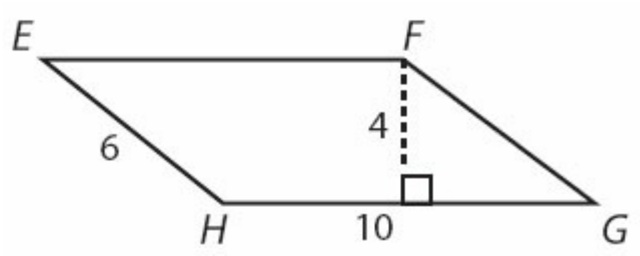
Polygons and Rectangular Solids

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

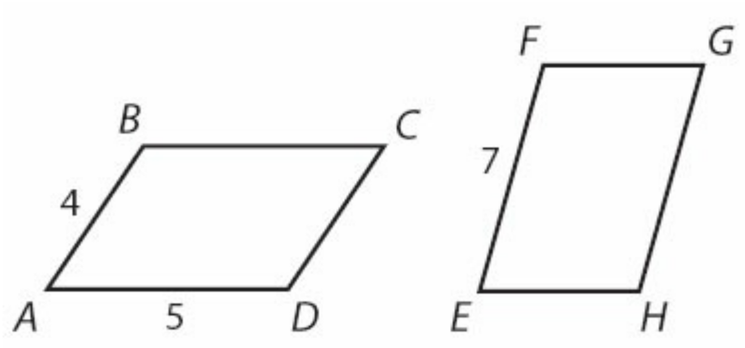
(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box, you are to enter your own answer in the box. For questions followed by a fraction-style numeric entry box, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is $\frac{1}{4}$, you may enter $\frac{25}{100}$ or any equivalent fraction.

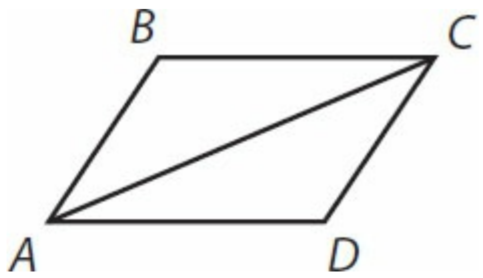
All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations, such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.



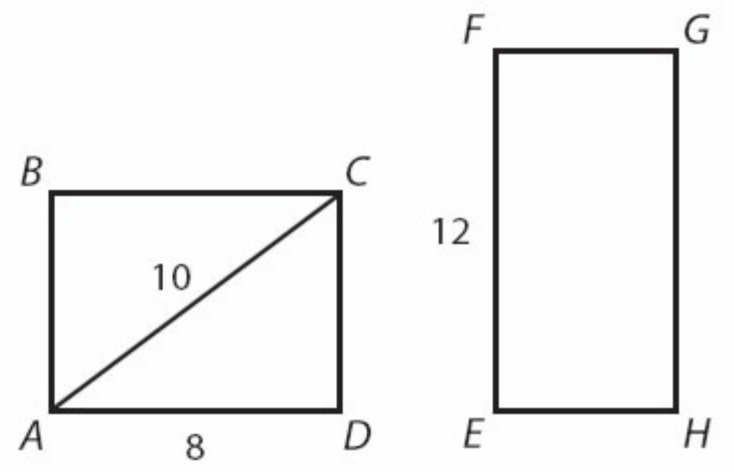
1. What is the area of parallelogram *EFGH*?



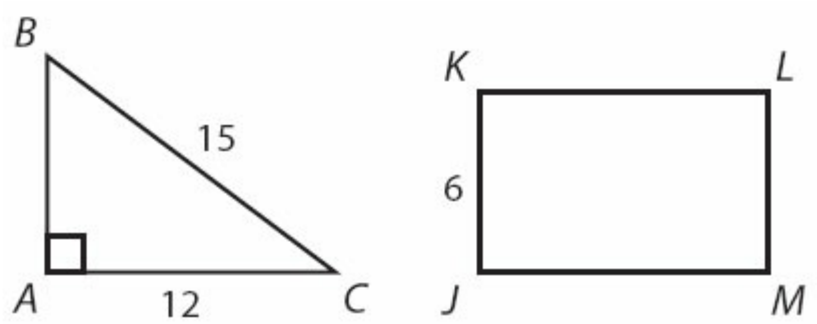
2. The two parallelograms pictured above have the same perimeter. What is the length of side *EH*?



