Probability, Combinatorics, and Overlapping Sets

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1. A number is randomly chosen from a list of 10 consecutive positive integers. What is the probability that the number is greater than the mean?

(A) 3/10
(B) 2/5
(C) 1/2
(D) 7/10
(E) 4/5

2. A number is randomly chosen from the first 100 positive integers. What is the probability that it is a multiple of 3?

(A) 32/100
(B) 33/100
(C) 1/3
(D) 34/100
(E) 2/3

3. A restaurant menu has several options for tacos. There are 3 types of shells, 4 types of meat, 3 types of cheese, and 5 types of salsa. How many distinct tacos can be ordered assuming that any order contains exactly one of each of the above choices?

4. A history exam features 5 questions. 3 of the questions are multiple-choice with four options each. The other two questions are true or false. If Caroline selects one answer for every question, how many different ways can she answer the exam?

5. A certain company places a six-symbol code on each of their products. The first two symbols are one of the letters A–E and the last four symbols are digits. If repeats are allowed on both letters and numbers, how many such codes are possible?

6. The probability is 1/2 that a coin will turn up heads on any given toss and the probability is 1/6 that a number cube with faces numbered 1 to 6 will turn up any particular number. What is the probability of turning up a heads and a 6?

(A) 1/36
(B) 1/12
(C) 1/6
(D) 1/4
(E) 2/3

7. An integer is randomly chosen from 2 to 20 inclusive. What is the probability that the number is prime?

8. Five students in a classroom are lining up one behind the other for recess. How many different lines are possible?

(A) 5
(B) 10
(C) 24
(D) 25
(E) 120

9. An Italian restaurant boasts 320 distinct pasta dishes. Each dish contains exactly one pasta, one meat, and one sauce. If there are 8 pastas and 4 meats available, how many sauces are there to choose from?

10. A 10-student class is to choose a president, vice president, and secretary from the group. Assuming that no person can occupy more than one post, in how many ways can this be accomplished?

11.

Quantity A	Quantity B
The number of 4-digit positive integers where all 4 digits are less than 5	625

12. BurgerTown offers many options for customizing a burger. There are 3 types of meats and 7 condiments: lettuce, tomatoes, pickles, onions, ketchup, mustard, and special sauce. A burger must include meat, but may include as many or as few condiments as the customer wants. How many different burgers are possible?

(A) 8!
(B) (3)(7!)
(C) (3)(8!)
(D) $(8)(2^7)$
(E) $(3)(2^7)$

13. The probability of rain is 1/6 for any given day next week. What is the chance it rains on both Monday and Tuesday?

(A) 1/36
(B) 1/12
(C) 1/6
(D) 1/3
(E) 2/3

14. How many five-digit numbers can be formed using the digits 5, 6, 7, 8, 9, 0 if no digits can be repeated?

(A) 64
(B) 120
(C) 240
(D) 600
(E) 720

15. A bag contains 3 red, 2 blue, and 7 white marbles. If a marble is randomly chosen from the bag, what is the probability that it is NOT blue?

16. A man has 3 different suits, 4 different shirts, 2 different pairs of socks, and 5 different pairs of shoes. In how

many ways can the man dress himself if he must wear 1 suit, 1 shirt, 1 pair of socks, and 1 pair of shoes?

$\boxed{}$

17. A state issues automobile license plates using two letters selected from a 26-letter alphabet, as well as four numerals selected from the digits 0 through 9, inclusive. Repeats are permitted. For example, one license plate combination could be GF3352.

Quantity A	Quantity B
The number of possible unique license plate combinations	6,000,000

18. A small nation issues license plates that consist of just one number (selected from the digits 0 through 9, inclusive) and four letters, selected from a 20-letter alphabet. Repeats are permitted. However, there is one four-letter combination that is not allowed to appear on license plates. How many allowable license plate combinations exist?

(A) 1,599,990
(B) 1,599,999
(C) 1,600,000
(D) 4,569,759
(E) 4,569,760

19. A bag contains 6 black chips numbered 1–6 respectively and 6 white chips numbered 1–6 respectively. If Pavel reaches into the bag of 12 chips and removes 2 chips, one after the other, without replacing them, what is the probability that he will pick black chip #3 and then white chip #3?

$$\frac{\boxed{}}{\boxed{}}$$

20. Tarik has a pile of 6 green chips numbered 1–6 respectively and another pile of 6 blue chips numbered 1–6 respectively. Tarik will randomly pick 1 chip from the green pile and 1 chip from the blue pile.

Quantity A	Quantity B
The probability that both chips selected by Tarik will display a number less than 4	1/2

21. A bag contains 6 red chips numbered 1–6 respectively and 6 blue chips numbered 1–6 respectively. If 2 chips are to be picked sequentially from the bag of 12 chips, without replacement, what is the probability of picking a red chip and then a blue chip with the same number?

22. In a school of 150 students, 75 take Latin, 110 take Spanish, and 11 take neither.

Quantity A	**Quantity B**
The number of students who take only Latin	46

23. How many 10-digit numbers can be formed using only the digits 2 and 5?

(A) 2^{10}
(B) (22)(5!)
(C) (5!)(5!)
(D) 10!/2
(E) 10!

24. A 6-sided cube has sides numbered 1 through 6. If the cube is rolled twice, what is the probability that the sum of the two rolls is equal to 8?

(A) 1/9
(B) 1/8
(C) 5/36
(D) 1/6
(E) 7/36

25. A coin with heads on one side and tails on the other has a 1/2 probability of landing on heads. If the coin is flipped 5 times, how many distinct outcomes are possible if the last flip must be heads? Outcomes are distinct if they do not contain exactly the same results in exactly the same order.

26. In a class of 25 students, every student takes either Spanish, Latin, or French, or two of the three, but no students take all three languages. 9 take Spanish, 7 take Latin and 5 take exactly two languages.

Quantity A	**Quantity B**
The number of students who take French	14

27. Bob has a 24-sided die with an integer between 1 and 24 on each face. Every number is featured exactly once. When he rolls, what is the probability that the number showing is a factor of 24?

28. A baby has x total toys. If 9 of the toys are stuffed animals, 7 of the toys were given to the baby by its grandmother, 5 of the toys are stuffed animals given to the baby by its grandmother, and 6 of the toys are neither stuffed animals nor given to the baby by its grandmother, what is the value of x?

29. How many integers between 2,000 and 3,999 have a ones digit that is a prime number?

30. How many integers between 2,000 and 6,999 are even and have a digit that is a prime number in the tens place?

31. A group of 12 people who have never met are in a classroom. How many handshakes are exchanged if each pair shakes hands exactly once?

(A) 12
(B) 22
(C) 66
(D) 132
(E) 244

32. A classroom has 12 girls and 20 boys. One quarter of the girls in the class have blue eyes. If a child is selected at random from the class, what is the probability that he/she is a girl who does not have blue eyes?

(A) 3/32
(B) 9/32
(C) 3/8
(D) 23/32
(E) 29/32

33. A coin with heads on one side and tails on the other has a 1/2 probability of landing on heads. If the coin is flipped three times, what is the probability of flipping 2 tails and 1 head, in any order?

(A) 1/8
(B) 1/3
(C) 3/8
(D) 5/8

(E) 2/3

34. A 6-sided cube has sides numbered 1 through 6. If the cube is rolled twice, what is the probability that at least one of the rolls will result in a number higher than 4?

(A) 2/9
(B) 1/3
(C) 4/9
(D) 5/9
(E) 2/3

35. Tiles are labeled with the integers from 1 to 100 inclusive; no numbers are repeated. If Alma chooses one tile at random, replaces it in the group, and chooses another tile at random, what is the probability that the product of the two integer values on the tiles is odd?

(A) 1/8
(B) 1/4
(C) 1/3
(D) 1/2
(E) 3/4

36. If the word "WOW" can be rearranged in exactly 3 ways (WOW, OWW, WWO), in how many ways can the word "MISSISSIPPI" be rearranged?

