Exponents and Roots

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box $\boxed{}$, you are to enter your own answer in the box. For questions followed by a fraction-style numeric entry box $\frac{\boxed{}}{\boxed{}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is $\frac{1}{4}$, you may enter $\frac{25}{100}$ or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations, such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

	Quantity A	Quantity B
1.	25^7	5^{15}

$216 = 2^x3^y$
x and y are integers.

	Quantity A	Quantity B
2.	x	y

	Quantity A	Quantity B
3.	$\sqrt{9}\sqrt{25}$	$\sqrt{15}$

	Quantity A	Quantity B
4.	$\sqrt{3}+\sqrt{6}$	$\sqrt{9}$

5. If $5{,}000 = 2^x5^y$ and x and y are integers, what is the value of $x + y$?

$\boxed{}$

6. If $3^2 9^2 = 3^x$, what is the value of x?

(A) 2
(B) 3
(C) 4
(D) 5
(E) 6

80 is divisible by 2^x.

	Quantity A	**Quantity B**
7.	x	3

8. If $17\sqrt[3]{m} = 34$, what is the value of $6\sqrt[3]{m}$?

$\boxed{}$

9. $\dfrac{\frac{1}{1}}{\frac{1}{5^{-2}}}$ is equal to which of the following?

(A) $\dfrac{1}{25}$
(B) $\dfrac{1}{5}$
(C) 1
(D) 5
(E) 25

10. $\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{4}}}}$ is equal to which of the following?

(A) $\sqrt{2}$
(B) 2

(C) $2\sqrt{2}$

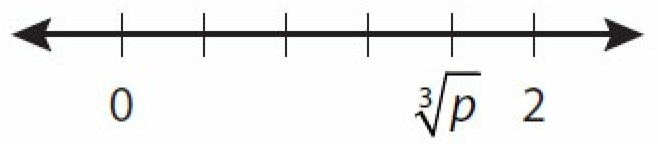
(D) 4

(E) $4\sqrt{2}$

	Quantity A	**Quantity B**
11.	$10^6 + 10^5$	$10^7 + 10^4$

12. For which of the following positive integers is the square of that integer divided by the cube root of the same integer equal to nine times that integer?

(A) 4
(B) 8
(C) 16
(D) 27
(E) 125

13. 

If the hash marks above are equally spaced, what is the value of p?

(A) $\frac{3}{2}$

(B) $\frac{8}{5}$

(C) $\frac{24}{15}$

(D) $\frac{512}{125}$

(E) $\frac{625}{256}$

14. What is the greatest prime factor of $2^{99} - 2^{96}$?

15. If $2^k - 2^{k+1} + 2^{k-1} = 2^k m$, what is the value of m?

(A) -1

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) 1

(E) 2

16. If $5^{k+1} = 2{,}000$, what is the value of $5^k + 1$?

(A) 399
(B) 401
(C) 1,996
(D) 2,000
(E) 2,001

17. If $3^{11} = 9^x$, what is the value of x?

18. If $\sqrt[5]{x^6} = x^{\frac{a}{b}}$, what is the value of $\frac{a}{b}$?

Give your answer as a fraction.

19. Which of the following is equal to $\dfrac{10^{-8}25^{7}2^{16}}{20^{6}8^{-1}}$?

(A) $\dfrac{1}{5}$
(B) $\dfrac{1}{2}$
(C) 2
(D) 5
(E) 10

20. If $\dfrac{5^7}{5^{-4}} = 5^a$, $\dfrac{2^{-3}}{2^{-2}} = 2^b$, and $3^8(3) = 3^c$, what is the value of $a + b + c$?

21. If 12^x is odd and x is an integer, what is the value of x^{12}?

22. What is the value of $\dfrac{44^{\frac{5}{2}}}{\sqrt{11^3}}$?

$$\frac{(10^3)(0.027)}{(900)(10^{-2})} = (3)(10^m)$$

	Quantity A	**Quantity B**
23.	m	3

24. Which of the following equals $\dfrac{2^2 + 2^2 + 2^3 + 2^4}{\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}-\sqrt{3}\right)}$?

