unknown positive number x is divisible by 6. How can you represent this on paper? There are many ways, depending on the problem. You could say that you know that x is a multiple of 6, or you could say that $x = 6 \times$ an integer. You could also represent the information with a factor tree. Careful though—although we've had a lot of practice drawing factor trees, there is one important difference now that we're dealing with an unknown number. We know that x is divisible by 6, but x may be divisible by other numbers as well. We have to treat what they have told us as incomplete information, and remind ourselves there are other things about x we don't know. To represent that on the page, our factor tree could look like this:

```
  x
 / \
6   ?
```

Now the question becomes—what else do we know about x? If a question on the GRE told you that x is divisible by 6, what could you definitely say about x? Take a look at these three statements, and for each statement, decide whether it *must* be true, whether it *could* be true, or whether it *cannot* be true.

I. x is divisible by 3

II. x is even

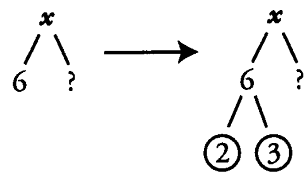
III x is divisible by 12

We'll deal with each statement one at a time. Let's begin with statement I—x is divisible by 3. One approach to take here is to think about the multiples of 6. If x is divisible by 6, then we know that x is a multiple of 6. Let's list out the first several multiples of 6, and see if they're divisible by 3.

	6	$6 \div 12 = 0.5$	
	12	$12 \div 12 = 1$	
x is a number on this list	18	$18 \div 12 = 1.5$	Some, but not all, of these numbers are also divisible by 12.
	24	$24 \div 12 = 2$	
	...	...	

At this point, we can be fairly certain that x is divisible by 3. In fact, listing out possible values of a variable is often a great way to begin answering a question in which you don't know the value of the number you are asked about.

But can we do better than say we're fairly certain x is divisible by 3? Is there a way to definitively say x *must* be divisible by 3? As it turns out, there is. Let's return to our factor tree, but let's make one modification to it.

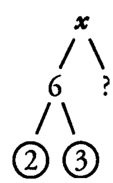


Remember, the ultimate purpose of the factor tree is to break numbers down into their fundamental building blocks: prime numbers. Now that the factor tree is broken down as far as it will go, we can apply the factor foundation rule. x is divisible by 6, and 6 is divisible by 3, so we can say definitively that x *must* be divisible by 3.

In fact, questions like this one are the reason we spent so much time discussing the factor foundation rule and the connection between prime factors and divisibility. Prime factors provide the foundation for a way to make definite statements about divisibility. With that in mind, let's look at statement II.

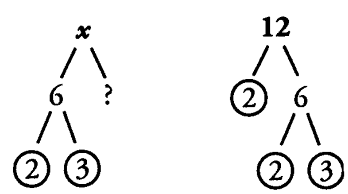
Statement II says x is even. This question is about divisibility, so the question becomes, what is the connection between divisibility and a number being even? Remember, an important part of this test is the ability to make inferences based on the given information.

What's the connection? Well, being even means being divisible by 2. So if we know that x is divisible by 2, then we can guarantee that x is even. Let's return to our factor tree.



We can once again make use of the factor foundation rule—6 is divisible by 2, so we know that x *must* be divisible by 2 as well. And if x is divisible by 2, then we know that x *must* be even as well.

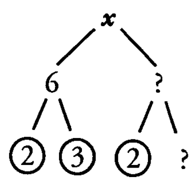
That just leaves the final statement. Statement III says x is divisible by 12. Let's look at this question from the perspective of factor trees. Let's compare the factor tree of x with the factor tree of 12.



What would we have to know about x to guarantee that it is divisible by 12? Well, when 12 is broken down all the way, we see that 12 is $2 \times 2 \times 3$. 12's building blocks are two 2's and a 3. For x to be divisible by 12, it would have to also have two 2's and one 3 among its prime factors. In other words, for x to be divisible by 12, it has to be divisible by *everything* that 12 is divisible by.

We need x to be divisible by two 2's and one 3 in order to say it *must* be divisible by 12. But looking at our factor tree, we only see one 2 and one 3. Because there is only one 2, we can't say that x *must* be divisible by 12. But then the question becomes, *could* x be divisible by 12? Think about the question for a second, and then keep reading.