37. If *a*, *b*, and *c* are integers randomly chosen from the set of prime numbers greater than 2 and less than 30, what is the probability that $ab + c$ is equal to 23?

38.

The probability of rain is 1/2 on any given day next week.

Quantity A	**Quantity B**
The probability that it rains on AT LEAST one out of the 7 days next week	127/128

39. Two fair dice with sides numbered 1 to 6 are tossed. What is the probability that the sum of the exposed faces on the dice is a prime number?

40. Jack has a cube with 6 sides numbered 1 through 6. He rolls the cube repeatedly until the first time that the sum of all of his rolls is even, at which time he stops. (Note: it is possible to roll the cube just once.) What is the probability that Jack will need to roll the cube more than 2 times in order to get an even sum?

(A) 1/8
(B) 1/4
(C) 3/8
(D) 1/2
(E) 3/4

41. Jan and 5 other children are in a classroom. The principal of the school walks in and chooses two children at random. What is the probability that Jan is chosen?

(A) 4/5
(B) 1/3
(C) 2/5
(D) 7/15
(E) 1/2

42. The probability that Gary will eat eggs for breakfast on any given day is 3/7. The probability that Gary will eat cereal for breakfast on any given day is 4/7. Gary never has both eggs and cereal for breakfast on the same day.

Quantity A	**Quantity B**
Probability that Gary eats eggs or cereal for breakfast on a particular day	1

43. The probability that Maria will eat breakfast on any given day is 0.5. The probability that Maria will wear a sweater on any given day is 0.3. The two probabilities are independent of each other.

Quantity A	**Quantity B**
The probability that Maria eats breakfast or wears a sweater	0.8

44. The probability of rain in Greg's town on Tuesday is 0.3. The probability that Greg's teacher will give him a pop quiz on Tuesday is 0.2. The events occur independently of each other.

Quantity A	**Quantity B**
The probability that either or both events occur	The probability that neither event occurs

45. The probability of event X occurring is the same as the probability of event Y occurring. The events occur independently of each other.

Quantity A	**Quantity B**
The probability that both events occur	The probability that neither event occurs.

46. A certain city has a 1/3 chance of rain occurring on any given day. In any given 3-day period, what is the probability that the city experiences rain?

(A) 1/3
(B) 8/27
(C) 2/3
(D) 19/27

(E) 1

47. Five students, Adnan, Beth, Carol, Dan, and Edmund are to be arranged in a line. How many such arrangements are possible if Beth is not allowed to stand next to Dan?

(A) 24
(B) 48
(C) 72
(D) 96
(E) 120

48. A polygon has 12 edges. How many different diagonals does it have? (A diagonal is a line drawn from one vertex to any other vertex inside the given shape. This line cannot touch or cross any of the edges of the shape. For example, a triangle has zero diagonals and a rectangle has two.)

(A) 54
(B) 66
(C) 108
(D) 132
(E) 144

49. A student council is to be chosen from a class of 12 students consisting of a president, a vice president, and 3 committee members. How many such councils are possible?

(A) $\frac{12!}{7!5!}$
(B) $\frac{12!}{7!3!}$
(C) $\frac{12!}{5!3!}$
(D) $\frac{12!}{7!}$
(E) 12!

50.

Quantity A	**Quantity B**
The number of possible pairings of 2 colors that can be selected from 5 possible options	The number of possible pairings of 8 colors that can be selected from 9 possible options

51.

Quantity A	**Quantity B**
The number of possible 4-person teams that can be selected from 6 people	The number of possible 2-person teams that can be selected from 6 people

52.

Quantity A	**Quantity B**

The number of ways 1st, 2nd, and 3rd place prizes could be awarded to 3 out of 6 contestants	The number of ways 1st, 2nd, 3rd, 4th, and 5th place prizes could be awarded to 5 contestants

53. An inventory of coins contains 100 different coins.

Quantity A	**Quantity B**
The number of possible collections of 56 coins that can be selected where the order of the coins does not matter	The number of possible collections of 44 coins that can be selected where the order of the coins does not matter

54. An office supply store carries an inventory of 1,345 different products, all of which it categorizes as "business use," "personal use," or both. 740 products are categorized as "business use" ONLY and 520 products are categorized as both "business use" and "personal use."

Quantity A	**Quantity B**
The number of products characterized as "personal use"	600

55. How many distinct 4-letter "words" can be made from the name "CHRISTYNA"? (A "word" is any arrangement of 4 letters regardless of whether it can be found in a dictionary.)

(A) 9
(B) 24
(C) 36
(D) 504
(E) 3,024

56. Seiko has a 6-sided number cube with sides labeled 1 through 6. If she rolls the cube twice, what is the probability that the product of the two rolls is less than 36?

(A) 1/6
(B) 1/3
(C) 2/3
(D) 5/6
(E) 35/36

57. There is an 80% chance David will eat a healthy breakfast and a 25% chance that it will rain. If these events are independent, what is the probability that David will eat a healthy breakfast OR that it will rain?

(A) 20%
(B) 80%
(C) 85%
(D) 95%
(E) 105%

58. The probability of rain is 1/2 for every day next week. What is the chance that it rains on at least one day during the workweek (Monday through Friday)?

(A) 1/2
(B) 31/32
(C) 63/64
(D) 127/128

(E) 5/2

59. Eight women and two men are available to serve on a committee. If three people are picked, what is the probability that the committee includes at least one man?

(A) 1/32
(B) 1/4
(C) 2/5
(D) 7/15
(E) 8/15

60. At Lexington High School, everyone takes at least one language — Spanish, French, or Latin — but no one takes all three languages. If 100 students take Spanish, 80 take French, 40 take Latin, and 22 take exactly two languages, how many students are there?

(A) 198
(B) 220
(C) 242
(D) 264
(E) 286

61.

Of 60 birds found in a certain location, 20 are songbirds and 23 are migratory. (It is possible for a songbird to be migratory, or not.)

Quantity A	**Quantity B**
The number of the 60 birds that are neither migratory nor songbirds	16

Probability, Combinatorics, and Overlapping Sets Answers

1. **(C).** In a list of 10 consecutive integers, the mean will be the average of the 5th and 6th numbers. Therefore, the 6th through 10th integers (five total integers) will be larger than the mean. Since probability is determined by the number of desired items divided by the total number of choices, the probability that the number chosen is higher than the mean is 5/10 = 1/2.

Another approach to this problem is to create a set of 10 consecutive integers; the easiest such list contains the numbers {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. The mean is one-half the sum of the first element plus the last element, or $\frac{1+10}{2} = 5.5$. Therefore, there are 5 numbers higher than the mean in the list: 6, 7, 8, 9 and 10. Again, the probability of choosing a number higher than the mean is 5/10 = 1/2.

2. **(B).** The first 100 positive integers comprise the set of numbers containing the integers 1 to 100. Of these numbers, the only ones that are divisible by 3 are {3, 6, 9, …, 96, 99}, which adds up to exactly 33 numbers. This can be determined in several ways. You could simply count the multiples of 3, but that's a bit slow. Alternatively, you can compute 99/3 = 33 and realize that there are 33 multiples of 3 up to and including 99. The number 100 is not divisible by 3, so the correct answer is 33/100.

Alternatively, you can use the "add one and you're done trick," subtracting the first multiple of 3 from the last multiple of 3, dividing by 3 and then adding 1: $\frac{(99-3)}{3} + 1 = 33$. Then, since probability is determined by the number of desired options divided by the total number of options, the probability that the number chosen is a multiple of 3 is 33/100.

3. **180.** This problem tests the fundamental counting principle, which states that the total number of choices is equal to the product of the independent choices. Therefore, the total number of tacos is (3)(4)(3)(5) = 180 tacos.

4. **256.** This question tests the fundamental counting principle, which states that the total number of choices is equal to the product of the independent choices. The five separate test questions give you five independent choices. For the three multiple choice questions there are four options each, whereas for the two true/false questions there are two options each. Multiplying the independent choices yields (4)(4)(4)(2)(2) = 256 different ways to answer the exam.