(A) 2
(B) 4
(C) 8
(D) 16
(E) 32

25. If $\dfrac{0.000027 \times 10^x}{900 \times 10^{-4}}$ = 0.03 × 10^{11}, what is the value of x?

(A) 13
(B) 14
(C) 15
(D) 16
(E) 17

26. Which of the following equals $\left(\sqrt[2]{x}\right)\left(\sqrt[3]{x}\right)$?

(A) $\sqrt[5]{x}$

(B) $\sqrt[6]{x}$

(C) $\sqrt[3]{x^2}$

(D) $\sqrt[5]{x^6}$

(E) $\sqrt[6]{x^5}$

$n = 0.00025 \times 10^4$ and $m = 0.005 \times 10^2$

	Quantity A	Quantity B
27.	$\frac{n}{m}$	0.5

28. If $2^2 < \frac{x}{2^6 - 2^4} < 2^3$, which of the following could be the value of x?

Indicate <u>all</u> such values.

- ☐ 24
- ☐ 64
- ☐ 80
- ☐ 128
- ☐ 232
- ☐ 256

29. Which of the following is equal to $x^{\frac{3}{2}}$?

(A) $x^2\sqrt{x}$

(B) $x\sqrt{x}$

(C) $\sqrt[3]{x^2}$

(D) $\sqrt[3]{x}$

(E) $(x^3)^2$

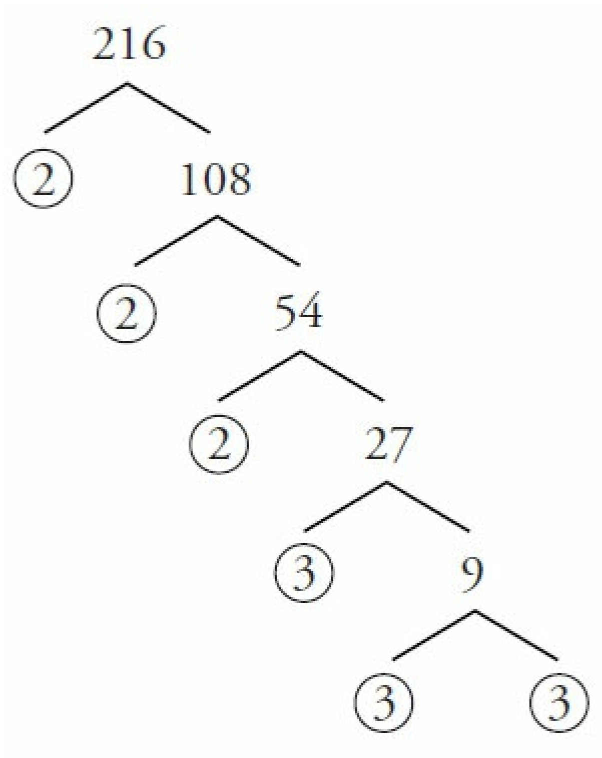
30. If $125^{14}48^8$ were expressed as an integer, how many consecutive zeros would that integer have immediately to the left of its decimal point?

(A) 22
(B) 32
(C) 42
(D) 50
(E) 112

Exponents and Roots Answers

1. **(B).** If a problem combines exponents with different bases, convert to the same base if possible. Since $25 = 5^2$, Quantity A is equal to $(5^2)^7$. Apply the appropriate exponent formula: $(a^b)^c = a^{bc}$. Quantity A is equal to 5^{14}, thus Quantity B is greater.

2. **(C).** Construct a prime factor tree for 216:

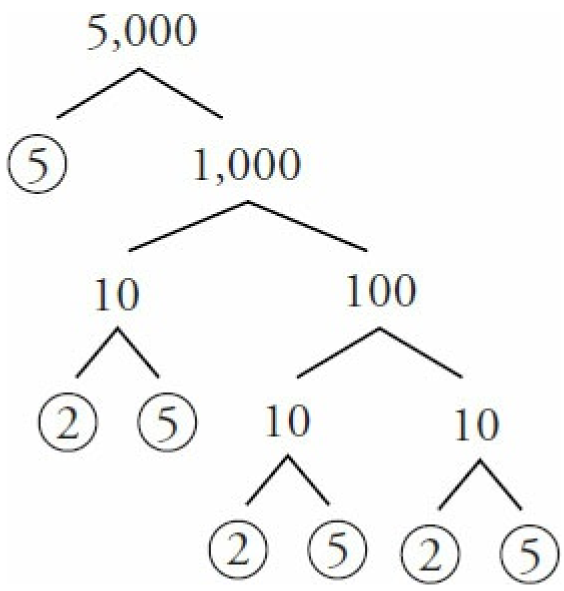


$216 = 2^3 3^3$, so $x = 3$ and $y = 3$.