3. In parallelogram *ABCD*, triangle *ABC* has an area of 12. What is the area of triangle *ACD*?



4. Rectangles *ABCD* and *EFGH* have equal areas. What is the length of side *FG*?

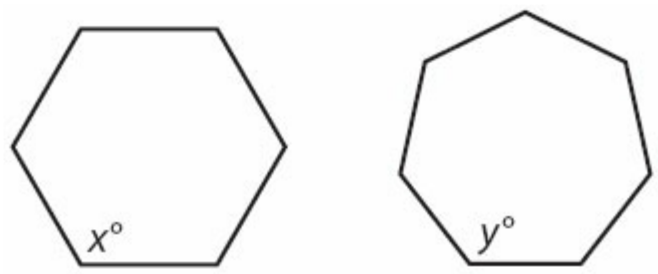


5. Triangle *ABC* and rectangle *JKLM* have equal areas. What is the perimeter of rectangle *JKLM*?

	Quantity A	Quantity B
6.	The area of a rectangle with perimeter 20	30

7. What is the area of a square with a diagonal measuring $6\sqrt{2}$?

	Quantity A	Quantity B
8.	The area of a parallelogram with a base of length 4 and height of 3.5	The area of a trapezoid with two parallel sides of lengths 5 and 9 and a height of 2



	Quantity A	Quantity B
9.	x	y

The perimeter of square W is 50% of the perimeter of square D.

	Quantity A	Quantity B
10.	The ratio of the area of square W to the area of square D	$\frac{1}{4}$

11. A 10-inch by 15-inch rectangular picture is displayed in a 16-inch by 24-inch rectangular frame. What is the area, in inches, of the part of the frame not covered by the picture?

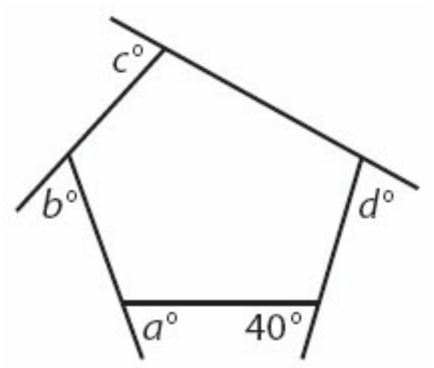
(A) 150
(B) 234
(C) 244
(D) 264
(E) 384

A rectangular box has edges of length 2, 3, and 4.

	Quantity A	Quantity B
12.	Twice the volume of the box	The surface area of the box

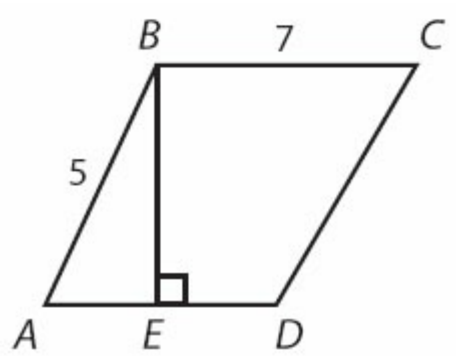
13. What is the maximum number of 2-inch by 2-inch by 2-inch solid cubes that can be cut from six solid cubes that are 1 foot on each side? (12 inches = 1 foot)

(A) 8
(B) 64
(C) 216
(D) 1,296
(E) 1,728



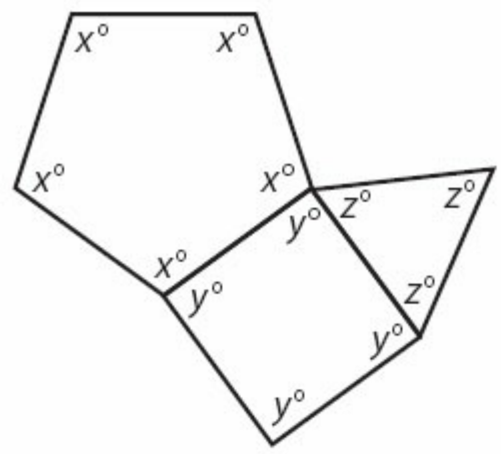
14. What is the value of $a + b + c + d$?

(A) 240
(B) 320
(C) 360
(D) 500
(E) 540



In the trapezoid above, $AE = ED = 3$ and BC is parallel to AD.