5. **250,000.** This question tests the fundamental counting principle, which states that the total number of choices is equal to the product of the independent choices. There are 5 choices for each of the letters at the beginning of the code (A, B, C, D, or E) and 10 choices for each of the 4 digits at the end of the code (0, 1, 2, 3, 4, 5, 6, 7, 8, or 9). Therefore, there are (5)(5)(10)(10)(10)(10) = 250,000 possible codes.

6. **(B).** The probability of independent events A AND B occurring is equal to the product of the probability of event A and the probability of event B. In this case, the probability of the coin turning up heads is 1/2 and the product of rolling a 6 is 1/6. Therefore, the probability of heads AND a 6 is equal to (1/2)(1/6) = 1/12. Alternatively, you could list all the possible outcomes: H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6. There are 12 total outcomes and only 1 with heads and a 6. Therefore, the desired outcome divided by the total number of outcomes is equal to 1/12.

7. **8/19.** Among the integers 2 through 20 inclusive there are 8 primes: 2, 3, 5, 7, 11, 13, 17, and 19. From 2 to 20 inclusive there are exactly 20 - 2 + 1 = 19 integers; remember to "add one before you're done" to include both endpoints. Alternatively, there are 20 integers from 1 to 20 inclusive, so there must be 19 integers from 2 to 20 inclusive. Since probability is defined as the number of desired items divided by the total number of choices, the probability that the number chosen is prime is 8/19.

8. **(E).** This problem describes a specific arrangement of people, so this is an ordering problem. The total number of ways to arrange n items in order is $n!$ (the exclamation point means "factorial"). Therefore, since there are 5 students to be arranged in a line, the total number of possible orderings is 5! = (5)(4)(3)(2)(1) = 120.

Alternatively, ask, "How many choices do I have for each place in the line?" Consider the first place in line. Since no students have been chosen yet, there are 5 total options for the first place in line. Similarly, for the second place there are 4 choices, because one student has already been chosen to occupy the first spot. Applying the same logic, there are 3 choices for the third place, 2 for the fourth, and 1 for the fifth. Using the fundamental counting principle, there are a total of (5)(4)(3)(2)(1) = 120 different lines.

9. **10.** This problem tests the fundamental counting principle, which states that the total number of choices is equal to the product of the independent choices. Let the number of sauces be represented by the variable S. The total number of possible pasta dishes can be represented by each separate choice multiplied together: $(8)(4)(S)$, or $32S$. The problem also indicates that the total number of pasta dishes must be equal to 320. Therefore, $32S = 320$ and $S = 10$.

10. **720**. One possible approach is to ask, "How many choices do I have for each of the class positions?" Begin by considering the president of the class. Since no one has been chosen yet, there are 10 students from whom to choose. Then, for the vice president there are 9 options because now one student has already been chosen as president. Similarly, there are 8 choices for the secretary. Using the fundamental counting principle, the total number of possible selections is (10)(9)(8) = 720.

Alternatively, you could use factorials. In this case order matters because you are choosing people for specific positions. This problem is synonymous to asking, "How many different ways can you line up 3 students as first, second, and third from a class of 10?" The number of ways to arrange the entire class in line is 10!. However, the problem is only concerned with the first 3 students in line, so exclude rearrangements of the last 7. The way in which these "non-chosen" 7 students can be ordered is 7!. Thus the total number of arrangements for 3 students from a class of 10 is $\frac{10!}{7!} = (10)(9)(8) = 720$.

11. **(B).** This is a combinatorics problem — to calculate Quantity A, make four "slots" (since the numbers are all four-digit numbers), and determine how many possibilities there are for each slot:

____ ____ ____ ____

Since all 4 digits must be less than 5, the possible options are limited to 0, 1, 2, 3 and 4. However, a 4-digit number cannot begin with 0 (the smallest four-digit number is 1,000). So, the first slot has only 4 possibilities, not 5:

$\underline{\ 4\ }\quad \underline{\ 5\ }\quad \underline{\ 5\ }\quad \underline{\ 5\ }$

(4)(5)(5)(5) = 500, which is less than 625. The answer is (B).

Note that the value in Quantity B comes from multiplying $5 \times 5 \times 5 \times 5$. Neglecting to note that 4-digit numbers cannot begin with 0 results in incorrect choice (C).

12. **(E).** This problem tests the fundamental counting principle, which states that the total number of choices is equal to the product of the independent choices. The key to this problem is realizing how many choices there are for each option. For the meat, there are obviously 3 choices. For each of the condiments there are exactly 2 choices: yes or no. The only real choice regarding each condiment is whether to include it at all. As there are 7 condiments, the total number of choices is $(3)(2)(2)(2)(2)(2)(2)(2) = (3)(2^7)$.

Note: the condiment options cannot be counted as 8! (0 through 7 = 8 options) because, in this case, the order in which the options are chosen does not matter; a burger with lettuce and pickles is the same as a burger with pickles and lettuce.

13. **(A).** For probability questions, always begin by separating out the probabilities of each individual event. Then, if you need all the events to happen (an "AND question"), multiply the probabilities together. If you only need one of the multiple events to happen (an "OR question"), add the probabilities together.

In this case, there are two events: rain on Monday and rain on Tuesday. The question asks for the probability that it will rain on Monday AND on Tuesday, so multiply the individual probabilities together:

$$\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

14. **(D).** This problem relies on the fundamental counting principle, which says that the total number of ways for something to happen is the product of the number of options for each individual choice. The problem asks how many five-digit numbers can be created from the digits 5, 6, 7, 8, 9, and 0. For the first digit, there are only five options (5, 6, 7, 8, and 9) because a five-digit number must start with a non-zero integer. For the second digit, there are 5 choices again, because now zero can be used but one of the other numbers has already been used, and numbers cannot be repeated. For the third number, there are 4 choices, for the fourth there are 3 choices, and for the fifth number there are 2 choices. Thus, the total number of choices is $(5)(5)(4)(3)(2) = 600$.

Alternatively, you can use the same logic and realize there are 5 choices for the first digit. (Separate out the first step because you have to remove the zero from consideration.) The remaining five digits all have an equal chance of being chosen, so choose four out of the remaining five digits to complete the number. The number of ways in which this second step can be accomplished is $(5!)/(1!) = (5)(4)(3)(2)$. Thus, the total number of choices is again equal to $(5)(5)(4)(3)(2) = 600$.

15. **5/6.** In the bag of marbles, there are 3 red marbles and 7 white marbles, for a total of 10 marbles that are NOT blue. There are a total of $3 + 7 + 2 = 12$ marbles in the bag. Since probability is defined as the number of desired items divided by the total number of choices, the probability that the marble chosen is not blue is $10/12 = 5/6$.

16. **120.** This problem utilizes the fundamental counting principle, which states that the total number of choices is equal to the product of the independent choices. Since the man must choose one suit, one shirt, one pair of socks, and one pair of shoes, the total number of outfits is the number of suits times the number of shirts times the number of socks times the number of shoes: $(3)(4)(2)(5) = 120$.

17. **(A).** This is a combinatorics problem. The license plates have 2 letters and 4 numbers, so make six "slots" and

determine how many possibilities there are for each slot. There are 26 letters in the alphabet and 10 digits to pick from, so:

$$\underline{26}\quad\underline{26}\quad\underline{10}\quad\underline{10}\quad\underline{10}\quad\underline{10}$$

Multiply 26 × 26 on your calculator to get 676. Add four zeroes for the four 10's to get 6,760,000. Quantity A is larger.

18. **(A).** First, calculate the total number of combinations, ignoring the one illegal "word." Since there are 10 possibilities for the first slot and 20 possibilities for the other 4 slots:

$$\underline{10}\quad\underline{20}\quad\underline{20}\quad\underline{20}\quad\underline{20}$$

Multiply to get 1,600,000.