The key to this question is the question mark that we put on x's factor tree. That question mark reminds us that we don't know everything about x. x could have other prime factors. What if one of those unknown factors was another 2? Then our tree would look like this:



So *if* one of those unknown factors were a 2, then x would be divisible by 12. The key here is that we have no way of knowing for sure whether there is a 2. x may be divisible by 12, it may not. In other words, x *could* be divisible by 12.

To confirm this, we can go back to the multiples of 6. We still know that x must be a multiple of 6, so let's start by listing out the first several multiples and see if they are divisible by 12.

x is a number on this list	6	$6 \div 3 = 2$	All of these numbers are also divisible by 3.
	12	$12 \div 3 = 4$	
	18	$18 \div 3 = 6$	
	24	$24 \div 3 = 8$	
	...	...	

Once again, we see that some of the possible values of x are divisible by 12, and some aren't. The best we can say is that x *could* be divisible by 12.

Check Your Skills

For these statements, the following is true: x is divisible by 24. For each statement, say whether it *must* be true, *could* be true, or *cannot* be true.

19. x is divisible by 6
20. x is divisible by 9
21. x is divisible by 8

Answers can be found on pages 48–49.

Let's answer another question, this time with an additional twist. Once again, there will be three statements. Decide whether each statement *must* be true, *could* be true, or *cannot* be true. Answer this question on your own, then we'll discuss each statement one at a time on the next page.

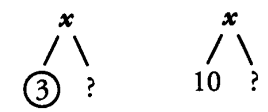
x is divisible by 3 and by 10.

I. x is divisible by 2

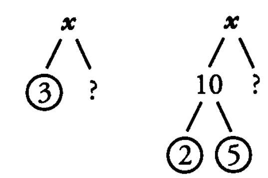
II. x is divisible by 15

III. x is divisible by 45

Before we dive into the statements, let's spend a moment to organize the information the question has given us. We know that x is divisible by 3 and by 10, so we can create two factor trees to represent this information

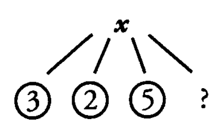


Now that we have our trees, let's get started with statement I. Statement I says that x is divisible by 2. The way to determine whether this statement is true should be fairly familiar by now—we need to use the factor foundation rule. First of all, our factor trees aren't quite finished. Factor trees should always be broken down all the way until every branch ends in a prime number. Really, our factor trees should look like this:



Now we are ready to decide whether statement I is true. x is divisible by 10, and 10 is divisible by 2, so we know that x is divisible by 2. Statement I *must* be true.

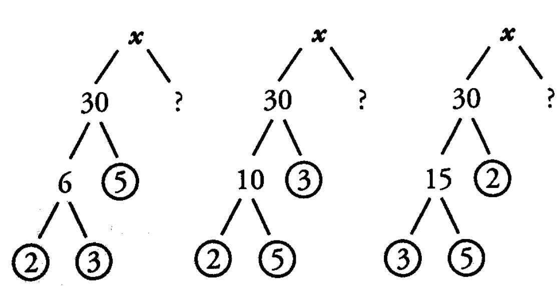
That brings us to statement II. This statement is a little more difficult. It also requires us to take another look at our factor trees. We have two separate trees, but they're giving us information about the same variable—x. Neither tree gives us complete information about x, but we do know a couple of things with absolute certainty. From the first tree, we know that x is divisible by 3, and from the second tree we know that x is divisible by 10—which really means we know that x is divisible by 2 and by 5. We can actually combine those two pieces of information and represent them on one factor tree.



Now we know three prime factors of x: 2, 3 and 5. Let's return to the statement. Statement II says that x is divisible by 15. What do we need to know to say that x *must* be divisible by 15? If we can guarantee that x has all the prime factors that 15 has, then we can guarantee that x is divisible by 15.

15 breaks down into the prime factors 3 and 5. So to guarantee that x is divisible by 15, we need to know it's divisible by 3 and 5. Looking back up at our factor tree, we see that x has both a 3 and a 5, which means that we know x is divisible by 15. Therefore, statement II *must* be true.

We can also look at this question more visually. Remember, prime factors are like building blocks—we also know that x is divisible by any combination of these prime factors. We can combine the prime factors in a number of different ways.

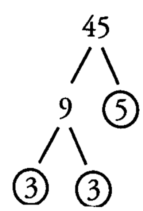


Each of these factor trees can tell us different factors of x. But what's really important is what they have in common. No matter what way you combine the prime factors, each tree ultimately leads to $2 \times 3 \times 5$, which equals 30. So we know that x is divisible by 30. And if x is divisible by 30, it is also divisible by everything 30 is divisible by. We know how to identify every number 30 is divisible by—we can use a factor pair table. The factor pair table of 30 looks like this.