	Quantity A	Quantity B
15.	The area of the trapezoid	35



	Quantity A	Quantity B
16.	$x + y + z$	270

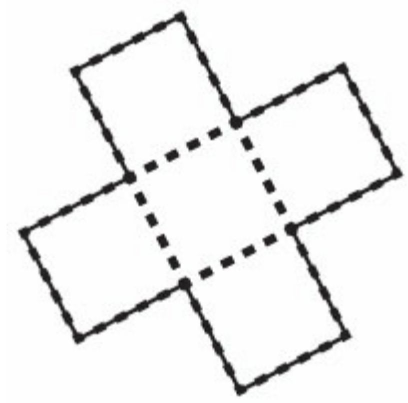
17. A 2-meter by 2-meter sheet of paper is to be cut into 2-centimeter by 10-centimeter rectangles. What is the maximum number of such rectangles that can be cut from the sheet of paper? (1 meter = 100 centimeters)

A parallelogram has two sides with length 10 and two sides with length 5.

	Quantity A	Quantity B
18.	The area of the parallelogram	30

19. What is the area of a regular hexagon of side length 4?

(A) $4\sqrt{3}$
(B) $6\sqrt{3}$
(C) $12\sqrt{3}$
(D) $24\sqrt{3}$
(E) $36\sqrt{3}$



The figure above is composed of 5 squares of equal area, as indicated by the dotted lines. The total area of the figure is 45.

	Quantity A	Quantity B
20.	The perimeter of the figure	48

21. A 2-foot by 2-foot by 2-foot solid cube is cut into 2-inch by 2-inch by 4-inch rectangular solids. What is the ratio of the total surface area of all the resulting smaller rectangular solids to the surface area of the original cube? (1 foot = 12 inches)

(A) 2 : 1
(B) 4 : 1
(C) 5 : 1
(D) 8 : 1
(E) 10 : 1

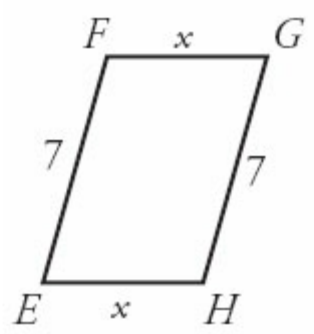
22. If a cube has the same volume (in cubic units) as surface area (in square units), what is the length of one side?

(A) 1
(B) 3
(C) $\frac{5}{3}$
(D) 6
(E) No such cube is possible.

Polygons and Rectangular Solids Answers

1. **40.** The area of a parallelogram is base × height. In this parallelogram, the base is 10 and the height is 4 (remember, base and height need to be perpendicular). So the area is $10 \times 4 = 40$.

2. **2.** First find the perimeter of parallelogram *ABCD*. If two sides have a length of 4, and two sides have a length of 5, the perimeter is $2(4 + 5) = 8 + 10 = 18$. That means parallelogram *EFGH* also has a perimeter of 18. Because *EF* is labeled 7, *GH* also is 7. The lengths of the other two sides are unknown but equal to each other, so for now say the length of each side is x. The parallelogram now looks like this:



Therefore, the perimeter is:

$$2(7 + x) = 18$$
$$2x + 14 = 18$$
$$2x = 4$$
$$x = 2$$

The length of side *EH* is 2.

3. **12.** One property that is true of any parallelogram is that the diagonal will split the parallelogram into two equal triangles. If triangle *ABC* has an area of 12, then triangle *ACD* must also have an area of 12.

4. **4.** Triangle *ACD* is a right triangle and two of the side lengths are labeled, so the length of *CD* can be determined. Either use the Pythagorean theorem or recognize that this is one of the Pythagorean triplets: a 6–8–10 triangle. The length of side *CD* is 6. The area of rectangle *ABCD* is $8 \times 6 = 48$.

The area of rectangle *EFGH* is also 48, so $12 \times FG = 48$. The length of side *FG* is 4.