Now, consider all the license plates that contain the forbidden word. Say, for example, that the forbidden word is GURG. That means that the plates 0GURG, 1GURG, 2GURG, 3GURG, etc. are forbidden, for a total of 10 forbidden plates. You can also express this mathematically as:

$$\underline{10}\quad\underline{1}\quad\underline{1}\quad\underline{1}\quad\underline{1}$$

Multiply to get 10 forbidden plates. Subtract: 1,600,000 - 10 = 1,599,990.

Alternatively, after finding the 1,600,000 figure, glance at the answer choices: the correct answer can't be (C), (D), or (E). Answer choice (B) subtracts just 1 plate, but more than 1 plate could have the forbidden "word." Therefore, (A) must be the correct answer.

19. $\mathbf{\frac{1}{121}}$ The probability of picking black chip #3 is $\frac{1}{12}$. Once Pavel has removed the first chip, only 11 chips remain, so the probability of picking white chip #3 is $\frac{1}{11}$. Multiply $\frac{1}{12}\times\frac{1}{11}=\frac{1}{121}$.

20. **(B).** In this problem, Tarik is NOT picking 1 chip out of all 12. Rather, he is picking 1 chip out of 6 green ones, and then picking another chip out of 6 blue ones. There are 3 green chips with numbers less than 4, so Tarik has a $\frac{3}{6}$ chance of selecting a green chip showing a number less than 4. Likewise, Tarik has a $\frac{3}{6}$ chance of selecting a blue chip showing a number less than 4. Therefore, Quantity A is equal to $\frac{3}{6}\times\frac{3}{6}=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$. Quantity B is larger.

21. $\mathbf{\frac{1}{22}}$. The trap answer in this problem is $\frac{1}{6}\times\frac{1}{6}=\frac{1}{36}$. This is NOT the answer to the question being asked—

rather, this is the answer to the question, "What is the probability of picking a red chip and then a blue chip that both have #3?" (or any other specific number). This is a more specific question than the one actually asked. In the question, asked, there are six possible ways to fulfill the requirements of the problem, not one, because the problem does not specify whether the number should be 1, 2, 3, 4, 5, or 6.

Thus, ANY of the 6 red chips is acceptable for the first pick. However, on the second pick, only the blue chip with the same number as the red one that was just picked is acceptable (you need whatever chip is the "match" for the first one picked). Thus:

$$\frac{6}{12} \times \frac{1}{11} = \frac{1}{2} \times \frac{1}{11} = \frac{1}{22}$$

22. **(B).** Use the overlapping sets formula for two groups: Total = Group 1 + Group 2 - Both + Neither. (Adding the two groups—in this case Latin and Spanish—double-counts the students who take both classes, so the formula subtracts the "both" students.)

$150 = 75 + 110 - B + 11$
$150 = 196 - B$
$46 = B$

Careful! This is not the value of Quantity A. Since 46 students take both Latin and Spanish, subtract 46 from the total who take Latin to find those who take only Latin:

$75 - 46 = 29$

Thus, Quantity A is equal to 29 and Quantity B is larger.

23. **(A).** This problem relies on the fundamental counting principle, which says that the total number of ways for something to happen is the product of the number of options for each individual choice. For any digit of the 10-digit number there are exactly two options, a 2 or a 5. Thus, since there are two choices for each digit and it is a 10-digit number, there are $(2)(2)(2)(2)(2)(2)(2)(2)(2)(2) = 2^{10}$ total choices.

24. **(C).** The probability of any event equals the number of ways to get the desired outcome divided by the total number of ways for the event to happen. Starting with the denominator, use the fundamental counting principle to compute the total number of ways to roll a cube twice. There are 6 possibilities (1, 2, 3, 4, 5, or 6) for the first roll and 6 for the second, giving a total of (6)(6) = 36 possibilities for the two rolls. For the numerator, determine the number of possible combinations that will add to 8. For example, you might roll a 2 the first time and a 6 the second time. The full set of options is (2,6), (3,5), (4,4), (5,3), and (6,2). Thus there are 5 possible combinations that sum to 8, yielding a probability of 5/36.

25. **16.** This problem utilizes the fundamental counting principle, which states that the total number of choices is equal to the product of the independent choices. For the first flip there are 2 options: heads or tails. Similarly, for the second flip there 1 2 options, for the third there are 2 options, for the fourth there are 2 options, and for the fifth there is only one option because the problem restricts this final flip to heads. Therefore, the total number of outcomes is (2)(2)(2)(2)(1) = 16. A good rephrasing of this question is, "How many different outcomes are there if the coin is flipped 4 times?" The fifth flip, having been restricted to heads, is irrelevant. Therefore, the total number of ways to flip the coin five times with heads for the fifth flip is equal to the total number of ways to flip the coin four times; either way, the answer is 16.

26. **(C).** The problem specifies that no one takes all three languages. In addition, a total of 5 people take 2 languages. Thus, 5 people have been double-counted. Because you know the total number of people who have been double-counted (5) and triple-counted (0), you can use the standard overlapping sets formula:

Total = Spanish + French + Latin - (Two of the Three) - 2(All Three)

25 = 9 + French + 7 - 5 - 2(0)
25 = 11 + French
14 = French

The two quantities are equal.

27. $\frac{1}{3}$ Probability equals the number of desired outcomes divided by the total number of possible outcomes. Among the integers 1 through 24 there are 4 factor pairs of 24: (1, 24), (2, 12), (3, 8), and (4, 6), for a total of 8 factors. The total number of possible outcomes when rolling the die once is 24. The probability that the number chosen is a factor of 24 is 8/24 = 1/3.

28. **17.** Use the overlapping sets formula for two groups: Total = Group 1 + Group 2 - Both + Neither. Here, the groups are "stuffed animal" and "given by the baby's grandmother." The problem indicates that the "both" category is equal to 5, and that the "neither" number is 6. The total is x.

Total = Group 1 + Group 2 - Both + Neither

$x = 9 + 7 - 5 + 6$
$x = 17$

29. **800.** This is a combinatorics problem. Make four "slots" (since the numbers are all four-digit numbers), and determine how many possibilities there are for each slot:

$\underline{\qquad}\quad\underline{\qquad}\quad\underline{\qquad}\quad\underline{\qquad}$

Since the number must begin with 2 or 3, there are 2 possibilities for the first slot. Because the ones digit must be prime and there are only 4 prime 1-digit numbers (2, 3, 5, and 7), there are 4 possibilities for the last slot.

$\underline{\quad 2 \quad}\quad\underline{\qquad}\quad\underline{\qquad}\quad\underline{\quad 4 \quad}$

The other slots have no restrictions, so put 10, since there are 10 digits from 0–9:

$\underline{\quad 2 \quad}\quad\underline{\quad 10 \quad}\quad\underline{\quad 10 \quad}\quad\underline{\quad 4 \quad}$

Multiply to get 800.

Alternatively, figure out the pattern and add up the number of qualifying 4-digit integers. In the first ten numbers, 2000–2009, there are exactly 4 numbers that have a prime units digit: 2002, 2003, 2005, and 2007. The pattern then repeats in the next group of ten numbers, 2010–2020, and so on. In any group of ten numbers, then, four qualify. In the first one hundred numbers, 2000–2099, there are ten groups of ten, or 10 × 4 = 40 numbers that have a prime units digit. In the first one thousand numbers, 2000–2999, there are ten groups of one hundred, or 100 × 4 = 400 numbers that have a prime units digit. There are a total of two groups of one thousand numbers (2000–2999 and 3000–3999), so there are a total of 400 × 2 = 800 numbers that have a prime units digit.