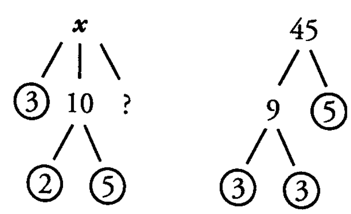
Small	**Large**
1	30
2	15
3	10
5	6

Again, statement II says that x is divisible by 15. We know x is divisible by 30, and 30 is divisible by 15, so x *must* be divisible by 15.

That brings us to statement III. Statement III says that x is divisible by 45. What do we need to know to say that x *must* be divisible by 45? Build a factor tree of 45, which looks like this.



45 is divisible by 3, 3 and 5. For x to be divisible by 45, we need to know that it has all the same prime factors. Does it?



45 has one 5 and two 3's. We know that x has a 5, but we only know that x has one 3. That means that we can't say for sure that x is divisible by 45. x *could* be divisible by 45, because we don't know what the question mark contains. If it contains a 3, then x is divisible by 45. If it doesn't contain a 3, then x is not divisible by 45. Without more information, we can't say for sure either way. So statement III *could* be true.

Now it's time to recap what we've covered in this chapter. When we deal with questions about divisibility, we need a quick, accurate way to identify *all* the factors of a number. A factor pair table provides a reliable way to make sure you find every factor of a number.

Prime factors provide essential information about a number or variable. They are the fundamental building blocks of every number. In order for a number or variable to be divisible by another number, it must contain all the same prime factors that the other number contains. In our last example, we could definitely say that x was divisible by 15, because x contained a 3 and a 5. But we could not say that it was divisible by 45, because 45 has a 5 and two 3's, but x only had a 5 and one 3.

Check Your Skills

For these statements, the following is true: x is divisible by 28 and by 15. For each statement, say whether it *must* be true, *could* be true, or *cannot* be true.

22. x is divisible by 14.
23. x is divisible by 20.
24. x is divisible by 24.

Answers can be found on page 49–50.

Fewer Factors, More Multiples

Sometimes it is easy to confuse factors and multiples. The mnemonic "Fewer Factors, More Multiples" should help you remember the difference. Factors divide into an integer and are therefore less than or equal to that integer. Positive multiples, on the other hand, multiply out from an integer and are therefore greater than or equal to that integer.

Any integer only has a limited number of factors. For example, there are only four factors of 8: 1, 2, 4, and 8. By contrast, there is an infinite number of multiples of an integer. For example, the first 5 positive multiples of 8 are 8, 16, 24, 32, and 40, but you could go on listing multiples of 8 forever.

Factors, multiples, and divisibility are very closely related concepts. For example, 3 is a factor of 12. This is the same as saying that 12 is a multiple of 3, or that 12 is divisible by 3.

On the GRE, this terminology is often used interchangeably in order to make the problem seem harder than it actually is. Be aware of the different ways that the GRE can phrase information about divisibility. Moreover, try to convert all such statements to the same terminology. For example, **all** of the following statements **say exactly the same thing**:

- 12 is divisible by 3
- 12 is a multiple of 3
- $\frac{12}{3}$ is an integer
- $12 = 3n$, where n is an integer
- 12 items can be shared among 3 people so that each person has the same number of items.
- 3 is a divisor of 12, or 3 is a factor of 12
- 3 divides 12
- $\frac{12}{3}$ yields a remainder of 0
- 3 "goes into" 12 evenly

Divisibility and Addition/Subtraction

If you add two multiples of 7, you get another multiple of 7. Try it: $35 + 21 = 56$. This should make sense: $(5 \times 7) + (3 \times 7) = (5 + 3) \times 7 = 8 \times 7$.

Likewise, if you subtract two multiples of 7, you get another multiple of 7. Try it: $35 - 21 = 14$. Again, we can see why: $(5 \times 7) - (3 \times 7) = (5 - 3) \times 7 = 2 \times 7$.

This pattern holds true for the multiples of any integer N. **If you add or subtract multiples of N, the result is a multiple of N.** You can restate this principle using any of the disguises above: for instance, if N is a divisor of x and of y, then N is a divisor of $x + y$.