5. **30.** To find the area of right triangle *ABC*, use the Pythagorean theorem to first find the length of side *AB*:

$$12^2 + AB^2 = 15^2$$
$$144 + AB^2 = 225$$

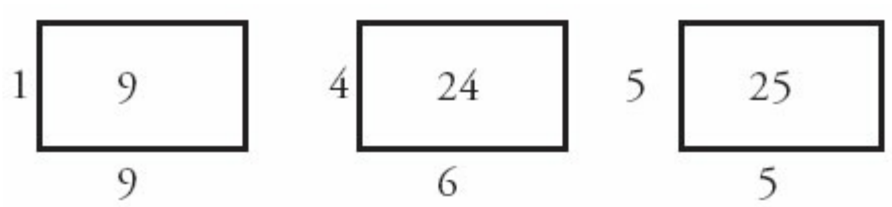
$AB^2 = 81$
$AB = 9$

(A 9–12–15 triangle is a 3–4–5 triangle, with all the measurements tripled.)

The area of triangle *ABC* is $\frac{1}{2}(12)(9) = \frac{108}{2} = 54$.

Rectangle *JKLM* also has an area of 54. One side of the rectangle is labeled, so solve for the other: $6 \times JM = 54$. The length of side *JM* is 9, so the perimeter of *JKLM* is $2(6 + 9) = 12 + 18 = 30$.

6. **(B).** While a rectangle with perimeter 20 could have many different areas, all of these areas are less than 30:



How can you be sure this will always be the case? It would be helpful to know the rule that the area of a rectangle with constant perimeter increases as length and width become more similar, and is maximized when the rectangle is a square. Thus, the 5-by-5 version of the rectangle represents the maximum possible area, which is still less than 30.

7. **36.** When a square is cut by a diagonal, two 45–45–90 triangles are created. Use the 45–45–90 formula (sides in the ratio $1 : 1 : \sqrt{2}$) to determine that the sides are equal to 6, and thus the area is $6 \times 6 = 36$. Alternatively, label each side of the square x (since they're the same) and use the Pythagorean theorem:

$$x^2 + x^2 = (6\sqrt{2})^2$$
$$2x^2 = 72$$
$$x^2 = 32$$
$$x = 16$$

If each side of the square is 6, the area is $6 \times 6 = 36$.

8. **(C).** The formula for area of a parallelogram is *base* × *height*, so Quantity A is $4 \times 3.5 = 14$.

The formula for area of a trapezoid is $A = \frac{(b_1 + b_2)}{2} \times h$, where b_1 and b_2 are the lengths of the parallel sides, so Quantity B is $\frac{(5+9)}{2} \times 2 = 14$.

The two quantities are equal.

9. **(D).** Do not assume that any polygon is a regular figure unless the problem explicitly or implicitly says so. (For instance, if *every* angle in the hexagon were labeled with the same variable, you could be sure the hexagon was regular.)

Using the formula $(n - 2)(180°)$ where n is the number of sides, calculate that the sum of the angles in the 6-sided figure is 720° and the sum of the angles in the 7-sided figure is 900°. However, those totals could be distributed any number of ways among the interior angles, so either x or y could be greater.

10. **(C).** If one square has twice the perimeter, it has twice the side length, so it will have four times the area. Why is this? Doubling only the length doubles the area. Then, doubling the width doubles the area *again*.

Alternatively, prove this with real numbers. Say square W has perimeter 8 and square D has perimeter 16. Thus, square W has side 2 and square D has side 4. The areas are 4 and 16, respectively. As a ratio, $\frac{4}{16}$ reduces to $\frac{1}{4}$. The two quantities are equal.

11. **(B).** The area of the picture is $10 \times 15 = 150$. The area of the frame is $16 \times 24 = 384$. Subtract to get the answer: $384 - 150 = 234$.

12. **(B).** The volume of a rectangular box is length × width × height. Therefore: $2 \times 3 \times 4 = 24$. Quantity A is double this volume, or 48.

The surface area of a rectangular box is 2(length × width) + 2(width × height) + 2(length × height). Therefore:
$2(6) + 2(12) + 2(8) = 52$.

Quantity B is greater.

13. **(D).** Each large solid cube is 12 inches × 12 inches × 12 inches. Each dimension (length, width, and height) is to be cut identically at 2 inch increments, creating 6 smaller cubes in each dimension. Thus, $6 \times 6 \times 6$ small cubes can be cut from each large cube. There are 6 large cubes to be cut this way, though, so the total number of small cubes that can be cut is $6(6 \times 6 \times 6) = 6 \times 216 = 1{,}296$.