3. **(A).** In Quantity A, $\sqrt{9}\sqrt{25} = 3 \times 5 = 15$. Since 15 is greater than $\sqrt{15}$, Quantity A is greater.

4. **(A).** You may *not* add $\sqrt{3}$ and $\sqrt{6}$ to get $\sqrt{9}$, but you can put each value in the calculator. $\sqrt{3} = 1.732 \ldots$ and $\sqrt{6} = 2.449 \ldots$, and their sum is about 4.18. Since Quantity B is $\sqrt{9} = 3$, Quantity A is greater.

5. **7.** Construct a prime factor tree for 5,000:



Thus, $5{,}000 = 2^3 5^4$, therefore $x = 3$ and $y = 4$, and the answer is $3 + 4 = 7$.

6. **(E).** In order to compare or combine exponents with different bases, convert to the same base if possible. Since $9 = 3^2$:

$$3^2(3^2)^2 = 3^x$$

Multiply exponents in accordance with the exponent formula, $(a^b)^c = a^{bc}$:

$$3^2 3^4 = 3^x$$

Add the exponents to multiply numbers that have the same base:

$$3^6 = 3^x$$

Therefore, $x = 6$.

7. **(D).** Construct a prime factor tree for 80; it has four factors of 2 and one factor of 5.

That doesn't mean x is 4, however! The problem does not say "80 is equal to 2^x". Rather, it says "divisible by."

80 is divisible by 2^4, and therefore also by 2^3, 2^2, 2^1, and 2^0 (any non-zero number to the 0th power equals 1). Thus, x could be 0, 1, 2, 3, or 4, and could therefore be less than, equal to, or greater than 3. Thus, the relationship cannot be determined.

8. **12.** This question looks much more complicated than it really is—note that $\sqrt[3]{m}$ is in both the given equation and the question. Just think of $\sqrt[3]{m}$ as a very fancy variable that you don't have to break down:

$$17\sqrt[3]{m} = 34$$

$$\sqrt[3]{m} = \frac{34}{17}$$

$$\sqrt[3]{m} = 2$$

Therefore, $6\sqrt[3]{m} = 6(2) = 12$.

9. **(A).** This question requires recognizing that a negative exponent in the denominator turns into a positive exponent in the numerator. In other words, the lowermost portion of the fraction, $\frac{1}{5^{-2}}$, is

equal to 5^2. The uppermost portion of the fraction, $\frac{1}{1}$, is just equal to 1.

Putting these together, the original fraction can be simplified. $\dfrac{\frac{1}{1}}{\frac{1}{5^{-2}}} = \dfrac{1}{5^2} = \dfrac{1}{25}$, which is the final answer.

10. **(B).** To solve, start at the “inner core”—that is, the physically smallest root sign:

$$\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{4}}}} =$$
$$\sqrt{2+\sqrt{2+\sqrt{2+2}}} =$$
$$\sqrt{2+\sqrt{2+2}} =$$
$$\sqrt{2+2} = 2$$

11. **(B).** Be careful! These quantities are not equal! When *multiplying* exponents with the same base, it is correct to add the exponents:

$10^6 \times 10^5 = 10^{11}$

However, numbers raised to powers cannot be directly combined by addition or subtraction. Instead, sum this way:

Quantity A = $10^6 + 10^5 = 1{,}000{,}000 + 100{,}000 = 1{,}100{,}000$

Quantity B = $10^7 + 10^4 = 10{,}000{,}000 + 10{,}000 = 10{,}010{,}000$

Thus, Quantity B is greater.

Alternatively, you can do some fancy factoring. The distributive property is a big help here: $ab + ac = a(b + c)$. In other words, factor out the a.

Factor out 10^5 in Quantity A:

$10^6 + 10^5 = 10^5(10^1 + 1) = 10^5(11) \cong 10^6$

Factor out 10^4 in Quantity B:

$10^7 + 10^4 = 10^4(10^3 + 1) = 10^4(1{,}001) \cong 10^7$

The approximation in the last step is just to make the point that you don’t have to be too precise: Quantity B is about 10 times greater than Quantity A.