30. **1,000.** This is a combinatorics problem. Make four "slots" (since the numbers are all four-digit numbers), and determine how many possibilities there are for each slot:

$$\underline{\qquad}\quad\underline{\qquad}\quad\underline{\qquad}\quad\underline{\qquad}$$

Since the number must begin with 2, 3, 4, 5, or 6, there are 5 possibilities for the first slot. Because the digit in the tens place must be prime and there are only 4 prime 1-digit numbers (2, 3, 5, and 7), there are 4 options for the tens place.

$$\underline{\quad 5 \quad}\quad\underline{\qquad}\quad\underline{\quad 4 \quad}\quad\underline{\qquad}$$

Since the number must be even, the final digit must be 0, 2, 4, 6, or 8—thus, there are 5 possibilities for the units digit. (Note that saying that a 4-digit number is even is simply saying that it ends in an even digit—for instance, 3,792 is even.) No restrictions were given for the hundreds slot, so there are 10 options, since there are 10 digits from 0–9.

$$\underline{\quad 5 \quad}\quad\underline{\quad 10 \quad}\quad\underline{\quad 4 \quad}\quad\underline{\quad 5 \quad}$$

Multiply to get 1,000.

Alternatively, figure out the pattern and add up the number of qualifying 4-digit integers. In the first one hundred numbers, 2,000–2,099, there are exactly four sets of ten numbers that have a prime tens digit: 202-, 203-, 205-, and 207-. Within each of those four sets, there are five possibilities for numbers that also end in an even digit. For example, the possibilities for the 202- set would be 2,020, 2,022, 2,024, 2,026, and 2,028. Out of the first one hundred numbers, then, there are 5 × 4 = 20 possibilities that have both a prime tens digit and end in an even digit.

From 2,000 to 6,999, there are 50 groups of 100 numbers (6,999 - 2,000 + 1 = 5,000, and 5,000 ÷ 100 = 50). Therefore, there are 20 × 50 = 1,000 numbers.

31. **(C).** Multiple approaches are possible here. One way is to imagine the scenario and count up the number of handshakes: you are the first person. How many hands do you need to shake? There are 11 other people in the room, so you need to shake hands 11 times. Now, move to the second person: how many hands must he shake? He has already shaken your hand, leaving him 10 others with whom to shake hands. The third person will need to shake hands with 9 others, and so on. Therefore, there are a total of 11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 handshakes. The fastest way to find the sum of a group of consecutive numbers is to take the average of the first and last terms and multiply it by the number of terms. The average is (11 + 1)/2 = 6 and there are 11 - 1 + 1 = 11 terms (find the difference between the terms and "add one before you're done"). The sum is 6 × 11 = 66.

Alternatively, rephrase the question as "How many different ways can any 2 people be chosen from a group of 12?" (This works because the problem ultimately asks you to "choose" each distinct pair of 2 people one time.) The key here is to realize that handshakes are independent of order, i.e., it doesn't matter if A shakes hands with B or if B shakes hands with A; it's the same outcome. Thus, you only care about how many pairs you can make. Any time you recognize a group of order-independent items being selected from a larger set, you can apply the formula $\frac{\text{total}!}{\text{in}!\,\text{out}!}$

to arrive at the total number of combinations. Thus: $\dfrac{12!}{2!10!} = \dfrac{12 \cdot 11}{2} = 66$.

32. **(B).** The probability of any outcome is equal to the number of desired outcomes divided by the total number of outcomes. There are 12 girls and 20 boys in the classroom. If one-quarter of the girls have blue eyes, then there are (12)(1/4) = 3 girls with blue eyes. Therefore, there are 12 - 3 = 9 girls who do NOT have blue eyes. The total number of ways in which you could choose a child is simply the total number of children in the class, namely 12 + 20 = 32. Therefore, the probability of choosing a girl who does not have blue eyes equals the number of girls without blue eyes divided by the total number of children, which is 9/32.

33. **(C).** There are only 2 possible outcomes for each flip and only 3 flips total. The most straightforward approach is to list all of the possible outcomes: {HHH, HHT, HTH, HTT, TTT, TTH THH, THT}. Of these 8 possibilities, 3 of the outcomes have one head and two tails, so the probability of this event is 3/8.

Alternatively, you can count the total number of ways of getting 1 head without listing all the possibilities. If the coin is flipped 3 times and you want only 1 head, then there are 3 possible positions for the single head: on the first flip alone, on the second flip alone, or on the third flip alone. Since there are 2 possible outcomes for each flip, heads or tails, there are (2)(2)(2) = 8 total outcomes. Again, the probability is 3/8.

Finally, you can compute the probability directly. The probability of flipping heads is 1/2 and the probability of flipping tails is also 1/2. The probability of getting heads in the first position alone, or HTT, is (1/2)(1/2)(1/2) = 1/8, where you multiply because you have heads AND tails AND tails. This represents the probability of heads in position 1, but heads could also be in position 2 alone or in position 3 alone. Since there are 3 possible positions for the heads, multiply by 3 to get the total probability (3)(1/8) = 3/8.

34. **(D).** Because this problem is asking for an "at least" solution, you can use the 1 - x shortcut. The probability that at least one roll results in a number higher than 4 is equal to 1 minus the probability that both of the rolls result in numbers 4 or lower. For one roll, there are 6 possible outcomes (1 through 6) and 4 ways in which the outcome can be 4 or lower, so the probability is 4/6 = 2/3. Thus, the probability that both rolls results in numbers 4 or lower is (2/3) (2/3) = 4/9. This is the result that you do NOT want; subtract this from 1 to get the probability that you do want. The probability that at least one of the rolls results in a number higher than 4 is 1 - (4/9) = 5/9.

Alternatively, write out the possibilities. The total number of possibilities for two rolls is (6)(6) = 36. Here are the ways in which at least one number higher than 4 can be rolled:

> 51, 52, 53, 54, 55, 56
> 61, 62, 63, 64, 65, 66
> 15, 25, 35, 45 (note: 55 and 65 have already been counted above)
> 16, 26, 36, 46 (note: 56 and 66 have already been counted above)

There are 20 elements (be careful not to double-count any options). The probability of at least one roll resulting in a number higher than 4 is 20/36 = 5/9.

35. **(B).** You need to use both probability and number properties concepts in order to answer this question. First, in order for two integers to produce an odd integer, the two starting integers must be odd. An odd times an odd equals an odd. An even times an odd, by contrast, produces an even, as does an even times an even.

Within the set of tiles, there are 50 even numbers (2, 4, 6, …, 100) and 50 odd numbers (1, 3, 5, …, 99). One randomly-chosen tile will have a 50/100 = 1/2 probability of being even, and a 1/2 probability of being odd. The probability of choosing an odd tile first is (1/2) and the probability of choosing an odd tile second is also (1/2), so the probability of "first odd AND second odd" is (1/2)(1/2) = 1/4.

Alternatively, you can recognize that there are only four options for odd/even pairs if two tiles are chosen: OO, OE, EO, EE. The only one of these combinations that yields an odd product is OO. Since all of these combinations are equally likely, and since OO is exactly one out of the four possibilities, the probability of choosing OO is 1/4.

36. **34,650.** This is a combinatorics problem, and the WOW example is intended to make it clear to you that any W is considered identical to any other W — switching one W with another would NOT result in a different combination, just as switching one S with another in MISSISSIPPI would not result in a different combination.

Therefore, solve this problem using the classic combinatorics formula for accounting for subgroups among which order does not matter:

<u>Total Number of Items!</u>
First Group! Second Group! Etc…

Because MISSISSIPPI has 11 letters, including one M, four S's, four I's, and two P's:

$$\frac{11!}{1!4!4!2!}$$

Now expand the factorials and cancel; use the calculator for the last step of the calculation:

$$\frac{11\times10\times9\times8\times7\times6\times5\times\cancel{4!}}{\cancel{1!}\cancel{4!}(4\times3\times2\times1)(2\times1)}=\frac{11\times10\times9\times\cancel{8}\times7\times\cancel{6}\times5}{(\cancel{4}\times\cancel{3}\times\cancel{2}\times\cancel{1})(\cancel{2}\times\cancel{1})} = 11 \times 10 \times 9 \times 7 \times 5 = 34{,}650$$

37. **0.** Because *a, b,* and *c* are prime numbers greater than 2, they must all be odd. So, $ab + c$ = (ODD)(ODD) + ODD. ODD times ODD yields another odd number, so the calculation simplifies to ODD + ODD, which will always yield an even answer.