14. **(B).** The interior figure shown is a pentagon, although an irregular one. The sum of the interior angles of any polygon can be determined using the formula $(n - 2)(180°)$, where n is the number of sides:

$$(5 - 2)(180°) = (3)(180°) = 540°$$

Using the rule that angles forming a straight line sum to 180°, the interior angles of the pentagon (starting at the top and going clockwise) are $180 - c$, $180 - d$, 140, $180 - a$, and $180 - b$. The sum of these angles can be set equal to 540:

$$\begin{aligned} 540 &= (180 - c) + (180 - d) + 140 + (180 - a) + (180 - b) \\ 540 &= 140 + 4(180) - a - b - c - d \\ 540 - 140 - 720 &= -(a + b + c + d) \\ -320 &= -(a + b + c + d) \end{aligned}$$

So, $a + b + c + d = 320$.

15. **(B).** While the figure may *look* like parallelogram, it is actually a trapezoid, as it has two parallel sides of unequal length ($AD = AE + ED = 6$ and $BC = 7$). The two parallel sides in a trapezoid are

referred to as the bases. The formula for the area of a trapezoid is $A = \frac{(b_1 + b_2)}{2} \times h$, where b_1 and b_2 are the lengths of the parallel sides and h is the height, which is the distance between the parallel sides (*BE* in this figure).

Triangle *ABE* is a 3–4–5 special right triangle, so *BE* is 4. (Alternatively, use the Pythagorean theorem to determine this.)

Thus, the area is $\frac{(6+7)}{2} \times 4 = 26$. Quantity B is greater.

16. **(B).** Each angle in the pentagon is labeled with the same variable, so this is a regular pentagon. Using the formula $(n - 2)(180°)$, where n is the number of sides, the sum of all the interior angles of the pentagon is $(3)(180°) = 540°$. Divide by 5 to get $x = 108$.

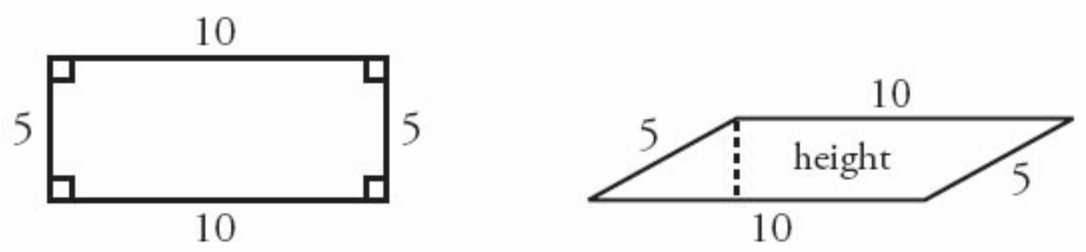
Now, the quadrilateral. All four-sided figures have interior angles that sum to 360°. Alternatively, use the formula $(n - 2)(180°)$ to determine this. Divide 360 by 4 to get $y = 90$.

Now, the triangle. It is equilateral, so $z = 60$. (The sum of angle measures in a triangle is always 180°; if the angles are equal, they will each equal 60°.)

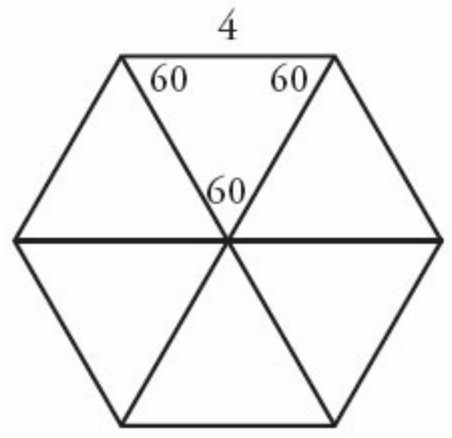
Thus, $x + y + z = 108 + 90 + 60 = 258$. Quantity B is greater.

17. **2,000.** Since the sheet of paper is measured in meters and the small rectangles in centimeters, first convert the measures of the sheet of paper to centimeters. The large sheet of paper measures 200 centimeters by 200 centimeters. The most efficient way to cut 2-centimeter by 10-centimeter rectangles is to cut vertically every 2 centimeter and horizontally every 10 centimeter (or vice versa; the idea is that all the small rectangles should be oriented the same direction on the larger sheet). Doing so creates a grid of $\frac{200}{2} \times \frac{200}{10} = 100 \times 20 = 2{,}000$ small rectangles.