12. **(D).** To solve this question, translate the text into an equation. Call “the square of that integer” x^2, “the cube root of the same integer” $\sqrt[3]{x}$, and “nine times that integer” $9x$:

$$\frac{x^2}{\sqrt[3]{x}} = 9x$$

Test the answers; doing so shows that choice (D) is correct:

$$\begin{aligned} \frac{27^2}{\sqrt[3]{27}} &= 9(27) \\ \frac{27^2}{3} &= 9(27) \\ 27^2 &= 9(27)(3) \\ 27 &= 9(3) \end{aligned}$$

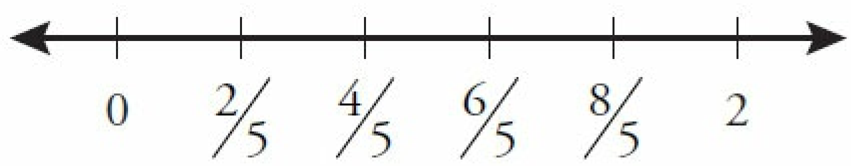
Choices (A) and (C) are not likely to be correct because the cube roots of 4 and 16, respectively, are not integers; test the others first:

Choice (B): $\frac{64}{2} = 9(8)$? No.

Choice (D): Correct as shown above.

Choice (E): $\frac{15{,}626}{5} = 9(125)$? No.

13. **(D).** To determine the distance between hash marks, divide 2 (the distance from 0 to 2) by 5 (the number of segments the number line has been divided into). The result is $\frac{2}{5}$. Therefore:



Note that 2 is equal to $\frac{10}{5}$, so the number line is labeled correctly.

Since $\sqrt[3]{p}$ marks the same hash mark on the number line as $\frac{8}{5}$:

$$\sqrt[3]{p} = \frac{8}{5}$$
$$p = \left(\frac{8}{5}\right)^3$$
$$p = \frac{512}{125}$$

The answer is (D). Watch out for trap answer choice (B), which represents $\sqrt[3]{p}$, not *p*.

14. **7**. You cannot subtract $2^{99} - 2^{96}$ to get 2^3! You cannot directly combine numbers raised to powers when adding or subtracting. (As it turns out, the difference between 2^{99} and 2^{96} is much, much greater than 2^3.) Instead, factor out the greatest common factor of 2^{99} and 2^{96} :

$$2^{99} - 2^{96} = 2^{96}\,(2^3 - 1) = 2^{96}\,(7)$$

Since $2^{99} - 2^{96}$ is equal to $2^{96}\,7^1$, its greatest prime factor is 7.

15. **(B).** First, factor 2^{k+1} into 2^k2^1 and 2^{k-1} into 2^k2^{-1}:

$$2^k - 2^k2^1 + 2^k2^{-1} = 2^km$$

Factor out 2^k from the left, then cancel 2^k from both sides:

$$\begin{aligned} 2^k(1 - 2^1 + 2^{-1}) &= 2^km \\ 1 - 2^1 + 2^{-1} &= m \\ 1 - 2 + \frac{1}{2} &= m \\ -\frac{1}{2} &= m \end{aligned}$$

16. **(B).** The key to solving this problem is to understand that 5^{k+1} can be factored into 5^k5^1. (Exponents are added when multiplying numbers with the same base, so the process can also be reversed; thus, any expression with the form x^{a+b} can be split into x^ax^b.) Thus:

$$5^{k+1} = 2{,}000$$

$$5^k5^1 = 2{,}000$$

Now divide both sides by 5:

$$5^k = 400$$

So, $5^k + 1 = 401$.

Notice that you can't solve for k itself—k is not an integer, since 400 is not a "normal" power of 5. But you don't need to solve for k. You just need 5^k.

17. **5.5.** Begin by converting 9 to a power of 3:

$$\begin{aligned} 3^{11} &= (3^2)^x \\ 3^{11} &= 3^{2x} \end{aligned}$$

Thus, $11 = 2x$ and $x = 5.5$.