Thus, it is impossible for $ab + c$ to be odd. The probability is zero. (Note: if you were ever asked to type a nonzero probability into a box, you would need to express it as a decimal between 0 and 1.)

38. **(C).** Since Quantity A is an "at least" problem, you can use the 1 - x shortcut. Rather than calculate the probability of rain on exactly 1 day next week, and then the probability of rain on exactly 2 days next week, and so on (after which you would still have to add all of the probabilities together!), instead calculate the probability of no rain at all on any day, and then subtract that number from 1. That will give the combined probabilities for any scenarios that include rain on at least 1 day.

Probability of NO rain for any of the 7 days = $\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{128}$

Subtract this probability from 1:

$$1 - \frac{1}{128} = \frac{128}{128} - \frac{1}{128} = \frac{127}{128}$$

Quantities A and B are equal.

39. **5/12.** First think about the prime numbers less than 12, the maximum sum of the numbers on the dice. These primes are 2, 3, 5, 7, 11.

The probability of rolling 2, 3, 5, 7, or 11 = the number of ways to roll any of these sums, divided by the total number of possible rolls. The total number of possible dice rolls is 6 × 6 = 36.

Sum of 2 can happen 1 way: 1 + 1
Sum of 3 can happen 2 ways: 1 + 2 or 2 + 1
Sum of 5 can happen 4 ways: 1 + 4, 2 + 3, 3 + 2, 4 + 1
Sum of 7 can happen 6 ways: 1 + 6, 2 + 5, 3 + 4, 4 + 3, 5 + 2, 6 + 1
Sum of 11 can happen 2 ways: 5 + 6, 6 + 5

That's a total of 1 + 2 + 4 + 6 + 2 = 15 ways to roll a prime sum.

Thus, the probability is 15/36 = 5/12.

40. **(B).** Jack will only continue to roll the cube if the sum of the individual rolls is odd. For the first roll, this will only occur if the number itself is odd; if Jack does not stop after the first roll, then, he must have rolled an odd number for the first roll. For the second roll, in order for the sum of the first and second to be odd, Jack must now roll an even (because odd + even = odd). You can rephrase the question: "What is the probability that Jack will roll an odd first and an even second?" The probability of event A AND event B equals the probability of A times the probability of B. Since the probability of odd = 1/2 and the probability of even = 1/2, the probability of the first number being odd AND the second number being even is (1/2)(1/2) = 1/4.

41. **(B).** The probability of any event equals the number of ways to get the desired outcome divided by the total number of outcomes.

Start with the denominator, which is the total number of ways that the principal can choose two children from the classroom. Use the fundamental counting principle. There are 6 possible options for the first choice and 5 for the second, giving (6)(5) = 30 possibilities. However, this double-counts some cases; for example, choosing Jan and then Robert is the same as choosing Robert and then Jan. Divide the total number of pairs by 2: $\frac{6 \cdot 5}{2} = 15$. Alternatively, use the formula for a set where the order doesn't matter: $\frac{\text{total!}}{\text{in! out!}}$. In this case: $\frac{6!}{2!4!} = \frac{6 \times 5 \times 4!}{(2)(4!)} = \frac{6 \times 5}{2} = 15$.

Now compute the numerator, which is the number of pairs that include Jan. Since the pair only includes two children and one is already decided (Jan), there are exactly 5 options for the other child. Thus, there are 5 total pairs that include Jan: Jan with each of the other students.

The probability of choosing a pair with Jan is 5/15 = 1/3.

As a final alternative, you may simply list all the pairs of students and count how many of them include Jan. Label the students in the class as J, 1, 2, 3, 4, and 5, where J is Jan. Then all the pairs can be listed as (J1), (J2), (J3), (J4), (J5), (12), (13), (14), (15), (23), (24), (25), (34), (35), and (45). (Be careful not to include repeats.) There are 15 total elements in this list and 5 that include Jan, yielding a probability of 5/15 = 1/3.

42. **(C).** The probability that Gary will eat eggs is 3/7. The probability that he will eat cereal is 4/7. In order to calculate the probability of eating one OR the other, add: 3/7 + 4/7 = 7/7 = 1. Quantities A and B are the same.

43. **(B).** The problem indicates that the events occur independently of each other. Therefore, in calculating Quantity A, the first step is to calculate the "or" situation, but you cannot stop there. When you add 0.5 + 0.3 = 0.8, you double-count the occurrences when both events occur. Next, you have to subtract out the probability of both events occurring in order to get rid of the "double counted" occurrences.

Notice that this is a Quantitative Comparison. At this point, you could conclude that, because the 0.8 figure includes at least one "both" occurrence, the real figure for Quantity A must be smaller than 0.8. Therefore, Quantity B must be larger.

To do the actual math, find the probability of both events occurring (breakfast AND sweater): (0.5)(0.3) = 0.15. Subtract the "AND" occurrences from the total "or" probability: 0.8 - 0.15 = 0.65

Quantity B is larger.

44. **(B).** The problem indicates that the events occur independently of each other. Therefore, in calculating Quantity A, do not simply add both events, even though it is an "or" situation. Adding 0.3 + 0.2 = 0.5 is incorrect because the probability that both events occur is counted twice. (Only add probabilities in an "or" situation when the probabilities are mutually exclusive.)

While Quantity A's value should include the probability that both events occur, make sure to count this probability only once, not twice. Since the probability that both events occur is 0.3(0.2) = 0.06, you must subtract this value from the "or" probability.

Quantity A: Add the two probabilities (rain OR pop quiz) and subtract BOTH scenarios (rain AND pop quiz):

$$0.3 + 0.2 - (0.3)(0.2) = 0.44$$

Quantity B: Multiply the probability that rain does NOT occur (0.7) and the probability that the pop quiz does NOT occur (0.8).

$$0.7(0.8) = 0.56$$

You could also note that, collectively, Quantities A and B include every possibility and are mutually exclusive of one another (Quantity A includes "rain and no quiz," "quiz and no rain," and "both rain and quiz," and Quantity B includes "no rain and no quiz"). Therefore, the values of Quantities A and B must add to 1. Calculating the value of either Quantity A or Quantity B would automatically tell you the value for the other quantity.

If you do this, calculate Quantity B first (because it's the easier of the two quantities to calculate) and then subtract

Quantity B from 1 in order to get Quantity A's value. That is, 1 - 0.56 = 0.44.

45. **(D).** The probabilities of both events are the same *as each other,* but that doesn't indicate anything about the value of those probabilities. For instance, if both probabilities are equal to 0.99, then the probability of both events occurring (Quantity A) is MUCH higher than the probability of neither occurring (Quantity B).

But what if the probability of each event is more like 0.000001 ("1 in a million")? Then the chance of neither occurring (Quantity B) would be much higher than the chance of both occurring (Quantity A).

In the case that the probabilities are each equal to 0.5, then — and only then — would the two quantities be equal.