18. **(D).** The formula for the area of a parallelogram is base × height, where height is the perpendicular distance between the parallel bases, not necessarily the other side of the parallelogram. However, if the parallelogram is actually a rectangle, the height *is* the other side of the parallelogram and is thereby maximized. So, if the parallelogram is actually a rectangle, the area would be equal to 50, but if the parallelogram has more extreme angle measures, the height could be very, very small, making the area much less than 30.



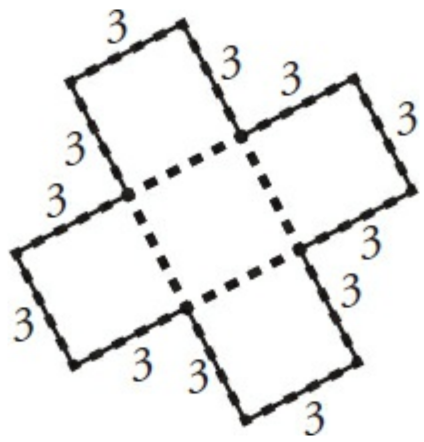
19. **(D).** Divide the hexagon with three diagonals (running through the center) to get six triangles. Since the sum of the angles in any polygon is $(n - 2)(180°)$, the sum for a hexagon is 720°. Divide by 6 to get that each angle in the original hexagon is 120°. When the hexagon is divided into triangles, each 120° angle is halved, creating two 60° angles for each triangle. Any triangle that has two angles of 60° must have a third angle of 60° as well, since triangles always sum to 180°. Thus, all six triangles are equilateral. Therefore, all three sides of each triangle are equal to 4.


4
60
60
60

For any equilateral triangle, the height equals half the side times $\sqrt{3}$. Therefore, the height is $2\sqrt{3}$. Since $A = \frac{1}{2}bh$, the area of each equilateral triangle is $A = \frac{1}{2}(4)(2\sqrt{3}) = 4\sqrt{3}$. Since there are six such triangles, the answer is $24\sqrt{3}$.

20. **(B).** If a figure with area of 45 is composed of 5 equal squares, divide to get that the area of each square is 9 and thus the side of each square is 3.

Don't make the mistake of adding up *every* side of every square to get the perimeter—only count lengths that are actually part of the perimeter of the overall figure. (Note that the central square does not have any lengths that are part of the perimeter), as shown below:



The perimeter is made of 12 segments, each with length 3. The perimeter is 36.

Incorrect choice (A) comes from reasoning that 5 squares have 20 total sides, each of length 3, and thus the combined length would be 60. Do not just subtract the four dotted line lengths, as each of these was actually counted twice, as part of the central square and one of the others. This mistake would incorrectly yield choice (C). The best approach here is to make a quick sketch of the figure, label the sketch with the given information, and count up the perimeter.

21. **(E).** To find the surface area of the original cube, first convert the side lengths to inches (it is NOT okay to find surface area or volume and then convert using 1 foot = 12 inches; this is only true for straight-line distances). The equation for surface area is $6s^2$, so, the surface area of the large original cube is $6(24 \text{ inches})^2 = 3{,}456$ square inches.

Each large solid cube is 24 inches × 24 inches × 24 inches. To cut the large cube into 2-inch by 2-inch by 4-inch rectangular solids, two dimensions (length and width, say) will be sliced every 2 inches, while one dimension (height, say) will be sliced every 4 inches. Thus, $\frac{24}{2} \times \frac{24}{2} \times \frac{24}{4} = 12 \times 12 \times 6 = 864$ small rectangular solids can be cut from the large cube.

The equation for the surface area of a rectangular solid is $2lw + 2wh + 2lh$. In this case, that is $2(2 \times 2) + 2(2 \times 4) + 2(2 \times 4) = 8 + 16 + 16 = 40$ square inches per small rectangular solid. There are 864 small rectangular solids, so the total surface area is $40 \times 864 = 34,560$ square inches.

Finally, the ratio of the total surface area of all the resulting smaller rectangular solids to the surface area of the original cube is the ratio of 34,560 to 3,456. This ratio reduces to 10 to 1.

22. **(D).** To solve this question, use the equations for the volume and the surface area of a cube:

Volume = s^3 Surface area = $6s^2$

If a cube has the same volume as surface area, set these equal:

$s^3 = 6s^2$
$s = 6$