18. $\frac{6}{5}$. The square root of a number equals that number to the $\frac{1}{2}$ power, so too is a fifth root the same

as a $\frac{1}{5}$ exponent. Thus:

$$\sqrt[5]{x^6} = (x^6)^{\frac{1}{5}} = x^{\frac{6}{5}}$$

Since $x^{\frac{6}{5}} = x^{\frac{a}{b}}$, $\frac{a}{b} = \frac{6}{5}$.

19. **(B).** Since $10^{-8} = \frac{1}{10^8}$ and $\frac{1}{8^{-1}} = 8^1$, first substitute to convert any term with negative exponents to one with a positive exponent:

$$\frac{10^{-8}25^7 2^{16}}{20^6 8^{-1}} = \frac{25^7 2^{16} 8^1}{10^8 20^6}$$

Then, convert the non-prime terms to primes, combining and canceling where possible:

$$\frac{25^7 2^{16} 8^1}{10^8 20^6} = \frac{\left(5^2\right)^7 2^{16}\left(2^3\right)^1}{\left(2^1 5^1\right)^8\left(2^2 5^1\right)^6} = \frac{5^{14}2^{16}2^3}{2^8 5^8 2^{12} 5^6} = \frac{5^{14}2^{19}}{2^{20}5^{14}} = \frac{1}{2}$$

20. **19.** To solve this problem, you need to know that to divide numbers with the same base, subtract the exponents, and to multiply them, add the exponents. Thus:

$\frac{5^7}{5^{-4}} = 5^{7-(-4)} = 5^{11}$, so $a = 11$.

$\frac{2^{-3}}{2^{-2}} = 2^{-3-(-2)} = 2^{-1}$, so $b = -1$.

$3^8(3) = 3^8(3^1) = 3^9$, so $c = 9$.

Therefore, $a + b + c = 11 + (-1) + 9 = 19$.

21. **0.** This is a bit of a trick question. 12^x is odd? How strange! 12^1 is 12, 12^2 is 144, 12^3 is 1,728 … every "normal" power of 12 is even. (An even number such as 12 multiplied by itself any number of times will yield an even answer.) These normal powers are 12 raised to a positive integer. What about negative integer exponents? They are all fractions of this form: $\frac{1}{12^{\text{positive integer}}}$.

The only way for 12^x to be odd is for x to equal 0. Any non-zero number to the 0th power is equal to 1. Since $x = 0$ and the question asks for x^{12}, the answer is 0.

22. **352.** A square root is the same as a $\frac{1}{2}$ exponent, so $\frac{44^{\frac{5}{2}}}{\sqrt{11^3}} = \frac{44^{\frac{5}{2}}}{\left(11^3\right)^{\frac{1}{2}}} = \frac{44^{\frac{5}{2}}}{11^{\frac{3}{2}}}$.

The common factor of 44 and 11 is 11, so factor the numerator:

$$\frac{44^{\frac{5}{2}}}{11^{\frac{3}{2}}} = \frac{11^{\frac{5}{2}}4^{\frac{5}{2}}}{11^{\frac{3}{2}}}$$

When dividing exponential expressions that have a common base, subtract the exponents:

$$\left(\frac{11^{\frac{5}{2}}}{11^{\frac{3}{2}}}\right)4^{\frac{5}{2}} = \left(11^{\frac{5}{2}-\frac{3}{2}}\right)4^{\frac{5}{2}} = \left(11^{\frac{2}{2}}\right)4^{\frac{5}{2}} = \left(11^{1}\right)4^{\frac{5}{2}}$$

Now simplify the 4 term, again noting that a $\frac{1}{2}$ exponent is the same as a square root:

$$\left(11^{1}\right)4^{\frac{5}{2}} = (11)\left(4^{\frac{1}{2}}\right)^{5} = (11)\left(\sqrt{4}\right)^{5} = (11)\left(2^{5}\right) = 352$$

23. **(B).** Since $(10^3)(0.027)$ is 27 and $(900)(10^{-2})$ is 9:

$$\frac{27}{9} = (3)(10^m)$$

$$3 = 3(10^m)$$

$$1 = 10^m$$

You might be a little confused at this point as to how 10^m can equal 1. However, you can still answer the question correctly. If m were 3, as in Quantity B, 10^m would equal 1,000. However, 10^m actually equals 1. So m must be less than 3.

As it turns out, the only way 10^m can equal 1 is if $m = 0$. Any non-zero number to the 0th power is equal to 1.