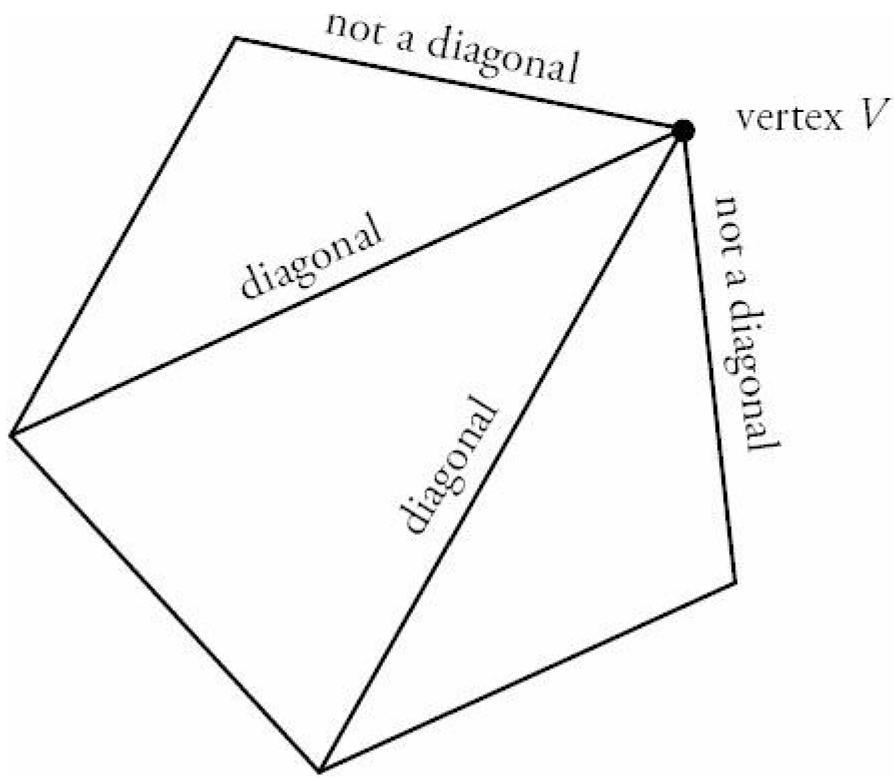
46. **(D).** In essence, the question is asking "What is the probability that one or more days are rainy days?" since any single rainy day would mean the city experiences rain. In this case, employ the 1 - x shortcut, where the probability of rain on one or more days is equal to 1 minus the probability of no rain on any day. Since the probability of rain is 1/3 on any given day, the probability of no rain on any given day is 1 - 1/3 = 2/3. Therefore, the probability of no rain on three consecutive days is (2/3)(2/3)(2/3) = 8/27. Finally, subtract from 1 to find the probability that it rains on one or more days: P(1 or more days) = 1 - P(no rain) = 1 - 8/27 = 19/27.

47. **(C).** The number of ways in which the students can be arranged with Beth and Dan separated is equal to the total number of ways in which the students can be arranged minus the number of ways they can be arranged with Beth and Dan together. The total number of ways to arrange 5 students in a line is 5! = 120. To compute the number of ways to arrange the 5 students such that Beth and Dan are together, group Beth and Dan as "one" person, since they must be lined up together. Then the problem becomes one of lining up 4 students, which gives 4! possibilities. However, remember that there are actually two options for the Beth and Dan arrangement: Beth first and then Dan or Dan first and then Beth. Therefore, there are (4!)(2) = (4)(3)(2)(1)(2) = 48 total ways in which the students can be lined up with Dan and Beth together. Finally, there are 120 - 48 = 72 arrangements where Beth will be separated from Dan.

Alternatively, compute the number of ways to arrange the students directly by considering individual cases. In this case, investigate how many ways there are to arrange the students if Beth occupies each spot in line and sum them to find the total. If Beth is standing in the first spot in line, then there are 3 options for the second spot (since Dan cannot occupy this position), 3 options for the next spot, 2 options for the next spot, and finally 1 option for the last spot. This yields (3)(3)(2)(1) = 18 total possibilities if Beth is first. If Beth is second, then there are 3 options for the first person (Dan cannot be this person), 2 options for the third person (Dan cannot be this person either), 2 options for the fourth person, and 1 for the fifth. This yields (3)(2)(2)(1) = 12 possibilities. In fact, if Beth is third or fourth in line, you arrive at the same situation as when Beth is second. Thus there will be 12 possible arrangements whether Beth is 2nd, 3rd, or 4th in line, yielding 36 total arrangements for these 3 cases. Using similar logic, the situation in which Beth is last in line is exactly equal to the situation where she is first in line. Thus, there are (18)(2) = 36 possibilities where Beth is first or last. In total, this yields 36 + 36 = 72 possible outcomes when considering all of the possible placements for Beth.

48. **(A).** A diagonal of a polygon is an internal line segment connecting any two unique vertices; this line segment does not lie along an edge of the given shape. Consider a polygon with 12 vertices. Construct a diagonal by choosing any two vertices and connecting them with a line. Remember that this is order independent; the line is the same regardless of which is the starting vertex. Therefore, this is analogous to choosing any 2 elements from a set of 12, and can be written as $\frac{12!}{10!2!}=\frac{12\times11\times10!}{10!(2)(1)}=\frac{12\times11}{2}=6\times11=66$. However, this method includes the vertices connected to their adjacent vertices, which form edges instead of diagonals. In order to account for this, subtract the number of edges on the polygon from the above number: 66 - 12 = 54.

Alternatively, if you choose a random vertex of the 12-sided shape, then there are 12 - 1 = 11 lines that can be drawn to other vertices since no line can be drawn from the vertex to itself. However, the lines from this vertex to the two adjacent vertices will lie along the edges of the polygon and therefore cannot be included as diagonals (see the figure of a pentagon below for an example).



Thus, there are 12 - 1 - 2 = 9 diagonals for any given vertex. Since there are 12 vertices, you might think that the total number of diagonals is equal to (12)(9) = 108. However, using this scheme you have counted each diagonal twice, using each side of the diagonal once as the starting point. Therefore, there are half this many different diagonals: 108/2 = 54.

49. **(B).** Consider each independent choice and then use the fundamental counting principle to calculate the total number of possibilities. Start by choosing the president, for which there are 12 total options. Next, for the vice president, there are 11 options. Finally, choose the committee of 3 from the remaining 10 students using the formula for a set where the order doesn't matter: $\frac{\text{total!}}{\text{in! out!}}$. The number of possible arrangements for the 3 committee members is $\frac{10!}{7!3!}$. Finally, using the fundamental counting principle, the total number of ways to choose a president, vice president, and committee of 3 is $\frac{12 \times 11 \times 10!}{7!3!}$ which can be rewritten as $\frac{12!}{7!3!}$.

50. **(A).** This is a classic combinatorics problem in which *order doesn't matter*—that is, the pairing "blue, green" is the same as "green, blue." A color is either "in" or "out." Use the standard "order doesn't matter" formula:

$$\frac{\text{Everything!}}{\text{Picked! NotPicked!}}$$

For Quantity A:

$$\frac{5!}{2!3!}=\frac{5\times4\times3!}{(2)(1)3!}=\frac{5\times4}{2}=10$$

For Quantity B:

$$\frac{9!}{8!1!}=\frac{9\times8!}{8!(1)}=9$$

Note that while the formula is necessary for Quantity A, you could reason your way to the value for Quantity B: every combination that selects 8 out of 9 colors will leave out exactly 1 color. Since there are 9 colors, there are 9 combinations.

51. **(C).** This is a classic combinatorics problem in which *order doesn't matter* — that is, picking Joe and Jane is the same as picking Jane and Joe. A person is either on the team or not. Use the standard "order doesn't matter" formula:

$$\frac{\text{Everything!}}{\text{Picked! NotPicked!}}$$

For Quantity A:

$$\frac{6!}{2!4!}=\frac{6\times5\times4!}{(2)(1)4!}=\frac{6\times5}{2}=15$$

For Quantity B:

$$\frac{6!}{4!2!}=\frac{6\times5\times4!}{4!(2)(1)}=\frac{6\times5}{2}=15$$

The quantities are equal. Note the first line of each Quantity: from that stage, you can already tell that the values will be the same.

This will always work—when order doesn't matter, the number of ways to pick 4 and leave out 2 is the same as the number of ways to pick 2 and leave out 4. Either way, it's one group of 4 and one group of 2. What actually happens to those groups (getting picked, not getting picked, getting a prize, losing a contest, etc.) is irrelevant to the ultimate solution.

52. **(C).** In this problem, order matters; if Jane comes in 1st place and Rohit comes in 2nd, there is a different outcome than when Rohit places 1st and Jane places 2nd. Use the fundamental counting principle to solve. To determine Quantity A make three slots (one for each prize). Six people are available to win 1st, and then five people could win 2nd, and four people could win 3rd:

$$\underline{\ 6\ }\quad \underline{\ 5\ }\quad \underline{\ 4\ }$$

Multiply: (6)(5)(4) = 120.

For Quantity B, make 5 slots, one for each prize. Five people can win 1st prize, then 4 people for 2nd prize, and so on:

$$\underline{\ 5\ }\quad \underline{\ 4\ }\quad \underline{\ 3\ }\quad \underline{\ 2\ }\quad \underline{\ 1\ }$$

Multiply (5)(4)(3)(2)(1) = 120. Quantities A and B are equal.

53. **(C).** This is a classic combinatorics problem in which *order doesn't matter* — in fact, the problem tells you that explicitly. Use the standard "order doesn't matter" formula:

$$\frac{\text{Everything!}}{\text{Picked! NotPicked!}}$$

For Quantity A:

$$\frac{100!}{56!44!}$$

Because the numbers are so large, there must be a way to solve the problem without actually simplifying (even with a calculator, this is unreasonable under GRE time limits). Try Quantity B and compare:

$$\frac{100!}{44!56!}$$

The quantities are equal. Note that this will always work — when order doesn't matter, the number of ways to pick 56 and leave out 44 is the same as the number of ways to pick 44 and leave out 56. Either way, it's one group of 56 and one group of 44. What actually happens to those groups (being part of a collection, being left out of the collection, etc.) is irrelevant to the ultimate solution.

54. **(A).** Use the overlapping sets formula for two groups: Total = Group 1 + Group 2 - Both + Neither. But first, add 740 ("business use ONLY") + 520 ("business use" and "personal use") to get 1260, the total number of products categorized as "business use."

Also note that the problem indicates that *all* of the products fall into one or both of the two categories, so "neither" in this formula is equal to zero.

Total = Business + Personal - Both + Neither
1,345 = 1,260 + P - 520 + 0
1,345 = 740 + P
605 = P

Quantity A is larger. Note that the question asked for the number of products characterized as "personal use" (which includes products in the "both" group). If the problem had asked for the number of products characterized as "personal use" ONLY, you would have had to subtract the "both" group to get 605 - 520 = 85. In this problem, however, Quantity A equals 605.

55. **(E).** Because all of the letters of the name "Christyna" are unique, there are 9 distinct choices of letters to form the 4-letter "word." In addition, order does matter: the "word" CHRI is different from the "word" CRHI. Use the fundamental counting principle to solve.

Begin by considering the first choice, for which there are 9 total options. Similarly, for the second choice there are 8 options, because one letter has already been chosen. Employing the same logic, there are 7 choices for the third letter and 6 choices for the fourth letter. Using the fundamental counting principle, (9)(8)(7)(6) = 3024 words.

Alternatively, try factorials. The total number of ways to arrange all 9 letters is 9! However, the problem is only concerned with "words" using four of these letters, meaning you must exclude rearrangements of the other 5. The number of ways in which you can order the 5 "non-chosen" letters is 5!. Thus the total number of "words" with 4 letters that can be made from the name "Christyna" is $\frac{9!}{5!}$ = (9)(8)(7)(6) = 3,024 "words."

56. **(E).** Use the 1 - *x* shortcut for this problem. How do you know to use this shortcut? In this case, you're being asked to solve for the probability that the product of the values from the two rolls will be less than 36. This will be time consuming to calculate directly because there are many different combinations that would produce a product less than 36 — (1)(6), (4)(5), (6)(3), and so on — the vast majority of combinations, actually! In fact, there's only one case when the product of the two rolls will *not* be less than 36: (6)(6). It is much easier to solve for this one value and then apply the 1 - *x* shortcut. In other words:

(The probability of rolling less than 36) + (The probability of rolling 36) = 1 $\longrightarrow$
(The probability of rolling less than 36) = 1 - (The probability of rolling 36)

This is an "AND question," because you need to get a 6 on the first roll *and* on the second roll. Multiply the two probabilities together:

$$\frac{1}{6}\times\frac{1}{6}=\frac{1}{36}$$

Subtract from 1:

$$1-\frac{1}{36}=\frac{35}{36}$$

57. **(C).** For probability questions, begin by separating out the probabilities of each individual event. Then, if all of the events must happen (an "AND question"), multiply the probabilities together. If only one of the multiple events needs to happen (an "OR question"), add. This question is an OR question, because it asks for the probability that David will eat a healthy breakfast *or* that it will rain.

At first glance, this may seem strange, because if you add the two probabilities together, you'll get something bigger than 100%, which is NEVER possible: 0.8 + 0.25 = 1.05. This figure double-counts the cases where David eats a healthy breakfast AND it rains. Subtract out these cases in order to find the desired value.

In order to calculate the probability that David will eat a healthy breakfast AND that it will rain, multiply the individual probabilities together:

$$0.8 \times 0.25 = 0.2$$

Finally, subtract to find the probability that David will eat a healthy breakfast OR that it will rain:

$$1.05 - 0.2 = 0.85, \text{ or } 85\%$$

58. **(B).** Because this question uses "at least" language, use the 1 - x shortcut. In this case, the only outcome you do *not* want is rain on zero days next week. It will be much faster to solve for that probability and subtract from 1 in order to find the probability of all of the other outcomes (that there will be rain on one or more days next week).

(The probability of rain on at least one day) + (The probability of no rain) = 1

How do you find the probability of no rain? You want no rain on Monday AND Tuesday AND Wednesday AND Thursday AND Friday. Multiply together the individual probabilities of no rain on each of the five days.

$$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{32}$$

Subtract this from 1 to get the desired answer:

$$1 - \frac{1}{32} = \frac{31}{32}$$

59. **(E).** Because this is an "at least" question, use the 1 - x shortcut.

(The probability of picking at least one man) + (The probability of picking no men) = 1

The probability of picking no men is an AND setup: woman AND woman AND woman.

For the first choice, there are 8 women out of 10 people: 8/10 = 4/5
For the second choice, there are 7/9 (because one woman has already been chosen)
For the third choice, there are 6/8 = 3/4

Multiply the three probabilities together to find the probability that the committee will be comprised of woman AND woman AND woman:

$$\frac{4}{5} \times \frac{7}{9} \times \frac{3}{4} = \frac{1}{5} \times \frac{7}{3} \times \frac{1}{1} = \frac{7}{15}$$

To determine the probability of picking at least one man, subtract this result from 1:

1 - 7/15 = 8/15

60. **(A).** This overlapping sets question can be solved with the following equation:

Total # of People = Group 1 + Group 2 + Group 3 - (# of people in 2 groups) - (2)(# of people in all 3 groups) + (# of people in no groups)

The problem indicates that everyone takes at least one language, so the number of people in no groups is zero. The problem also indicates that nobody takes all three languages, so that value is also zero.

Total # of Students = 100 + 80 + 40 - 22 - (2)(0) + 0 = 198.

61. **(A).** It is not possible to solve for a single value for Quantity A, but it is possible to tell that Quantity A is greater than 17. Since 20 birds are songbirds and 23 are migratory, the total of these groups is 43, which is less than 60. It is possible for the overlap (the number of migratory songbirds) to be as little as 0, which would result in 20 songbirds, 23 non-songbird migratory birds, and 60 - 20 - 23 = 17 birds that are neither songbirds nor migratory.

It is also possible that there could be as many as 20 birds that overlap the two categories. (Find this figure by taking the number of birds in the smaller group; in this case, there are 20 songbirds). In the case that there are 20 migratory songbirds, there would also be 3 migratory birds that are not songbirds, in which case there would be 60 - 20 - 3 = 37 birds that are neither migratory nor songbirds.

Thus, the number of birds that are neither migratory nor songbirds is at least 17 and at most 37. No matter where in the range that number may be, it is greater than Quantity B, which is only 16.