24. **(D).** You could factor 2^2 out of the numerator, but the numbers are small enough that you might as well just say that the numerator is $4 + 4 + 8 + 16 = 32$.

FOIL the denominator:

$$(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})$$

$$\sqrt{25}+\sqrt{3}\sqrt{5}-\sqrt{3}\sqrt{5}-\sqrt{9}$$

$$\sqrt{25}-\sqrt{9}$$

$5 - 3 = 2$

$\frac{32}{2}$ = 16 is the final answer.

25. **(A).** One good approach is to convert 0.000027, 900, and 0.03 to powers of 10:

$$\frac{27\times10^{-6}\times10^{x}}{9\times10^{2}\times10^{-4}}=3\times10^{-2}\times10^{11}$$

Now combine the exponents from the terms with base 10:

$$\frac{27\times10^{-6+x}}{9\times10^{-2}}=3\times10^{9}$$

Since $\frac{27}{9}$ = 3, cancel the 3 from both sides, then combine powers of 10:

$$\begin{aligned}\frac{10^{-6+x}}{10^{-2}} &= 10^9\\ 10^{-6+x-(-2)} &= 10^9\\ 10^{-4+x} &= 10^9\end{aligned}$$

Thus, $-4 + x = 9$, and $x = 13$.

26. **(E).** A good first step is to convert to fractional exponents. A square root is the same as the $\frac{1}{2}$ power and a cube root is the same as the $\frac{1}{3}$ power:

$$x^{\frac{1}{2}}x^{\frac{1}{3}}=x^{\frac{1}{2}+\frac{1}{3}}=x^{\left(\frac{3}{6}+\frac{2}{6}\right)}=x^{\frac{5}{6}}=\sqrt[6]{x^5}$$

27. **(A).** To simplify 0.00025×10^4, move the decimal in 0.00025 four places to the right to get 2.5. To simplify 0.005×10^2, move the decimal in 0.005 two places to the right to get 0.5. Thus, $n = 2.5$, $m = 0.5$, and $\frac{n}{m}=\frac{2.5}{0.5}$ = 5.

28. **232 and 256 only.** The inequality could be simplified using exponent rules, but all the numbers are small enough either to have memorized or to quickly calculate:

$$2^2<\frac{x}{2^6-2^4}<2^3$$

$$4<\frac{x}{64-16}<8$$

$$4<\frac{x}{48}<8$$

To isolate x, multiply all three parts of the inequality by 48:

$192 < x < 384$

The only choices in this range are 232 and 256.

29. **(B).** Since a number to the $\frac{1}{2}$ power equals the square root of that number, $x^{\frac{3}{2}}$ could also be written as $\sqrt{x^3}$. This, however, does not appear in the choices. Note, however, that $\sqrt{x^3}$ can be simplified:

$$\sqrt{x^2 \times x}$$

$$\sqrt{x^2} \times \sqrt{x}$$
$$x\sqrt{x}$$

This matches choice (B). Alternatively, convert the answer choices. For instance, in incorrect choice (A), $x^2\sqrt{x} = x^2x^{\frac{1}{2}} = x^{\frac{5}{2}}$. Since this is not equal to $x^{\frac{3}{2}}$, eliminate (A). Correct choice (B) can be converted as such: $x\sqrt{x} = x^1x^{\frac{1}{2}} = x^{\frac{3}{2}}$.

31. **(B).** Exponents questions usually involve prime factorization, because you always want to find common bases, and the fundamental common bases are prime numbers. Test some values to see what leads to zeros at the end of an integer.

$10 = 5 \times 2$
$40 = 8 \times 5 \times 2$
$100 = 10 \times 10 = 2 \times 5 \times 2 \times 5$
$1,000 = 10 \times 10 \times 10 = 2 \times 5 \times 2 \times 5 \times 2 \times 5$

Ending zeros are created by 10's, each of which is the product of one 2 and one 5. So, to answer this question, determine how many pairs of 2's and 5's are in the expression:

$$125^{14}48^8 = (5^3)^{14} \times (2^4 \times 3)^8 = 5^{42} \times 2^{32} \times 3^8$$

Even though there are 42 powers of 5, there are only 32 powers of 2, so you can only form 32 pairs of one 5 and one 2.